The introduction, analysis, and simulation of a new multilevel lot sizing heuristic for MRP inventory systems

Brian Lee Heemsbergen

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The introduction, analysis, and simulation of a new multilevel lot sizing heuristic for MRP inventory systems

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Iowa State University, 1989
The introduction, analysis, and simulation of a new multilevel lot sizing heuristic for MRP inventory systems

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# TABLE OF CONTENTS

**ABSTRACT** ................................................................. ix  

I. INTRODUCTION ............................................................. 1  

II. LITERATURE REVIEW ..................................................... 5  
   A. The Independent Demand Environment ............................... 5  
   B. The Single Level Discrete Demand Environment .................. 6  
   C. The Multilevel Discrete and Dependent Demand Environment .. 8  
      1. Discrete time and discrete demand ............................ 11  
      2. Setup costs ..................................................... 13  
         a. Standard setup cost definition ............................. 13  
         b. Setup cost using group technology ......................... 14  
         c. Setup cost using capacity implications .................... 15  
         d. Additional setup costs arrangements ....................... 16  
      3. Holding costs ................................................... 16  
         a. Units of holding cost ........................................ 17  
         b. Method of calculating inventory ............................ 19  
         c. Total versus echelon holding cost ......................... 20  
      4. Master schedule construction .................................... 21  
      5. Capacity restrictions ........................................... 22  
      6. Product structures and cost ratios ............................ 23  
         a. Product structure types ..................................... 24  
         b. Component quantities ....................................... 25  
         c. Cost ratios .................................................. 26  
      7. Scrap, rework, and safety stock .................................. 27  
      8. Component lead times ............................................. 28
9. Rolling horizon effects ........................................... 30
10. System nervousness .............................................. 30
11. The integer principle ........................................... 31
12. Collapsing the problem space .................................. 34
13. Lot sizing models ................................................. 35
    a. Multiple passes with a single level heuristic ........... 35
    b. Modified single level heuristics ......................... 36
    c. Optimum solutions ........................................... 38
14. Real world results .................................................. 41
D. Additional Multilevel Discrete Demand Environments ........ 41
    1. KANBAN system ................................................ 41
    2. Just in time (JIT) system ................................... 42
    3. OPT ............................................................. 42
III. PROBLEM DEFINITION ............................................... 44
IV. DEVELOPMENT OF MRP MODEL ASSUMPTIONS ..................... 46
A. Assumptions Adopted Without Analysis ......................... 46
    1. Discrete time and discrete demand ......................... 47
    2. Setup cost .................................................... 47
    3. Master schedule construction ................................ 47
    4. Capacity restrictions ......................................... 48
    5. Scrap, rework, and safety stock ............................. 48
    6. Component lead times ......................................... 48
    7. System nervousness ........................................... 49
    8. Product structures and cost ratios .......................... 49
B. Assumptions Adopted After Analysis ............................. 49
1. The integer principle ........................................ 50
2. Holding costs .................................................... 53
   a. An argument for replacement costing ..................... 54
   b. An argument for incremental costing ..................... 55
   c. Conclusions .................................................. 56
3. Collapsing the problem ....................................... 57
   a. Collapsing the problem using holding costs ............. 58
   b. Collapsing the problem using cost ratios ............... 62
   c. Summary ...................................................... 63
4. Rolling horizon effects ...................................... 64
   a. The rolling schedule environment and optimal solutions ............. 65
   b. The rolling schedule environment and heuristics .......... 71
   c. The rolling horizon implementation ....................... 78
V. MODEL SIMULATION ........................................... 79
   A. Simulation Development and Data Presentation ............. 79
      1. Master production schedule construction ................ 79
      2. Parameter development .................................... 80
      3. Simulation data representation .......................... 82
   B. Single Level Wagner Whitin Algorithm ...................... 84
   C. Lot For Lot Heuristic ....................................... 86
   D. Groff Marginal Cost Rule Heuristic ........................ 88
   E. Incremental Order Quantity Heuristic ....................... 90
   F. McLaren Modified Single Level Heuristic ................... 93
   G. The New Integer Lot Sizing (ILS) Heuristic ................ 95
1. Overall performance comparison .................. 96
2. ILS methodology ..................................... 99
3. Conclusions ........................................... 103

VI. CONCLUSIONS ........................................ 105
VII. BIBLIOGRAPHY ....................................... 108
LIST OF TABLES

TABLE 2.1 - Sample Demand Data .......................... 12
TABLE 2.2 - Monthly Periods .............................. 12
TABLE 2.3 - Weekly Periods ............................... 12
TABLE 2.4 - GT Example Data ............................. 14
TABLE 2.5 - Multiple Passes with Single Level Heuristics ...... 37
TABLE 2.6 - Modified Single Level Heuristics ............... 39
TABLE 2.7 - Optimum Solutions ........................... 40
TABLE 4.1 - Example Parameters .......................... 66
TABLE 4.2 - Possible Lot Sizing Plans .................... 67
TABLE 4.3 - Costs for Various Options .................... 69
TABLE 4.4 - Costs for Fixed Horizon and Global Optimum Solutions ... 71
TABLE 4.5 - Effects of a Large Planning Horizon (P) ........ 74
TABLE 4.6 - Effects of a Medium Planning Horizon (P) ....... 75
TABLE 4.7 - Effects of a Small Planning Horizon (P) ........ 76
TABLE 5.1 - Demand Array Statistics ...................... 80
TABLE 5.2 - Simulation Holding Costs ..................... 82
TABLE 5.3 - Locations of Poor ILS Heuristic Performance  .......... 97
LIST OF FIGURES

FIGURE 2.1 - Sequential Product Structure .................................... 24
FIGURE 2.2 - Assembly Product Structure ........................................ 25
FIGURE 2.3 - Lot For Lot Sequencing .............................................. 32
FIGURE 2.4 - Joint Sequencing ...................................................... 33
FIGURE 2.5 - Split Sequencing ..................................................... 33
FIGURE 2.6 - Noninteger Sequence ................................................. 34
FIGURE 4.1 - Lot For Lot Sequencing .............................................. 50
FIGURE 4.2 - Joint Sequencing ...................................................... 51
FIGURE 4.3 - Split Sequencing ..................................................... 51
FIGURE 4.4 - Noninteger Sequence ................................................. 52
FIGURE 4.5 - Joint Sequencing With Two Options ......................... 58
FIGURE 4.6 - Constant Demand Discrete Lot Sizing Environment .......... 73
FIGURE 5.1 - Assembly Production Set Structure ........................... 81
FIGURE 5.2 - WW Algorithm and Sequential Set 1 .......................... 85
FIGURE 5.3 - WW Algorithm and Sequential Set 2 .......................... 85
FIGURE 5.4 - WW Algorithm and Assembly Set ............................... 86
FIGURE 5.5 - LFL Heuristic and Sequential Set 1 ............................ 87
FIGURE 5.6 - LFL Heuristic and Sequential Set 2 ............................ 87
FIGURE 5.7 - LFL Heuristic and Assembly Set ................................. 88
FIGURE 5.8 - GMR Heuristic and Sequential Set 1 .......................... 89
FIGURE 5.9 - GMR Heuristic and Sequential Set 2 .......................... 89
FIGURE 5.10 - GMR Heuristic and Assembly Set ............................. 90
FIGURE 5.11 - IOQ Heuristic and Sequential Set 1 .......................... 91
FIGURE 5.12 - IOQ Heuristic and Sequential Set 2 .......................... 91
FIGURE 5.13 - IOQ Heuristic and Assembly Set ......................... 92
FIGURE 5.14 - MCL Modified GMR Heuristic and Sequential Set 1 ........ 94
FIGURE 5.15 - MCL Modified GMR Heuristic and Sequential Set 2 ........ 94
FIGURE 5.16 - MCL Modified GMR Heuristic and Assembly Set .......... 95
FIGURE 5.17 - ILS Interaction Number One .................... 100
FIGURE 5.18 - ILS Interaction Number Two .................... 101
FIGURE 5.19 - ILS Interaction Number Three .................... 102
FIGURE 5.20 - ILS Interaction Number Four .................... 102
The problem of when to build a lot of component parts, and how many to include in the lot so as to minimize overall production costs, is a fundamental problem facing all repetitive manufacturing environments. Recently this problem has been addressed through the use of Material Requirements Planning (MRP). However, MRP is not in itself a complete answer, since it does not provide a built in method to determine lot sizes and thus optimize the total cost.

A number of authors have studied the problem of developing lot sizes in MRP systems, however a consensus has been reached on very little. One reason that few of the results agree is, simply, they lack a basic examination of the actual problem structure. By examining this structure, several currently unrecognized or misunderstood features can be developed. This dissertation begins by developing these features and studying how they impact upon the lot sizing solutions.

These features allow a relatively simple, yet robust, new method of determining lot sizes to be developed. This new method is called the Integer Lot Sizing (ILS) heuristic. While, in some ways, the ILS heuristic is similar to previously explored lot sizing methods, in many aspects the ILS heuristic is unique. By utilizing the newly recognized features of MRP systems, the ILS heuristic develops costs which approach the optimum while utilizing an acceptable amount of computational resources.
I. INTRODUCTION

The problem of when to build a lot of component parts, and how many to include in the lot when it is built so as to minimize the overall production costs, is one of the most fundamental problems facing any repetitive manufacturing environment. Enormous amounts of effort and resources have gone into the study of this problem during this century. Indeed this problem forms one of the basic areas of the entire Industrial Engineering profession.

The earliest work on this problem utilized statistical methods to minimize the costs associated with each of the various components which go into a finished product. The application of these methods was performed on each component separately. The overall costs of operation were then imagined to be minimized due to the minimization of costs associated with each of the respective component parts. While these statistical methods have proven valuable for the independent demand environment, they do not accurately represent the dependent and discrete demand situation which is present in a repetitive manufacturing environment.

In 1958 Wagner and Whitin introduced a new means of analyzing a problem which contains discrete demands. They developed a methodology which produces the optimum (lowest) total cost solution for a single component over a given number of discrete time periods, when the demands for the component are known with certainty for each of the discrete time
periods. The method they developed has become known as the Wagner-Whitin (WW) algorithm.

Since the 1960s, a number of authors have presented heuristic methods which solve the same problem as the WW algorithm. These methods do not guarantee the optimum total cost. However, they (hopefully) develop a total cost which is close to optimum, while using only a small portion of the computational resources required by the WW algorithm. The utility of the heuristic methods ranges from poor to excellent.

While addressing the discrete nature of the demands in a repetitive manufacturing environment, neither the WW algorithm nor the heuristic methods addressed the dependent nature of the demands. However, in 1975 Orlicky presented a methodology which can be used to model the dependent demand environment. This methodology is generally known as Material Requirements Planning or MRP.

However, the MRP methodology is not in itself a complete answer to the underlying problems. MRP does not provide a built in method to determine lot sizes and thus optimize the total cost developed during the production of a product. Orlicky simply applies the WW algorithm and some of the heuristic methods developed earlier to the individual component demands developed in the MRP system. In the end he states "There does not appear to be one best lot sizing algorithm that could be selected ...".

Since 1975 a number of authors have studied the problem of developing lot sizes in MRP systems. The work of these authors has ranged from
simple comparisons of the methods proposed by Orlicky, to the presentation of new methods, to the extension of the WW algorithm so that it can more accurately model the dependent demand environment. While this work forms an impressive array of ideas and systems, a consensus is reached on very little.

One reason that few of the results from preceding studies agree is, simply, they lack a basic examination of the actual problem structure presented by a MRP system. By examining the basic problem structure, several currently unrecognized or misunderstood features can be developed. This dissertation begins by developing these features and studying how they impact upon the possible lot sizing solutions. These features represent, both, the discrete and dependent nature of component demands which are always present in an MRP system.

These features allow a relatively simple, yet robust, new method of determining lot sizes to be developed. This new method is called the Integer Lot Sizing (ILS) heuristic. While, in some ways, the ILS heuristic is similar to previously explored lot sizing methods, in many aspects the ILS heuristic is unique. By utilizing the newly recognized features of MRP systems, the ILS heuristic develops costs which approach the optimum while utilizing an acceptable amount of computational resources.

The basic thrust of this dissertation is, therefore, divided into two successive areas. First, and perhaps foremost, a more comprehensive understanding of MRP systems is developed, to provide a framework for
subsequent work. Secondly, a new lot sizing method, the ILS heuristic, is developed. The ILS heuristic is based upon the MRP features developed in this dissertation and, hopefully, represents a step forward in the continuing struggle to more efficiently operate a repetitive manufacturing environment.
II. LITERATURE REVIEW

The amount of study that has gone into the problem of inventory control and the quantity of literature that has been generated by this study is, simply, enormous. The literature ranges from introductory texts which illustrate basic inventory control models using highly simplified abstractions, to extremely complex dissertations which utilize mathematical optimization techniques, to user manuals which describe entire computer languages which are designed to simulate actual manufacturing situations. While it is well beyond the scope of this literature review to examine all the preceding literature, the preceding literature can easily be divided into three basic areas. Each of these three areas is discussed in a subsequent section. The three general areas of study into inventory control problems are:

- The independent demand environment.
- The single level discrete demand environment.
- The multilevel discrete and dependent demand environment.

A. The Independent Demand Environment

The independent demand environment, simply, describes a setting in which demand for a product or service is created in isolation of the actions taken by the party which will satisfy the demand. Thus the producer of a product or service in this environment must plan for unknown and perhaps highly variable demand. The resulting demand pattern can, of course, be influenced by outside forces such as weather, competition, or
economic setting. Examples of independent demand include sales of gasoline at a service station, sales of automobiles at a car dealership, or service calls to the local plumber.

Studies of the independent demand environment include models for both, discrete and continuous demand. That is, the demand can be in unit quantities (discrete) or in indivisible quantities (continuous). While the results generated by these studies are not directly applicable to the subject of this dissertation, extensions of the results are sometimes useful. These extensions are used and noted were they occur.

This dissertation makes no general attempt to describe or categorize the independent demand environment studies and the resulting literature, except to note that they do exist and form a valid and useful subset of the work that has been done on the inventory control problem. Descriptions of the results of studies into the independent demand problem are included in most inventory control texts, samples of which are readily available [22, 53, 95, 122, 241].

B. The Single Level Discrete Demand Environment

The single level discrete demand environment is categorized by a series of time phased independent discrete demands, a set of cost parameters, and the operating costs which occur from various production timings. The objective of studies into this environment is to develop a system which minimizes the resulting costs for the given set of inputs. Studies into this problem are numerous [4, 7, 9, 11, 13, 14, 27, 37, 38,
These studies contain, both, heuristic and optimal solutions. A complete review of this environment would form an extensive body of work. Since this dissertation focuses on the multilevel problem, such a review is not performed. Rather, this dissertation relies largely upon the results of one previous study [127] into the single level discrete demand environment. This dissertation is, then, an extension of the previous study, with this extension performed by analyzing the results of the previous study and determining how they interact with the multilevel environment. Of note, a second, more recent, comprehensive study of this environment [208, 209] found results which are remarkably similar to those found by the primary study.

The results of the primary single level study are:

- The single level environment reacts in an understandable and predictable manner to various inputs.
- The overall performance of the different heuristics tested varied markedly, with only four yielding acceptable overall results. The best overall heuristic was the Groff marginal cost rule.
- The performance of the heuristics was based more upon the logic of the heuristic than upon the various input parameters.
• The performance of the best heuristic methods was close to optimum for most input parameters and differed from optimum by an acceptable margin for all input parameters.

• The method used to calculate holding costs can affect the performance of some, but not all, of the heuristics.

C. The Multilevel Discrete and Dependent Demand Environment

The multilevel discrete and dependent demand environment defines the type of inventory control problem which is common in repetitive manufacturing. In the early 1970s a system of maintaining order in a discrete and dependent demand environment was developed [193, 199]. This system became known as materials requirements planning or MRP. In this dissertation, the terms 'multilevel discrete and dependent demand environment' and 'MRP environment' are synonymous.

An MRP environment is categorized, first, by a series of time phased, independent discrete demands for a finished product. This demand series is generally known as the master production schedule (MPS). The MPS shows the quantity of finished products that are required for each discrete time period into the future, up to some maximum future date. The amount of time that the master production schedule covers is known as the planning horizon.

The demands for the finished product, in turn, create demands for the component parts of the finished product. The component demands are said to be dependent upon the demand for the finished product. The components
are required in integer multiples of the demands for the finished product. That is, 1 or 2 or 3 or etc. of each component item is required for each finished product. These component demands are also time phased. However, they are required at an earlier time than the corresponding finished product demands.

The component demands will, in turn, create demands for their components. Thus, the demands for the finished product, in essence, cascade through a series of levels of component parts, until finally the demands for the finished product are broken down into the corresponding demands for raw materials. A brief example can, perhaps, best illustrate this process. This example is referred to throughout this literature review.

Suppose that a factory is building toy wagons. Each wagon consists of one painted box, one painted handle, and four painted wheel-tire subassemblies. If the demand for the finished product, the toy wagon, is known to be 50 units in week 10, the demand for the components that are required is also known. However, these components need to be ready in week 10, so they need to be assembled in week 9. The component demands for week 9 in this simple example would be: 50 painted boxes, 50 painted handles, and 200 painted wheel-tire subassemblies.

These component demands in week 9 would, in turn, generate subcomponent demands in week 8. The entire system would finally result in showing the time phased demands for the raw materials; the sheet steel to build the boxes, the tube steel to build the handles, the paint, etc. If
the master production schedule also shows demand for the toy wagon in week 11, this demand could also be cascaded through the system and would result in additional time phased demands for all the components.

However, simply cascading the component demands, probably, does not result in a workable, let alone optimum, solution. Such a solution would require that each component be built in each period where there was a demand. Thus, these cascaded demands need to be grouped together into workable and expedient sizes. This grouping is done by lot sizing the demands. Lot sizing logic is included in all MRP systems.

Since the advent of MRP, a number of authors have noted problems and inconsistencies in the MRP lot sizing logic. Each of these areas of contention is discussed in a separate section. The areas of contention surrounding MRP are:

- The components of the setup cost and how it should be calculated.
- The components of the holding cost and how it should be calculated.
- The manner in which test demand arrays should be constructed.
- The manner in which capacity restrictions and interactions affect test models.
- The manner in which product structures and cost ratios for the components are modeled.
- The manner in which scrap and/or rework is handled in the model.
- The manner in which component lead times should be included in test models.
- The impact of a rolling horizon on model performance.
• The system nervousness that is created as a result of changes in
  the master schedule or changes in the status of component parts.
• The validity of the integer principle for calculating lot sizes.
• The ability to collapse the problem into a less complex form.
• The manner in which lot sizes are calculated.
• The results of real world applications of the MRP model.

1. Discrete time and discrete demand

The description of discrete time and discrete demand is one of the
few areas of MRP that has a general agreement of validity and meaning.
The total demand for finished products is simply divided into the
quantities which represent the demand during a series of discrete, and
usually equivalent, time intervals. Thus, time does not flow
continuously, but rather in equivalent jumps. Also, the demands are not
imagined to occur throughout each discrete time interval, but rather to
occur in entirety at the start of the respective time interval. By
imagining the entire demand to occur at the start of each period, any
underlying demand patterns are also satisfied.

For example, returning to the toy wagon, imagine that the demands
shown in Table 2.1 are forecast to occur. If a discrete time interval of
1 month is chosen, these demands would appear as shown in Table 2.2. If a
discrete time interval of 1 week is chosen, these demands will appear as
shown in Table 2.3. In all three tables, the total demand is the same.
The demands have simply been fitted into a system utilizing different
discrete time intervals.
TABLE 2.1 - Sample Demand Data

<table>
<thead>
<tr>
<th>DEMAND</th>
<th>DEMAND DATE</th>
<th>DEMAND</th>
<th>DEMAND DATE</th>
<th>DEMAND</th>
<th>DEMAND DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1/5</td>
<td>5</td>
<td>2/3</td>
<td>22</td>
<td>3/3</td>
</tr>
<tr>
<td>11</td>
<td>1/8</td>
<td>12</td>
<td>2/5</td>
<td>14</td>
<td>3/8</td>
</tr>
<tr>
<td>18</td>
<td>1/12</td>
<td>29</td>
<td>2/9</td>
<td>7</td>
<td>3/10</td>
</tr>
<tr>
<td>25</td>
<td>1/15</td>
<td>6</td>
<td>2/12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1/18</td>
<td>8</td>
<td>2/16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1/20</td>
<td>11</td>
<td>2/19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1/26</td>
<td>21</td>
<td>2/23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1/28</td>
<td>17</td>
<td>2/26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2.2 - Monthly Periods

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>JAN.</th>
<th>FEB.</th>
<th>MAR.</th>
<th>APR.</th>
<th>MAY</th>
<th>JUNE</th>
<th>JULY</th>
<th>AUG.</th>
<th>SEP.</th>
<th>OCT.</th>
</tr>
</thead>
</table>

TABLE 2.3 - Weekly Periods

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>1/4</th>
<th>1/11</th>
<th>1/18</th>
<th>1/25</th>
<th>2/1</th>
<th>2/8</th>
<th>2/15</th>
<th>2/22</th>
<th>2/29</th>
<th>3/7</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEMAND</td>
<td>18</td>
<td>43</td>
<td>23</td>
<td>23</td>
<td>17</td>
<td>35</td>
<td>19</td>
<td>38</td>
<td>22</td>
<td>21</td>
</tr>
</tbody>
</table>

The length of time forming a discrete time interval is important in an actual MRP system [49]. However, in a simulation the time interval does not need to be defined, if all time relative units are defined with respect to this unknown, discrete time interval [253]. Thus, demand can be defined in units/period rather than units/day or units/year, and costs are defined in $/period rather than $/day or $/year. The results of a simulation utilizing such an undefined discrete time interval should be valid for real world situations with an actual discrete time interval.
2. Setup costs

Every study into the multilevel discrete and dependent demand environment requires a definition for the manner in which the setup cost is calculated and what the setup cost describes. In most of the prior literature, these definitions are implicit. If the definition is given explicitly it, almost exclusively, utilizes the standard definition given below. However, a few studies use the nonstandard definitions that are described in later sections.

a. Standard setup cost definition In the standard setup cost definition, the setup cost simply refers to that cost which is incurred each time a machine or process is changed from the production of one component to the production of a different component. The setup cost includes the cost of lost production while the machine (process) is changed, the cost of the operator time while the machine (process) is changed if the operator is idled, the cost of any special setup personnel that are required, the cost of any materials that are utilized or scrapped as a result of the change, and the cost of initiating the change with respect to paper work and overhead functions [8, 160, 193].

The setup cost is listed in $/setup. The setup cost is fixed with respect to both the time that it is performed and with respect to the number of parts which will pass through the machine (process) using the new setup. To calculate the total setup cost which is incurred, simply multiply the number of setups by the setup cost.
b. Setup cost using group technology  

The first nonstandard setup cost definition describes the possible reductions in the setup cost which can be achieved by specifically arranging the setup order [54, 108, 120, 222, 228]. This is performed by grouping the parts which pass over a machine (process) into families and producing the family grouping together. The use of families of parts is a standard procedure in group technology (GT) research and applications. An example can perhaps best illustrate how families of parts can be used to reduce setup costs.

Imagine a lathe which machines a high grade finish upon bar stock. The various parts which flow over the lathe are different in finished length and diameter. Table 2.4 illustrates an imaginary sample of such parts. Now also imagine that the setup of the machine requires 10 minutes to change the finished length and 15 minutes to change the finished diameter, and that setup time can be used to represent the setup cost. Finally, imagine that the machine is initially setup to produce part A, and that a single lot of each part is required.

<table>
<thead>
<tr>
<th>PART</th>
<th>LENGTH</th>
<th>DIAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6 inches</td>
<td>3 inches</td>
</tr>
<tr>
<td>B</td>
<td>4 inches</td>
<td>3 inches</td>
</tr>
<tr>
<td>C</td>
<td>2 inches</td>
<td>3 inches</td>
</tr>
<tr>
<td>D</td>
<td>6 inches</td>
<td>2 inches</td>
</tr>
<tr>
<td>E</td>
<td>4 inches</td>
<td>2 inches</td>
</tr>
<tr>
<td>F</td>
<td>2 inches</td>
<td>2 inches</td>
</tr>
</tbody>
</table>

It is easy to see from these data that the diameter should only be changed once, since this change requires more time than a change in
length. The processing sequence should be A-B-C-D-E-F, resulting in a total setup time of 65 minutes to cycle through all the parts. It is also easy to see that changing both dimensions at each setup will result in a maximum of setup time. This sequence is A-E-C-D-B-F, and results in a total setup time of 125 minutes or 92% more time than the optimum.

While this simple example might not be realistic, it does illustrate that the order in which setups are performed can impact upon setup cost and through this, the total cost. In real world situations the parts are, probably, more diverse, with many more changing dimensions and requirements. However, as illustrated here, the use of GT in a multilevel environment can result in a decrease in total costs.

c. Setup cost using capacity implications  A third manner in which setup costs can be handled uses capacity implications in determining the applicable setup cost [4, 8, 10, 45, 145, 146, 147, 197, 204, 224, 225, 226, 227, 243, 244]. In this method a production and manning level is assumed fixed for a finite period. Then all setups which can be performed within this production and manning level do not incur any variable cost. The cost of these setups only includes the cost of lost production, the cost of any materials that are utilized or scrapped as a result of the change, and the cost of initiating the change with respect to paper work and overhead functions. The cost of the operator and any setup specialists are not included in the cost of a setup, if the setup is performed within the assumed standard production and manning level.
If the setups can not be performed within the standard production and manning levels, the overtime cost of the operator and any setup specialists is added to the setup cost, forcing larger lot sizes in most cases. This in turn results in less setups and more actual production. If the setups can not be performed even utilizing overtime, a very high value is added to the per lot setup cost in an attempt to force the lot sizes to even larger numbers.

d. Additional setup costs arrangements

Two additional setup cost arrangements are occasionally used. The first is based upon the variability of setup costs with time. This could describe a situation where the season or production level influences the setup cost. The second is based upon the variability of setup costs with the quantity that is produced during the setup. This could describe a situation where tooling changes or quality control functions need to be performed on a regular basis. No literature reviewed used either of these setup cost arrangements. They are included, simply, for completeness.

3. Holding costs

The holding cost describes the cost which is incurred as a result of maintaining value in inventory [194, 199]. However, there are several differences in the manner in which previous studies define holding costs. These differences range all the way from the units that are used to calculate holding cost, to what exactly the holding cost represents. Each of these areas is covered in a following section.
a. Units of holding cost

The total holding cost is calculated using the inventory counts for each item over the period of time in question. The discrete nature of time in an MRP system allows the inventory counts for each item for each period to be known at the end of each MRP run. The total holding cost can then be calculated by using either of two sets of units.

Equation 2.1 shows the calculation of total holding cost by basing the holding cost directly on the inventory counts [49, 127, 129, 130, 160]. In Equation 2.1, the inventory count in each period is multiplied by the cost of holding that unit in inventory for that period. Generally, the only time variant value in the equation is the inventory count, that is, \( H \) is held constant with respect to time as shown in Equation 2.2.

\[
THC = \sum_{i=1}^{P} \sum_{j=1}^{T} H[i,j] \cdot I[i,j] \quad (2.1)
\]

Or

\[
THC = \sum_{i=1}^{P} \sum_{j=1}^{T} H[i] \cdot I[i,j] \quad (2.2)
\]

Where:

- \( H[i,j] \) = holding cost [part \( i \), period \( j \)] in $/unit-inventory;
- \( H[i] \) = holding cost [part \( i \)] in $/unit-inventory time independent;
- \( I[i,j] \) = the inventory count [part \( i \), period \( j \)] in units;
- \( P \) = the total number of parts;
- \( T \) = the number of discrete time intervals;
- \( THC \) = the total inventory holding cost in $.
Equation 2.3 shows the calculation of total holding cost by basing the holding cost on the inventory value [106]. In Equation 2.3, the inventory count in each period is multiplied by the cost of the item in that period, and this total is multiplied by the cost of maintaining inventory value for that part for that period. Again, $H$ and $C$ are usually held fixed with respect to time as shown in Equation 2.4.

\[
THC = \sum_{i=1}^{P} \sum_{j=1}^{T} H[i,j] \cdot I[i,j] \cdot C[i,j] \tag{2.3}
\]

Or

\[
THC = \sum_{i=1}^{P} \sum_{j=1}^{T} H[i] \cdot I[i,j] \cdot C[i] \tag{2.4}
\]

Where:

$H[i,j]$ = holding cost [part $i$, period $j$] in $$/inventory;

$H[i]$ = holding cost [part $i$] in $$/inventory time independent;

$I[i,j]$ = the inventory count [part $i$, period $j$] in units;

$C[i,j]$ = the cost [part $i$, period $j$] in $$/inventory/unit;

$C[i]$ = the cost [part $i$] in $$/inventory/unit time independent;

$P$ = the total number of parts;

$T$ = the number of discrete time intervals;

$THC$ = the total inventory holding cost in $.$

Equations 2.1 through 2.4 result in the same total holding cost if the values assigned are consistent. The choice of the units to use can be based upon convenience. However, care should be taken when the results developed in different studies are compared that the definitions used in each study are consistent.
b. Method of calculating inventory

Three methods of calculating inventory counts are in use; the end of period (EOP), the average inventory level (AIL), and the generated part period (GPP) methods. These three methods can give different results for the same input data.

However, it has been shown [129, 130] that these three methods are all linearly related. This is shown in Equations 2.5, 2.6, and 2.7. Thus, the choice of a method to use to determine inventory counts can be based upon convenience. However, when examining the results of past studies care should be taken to note which of the three methods was used, so that results can be accurately analyzed.

\[
\text{Inventory Count}(\text{GPP}) = \text{Inventory Count}(\text{EOP})
\] (2.5)

And

\[
\text{Inventory Count}(\text{AIL}) = \text{Inventory Count}(\text{EOP}) + \kappa
\] (2.6)

With:

\[
\kappa = \sum_{i=1}^{T} D[i] + 2
\] (2.7)

Where:

\[
\kappa = \text{difference in inventory count for the EOP and AIL methods;}
\]

\[
T = \text{the number of discrete time intervals;}
\]

\[
D[i] = \text{the demand in period } i.
\]

It should be noted that the difference in inventory count for the AIL method \(\kappa\) is constant for a given total demand. If the other factors (\(H\) and \(C\)) are held constant over time (as shown in Equations 2.2 and 2.4), this means the difference in total holding cost is also a constant amount higher when the AIL method is used.
c. Total versus echelon holding cost

Previous literature is divided into two camps with regards to how to value inventory for the purpose of lot sizing. The opinion of the first camp is that the inventory value of an item is, simply, the item's replacement value [25, 74, 77, 188, 246, 252, 255]. This idea produces inventory values which are, generally, monotonically increasing for an item as the item passes from raw material to finished good. That is, as an item becomes closer to the finished product, the inventory value, and thus holding cost used for the item increases.

The second camp believes that the incremental increase in the value of an item represents the correct value to be used in lot sizing equations [1, 2, 3, 25, 64, 74, 77, 188, 196, 217, 236, 237, 252, 255, 258]. This incremental increase is termed the echelon cost of the item. The echelon cost does not monotonically increase, but rather is a function of the amount of work or material value that is added to the item at each level.

The logic behind the use of the echelon cost is rather simple. As each item is processed, the only change in the inventory value is represented by the incremental increase in value of the item. The total value of the component items is a sunk cost, and, thus, should have no impact upon the lot sizing decision. The use of the echelon cost in lot sizing equations creates larger lots, since less holding cost is encountered for a given size of lot.
4. Master schedule construction

The master schedule, or master demand array, in an MRP environment describes the demand inputs to the system, both with respect to quantity and timing, as shown in Tables 2.2 and 2.3. In order to simulate an MRP type environment, sample master schedules must be used. Previous studies have created sample master schedules from postulated data [37, 46, 67, 68, 69, 76, 83, 111, 187, 266, 267, 271]. These postulated distributions are generally made through the use of random numbers.

Postulated data can be produced to represent a number of different real world distributions. Two of the possible types of postulated distributions are:

- Distributions which simulate random arrivals from a single calling distribution.
- Distributions which simulate random arrivals from two or more calling distributions with only one active at a time.

The first type of postulated distribution represents demands which arise from a single source. The source of the demands is active at all times and the distribution of demands remains the same over time.

The second type of postulated distribution represents demands which come from two sources. First one source is active for a period of time and then the second source is active. This could represent the situation where product is made for both customer order and finished goods inventory. When the product is made for finished goods inventory, a
higher rate demand distribution would be in effect. This demand
distribution would remain in effect for a period of time and then the
lower rate distribution would remain in effect for a subsequent period of
time.

5. Capacity restrictions

A large number of previous studies into the MRP environment do not
consider capacity restrictions. They assume that capacity is unlimited
and investigate other aspects of the environment. Of course any real
world situation would have some type of capacity restriction, and not
including them in a simulation would represent a simplification of the
problem.

Of those studies which do include capacity restrictions, a number of
different treatments are available. The inclusion of any type of capacity
restriction represents the addition of more data into the simulation. So
not only is the simulation itself more complicated, but the design of the
simulation data is also complicated by the requirement for the design of
these additional data.

The most basic type of capacity restriction is the limiting of the
amount of production which can be performed in any one period [5, 10, 16,
18, 24, 26, 28, 29, 31, 32, 33, 43, 75, 89, 93, 102, 123, 151, 160, 161,
162, 165, 166, 167, 189, 191, 192, 202, 224, 242, 243, 244]. This places
a constraint on the number of items of each type which can be produced in
any period. In order to simulate this type of constraint, machine and
setup times for each item in the system must be developed, as well as establishing a maximum of production time for each machine. The maximum production time is sometimes allowed to fluctuate with overtime surcharges.

A second type of capacity restriction is the limiting of total inventory value in the system [4, 5]. This places an economic constraint on the system. In order to simulate this type of constraint, the per item cost of all items must be known, as well as the ceiling for inventory value. The ceiling is sometimes allowed to increase under a specified cost penalty.

A third type of capacity restriction is the limiting of total inventory space requirements. In order to simulate this constraint, the size of each item must be known, as well as the total amount of space available. Extra space can sometimes be rented at a increased cost.

Of course any of these restrictions can be used in combination with other of the restrictions. In a real world situation, all of the restrictions probably apply, and a model which includes all three would be the most realistic. However, a model including all three is much more complex, is much harder to develop realistic data for, and is also much harder to validate.

6. Product structures and cost ratios

The product structure defines the manner in which component parts are utilized in producing parent items. In an MRP environment, the product
structure is, generally, called the bill of materials (BOM). The BOM contains information on both, the structure type and the component quantities. The cost ratio of an item defines the relationship between setup and holding costs for the item. They are included in the same discussion because their use in a simulation is highly intertwined.

a. Product structure types

There are two main types of product structures [15, 25, 26, 40, 41, 43, 44, 159, 194, 195, 196, 206, 210, 217, 236, 237, 244, 246, 247, 248, 258]. Figure 2.1 shows a sequential product structure and Figure 2.2 shows an assembly structure. In both figures, the finished product is at the top, and the components flow into this finished product.

The sequential product structure defines a product which is produced without branching subcomponents. This product is produced from a single piece of raw material and modified into the finished product through a series of production steps. The assembly product structure defines an
FIGURE 2.2 - Assembly Product Structure

item which is produced by assembling a group of components into a finished product. Each of the components can in turn have sequential or assembly types of structures. There are, of course, an unlimited number of assembly type structures possible.

Previous studies have generally investigated both types of product structures. Usually, preliminary investigations are performed on the sequential structure and then the results of these investigations are extended to see how they perform in the more complex assembly structure. There are, however, a group of studies which have limited their investigation to the sequential structure. In general, the results of these studies cannot be extended into an assembly structure.

b. Component quantities In addition to the type of structure, the quantity of each component that is required per finished product must be included in the product structure data. In early studies, the quantity of each type of component that is required was assumed to be an important
design criterion for the simulation. However, in recent studies, this area has been greatly simplified by assuming that all components are required in quantities of 1 and then allowing only the cost ratios to change [1, 43, 179, 180, 181, 194, 195, 196].

For example, returning to the toy wagon which was described earlier, each finished wagon requires four wheel/tire subassemblies. However, the quantity questions can be simplified by requiring only one set of four of these wheel/tire assemblies per finished wagon. Then all definitions should be created on the basis of this set as opposed to the individual component items.

c. Cost ratios The cost ratio of an item is determined as shown in Equation 2.8. The cost ratio defines the ratio of setup cost to holding cost [40, 41, 44, 67, 68, 69, 76, 159, 196, 246, 271]. The cost ratio is a primary input in most lot sizing methods. In most previous studies, the cost ratio is time independent, and this simplifies the cost ratio to Equation 2.9. Of course if the component quantities are all assumed to be one as described in the previous section, the cost ratio should be listed in term of sets rather than in terms of actual component items. The holding cost shown in Equations 2.8 and 2.9 use the same definitions as Equations 2.1 and 2.2.

\[
\begin{align*}
R[i,j] &= S[i,j] + H[i,j] \quad (2.8) \\
R[i] &= S[i] + H[i] \quad (2.9)
\end{align*}
\]

Where:

\[S[i,j] = \text{setup cost [part } i, \text{period } j \text{] in } \$/\text{setup};\]
\[ H[i,j] = \text{holding cost [part } i, \text{period } j \text{] in } \$/\text{unit-inventory}; \]

\[ R[i,j] = \text{the cost ratio [part } i, \text{period } j \text{] in units/setup}; \]

\[ S[i] = \text{setup cost [part } i \text{] in } \$/\text{setup time independent}; \]

\[ H[i] = \text{holding cost [part } i \text{] in } \$/\text{unit-inventory time independent}; \]

\[ R[i] = \text{the cost ratio [part } i \text{] in units/setup time independent}. \]

Very little has been said about the values of the cost ratios. They can increase or decrease throughout the product structure. While cost ratios are used in lot sizing methods, they are not generally used in the overall objective function. Usually, the holding and setup costs are simulation inputs and the cost ratios are calculated from these values.

7. Scrap, rework, and safety stock

Scrap and/or rework define the situation where the number of items put into a process is more than the number of items which are produced by the process. Scrap differs from rework in that the parts are lost for good, whereas in rework additional processing can return the parts back to the system, generally in a later period. Scrap/rework in a real world situation can be the result of bad design, processing, or material. A number of previous simulations have investigated the implications of scrap and rework [116, 135, 158, 186]. Generally, these studies assume a yield distribution for each item and use random number generators to develop scrap/rework quantities for each lot.

When scrap/rework is present in a system, the number of finished components that each process will produce is unknown. This introduces a
randomness into the environment. Usually, this randomness is handled by introducing safety stock into the system [5, 49, 52, 66, 70, 82, 83, 96, 116, 135, 141, 152, 160, 161, 251, 254, 262, 264, 270]. Safety stock is inventory that is held in order to satisfy the randomly scrapped items so that the master production schedule can be satisfied.

Safety stock costs money through the cost of holding the items in inventory when they are not required. On the other hand, not having enough safety stock on hand can produce stockouts. These stockouts are usually assigned a stockout cost, either in terms of a per item short cost or a per stockout cost. The optimum safety stock is determined by minimizing the safety stock holding cost and stockout costs.

Safety stock can also be used to satisfy demands which arise outside the normal MRP system. These could be such things as replacement parts, parts used for testing, etc. Very few previous simulations have used demands which are outside the normal MRP system, and these types of safety stocks are rarely investigated.

8. Component lead times

Component lead times describe the amount of time that is required for a component item to be processed at the preceding stage [28, 66, 67, 68, 69, 126, 143, 172, 173, 212, 214, 240, 268, 269]. For example, returning to the toy wagon, if the lead time for handles is 1 week and 100 wagons are to be assembled in week 10, the handles must be started in week 9. If
the component lead time was 2 weeks, the handles would need to be started in week 8, etc.

Component lead times are used to cover the amount of time that is actually spent in processing, the amount of time that the components spend in a queue awaiting processing, and any transportation time. In most previous studies, the component lead times have been a major design criterion for the simulation. However, some recent studies have assigned all component lead times to zero [1, 15, 40, 41, 44, 49, 179, 180, 181, 203, 218].

The logic behind assigning all component lead times to zero is straightforward. In MRP, each component lead time is assigned a value by the administrator. Thus, since all items spend a component lead time at the process, the costs associated with the component lead time are nonvariable and do not need to be optimized. These costs are simply excluded from the simulation.

However, other studies attempt to design logic which minimizes component lead time by minimizing the queue time [1, 16, 29, 31, 45, 48, 60, 66, 91, 124, 146, 147, 173, 189, 192, 212, 225, 226, 240, 244, 245, 264, 268, 269]. The results of these studies are schedule based systems which determine the optimum lead times as well as the optimum lot sizes. Optimizing the lead times complicates the simulation to a large degree and requires that capacity utilization data also be included in the simulation.
9. **Rolling horizon effects**

In an MRP environment, as time passes, the current demands become history and new demands are added to the far end of the master schedule. This process is known as a rolling horizon. In a rolling horizon environment only the decision about the current period is implemented, the horizon is then rolled forward one period, and the decision process reperformed. Previous studies have handled the rolling horizon nature of MRP differently.

A number of studies have, simply, ignored the rolling horizon aspect and treated the problem as a fixed horizon problem. This simplifies the problem considerably, however the results developed in these studies might not be directly applicable to a real world problem. A number of other studies have examined the effect that the length of the planning horizon has on the performance of the various lot sizing methods [4, 9, 38, 39, 50, 62, 134, 149, 150, 234]. Finally, some studies have attempted to negate the rolling horizon by utilizing assumed demands at the end of the master schedule [65].

10. **System nervousness**

Whenever the master schedule is changed in an MRP system, the changes cascade down through the demands for the various components. The effect of these changes is known as system nervousness. A system is highly nervous if a small change in the master schedule results in numerous changes in the component lot sizes and/or lot timings. A number of
authors have investigated system nervousness [35, 36, 63, 71, 77, 94, 116, 131, 141, 155, 169, 175, 229, 251, 261, 270].

One common recommendation of preceding studies is to freeze the first few periods in the master schedule, only allowing changes in periods farther in the future. Other studies attempt to locate changes that can be made without creating a large cascading effect. A few authors study the interaction between changing lot sizes and changing lot timing. Finally, some authors investigated the effects and costs of maintaining safety stocks in order to minimize system nervousness.

11. The integer principle

As items flow through the MRP system, they flow in the form of lots. As these lots flow into and out of each level in the BOM, they can interact with lots of the same items which are adjacent in time. The integer principle outlines these possible interactions. According to the integer principle, lots are only joined in integer numbers. Wagner and Whitin proved the optimality of the integer principle for single level discrete demands (theorem 1) [253]. A recent extension of this proof was created for the MRP environment [64, 265], while other authors have designed cases where the integer principle is purported to not work [238]. Most of the recent studies have utilized the integer principle [15, 31, 64, 72, 77, 79, 179, 180, 181, 196, 203, 238, 265].

Using the integer principle, only three types of interactions are allowed, as shown in Figures 2.3, 2.4, and 2.5. In these figures, each
horizontal line represents one stage of production, the instantaneous state changes which occur at the discrete time boundaries are the vertical lines, time is on the horizontal axis, and the nodes represent lots. The finished product is at the top of the figure, and the items flow from bottom to top (raw material to finished good) and from left to right (increasing time).

FIGURE 2.3 - Lot For Lot Sequencing

Figure 2.3 shows lots flowing in a lot for lot (LFL) fashion through the system. Once a lot size is determined, it remains in effect as long as this LFL sequencing pattern is used.

Figure 2.4 shows lots in a joint sequencing pattern. Here two adjacent lots of parent items are joined into one lot of subcomponent items. This could occur at any level and more than two lots could be joined. The term joint sequence is derived by looking at the lots from the top of the system (finished product) toward the bottom (raw materials).
Figure 2.5 shows lots in a split sequencing pattern. Here a single lot of parent items are split into two lots of subcomponent items. Again, this splitting can occur on any level and in quantities other than one into two. Again, the term split sequencing can be visualized by looking at the lots from top to bottom.
Figure 2.6 shows a situation which does not follow the integer principle. Here the lots both join and split at the same time. The integer principle states that this would never occur in an optimum solution to the MRP lot sizing problem.

12. Collapsing the problem space

A recent study has attempted to expand the integer principle to include a method of collapsing the solution space by finding nodes that will never react other than in a LFL manner [15]. The key to finding these nodes is through the cost ratio (Equations 2.8 and 2.9). According to their study, if the cost ratio decreases from parent to component, the lots will never be joined as shown in Figure 2.4.

When this situation occurs, the component items can be collapsed into the parent item for lot sizing purposes. The setup costs and holding costs for the levels are then added to form a single setup cost and
holding cost for the collapsed group of items. Their study included tests which allow, both, sequential and assembly structures to be collapsed. The authors used the echelon method for calculating holding costs.

13. Lot sizing models

Lot sizing the component demands in MRP controls the manner in which the joining and splitting of lots shown in Figures 2.3, 2.4, and 2.5 is performed. The object of lot sizing is the minimization of variable costs. The variable costs are, generally, assumed to be the setup cost, holding cost, safety stock cost, and shortage cost. This section describes the major methods that have been used in the attempt to minimize these variable costs [81].

a. Multiple passes with a single level heuristic The simplest manner in which lot sizes are developed in a MRP environment is to use one of the plentiful single level heuristics on each individual level. In this dissertation, this method is called the multipass single level (MPSL) method. The MPSL method was presented along with the original description of MRP [194]. A number of studies have evaluated which single level heuristics work best in the multilevel environment [25, 26, 30, 67, 68, 69, 79, 82, 138, 159, 173, 179, 180, 181, 185, 246, 258, 271].

The most important studies which analyze the MPSL method, and their results are shown in Table 2.5. The columns in Table 2.5 show if the studies were capacity constrained, if the end item demands varied, if the study used echelon or replacement (total) holding costs, if serial and/or
assembly product structure types were investigated, if the results were compared to the optimum solution, if a new heuristic was presented, and if the study used the integer principle. The final column lists the recommendation that the authors present as a result of the study. Only the primary author is listed in Table 2.5, for a complete reference please see the bibliography.

As Table 2.5 shows, the studies which investigate the use of MPSL method reach no general consensus. Usually, these studies recommend certain heuristics for low level items and other heuristics for high level items. The recommended heuristics often change when the input parameters change. Thus, one combination of heuristics might be recommended for highly variable demand and a different set for demands which are fairly constant. Other factors such as the cost ratios, the type of structure, or capacity constraints can also affect the recommendation. Of those studies that made a single recommendation, the periodic order quantity (POQ) heuristic was recommended, the single level Wagner-Whitin (SL-WW) algorithm was recommended, and two studies recommended new heuristics.

b. Modified single level heuristics A different attack on the MRP lot sizing problem entails the development of new multilevel heuristics [2, 40, 41, 43, 44, 57, 74, 76, 90, 115, 138, 171, 196, 202, 207, 258]. These heuristics tend to be modifications of the heuristics which work well on the single level problem. In this dissertation, these are called modified single level (MSL) heuristics. Usually, the MSL heuristics modify the cost ratio in some manner in the attempt to achieve
### TABLE 2.5 - Multiple Passes with Single Level Heuristics

<table>
<thead>
<tr>
<th>STUDY</th>
<th>CAPACITY</th>
<th>VARIABLE</th>
<th>HOLDING COST</th>
<th>STRUCTURE</th>
<th>OPTIMAL</th>
</tr>
</thead>
<tbody>
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<td>Benton [24, 25]</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Biggs [29]</td>
<td>yes</td>
<td>yes</td>
<td>?</td>
<td>?</td>
<td>yes</td>
</tr>
<tr>
<td>Choi [62 - 64]</td>
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<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Collier [71]</td>
<td>yes</td>
<td>yes</td>
<td>?</td>
<td>?</td>
<td>no</td>
</tr>
<tr>
<td>Crowston [74]</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>LaForge [146, 228]</td>
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<td>yes</td>
<td>no</td>
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<td>no</td>
</tr>
<tr>
<td>Melnyk [157]</td>
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<td>yes</td>
<td>?</td>
<td>?</td>
<td>yes</td>
</tr>
<tr>
<td>Moilly [163 - 165]</td>
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<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Yelle [253]</td>
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<td>yes</td>
<td>?</td>
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### e Level Heuristics

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<th>ASSEMBLY</th>
<th>OPTIMAL</th>
<th>NEW</th>
<th>INTEGER</th>
<th>PRINCIPLE</th>
<th>RESULTS</th>
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<td>no</td>
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<td>various</td>
</tr>
<tr>
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<td>yes</td>
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more cost effective solutions. They differ from the single level heuristics in that they utilize information from multiple levels in each pass through the master production schedule.

The most important studies which include MSL heuristics and their results are shown in Table 2.6. The columns in Table 2.6 are the same as the columns in Table 2.5, with one exception. The next to last column shows if the study included MPSL heuristics.

As shown in Table 2.6, the MSL heuristics dominate the MPSL heuristics. All of the studies but three find that the MSL heuristics performs in a more effective manner. One study recommends that the optimum solution be utilized, while two studies state that the results still depend upon input parameters.

c. Optimum solutions The final method of attacking the MRP lot sizing problem is the development of models which provide an optimal solution [3, 33, 57, 72, 79, 170, 179, 180, 181, 201, 203, 217, 231, 232, 236, 239, 247, 248]. Generally, these models work on a fixed horizon problem, however some make attempts to modify the end of horizon data for better solutions. These models work in a number of different manners. The most important studies and their results are shown in Table 2.7.

A number of the studies which describe MPSL and MSL heuristics include an optimum solution method. These methods are, usually, one of the methods shown in Table 2.7. Of these methods, a number work only on the constant demand problem or only on the serial structure.
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14. Real world results

The final group of studies present evaluations of actual MRP systems. These studies tend to be general and opinionated. A number of these studies present favorable findings [55, 59, 92, 124, 190, 193, 195], while a number of other studies present poor findings [59, 80, 107, 156, 168, 211, 213, 215, 263]. The general consensus is that MRP has not worked as well as originally hoped, however it has worked better than any other system currently available.

D. Additional Multilevel Discrete Demand Environments

Three additional environments have been proposed in recent years in an attempt to replace MRP systems. These are the KANBAN system, the OPT system, and the just in time (JIT) system. While each of these systems have their place, none have replaced MRP systems as the planning norm.

1. KANBAN system

The KANBAN system works through a series of cards which represent lots [12, 23, 34, 61, 85, 117, 121, 154, 163, 178, 205, 216]. When a lot is used, a card is freed and sent to the preceding station in order to initiate a new lot of the same item. Thus, the KANBAN system attempts to keep a certain level of inventory moving through the system at all times.

This process works fine for stable production levels. However, when the production levels change, what number of KANBANS should be on the
floor? In general, some type of planning system must coordinate the KANBAN system, and often it is MRP.

2. Just in time (JIT) system

Just In Time (JIT) is not really an environment, but rather a philosophy [6, 12, 21, 34, 56, 61, 85, 98, 101, 109, 119, 121, 125, 157, 163, 197, 198, 200, 216, 223, 227, 243, 249, 250, 252, 255]. JIT states that no inventory at all should be held. Of course this is quite impossible in most manufacturing situations. What JIT really does is highlight those areas that have the best cost to return ratios.

For example, in order to reduce inventory JIT tends to focus on reducing setup costs so that lots can be smaller. However, if the lower setup costs had been created without JIT, an MRP system should also have produced smaller lot sizes.

3. OPT

OPT is a proprietary product that has received a lot of attention in recent years [101, 109, 110, 137, 198, 235, 249]. OPT focuses on identifying the bottlenecks in the production system and maximizing the production through these bottlenecks. The OPT system is, thus, similar to JIT in that it focuses where the highest yields are.

In other manners, OPT is similar to MRP. Both explode the BOM into component demands and time phase these demands. OPT is perhaps more like an example of the optimum solutions for the MRP environment described in a
preceding section than is sometimes advertised. However, the logic of the system is maintained as a trade secret and no real comparisons have been published.
III. PROBLEM DEFINITION

The problem addressed in this dissertation is the optimization of a simulated MRP environment with respect to variable lot sizing costs. This overall problem is attacked by subdividing it into three major elements. These three elements are:

- The development of an underlying MRP model
- The construction of a simulation of this underlying MRP model.
- The optimization of the lot sizing logic used in the MRP model.

The first step to be performed in any simulation is the development of the simulation model [17, 19, 20, 73, 78, 86, 132, 136, 140, 144, 164, 176]. This model should accurately reflect the real world that it is attempting to replicate, while omitting those portions of the real world problem that are extraneous to the area of analysis. As shown in the preceding chapter, there have been a large number of differing MRP model assumptions used in previous studies of the MRP environment. The next chapter describes the assumptions under which the model developed in this research operates.

The second problem to be addressed is the construction of a simulation of the developed MRP model. This simulation should allow the model to be tested on a variety of input data. Since the simulation performed in this dissertation is digital, this portion reduces to basically a software development and testing problem. This element of the overall problem analysis is not discussed directly in this dissertation.
The third step is the optimization of the lot sizing logic used in the MRP model. In order to judge the efficacy of the lot sizing logic, the results are compared with those developed by previous lot sizing methods. Thus, lot sizing methods presented in previous research are also simulated, with the results of the various lot sizing methods compared and any weakness of the new lot sizing logic explored. This third element of the overall problem analysis is contained in Chapter V.
IV. DEVELOPMENT OF MRP MODEL ASSUMPTIONS

As shown in the literature review, previous studies into the MRP environment have utilized a large number of differing assumptions. This chapter examines these assumptions and describes how they are utilized in this dissertation. This description can include an analysis of the effects that the assumptions can have on the developed MRP model and resulting simulation.

The first section in this chapter lists those areas in which a specific assumption is adopted without a thorough analysis. These represent assumptions common to the majority of previous studies, or assumptions which cover areas not investigated in this dissertation. The second section lists those areas in which various possible assumptions are investigated. In general, an analysis of the various possible assumptions which cover an area is performed with the result being the adoption of one of these possible assumptions for the MRP model developed in this dissertation.

A. Assumptions Adopted Without Analysis

This section lists those areas in which a specific assumption is adopted without an indepth analysis. These assumptions are either utilized by a large percentage of previous studies or describe portions of the MRP environment which are not analyzed in this dissertation.
1. Discrete time and discrete demand

The demand data in this dissertation are presented in the terms of units per period. The length of the period remains undefined. All input data and results are presented in this form.

Note, that the use of an indefinite period does not mean that the choice of a discrete time period is unimportant in a real world environment. The use of an undefined period is simply a simulation abstraction that allows results to be developed independent of a fixed time unit base. All real world implementations should analyze the problem of setting a period length before attempting to utilize these simulation results.

2. Setup cost

The setup cost definition utilized in this dissertation is the same as the standard definition described in the literature review. The setups are time independent. No group technology or capacity implications are examined.

3. Master schedule construction

The master schedules utilized in this dissertation are developed using computer generated random numbers. A random number generator which satisfies common statistical tests is utilized. The demand distributions represent, both single sources of demand and more complex demand patterns.
4. Capacity restrictions

No capacity restrictions are included in this dissertation. Production quantities, capital, and inventory space are all assumed infinite. This simplifies, both, the development of the model and the construction of the simulation data. The results of the simulation need to be reconciled with the restricted real world environment prior to their actual usage.

5. Scrap, rework, and safety stock

The problems associated with scrap, rework, and safety stock are not addressed in this dissertation. Like capacity restrictions, the inclusion of these factors greatly complicates the model and requires that time be taken away from the study of other areas. Again, the results developed in the simulation need to reconciled with the real world before the results can actually be applied.

6. Component lead times

The component lead times in the simulation are all assumed to be zero as is common with most recent simulations. This has no effect on the simulation results. However, real applications should take great care in setting the lead times. As discussed by a number of authors, the determination of lead times is a parameter that greatly affects the results of real world MRP systems.
7. System nervousness

The issue of system nervousness is not included in the simulation model. This issue requires that the safety stock and capacity issues also be addressed. This, in turn, requires that data be developed on scrap/rework rates, shortage costs, and system capacity.

8. Product structures and cost ratios

Both the sequential and assembly product structure are utilized in the simulation. For both structures, the components are always required in one to one ratios (sets).

Cost ratios are used only indirectly in this dissertation. As shown in the later section which describes collapsing the problem (IV.B.3.b), cost ratios can be used to determine inventory levels which are never handled by the optimum solution in other than a lot for lot manner. These inventory levels can then be collapsed into adjacent levels. In general, cost ratios which result in such collapsed problems are not analyzed.

B. Assumptions Adopted After Analysis

This section lists areas in which various assumptions are investigated in conjunction with the development of the MRP model. In general, these areas need to be investigated because previous studies have used differing and opposing assumptions in developing an MRP model. After investigating the various assumptions possible for an area, an assumption is adopted for the remainder of this study.
1. The integer principle

The integer principle describes the manner in which lots of items are joined and split as they pass through the various inventory levels. The three types of interactions allowed by the integer principle are shown in Figures 4.1, 4.2, and 4.3. An interaction of lots which does not follow the integer principle is shown in Figure 4.4. All four of these figures were originally shown in Chapter II.

A simple examination of Figure 4.4 shows that handling lots in a manner not consistent with the integer principle must result in excess costs. The number of setups and the amount of holding costs generated would both be reduced by producing all the parts in levels $x+2$ and $x+3$ in a single lot and timing this lot consistent with the second lot. This would result in a joint sequence as depicted in Figure 4.2.
A review of studies which do not use the integer principle, reveals that these studies fall into two types. First, those studies which were performed prior to the advent of the integer principle. Second, studies which are not discrete in nature. Since this dissertation is discrete the integer principle is obeyed.
An additional fact related to the integer principle can also be observed by examining Figure 4.3 (split sequencing). This fact is simply that split sequencing of lots will always result in excess costs. For example, in Figure 4.3 the number of setups and the amount of holding costs are both higher because there are two lots at levels x+2 and x+3. The fact that split sequencing always results in excess costs is not generally recognized.

Thus, there are only two possible methods in which lots can interact as they pass through the inventory system and still result in a minimum of total costs. These are the lot for lot method as shown in Figure 4.1 and the joint sequencing shown in Figure 4.2. All other interactions must result in higher total costs.

Before leaving this subject, a brief caveat is probably required. Simply put, this warning is that factors other than minimizing total cost
could require that split or noninteger lot sequencing be used in an actual MRP system. Some possible reasons include capacity restrictions, redrawn master schedules, scrap problems, and inventory space restrictions. However, since these areas are used in this dissertation, only the joint and lot for lot sequencing patterns will be studied.

2. Holding costs

As described in the literature review, there are a number of conflicting ideas surrounding the use and meaning of holding costs. The first area of conflict is the method used to determine inventory levels. However, as shown earlier, the three major methods (end of period, generated part period, and average inventory level) are all linearly related and thus somewhat interchangeable. This dissertation utilizes the end of period (EOP) method for both optimization and cost development.

As also shown earlier, there have been two methods in which the inventory levels have been changed into holding costs. These methods are shown in Equations 2.1 through 2.4, and represent holding costs based upon inventory counts or inventory value. Also, the holding cost can be time dependent or time independent. In this dissertation, the inventory cost is time independent and is based upon the inventory count. Thus, holding costs are developed as shown in Equation 2.2.

Finally, there have been two major methods in which the per part inventory holding cost (H in Equations 2.1 through 2.4) has been valued. These are the replacement cost and incremental (echelon) cost. The
remainder of this section analyzes which of these two methods is better and how these methods are used in this study. However prior to this investigation an additional caveat is probably required.

This dissertation examines only variable holding costs. As described earlier, component lead times are set to zero throughout this study. In a real MRP system the lead times would of course not be zero and these lead times would develop actual holding costs. However, these holding costs are nonvariable with respect to lot sizing and are, thus, of no interest to this study.

Of final note is the fact that, in this study, the lot for lot method of sequencing never generates any holding costs because of the zero lead times. Thus, holding costs are generated only when the joint lot sequencing pattern is used. These holding costs then represent only the variable holding costs of a real world MRP system.

a. An argument for replacement costing As stated above, variable holding costs are created, only, when the joint lot sequencing pattern (Figure 4.2) is used. By studying this figure, it is easy to see the argument for using replacement cost in calculating the holding cost.

When a multipass single level (MPSL) heuristic is used to lot size in an MRP environment, the finished product level is the first level processed. The next level for which lots are developed contains the immediate subcomponents. Thus in Figure 4.2, level x is lot sized first with the resulting lots becoming the demands in level x+1, etc. If the
joint sequencing pattern is formed in this situation, the result is that items are moved back in time (earlier) in the inventory system.

For example, in Figure 4.2 at level \( x+2 \) and level \( x+3 \) all items are processed in the same lot. Thus the processing for the items from the second lot in levels \( x \) and \( x+1 \) has been moved to an earlier time than would be absolutely necessary. The end result is that these items have been held in inventory for an excess amount of time, with the amount of time equal to the time difference between the two lots. Holding these parts in inventory in turn generates variable holding costs. The amount of holding costs is related to the replacement cost of the items at level \( x+2 \), with this relationship described by the holding cost (\( H \)) parameter.

This logic for using the replacement cost for an item in calculating the holding costs is a direct extension of the earlier single level studies. The logic is valid if the lot sizes are decided by working level by level from the top (finished product) of the MRP system. Once an item is moved backward in the inventory system, the item can never be moved forward to the original time sequence. Thus, all subcomponents (including raw material purchases) would have to be moved backwards with the end result being increased holding costs proportional to the replacement cost of the lot sized component, again with this relationship described by \( H \) in Equation 2.2.

b. An argument for incremental costing A somewhat similar, but opposite, argument is available for using the incremental cost of an item in constructing the \( H \) parameter and thus calculating the variable holding
costs. This argument assumes that the lot sizes are calculated from the bottom (raw material level) up through the MRP system. Again, variable holding costs are generated, only, when the joint lot sequencing pattern (Figure 4.2) is used.

Imagine for example, that the lot size at levels x+3 and x+2 is known in Figure 4.2 and the lot size at level x+1 is to be determined. The lot sizing process should offset the increased setup costs with the savings in holding costs which would be generated by splitting (joining when looking from the top) the lot. The viable holding cost to use would then reflect the incremental difference between holding the parts at level x+2 and level x+1. Since the parts are already at level x+2 no holding cost savings can be achieved for the parts at that level.

Most previous studies that have utilized the echelon cost have used this cost for only the level at which the lot sizing is being performed. However, by studying Figure 4.2 it can be seen that viable holding cost is actually the difference between the replacement cost at the level at which the lot sizing is being performed (level x+2) and the finished product (level x). This is due to the fact that the items must be held in inventory and the only change is whether they should be held at the intermediate level (x+2) or a higher level (x). Note that the difference in replacement value at the two levels is, probably, related to the sum of the echelon costs between the two levels.

c. Conclusions As described above, there are viable arguments for using both the replacement cost and incremental cost in the
development of the H parameter used to calculate lot sizes. The choice of which to use depends upon which way the model is being worked. If the lot sizes are calculated from the top (finished product) down, the replacement cost should be used. If the lot sizes are calculated from the bottom (raw material) up, the incremental holding cost should be used. In either case, only the joint lot sequencing pattern, Figure 4.2, creates variable holding costs.

When the holding costs which are generated by a lot sizing plan are calculated, the replacement cost value for each item should always be used, since the items held in inventory represent an inventory value equal to the replacement cost. This is true whether the parts are held due to lot sizing calculations performed from the bottom or top of the MRP system.

In this dissertation, all lot sizing models operate from the top to the bottom of the MRP system. Thus, only replacement cost valuation is included in the study. Of course, the holding costs for the developed lot sizing plans are calculated using replacement cost valuation.

3. Collapsing the problem

The idea of collapsing the problem is relatively new to the study of MRP environments. In the past, problems have been collapsed using the cost ratio information from the various inventory levels. However, as shown here a problem can also be collapsed using only the holding cost information. Both of these methods of collapsing the problem, as well as
a summary of the manner in which this information is used in this dissertation, are included in the following sections.

a. Collapsing the problem using holding costs The idea of collapsing the problem using only holding costs is new to this dissertation. This idea depends upon the use of set holding costs. That is, holding costs are based upon the set of items which are required for one finished product rather than based upon each of the individual items themselves. The use of set holding costs is not new and was described in the literature review in Section II.C.6.b.

In order to show how the problem can be collapsed using only holding costs, Figure 2.4 (or 4.2) is again repeated and is shown as Figure 4.5. This time two alternate lot sizing plans are shown as the two sets of dashed lines. These two lot sizing plans are to join the two lots at level x+1 or level x+2.

![Diagram of joint sequencing with two options]

FIGURE 4.5 - Joint Sequencing With Two Options
The difference in total costs for these two lot sizing plans is easily determined. First, the total costs for all levels other than levels x+1 and x+2 are the same for either plan. Also, since the lead time offsets are the same for all lots passing through the system, the lots are the same number of periods apart at both levels x+1 and x+2. Finally, there is the same number of sets of parts going through both levels.

Thus, the total cost of joining the two lots at level x+1 is simply the number of sets of parts times the number of periods apart the two lots are times the holding cost parameter H[x+1] plus a setup at both levels x+1 and x+2 plus the total cost for all other levels. The cost of joining the two lots at level x+2 is the same except the holding cost parameter to use is H[x+2] and there is an additional setup at level x+1. These two formulas are shown in Equations 4.1 and 4.2 with the constants included as ρ and κ.

\[
TC[x+1] = H[x+1] \cdot \rho + S[x+1] + S[x+2] + \kappa \tag{4.1}
\]

And

\[
TC[x+2] = H[x+2] \cdot \rho + 2 \cdot S[x+1] + S[x+2] + \kappa \tag{4.2}
\]

Where:

- TC[x+1] = the total cost of joining lots at level x+1;
- TC[x+2] = the total cost of joining lots at level x+2;
- H[x+1] = the holding cost for level x+1 in $/unit-sets inventory;
- H[x+2] = the holding cost for level x+2 in $/Unit-sets inventory;
- ρ = the number of sets times the periods the lots are apart;
- S[x+1] = the setup cost at level x+1 in $/setup;
\[ S[x+2] = \text{the setup cost at level } x+2 \text{ in } \$/\text{setup}; \]
\[ \kappa = \text{the constant total cost for all other levels.} \]

Now, simply, imagine that \( TC[x+2] \) is less than \( TC[x+1] \) as shown in Equation 4.3. This reduces to Equation 4.4 when the common factors are removed. Finally, since \( S[x+1] \) is strictly nonnegative and is on the small side of the inequality and \( \rho \) is a constant; the inequality can be reduced to Equation 4.5.

\[ TC[x+2] < TC[x+1] \quad (4.3) \]

Or

\[ H[x+2] \cdot \rho + S[x+1] < H[x+1] \cdot \rho \quad (4.4) \]

Yields

\[ H[x+2] < H[x+1] \quad (4.5) \]

Where:

\( TC[x+1] = \text{the total cost of joining lots at level } x+1; \)
\( TC[x+2] = \text{the total cost of joining lots at level } x+2; \)
\( H[x+1] = \text{the holding cost for level } x+1 \text{ in } \$/\text{unit-sets inventory}; \)
\( H[x+2] = \text{the holding cost for level } x+2 \text{ in } \$/\text{Unit-sets inventory}; \)
\( \rho = \text{the number of sets times the periods the lots are apart; } \)
\( S[x+1] = \text{the setup cost at level } x+1 \text{ in } \$/\text{setup.} \)

Thus, in order for the total cost to be reduced by joining the lots at level \( x+2 \) instead of \( x+1 \), the set holding cost must also be smaller at level \( x+2 \). If the set holding cost is not smaller at level \( x+2 \), the optimum solution will never choose to join the lots at level \( x+2 \) and the problem can be collapsed. This same proof can be extended to all levels.
The end result of this proof is that all viable levels must have decreasing set holding costs from top (finished product) to bottom (raw material). If a level does not have a decreasing set holding cost, that level can be collapsed and removed from the lot sizing problem.

For an assembly problem, each subtree needs to be analyzed separately. That is, if a parent part has two child parts, collapsing one of the children does not require collapsing both of the children. It would be possible for one of the subtrees to join lots while the other subtree does not.

Before leaving two caveats are, probably, required. First, this dissertation does not consider scrap or rework. Thus, if scrap and/or rework are included, the set holding costs should be adjusted accordingly before the problem is collapsed. This should be relatively simple since scrap/ rework is usually treated using yield constants for the various levels. Thus if the yield is 90% for a level, the set holding cost for all lower levels should reflect the fact that 11.1% more parts are included in the set on average than are actually required in the finished product.

Second, decreasing set holding costs are probably common in actual inventory systems. As items flow from raw materials to finished goods, they tend to get larger and more valuable and thus have a higher set holding cost.
b. Collapsing the problem using cost ratios

The idea of collapsing the problem using cost ratios is not new and was reviewed in Section II.C.12. This previous study used the echelon method in determining holding costs. However, the use of echelon holding costs will again depend upon the manner in which the lot sizes are developed, as described in Section IV.B.2.

A proof that cost ratios can be used to collapse the problem is easy to develop and is shown in Equations 4.6 through 4.11. This proof, again, uses Figure 4.5. However, this time the lots are imagined joined at level x+2.

The lots will be joined at level x+2, only, if the setup cost at level x+2 is higher than the cost of holding the parts in inventory at level x+2, as shown in Equation 4.6. Solving Equation 4.6 for \( p \) gives Equation 4.7. In order for the lots to be joined at level x+2, they must not have been joined at level x+1, which in turn means that the inequality shown in Equation 4.8 must be true. Again, solving for \( p \) yields Equation 4.9.

Equation 4.10 can be developed by joining Equations 4.7 and 4.9. Finally, Equation 4.11 shows that the cost ratios must be increasing. The development of Equation 4.11 is possible since \( H[x+1] \) and \( H[x+2] \) are strictly positive and \( H[x+1] \) must be larger than \( H[x+2] \) as shown in the previous section.

\[
S[x+2] > H[x+2] \cdot \rho \quad (4.6)
\]

Or
\[ S[x+2] + H[x+2] > \rho \] (4.7)

And
\[ H[x+1] \cdot \rho > H[x+2] \cdot \rho + S[x+1] \] (4.8)

Or
\[ \rho > S[x+1] + (H[x+1] - H[x+2]) \] (4.9)

Thus
\[ S[x+2] + H[x+2] > \rho > S[x+1] + (H[x+1] - H[x+2]) \] (4.10)

Or
\[ S[x+2] + H[x+2] > S[x+1] + H[x+1] \] (4.11)

Where:
- \( S[x+1] \) = the setup cost at level \( x+1 \) in \$/setup;
- \( S[x+2] \) = the setup cost at level \( x+2 \) in \$/setup;
- \( H[x+1] \) = the holding cost for level \( x+1 \) in \$/unit-sets inventory;
- \( H[x+2] \) = the holding cost for level \( x+2 \) in \$/unit-sets inventory;
- \( \rho \) = the number of sets times the periods the lots are apart.

Equation 4.11 can, of course, be applied to any level in a MRP system. Thus, Equation 4.11 shows that the cost ratios must be increasing from the top levels to the bottom levels throughout the MRP system or levels can be collapsed. If an assembly system is used, each subtree should, again, be treated independently.

c. **Summary** This section has shown that levels in an MRP system can often be collapsed into adjoining levels for lot sizing purposes. In general, the collapsing of the problem can be done before any lot sizing is performed. The result is that the number of levels which need to be
examined for lot sizing purposes is often less than the total number of levels in the MRP system.

The final question surrounding the idea of collapsing levels out of the lot sizing problem is what to do with the parameters for the collapsed level. First, when a level is collapsible, the level will always react in a lot for lot manner. Parts will never be held in inventory at the level. Thus, the level can be removed from the lot sizing system and simply treated as a portion of the parent level. Since parts will never be held in inventory, the inventory holding cost for the level can be discarded if the full cost method of valuing inventory is used. If the echelon method of valuing inventory is used, care should be taken that the echelon costs for the parent and child levels reflect the value of the collapsed level.

The setup cost for the collapsed level, on the other hand, always needs to be included in the setup cost for parent items. Once a lot is formed at the parent level, this lot will also pass through the collapsed level in a lot for lot manner. This is true no matter what lot sizing method is used.

Collapsible levels are not included in the remaining portion of this study. All data can be imagined as either previously collapsed or collapse proof.

4. Rolling horizon effects

As shown earlier, a rolling horizon environment describes the manner in which the passage of time is handled in an MRP system. As time passes,
the current demands become history and additional demands are added to the end of the planning horizon. All MRP systems are subject to a rolling horizon, unless the total demand for the life of the product is known at the start of production.

There are two main issues involved in analyzing the rolling horizon environment. The first issue is the effect of a rolling horizon on the performance of optimal solutions. The second issue is the effect of a rolling horizon on the performance of heuristic solutions. Both of these issues are discussed in a separate section. Also included is a summary of the manner in which rolling horizons are treated in this dissertation.

a. The rolling schedule environment and optimal solutions

An optimal solution to an MRP type of problem guarantees optimality only for a fixed horizon problem. That is, the optimal solution must be developed using known data and in an MRP system these known data are available only for the period of time contained in the planning horizon. As this horizon moves (rolls), new data are added to the end. These new data can in turn make previous decisions nonoptimal.

In this section, a simple example is developed which shows how an optimal solution can easily be turned nonoptimal in a rolling horizon environment. This example uses a single level problem and constant per-period demand, although the process can easily be seen to extend to variable demand and the multilevel problem. The example also uses constant setup and holding costs consistent with the assumptions used throughout this dissertation. The example uses the data in Table 4.1.
TABLE 4.1 - Example Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-lot Setup Cost</td>
<td>$35.00</td>
</tr>
<tr>
<td>Per-part Per-period Holding Cost</td>
<td>$1.00</td>
</tr>
<tr>
<td>Constant Per-period Demand</td>
<td>10</td>
</tr>
<tr>
<td>Length of the Demand Array</td>
<td>6</td>
</tr>
<tr>
<td>Length of the Planning Horizon</td>
<td>4</td>
</tr>
</tbody>
</table>

The only six possible lot sizing plans using a planning horizon of four periods are shown in Table 4.2. In Table 4.2 each possible lot sizing plan is given an option number. An optimal solution to this problem would of course choose the minimum cost solution from these six options. The six possible options are:

- **Option 1** — produce the four period planning horizon in four lots. Each lot includes the demand from only one period.
- **Option 2** — produce the four period planning horizon in two lots. Each lot includes the demand from two periods.
- **Option 3** — produce the four period planning horizon in two lots. The first lot includes the demand from three periods. The second lot includes the demand from only one period.
- **Option 4** — produce the four period planning horizon in one lot. The lot includes the demand from all four periods.
- **Option 5** — produce the four period planning horizon in three lots. One lot includes the demand from two periods. The other two lots include the demand from only one period each.
• Option 6 -- produce the four period planning horizon in two lots. The first lot includes the demand from one period. The second lot includes the demand from three periods.

### TABLE 4.2 - Possible Lot Sizing Plans

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Option 1 Lot Sizing Plan</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Option 2 Lot Sizing Plan</td>
<td>20</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option 3 Lot Sizing Plan</td>
<td>30</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Option 4 Lot Sizing Plan</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option 5 Lot Sizing Plan</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Option 6 Lot Sizing Plan</td>
<td>10</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Option 5 is trivial. If Option 5 develops a lower total cost than Option 1, the total cost for Option 5 must be higher than the total cost for Option 2. In other words, for constant per-period demand, setup cost, and holding cost; if it is cheaper to join two one period lots into one two period lot, it must be cheaper still to join four one period lots into two two period lots. Option 5 is not investigated.

Regarding Option 6, given constant per-period demand, setup cost, and holding cost; the total cost for the fixed horizon problem developed by Option 6 is the same as that developed by Option 3. In a rolling schedule environment, however, Options 3 and 6 would develop very different global results. Option 3 would develop lot sizes with 30 items in each lot,
while Option 6 would develop lot sizes with 10 items in each lot. Thus, Option 6 would develop lot sizes the same as Option 1. Because of this, Option 6 is not investigated.

The real end result of this cost equivalence for Options 3 and 6 is that depending upon the particular optimal solution implementation, either Option 3 or Option 6 could be selected if they both developed a lower cost than all the other possible options. That is, for constant per-period demand, setup cost and holding cost; if the minimum cost solution is to use two lots of differing length, the optimal solution could logically select either the smaller or the larger lot size to be produced first. In the rolling schedule environment, where only the first lot is implemented and then the planning horizon is rolled, this could have a great effect on the global total cost which is developed. The choice by the optimal solution of the smaller or larger lot first is not investigated further in this dissertation.

The costs which are developed by the remaining options (1, 2, 3, and 4) are shown in Table 4.3. The holding cost shown in Table 4.3 are calculated using the end of period (EOP) method.

Table 4.3 shows that Option 2 develops the minimum cost solution for the four period planning horizon. Of course any type of optimal solution will select Option 2 for this four period problem. However, as will soon be shown, the global optimum lot size is to use three periods of demand in each lot.
TABLE 4.3 - Costs for Various Options

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Sizing Plan</td>
<td>10</td>
</tr>
<tr>
<td>Ending Inventory</td>
<td>0</td>
</tr>
<tr>
<td>EOP Holding Cost</td>
<td>0</td>
</tr>
<tr>
<td>Setup Cost</td>
<td>35</td>
</tr>
<tr>
<td>Sum Total Cost</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Option 3</th>
<th>Option 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Sizing Plan</td>
<td>30</td>
</tr>
<tr>
<td>Ending Inventory</td>
<td>20</td>
</tr>
<tr>
<td>EOP Holding Cost</td>
<td>20</td>
</tr>
<tr>
<td>Setup Cost</td>
<td>35</td>
</tr>
<tr>
<td>Sum Total Cost</td>
<td>55</td>
</tr>
</tbody>
</table>

When this solution for the four period planning horizon is used in the rolling schedule environment, the first lot containing two periods of demand is fixed. These first two periods are then removed from the planning horizon and the next two periods added. This, again, results in a planning horizon of four periods and exactly the same problem. Again, the optimal solution is Option 2.

When the second lot is removed from the planning horizon no additional periods are available to bring the planning horizon back up to four periods. The planning horizon then consists of only two periods.
(periods five and six). This leaves only two options available, i.e., use a lot containing both periods or use two lots containing one period each. By examining the costs developed in the first two periods of Option 1 and Option 2 in Table 4.3, it can be seen that including both periods five and six in one lot is the minimum cost solution.

Thus, for this planning horizon and set of parameters, the optimal solutions to the series of fixed horizon problems select an overall lot sizing plan containing three lots; each of which contains the demand from two periods. This results in a nonoptimal global solution.

Table 4.4 shows the overall solution developed by the optimal fixed horizon problem and the actual overall optimum solution. The optimal fixed horizon solution develops an global cost of $135. The actual global optimal solution develops an overall cost of $130. Thus, the optimal fixed horizon solution is nonoptimal by 3.8% in this example. The holding costs are calculated using the EOP method in Table 4.4.

Of course if the six period problem was formulated without a rolling schedule environment, the problem becomes a fixed horizon problem and the correct solution would be chosen. However, when an optimal solution is used in conjunction with a rolling horizon, the solution chosen is optimal only with respect to the current planning horizon. These local optima do not necessarily result in a global optimum. This example develops one instance where the use of the fixed horizon optimum solution in conjunction with a rolling schedule environment results in a nonoptimal global solution.
TABLE 4.4 - Costs for Fixed Horizon and Global Optimum Solutions

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixed Horizon Optimum Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Sizing Plan</td>
</tr>
<tr>
<td>Ending Inventory</td>
</tr>
<tr>
<td>EOP Holding Cost</td>
</tr>
<tr>
<td>Setup Cost</td>
</tr>
<tr>
<td>Sum Total Cost</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Global Optimum Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Size</td>
</tr>
<tr>
<td>Ending Inventory</td>
</tr>
<tr>
<td>EOP Holding Cost</td>
</tr>
<tr>
<td>Setup Cost</td>
</tr>
<tr>
<td>Sum Total Cost</td>
</tr>
</tbody>
</table>

b. The rolling schedule environment and heuristics

This section examines the effects of a rolling horizon on heuristic solutions. Again, the examples developed in this section focus on single level problems but it is easy to extend the example to the multilevel environment.

The heuristics that are used to operate in the single level discrete demand lot sizing environment do not suffer from the same problem as optimal solution in a rolling schedule environment. The heuristics do not consider the entire demand array at once. Rather, they consider the demands sequentially.
The heuristics start with the first period in the demand array and continue to add periods until the first lot is complete. When the first lot is complete, demands are added into the second lot until the second lot is complete, etc. In general, only one lot is considered at a time. Once a lot size is determined, no changes are made to that lot.

Each heuristic uses a separate decision rule to determine when a lot is complete, with the decision rule processing the demand array sequentially. Thus, it would seem that the effects of a rolling schedule environment on the performance of the various heuristics would form the basis for a valid investigation. However, these effects tend to trivialize the performance evaluation.

An example using constant per-period demand is, again, included to illustrate the manner in which the rolling schedule environment can trivialize a heuristic performance evaluation. Figure 4.6 is included for the purpose of this example. Figure 4.6 graphically shows the single level discrete demand lot sizing environment given constant per-period demand, constant setup cost, and constant holding cost [128].

For constant per-period demand, setup cost, and holding cost, without a rolling schedule environment, it can easily be shown that a number of heuristics will always develop an optimal solution. These same heuristics will develop the global optimum solution even with a rolling horizon given that the planning horizon is larger than that required by the global optimum. Some of the heuristics which develop global optima include the part-period algorithm [84], the Silver-Meal heuristic [221], and the Groff
marginal cost rule [118]. In Figure 4.6 this optimum number of periods is shown as β.

In addition, imagine that two other heuristics are used to lot size the demand array without using a rolling schedule environment. Imagine that one heuristic chooses a larger sized lot and one heuristic chooses a smaller sized lot. Call the number of periods in the smaller sized lot α. Call the number of periods in the larger sized lot γ. Both α and γ are also shown in Figure 4.6.
The total cost for a lot sizing plan using either \( a \) or \( \gamma \) periods per-lot is higher than the total cost for the lot sizing plan using \( \beta \) periods in each lot as shown in Figure 4.6. The magnitude of the difference in total costs is of no concern in this analysis. The only important fact is that the costs are higher for all lot sizes other than the optimum.

If the planning horizon in use for the performance evaluation is larger than \( \gamma \) (the largest number of periods to join into each lot for the three heuristics), the rolling schedule environment has no effect on the performance analysis. Each of the heuristics develops lot sizing plans the same under this rolling schedule environment as the lot sizing plans developed by the heuristic without the rolling schedule environment. This is shown in Table 4.5. In Tables 4.5 through 4.7, \( P \) represents the length of the planning horizon.

<table>
<thead>
<tr>
<th>Planning Horizon</th>
<th>Heuristic Lot Sizes</th>
<th>Rolling Schedule Lot Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heuristic Lot Sizes</td>
<td>( a )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Rolling Schedule Lot Sizes</td>
<td>( a )</td>
<td>( \beta )</td>
</tr>
</tbody>
</table>

If the planning horizon (\( P \)) is less than \( \gamma \) periods and greater than or equal to \( \beta \) periods, the rolling schedule environment only affects the performance of the heuristic which attempts to join \( \gamma \) periods into each lot. This heuristic is not able to join as many periods into each lot as the heuristic desires. The planning horizon overrides the decision rule for the heuristic.
As shown in Figure 4.6, the total cost for the affected heuristic decreases and becomes closer to the optimum. When the planning horizon \( (P) \) is decreased to the optimum number of periods to join into each lot \( (\beta) \), the solution developed by the heuristic which attempts to join \( \gamma \) periods into each lot becomes optimal. This results because only \( \beta \) periods can be joined into each lot due to the planning horizon. The performance of the heuristic which attempts to join \( \alpha \) periods in each lot is unaffected and remains nonoptimal. This is shown in Table 4.6.

**TABLE 4.6 - Effects of a Medium Planning Horizon \((P)\)**

<table>
<thead>
<tr>
<th>Planning Horizon</th>
<th>Heuristic Lot Sizes</th>
<th>Rolling Schedule Lot Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( P )</td>
<td>( \beta \gamma )</td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 \( \leq \) Periods In Each Lot \( < \infty \)

For all planning horizons smaller than \( \beta \) periods, the optimal solution is unavailable. Those heuristics which develop the optimal solution when not in a rolling schedule environment are forced by the planning horizon to select lot sizes that are too small. The solution chosen is the best available but is nonoptimal when the entire demand array is considered.

Also, when the planning horizon is less than or equal to \( \beta \) periods, the heuristic which attempts to join \( \gamma \) periods into each lot, if not for the rolling schedule environment, chooses the best possible solution available \( (P) \). Thus, for all planning horizons less than or equal to \( \beta \) periods, those heuristics which would chose too large of lot sizes, if not
for the rolling schedule environment, are limited by the planning horizon to choosing the best solution available.

When the planning horizon is decreased to $\alpha$ periods (or all smaller values), the solutions developed by all three heuristics are equivalent. The planning horizon ($P$) is then the number of periods joined into each lot for all three heuristics. This is shown in Table 4.7.

TABLE 4.7 - Effects of a Small Planning Horizon ($P$)

<table>
<thead>
<tr>
<th>Planning Horizon</th>
<th>Heuristic Lot Sizes</th>
<th>Rolling Schedule Lot Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$a\beta\gamma$</td>
<td>$\gamma$</td>
<td></td>
</tr>
</tbody>
</table>

1 \leq \text{Periods In Each Lot} < \infty

Thus, the use of a rolling schedule environment can only decrease the variation in the performance exhibited by the various heuristics, at least for constant per-period demand. As the planning horizon becomes smaller and approaches the optimum number of periods of demand to join into each lot, the performance of those heuristics that attempt to create lots that are too large is improved. When the planning horizon is decreased further, the optimum number of periods to join into each lot is no longer available, and the performance of the best heuristics approaches the performance of those heuristics which join too few periods of demand into each lot. Finally, when the planning horizon becomes very small, the performance of all heuristics is equivalent.
While this analysis is developed using constant per-period demand, it is easy to see that the general results still apply if the per-period demand varies. Some heuristics still choose lots that are too large and some heuristics still choose lots that are too small. The use of a planning horizon still decreases the variation in performance exhibited by the various heuristics.

The object of any performance analysis should, of course, be to highlight the differences in the performance of the various alternatives. Thus, the use of a rolling schedule environment to analyze the performance of various heuristics is counter productive. The use of a rolling schedule environment can only limit the apparent difference in the performance of the various heuristics.

In a real world application where a planning horizon must be used, the best choice for a heuristic is the heuristic that performs best without a planning horizon. If the planning horizon is larger than the number of periods that would be chosen by this heuristic, the planning horizon has no effect. If the planning horizon is smaller than the number of periods which would be chosen by this heuristic, the heuristic chooses the best solution possible (the maximum allowed by the planning horizon). Thus, the effects of a rolling schedule environment need not be considered when choosing a heuristic for real world applications.

While this examination focuses on the single level environment, it is easy to see that the effects would also carry over into the multilevel environment. Heuristics must still tend to lot size too small, too large,
or just about correct. If a rolling horizon environment is added to the comparison of these heuristics, the only effect must be to decrease the variability of the various heuristics. The best choice for all lengths of planning horizons must still be the heuristic that operates best outside the rolling horizon environment.

c. The rolling horizon implementation

As described in the previous two sections, the decision on whether to use or not use a rolling schedule environment in an MRP analysis is not as straightforward as it first might seem. If optimum solutions are examined, not using a rolling schedule will result in a bias in their favor. If heuristics are examined using a rolling schedule, the rolling horizon will tend to trivialize the simulation results.

In this dissertation, the goal is to develop and test a heuristic method of lot sizing an MRP environment. Thus, in this dissertation, a rolling horizon is not used. Rather long fixed length demand arrays are analyzed.
V. MODEL SIMULATION

The simulation contained in this dissertation investigates six different MRP lot sizing methods. These different methods include four single level heuristics, one modified single level heuristic using the McLaren setup cost modification, and the new heuristic described in this dissertation. The first section in this chapter outlines the manner in which the simulation is performed. The remaining sections each describe a separate lot sizing method and contains the accompanying simulation results.

A. Simulation Development and Data Presentation

The simulation contained in this dissertation was developed and run on IBM-PC compatible microcomputers, using the MicroSoft C V5.1 compiler. The simulation uses two types of input parameters, master production schedule (MPS) data and cost parameter data. Both of these, along with the method used to display simulation output, are described in a following section.

1. Master production schedule construction

The master production schedule (MPS) data were developed using the random number generators contained in the SIMSCRIPT II.V language. A total of 15 demand arrays were used, with each of the demand arrays 360 periods long, and with each demand array having a mean of approximately 100 units per period. Table 5.1 summarizes the demand arrays.
TABLE 5.1 - Demand Array Statistics

<table>
<thead>
<tr>
<th>Demand Array</th>
<th>Demand Mean</th>
<th>Coefficient of Variation</th>
<th>High Demand</th>
<th>Low Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>D[1]</td>
<td>100.0</td>
<td>0.0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>D[2]</td>
<td>100.3</td>
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</tr>
<tr>
<td>D[3]</td>
<td>97.2</td>
<td>0.465</td>
<td>179</td>
<td>20</td>
</tr>
<tr>
<td>D[4]</td>
<td>99.7</td>
<td>0.121</td>
<td>131</td>
<td>62</td>
</tr>
<tr>
<td>D[5]</td>
<td>97.8</td>
<td>0.449</td>
<td>197</td>
<td>3</td>
</tr>
<tr>
<td>D[6]</td>
<td>100.4</td>
<td>0.703</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>D[7]</td>
<td>104.2</td>
<td>1.370</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
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</tr>
<tr>
<td>D[9]</td>
<td>98.5</td>
<td>0.538</td>
<td>180</td>
<td>20</td>
</tr>
<tr>
<td>D[10]</td>
<td>100.0</td>
<td>0.500</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>D[11]</td>
<td>98.9</td>
<td>0.506</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>D[12]</td>
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<td>0.544</td>
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<td>10</td>
</tr>
<tr>
<td>D[13]</td>
<td>95.1</td>
<td>0.575</td>
<td>189</td>
<td>10</td>
</tr>
<tr>
<td>D[14]</td>
<td>102.1</td>
<td>0.686</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>D[15]</td>
<td>100.9</td>
<td>0.728</td>
<td>200</td>
<td>0</td>
</tr>
</tbody>
</table>

2. Parameter development

In this dissertation, three different sets of model parameters are used to investigate the various lot sizing methods. The first two sets consist of sequential production and accompanying cost parameters, with the third set using assembly production (see Figures 2.1 and 2.2). Not only were data developed for each complete parameter set, but also for numerous portions of each set.

The first sequential production set consists of up to six levels (nodes), with data developed as each level (node) is added. The second sequential production set consists of up to twenty levels, with data developed, only, for four, eight, twelve, sixteen, and twenty levels. The assembly production set consists of up to three levels and eight nodes, with data developed as each node is added. Figure 5.1 shows this assembly...
production set, with the nodes numbered in the order in which they are added.

Because of the desire to avoid using a set of cost parameters in which one or more levels could be collapsed (see Section IV.B.3), the set holding costs for the various levels decrease from top (finished product) to bottom (raw material), and were held constant for all iterations, while the setup cost was varied. These holding costs are shown in Table 5.2.

In addition, the setup cost was the same for all levels during each iteration. However, 26 iterations of each model were run with the setup cost varying from 10 to 10010 in steps of 400. Thus, for example, for sequential set one, a total of 156 iterations was run (combinations of 1, 2, 3, 4, 5, and 6 levels and 26 holding cost values) for each of the 15 demand arrays tested. For sequential set two a total of 130 iterations was run for each demand array and for the assembly set a total of 208 iterations was run for each demand array.
### TABLE 5.2 - Simulation Holding Costs

<table>
<thead>
<tr>
<th>NODE</th>
<th>SEQUENTIAL SET 1</th>
<th>SEQUENTIAL SET 2</th>
<th>ASSEMBLY SET 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>40</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>38</td>
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<td>3</td>
<td>8</td>
<td>36</td>
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<tr>
<td>4</td>
<td>4</td>
<td>34</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>28</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>26</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>22</td>
<td></td>
</tr>
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<td>11</td>
<td></td>
<td>20</td>
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<td>12</td>
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<td>16</td>
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<tr>
<td>14</td>
<td></td>
<td>14</td>
<td></td>
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<td>15</td>
<td></td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

3. Simulation data representation

All resulting simulation data are presented in three dimensional surface plots produced on a Xerox 4050 laser printer using the SASGRAPH plotting package. In general, these plots represent the average results developed over all 15 of the demand arrays. Equation 5.1 shows the manner in which the data shown in the plots was developed.

\[
Z(x,y) = \frac{k}{\sum_{i=1}^{k} TC[???]_i(x,y,i)}
\]

\[
Z(x,y) = \frac{k}{\sum_{i=1}^{k} TC[ILS]_i(x,y,i)}
\]

Where:

\[
Z(x,y) = \text{the performance in percentage degradation};
\]
\( x \) = the setup cost (X axis);
\( y \) = the number of levels or nodes (Y axis);
\( k \) = the number of demand arrays (15);
\( \text{TC}[?](x,y,i) \) = total cost[? method] for given \( x \), \( y \), and \( i \);
\( \text{TC}[\text{ILS}](x,y,i) \) = total cost[ILS] for given \( x \), \( y \), and \( i \).

Although the plots are all the same physical size, the units on the performance axis \((z)\) are considerably different for the various plots. Therefore care should be taken when comparing different plots in order to correctly identify the relative performance.

Originally, an attempt was made to develop the optimum multilevel solution for a number of models and sets of parameters. However, developing the optimum solution proved too computer intensive to be of much value. Thus, the individual plots represent the performance of the various lot sizing methods divided by the performance of the new multilevel heuristic over the same set of data.

In the figures, the bottom right axis shows the setup cost and increases from left to right. The bottom left axis shows the number of levels (nodes) in the data and increases from right to left. The vertical axis shows the percentage cost increase (total cost for the method divided by the total cost for the new heuristic).

Each of the figures represents the average total cost over all 15 demand arrays of 360 period each, for each number of levels (nodes) and 26 sets of cost parameters. Thus, each figure depicting sequential set one
contains 156 data points (6*26), each of which contains the information from 5400 periods (15*360).

Of final note is the fact that the figures developed during this analysis form but three planes through the solution space for the various lot sizing methods. In these planes, the setup costs were constant for all levels, with holding cost as shown in Figure 5.2. Of course, an infinite number of other planes could also be studied. It is, simply, hoped that the results developed in this analysis are representative of the general results available from each of the various lot sizing models.

B. Single Level Wagner Whitin Algorithm

The Wagner Whitin (WW) Algorithm is a well-known method of producing the optimum solution to a single level lot sizing problem. However, when this method is used sequentially on a multilevel problem, the overall results are not optimum, as shown in Figures 5.2 through 5.4.

The overall nonoptimality of the WW algorithm occurs due to the inability of a single level algorithm to utilize the possible savings which can be achieved by optimizing multiple levels at one time. While the overall results achieved by the WW algorithm probably depend a great deal upon the cost ratios of the various levels, as shown in Figure 5.3, as the number of levels increases the overall performance of the WW algorithm degrades rapidly. At least the overall performance does not appear to continue to degrade with increases in setup costs for a given number of levels.
FIGURE 5.2 - WW Algorithm and Sequential Set 1

FIGURE 5.3 - WW Algorithm and Sequential Set 2
The lot for lot (LFL) heuristic performed the worst of all the lot sizing methods studied. The results developed by this method are shown in Figures 5.5 through 5.7. This poor showing is not surprising since the LFL model also performs badly in the single level environment.

Interpretation of the results developed by the LFL heuristic is rather straightforward. As the setup cost increases, the LFL heuristic does not attempt to balance setup and holding costs by joining multiple lots. If the number of levels in the system is increased, the LFL heuristic also suffers from the same myopia. Only if setup costs are exceedingly low does the LFL heuristic perform satisfactorily. In fact,
FIGURE 5.5 - LFL Heuristic and Sequential Set 1

FIGURE 5.6 - LFL Heuristic and Sequential Set 2
FIGURE 5.7 - LFL Heuristic and Assembly Set

these figures graphically illustrate the extreme importance of setup cost reductions in the (so called) lot for lot manufacturing system.

D. Groff Marginal Cost Rule Heuristic

The Groff Marginal Cost Rule (GMR) was included in this analysis since this heuristic performs very well on the single level lot sizing problem. However, as shown in Figures 5.8 through 5.10, the heuristic suffers continuous and large scale performance degradation as the number of levels increases.

The poor performance of the GMR heuristic closely models that of WW algorithm. This is to be expected, since the performance of the GMR heuristic closely approaches that of the WW algorithm for single level
FIGURE 5.8 - GMR Heuristic and Sequential Set 1

FIGURE 5.9 - GMR Heuristic and Sequential Set 2
FIGURE 5.10 - GMR Heuristic and Assembly Set

problems, and since both are entirely myopic to multiple level interactions. Only on problems with a single level does the WW algorithm markedly and consistently outperform the GMR heuristic. Also, as with all the single level methods, the GMR heuristic performs very well for very small setup costs.

E. Incremental Order Quantity Heuristic

The Incremental Order Quantity (IOQ) heuristic was included in this analysis since a number of authors have claimed satisfactory results for this heuristic in a multilevel environment. The results obtained by the heuristic are shown in Figures 5.11 through 5.13.
FIGURE 5.11 - IOQ Heuristic and Sequential Set 1

FIGURE 5.12 - IOQ Heuristic and Sequential Set 2
The performance of the IOQ heuristic on single level lot sizing problems degrades rapidly as the setup cost rises. Quite simply, the IOQ heuristic creates too large of lot sizes. However, these excess sized lots work better as the number of levels increases. In fact, for the three level serial problem and relatively high setup costs, the IOQ heuristic performs as well any of the methods examined. However, as shown in Figure 5.12, as the number of levels increases, even the (larger) lot sizes developed by the IOQ heuristic become too small.

This illustrates a particular problem with extending single level results into the multilevel environment. As stated earlier, the performance of the GMR heuristic closely approaches optimum for single level problems, while the IOQ heuristic does not perform very well at all.
However, when these two heuristics are used in a small sized multilevel environment the performance ranking is reversed.

F. McLaren Modified Single Level Heuristic

The McLaren setup cost modification (MCL) can be used to modify a single level heuristic for the multilevel environment. The MCL modification multiplies the cost ratio for a level by the sum of the square roots of cost ratios of all the children of the level. Thus, the setup cost used by any level (but the raw material) appears larger after the MCL modification is applied. In this dissertation the MCL modification is applied to the GMR heuristic. A number of authors have touted the MCL heuristic as very good in the multilevel environment. The results obtained by the GMR heuristic using the MCL modification are shown in Figures 5.14 through 5.16.

The performance of the MCL modified GMR heuristic was disappointing. Although the MCL modification improved the performance of the GMR heuristic, the performance continues to degrade rapidly as the number of levels in the system rises. This should probably be expected since the setup cost modification includes information from, only, the children of the level being lot sized. As more levels are added, this modification does not increase the lot sizes fast enough.
Percentage Cost Increase

Nodes

Setup Cost

FIGURE 5.14 - MCL Modified GMR Heuristic and Sequential Set 1

Percentage Cost Increase

Nodes

Setup Cost

FIGURE 5.15 - MCL Modified GMR Heuristic and Sequential Set 2
G. The New Integer Lot Sizing (ILS) Heuristic

A new multilevel lot sizing heuristic, called the integer lot sizing heuristic (ILS), has been constructed in conjunction with this dissertation. The performance of this new ILS heuristic has been reported throughout this chapter, in that the performance of the other lot sizing methods have been shown relative to the performance of the new heuristic. As shown in the figures accompanying the description of the other methods, the ILS heuristic consistently develops total costs less than the other methods, with this performance particularly noticeable as the number of levels in the system increases.
1. Overall performance comparison

The performance of the ILS heuristic does not completely dominate the performance of the other lot sizing methods evaluated. Table 5.3 shows the 133 plotted locations in which other methods develop averaged costs less than those developed by the ILS heuristic. Of these 133 locations, 50 represent locations comparing the WW algorithm and the ILS heuristic on a single level problem. Since the WW algorithm develops the optimum cost for these single level problems, there are only 83 locations in which the ILS heuristic was outperformed by a method which does not guarantee an optimum solution.

During the course of the study 2470 total cost comparisons were made. These comparisons were comprised of 156 for sequential set one, 130 for sequential set two, and 208 for the assembly set, with each of these made for 5 methods (WW, LFL, GMR, IOQ, MCL). Thus, the ILS heuristic was outperformed on the averaged data only 5.38% of the time (133+2470), or 3.43% of the locations other than WW and single level (83+2420).

Also of note are the areas in which the ILS heuristic was outperformed. These tend to be areas with a small number of levels. For example, no other method outperformed the ILS heuristic for any location on sequential set two, with sequential set two starting with four levels and ending with twenty levels. Finally, the performance of the ILS heuristic was never bettered by over 4% for any location.
TABLE 5.3 - Locations of Poor ILS Heuristic Performance

<table>
<thead>
<tr>
<th>DATA SET</th>
<th>METHOD</th>
<th>SETUP COST</th>
<th>LEVELS</th>
<th>PERFORMANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial 1</td>
<td>WW</td>
<td>4 - 100</td>
<td>1</td>
<td>98.00564*</td>
</tr>
<tr>
<td>Serial 1</td>
<td>ioq</td>
<td>8</td>
<td>1</td>
<td>99.99385</td>
</tr>
<tr>
<td>Serial 1</td>
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<td>12</td>
<td>1</td>
<td>99.98919</td>
</tr>
<tr>
<td>Serial 1</td>
<td>ioq</td>
<td>16</td>
<td>1</td>
<td>99.96807</td>
</tr>
<tr>
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<td>ioq</td>
<td>20</td>
<td>1</td>
<td>99.96800</td>
</tr>
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<td>3</td>
<td>99.60660</td>
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<td>100</td>
<td>3</td>
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</tr>
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<td>99.99862</td>
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<td>8</td>
<td>2</td>
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<td>3</td>
<td>99.96067</td>
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</tr>
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<td>99.84535</td>
</tr>
<tr>
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<td>mcl</td>
<td>4</td>
<td>4</td>
<td>99.98447</td>
</tr>
<tr>
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<td>4</td>
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<tr>
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</tr>
<tr>
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2. ILS methodology

The good performance of the ILS heuristic is to be expected, since the heuristic uses one of the main MRP features developed in this dissertation. That is, the ILS heuristic recognizes the fact that lots should only be joined according to the integer principle. In fact, the ILS heuristic was constructed to specifically recognize that only the joining of lots, as shown in Figure 2.2, can be used to develop lot sizes in the optimum cost solution.

In order to describe the method in which the ILS heuristic operates, Figures 5.17 through 5.20 have been developed. These figures follow the same conventions developed earlier, i.e., that the finished product is at the top and time flows from left to right. The figures show a four level system and display only the number of periods currently being analyzed.

The ILS heuristic operates by sequentially analyzing the demands from left to right (increasing time). Thus, the ILS heuristic operates much like a single level heuristic in that decisions are made with respect to only one lot at a time. When a lot is closed, no changes to that lot will ever be made. The ILS heuristic will, then, open a new lot and proceed with the new lot until it is also closed. The ILS heuristic differs from single level heuristics in that the interaction between levels is recognized.

In order to start the ILS heuristic, two periods of demand are needed. These two periods are shown in Figure 5.17. In Figure 5.17, the
FIGURE 5.17 - ILS Interaction Number One

The ILS heuristic determines at which level the greatest cost savings can be achieved by joining the lots. The applicable costs at a level are the sum of the setup costs which can be saved and the holding cost which would be created. Since the second series of lots in Figure 5.17 is being processed in a lot for lot fashion, the possible setup costs savings for a level are defined by the sum of setup costs from the level being analyzed through all lower levels. In the example developed here, the greatest savings are imagined at level x+2, with the resulting lot sizing plan shown in Figure 5.18.
The ILS heuristic, then, continues to examine the lot sizing plan for additional possible savings. This is shown by the dashed lines in Figure 5.18. Here the possible setup cost savings include only those levels which are not already joined. The holding cost penalty for joining at a higher level should also take into account the fact that the lots are already joined at a lower level. In the example, if additional savings are possible by joining at level \( x+1 \), the lots would be joined at this level rather than level \( x+2 \). This is shown by the first two periods of demand in Figure 5.19.

When all possible saving are realized from the first two periods of demand, a third period is added as, also, shown in Figure 5.19. This third period can be joined into, only, the adjacent periods as shown by the dashed lines. Adding this third period into the lot is performed in much the same manner as adding the second period. However, care needs to be taken to utilize the correct number of periods of time between adjacent
FIGURE 5.19 - ILS Interaction Number Three

lots. For example, in Figure 5.19, the number of periods between adjacent lots at levels \( x \) and \( x+1 \) is one period while the number of periods between adjacent lots at levels \( x+2 \) and \( x+3 \) is two periods.

FIGURE 5.20 - ILS Interaction Number Four

When all possible savings for the third period are realized, a fourth period is added as shown in Figure 5.20. Again, the lot can only be added into the adjacent lots as shown by the dashed lines. At some point in
time no savings can be realized by adding the new lot into an adjacent lot. The ILS heuristic will then determine to close the old series of lots and open a new series. This is shown by the diagonal line in Figure 5.20.

This new lot will, again, always, be processed in a lot for lot fashion. When the next lot is added the system will, again, be as shown in Figure 5.17 and the ILS heuristic will process this new series of lots in the same manner as described above.

3. Conclusions

The performance of the ILS heuristic was consistently better on the data analyzed in this dissertation than the performance of the other methods tested. However, the ILS heuristic also takes more computer resources to operate than the other heuristic methods, with the ILS heuristic requiring approximately ten times the CPU time of the simpler, single level, methods. These extra CPU resources are required since the ILS heuristic uses information from all levels during the construction of the lots.

This relatively small computational penalty allows the ILS heuristic to develop lots which depend not only on the data for a single level, but rather on the data for the entire MRP system. As the MRP system becomes more complex, the inclusion of this additional information becomes more valuable. Thus, the greatest difference in the performance of the ILS heuristic and other methods occurs on the data sets with the largest
number of levels and nodes. Finally, since real world MRP systems tend to be much more complex than those analyzed in this dissertation, the use of the ILS heuristic should prove valuable in actual MRP systems.
VI. CONCLUSIONS

In the widely respected book on material requirements planning, Orlicky states "When it comes to selecting a lot-sizing technique ... it is the authors opinion that neither detailed studies nor exhaustive debates are warranted - in practice, one discrete lot-sizing algorithm is about as good as another." However, as shown in this dissertation, better lot sizing methods can be developed when these methods utilize a realistic understanding of the interactions between the numerous levels which are always present in a real world MRP system.

The information presented in this dissertation falls into two major categories. First, and perhaps foremost, a better understanding of multilevel discrete demand (MRP) lot sizing problem is developed. Second, a new heuristic for MRP lot sizing is designed and tested with the results developed by the new heuristic compared with the results developed by other, older methods. This new heuristic is called the integer lot sizing (ILS) heuristic and uses some of the MRP features described in this dissertation.

The MRP features described in this dissertation include the following:

- The viability of the integer principle for multilevel lot sizing.
- The ability to collapse a multilevel lot sizing problem.
- The effects of a rolling horizon on a multilevel simulation.
The fact that the integer principle can be utilized to describe the interaction of lots in a multilevel environment has been previously recognized. However, this dissertation presents a graphical proof for the integer principle and shows that only two types of interactions are actually used in an optimal solution. These are the lot for lot and joint interactions.

The ability to collapse levels out of a multilevel system using cost ratio information was also previously described. However, a new proof for this is developed and presented. In addition, a method of collapsing levels using, only, holding costs is presented. The ability to collapse levels using holding cost information has never before been recognized.

Previous simulations into the multilevel environment have often used rolling horizons. However, as described in this dissertation, when heuristics are analyzed, the use of rolling horizons can, only, trivialize the results. As shown in this dissertation, the best performing heuristic method in a fixed horizon problem must also be the best performing heuristic in a rolling horizon.

Finally, this dissertation develops a new multilevel lot sizing heuristic called the integer lot sizing (ILS) heuristic. The ILS heuristic recognizes the interaction between the levels present in all MRP systems. The ILS heuristic consistently outperforms the other lot sizing methods tested, with this performance advantage most noticeable as the number of levels in the system grows.
It is hoped that the work performed in this dissertation can help to add insight into the problem of developing workable lot sizes in a multilevel MRP environment. While, perhaps, not an end unto itself, it can at least be hoped that this work can provide a beginning for a more complete understanding of the type of environment present in most repetitive manufacturing systems.
VII. BIBLIOGRAPHY


