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INTERFACE WAVES ON INTERFERENCE-FIT FASTENERS

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INTRODUCTION

The Aerospace Research Laboratories, like the Rockwell Science Center, are located on top of a hill. Down below the hill are several of the Air Force development laboratories. Just as Mr. Caustin comes up to the Science Center, people come up from the development laboratories and keep us very much in touch with the real world. We occasionally go down and talk to them, too. The interaction between the two places leads to some interesting things. In fact, I'd like to talk to you about one such interaction today.

A while ago some people came up from the Materials Laboratory and said how much they would like to detect flaws under installed interference-fit fasteners. Perhaps the flaws would be cracks, but perhaps also they would be very badly flawed installation in which the inner plate was not even making an interference fit with the fastener. They described some techniques that were being used. As Mr. Caustin indicated, there are acoustic techniques which allow you to detect fairly small cracks, but the one which seems to be working the best involved getting just the right orientation between a pitching transducer, a catching transducer, and the crack. One must move an array of this type all the way around the fastener in order to inspect it. It takes quite a bit of time. The Materials Laboratory people were saying that, not only would they like to detect both small and very significant flaws on installed interference-fit fasteners, but they would also like to be able to do this pretty quickly.

I said that I thought there might be some elastic modes that one could think of exciting around the fastener-part structure, and that I'd spend some time and look at some of those.
Well, one mode that seemed to be fairly interesting involved the possibility of using the fastener-part interface as a wave guide. It turns out to be possible to propagate guided elastic waves along the fastener-part interface, at least on paper. Dennis Corby, of AFML, is going to find out how possible it really is. We think these waves may perhaps give a useful, fairly quick technique for checking for very badly flawed installations of interference-fit fasteners.

Guided elastic waves are fairly well known. I suppose the best known guided elastic waves are the Stoneley waves which propagate along the plane interface between elastic half spaces of different properties. If the waves propagate in the Z direction on an interface in the Y, Z-plane, the displacement fields are:

\[ y > 0: U = \{ A_1 e^{-KA_1 x}, 0, i e^{-KA_1 x} \} + e_2 \{ e^{-KB_1 x}, 0, i e^{-KB_1 x} \} e^{i(c t - k z)} \]

\[ y < 0: U = \{ C_3 e^{-KA_2 x}, 0, i e^{-KA_2 x} \} + e_4 \{ e^{-KB_2 x}, 0, -i e^{-KB_2 x} \} e^{i(c t - k z)} \]  

\[ A = \frac{\sqrt{1 - \frac{c^2}{a^2}}}{1 - \frac{c^2}{b^2}} \]

\[ B = \frac{\sqrt{1 - \frac{c^2}{b^2}}}{1 - \frac{c^2}{a^2}} \]

There is exponential decay of the displacement field amplitudes going into either material i.e., for \( |x| \) increasing.

It is easy to verify that wave fields given by eqn. (1) are solutions of the elastic equations. If you ask for continuity of tractions and displacements across the interface, you come to the dispersion relation for Stoneley waves:

\[
\begin{vmatrix}
2\rho_1 b_1^2 A_1 & \rho_1 (2b_1^2 - c^2) & 2\rho_2 b_2^2 A_2 & \rho_2 (2b_2^2 - c^2) \\
\rho_1 (c^2 - 2b_1^2) & -2\rho_1 b_1^2 b_1 & -\rho_2 (c^2 - 2b_2^2) & 2\rho_2 b_2^2 B_2 \\
1 & B_1 & -1 & -B_2 \\
A_1 & 1 & A_2 & 1
\end{vmatrix} = 0
\]

\[ (1) \]

\[ (2) \]
One could try to use Stoneley waves for testing interference-fit 
fasteners. He could say, "I will try to have some waves whose wave length is 
very small in comparison with the radius of the fastener, and maybe these 
guys will run along the fastener-part interface just like Stoneley waves do". 
One might think of exciting these fellows with mode conversions starting with 
a surface wave on the outer part. If he did that, though, he would still have 
the problem of going all the way around the fastener, making inspections at 
several different azimuthal stations. It might be more interesting to 
think about something like the axisymmetric propagation sketched in Fig. 1. 
If guided, axisymmetric waves could be excited on the fastener-part inter­
face, they have some advantages over longitudinal or shear waves induced in 
the part. For one thing, the amplitudes decay away from the fastener, so 
you keep the energy where you want it, that is, close to the interface where 
I am told many of the cracks that you want to find occur. If you had the 
badly-flawed situation in which you didn't have an interference fit between 
the fastener and part, then you would be likely to find out about that right 
away.

So what one wants to ask is: Are there, in fact, guided elastic waves 
that propagate along a cylindrical interface? (I am going to leave out the 
taper of the interference-fit fasteners.) Well, as a matter of fact, there 
are. It isn't difficult to verify that the displacement fields

\[ \mathbf{U} = \begin{cases} 
  c_1 \left\{ A_1 K_1(A_1 Kr), 0, iK_0(A_1 Kr) \right\} + c_2 \left\{ K_1(B_1 Kr), 0, iB_1 K_0(B_1 Kr) \right\} e^{iK(ct-Z)} \\
  & \text{for } r \geq R 
\end{cases} \]

and

\[ \mathbf{U} = \begin{cases} 
  c_3 \left\{ -A_2 I_1(A_2 Kr), 0, iI_0(A_2 Kr) \right\} + c_4 \left\{ -I_1(B_2 Kr), 0, iB_2 I_0(B_2 Kr) \right\} e^{iK(ct-Z)} \\
  & \text{for } r \geq R 
\end{cases} \]  

are exact solutions of the elastic equations, in cylindrical coordinates 
\((r, \theta, Z)\). The things which were exponentials in the Stoneley waves are 
replaced by modified Bessel functions of the orders zero and one, of just the 
right types.
Fig. 1. Axisymmetric interface wave on interference-fit fastener
The modified Bessel functions, the K-fellows, are monotonically decreasing functions, essentially exponentially decreasing. I will show their asymptotic behavior in just a minute. The modified Bessel functions of the I-type are monotonically increasing functions, essentially exponentially increasing, so you get exponential decrease away from the fastener-part interface going both ways. If you ask then for continuity of tractions and displacement fields at the cylindrical interface, you are led to the dispersion relation given in eqn. 4. It is certainly messier than Stoneley's but it looks a good deal more forbidding than it actually is. The modified Bessel functions are extremely well-known special functions.

There are some interesting things connected with this new dispersion relation. For one thing, these waves are dispersive, while Stoneley waves are not. There are also possibilities that you don't have in the Stoneley case. For example, if the shear wave speed is slower in the fastener than in the parts, you have the possibility of waves which behave like ordinary Bessel functions in the fastener but which decay exponentially in the parts. I don't know whether that is a terribly interesting case. It is interesting to me as an applied mathematician, but I am not too sure how interesting it is for NDE.

The main thing is, waves of this type do, in fact, propagate, and I will show you some examples in what I think are maybe realistic engineering materials.

I promised a minute ago to show the asymptotic behavior of the modified Bessel functions involved. It is this:

\[ I_j(Z) \sim \frac{e^Z}{\sqrt{2\pi Z}} \quad K_j(Z) \sim \sqrt{\frac{\pi}{2Z^2}} e^{-Z} \]  

when \( Z \gg 1 \). If you say, Okay, I am going to look for solutions of dispersion relation from eqn. 4 in the neck of the wave number speed-woods where the arguments of all the modified Bessel functions are large, (and that is not necessarily an obvious limit, because the quantities \( A \) and \( B \) can have small magnitudes) you find that eqn. 4 reduces to the Stoneley dispersion relation given in eqn. 2. This is nice because it guarantees that in all cases where Stoneley waves propagate, there are also solutions of eqn. 4 if you go to large enough wave number.
\[
\begin{align*}
&2\rho_1 b_1^2 A_1 K_1(A_1 \xi) & \rho_1 (2b_1^2 - c^2) K_1(B_1 \xi) & 2\rho_2 b_2^2 A_2 I_1(A_2 \xi) & \rho_2 (2b_2^2 - c^2) I_1(B_2 \xi) \\
\left[ \rho_1 (c^2 - 2b_1^2) K_0(A_1 \xi) \right] & -2\rho_1 b_1^2 B_1 k_0(B_1 \xi) & - \left[ \rho_2 (c^2 - 2b_2^2) I_0(A_2 \xi) \right] & 2\rho_2 b_2^2 B_2 I_0(B_2 \xi) \\
-2\rho_1 b_1^2 A_1 K_1(A_1 \xi) & + K_1(B_1 \xi) & \frac{2\rho_2 b_2^2 A_2 I_1(A_2 \xi)}{\xi} & - \frac{I_1(B_2 \xi)}{\xi} \\
K_0(A_1 \xi) & B_1 K_0(B_1 \xi) & -I_0(A_2 \xi) & -B_2 I_0(B_2 \xi) \\
A_1 K_1(A_1 \xi) & K_1(B_1 \xi) & A_2 I_1(A_2 \xi) & I_1(B_2 \xi)
\end{align*}
\]
\[= 0\]  
(4)
There is another connection with other things which is worth making. If one sets the density in the outer material equal to zero, with a few fairly straightforward manipulations, he can recover the Pochammer-Chree dispersion relation from eqn. 4. The waves characterized by eqn. 4 can be viewed either as Stoneley waves on a cylindrical interface or as Pochammer-Chree waves with something else on the outside other than a vacuum.

If one were going to use the waves characterized by eqn. 4 in testing, he would want to know what the displacement fields look like, to suggest ways of exciting them. Let's take a look.

One example I've worked out is the case of a steel fastener in an aluminum part (Fig. 2). The propagation goes along with essentially a radial displacement, strongly peaked just inside the fastener. This isn't really a very surprising sort of displacement field. It is very reminiscent of the Raleigh wave displacement field, as we will see, the stress field is very much like that of the usual surface waves.

One thing which is interesting about these waves is the distribution of elastic energy propagation (Fig. 3). By far, the preponderance of the energy carried by this wave is carried inside the fastener. Everybody will quickly notice that the energy propagation is discontinuous at the fastener-part interface. That is because there is significant energy associated with the component of the traction which we don't make continuous across the interface.

Turning to another system, I wanted to talk about titanium fasteners in aluminum parts for obvious reasons. So, I went to ask Mike Buckley what the wave speeds were for titanium, and he told me, and I asked Dennis Corbly, and he told me, and I asked Grover Hardy, and he told me. In fact, Grover told me three different sets of wave speeds. Now, I am just a simple mathematician trying to get along. I decided to pretend, just as an interesting case to look at, that both aluminum and titanium had exactly the same shear given. The results are shown in Fig. 4. Probably because of the smaller ratio of the densities in this case the fields are somewhat less concentrated in the fastener.
STEEL
\[ a = 5.79 \times 10^5 \text{ cm/s} \]
\[ b = 3.10 \times 10^5 \text{ cm/s} \]
\[ \rho = 7.91 \text{ g/cm}^3 \]

ALUMINUM
\[ a = 6.35 \times 10^5 \text{ cm/s} \]
\[ b = 3.16 \times 10^5 \text{ cm/s} \]
\[ \rho = 2.7 \text{ g/cm}^3 \]

Fig. 2. Radial (upper curve) and axial (lower curve) components of displacement field vs. normalized radius (radius in units of fastener radius) for steel fastener in aluminum parts.
Fig. 3 Intensity of elastic wave energy vs. normalized radius, steel-aluminum system
Fig. 4 Radial (upper curve) and axial (lower curve) components of displacement field vs. normalized radius for "titanium" fastener in "aluminum" parts

"TITANIUM"

a = $6.3 \times 10^5$ cm/s

b = $3.3 \times 10^5$ cm/s

$\rho = 4.54 \text{ g/cm}^3$

"ALUMINIUM"

a = $6.3 \times 10^5$ cm/s

b = $3.3 \times 10^5$ cm/s

$\rho = 2.7 \text{ g/cm}^3$
I happen to have stress fields for the "aluminum-titanium" fellows (Fig. 5). The figure shows tractions on a surface of $Z$ equals constant, i.e., perpendicular to the axis of the cylinder. The radial component of traction dominates. Again, the similarity with classical surface waves is pretty marked.

Once one sees these pictures, he thinks of things he might to do to try to excite the waves. Although certainly a good deal remains to be done, it should be possible to give analytical characterizations of signals you would get back from interactions of these waves with flaws, to see what you can detect, in a sense, by shouting down the fastener part-interface as an elastic wave guide.

References


Fig. 5 Radial (A) and axial (B) components of surface traction vs. normalized radius, "titanium"-"aluminum" system.
DISCUSSION

PROF. HENRY BERTONI (Polytechnic Institute of New York): It is difficult to find Stoneley waves on a plane interface when the mass densities of the two materials are rather close together. It is only in a very narrow range of velocity differences that the Stoneley wave will exist.

DR. LEE: Yes.

PROF. BERTONI: Do you have to have rather dissimilar densities for the fastener and the part to get this?

DR. LEE: Yes. I am glad you asked me that. I have been meaning to speak to that point. Let me amplify your point by saying it is well known that the region of existence of real Stoneley waves, Stoneley waves that go without attenuation, is pretty small. It is mostly the shear speeds that need to be about the same in the two materials. It is very interesting what goes on if, in fact, you are outside of the region of existence for real Stoneley waves. I have some results that I just finished on that. What happens is you wind up with complex axial wave numbers, at least in some cases. For example, one of the sets of Ti-Al parameters that Grover Hardy gave me happens to lie outside the existence region for real waves. There appear to be solutions of (4) corresponding to very weakly attenuated waves. I am very anxious to work through this and see how they go. The best answer I can give you now is even if you happen to be outside the region of existence for real waves in this case, you may still have a chance to use these fellows for testing.

PROF. BERTONI: Another question on this.

DR. LEE: Yes.

PROF. BERTONI: I am surprised when you took the velocity to be the same for titanium and aluminum that you got a guided wave. Is this a case that the curvature is different? I would expect on a plane surface you wouldn’t get it.
DR. LEE: No, as a matter of fact, you do get it, and that is one of Stoneley's cases in his 1924 paper which occurs when the two speeds are exactly the same in both materials.

DR. JOHN TIEN (Henry Crumb School of Mines, Columbia University): Are you planning to do any experiments?

DR. LEE: Yes, Dennis Corbly says he is going to do some experiments. I am not going to. I wouldn't know how.

DR. BRUCE THOMPSON (Science Center, Rockwell International): The modes you were describing then were symmetric about the axis?

DR. LEE: Yes.

DR. BRUCE THOMPSON: Have you looked at any of the higher order modes and see how separated or close they are?

DR. LEE: Yes.

DR. BRUCE THOMPSON: I think it is also interesting to look at modes which are propagating around the fastener. Grover Hardy says he is pretty sure he has seen mode conversion between, say, grazing type acoustic waves and modes running around the fastener.

DR. LEE: Yes, I have. In the time available, I didn't think I would try to squeeze that in.

MR. ROBERT CRANE (Air Force Materials Laboratory, WPAFB): Have you looked at what would happen to the displacement field in the crack in the material?

DR. LEE: Yes, that is a good question, too. A very interesting question which I think is the one you are raising, is just how sensitive is this business to something like a penny-shape crack running radially away from the fastener. I don't have that worked out. The one thing that is encouraging is that, because of the curvature, this wave propagation is accompanied by a hoop stress, and I think you have a chance to interfere with the crack.
DR. PAUL PACKMAN (Vanderbilt University): The problem that Bob raises is that, really, with an interference fastener, you have a residual compressive stress state associated across the interface and the crack resulting in a slight compressive field. There would be no change because the crack is actually squeezed together for a goodly portion.

DR. LEE: Yes.

DR. PACKMAN: It seems to me there would be a minimal change in the compliance of the system for the wave crossing.

DR. LEE: Yes, I think so too.

DR. PACKMAN: Across the interface?

DR. LEE: Yes, I think, too, the waves don't exactly cross the interface.

DR. PACKMAN: It flows along it.

DR. LEE: Yes. I think that a crack held closed by residual stress is probably an awkwardness for any sort of acoustic crack detection.

MR. CORBLY (Air Force Materials Laboratory, WPAFB): The thing I just wanted to raise a little ruckus about concerns the stress that goes around the interface fastener. I think Paul assumed it was going to be compressive. That is not a subject for this meeting, but I still don't think we can say--

DR. LEE: Shall we take votes on what the residual stress actually is? Some people at the Math Lab at ARL hammered pretty hard on a rational solution for a circular hole in a finite circular plate. You can make it compressive if you load up and get plastic deformation. Of course, it is going to be tensile, too, in many cases. I certainly go along with what I think Dennis is saying. It is not terribly clear to me at least just what is the residual stress will be in an installed fastener that you are going to be wanting to inspect.

PROF. GORDON KINO (Stanford University): I was wondering whether the first mode, particularly the dipole mode, has a low frequency cutoff, because
if so, you propagate it along the sheet. You would excite this dipole mode. You would see a cutoff which might vary with the presence of the fracture.

DR. LEE: That is an interesting point. I don't have that worked out. The things that I have written down are the dispersion relations for the fellows going around the fastener, and I haven't finished exploring.