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R. King
Stanford University

George Herrmann
Stanford University

Gordon S. Kino
Stanford University

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Abstract

Acoustic measurements, using longitudinal waves in plane specimens, based on the theory of acoustoelasticity, permit the determination of the sum of the principal stresses ($\sigma_1 + \sigma_2$). By automatic scanning, we are able to make such measurements throughout a region of interest. In this paper we shall be concerned with the application of this acoustoelastic stress analysis to fracture mechanics. Specifically, the energy release rates for extension and rotation of a crack will be determined experimentally (J integral for extension, L integral for rotation) followed by a numerical adjustment procedure which may be called the *rescaling* technique. If desired the stress intensity factors at a crack tip may also be evaluated. This procedure was applied to three different specimen configurations, and the results compare favorably with purely theoretical predictions.

Keywords

Nondestructive Evaluation

Disciplines

Materials Science and Engineering

ACOUSTIC NONDESTRUCTIVE EVALUATION OF ENERGY
RELEASE RATES IN PLANE CRACKED SOLIDS

R. King, G. Herrmann and G. Kino
Stanford University
Stanford, California 94305

ABSTRACT

Acoustic measurements, using longitudinal waves in plane specimens, based on the theory of acoustoelasticity, permit the determination of the sum of the principal stresses ($\sigma_1 + \sigma_2$). By automatic scanning, we are able to make such measurements throughout a region of interest.

In this paper we shall be concerned with the application of this acoustoelastic stress analysis to fracture mechanics. Specifically, the energy release rates for extension and rotation of a crack will be determined experimentally (J integral for extension, L integral for rotation) followed by a numerical adjustment procedure which may be called the *rescaling* technique. If desired the stress intensity factors at a crack tip may also be evaluated. This procedure was applied to three different specimen configurations, and the results compare favorably with purely theoretical predictions.

INTRODUCTION

In this paper a summary is presented of recent efforts at Stanford University on applying acoustoelastic stress analysis using ultrasonics to the evaluation of conservation integrals in fracture mechanics. A brief review is included of the theory of acoustoelasticity and the experimental apparatus and techniques used to make stress measurements with ultrasonics. After discussing the practical importance of nondestructively evaluating three conservation integrals, the so-called J, L, and M integrals, attempts at performing such evaluation using ultrasonic stress measurements are described.

ULTRASONIC STRESS ANALYSIS

The application of acoustoelasticity to stress analysis using ultrasonic measurements has been discussed in detail in the literature,¹⁻⁵ including previous work in this field at Stanford. The important features of ultrasonic stress measurements in plane specimens are summarized here.

Due to nonlinear deformation, the velocity of an acoustic wave travelling through a stressed solid is dependent on the state of deformation, and hence through a constitutive law, on the state of stress in the solid. For the case of a longitudinal wave propagated at normal incidence in a plane specimen, the relation between wave speed and stress is

$$\frac{V - V_0}{V_0} \cong \frac{\Delta V}{V_0} = B(\sigma_1 + \sigma_2) \quad (1)$$

where V and V_0 are the velocities of the wave in the stressed and unstressed states, respectively, $(\sigma_1 + \sigma_2)$ is the planar first stress invariant, and the proportionally constant B is a material property which depends on the elastic constants of the material, including the third order (Murnaghan) constants. B is calibrated directly for a given material by using a uniaxial tension test. With knowledge of B , relative velocity measurements at many points will enable determination of $(\sigma_1 + \sigma_2)$ throughout a specimen. A diagram of the device we have used for performing such measurements is shown in Fig. 1.

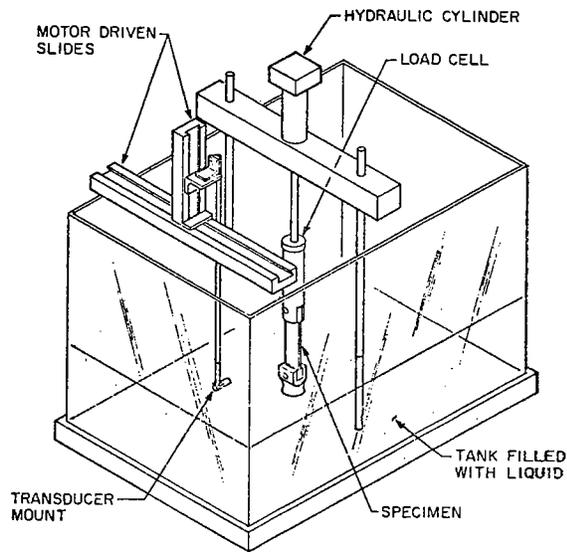


Figure 1. Acoustoelastic scanning device

For the case of shear waves at normal incidence to a plane specimen, the relevant acoustoelasticity relation is

$$\frac{V_1 - V_2}{V_0} = C(\sigma_1 - \sigma_2) \quad (2)$$

where V_1 and V_2 are the velocities of waves in the stressed medium polarized in the x_1 and x_2 directions, and V_0 is the velocity of the incident wave in the unstressed medium. Thus shear waves permit evaluation of the difference of the in plane principal stresses. In addition, principal directions can readily be found, using shear waves, from a plot of amplitude of received signal versus polarization angle. The constant C in Eq. (2) is also a material property and, as with B , is calibrated using a uniaxial tension test. By simultaneous application of longitudinal and shear wave measurements at many points, all components of the stress tensor can be evaluated throughout a region in plane specimens. A device similar in principle to that shown in Fig. 1 has been constructed for scanning

shear wave measurements, which uses direct contact to couple the waves. Preliminary work is underway in calibrating the constant C using this device, but two-dimensional stress analysis has not yet been successfully performed.

CONSERVATION INTEGRALS

J, L, and M Integrals

The well known J integral is actually one of a series of path-independent conservation integrals which exist in elasticity.⁶ Three of these, the so-called J, L, and M integrals, have the potential for practical applicability in fracture mechanics. A theoretical discussion of conservation integrals for plane cracked bodies is made elsewhere in these proceedings,⁷ so a brief description of their definition and practical utility will suffice here. The J, L, and M integrals are defined as follows:

$$J = \int_C (Wn_i - T_k U_{k,i}) ds$$

$$L = \int_C e_{3ij} (Wx_j n_i - T_i U_j - T_k U_{k,i} x_j) ds$$

$$M = \int_C (Wx_i n_i - T_k U_{k,i} x_i) ds$$

where W is the strain energy density, T_k is the traction vector acting on the outer side of C, U_k is the displacement vector, and e_{ijk} is the permutation tensor. For the J integral, C is a contour in the $x_1 - x_2$ plane around the tip of a crack, while for L and M, C completely encloses the crack. These integrals are physically interpreted as energy release rates with respect to translation of the tip of the crack for J and with respect to rotation and self-similar expansion of the entire crack for L and M respectively.

The practical significance of the J integral in fracture mechanics is that comparison of J versus a critical value of J (J_{IC}) provides a useful fracture criterion⁸ which remains valid even when general yielding occurs as long as there is no unloading. Thus, if J_{IC} is known for a material, the ability to nondestructively evaluate J will allow assessment of the structural integrity of a cracked element. In situations governed by Linear Elastic Fracture Mechanics (LEFM), the J integral is the same as the crack extension force G_I . Thus

$$J = G_I \sim k_I^2 \quad (3)$$

In linear elastic cases, use of the J integral has certain advantages over direct evaluation of the stress intensity factor, such as the ability, through path-independence, to obtain knowledge of the near tip stress fields from information along a contour further away from the crack and the smoothing effect of integration on noise in numerical or experimental data.

The M integral is useful because J can be determined from M. For instance, it is easily shown through path-independence arguments that for an interior crack of half length a,

$$M = 2aJ \quad (4)$$

It is sometimes more convenient to evaluate M using a closed contour rather than evaluate J along a contour around the tip of a crack, and M can be applied in certain situations involving loading on the crack faces where path-independence of J would no longer hold.

The practical importance of the L integral will arise in mixed mode cases. Since extension of a crack in mixed mode deformation does not occur along its original length but rather at some angle to it, the J integral alone is insufficient to predict onset of crack extension. The L integral may provide the additional information needed in such cases.

Experimental Evaluation of Conservation Integrals

The customary techniques for measuring the J integral involves direct determination of the energy release rate with respect to crack extension using compliance measurements.^{8,10} These methods are not suitable for nondestructive evaluation of J in structural elements but rather are designed for laboratory determination of J_{IC} . In contrast, the approach we have used for evaluating J, L, and M is to determine the value of the integrand at points along a contour and then numerically integrate. The difficulty of this approach is evident if the conservation integrals are shown in expanded form. For instance, in plane stress if the material along C is linear elastic, the J integral becomes

$$J = \int_C \frac{1}{2E} (\sigma_{yy}^2 - \sigma_{xx}^2) dy + \frac{\sigma_{xy}}{E} (\sigma_{xx} + \sigma_{yy}) dx + \omega_{xy} (\sigma_{xy} dy - \sigma_{yy} dx) \quad (5)$$

It is seen that evaluation of the integrand requires knowledge of all the components of the stress tensor⁹ as well as the rotation ω_{xy} . This is also true for L and M. Three different avenues have been explored for obtaining this information. They are:

1. Use of both shear and longitudinal wave measurements;
2. Use of longitudinal waves and special contours along which the integrand simplifies;
3. Use of longitudinal waves and "rescaling".

Description of each of the methods will follow.

Use of Shear and Longitudinal Waves

As discussed above, simultaneous application of shear and longitudinal wave measurements permits determination of all three components of the plane stress tensor. As seen in Eq. (5), it remains to determine ω_{xy} in order to evaluate J, L, or M. A numerical technique was presented in the 1979 ARPA/AFML Proceedings¹¹ for evaluating ω_{xy} using the known stress components and forward integration of the compatibility relations. This method has been successfully applied to the evaluation of both

J and M on theoretical data. In addition, since shear wave data were not available, experimental values of σ_{xx} , σ_{yy} , and σ_{xy} were simulated by introducing noise into the theoretical data, and again J and M were successfully evaluated. No further progress has been made on this approach since scanning shear wave data is still not available.

Special Contours

In certain cases, special contours can be found along which the integrand of J, L, or M simplifies considerably. The successful application of this approach on three different specimen configurations has been described in Ref. 12 and will be summarized here. The center-cracked panel specimen shown in Fig. 2 is chosen for illustration purposes because it shows both the utility of the M integral and the simplification along a special contour. The contour used proceeded vertically along the edges of the specimen and horizontally a slight distance from the shoulder as shown. By symmetry it is only necessary to consider one quadrant. Thus

$$M = 4(M_{AB} + M_{BC}) \quad (6)$$

where M_{AB} and M_{BC} are the contributions to M of paths AB and BC, respectively. Note that while the J integral would not be useful along a closed path such as this one (it would vanish identically), the M integral gives a useful result (see Eq. (4)).

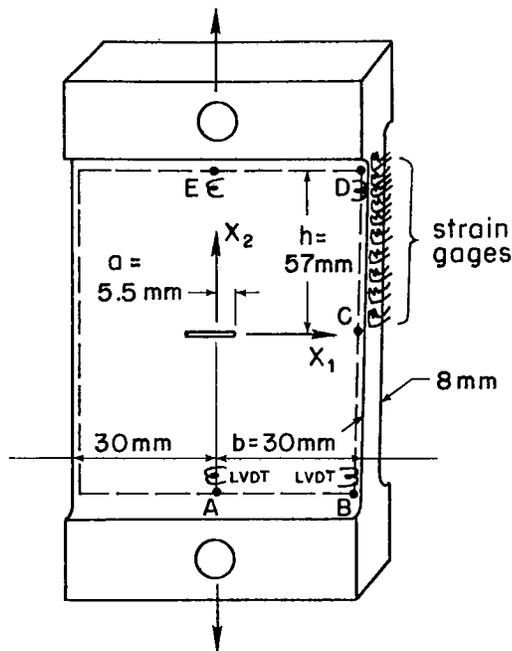


Figure 2. Center-cracked specimen used for M integral experiment.

On the traction-free vertical edge BC, $W = (E/2)\epsilon_{yy}^2$ and $x_1 n_1 = b$, so

$$M_{BC} = \frac{bE}{2} \int_0^h \epsilon_{yy}^2 dy = \frac{b}{2E} \int_0^h \sigma_{yy}^2 dy \quad (7)$$

which can be evaluated using either strain gages or longitudinal wave ultrasonic measurements. Evaluation of M_{AB} is slightly more complicated, but it is shown in Ref. 12 that M_{AB} is approximately given by

$$M_{AB} = \frac{P}{A} - \frac{hbP}{AE} + \frac{4b}{3} [U_y(b,h) - U_y(0,h)] \quad (8)$$

where P is the applied load and A is the cross-sectional area of the specimen. Thus evaluation of M_{AB} requires only two displacement measurements, which is accomplished using linear variable differential transformers (LVDTs). With the specimen loaded in tension to 30000N, the M integral was experimentally found to be 11.52N. The theoretical value for M was found in plane stress using

$$M = 2aJ = 2a \frac{K_I^2}{E} \quad (9)$$

to be 11.23N. (The discrepancy is 3%.) This first experiment used strain gages to evaluate M_{BC} . The two other specimens considered were an edge cracked specimen loaded in uniaxial tension and a specimen with free edges and an edge notch into which a wedge was forced. The J integral was evaluated for the former specimen and the M integral for the latter with agreement with expected values within 3% and 12%, respectively. Again the strain gages were employed in these experiments.

Attempts were made to repeat each of these experiments using longitudinal wave measurements with disappointing results. The difficulty is that evaluation of the integrand along paths such as M_{BC} above requires, theoretically, measurements exactly on the edge of the specimen. In practice, measurements are made slightly inside the edge, and the velocity measurements are extrapolated to the edge. The results were erratic and depended to a great extent on the extrapolation scheme used. We feel this is because the theory behind our measurement (Eq. (1)) is not valid near the edge of a specimen, and there are effects such as diffraction which must be accounted for. We concluded that this approach is quite useful in conjunction with strain gage measurements but will not be a fruitful application of ultrasonic measurements unless the difficulties described are overcome.

Longitudinal Wave Measurements and "Rescaling"

A method has been derived for evaluation of conservation integrals solely from knowledge of $(\sigma_{xx} + \sigma_{yy})$ in a region in the vicinity of a crack. This method is based on the following postulate: in the region in which data is taken, it is assumed that the deformation fields in the body vary with position in a geometrically similar manner to the stresses in an analogously loaded infinite plate with an identical crack. This assumption is conceptually similar to that originally made by Theocaris and Gdoutos¹³ in photoelastically evaluating K_I , which was also used by Hunter¹¹ to evaluate K_I from ultrasonic data. One expects this assumption to be a good one as long as the data is taken sufficiently far from the boundaries and close enough to the crack. Mathematically, the assumption is stated as follows: representing stresses in the infinite plate by a superscript "0",

the infinite plate solution can be expressed as

$$\sigma_{ij}^0(x,y) = a_1 f_{ij}^0(x,y) ; \omega_{xy}^0(x,y) = a_1 g_{ij}^0(x,y) \quad (10)$$

Assume in the region of interest in the finite body containing a crack that

$$\sigma_{ij}(x,y) = a_2 f_{ij}(x,y) , \omega_{xy}(x,y) = a_2 g_{ij}(x,y) \quad (11)$$

If $(\sigma_{xx} + \sigma_{yy})$ has been measured, then a_2 can be determined:

$$a_2 = \frac{a_1(\sigma_{xx} + \sigma_{yy})}{(\sigma_{xx} + \sigma_{yy})^0} \quad (12)$$

With measured values of $(\sigma_{xx} + \sigma_{yy})$ available at many points, the value of a_2 which will best fit the infinite plate solution to the measured data can be determined. This has been dubbed the "rescaling" method because it involves determination of a multiplicative constant used to "rescale" the infinite plate stresses and rotations. In situations involving more complicated far-field loading, a composite infinite plate solution made up of several superimposed solutions will be needed, and simultaneous adjustment of several multiplicative constants to best fit the composite solution to measured data will be necessary. An example of this is shown below.

This technique has been successfully applied on three different specimen geometries. Each specimen was made of aluminum 6061-T6 for which the B constant in Eq. (1) had previously been calibrated. The experimental procedure in each case was to make a velocity scan in a region of the specimen with no load applied and repeat the scan underload in order to evaluate relative velocity change with stress from which $(\sigma_{xx} + \sigma_{yy})$ could be evaluated using Eq. (1).

The first specimen to be considered was the edge cracked panel shown in Fig. 3 to which uniaxial tension was applied. The ultrasonic scanning was performed in the 15 mm square region shown in the vicinity of the crack, with the specimen unloaded and with a load of 40000N applied. A computer program which makes use of the rescaling method and the elasticity solution for an infinite plate with semi-infinite edge crack under far field tension¹⁴ was run on the experimental data in order to evaluate the J integral. The resulting value of J was 6.35 N/mm, which compares with the theoretical value of J for this specimen and loading (J = 5.83 N/mm) within 9%.

In a second experiment, the J integral was evaluated for the center cracked panel shown in Fig. 4. This is the same specimen used in evaluating the M integral by the "special contour" method discussed above (Fig. 2). With $(\sigma_{xx} + \sigma_{yy})$ evaluated experimentally in the region shown, the rescaling method was used in conjunction with the solution for an infinite center cracked panel under remote tension.¹⁵ The resulting value for J was 1.91 N/mm, which agreed with the theoretical value (J = 1.77 N/mm) within 8%.

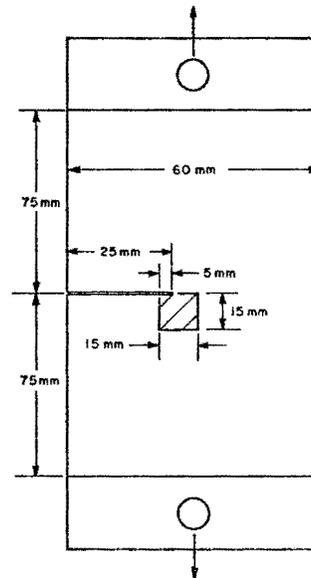


Figure 3. Edge-cracked panel used for acoustic J integral experiment.

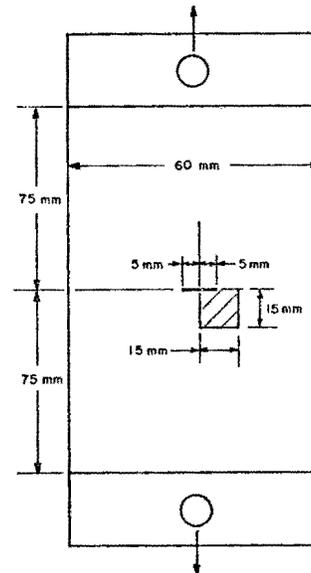


Figure 4. Center-cracked specimen used for acoustic J integral experiment.

Finally, the specimen with slanted central crack shown in Fig. 5 was considered. When this specimen is subject to uniaxial tension, the tractions in a coordinate system normal and tangential to the crack depicted in Fig. 6 result. Thus an infinite plate solution involving far field biaxial tension and shear is needed and obtained by superposition from basic solutions.¹⁴ Simultaneous adjustment of 3 parameters is required to apply rescaling. Both the J integral and the L integral were evaluated in this fashion. The result for J was 3.86 N/mm, which agrees with the theoretical value $(J = (K_{II}^2 + K_{III}^2)/E = 3.63 \text{ N/mm})$, within 6%. The L integral was experimentally found to be 44.65 N. The theoretical value is found using the relation⁸

$$L = \frac{-2aK_{II}}{E} (K_I + \sigma_{11}\sqrt{\pi a}) \quad (13)$$

to be 36.3. The discrepancy between these two values is 22%. The reason for the larger error in evaluating L has not yet been ascertained.

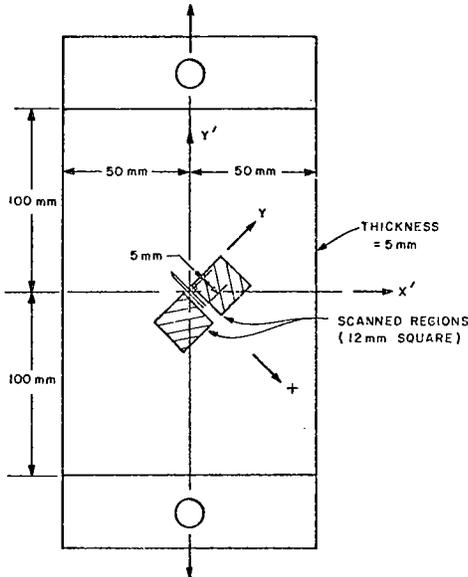


Figure 5. Specimen with slanted central crack used for evaluating J and L.

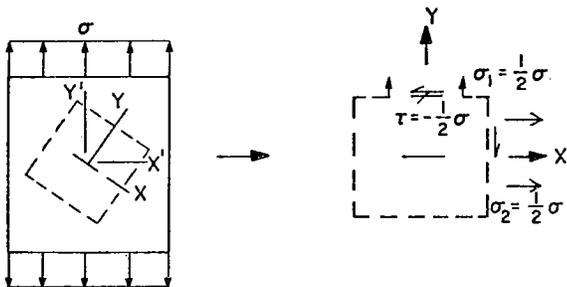


Figure 6. Remote loading of crack in specimen with slanted crack.

CONCLUSIONS

Various approaches have been presented for nondestructive evaluation of conservation integrals in cracked bodies. The most versatile of these appears to be the simultaneous use of shear and longitudinal waves, but this requires the yet-to-be developed capability of shear wave scanning.

The best currently available method appears to be the use of longitudinal waves in conjunction with "rescaling". It was seen that this method worked well on 3 different specimens including one involving complicated mixed mode deformation. With improvement in measurement technology it is hoped this method will be applicable to field as well as laboratory situations.

All the experiments presented were restricted to linear elastic fracture mechanics. A fruitful area for further study should be the application of the techniques presented to situations involving

the techniques presented to situations involving large scale elastic-plastic deformation.

Acknowledgements

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SUMMARY DISCUSSION

Otto Buck, Chairman (Rockwell Science Center [now Ames Laboratories]): Any questions?

Kamel Salama (University of Houston): In determining the J integral, is this crack 15 millimeters long?

Richard King (Stanford University): The crack is 10 millimeters long.

Kamel Salama: And you have to determine if the stress is 15 millimeters on its side?

Richard King: Yes.

Kamel Salama: How many measurements do you need to take in this square?

Richard King: No systematic study has been made of how few measurements you can get away with. We actually just did it at 1 millimeter point spacing so we took 225 data points; but I'm sure you can get by with much fewer than that.

Neil Paton (Science Center): The theory assumes that you have an elastically isotropic material, and the material you have is probably not exactly isotropic. Did you measure what the departure from isotropy was and could that explain the discrepancy between the calculated values and measured values?

Richard King: That might be part of it, yes. If you measure that B constant for specimens pulled parallel to the grain and against the grain, you get different values. As a matter of fact, it can differ by as much as 30 percent, I think, and that has not been taken into account. Our specimens are pulled along the rolling direction, and we use that B for the specimens pulled along the rolling direction. And yes, if we do take that anisotropy into account, that might help.

Another thing we're just recently looking into is inhomogeneity in the B constant. We assume it's homogeneous throughout the specimen and we can just get by with one uniaxial tension measurement; and actually we found it does vary a little by maybe 10 percent throughout a nominally nonhomogeneous specimen.

Gary Hawk (Aerospace Corporation): How large a velocity change do you measure?

Richard King: Very small indeed. Relative velocity changes, I think, are down to one part 10 to the fourth.

Chris Fortunko (NBS): (Inaudible)

Richard King: I'm not sure. Well the B constant, and you can work backwards from that. It's of the order of 10 times 10 to the minus six per megapascal.

Roger Chang (Science Center): (Inaudible) Very small cracks, say, 50 microns?

Richard King: I'm not certain. We haven't given any thought to that.

Gordon Kino (Stanford University): We scale it up in frequency, and we look at small samples, yes. It should apply. But the definition in the present system is on the order, at best, of a millimeter and possibly two millimeters.

Otto Buck: I have a short question that goes back to a question after the first one. What would we do experimentally in case of a partial crack? Can you imagine doing thermography; doing it in case of a partial crack?

Gordon Kino: We're trying. Let's put it that way.

Otto Buck: Very good (laughter). Thank you so much.

Richard King: Thank you.

Otto Buck: That's very informative.