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Abstract
We report calculations of the impedance of a long solenoid which surrounds a cylinder of conducting material containing a radial surface crack. The calculation is accomplished by two independent methods. The first method expresses the field in the interior of the "cracked" cylinder as an infinite series of cylindrical Bessel functions. The coefficients in the series are determined in principle by boundary conditions; the most significant terms are calculated by solving the finite set of equations obtained by truncation of the series. The second method, applicable to any uniform geometric cross-section, obtains the impedance from the normal derivative of the field on the boundary of the conductor. This normal derivative satisfies a (boundary) Fredholm integral equation of the first kind; a solution is obtained by discretizing and solving the resulting linear system of algebraic equations. The impedance is calculated for a wide range of values of the ratios of crack depth-to-radius and radius-to-skin depth. The results are displayed in graphical form giving the fractional charges of the real and imaginary parts of the complex impedance induced by the presence of the crack.

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AC MAGNETIC FIELDS IN THE VICINITY OF A CRACK CALCULATED BY ANALYTIC AND NUMERICAL METHODS

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ABSTRACT

We report calculations of the impedance of a long solenoid which surrounds a cylinder of conducting material containing a radial surface crack. The calculation is accomplished by two independent methods. The first method expresses the field in the interior of the "cracked" cylinder as an infinite series of cylindrical Bessel functions. The coefficients in the series are determined in principle by boundary conditions; the most significant terms are calculated by solving the finite set of equations obtained by truncation of the series. The second method, applicable to any uniform geometric cross-section, obtains the impedance from the normal derivative of the field on the boundary of the conductor. This normal derivative satisfies a (boundary) Fredholm integral equation of the first kind; a solution is obtained by discretizing and solving the resulting linear system of algebraic equations. The impedance is calculated for a wide range of values of the ratios of crack depth-to-radius and radius-to-skin depth. The results are displayed in graphical form giving the fractional charges of the real and imaginary parts of the complex impedance induced by the presence of the crack.

We consider an infinitely long conducting cylinder with a thin radial crack as shown in Fig. 1 and Fig. 2. The goal of the calculation is to obtain the familiar impedance diagram, i.e., a plot of imaginary versus real part of the impedance of a surrounding coil (with unity filling factor), when the ratio of radius to electromagnetic skin-depth is varied. The eddy current equation shown in Fig. 3 must be solved for the field in order to obtain the impedance.

An analytic solution can be obtained by a generalization of a closed form expression for the case when the crack depth is equal to the radius. To apply the boundary conditions on the circumference, it is necessary to transform coordinates through use of the Bessel function addition theorem. The form of the solution is shown in Fig. 4. The coefficients A_n are obtained by truncating the series at 30 to 40 terms and solving numerically. Accuracy of the impedance is estimated as better than 0.05% in the range covered.

An alternate approach is to use Green's theorem to recast the differential equation of Fig. 1 into integral equation form. This approach has the advantage of being applicable to any shape of cross-section, although here it is used only for the circular cylinder with a crack. In Fig. 5 we show the integral equations which apply with and without the crack. Subtraction of the two provides simplification and yields an equation for the perturbation of the normal derivative of the field at the surface, induced by the crack. The change of normal derivative is calculated numerically by discretizing, applying approximation techniques, and solving the resultant linear equations. With 64 collocation points agreement with the eigenfunction method is within 1%.

The resulting impedance plot is shown in Fig. 6, where arrows indicate the shift of points, induced by the crack, corresponding to representative values of a/δ (the ratio of radius to skin-depth). We see that the shape of the impedance curve is changed only slightly, but that representative points are shifted significantly along the curve. At low frequencies the crack impedes the flow of penetrating eddy currents, decreasing the loss; at high frequencies the current is confined to the surface and the crack increases the loss through the increased surface area. At all frequencies the inductance is increased by the crack, corresponding to enhanced magnetic flux penetration. In Fig. 7 and Fig. 8 we show numerical results for the fractional change of the real and imaginary parts of the impedance as a function of crack depth for a representative range of values of a/δ.

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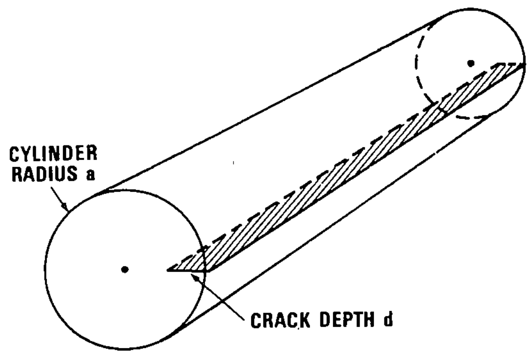

Fig. 1. Segment of an infinite conducting cylinder with a radial surface crack. The cylinder is placed in a coaxial solenoid and the change of impedance due to the crack is calculated.

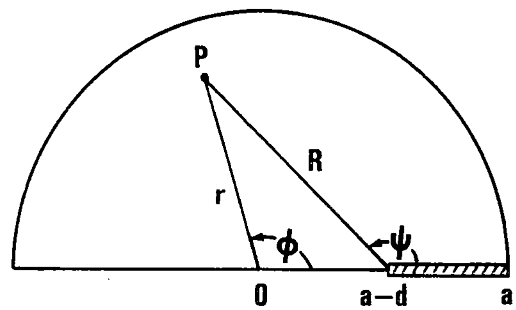

Fig. 2. Cross-section of the cylinder showing coordinate systems used in the analysis. Point P is a general point which may be expressed by (r, ϕ) measured from the center, or by (R, ψ) from the crack tip.

$$(\nabla^2 + k^2)\, H(r) = 0$$

$$k^2 = i\sigma\omega\mu$$

$$k = (1 + i)/\delta$$

H = magnetic field
σ = conductivity
ω = angular frequency
μ = permeability
δ = skin depth
k = propagation constant

Time factor = $e^{-i\omega t}$

Boundary condition: $H = H_o$ on surface

Impedance:

$$Z = \frac{Lk^2n'^2}{\sigma}\iint r\,dr\,d\phi\; H(r)/H_o$$

or

$$Z = \frac{Ln'^2}{\sigma}\frac{1}{H_o}\oint \frac{\partial H}{\partial n_o}\, dS_o$$

L = length of solenoid

n' = reciprocal of pitch

Fig. 3. Eddy current differential equation and related quantities for two-dimensional problems.

Eigenfunction Expansion

$$H(\vec{r})/H_0 = \cos(kR \sin\psi) + \sum_{n=0}^{\infty} A_n \frac{J_{n+\frac{1}{2}}(kR)}{J_{n+\frac{1}{2}}(ka)} \sin(n+\tfrac{1}{2})\psi$$

$$= \cos(kr \sin\phi) + \begin{cases} \displaystyle\sum_{\ell=-\infty}^{\infty} \sum_{n=0}^{\infty} A_n(-1)^n \frac{J_{n+\frac{1}{2}+\ell}(k(a-d))}{J_{n+\frac{1}{2}}(ka)} J_\ell(kr)\cos\ell\phi & \text{for } r < a-d \\ \displaystyle\sum_{\ell=-\infty}^{\infty} \sum_{n=0}^{\infty} A_n \frac{J_{\ell-n}(k(a-d))}{J_{n+\frac{1}{2}}(ka)} J_{\ell+\frac{1}{2}}(kr)\sin(\ell+\tfrac{1}{2})\phi & \text{for } r > a-d. \end{cases}$$

Fig. 4. Form of solution for a crack in a circular cylinder. The second version, obtained by the application of the Bessel function addition theorem, allows application of the boundary conditions at r = a.

Integral Equation Formulation

With crack:

$$H(\vec{r}) = \frac{1}{4\pi}\int_{\ominus} G(r,S_0)\frac{\partial H}{\partial n_0}\, dS_0 - \frac{1}{4\pi}\int_{\ominus} \frac{\partial G(r,S_0)}{\partial n_0} H(S_0)\, dS_0$$

where $G(r,r') = i\pi H_0^{(1)}(k|r-r'|)$

without crack:

$$H(r) = \frac{1}{4\pi}\int_{\bigcirc} G(r,S_0)\frac{\partial H}{\partial n_0}\, dS_0 - \frac{1}{4\pi}\int_{\bigcirc} \frac{\partial G(r,S_0)}{\partial n_0} H(S_0)\, dS_0$$

$$= H_0 J_0(kr)/J_0(ka)$$

Subtract, let $r \to S$, simplify:

$$\frac{1}{4\pi}\int_{\ominus} G(S, S_0)\, \Delta \frac{\partial H}{\partial n_0}\, dS_0 = \begin{cases} 0 & S \text{ on circle} \\ H_0\left[1 - \dfrac{J_0(kr)}{J_0(ka)}\right] & S \text{ on crack} \end{cases}$$

Fig. 5. Integral equation formulation for two-dimensional eddy current problems. In the last equation Δ represents the perturbation of the normal derivative of H, on the boundary of the conductor, produced by the crack.

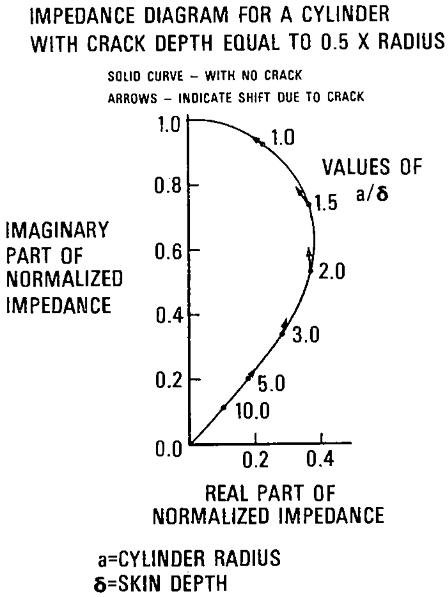

Fig. 6. Impedance diagram for a demonstrative case. Arrows drawn to scale, show the shifts of selected a/δ points induced by the presence of the crack.

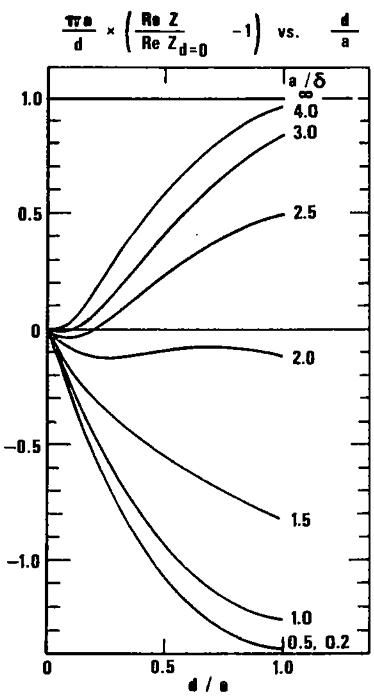

Fig. 7. Fractional change due to the crack, of the real part of the impedance as a function of d/a for selected values of a/δ.

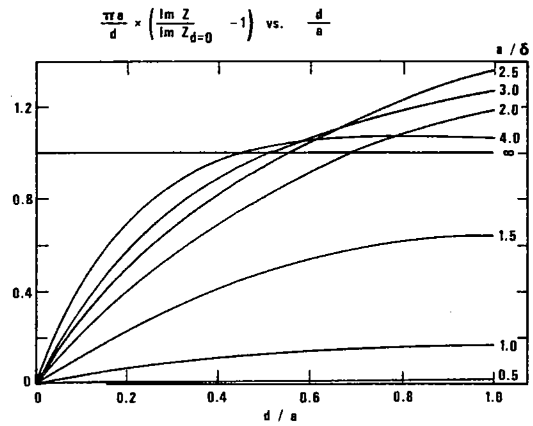

Fig. 8. Fractional change, due to the crack, of the imaginary part of the impedance as a function of d/a for selected values of a/δ.