Intermediation, Bubbles, and Pareto Efficiency in Economies with Production

Mark Pingle
Iowa State University

Leigh Tesfatsion
Iowa State University, tesfatsi@iastate.edu

Follow this and additional works at: http://lib.dr.iastate.edu/econ_las_economicreports

Part of the Business and Corporate Communications Commons, Growth and Development Commons, Industrial Organization Commons, Models and Methods Commons, and the Public Economics Commons

Recommended Citation
http://lib.dr.iastate.edu/econ_las_economicreports/31

This Report is brought to you for free and open access by the Economics at Iowa State University Digital Repository. It has been accepted for inclusion in ISU Economic Report Series by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Intermediation, Bubbles, and Pareto Efficiency in Economies with Production

Abstract
In a recent study, Tirole (1985) extends Diamond's (1965, pp. 1130-1135) well-known overlapping generations model of a private production economy by permitting consumption loans. That is, in addition to financing the capital investment of firms, the savings of one generation can be used to finance the consumption of agents in other generations whose consumption demands are in excess of their endowments. Tirole then shows that the resulting production-consumption loan economy fails to satisfy the First Welfare Theorem. Specifically, as reviewed in Section 2, below, two stationary competitive equilibria exist for this economy: a Pareto inefficient equilibrium $e$ with no consumption loans; and a Pareto efficient "golden-rule" equilibrium $e^*$ in which consumption loans are made...

Disciplines
Business and Corporate Communications | Growth and Development | Industrial Organization | Models and Methods | Public Economics
Intermediation, Bubbles, and Pareto Efficiency in Economies with Production

by

Mark Pingle
and
Leigh Tesfatsion
Economic Report Series No. 24
April, 1991
Intermediation, Bubbles, and Pareto Efficiency in Economies With Production

Mark Pingle
Department of Economics
University of Nevada, Reno, NV 89557

Leigh Tesfatsion
Department of Economics and Department of Mathematics
Iowa State University, Ames, IA 50011-1070

ABSTRACT

Tirole (1985) extends Diamond’s well-known model of an overlapping generations economy with production by permitting consumption loans (bubble asset investment) and shows that the First Welfare Theorem still fails to hold. This paper suggests that the First Welfare Theorem fails for the Diamond-Tirole model because intermediation is modelled as a purely passive coordination activity. When the Diamond-Tirole model is extended to include an active earnings-driven corporate intermediary owned by consumer-shareholders, the efficiency properties of the model are dramatically altered. For example, if the intermediary maximizes the minimum per-share dividend distributed to shareholders over time, every competitive equilibrium is Pareto efficient. The key reason for this result is that a solution exists for the intermediary’s dividend distribution problem if and only if price conditions hold which are analogous to the Cass-Balasko-Shell transversality conditions shown by Balasko and Shell (1980) to be necessary and sufficient for Pareto efficiency in the context of a pure exchange overlapping generations model.

KEYWORDS Intermediation, Bubbles, Pareto Efficiency, Overlapping Generations, Production Economies.

*A previous version of this paper was presented at the American Mathematical Society Meeting, South Bend, Indiana, March 1991, and at the Department of Economics, University of Iowa, Iowa City, Iowa, April 1, 1991. The authors are grateful to meeting and seminar participants for helpful comments. Please address correspondence to Leigh Tesfatsion.
1. INTRODUCTION

In a recent study, Tirole (1985) extends Diamond's (1965, pp. 1130-1135) well-known overlapping generations model of a private production economy by permitting consumption loans. That is, in addition to financing the capital investment of firms, the savings of one generation can be used to finance the consumption of agents in other generations whose consumption demands are in excess of their endowments. Tirole then shows that the resulting production-consumption loan economy fails to satisfy the First Welfare Theorem. Specifically, as reviewed in Section 2, below, two stationary competitive equilibria exist for this economy: a Pareto inefficient equilibrium $e$ with no consumption loans; and a Pareto efficient "golden-rule" equilibrium $e^*$ in which consumption loans are made.

This paper suggests that the failure of the Diamond-Tirole economy to satisfy the First Welfare Theorem can be attributed to the passive role assigned to intermediation. The only intermediary in the Diamond-Tirole economy is an implicitly present Walrasian Auctioneer concerned with coordination but not with optimization. As detailed in Section 3, below, private agents in the Diamond-Tirole economy do not exploit earnings opportunities which arise from suboptimal intertemporal trade and credit arrangements.

To explore this issue further, the Diamond-Tirole economy is generalized in Section 4 to include the explicit presence of a corporate intermediary owned by consumer-shareholders. The efficiency properties exhibited by the resulting "Brokered Economy" then depend on the exact modelling of the intermediary's objective. One possibility is that the Brokered Economy intermediary behaves as a Walrasian Auctioneer, i.e., a price-setting agent con-

\[1\] A similar claim is made by Pingle and Tesfatsion (1991) for Samuelson's (1958) pure exchange economy, a special case of the Diamond-Tirole economy.

\[2\] Bernanke and Gertler (1985, 1986) also introduce intermediaries ("banks" or "insider investment coalitions") into an overlapping generations model. However, they focus on the potential role of these intermediaries in reducing deadweight losses due to principal-agent problems in the loan market for investment projects. They do not permit intermediaries to issue consumption loans. The closest forerunner to the present paper is apparently E. Thompson (1967). Thompson argues that genuinely perfect competition requires the introduction of a market for private debt instruments (e.g., a corporate pension fund), a market omitted from models such as Samuelson (1958).
cerned only with trade and credit coordination. In Part A of Section 5 it is shown that the Brokered Economy reduces to the Diamond-Tirole Economy in this case, and hence supports the Pareto inefficient competitive equilibrium $\bar{e}$. Another possibility, however, is that the intermediary is an active optimizing agent with an earnings objective. In this case, it is shown that the Pareto inefficient outcome $\bar{e}$ cannot be supported as a competitive equilibrium for the Brokered Economy under any reasonable specification for the intermediary’s earnings objective; for, given $\bar{e}$, the intermediary would correctly perceive the possibility of increasing his net earnings in every period.

A number of subtle issues arise, however, when an attempt is made to specify a reasonable earnings objective for the corporate intermediary, given that the usual competitive assumption of price-taking is to be retained for the intermediary as well as for producers and consumers. These issues are taken up in Part B of Section 5. It is shown, for example, that the very meaning of “profits” is ambiguous. Nevertheless, if the intermediary maximizes the minimum per capita dividend distributed to shareholders over time, the Pareto efficient allocation associated with the golden-rule competitive equilibrium $e^*$ for the Diamond-Tirole economy is the unique stationary equilibrium allocation for the Brokered Economy

More generally, given this dividend objective for the corporate intermediary, it is shown that the market value of the intermediary is maximized at each point in time, and every Brokered Economy equilibrium is Pareto efficient. The crucial fact used to establish this Brokered Economy first welfare theorem is that the solution set for the intermediary’s optimization problem is nonempty if and only if certain price conditions hold. These price conditions are analogous to the well-known Cass-Balasko-Shell price conditions determined in Balasko and Shell (1980) to be necessary and sufficient for Pareto efficiency in the context of a pure exchange overlapping generations economy. The important point here is that an economic interpretation is provided for these price conditions; they are the “transversality conditions” for the optimizing corporate intermediary.
Overall, our findings suggest that the inefficiency exhibited by commonly used models for dynamic open-ended competitive economies can, to a large extent, be accounted for by the fact that intermediary behavior is modelled as a purely passive coordination activity. As demonstrated here for the Diamond-Tirole economy, this inefficiency might well be significantly reduced or eliminated if intermediation were to be more realistically modelled as an activity oriented toward the exploitation of earnings opportunities.

2. THE DIAMOND-TIROLE ECONOMY

A. The Basic Diamond-Tirole Model

The basic model of this paper is Diamond's classic framework (1965), generalized as in Tirole (1985) to include the possibility that consumer savings are invested in a "bubble asset" as well as in capital. Savings invested in the bubble asset are used to finance consumption loans.

Consider an overlapping generations economy in which each consumer lives for just two periods, "youth" and "old age." The generation of consumers born at the beginning of period $t$ consists of $L_t$ consumers and is referred to as "generation $t." Population grows at the rate $n \geq 0$, so that $L_{t+1} = (1 + n)L_t$. The economy begins in period 1 with $L_0 > 0$ old consumers of generation 0 and $L_1 = (1 + n)L_0$ young consumers of generation 1.

There is a single resource which may either be consumed or used in production as capital. Adopting the conventional time dating of goods, the resource during period $t$ will be referred to as "good $t." Each young consumer in each generation $t \geq 1$ inelastically supplies one unit of labor in return for a real (resource) wage $w_t$. Wage income is used by young consumers to provide young-age consumption $c_t^1$ and savings $s_t$. Old-age consumption $c_{t+1}^2$ is provided entirely from savings and accumulated interest; old consumers do not work.

Consumer saving takes two forms: investment in capital; and investment in a bubble asset.
which pays no dividends.\textsuperscript{3} The bubble asset can be thought of as vouchers representing real purchasing power—for example, real money balances or bonds. If capital and the bubble asset are both to be held in competitive equilibrium, the bubble asset must bear the same yield as capital. Thus, it is assumed that saving in either form has a common rate of return $r_{t+1}$ over each period $t \geq 1$.

Preferences of consumers in every generation $t \geq 1$ are assumed to be identical. The objective of each generation $t$ young consumer is to maximize his lifetime utility $U(c^1_t, c^2_{t+1})$, where $U(\cdot)$ has the usual curvature properties.\textsuperscript{4} Given any $w_t > 0$ and $r_{t+1} > -1$, the problem faced by this young consumer takes the form

$$\begin{align*}
\max_U(c^1_t, c^2_{t+1})
\end{align*}$$

with respect to $(s_t, c^1_t, c^2_{t+1})$ subject to the budget and nonnegativity constraints

\begin{align*}
&c^1_t = w_t - s_t;
&c^2_{t+1} = [1 + r_{t+1}]s_t;
&c^1_t \geq 0, c^2_{t+1} \geq 0.
\end{align*}

Let the solution to this problem be denoted by

$$\begin{align*}
x(w_t, r_{t+1}) = (s(w_t, r_{t+1}), c^1(w_t, r_{t+1}), c^2(w_t, r_{t+1})).
\end{align*}$$

Each old consumer in the initial period 1 is assumed to be entitled to a principal-plus-interest payment $[1 + r_1]s_0$, where the savings level $s_0$ satisfies $s_0 = s(w_0, r_1)$, and the savings function $s(\cdot)$ coincides with the savings function determined for each generation $t \geq 1$ agent in (2). The consumption of each old consumer in period 1 is therefore $c^2_1 = [1 + r_1]s(w_0, r_1)$.

\textsuperscript{3}An asset is said to exhibit a "bubble" at time $t$ if its price at time $t$ differs from its fundamental value, determined as the present value of its current and future dividends. A "bubble asset" is any asset on which a bubble may form. When an asset pays no dividends, it necessarily exhibits a bubble whenever its price is positive. Tirole (1985) does not incorporate a market for stock shares into his model, hence bubbles on capital holdings are not considered.

\textsuperscript{4}Specifically, it is assumed that $U(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly quasi-concave, with $U(0, c^2_{t+1}) = U(c^1_t, 0) = U(0, 0)$. 

Output in the Diamond-Tirole Economy is produced at the beginning of each period using capital and labor inputs in accordance with the production relation \( Y = F(K, L) \). The production function \( F(\cdot) \) is assumed to exhibit constant returns to scale, and to satisfy the usual continuity and curvature restrictions.\(^5\) Letting \( k \equiv K/L \) and \( y \equiv Y/L \) denote the capital-labor ratio and the output-labor ratio, respectively, the production relation can be expressed in per-capita (i.e., per-worker) form as \( y = F(k, 1) \equiv f(k) \).

In each period \( t \geq 1 \) the producer must pay the rental rate \( r_t \) on capital employed and the wage rate \( w_t \) to laborers employed. The price-taking producer selects levels of capital and labor inputs to maximize profits. Formally, the producer's problem may be stated as

\[
\max_{K \geq 0, L \geq 0} [F(K, L) - r_t K - w_t L] .
\]

In order for a vector \((K_t, L_t)\) of capital and labor inputs to solve problem (3), it is both necessary and sufficient that the capital-labor ratio \( k_t = K_t/L_t \) satisfy

\[
(4) \quad r_t = f'(k_t) ;
\]

\[
(5) \quad w_t = f(k_t) - f'(k_t)k_t .
\]

Conditions (4) and (5) generate the well-known "factor-price frontier" relationship between the wage rate \( w_t \) and the interest rate \( r_t \). For any given interest rate \( r_t > 0 \), let \( k(r_t) \) denote the capital-labor ratio \( k_t \) which uniquely satisfies condition (4). Substituting \( k(r_t) \) into condition (5), the wage rate \( w_t \) which satisfies condition (5) is then uniquely determined as a strictly decreasing function of \( r_t \). Hereafter this wage rate will be denoted by \( w(r_t) \).

In each period \( t \geq 1 \) the supply of capital consists of aggregate savings \( S_{t-1} = L_{t-1}s_{t-1} \) less that part of savings held in the form of the bubble asset. Let \( B_{t-1} = L_{t-1}b_{t-1} \) denote aggregate bubble asset holdings and \( K_t \) denote the producer’s aggregate demand for capital.

\(^5\)More precisely, the following restrictions are imposed on \( F(\cdot) \): twice continuous differentiability and strict concavity over the positive orthant \( \mathbb{R}^2_+ \), with \( F_K > 0, F_L > 0, \) and \( F_{KK} < 0; \) continuity over the nonnegative orthant \( \mathbb{R}^2_+ \); and, for each \( L > 0, F_K(K, L) \to 0 \) as \( K \to +\infty \) and \( F_K(K, L) \to +\infty \) as \( K \to 0 \).
Then, in per capita terms, supply equals demand in the capital market when

\[(6) \quad s_{t-1} - b_{t-1} = [1 + n]k_t . \]

As in Tirole (1985, p. 1503), the following restriction is imposed on the growth of the aggregate bubble asset holdings \(B_t\):

\[(7) \quad B_t = [1 + r_t]B_{t-1} , \]

or, in per-capita terms,

\[(8) \quad b_t = [(1 + r_t)/(1 + n)]b_{t-1} . \]

The implications of restriction (7) are examined in Section 3, below.

Young consumers in each period \(t > 1\) supply labor inelastically, in total amount \(L_t\). Supply equals demand in the labor market when the producer chooses to employ this labor supply. Hereafter it is assumed without further comment that supply equals demand in the labor market in each period \(t\), so that \(L_t\) denotes the period \(t\) work force as well as the period \(t\) population of young consumers.

Since capital in the Diamond-Tirole Economy does not depreciate, the total supply of product available during any period \(t \geq 1\) is \(Y_t + K_t\). The total demand for product includes capital demand for the following period, \(K_{t+1}\), and aggregate consumption for the current period, \(L_t c_t^1 + L_{t-1} c_t^2\). In per capita terms, supply equals demand in the product market for period \(t\) when

\[(9) \quad y_t + k_t = [1 + n][k_{t+1} + c_t^1 + c_t^2]/[1 + n] . \]

\(^6\)In the standard Arrow-Debreu general equilibrium model, market clearing conditions typically only require that supply be at least as great as demand in quantity terms. Walras' law then implies that any good in excess supply must have a zero price. However, Walras' law typically does not hold for overlapping generations models (see, e.g., Wilson (1981)), and market clearing conditions have traditionally been stated in a stronger form requiring directly that supply equal demand in each market in real value terms. See, for example, Diamond (1965), Gale (1973), Balasko and Shell (1980), Scheinkman (1983), Tirole (1985), Weil (1987), and Aliprantis et al. (1990).
Finally, following Tirole (1985, p. 1505), the economy is initialized by assuming that the capital-labor ratio $k_0 > 0$ and the per capita bubble asset holdings $b_0 \geq 0$ are historically given. The capital-labor ratio $k_0$ in turn determines the initial interest rate $r_0$ and the initial wage rate $w_0$ in accordance with the marginal productivity conditions

\[(10) \quad r_0 = f'(k_0) \; ; \]
\[(11) \quad w_0 = f(k_0) - f'(k_0)k_0 . \]

The Diamond-Tirole Economy can then be reduced to a pair of difference equations in the "state variables" $k_t$ and $b_t$ over times $t \geq 1$,

\[(12) \quad [1 + n]k_t = s \left( f(k_{t-1}) - f'(k_{t-1})k_{t-1} \right) - b_{t-1} ; \]
\[(13) \quad b_t = \left( [1 + f'(k_t)] / [1 + n] \right) b_{t-1} \]

starting from the exogenously determined initial values $k_0$ and $b_0$.

B. Competitive Equilibrium and Efficiency

Following Diamond (1965) and Tirole (1985), a competitive equilibrium will now be defined for the Diamond-Tirole Economy in terms of optimality conditions for the consumers and the producer, the capital market clearing condition, and the growth restriction on bubble asset holdings.\(^7\)

\(^7\)Given consumer and producer optimization, and the capital market clearing condition (6), the bubble asset growth restriction (8) is equivalent to the product market clearing condition (9). To see this, consider the following sequence of implications in both the forward and reverse directions:

\[ (1 + n)k_{t+1} + c_t^1 + c_t^2 / [1 + n] = y_t + k_t ; \]
\[ (1 + n)k_{t+1} + [w_t - s_t] + [1 + r_t]s_{t-1} / [1 + n] = y_t + k_t ; \]
\[ (1 + n)k_{t+1} + [w_t - s_t] + [1 + r_t]s_{t-1} / [1 + n] = w_t + [1 + r_t]k_t ; \]
\[ (1 + n)k_{t+1} - s_t + [1 + r_t]s_{t-1} / [1 + n] = [1 + r_t]k_t ; \]
\[ (1 + n)k_{t+1} - [b_t + [1 + n]k_{t+1}] + [1 + r_t][b_{t-1} + [1 + n]k_t] / [1 + n] = [1 + r_t]k_t ; \]
\[ -b_t + [1 + r_t]b_{t-1} / [1 + n] = 0 . \]

7
DEFINITION 1: [Diamond-Tirole Equilibrium] Given initial values $k_0 > 0$ and $b_0 \geq 0$ for capital and bubble asset holdings, a sequence $(s_t, c^1_t, c^2_t, k_t, b_t, r_t, w_t : t \geq 1)$ of savings levels $s_t$, consumption levels $c^1_t$ and $c^2_t$, capital-labor ratios $k_t$, per capita bubble asset levels $b_t$, interest rates $r_t > 0$, and wage rates $w_t > 0$ is a competitive equilibrium $e(k_0, b_0)$ for the per capita Diamond-Tirole Economy if and only if it satisfies the following four conditions:

- **[Consumer Optimization]** In each period $t \geq 1$, the young consumer's choice vector $x_t = (s_t, c^1_t, c^2_t)$ solves the lifetime utility maximization problem (1) conditional on $w_t$ and $r_{t+1}$, i.e., $x_t = x(w_t, r_{t+1})$; and each old consumer in period 1 consumes $c^2_t = [1 + r_t]s_0$ with $s_0 = s(f(k_0) - f'(k_0)k_0, r_1)$.

- **[Producer Optimization]** In each period $t \geq 1$, the producer's capital-labor ratio choice $k_t$ solves the necessary and sufficient conditions (4) and (5) for profit maximization conditional upon $w_t$ and $r_t$.

- **[Capital Market Clearing]** In each period $t \geq 1$, condition (6) holds.

- **[Bubble Asset Growth Restriction]** In each period $t \geq 1$, per capita bubble asset holdings grow in accordance with condition (8).

A competitive equilibrium $e(k_0, b_0)$ for the Diamond-Tirole Economy will be called a stationary competitive equilibrium if $k_t = k_0$ and $b_t = b_0$ for each period $t \geq 1$.

Given various regularity conditions, Tirole (1985) proves that the Diamond-Tirole Economy has two distinct stationary competitive equilibria: a Pareto inefficient equilibrium characterized by the interest rate $r = \bar{r}$ and zero bubble asset holdings; and a Pareto efficient "golden-rule" equilibrium characterized by the biological interest rate $r = n$ and positive

---

8As in Diamond (1965), Tirole (1985) assumes that there exists a unique $\bar{r}$ satisfying $s(w(\bar{r}), r) = [1 + n]e(r)$, with $\bar{r} < n$. Also, as detailed in Section A of the Appendix, Tirole imposes several additional technical regularity conditions on the savings function $s(\cdot)$ and the production function $f(\cdot)$. 

---
bubble asset holdings. Consequently, the simple inclusion of a bubble asset (e.g., fiat money or government bonds) is not enough to guarantee that all competitive equilibria are Pareto efficient.

For later purposes, it is useful to provide here a brief review of the efficiency and stability properties which Tirole establishes for these two stationary competitive equilibria.

Given the stationary interest rate $r = \bar{r}$, the optimal consumer and producer choice variables and the wage rate are stationary, and are given by $\bar{x} = x(w(\bar{r}), \bar{r})$, $\bar{k} = k(\bar{r})$, and $\bar{w} = w(\bar{r})$. It is easily established that the sequence $e(k, 0) = (\bar{x}, \bar{k}, 0, \bar{r}, \bar{w} : t \geq 1)$ satisfies the conditions in Definition 1 characterizing a stationary competitive equilibrium. Hereafter this bubbleless stationary competitive equilibrium will be abbreviated by $\bar{e}$.

The allocation achieved under the bubbleless stationary equilibrium $\bar{e}$ is not Pareto efficient. Since $\bar{r} < n$, a suitably small reduction in the equilibrium capital-labor ratio $\bar{k}$ in any given period $t*$, offset by a corresponding increase in bubble asset holdings, permits an increased stationary level of "net output" $y - nk$ in all periods $t \geq t*$. This in turn implies that young and old age per-capita consumptions can also be increased in all periods $t \geq t*$; see condition (9).

Given the stationary interest rate $r = n$, the optimal consumer and producer choice variables and the wage rate take on the stationary values $x^n = x(w(n), n)$, $k^n = k(n)$, and $w^n = w(n)$. Define $b^n = s(w(n), n) - (1 + n)k(n)$. Although the nonnegativity of $b^n$ is not established by Tirole (1985), it can be shown that $b^n$ is actually strictly positive. It is then easily verified that the sequence $e^n(k^n, b^n) = (x^n, k^n, b^n, n, w^n : t \geq 1)$ satisfies all of the conditions in Definition 1 characterizing a stationary competitive equilibrium. The allocation generated under $e(k^n, b^n)$ is the Pareto efficient allocation yielding maximum net output $y - nk$ in each period $t \geq 1$. Hereafter the equilibrium $e^n(k^n, b^n)$ will be abbreviated by $e^n$.

---

9Since the proof is not entirely straightforward, it is given in Section A of the Appendix.
Finally, given any initial capital-labor ratio $k_0 > 0$, there exists a maximum feasible value $\hat{b}_0$ for the initial bubble asset holdings such that the following two results are true. First, given any $b_0$ in $[0, \hat{b}_0)$, there exists a unique competitive equilibrium $e(k_0, b_0)$; this equilibrium in Pareto inefficient, and it converges to the Pareto inefficient bubbleless stationary competitive equilibrium $\bar{e}$. Second, there exists a unique competitive equilibrium $e(k_0, \hat{b}_0)$; this equilibrium is Pareto efficient, and it converges to the Pareto efficient stationary competitive equilibrium $\bar{e}^n$.

3. PASSIVE MEDIATION IN THE DIAMOND-TIROLE ECONOMY

The Diamond-Tirole Economy does not include an explicit intermediating institution. Nevertheless, it is still possible to consider intermediation as a distinct function performed separately from production. The production function is that of transforming the capital and labor inputs into output. The intermediation function is that of obtaining savings for investment and consumption loans, and of fulfilling outstanding savings contract obligations. Hereafter we use the term "intermediary" to describe the unit performing the intermediation function.

The Diamond-Tirole Economy differs from the Diamond (1965) model of a private production economy in one crucial respect: The market clearing condition for capital is relaxed to allow for consumption loans. Specifically, as depicted in Figure 1, the Diamond market clearing condition $S_t = K_{t+1}$ is replaced by the condition $S_t = B_t + K_{t+1}$. From the viewpoint of young consumers, $B_t$ represents the portion of their savings held in the form of a bubble asset, e.g., fiat money or bonds. From the viewpoint of the intermediary, $B_t$ represents the portion of his deposits not invested in capital.

The equilibrium $e(k_0, b_0)$ fails to be Pareto efficient because it is characterized by capital overaccumulation in every period—cf. Tirole (1985, Figure 1, p. 1505). A suitably small increase in the bubble asset holdings $b_{t-1}$ offset by a decrease in the capital labor ratio $k_t$ for any period $t$ permits an increase in period $t$ net output $y_t - n k_{t+1}$, and hence also in the per capita consumptions for the young and old consumers in all periods $t' \geq t$. 

---

10The equilibrium $e(k_0, b_0)$ fails to be Pareto efficient because it is characterized by capital overaccumulation in every period—cf. Tirole (1985, Figure 1, p. 1505). A suitably small increase in the bubble asset holdings $b_{t-1}$ offset by a decrease in the capital labor ratio $k_t$ for any period $t$ permits an increase in period $t$ net output $y_t - n k_{t+1}$, and hence also in the per capita consumptions for the young and old consumers in all periods $t' \geq t$. 

10
The Diamond-Tirole intermediary is thus potentially able to use $B_t$ to finance consumption loans. However, as seen in Section 2, the passive financing of consumption loans is not enough, in and of itself, to guarantee a Pareto efficient outcome.

The Diamond-Tirole intermediary in period $t$ is obliged to make a principal-plus-interest payment $[1 + r_t]S_{t-1}$ to generation $t - 1$ old consumers. The intermediary also receives from the producer a demand for capital, $K_{t+1}$. Incoming receipts consist of the savings deposits $S_t$ received from young consumers and the principal plus interest repayment $[1 + r_t]K_t$ received from the producer for capital borrowed in period $t - 1$. The net earnings of the intermediary in period $t$ are therefore given by

$$
\Pi_t = S_t + [1 + r_t]K_t - [1 + r_t]S_{t-1} - K_{t+1}
$$

$$
= [S_t - K_{t+1}] - [1 + r_t][S_{t-1} - K_t]
$$

$$
= B_t - [1 + r_t]B_{t-1}.
$$

The restriction (7) imposed by Tirole on the growth of the bubble asset holdings $B_t$ now takes on a new light. Given this restriction, the intermediary’s net earnings (14) are forced to be zero in every period $t \geq 1$. In imposing condition (7), Tirole (1985, p. 1503) correctly notes that physical capital and the bubble asset must earn the same yield in order for both assets to be held in equilibrium. However, the equality of these yields does not, in and of itself, imply that condition (7) must hold. Rather, condition (7) holds if and only if an additional special-case restriction is invoked: namely, the bubble asset remains in fixed supply.

To understand this, let $\Theta_{t-1}$ denote the quantity of the bubble asset supplied by the intermediary in period $t - 1$, and let $p^\theta_{t-1}$ denote the price of the bubble asset during period $t - 1$ measured in units of good $t - 1$. Because the bubble asset is used exclusively to finance consumption loans, it follows that $B_{t-1} = p^\theta_{t-1}\Theta_{t-1}$. For the bubble asset to be held at the
same time capital loans are being made, the bubble asset must earn the same rate of return as capital, meaning that \( p_t^s / p_{t-1}^s = [1 + r_t] \). Consequently, the following sequence of equalities must hold in the forward and reverse directions:

\[
\Pi_t = B_t - [1 + r_t]B_{t-1}
= p_t^s \Theta_t - [1 + r_t]p_{t-1}^s \Theta_{t-1}
= p_t^s [\Theta_t - \Theta_{t-1}].
\]

As condition (15) makes clear, whether or not Tirole’s condition (7) holds, and hence whether or not the intermediary has nonzero net earnings, depends upon whether or not the supply of the bubble asset is fixed.

As shown in Section 2, Footnote 7, Tirole’s condition (7) is equivalent to the assumption that supply equals demand in the product market. Thus, if Tirole’s condition need not hold, product market clearing in this strong supply-equal-demand form need not hold either (cf. Footnote 6). In fact, the intermediary’s ability to achieve positive net earnings depends upon his ability to generate an excess supply in the product market. When there is an excess demand in the product market, \( B_t \) is strictly less than \([1 + r_t]B_{t-1}\) and the intermediary’s net earnings \( \Pi_t \) are negative. This situation cannot arise in competitive equilibrium because the intermediary is unable to fulfill all contract obligations when net earnings are negative. However, when there is an excess supply in the product market, i.e., when \( B_t \) is strictly greater than \([1 + r_t]B_{t-1}\), the intermediary’s net earnings are positive. In this case, the intermediary can meet all contract obligations. Having met all outstanding obligations, the intermediary is entitled to keep, as profit, the excess supply generated.

In short, once the assumption of a fixed bubble asset supply is relaxed, the intermediary is able to achieve positive net earnings. Is it reasonable to suppose that the coordination of trade and credit transactions remains his sole objective? The intermediary’s stance toward net earnings must now be clarified in order for the model to be complete.
The next Section sets out a generalization of the Diamond-Tirole Economy—referred
to as the "Brokered Economy"—which explicitly includes a corporate intermediary with
an earnings objective. The intermediary is not required to keep the bubble asset in fixed
supply. The efficiency implications of active earnings-driven intermediation are explored for
the Brokered Economy in Section 5.

4. THE BASIC BROKERED ECONOMY

Consider an economy, hereafter referred to as the "Brokered Economy," where all
trade outside the labor market is mediated by an intermediary. Consumers have the opport-
unity to purchase two assets from the intermediary: (1) shares of stock in the intermediary;
and (2) bonds. There are no risk differences between stock shares and bonds, so that con-
sumers base their asset choices only upon expected rate of return.

Each young consumer in generation \( t \geq 1 \) demands \( \theta_t^d \) shares of stock and \( \lambda_t^d \) bonds, to be
sold in the subsequent period. The period \( t \) prices of stock shares and bonds in terms of good
\( t \) are \( p_t^s \) and \( p_t^b \), \( t \geq 1 \), and are taken as given by consumers. The intermediary distributes
all net earnings back to consumers as dividends in accordance with the following dividend
policy: Each share of stock purchased during period \( t \) entitles the owner to a dividend \( d_{t+1} \)
in period \( t + 1 \).

Let \( d_{t+1}^e \) denote the dividend per share expected by a representative young consumer in
generation \( t \geq 1 \). The utility maximization problem of this price-taking young consumer
then takes the form

\[
\max U(c_t^1, c_{t+1}^2)
\]

with respect to \( (\theta_t^d, \lambda_t^d, c_t^1, c_{t+1}^2) \) subject to the budget and nonnegativity constraints

\[
c_t^1 = w_t - p_t^s \theta_t^d - p_t^b \lambda_t^d;
\]

\[
c_{t+1}^2 = p_{t+1}^s \theta_t^d + p_{t+1}^b \lambda_t^d + d_t^e \theta_t^d;
\]

\[
13
\]
\[ c_t^1 \geq 0, \ c_t^2 \geq 0. \]

No sign restrictions are placed on \( \theta_t^q \) or \( \lambda_t^q \), implying that short sales are allowed for both stock shares and bonds. Consequently, assuming all prices are positive, no finite solution exists for problem (16) unless stock shares and bonds have a common rate of return, i.e., unless

\[ (17) \quad \frac{[p_{t+1}^\theta + d_{t+1}^\theta]}{p_t^\theta} = \frac{p_{t+1}^\lambda}{p_t^\lambda} = [1 + r_{t+1}] \]

for some \( r_{t+1} > -1 \). Given positive prices, condition (17), and the assumed regularity conditions on preferences (Footnote 4, Section 2), there exists a determinate solution for the planned savings level \( s_t \equiv [c_t^1 - w_t] \) and consumption levels \( (c_t^1, c_{t+1}^2) \) as a function of the wage \( w_t \) and the common rate of return \( r_{t+1} \). However, the consumer is indifferent among all stock-bond combinations \( (\theta_t^q, \lambda_t^q) \) which yield this planned savings level. Let this determinate solution for the savings and consumption levels \( (s_t, c_t^1, c_{t+1}^2) \) be denoted by \( (18) \)

\[ (18) \quad (s(w_t, r_{t+1}), c_t^1(w_t, r_{t+1}), c_{t+1}^2(w_t, r_{t+1})) \]

In the initial period 1, each old consumer owns a nonnegative number \( \theta_0 \) of stock shares and a nonnegative number \( \lambda_0 \) of bonds. Thus, the aggregate quantities of stock shares and bonds in existence in period 1 are \( \Theta_0 \equiv L_0 \theta_0 \) and \( \Lambda_0 \equiv L_0 \lambda_0 \). Each old consumer is entitled to the period 1 dividend \( d_t \theta_0 \) and also earns income in period 1 by selling the stock shares and bonds owned. Thus, each old consumer in period 1 plans to consume

\[ (19) \quad c_t^2 = p_t^\theta \theta_0 + p_t^\lambda \lambda_0 + d_t \theta_0. \]

It will be assumed that this planned consumption derives from an (unmodelled) time 0 choice problem of the form (16), and that a viability condition of the form (17) holds also

\[ ^{11} \text{The savings function } s(\cdot) \text{ in (18) coincides with the savings function for the Diamond-Tirole Economy derived in Section 2. Recalling the definitions for the factor-price frontier functions } w(\cdot) \text{ and } k(\cdot) \text{ given in Part A of Section 2, it will be assumed here as for the Diamond-Tirole Economy that there exists a unique } \tilde{r} \text{ satisfying } s(w(\tilde{r}), \tilde{r}) = [1 + n]k(\tilde{r}), \text{ with } \tilde{r} < n. \text{ Also, as noted in Part B of Section 2, Diamond (1965) and Tirole (1985) impose several additional technical regularity conditions on the savings function } s(\cdot) \text{ and the production function } f(\cdot). \text{ These regularity conditions are also assumed to hold for the Brokered Economy. A detailed statement of these regularity conditions can be found in Part A of the Appendix.} \]
for $t = 0$. Consequently, the planned consumption of old agents in period 1 can equivalently be expressed in the form

$$c_t^2 = [1 + r_t] s(w_0, r_t),$$

where $w_0 = f(k_0) - y'(k_0)k_0$ is the wage received by the old agent in youth, and the savings function $s(\cdot)$ coincides with the savings function determined for each generation $t \geq 1$ agent in (18).

The problem facing the producer in the Brokered Economy is identical to that facing the producer in the Diamond-Tirole Economy. An initial positive capital-labor ratio $k_0$ is assumed to be historically given. In each period $t \geq 1$, the profit-maximizing producer has the opportunity to rent capital from the intermediary to be used as an input to production along with the labor $L_t$ supplied inelastically by young consumers. The rental capital $K_t^d$ which the producer plans to employ during period $t$ is demanded from the intermediary during period $t - 1$. As in Tirole (1985), it will be assumed that the rate of interest charged to the producer for the rental of this capital is the same as the rate of return $r_t$ on savings. Consequently, the producer plans to pay the intermediary a principal-plus-interest payment $[1 + r_t]K_t^d$ during period $t$.

The intermediary's period $t$ net earnings $\Pi_t$ are equal to the quantity of good $t$ remaining in the intermediary's possession after all contract obligations are fulfilled. These net earnings are determined by a consideration of the intermediary's stock, bond, and capital transactions. Let $\Theta_t^s \equiv L_t\theta_t^s$ and $\Lambda_t^s \equiv L_t\lambda_t^s$ denote the total amounts of stock shares and bonds which the intermediary plans to supply to consumers in period $t$, and let $K_t^r$ denote the amount of rental capital which the intermediary plans to supply to the producer for use in period $t$. As a result of stock share sales and purchases, the intermediary in period $t$ plans to receive $p_t^g\Theta_t^r$ units of good from generation $t$ young consumers and to deliver $p_t^g\Theta_{t-1}^r$ units of

\footnote{It is assumed throughout the remainder of the paper that initial supplies for stock shares, bonds, and per capita capital coincide with the historically given values; i.e., $\Theta_0^s \equiv \Theta_0$, $\Lambda_0^s \equiv \Lambda_0$, and $K_0^r/L_0 \equiv k_0$.}
good \( t \) to generation \( t - 1 \) old consumers. Moreover, as a result of bond sales and purchases, the intermediary in period \( t \) plans to receive \( p_t^\Lambda_t^i \) units of good from generation \( t \) young consumers and to deliver \( p_t^\Lambda_{t-1}^i \) units of good \( t \) to generation \( t - 1 \) old consumers. Finally, as a result of capital rental transactions, the intermediary in period \( t \) plans to receive a principal-plus-interest payment \([1 + r_t]K_t^i\) from the producer and deliver capital \( K_{t+1}^i\) to the producer to be employed in the subsequent period \( t + 1 \). Consequently, the intermediary’s planned period \( t \) net earnings take the form

\[
\Pi_t = p_t^\Theta_t^i - \Theta_{t-1} + p_t^\Lambda_t^i - \Lambda_{t-1} - [K_{t+1}^i - (1 + r_t)K_t^i], \quad t \geq 1.
\]

In the Diamond-Tirole Economy, only one asset—the “bubble asset”—is used to finance consumption loans. In the Brokered Economy there are two assets which the intermediary could use to finance consumption loans: namely, stocks and bonds. To achieve a more direct comparison with the Diamond-Tirole Economy, it is useful to place a financing restriction on the intermediary:

\[
K_t^i = p_{t-1}^\Lambda_{t-1}^i \quad \text{and} \quad B_{t-1} = p_{t-1}^\Theta_{t-1}^i, \quad t \geq 1.
\]

Condition (22) guarantees that the intermediary finances his capital loans \( K_t^i \) solely by means of bond transactions and his consumption loans \( B_{t-1} \) solely by means of stock transactions. Given the viability condition (17) on prices and interest rates and the financing condition (22), the intermediary’s planned period \( t \) net earnings (21) further reduce to

\[
\Pi_t = p_t^\Theta_t^i - \Theta_{t-1} - [p_t^\Lambda_t^i / p_{t-1}^\Theta_{t-1}^i]B_{t-1}
\]

\[
= d_t^i \Theta_{t-1}^i + B_t - [1 + r_t]B_{t-1}, \quad t \geq 1.
\]

By assumption, the intermediary plans to distribute his period \( t \) net earnings \( \Pi_t \) as dividends to generation \( t - 1 \) old shareholders, so that \( \Pi_t = d_t \Theta_{t-1}^i \). Consequently, it follows
from (23) that Tirole's bubble asset growth restriction (7) holds *ex post* for the Brokered Economy if young consumers correctly anticipate their per share dividend payments and all of the intermediary's planned stock, bond, and capital transactions are realized. Note, however, that Tirole's restriction is not an *ex ante* restriction on the intermediary's behavior. Moreover, in the Brokered Economy, Tirole's restriction does not imply that the intermediary's net earnings are zero. Rather, assuming positive prices, it follows from (23) that the net earnings of the intermediary are zero if and only if the bubble asset is in fixed supply.

The Brokered Economy is not yet complete. An objective for the intermediary needs to be specified, along with market clearing conditions. In standard overlapping generations models, these two specifications have traditionally been equated; the intermediary is assumed to be a passive Walrasian Auctioneer concerned only with trade and credit coordination. As will now be clarified, the alternative specification of an active intermediary motivated by earnings rather than coordination has immediate and dramatic implications for the efficient operation of the economy.

5. MODELLING THE INTERMEDIARY: EFFICIENCY IMPLICATIONS

A. The Intermediary as a Passive Coordinator

Suppose the Brokered Economy intermediary behaves as a passive Walrasian Auctioneer. That is, he sets stock share prices, bond prices, and interest rates, and then stands ready to mediate all resulting trade and credit transactions. Specifically, trades for existing stock shares and goods are brokered, and savings are channeled into capital investment and consumption loans. The sole objective of the intermediary is to coordinate these transactions so that supply equals demand in all markets.

The stock shares $\Theta^*_t$ supplied by the intermediary in period $t$ are given by the stock shares $\Theta^*_{t-1}$ which generation $t - 1$ old agents sell back to the intermediary in period $t$. Bonds $\Lambda^*_t$ are issued as needed to meet demand $\Lambda^*_t$. The capital and product markets are the same as
for the Diamond-Tirole Economy. In analytical terms, then, the intermediary's coordination objective implies that he sets prices and interest rates in each period \( t \geq 1 \) so that, in per capita terms, the stock, bond, capital, and product markets clear as follows:

\[
\begin{align*}
(24) & \quad p_t^\theta \theta^t_t \equiv p_t^\theta[\theta^t_{t-1}/(1 + n)] = p_t^\theta \theta^d_t, \quad t \geq 1; \\
(25) & \quad p_t^\lambda \lambda^t_t = p_t^\lambda \lambda^d_t, \quad t \geq 1; \\
(26) & \quad k_t^d \equiv [s_{t-1} - b_{t-1}]/(1 + n) = k_t^d, \quad t \geq 1; \\
(27) & \quad y_t + k_t^d = [1 + n]k_{t+1}^d + c_t^1 + c_t^2/[1 + n], \quad t \geq 1.
\end{align*}
\]

Given any positive price and interest rate sequences satisfying the viability condition (17), it follows from (23) that the intermediary's per capita net earnings \( \pi_t \equiv \Pi_t/L_t \) take the form

\[
\pi_t = p_t^\theta [\theta^t_t - \theta^t_{t-1}/(1 + n)] \\
= b_t - [p_t^\theta/(1 + n)p^t_{t-1}]b_{t-1} \\
= d_t^e[\theta^t_{t-1}/(1 + n)] + b_t - [(1 + r_t)/(1 + n)]b_{t-1}.
\]

Condition (24) implies that these net earnings are zero in each period \( t \geq 1 \). Since dividend payments consist entirely of the intermediary's net earnings, per share dividend expectations are fulfilled only if these expectations are zero.

The optimization problems faced by the producer and consumers in the Brokered Economy coincide with the optimization problems faced by the producer and consumers in the Diamond-Tirole Economy once the identification \( s_t \equiv [c_t^1 - w_t] \) is made for the Brokered Economy and the viability condition (17) is assumed. Also, using the capital market clearing condition (26), and assuming consumer and producer optimization, it is straightforward to show (cf. Footnote 7) that the product market clearing condition (27) is equivalent to Tirole's bubble asset growth restriction (8).
In summary, when the market clearing conditions (24) through (27) are imposed on the Brokered Economy, the resulting "Walrasian" Brokered Economy is essentially equivalent to the Diamond-Tirole Economy. One implication of this equivalence is that the Walrasian Brokered Economy supports the Pareto inefficient outcome $e$ as a competitive equilibrium. Another implication of this equivalence is that the intermediary implicitly present in the Diamond-Tirole economy is a passive agent who does not seek out earnings opportunities. When compared to intermediaries observed in society, such behavior must be considered peculiar. Why would an intermediary restrict itself to zero net earnings when positive net earnings are possible?

As indicated by (28), the Brokered Economy intermediary must be able to issue additional shares of stock (bubble asset) if he is to obtain positive net earnings. However, there does not seem to be any reason why stock shares should be in fixed supply. Instead, one would think the intermediary would take advantage of the opportunity to increase net earnings by issuing additional stock shares up to the point where this dilution had perceived adverse consequences on net earnings.

B. The Intermediary as an Active Earnings Seeker

Suppose the passive trade-coordinating Walrasian intermediary is replaced by an active earnings-driven intermediary who is able to issue additional shares of stock. Two basic possibilities can be considered: the earnings-driven intermediary is a price-setter; or the earnings-driven intermediary is a quantity-setter who takes prices as given.

The former possibility is extremely attractive. After all, the original motivation for explicitly introducing the intermediary was to put a corporate business suit on the Walrasian auctioneer, and the auctioneer is most certainly a price-setter rather than a price-taker.

$^{13}$Specifically, any equilibrium $(s_i, c_t^1, c_t^2, k_t, b_t, r_t, w_t; t \geq 1)$ for a Diamond-Tirole Economy with initial conditions of the form $k_0 > 0$ and $b_0 \geq 0$ is an equilibrium sequence of real outcomes for a Walrasian Brokered Economy with initial conditions of the form $k_0 > 0$ and $(b_0, b_0, \lambda_0) \geq 0$, and vice versa, where $s_t \equiv [c_t^1 - w_t]$ for the Brokered Economy.
Nevertheless, dropping the price-taking assumption represents a radical departure from the Diamond-Tirole Economy and is best left to a separate study.

How, then, might a plausible earnings objective be specified for the Brokered Economy intermediary, assuming that he is to be modeled as a price taker?

In order to adhere as closely as possible to the Diamond-Tirole framework, the supply-equal-demand equilibrium conditions (25), (26), and (27) for the bond, capital, and product markets in the Walrasian Brokered Economy will be retained for the general Brokered Economy. However, the stock market clearing condition (24) with fixed stock share supply will be replaced by the weaker requirement

\[ p_t^0 [\theta_t^i - \theta^{r}_t] = 0, \quad t \geq 1. \]

Thus, the real value of excess stock share supply must still equal zero in every period \( t \), as an equilibrium condition. However, the supply of stock shares issued by the intermediary to young consumers in period \( t \) is no longer constrained to equal the number of shares which the intermediary buys back from the old generation \( t - 1 \) shareholders. Rather, this supply is now to be determined by the behavioral specification of an earnings objective for the price-taking intermediary.

An immediate and important implication of this change in model specification is that the Pareto inefficient allocation associated with the bubbleless Diamond-Tirole stationary competitive equilibrium \( \bar{e} \) cannot be supported as a competitive equilibrium for the Brokered Economy under any reasonable specification for the intermediary’s earnings objective. The equilibrium \( \bar{e} \) is characterized by the stationary rate of return \( \bar{r} < n \) and zero net earnings.

---

14 As noted in Section 3, excess supply in the product market could feasibly occur in the Diamond-Tirole Economy because the intermediary could accumulate positive net earnings (positive amounts of good) which he does not distribute back to consumers. Consequently, the supply-equal-demand equilibrium condition for the Diamond-Tirole product market is overly restrictive. In the Brokered Economy, however, all net earnings are distributed back to consumers in the period in which they are accumulated. Thus, all goods produced in any period \( t \) are acquired by consumers through wages, dividends, and net receipts from stock and bond transactions. The supply-equal-demand condition (27) is therefore an appropriate product market clearing condition for the Brokered Economy.
in each period \( t \geq 1 \). In this equilibrium the intermediary issues no consumption loans. However, given \( \bar{\alpha} \), the intermediary in the Brokered Economy would perceive the possibility of increasing his net earnings \( \Pi_t \) in each period \( t \) by switching to an intermediation plan under which some consumption loans are issued.

To see this, suppose the price-taking intermediary decides to sell an additional fixed real amount of stock \( \Delta v = p_t^0 \Delta \theta_t^0 \) to each young consumer in each generation \( t \geq 1 \) at the given rate of return \( \bar{\alpha} \), where all receipts from these stock sales are to be used to finance consumption loans. In the initial period 1, the intermediary would then expect to obtain the positive net earnings increment \( L_1 \Delta v \), for the liability incurred by the additional stock sale does not come due until the following period. Moreover, in every subsequent period \( t \geq 2 \) the intermediary would expect to obtain the net earnings increment \( L_t \Delta v - (1 + \bar{\alpha}) L_{t-1} \Delta v \), or \( [n - \bar{\alpha}] L_{t-1} \Delta v \), which is positive since \( n > \bar{\alpha} \). Thus, anticipated net earnings under this alternative intermediation plan would be higher in each period \( t \geq 1 \).

In short, under any reasonable specification of an earnings objective, the intermediary in the Brokered Economy would desire to sell more stock shares at \( \bar{\alpha} \) than consumers would desire to purchase. It follows that \( \bar{\alpha} \) cannot be a stationary equilibrium rate of return for the Brokered Economy.

Nevertheless, the concrete specification of a reasonable earnings objective for the price-taking Brokered Economy intermediary is not a simple matter. The usual objective assumed for a price-taking corporation in standard general equilibrium contexts is the maximization of present-value profits. This specification is problematic for a price-taking corporation in an overlapping generations economy. The crux of the difficulty is an ambiguity in the very meaning of “profits.” Financial flows are staggered for the Brokered Economy intermediary; revenues and costs associated with any one transaction are separated in time. The period \( t \) profit recognized by the intermediary thus depends upon whether the intermediary accounts for profit using “accrual-basis” or “cash-basis” accounting.
Under accrual-basis accounting, the revenues from a particular transaction are only recognized in a profit calculation if all of the associated costs of the transaction are also recognized, and vice versa. Accrual-basis profit on any transaction made by the intermediary must equal zero, for each transaction generates a revenue which is exactly offset by a cost in present value terms. In contrast, under cash-basis accounting, period t profit is recognized as the net value of period t economic flows. Thus, the intermediary’s period t cash-basis profit is equal to his net earnings \( N_t \), as defined by (21). Period t cash-basis profit is the quantity of good \( t \) which the intermediary controls during period t in excess of that required to fulfill all period t contract obligations. To avoid confusion between accrual-basis and cash-basis profit, the term “net earnings” will hereafter be used in place of “cash-basis profit.”

The “profit” opportunities perceived by the Brokered Economy intermediary under the two different accounting methods are dramatically different. For example, if the intermediary were interested in maximizing present value accrual-basis profit, he would be indifferent among all feasible intermediation plans; for each such plan generates zero present value accrual-basis profit. Nevertheless, different feasible intermediation plans generate different streams of net earnings (potential dividend payments). Consequently, if the intermediary focuses on net earnings, i.e., on dividend payments to shareholders, he will presumably not be indifferent regarding his choice of a feasible intermediation plan.

We conclude from these observations that the objective of the Brokered Economy intermediary is more plausibly specified as a concern for net earnings (dividend payments to shareholders) rather than as a concern for accrual-basis profits. As a preliminary step in formulating this objective, we first provide a more careful definition for an “intermediation plan.” Attention is restricted to plans conditioned on price and interest rate sequences satisfying the viability condition (17) in each period t. The intermediary’s behavior in the face of nonviable price and interest rate sequences is irrelevant, since such sequences will never occur in equilibrium.
Given any viable price and interest rate sequences, it follows from (28) that the intermediary's period $t$ per capita net earnings $\pi_t$ can be expressed as a function

\[(30) \quad \pi_t(b, p^\theta) = b_t - [p_t^\theta/(1 + \eta)p_{t-1}^\theta]b_{t-1}\]

of the sequence $b = (b_1, b_2, \ldots)$ of per capita bubble asset investments (consumption loans) made by the intermediary, with $b_0 \geq 0$ historically given, and the sequence $p^\theta = (p_0^\theta, p_1^\theta, \ldots)$ of stock share prices the intermediary takes as given. In each period $t \geq 1$, the intermediary chooses the number of stock shares and bonds to supply to consumers in order to finance his capital and bubble asset investments. However, it is clear from (30) that the sequence $b$ of per capita bubble asset investments chosen by the intermediary is of primary importance in the determination of his net earnings. This sequence will be called the intermediary's *intermediation plan*.

Given any viable price sequence $p^\theta$, what constitutes a "feasible" intermediation plan? The intermediary must fulfill all contract obligations. For all contract obligations to be fulfilled, the intermediary must receive enough good in each period to meet all outstanding obligations. This implies that the intermediary must choose an intermediation plan $b$ such that $\pi_t(b, p^\theta) \geq 0$ holds for all $t \geq 1$.

In the absence of any additional constraint on the intermediation plans, however, the intermediary would *incorrectly* perceive the possibility of obtaining arbitrarily large net earnings in each period $t \geq 1$ by rolling over ever-larger amounts of debt. Rationally perceived quantity constraints ("no-Ponzi-game" conditions) are commonly used in overlapping generations contexts to prevent price-taking altruistic consumers with bequest motives from choosing nonfeasible "optimal" paths with exploding debt; see, e.g., Blanchard and Fischer (1989, pp. 49-50). Apparently this kind of constraint also needs to be imposed on the Brokered Economy intermediary.

We therefore assume that the intermediary restricts his choice of an intermediation plan
to the set $B$ of per capita bubble asset sequences $b$ which do not diverge to infinity.\textsuperscript{15}

As established in Section B of the Appendix, this assumption is rather innocuous; for the intermediary can recognize \textit{a priori} that a per capita bubble asset sequence $b$ which diverges to infinity could never be actualized in any equilibrium for the Brokered Economy.

In summary, given a viable price sequence $p^\theta$, the set $F(p^\theta)$ of feasible intermediation plans $b$ is defined to be the collection of all nondivergent intermediation plans yielding nonnegative net earnings in each period $t \geq 1$. Formally,

$$(31) \quad F(p^\theta) = \{b \in B : \pi_t(b, p^\theta) \geq 0 \text{ for all } t \geq 1\}.$$  

The intermediary has considerable flexibility in distributing net earnings over time. However, each shareholder cares only about the net earnings distributed during his lifetime. How is the distribution of net earnings to be determined?

Here we consider one illustrative example of a dividend distribution objective for the intermediary which takes into account the varied interests of all shareholders. Specifically, we consider a version of Rawls' (1971) max-min "Difference Principle." We assume that the intermediary seeks to maximize the minimum per capita dividend $\pi_t$ distributed to shareholders over time. As will be shown below (Corollary 1), this dividend distribution objective implies that the market value of the corporate intermediary in any competitive equilibrium attains its maximum feasible value in each period $t \geq 1$.

Formally, the dividend distribution objective of the intermediary takes the form

$$\max_{b \in F(p^\theta)} \inf_{t \geq 1} \pi_t(b, p^\theta).$$

The (possibly empty) solution set for problem (32) will be denoted by $B(p^\theta)$.

A definition of competitive equilibrium will now be given for the Brokered Economy

\textsuperscript{15}A real sequence $(b_1, b_2, \ldots)$ "diverges to infinity" if, for every real number $v$, there exists a subscript $t$ such that $|b_t| \geq v$ for all $s \geq t$. The set $B$ of nondivergent sequences thus contains bounded sequences as well as, e.g., cyclic sequences with ever increasing amplitudes.
which includes the dividend objective of the intermediary as well as the utility objectives of consumers and the profit objective of the producer.

DEFINITION 2: [Brokered Economy Equilibrium] Let initial values $k_0 > 0$ and $(b_0, \theta_0, \lambda_0) \geq 0$ for per capita capital, per capita bubble asset, per capita stock shares, and per capita bonds be given. A sequence $(v_t, d_t^p, k_t^d, m_t, p_t: t \geq 1)$ consisting of consumer choice vectors $v_t = (\theta_t^i, \lambda_t^i, c_t^i, c_{t+1}^i)$, expected per share dividends $d_t^p$, producer per capita capital demands $k_t^d$, intermediary choice vectors $m_t = (\Theta_t^i, \Lambda_t^i, k_t^i, b_t)$, and price-interest rate vectors $p_t = (p_{t-1}^\theta, p_{t-1}^\lambda, r_t, w_t) > 0$ is a Brokered Economy equilibrium $e(k_0, b_0, \theta_0, \lambda_0)$ if the following seven conditions are met:

- **[Consumer Optimization]** In each period $t \geq 1$, $(\theta_t^i, \lambda_t^i, c_t^i, c_{t+1}^i)$ solves the lifetime utility maximization problem (16) conditional on $w_t, p_t^\theta, p_t^\lambda, p_{t+1}^\theta, p_{t+1}^\lambda, d_{t+1}^p$; and the consumption $c_t^i$ of each generation 0 old consumer in period 1 satisfies $c_t^i = [1 + r_1]s(f(k_0) - f'(k_0)k_0, r_1)$ with the savings function $s(\cdot)$ determined as in (20).

- **[Producer Optimization]** In each period $t \geq 1$, the producer’s per capita capital demand $k_t^d$ solves the necessary and sufficient conditions (8) and (9) for profit maximization conditional on $w_t$ and $r_t$.

- **[Intermediary Optimization]** The intermediary chooses a feasible intermediation plan $b$ which maximizes the minimum per capita dividend distributed over time; i.e., $b \in B(p^\theta)$.

- **[Market Clearing Conditions]** In each period $t \geq 1$, the bond, capital, and product markets satisfy the market clearing conditions (25), (26), and (27), and the stock market satisfies the market clearing condition (29).

- **[Fulfilled Expectations]** In each period $t \geq 1$, the expected per share dividend $d_t^p$ coincides with the actual per share dividend.
• [Price-Interest Rate Viability Condition] The rate of return paid by borrowers is equal to the rate of return received by lenders; i.e., condition (17) holds for all $t \geq 0$.

• [Financing Condition] Capital loans are only financed using bond transactions, and consumption loans are only financed using stock share transactions; i.e., condition (22) holds for all $t \geq 1$.

A Brokered Economy equilibrium $e(k_0, b_0, \theta_0, \lambda_0)$ will be called a stationary Brokered Economy equilibrium if $k_t = k_0$ and $b_t = b_0$ for all $t \geq 1$.

Given the defining conditions for a Brokered Economy equilibrium, the stationarity restriction on the per capita capital stock $k_t$ is equivalent to assuming the stationarity of the interest rate $r_t$. Stationarity of $r_t$ and $k_t$ in turn implies the stationarity of the real wage $w_t$ and the savings and consumption levels $(s_t, c_t^1, c_t^2)$ chosen by consumers in each period $t \geq 1$, where $s_t \equiv [c_t^1 - w_t]$.

Using the definition for a Brokered Economy equilibrium, the following three theorems are obtained. Proofs are given in Section C of the Appendix.

THEOREM 1: Given any positive price sequence $p^\theta$ and any initial per capita bubble asset level $b_0 \geq 0$, the solution set $B(p^\theta)$ for the intermediary's optimization problem (32) is nonempty if and only if the sequence $(S_t)$ diverges to infinity and the sequence $(q_t q_{t-1} \cdots q_1 b_0)$ does not diverge to infinity, where

$\begin{align*}
q_t &\equiv p_t^\theta/(1 + n)p_{t-1}^\theta, \quad t \geq 1, \\
S_t &\equiv 1 + q_t + q_t q_{t-1} + \cdots + [q_t q_{t-1} \cdots q_1] b_0, \quad t \geq 2,
\end{align*}$

with $S_1 \equiv 1$.

The price conditions appearing in Theorem 1 are the Brokered Economy analogs of the well-known Cass-Balasko-Shell price conditions shown by Balasko and Shell (1980, Proposi-
tion 5.6, pp. 296-297) to be necessary and sufficient for Pareto efficiency in the context of a pure exchange overlapping generations economy. Balasko and Shell do not provide an economic interpretation for their price conditions. The interesting point here is that the analogs of the Balasko-Shell price conditions now appear as necessary and sufficient conditions for the earnings-driven intermediary to have a solution to his dividend distribution problem. Theorem 1 is crucial for the proof of the next theorem.

**THEOREM 2:** There exists at least one stationary equilibrium for the Brokered Economy. Every stationary equilibrium generates the same real outcome \((s, c^1, c^2, k, b, r, w)\), where \(s = \left[ c^1 - w \right] \); namely, the Pareto efficient real outcome \(e^n(k^n, b^n)\) constituting the golden-rule equilibrium for the Diamond-Tirole Economy. In particular, the unique stationary equilibrium interest rate \(r\) for the Brokered Economy is given by \(r = n\).

Theorem 2 states that the Pareto efficient golden-rule equilibrium for the Diamond-Tirole Economy is the unique real outcome possible for the Brokered Economy in any stationary equilibrium. In particular, the Pareto inefficient stationary equilibrium \(\bar{e}\) for the Diamond-Tirole Economy is not supported as a stationary equilibrium real outcome for the Brokered Economy. As noted in Part B of Section 2, all Pareto inefficient equilibria for the Diamond-Tirole Economy converge to \(\bar{e}\); hence one might surmise that the elimination of this Pareto inefficient outcome would lead to the restoration of a First Welfare Theorem. The next theorem shows that this is indeed the case.

**THEOREM 3:** Every Brokered Economy equilibrium is Pareto efficient.

Theorem 3 establishes that the inefficiency observed in the Diamond-Tirole Economy is completely eliminated if the intermediary is modelled as an earnings-driven corporation which attempts to maximize the minimum per capita dividend distributed to shareholders
over time. The final result follows immediately from the proof of Theorem 3.

**COROLLARY 1:** Given any equilibrium for the Brokered Economy, the market value $B_t = p_t^t \Theta_t$ of the corporate intermediary in each period $t \geq 1$ is positive and takes on its maximum feasible value. Moreover, the stock share price $p_t^t$ in each period $t \geq 1$ differs from its fundamental value—i.e., the present value of the current and future per share stock dividends $(d_r : r \geq t)$—if and only if the sequence $([q_t q_{t-1} \cdots q_1] b_0)$ does not converge to zero as $t \to \infty$, where $q_t$ is defined as in (33).

It follows from the first statement of Corollary 1 that the max-min dividend objective of the intermediary is consistent with the goal of market value maximization. The second statement establishes a necessary and sufficient condition for the existence of a bubble on the stock shares issued by the intermediary.

6. CONCLUDING REMARKS

This paper establishes that the inefficiency properties exhibited by the Diamond-Tirole Economy under standard competitive assumptions are dramatically altered when a price-taking earnings-driven corporate intermediary is introduced into the model. In particular, the Pareto inefficient stationary equilibrium for the Diamond-Tirole Economy is no longer tenable. These findings indicate that the inefficiency observed in previous overlapping generations models with production may not be the reflection of an intrinsic market failure. Rather, the observed inefficiency could be due to an incompleteness in the model specification—the presumed inability of private agents to exploit the earnings opportunities associated with incurring and forever rolling over debt.

For future studies, however, we believe that modelling intermediaries as price-setting agents will be more fruitful. The Walrasian Auctioneer is, after all, a price-setting agent. However, the motives of the intermediary must be extended beyond those currently associated with the Walrasian Auctioneer. Real-world intermediaries are not merely trade and
credit coordinators. They set prices, interest rates, and other borrowing and lending terms with the primary objective of obtaining earnings. In short, real-world counterparts of the Walrasian Auctioneer are dressed in corporate business suits.

Our work also suggests that some restrictions on free entry and debt accumulation will have to be imposed on the intermediation sector in order to give private agents both the incentive and the ability to create and maintain viable corporate intermediary institutions. Real-world intermediaries such as banks and savings and loan institutions are already subject to such restrictions, e.g., charter and reserve requirements. Future studies along the lines suggested in this paper might clarify the reasons for the historical emergence of these regulations.

REFERENCES


appear.


Figure 1: The Diamond-Tirole Economy
APPENDIX

A. Proof that $b^n > 0$

As noted in Section 2, Part B, Diamond (1965) and Tirole (1985) assume that the equation $0 = z(r)$ has a unique solution $\tilde{r}$, where

$$z(r) \equiv \frac{[L_1s(w(r), r) - L_t k(r)]/L_t = s(w(r), r)/[1 + n] - k(r)}{\text{denotes the excess supply of savings per young consumer when each optimizing consumer and producer is faced with the interest rate } r. \text{ In addition, in order to focus on the case where inefficiency is possible, they assume that } \tilde{r} < n.}$$

It will first be shown that, under the Diamond-Tirole assumptions, the excess supply of savings $z(r)$ is an increasing function of $r$ at $r = \tilde{r}$; i.e., $0 < z'(\tilde{r})$. Using this result, it will then be shown that

$$0 < z(r) \text{ for all } r \text{ satisfying } \tilde{r} < r.$$  

Hence, in particular, $0 < z(n) = b^n/[1 + n]$.

Given the stationary interest rate $\tilde{r}$, producer optimization in the Diamond-Tirole Economy implies that the capital-labor ratio $k$ and the wage rate $w$ satisfy $\tilde{r} = f'(k)$ and $w = f(k) - f'(k)k$. Consequently, it follows from Section 2, Part A, that $k = k(\tilde{r})$ and $w = w(\tilde{r})$ with

$$k'(\tilde{r}) = 1/f''(k(\tilde{r}));$$  

$$w'(\tilde{r}) = -k(\tilde{r}) \cdot f''(k(\tilde{r})) \cdot k'(\tilde{r}).$$

Using conditions (37) and (38), one obtains

$$z'(\tilde{r}) = \frac{s_\omega(w(\tilde{r}), \tilde{r})w'(\tilde{r}) + s_r(w(\tilde{r}), \tilde{r})/[1 + n] - k'(\tilde{r})}{[s_r(w(\tilde{r}), \tilde{r}) - s_\omega(w(\tilde{r}), \tilde{r})k(\tilde{r})f''(k(\tilde{r}))k'(\tilde{r})]/[1 + n] - k'(\tilde{r})} = \frac{s_r(w(\tilde{r}), \tilde{r}) - s_\omega(w(\tilde{r}), \tilde{r})k(\tilde{r})f''(k(\tilde{r}))k'(\tilde{r})]/[1 + n] - k'(\tilde{r})}{[s_r(w(\tilde{r}), \tilde{r}) - s_\omega(w(\tilde{r}), \tilde{r})k(\tilde{r})k'(\tilde{r})]/[1 + n] - k'(\tilde{r})}.$$  

(39)
Diamond and Tirole place a condition on the relative slopes of the capital market supply and demand curves along any competitive equilibrium path; see (Diamond, 1965, eq.(11), p. 1133). In particular, for the unique stationary competitive equilibrium $\bar{e} = e(\bar{k}, 0)$ associated with $\bar{f}$, this condition reduces to

$$\left[ f''(\bar{k}) k(\bar{k}) s_w(\bar{w}, \bar{r}) \right]/\left[ 1 + n - f''(\bar{k}) s_r(\bar{w}, \bar{r}) \right] < 0 .$$

Moreover, Diamond and Tirole assume that consumption is a normal good in each period of life for each agent, which implies $0 < s_w < 1$; see (Diamond, 1965, footnote 1). The numerator in condition (40) is therefore negative. Thus, in order for condition (40) to hold, the denominator must be positive; i.e.,

$$\left[ 1 + n - f''(\bar{k}) s_r(\bar{w}, \bar{r}) \right] > 0 .$$

Diamond (1965) also assumes that $\bar{e}$ is locally stable, and he gives a condition (eq.(12), p. 1134) which is necessary for this to be the case; namely,

$$\left| -k(\bar{k}) f''(k(\bar{k})) s_w(w(\bar{k}), \bar{r}) \right|/[1 + n - f''(k(\bar{k})) s_r(w(\bar{k}), \bar{r})] \leq 1 .$$

A sufficient condition for the local stability of $\bar{e}$ is that condition (42) hold with strict inequality, a condition assumed by Tirole (1985, p. 1502) in his extension of the Diamond framework. Suppose that this slightly stronger sufficient condition holds.

Then, using conditions (37), (39), (41), and (42) with strict inequality, one obtains the following series of implications:

$$[-k(\bar{k}) f''(k(\bar{k})) s_w(w(\bar{k}), \bar{r})] < [1 + n - f''(k(\bar{k})) s_r(w(\bar{k}), \bar{r})];$$

$$0 < [k(\bar{k}) f''(k(\bar{k})) s_w(w(\bar{k}), \bar{r}) + 1 + n - f''(k(\bar{k})) s_r(w(\bar{k}), \bar{r})];$$

$$0 < [-k'(\bar{k}) f''(k(\bar{k})) s_w(w(\bar{k}), \bar{r}) - k'(\bar{r})[1 + n] + k'(\bar{k}) f''(k(\bar{k})) s_r(w(\bar{k}), \bar{r})];$$

$$0 < [s_r(w(\bar{k}), \bar{r}) - s_w(w(\bar{k}), \bar{r}) k(\bar{r})]/[1 + n] - k'(\bar{r});$$

$$0 < z'(\bar{r}).$$
As previously noted, \( \bar{r} \) satisfies \( 0 = z(\bar{r}) \). It follows from (43) that, for some positive \( \epsilon \), \( 0 < z(r) \) for all \( r \) in the interval \( (\bar{r}, \bar{r} + \epsilon) \). Suppose that \( z(r^*) \leq 0 \) for some \( r^* \) satisfying \( \bar{r} + \epsilon < r^* \). By continuity of \( z(\cdot) \), this would imply the existence of some \( r_0 \) lying between \( \bar{r} + \epsilon \) and \( r^* \) which satisfies \( 0 = z(r_0) \), a contradiction of the assumption that there exists a unique solution to the equation \( 0 = z(r) \).

It follows that condition (36) must hold for the Diamond-Tirole Economy. \( \square \)

B. Proof that Equilibrium Intermediation Plans Cannot Diverge

Suppose \( b \) is an equilibrium intermediation plan for the Brokered Economy which diverges to infinity. Since the historically given value \( b_0 \) is nonnegative by assumption, it follows from the expression (28) for the intermediary’s per capita net earnings \( \pi_t \), and the necessity to have \( \pi_t \geq 0 \) to fulfill all contracts, that the sequence \( b \) is nonnegative. Consequently, using (26), the nonnegative elements \( b_t = [s_t - (1 + n)k_{t+1}] \) of the sequence \( b \) must become infinitely large as \( t \) approaches infinity.

The savings \( s_t \) of a representative generation \( t \) young agent must be nonnegative and less than his endowment \( w_t \) in order for the agent to consume a nonnegative amount of good when young and when old. Moreover, since the interest rate along any equilibrium path is positive by assumption, it follows from the producer optimization condition (4) that the capital-labor ratio \( k_t \) along any equilibrium path must also be positive. Using these observations, together with the producer optimization condition (5), one obtains

\[
0 \leq s_t/k_t \leq w_t/k_t = f(k_t)/k_t - f'(k_t) < f(k_t)/k_t \quad \text{for all } t \geq 1.
\]

Since the capital-labor ratio \( k_t \) is positive along any equilibrium path, \( b_t \) can only become infinitely large if \( s_t \) and thus \( w_t \) both become infinitely large. By conditions (4) and (5), and the production function restrictions listed in footnote 5, for \( w_t \) to become infinitely large as \( t \) approaches infinity, the interest rate \( r_t \) must approach zero and the capital-labor ratio \( k_t \).
must become infinitely large as $t$ approaches infinity. Consequently, by strict concavity of
the production function $f(\cdot)$, and the assumption that $f'(k)$ approaches zero as $k$ approaches
infinity (cf. footnote 5), the right-hand average product of capital in (44) must approach zero
as $t$ approaches infinity, implying that the ratio $s_t/k_t$ does also.

Finally, consider the expression

$$s_{t-1}/k_{t-1} - [1 + n]k_t/k_{t-1} = b_{t-1}/k_{t-1} \geq 0.$$  

Since $s_{t-1}/k_{t-1}$ approaches zero as $t$ approaches infinity, the (positive) term $[1 + n]k_t/k_{t-1}$
must also approach zero as $t$ approaches infinity in order to have the right term remain
nonnegative for all $t$. Consequently, $k_t < k_{t-1}$ for all sufficiently large $t$. But this contradicts
the fact, established above, that $k_t$ becomes infinitely large as $t$ approaches infinity.

It follows that no equilibrium intermediation plan $b$ for the Brokered Economy can diverge
to infinity.  

C. Proofs for Section 5 Theorems

PROOF OF THEOREM 1: NECESSITY Suppose $(S_t)$ does not diverge to infinity. Let $Q_t \equiv [q_t, \ldots, q_1]b_0$ for each $t \geq 1$, where $b_0 \geq 0$ denotes initial per capita bubble asset
holdings; and consider the intermediation plan $b$ defined by

$$b_t = M + q_t b_{t-1} = S_t M + Q_t, \quad t \geq 1,$$

where $M$ is an arbitrary positive constant. Since $q_t > 0$ for each $t \geq 1$, $0 \leq Q_t \leq S_t q_1 b_0$ for
each $t \geq 1$. It then follows from (46) that $0 \leq b_t \leq S_t [M + q_1 b_0]$ for each $t \geq 1$. Consequently,
the nondivergence of $(S_t)$ implies that $b$ lies in the set $B$ of nondivergent sequences. Finally,
it follows from (46) and (28) that the intermediary’s per capita net earnings $\pi_t$ satisfy $\pi_t = M$ for each $t \geq 1$. Since $M$ can be chosen arbitrarily large, this implies that the solution set
for the intermediary’s optimization problem (32) is empty.
Thus, the solution set for the intermediary's optimization problem (32) is nonempty only if \((S_t)\) diverges to infinity.

By assumption, \(b_0 \geq 0\). Given any feasible intermediation plan \(b\), it follows by (28) and the definition of feasibility that \(\pi_t = b_t - q_t b_{t-1} \geq 0\) for each \(t \geq 1\). Thus, \(b_t \geq Q_t\) for each \(t \geq 1\). If the nonnegative sequence \((Q_t)\) were to diverge to infinity, \(b\) would diverge to infinity and hence be infeasible, a contradiction. Consequently, the set \(F(p^\theta)\) of feasible intermediation plans is nonempty only if \((Q_t)\) does not diverge to infinity.

It follows that the solution set for the intermediary's optimization problem (32)—a subset of \(F(p^\theta)\) by construction—is nonempty only if \((Q_t)\) does not diverge to infinity.

**PROOF OF THEOREM 1: SUFFICIENCY** Suppose \((S_t)\) diverges to infinity but \((Q_t)\) does not diverge to infinity. Let \(\hat{M}\) denote the largest level of per capita net earnings which the intermediary can achieve in each period \(t \geq 1\) by means of some feasible intermediation plan \(b\). Using (28), this means that \(b\) must satisfy \(\pi_t = b_t - q_t b_{t-1} \geq \hat{M}\) for all \(t \geq 1\), or

\[
(47) \quad b_t \geq S_t \hat{M} + Q_t, \quad t \geq 1.
\]

Since the nonnegative sequence \((S_t)\) diverges to infinity, and \(Q_t\) is nonnegative for all \(t \geq 1\), \(b\) lies in the set \(B\) of nondivergent sequences only if \(\hat{M} \leq 0\). Thus, the max-min level of per capita net earnings which the intermediary can feasibly achieve is at most 0.

To establish that the solution set for the intermediary's optimization problem (32) is nonempty, it thus suffices to show that there exists a feasible intermediation plan which yields zero per capita net earnings in each period \(t \geq 1\). Since \((Q_t)\) does not diverge to infinity, such an intermediation plan is found by taking \(b_t = q_t b_{t-1}\) for all \(t \geq 1\). \(\square\)

**LEMMA 1:** If the sequence \((S_t)\) diverges to infinity, the max-min level of per capita net earnings which the intermediary can feasibly achieve is zero.

**PROOF:** As established in the proof of sufficiency for Theorem 1, the divergence of \((S_t)\) implies that \(\hat{M} = 0\) is the maximum level of per capita net earnings which the intermediary
can feasibly achieve in each period $t \geq 1$. □.

**Lemma 2:** Any equilibrium sequence $(s_t, c^1_t, c^2_t, k_t, b_t, r_t, w_t; t \geq 1)$ of real outcomes for a Brokered Economy with initial conditions of the form $k_0 > 0$ and $(b_0, \theta_0, \lambda_0) \geq 0$ is an equilibrium for a Diamond-Tirole Economy with initial conditions of the form $k_0 > 0$ and $b_0 \geq 0$, where $s_t \equiv [c^1_t - w_t]$ for the Brokered Economy.

**Proof of Lemma 2:** Let $k_0 > 0$ and $(b_0, \theta_0, \lambda_0) \geq 0$ be given initial conditions for a Brokered Economy. Consider the general definition for a Brokered Economy equilibrium presented in Section V. Using the various conditions listed in this definition, particularly the capital market clearing and fulfilled expectations conditions, it is straightforward to show that any Brokered Economy equilibrium paths for $k_t$ and $b_t$ must satisfy the two basic state equations (12) and (13) for the Diamond-Tirole Economy; hence they must also be equilibrium paths for the Diamond-Tirole Economy.

Given the viability condition on interest rates and prices appearing in the definition of a Brokered Economy equilibrium, the Brokered Economy equilibrium conditions for consumer and producer optimization reduce to the conditions for consumer and producer optimization appearing in the definition of a Diamond-Tirole equilibrium. Consequently, given any paths for $k_t$ and $b_t$ which are equilibrium paths for both the Brokered Economy and the Diamond-Tirole Economy, the equilibrium paths uniquely generated for the variables $(s_t, c^1_t, c^2_t, r_t, w_t)$ by means of these consumer and producer optimization conditions will be the same. □

**Lemma 3:** In order to have a stationary Brokered Economy equilibrium, it is necessary that $r_t = n$ for each $t \geq 1$.

**Proof:** Suppose $\mathbf{e}(k_0, b_0, \theta_0, \lambda_0)$ is a stationary Brokered Economy equilibrium. By definition of stationarity, $k_t = k_0$ for all $t \geq 0$. Producer optimization then implies that $r_t = f'(k_0)$ for all $t \geq 1$. Let $r \equiv f'(k_0)$. It then follows from the price-interest rate viability condition appearing in the definition for a Brokered Economy equilibrium that any sequence of stock...
share prices in a stationary equilibrium must satisfy

(48) \[ q_t \equiv \frac{p_t^o}{1+n}p_{t-1}^o = a - \frac{z_t^e}{(1+n)}, \quad t \geq 1, \]

where \( z_t^e \equiv d_t^e/p_{t-1}^o \) denotes the period \( t \) price-normalized expected per share dividend, and \( a \equiv [1 + r]/[1 + n] > 0 \).

By assumption, the equilibrium stock share price \( p_{t-1}^o \) is positive for all \( t \geq 1 \). Moreover, since equilibrium per capita net earnings \( \pi_t \) must be nonnegative in each period \( t \geq 1 \), it follows by fulfilled expectations that the equilibrium expected per share dividend \( d_t^e \) must be nonnegative for each \( t \geq 1 \). Thus, \( z_t^e \geq 0 \) for all \( t \geq 1 \), implying that \( q_t \leq a \) for all \( t \geq 1 \).

By Theorem 1, the sequence \( (S_t) \) defined as in (34) must diverge to infinity in order for a solution for the intermediary's problem to exist, and hence in order for a Brokered Economy equilibrium to exist. Since \( q_t \leq a \) for all \( t \geq 1 \), the divergence of \( (S_t) \) implies the divergence of \( (S_t) \), where

(49) \[ S_t \equiv 1 + a + a^2 + \ldots + a^{t-1}, \quad t \geq 1. \]

However, the sequence \( (S_t) \) diverges only if \( a \geq 1 \). Thus, \( r \geq n \) must hold.

Next, suppose \( a > 1 \), or that \( r > n \). In any Brokered Economy equilibrium, the actual per share dividend \( d_t \) in each period \( t \) must equal the expected per share dividend \( d_t^e \). Hence, since all net earnings are distributed as dividends to old agents in each period \( t \), \( \pi_t = d_t^e/[(1 + n)] \) for all \( t \geq 1 \). It follows from (28) and the definition of \( a \) that \( 0 = b_t - ab_{t-1} \) for all \( t \geq 1 \). But \( a > 1 \) then implies that \( b_t \) diverges to infinity (and hence is infeasible) unless \( b_t \equiv 0 \). Given \( a > 1 \) and \( b_t \equiv 0 \), it follows from the capital market clearing condition (26) and producer optimization that there exists an \( r > n \) satisfying \( s(w(r), r) = [1 + n]k(r) \), a violation of a basic Diamond-Tirole assumption retained for the Brokered Economy (see Section II, Part B). Thus, \( r = n \) must hold. \( \Box \)

PROOF OF THEOREM 2 As established by Tirole (1985) and reviewed in Section 2, the Diamond-Tirole Economy has a unique equilibrium associated with the stationary interest
rate \( r_t = n \): namely, the golden-rule equilibrium \( e^* (k^n, b^n) \). Consequently, it follows from Lemmas 2 and 3 that the real outcome associated with any stationary Brokered Economy equilibrium must coincide with this golden-rule equilibrium.

To complete the proof, it suffices to show that the Brokered Economy has a stationary equilibrium with real outcome given by \( e^* \).

Suppose, then, that the real outcome \((s_t, c^1_t, c^2_t, k_t, b_t, r_t, w_t)\) for the Brokered Economy is given by the stationary golden-rule outcome \( e^* \) in each period \( t \geq 1 \). Thus, in particular, \( r_t = n, k_t = k^n, \) and \( b_t = b^n \) for each \( t \geq 1 \). Suppose in addition that the expected per share dividend satisfies \( d^*_t = 0 \) for each \( t \geq 1 \). Let arbitrary initial stock and bond levels \( \theta_0 \geq 0 \) and \( \lambda_0 \geq 0 \) be given. If these initial values are positive, then, using the financing condition appearing in the definition for a Brokered Economy equilibrium, determine initial price levels for stock shares and bonds by \( p^\theta_0 = b^n/\theta_0 \) and \( p^\lambda_0 = [1 + n]k^n/\lambda_0 \). Otherwise, let these initial price levels take on arbitrary positive values.

Given the stationary (positive) golden-rule outcomes \( k^n \) and \( b^n \), together with the positive initial price levels \( p^\theta_0 \) and \( p^\lambda_0 \), determine values for the stock price \( p^\theta_t \), bond price \( p^\lambda_t \), stock level \( \theta_t \), and bond level \( \lambda_t \) in each period \( t \geq 1 \) by the following four auxiliary relations derived from the viability and financing conditions, together with the assumptions \( r_t = n \) and \( d^*_t = 0 \):

\[
\begin{align*}
  p^\theta_t &= [1 + n]p^\theta_{t-1} ; \\
  p^\lambda_t &= [1 + n]p^\lambda_{t-1} ; \\
  \theta_t &= b^n/p^\theta_t ; \\
  \lambda_t &= [1 + n]k^n/p^\lambda_t .
\end{align*}
\]

By construction, the sequences described above satisfy all of the defining conditions for a stationary Brokered Economy equilibrium with the possible exceptions of intermediary optimization and fulfilled dividend expectations. To finish the proof, it remains to show
that the stationary intermediation plan $b_t \equiv b^n$ lies in the solution set for the intermediary's optimization problem (32); and, making use of this plan, the per share dividend expectations $d_t^* = 0$ are fulfilled.

Condition (50) implies that $q_t \equiv p_t^n/(1 + n)p_{t-1}^n$ is identically equal to 1. Thus, the sequence $(S_t)$ defined as in (34) diverges to infinity. Divergence of $(S_t)$ implies, by Lemma 1, that the max-min level of per capita net earnings $\pi_t$ which the intermediary can feasibly achieve is zero. Consequently, the solution set for the intermediary's optimization problem (32) consists of all feasible intermediation plans since, by definition, each such plan must yield at least zero per capita net earnings in each period $t \geq 1$. The stationary intermediation plan $b_t \equiv b^n$ is therefore an element of this solution set; for it is clearly a nondivergent plan, and it yields $\pi_t = [1 - q_t]b^n = 0$ in each period $t \geq 1$. Finally, since the intermediary making use of the plan $b_t \equiv b^n$ has zero net earnings in each period $t \geq 1$, he distributes zero per share dividends in each period $t \geq 1$, implying that the per share dividend expectations $d_t^* = 0$ are fulfilled. □

**PROOF OF THEOREM 3:** Suppose $v^* = (k_t^*, b_t^*: t \geq 1)$ is a sequence of values for $k_t$ and $b_t$ corresponding to a Pareto inefficient equilibrium for the Brokered Economy. As seen in the proof of Theorem 2, $v^*$ must solve the Diamond-Tirole state equations (12) and (13).

As established by Tirole (1985, Propositions 1 and 2, pp. 1504 and 1507), the solutions to the state equations (12) and (13) can be partitioned into two subsets: the Pareto efficient solutions which converge to the golden-rule equilibrium state vector $(k^n, b^n)$; and the Pareto inefficient solutions which converge to the bubbleless state vector $(\bar{k}, 0)$. It follows from Tirole's results that the Pareto inefficient solution $v^*$ must converge to $(\bar{k}, 0)$, hence the interest rate $r_t^*$ associated with $v^*$ must converge to $\bar{r} \equiv f'(\bar{k})$.

An assumption made by both Diamond (1965) and Tirole (1985), and maintained throughout the current paper, is that $\bar{r} < n$. Consequently, $r_t^* \to \bar{r}$ implies that there exists some
$T \geq 2$ and some $\epsilon > 0$ such that $r_t^* \leq r^* \equiv n - \epsilon$ for all $t \geq T$. Let $u_t \equiv [1 + r_t^*] /[1 + n]$ and $u \equiv [1 + r^*] /[1 + n]$.

Making use of the viability condition appearing in the definition for a Brokered Economy equilibrium, and the nonnegativity of equilibrium per share dividend expectations, it follows as in (48) that the (positive) stock share prices $p_t^o$ along the equilibrium path $v^*$ must satisfy

$$(51) \quad q_t \equiv p_t^o /(1 + n)p_{t-1}^o \leq u_t \leq u < 1$$

for all $t \geq T$. But the relation (51) implies that $(S_t)$ defined as in (34) is a bounded sequence. Specifically, for each $t > T + 2$, the nonnegative sequence term $S_t$ is bounded above by

$$(52) \quad S_t^* \equiv 1 + u + u^2 + \ldots + u^r + u^*[T - 1]c_{\text{max}},$$

where $r \equiv t - T$, and $c_{\text{max}}$ denotes the largest of the $T - 1$ products $c_s$, $s = 2, \ldots, T$, defined by $c_s \equiv [q_T \cdots q_s]$. Since $u^r \rightarrow 0$ as $r \rightarrow +\infty$, the sequence $(S_t^*)$ converges to $1/[1 - u]$. It then follows from Theorem 1 that the solution set $B(p^o)$ for the intermediary's optimization problem (32) is empty, which contradicts the assumption that $v^*$ is a Brokered Economy equilibrium.

It follows that all Brokered Economy equilibria must be Pareto efficient. □

PROOF OF COROLLARY 1: Tirole (1985, p. 1504) establishes for the Diamond-Tirole Economy that, given any capital-labor ratio $k > 0$, there exists a maximum feasible positive value $\hat{b} = \hat{b}(k)$ for per capita bubble asset holdings consistent with equilibrium. Starting with $(k, \hat{b})$ as initial conditions in any period $\tau \geq 1$, the unique solution to the stationary state equations (12) and (13) is a Pareto efficient Diamond-Tirole equilibrium path for $(k_t, b_t)$ over periods $t \geq \tau$ which converges to the golden-rule outcome $(k^n, b^n)$. Starting with $(k, b)$, $b \in [0, \hat{b})$, as initial conditions in any period $\tau \geq 1$, the unique solution to these state equations is a Pareto inefficient Diamond-Tirole equilibrium path for $(k_t, b_t)$ over periods $t \geq \tau$ which converges to $(\bar{k}, 0)$. 
As noted in the proof of Theorem 2, every Brokered Economy equilibrium path for \((k_t, b_t)\) must solve the state equations (12) and (13). However, as established in Theorem 3, only the Pareto efficient solution paths for these state equations are tenable equilibrium outcomes for the Brokered Economy. The proof of the first corollary statement then follows immediately once it is recalled that \(b_t = B_t / L_t\), where \(L_t = [1 + n]^t L_0\) denotes the number of agents born in period \(t\) for exogenously given values \(n \geq 0\) and \(L_0 > 0\).

Now let \(R_T\) denote the product of the interest factors \([1 + r_j]\) for periods \(j = 1, \ldots, T\). The tail term for periods \(t \geq T\) in the calculation of the fundamental value of the initial stock share price \(p_0^\circ\) is then given by

\[
\frac{p_T^\circ}{R_T^\circ} = \frac{p_T^\circ}{[1 + n]^T} \times \frac{[1 + n]^T}{R_T} = \frac{p_T^\circ}{[1 + n]^T} \times \frac{b_0}{b_T} = [(q_T \ldots q_1)b_0] \times \frac{p_0^\circ}{b_T}.
\]

The price \(p_0^\circ\) equals its fundamental value if and only if the tail term (53) converges to zero as \(T \to \infty\). However, it follows from previous observations that \(b_T\) must be positive and convergent to \(b^n > 0\) along any equilibrium path for the Brokered Economy. Consequently, the second corollary statement is also established.