Quantitative Measurement of Crack Parameters Using Microwave Eddy-Current Techniques

A. J. Bahr
SRI International

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The objectives of this work were to develop an electromagnetic scattering model that can be used to predict the microwave scattering from a crack in a metallic surface, and to evaluate the potential of using microwave scattering measurements to determine the dimensions of the crack. The initial approach to this problem has been to model the surface crack as a section of rectangular waveguide shorted at one end. Theoretical and experimental results are presented for this case, and the potential for obtaining a quantitative evaluation of the crack dimensions is discussed.

INTRODUCTION

In the low-frequency eddy-current testing of metals, currents are caused to flow in the test specimen by placing it in the magnetic field of an induction coil. The flow of currents is affected by the electrical properties, the shape of the test specimen, and the presence of discontinuities and defects. In turn, these currents react back on the exciting coil and affect its impedance. Thus, the presence of a defect is determined by monitoring the test coil impedance.

Such eddy-current tests are typically conducted at frequencies of less than 1 MHz when induction fields predominate and the electromagnetic wavelength is greater than 300 m. Because the wavelength of the interrogating fields determines sensitivity and resolution, the need to find and characterize smaller and smaller flaws suggests that the use of higher (microwave) frequencies may prove advantageous in eddy-current inspection. In using microwave frequencies, the radiation fields associated with the sensors become an important consideration, and the physics involved is best described in terms of fields and waves. For example, the effect of a defect should be thought of as producing a change in the scattering of electromagnetic waves from the metal surface. It should also be noted that, since the use of microwave frequencies causes the currents induced in the test object to flow essentially on the surface (i.e., the skin depth is typically less than 1 μm at 100 GHz), microwave eddy-current techniques are limited to surface inspection in metals.

The use of microwave frequencies in the range 10 to 30 GHz for the detection and characterization of thin slits and cracks in metal surfaces has been studied previously by several workers. Their results showed that good sensitivity to small cracks could be obtained. Cracks as small as 2 μm wide and 25 μm deep were detected. There was also a clear correlation between crack depth and scattered energy. In view of these encouraging results, work was initiated in SRI's Remote Measurements Laboratory to study the use of microwave eddy-current techniques at frequencies around 100 GHz. In addition, development of an electromagnetic scattering model for predicting the microwave scattering from a crack in a metallic surface was undertaken so that the potential of such measurements for determining the dimensions (particularly the depth) of a crack could be evaluated.

As a first approximation, a crack was modeled as a slot with a rectangular cross section and straight sides. This simplification permitted the development of an analytic theory for the scattering from such a slot, which in turn allowed studies of the relation between scattered power, crack dimensions, and frequency. The derivation of this analytic theory is outlined in Section II. To provide a test comparison for the theory, measurements were made of the scattering from small slits machine in an aluminum plate. These experimental results are presented in Section III, and Section IV summarizes the conclusions derived from this work.

ELECTROMAGNETIC SCATTERING FROM A RECTANGULAR SLOT

Consider a rectangular slot having length a, width b, and depth d. The geometry and coordinate system for this slot are shown in Fig. 1. The transmitted and received waves propagate in the half-space z > 0.

The steps that were followed in the analysis are shown in Fig. 2. The analysis is written in terms of a slot admittance, which is a parameter that depends mostly on the dimensions of the slot and on the frequency, and only slightly on the method of electromagnetic inspection.

Internal Fields—If b/a << 1, the fields in the slot can be expressed in terms of transverse electric (TE) waveguide modes having no variation in the y direction. Since the tangential electric field must be zero at the bottom of the slot, suitable expressions for the interior electric (E) and magnetic (H) fields are: 

\[ E_z = \frac{4\pi}{\lambda} \frac{d}{z} \sin \left( \frac{\pi d}{\lambda} z \right) \text{TE}^0 \]

\[ H_y = \frac{4\pi}{\lambda} \frac{d}{z} \cos \left( \frac{\pi d}{\lambda} z \right) \text{TE}^0 \]
EXPAND THE FIELDS INSIDE THE RECTANGULAR
CAVITY IN TERMS OF WAVEGUIDE MODES (TE
MODES ARE SUFFICIENT IF THE SLOT OPENING
IS LONG AND NARROW)

APPROXIMATE THE FIELDS IN THE SLOT OPENING
BY THE APPROXIMATE SOLUTION FOR A NARROW
RECTANGULAR APERTURE IN A METAL SCREEN

MATCH THE INTERNAL FIELDS TO THE FIELDS IN
THE SLOT OPENING SO THAT THE COEFFICIENTS
OF THE WAVEGUIDE MODES ARE EXPRESSED IN
TERMS OF THE MAGNITUDE OF THE FIELDS IN
THE SLOT OPENING

DEFINE AND CALCULATE A CAVITY ADMITTANCE
AND A RADIATION ADMITTANCE FOR THE SLOT

RELATE THE SCATTERED POWER MEASURED AT
THE RECEIVER TO THE TOTAL ADMITTANCE OF
THE SLOT

Aperture Field—If \( d/b > 1 \), the distribution of
electric field in the slot opening should be
accurately approximated by the solution for the
fields excited in a narrow aperture in a metallic
screen. An approximate expression for such an
aperture distribution is:

\[
E_y(x,z,k) = \frac{V_0}{b} \sin \left[ k \left( \frac{z}{2} - \frac{d}{2} \right) \right]
\]  

where \( V_0 \) is an undetermined frequency-independent
amplitude coefficient.

Amplitude Coefficients—By equating Equations
(1) and (5), and by using the orthogonality prop­
ties of the trigonometric functions, one finds
that for \( n \) odd,

\[
Q_n = \frac{V_0}{b} (-1)^{n-1} \frac{k a \cos (ka/2)}{(\Gamma_n^2) \sin \left( \Gamma_n \frac{d}{a} \right)}
\]  

and for \( n \) even,

\[
Q_n = 0
\]  

Cavity Admittance—As is shown in Appendix A,
a cavity admittance can be defined for the slot by
the expression

\[
Y_c = j \frac{\text{Im} \int_{-a}^{a} dx \int_{0}^{b} dy \left( \vec{E} \times \vec{H} \right)_{z=0} \cdot \hat{a}_z}{|V|^2}
\]  

where the definition of voltage

\[
V = -V_0 \left( \frac{ka}{4} \right) \left( \sin \frac{ka}{4} \right)^2
\]

is consistent with (5) and plane-wave excitation.
Substituting (1) and (2) into (7) gives

\[
|V|^2 Y_c = j \frac{\text{Im} \left( \frac{ab}{2k \eta_0} \right)}{\sum_{n=1}^{\infty} \Gamma_n Q_n a \sinh \left( \frac{\Gamma_n d}{a} \right) \cosh \left( \frac{\Gamma_n a}{4} \right)}
\]

Finally, substituting (6) and (8) into (9) and
truncating the series at three terms gives
\[ Y_c = -\frac{1}{2} \frac{a}{b} \frac{(ka)^3 \cos^2 (ka/2)}{n_0 \sin (ka/2)} \sum_{n=1,3,5} \frac{1}{\left( \tanh \left( \frac{\alpha}{n} \right) \right)^3} \tan \left( \frac{\beta}{n} \right) \]  

(10)

This expression holds for the frequency range \( 0 < ka < 3n \). Note that for \( ka > n \),

\[ \left( \frac{\alpha}{n} \right)^3 \tanh \left( \frac{\alpha}{n} \right) = \left( \frac{\beta}{n} \right) \tan \left( \frac{\beta}{n} \right) \]  

(11)

where \( \alpha = j \beta \).

5. Radiation Admittance--From Appendix A, the radiation admittance of the slot aperture can be written

\[ \frac{1}{2} |V|^2 \hat{V}_r = \frac{1}{2} \int_a^b dx \int_0^b dy \left( \frac{\hat{E} \times \hat{H}}{2} \right)_{z=0-} \cdot \hat{A} \]  

(12)

Assuming an aperture field of the form given in (5), Rhodes\(^5,6\) has calculated this radiation admittance for the case \( k_b << 1 \):

\[ \text{Re}(Y_r) = \frac{(ka)^2}{16 \pi \alpha^4 \sin^4 (ka/2)} \left[ \text{Cin} (ka) ight. \\
\left. + \left[ \text{Cin} (ka) - 1/2 \text{Cin} (2ka) \right] \cos ka \right. \\
\left. - \left[ \text{Si} (ka) - 1/2 \text{Si} (2ka) \right] \sin ka \right] \]  

(13a)

\[ \text{Im}(Y_r) = \frac{(ka)^2}{16 \pi \alpha^4 \sin^4 (ka/2)} \left[ \text{Si} (ka) \\
+ \left[ \text{Si} (ka) - 1/2 \text{Si} (2ka) \right] \cos ka \right. \\
\left. + \left[ \text{Cin} (ka) - 1/2 \text{Cin} (2ka) \right. \\
\left. \times \ln \left( \frac{e^{3/2} \alpha}{2b} \right) \sin ka \right] \right] \]

(13b)

where

\[ \text{Cin} (x) = \int_0^x \frac{1 - \cos u}{u} \, du \]

(14a)

and

\[ \text{Si} (x) = \int_0^x \frac{\sin u}{u} \, du \]  

(14b)

Equation (13) differs slightly from that given by Rhodes because of the differing definitions of voltage [see Eq. (A-6a)].

Cross-Polarization Scattering Coefficient--Assume that the slot is excited by an illuminating mode designated by subscript \( V \), and that it is desired to calculate the energy scattered into an orthogonal mode which is designated by subscript \( H \). In Appendix B it is shown that the relevant scattering coefficient is given by the expression

\[ \Gamma_{HV} = \frac{1}{4 P_s} \int_a^b dx \int_0^b dy \left( \frac{\hat{E}_V \times \hat{H}_H}{2} \right) \cdot \left( -\hat{a} \right) \]  

(15)

where \( P_s \) is the average power available from the source. Assuming plane-wave excitation and substituting (5) and (A-6) into (15), one obtains the result that

\[ \Gamma_{HV} = \frac{I_{HV}}{4 P_s} \]  

(16)

Since by definition,

\[ \hat{V}_V = \frac{I_V}{Y_s} \]  

(17)

one has

\[ \Gamma_{HV} = \frac{I_{HV}}{4 P_s Y_s} \]  

(18)

For the particular case of plane-wave backscattering, the currents can be written explicitly as

\[ I_H = 2 H_0 a \cdot \sin \theta \]

(19a)

and

\[ I_V = 2 H_0 a \cdot \cos \theta \]

(19b)

where \( H_0 \) is the magnitude of the magnetic field in the incident plane wave at \( z = 0 \), and \( \theta \) is the angle between \( E_V \) and the \( y \) axis. The incident magnetic field at the slot is related to the available power--i.e.,

\[ H_0^2 = K_p \frac{P_s}{s} \]  

(20)

where \( K_p \) must be determined by calibration, or perhaps by theory. Substituting (19) and (20) into (18) gives the final result

\[ \Gamma_{HV} = -\frac{1}{2} \frac{K_p a^2 \sin 2\theta}{Y_s} \]  

(21)

Cross-Polarized Scattered Power--The cross-polarized scattered power, \( P_{HV} \), is given by

\[ P_{HV} = |\Gamma_{HV}|^2 P_s \]  

(22)

Substitution of (21) into (22) gives

\[ P_{HV} = \frac{1}{4} a^4 \frac{K_p a^2 \sin^2 2\theta}{Y_s} P_s \]  

(23)

It is convenient to calculate the following normalized power:

\[ \left( P_{HV} / P_s \right) = \frac{1}{4} \frac{a^2 \sin^2 2\theta}{n_0 \frac{1}{2} |Y_s|^2} \]  

(24)
For the long-wavelength (Rayleigh) region \( (ka \ll 1) \) \(|T_c| \gg |T_s| \), so (24) becomes

\[
\frac{P_{HV}/P_s}{(1/a^2 b^2 \cos^2 \theta_0 R_s^2 \sin^2 2\theta)} = \frac{\sin^2(ka/4)}{(ka/\pi)^6} \tan^2(\pi d/a).
\]

Equation (24) is shown plotted in Fig. 3 as a function of normalized frequency, \( 0.35 \leq ka/\pi \leq 2.1 \), for \( a/b = 10 \) and \( 0.1 \leq d/a \leq 1 \). The resonant nature of the scattering is evident for \( ka/\pi > 1 \), particularly for the deeper slots. Resonance occurs when the net magnetic energy stored in the cavity balances the net stored electric energy outside the aperture. The difference in scattering cross section for slots of different depths is greatest in the resonance region. However, the cross section is a multivalued function of slot depth at a given frequency; consequently, a scattering measurement made at one frequency is ambiguous.

In the long-wavelength (Rayleigh) region, the scattering is much smaller than in the resonance region, but the scattering cross section is a monotonous function of slot depth. A plot of Eq. (25) which illustrates this behavior is shown in Fig. 4 for \( 0.01 \leq ka/\pi \leq 0.1 \) and \( 0.1 \leq d/a \leq 1 \). It can be seen from Fig. 4 that the scattering becomes insensitive to slot depth when the depth is about the same dimension as the slot length. This effect is a manifestation of the fact that the electromagnetic fields do not propagate inside the slot in this frequency region, and so they cannot fully penetrate a deep slot.

**EXPERIMENT**

Examination of the theory outlined in the previous section shows that if one knows the dimensions \( a \) and \( b \) of the slot cross section, and the calibration constant \( n_0 R_s \sin 2\theta \), it should be possible to determine the slot depth, \( d \), from a measurement of \( P_{HV} \). This possibility was tested using the microwave-backscatter measurement system whose schematic diagram is shown in Fig. 5. This system uses an orthomode coupler to discriminate against copolarized backscatter, and a superheterodyne detection system. The IF attenuator (IF frequency = 400 MHz) permits precision measurements to be made of changes in backscattered power. The antenna used was a lens-focused horn with a beamwidth at its focal point of about 3.5 mm at the operating frequency of 100 GHz.

An aluminum plate with six slots of different sizes electrodischarge machined into its surface was prepared according to the layout shown in Fig. 6. Slots 1, 2, and 3 have a cross section \((a \times b)\) of 0.100 in. \( \times \) 0.010 in. (2.5 mm \( \times \) 0.25 mm); slots 4, 5, and 6 have a cross section of 0.050 in. \( \times \) 0.010 in. (1.25 mm \( \times \) 0.25 mm). Thus,

* This test plate was prepared under the direction of Dr. O. Buck of the Rockwell International Science Center, Thousand Oaks, California.
Fig. 5 Microwave-backscatter measurement system

Fig. 6 Layout of slotted aluminum plate
(Slots are aligned in the X direction)

\( \frac{a}{b} = 10 \) for the first set of slots, and \( \frac{a}{b} = 5 \) for the second set of slots. Also, at 100 GHz, \( \frac{k}{\lambda} = 1.7 \) for the first set, and \( \frac{k}{\lambda} = 0.85 \) for the second set. Finally, slots 1 and 4 were specified to be 0.010 in. (0.25 mm) deep, slots 2 and 5 0.020 in. (0.5 mm) deep, and slots 3 and 6 0.040 in. (1 mm) deep.

Actually, the electro-discharge machining process does not produce slots with a flat bottom, nor can the desired depth be achieved perfectly. To see the shapes of the slots and measure their depths, replicas of the slots were made by filling the slots with a rubbery compound (Dow-Corning RTV-Type E) and peeling it off the plate after the compound had set. Scanning-electron micrographs of the replica of slot no. 1 are shown in Fig. 7.

(a) 19X
(b) 100X

Fig. 7 Scanning-electron micrographs of
Dow-Corning RTV (Type E) replica of slot No. 1
\( a = 2.5 \) mm, \( b = 0.25 \) mm, \( d - 0.25 \) mm

The roundness of the bottom of the slot can be clearly seen. By measuring the image in the photograph showing the edge view of the slot [Fig. 7(b)], the actual depth of the slot (for example, to its deepest point) can be determined. Similar scanning-electron micrographs for slot no. 3 (the largest slot) are shown in Fig. 8. In this case, the depth of the slot was found to be 10% larger than specified. All of the slots were examined in this way.

(a) 18X
(b) 50X

Fig. 8 Scanning-electron micrographs of
Dow-Corning RTV (Type E) replica of slot No. 3
\( a = 2.5 \) mm, \( b = 0.25 \) mm, \( d = 1.12 \) mm
The measured cross-polarized power backscattered from the slots (the slots were oriented at 45° to the incident electric field) is shown as a function of beam position in Figs. 9 and 10 at its focal plane using an open-ended W-band waveguide sampling antenna, and this pattern is shown superimposed in Fig. 9 for comparison. The beam pattern appears about twice as large as actual.

**Fig. 9** Cross-polarized backscattered power from slotted aluminum plate versus beam position

**Fig. 10** Cross-polarized backscattered power from slotted aluminum plate versus beam position

(Gain increased by a factor of 4 over that used on Fig. 9)

Figure 10 is the same as Fig. 9 except that the gain in Fig. 10 was increased by a factor of 4, so that the magnitude of the signal-to-clutter ratio would be more evident. Also, the beam pattern of the focused interrogating beam was measured size because it is produced by convolution with the relatively large sampling antenna. In any event, since the actual beamwidth is larger than the slot cross-sectional dimensions, the microwave C scan does not reproduce the detailed shape of the slot.
cross section. Of course, this is the desired situation when one wants to deduce the slot depth from a single measurement of scattering cross section.

A quantitative measure of the scattering cross section for five of the six slots was obtained by determining the peak value of the backscattered power produced by these slots relative to that produced by the largest slot (slot 3). In other words, slot 3 was used as a reference scatterer or "standard." Photographs showing peak relative backscattered power versus beam position for all six slots are shown in Fig. 11. The relative values of the peak amplitudes are indicated in the figure. Figure 11(b) was obtained with the gain increased by 7 dB over that used to obtain Fig. 11(a). It can be seen in Fig. 11(b) that the smallest slot (slot 4) produces a signal whose amplitude is on the order of that of the clutter signal. These data were taken without expending any effort to improve the signal-to-clutter ratio for the system.

A comparison between theory and experiment is shown in Fig. 12. Since the objective of the measurement is to determine slot depth (assuming a and b are known), the normalized slot depth is plotted as a function of normalized backscattered power (relative to slot 3). Figure 12(a) shows the above-resonance case where \( \frac{ka}{\pi} = 1.7 \) (slots 1 and 2). The multivalued nature of the theoretical curve precludes obtaining an unambiguous value for slot depth in this frequency times slot length range. However, in this experiment the slot depths are known, so the experimental data can be plotted, as in the figure, for comparison with the theory. The measured values (including clutter) are shown connected by a dashed line. The measured values of backscattered power are about 2 dB larger than the theoretical values, but the slope of data agrees well with theory.

In general, the clutter is composed of a combination of incoherent (noise) signals and coherent leakage signals. Sources of coherent leakage signals are located both inside and outside the measurement system. The worst case occurs when all of the clutter is coherent, since the clutter signal can then either add to or subtract from the
desired signal. The degree of coherence in the measured clutter is not known, but the corresponding uncertainty in the measured data that would occur if the clutter observed in this case [see Fig. 11(b)] were entirely coherent is indicated in Fig. 12 by the error bars drawn through the measured data points. It can be seen that the resulting uncertainty is quite large. On the other hand, if the measured clutter were composed entirely of noise, the true values of the measured backscattered power would only be slightly smaller than the observed values. It is clear from these results that coherent clutter should be minimized as much as possible.

The below-resonance case, where \( \frac{ka}{\pi} = 0.85 \) (slots 4, 5, and 6), is shown in Fig. 12(b). In this case, the theoretical curve is not multivalued, so there is a one-to-one relationship between scattered power and slot depth. Again, the measured data are shown connected by a dashed line, and the error bars show the uncertainty in the measurement that would exist if the observed clutter were coherent. The discrepancy between theory and experiment is larger in this case; the measured values of backscattered power are about 7 dB larger than the theoretical values. This discrepancy could be the result of inaccuracies in the theory caused by a failure to satisfy the approximation conditions (e.g., \( a/b \) is too small, the bottom of the slot is rounded, etc.), as well as being caused by measurement inaccuracies produced by the lower signal-to-clutter ratios associated with these smaller slots. It is interesting to note, however, that, again, the slope of the data agrees well with theory. In any event, the discrepancy in this case is large enough to prevent the determination of slot depth to reasonable accuracy using the measured data and theory described in this paper. The quantitative determination of slot depth from microwave backscatter measurements will require improvements in both theory and measurement technique.

IV SUMMARY

An approximate theory for the electromagnetic cross-polarized power backscattered from a rectangular slot has been developed. This theory, though approximate, reveals all of the essential features of the backscattering as a function of frequency and slot dimensions. For example, when the slot length is greater than one-half wavelength, resonances can occur, and this situation makes the determination of slot depth from measured backscattered power ambiguous. For smaller slot lengths, there is a one-to-one relation between slot depth and backscattered power, but the amplitude of the backscattered energy is smaller and slot depths that are greater than one slot length are not well resolved.

Experimental results obtained using electro-discharge machined slots in an aluminum plate were found to be in reasonable agreement with theory for the above-resonance case, with a slot length-to-width ratio of 10. Results obtained with smaller slots that had a smaller length-to-width ratio did not agree as well with the theory, except that the slope of the data did follow that of the theoretical curve. It was also found that signal-to-coherent-clutter power ratios of less than three can produce large uncertainties in the measurement of slot depth, particularly if the backscatter is small, as in the below-resonance case.

It can be concluded that the potential of obtaining a quantitative determination of crack depth from measurements of microwave backscatter in the below-resonance region does exist, but that further work is needed to improve the theory and the measurement technique. In particular, the theory should be extended to account for realistic crack geometries, probably by using finite-element techniques, and a measurement technique should be developed that minimizes the observed clutter level.

APPENDIX A

Slot Admittance—Consider an infinitesimal volume, \( \Delta V \), whose cross-sectional dimensions \( a \) and \( b \) are the same as those of the slot, and that extends a depth, \( \Delta z \), into the slot. Application of the Poynting energy theorem to this volume gives

\[
\frac{1}{\Delta V} \int_{\Delta V} \mathbf{E} \times \mathbf{H} \, dz = \frac{1}{2} \int_{\Delta V} \mathbf{E}^2 + \mathbf{H}^2 \, dx \, dy
\]

Here \( \Delta z \) is a unit vector pointing along the positive \( z \) axis, \( \omega \) is the radian frequency, \( \mu_0 \) and \( \epsilon_0 \) are the permeability and permittivity of free space, respectively, and \( \mathbf{J} \) is the current density driving the fields inside the volume. The physical meaning of Eq. (A-1) can be stated in words:

The complex power radiated from the slot plus the complex power entering the slot equals the negative time rate of change of the difference between magnetic and electric energies stored in \( \Delta V \) plus the complex power delivered to \( \Delta V \) by the driving fields.

The driving fields are those fields that exist in the absence of the slot. For simplicity, let the incident wave be plane and have an \( x \) component of magnetic field equal to \( H_{ox} \). The magnetic field due to this wave at the unslotted metal surface is equal to \( 2H_{ox} \) because the wave is completely reflected. Hence the effective driving current is

\[
\mathbf{J} = 2\mathbf{a}_z \times \mathbf{a}_x \mathbf{H}_{ox} \delta(z + \frac{a}{2}) = 2\mathbf{a}_z \mathbf{H}_{ox} \delta(z + \frac{a}{2})
\]

where \( \delta(z) \) is the Dirac delta function. One may think of a current generator \( I \) driving the slot to produce a voltage \( V \) across the slot such that

\[I = 0, \quad V = 0\]

It is assumed that the conductivity of the metal walls is infinite.
Substituting (A-2) and (5) into (A-3), one obtains

\[
\frac{1}{2} V^* = \frac{1}{2} \int \int \int \mathbf{E} \cdot \mathbf{E} \, dV
\]

(A-3)

Substituting (A-2) and (5) into (A-3), one obtains

\[
\frac{1}{2} V^* = \frac{1}{2} \int_a^b \int_0^b \int_0^d 0 \, dx \, dy \, dz
\]

(A-4)

\[
2H_{0x}^* \delta \left( z + \frac{a}{2} \right) \frac{V}{b} \sin \left( k \frac{a}{2} - \frac{x - a}{2} \right)
\]

or

\[
\frac{1}{2} V^* = \frac{1}{2} \left[ \frac{-V}{b} \left( \frac{\sin \frac{ka}{4}}{\frac{ka}{4}} \right) \right] \left[ 2H_{0x}^* a \right]
\]

(A-5)

Hence, by definition,

\[
V = -V_0 \left( \frac{\sin \frac{ka}{4}}{\frac{ka}{4}} \right)^2
\]

(A-6a)

and

\[
I = 2H_{0x} a
\]

(A-6b)

The definition of slot admittance, \( Y_s \), follows immediately in the usual way, viz.,

\[
Y_s = I/V
\]

(A-7)

Now let \( \Delta V \to 0 \) so that the stored-energy term in (A-1) goes to zero. Using (A-3) and (A-7), (A-1) then becomes

\[
\frac{1}{2} |V|^2 Y_s^* = \frac{1}{2} \int_a^b \int_0^b \int_0^d \left( \mathbf{E} \times \mathbf{H}^* \right)_{z=0-} \, dz
\]

(A-8)

\[
+ \frac{1}{2} \int_a^b \int_0^b \int_0^d \left( \mathbf{E} \times \mathbf{H}^* \right)_{z=0-} \, dz
\]

or by definition

\[
\frac{1}{2} |V|^2 Y_s^* = \frac{1}{2} |V|^2 Y_r^* + \frac{1}{2} |V|^2 Y_c^*
\]

(A-9)

Here \( Y_r \) is the radiation admittance of the aperture and \( Y_c \) is the cavity admittance. \( Y_r \) has been calculated for a narrow rectangular aperture by Rhodes.\(^5,6\) Since the cavity is assumed lossless, the cavity admittance can be written as

\[
Y_c = j B_c
\]

(A-10)

\[
j \text{Im} \int_a^b \int_0^b \int_0^d \left( \mathbf{E} \times \mathbf{H}^* \right)_{z=0-} \, dz
\]

\[
= \frac{1}{|V|^2}
\]

It is the computation of this cavity admittance with which this paper is concerned.

**APPENDIX B**

Cross-Polarization Scattering Coefficient\(^*\)--Let \( \mathbf{E}_V, \mathbf{H}_V, \mathbf{H}_H \) be two independent sets of solutions to Maxwell's equations for the interior of a closed surface \( S \). In particular, the \( \"V\" \) fields exist in the presence of a crack in the test piece being inspected, and the \( \"H\" \) fields exist when no crack is present. The geometry for this situation is depicted in Fig. B-1. In addition, the \( V \) and \( H \) fields are orthogonal within the transmit/receive waveguide (e.g., for linear polarization, they are cross polarized). Since \( S \) is assumed to be devoid of sources, the Lorentz reciprocity theorem\(^3\) can be written as

\[
\oint (\mathbf{E}_H \times \mathbf{H}_V - \mathbf{H}_V \times \mathbf{E}_H) \cdot \mathbf{n} \, dS = 0
\]

(B-1)

where \( \mathbf{n} \) is a unit vector that points out of the volume enclosed by \( S \). Now, the tangential electric field is zero on all metallic surfaces, including the crack-mouth surface, \( S_c \), when no crack is present. Hence, (B-1) becomes

\[
\int \int \left( \mathbf{E}_H \times \mathbf{H}_V - \mathbf{H}_V \times \mathbf{E}_H \right) \cdot \mathbf{n} \, dS = 0
\]

(B-2)

\[
\int \int \left( \mathbf{E}_V \times \mathbf{H}_H \right) \cdot \mathbf{n} \, dS = 0
\]

\[
\int \int \left( \mathbf{E}_V \times \mathbf{H}_H \right) \cdot \mathbf{n} \, dS = 0
\]

**Fig. B-1** Geometry used in deriving cross-polarization scattering coefficient

\( * \) This derivation was provided by B. A. Auld of Stanford University.
\[ \mathbf{E} = j\omega \mathbf{r} \times (\mathbf{t} \times \mathbf{A}) \]  
\[ \mathbf{H} = -j \frac{\omega}{\eta_0} (\mathbf{t} \times \mathbf{A}) \]

where \( \mathbf{A} \) is a vector potential function and \( \mathbf{t} \) is unit vector pointing from the origin of coordinates to the point where the fields are calculated. Using (B-3) it is easy to show that

\[ \mathbf{\check{E}}_H \times \mathbf{\check{H}}_V = \mathbf{\check{E}}_V \times \mathbf{\check{H}}_H \]  
\[ \text{on } S_w, \]  
and hence the integral over \( S_w \) is zero.

Now consider the surface, \( S_w \), that cuts through the waveguide. The fields there can be written in terms of a wave traveling outward from the source and complex scattering (reflection) coefficients, \( \Gamma \), that are a measure of the waves being scattered back to the source. The expressions for the fields on \( S_w \) are:

\[ \mathbf{\check{E}}_H = (1 + \Gamma_{HH}) a_H^{++} \mathbf{H}_H^+ \]  
\[ \mathbf{\check{H}}_H = (1 - \Gamma_{HH}) a_H^{+\dagger} \mathbf{H}_H^+ \]  
(8-5a)

\[ \mathbf{\check{E}}_V = (1 + \Gamma_{VV}) a_V^{++} \mathbf{V}_V^+ \]  
\[ \mathbf{\check{H}}_V = (1 - \Gamma_{VV}) a_V^{+\dagger} \mathbf{V}_V^+ \]  
(8-5b)

and

\[ \mathbf{\check{E}}_V = (1 + \Gamma_{HV}) a_{HV}^{++} \mathbf{V}_H^+ \]  
\[ \mathbf{\check{H}}_H = (1 - \Gamma_{HV}) a_{HV}^{+\dagger} \mathbf{V}_H^+ \]  
(8-6a)

\[ \mathbf{\check{E}}_H = (1 + \Gamma_{VH}) a_{ VH}^{++} \mathbf{V}_H^+ \]  
\[ \mathbf{\check{H}}_V = (1 - \Gamma_{VH}) a_{ VH}^{+\dagger} \mathbf{V}_H^+ \]  
(8-6b)

where \( a_H, a_V \) are the amplitudes of the waves emanating from the source. Without loss of generality, let \( a_H = a_V = a \). Substituting (8-5) and (8-6) into the integral over \( S_w \) and using the orthogonality of the \( H \) and \( V \) modes gives

\[ \int \int \left( \mathbf{\check{E}}_H \times \mathbf{\check{H}}_V - \mathbf{\check{E}}_V \times \mathbf{\check{H}}_H \right) \cdot \mathbf{n} \, dS \]
\[ = \int \int -2 \Gamma_{HV} a^2 \left( \mathbf{\check{E}}_H^+ \times \mathbf{\check{H}}_V^+ \right) \cdot \mathbf{n} \, dS \]  
\[ \text{on } S_w \]  
(8-7)

Since the modes are assumed to be propagating in the waveguide, \( a, \mathbf{a}_H, \) and \( \mathbf{a}_H^\dagger \) can be chosen to be real and so

\[ \int \int a^2 \left( \mathbf{a}_H^\dagger \times \mathbf{a}_H^+ \right) \cdot \mathbf{n} \, dS = 2 P_s, \]  
(8-8)

where \( P_s \) is the average power available from the source. Substitution of (8-7) and (8-8) into (8-2) gives the final result for the cross-polarization scattering coefficient:

\[ \Gamma_{HV} = \frac{1}{4 P_s} \int \int \left( \mathbf{\check{E}}_V \times \mathbf{\check{H}}_H \right) \cdot \mathbf{n} \, dS \]  
(8-9)

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REFERENCES


DISCUSSION

Jerry Tiemann (General Electric): I'm going to ask two or three questions. The first question is: What is the cause of the coherent clutter?

Alfred J. Bahr (Stanford Research Institute): The major cause in this particular system was leakage through the orthomode coupler and that is being worked on now. It is a matter of tuning that coupler to improve its isolation. There is a second cause that could be external, and that would be curvature of the specimen.

Jerry Tiemann: Okay, that brings me to the question as to whether the accuracy of alignment causes clutter. The other one is: Do you think it would be practical to apply this to the kind of engine disk contours that are found which are, in fact, multi-curved?

Alfred J. Bahr: The alignment is not too difficult using the focused horn. If you use a near field probe like an open ended wave guide, or any near field probe, it is more difficult. In the case of ceramics, which we have also looked at, it is a very big problem to align the receiver and the transmitter in the transmission measurement. In the back scatter measurement, the depth of field is several wave lengths and you can level your sample.

I don't have a good feel for the level of clutter caused by curvature yet. It's something you could calculate, but I haven't done it and so I'm not sure what limitations exist relative to real world geometries other than to say that probably the curvature would have to be small compared to the wave lengths. I don't have any better answer than that. That is an important question.

Harish Dalal (SKF Industries): Is the curvature that you mentioned the curvature of the sample?

Alfred J. Bahr: Yes.

Harish Dalal: The question I have relates to ceramics. Do you have any feel for the limit of the depth of flaw that you can detect using this technique?

Alfred J. Bahr: The depth of what, sir?

Harish Dalal: The depth of a crack. What is the minimum depth you might be able to detect?

Alfred J. Bahr: In ceramics, I don't know yet. We can detect voids in ceramics that are smaller than a wave length.

William Lord, Chairman (Colorado State University): Before we adjourn I would like to make one final comment on this afternoon's session. I think the speakers have shown that there are some exciting things happening in electromagnetic methods of non-destructive testing, both with regard to the development of new or improved techniques and in the modeling of field defect interactions.

I have had a long standing interest in the prediction of magnetic fields in ferromagnetic materials and over the years I've attended many of the ARPA/AFML conferences. I must say, I have been rather envious of the amount of progress that has been made in the ultrasonics area as part of this program. I've always felt that the research philosophy which was developed in this program was directly applicable to electromagnetic methods of nondestructive testing as well, including the more traditional leakage field and residual methods. Perhaps the time is ripe for someone to coordinate all of these isolated activities into one coherent program.