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AN ANALYTICAL MODEL OF EDDY CURRENT FERRITE-CORE PROBES

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ABSTRACT. An analytical model of an axisymmetric eddy current probe with a cylindrical ferrite core above a layered conductive half-space is developed. Initially we consider the magnetic vector potential of a circular filament coaxial with a ferrite core over a layered conducting half-space. The principle of superposition is then used to derive close-form expressions for both the electromagnetic field and the impedance of a coil from the filament field. Rather than locating the probe in infinite space, it is confined coaxially within a circularly cylindrical boundary on which the vector potential field is zero. The radius of this artificial boundary is large in order to ensure that does not interfere substantially with the field near the probe. By using a truncated region in this way, the vector potential in the probe region can be expanded as a series rather than an integral form. Thus the solution of the problem amounts to finding the expansion coefficients in the series. The numerical predictions of probe impedance have been compared with experimental data showing good agreement.

Keywords: Eddy Current, Ferrite Core
PACS: 41.20.Gz

INTRODUCTION

Eddy current testing is widely applied for the detection of surface cracks and subsurface flaws in conducting materials. Besides, eddy current probes have been used to determine the nonconductive coating thickness and physical properties of coated metals. A cylindrical ferrite core encircled by a coaxial rectangular cross-section coil is usually used to confine and enhance the magnetic field. The core can also be used to improve the signal-to-noise ratio, which is of particular benefit to measure magnetic substrate metals at relatively low frequencies [1]. The field of an air-cored probe above a single or layered conducting half-space has been studied in depth by using a number of approaches such as the analytical Dodd and Deeds model [2]. For the problem of a ferrite-cored coil above a layered half-space, numerical [3] and empirical [4] methods have been applied. In addition a semi-analytical model of an axisymmetric ferrite-cored probe in the presence of a two layered conductive half-space has been developed using truncated region eigenfunction expansions (TREE) method [5].

Here, the TREE method is used to derive the field and impedance of the axially symmetric ferrite core coil with rectangular cross-section above a multi-layered conductive half space, Figure 1. Rather than making a direct analysis on the full problem...
as before [5], the present development starts with a simple case, studying the field of a circular filamentary coil encircling an infinite long cylindrical ferrite core. Then we deal with more complicated cases via intermediate results. This step-by-step approach leads to expressions for the field and impedance of the probe including the contributions of the coil, the effects of the ferrite core and the multi-layered conductor separately.

**ANALYSIS**

An infinite circularly cylindrical ferrite core encircled by a coaxial current filament, Figure 2, has a magnetic vector potential outside of the ferrite core given by

\[ A_\phi (z, \rho) = \mu_0 I \left \langle F_1 (q \rho) \left [ \frac{1}{2} q^{-1} e^{-q |z - z_0|} D^{-1} \right ] \rho_0 F_1 (q \rho_0) \right \rangle . \]  

(1)

in a domain truncated radially at \( \rho = b \), where \( A_\phi (z, b) = 0 \). The vector Dirac notation, viewed from right to left, consists of a column vector multiplied by a matrix and by a row vector to form a scalar product. \( F_1 (q \rho) \) is defined by \( F_n (q \rho) = A_c (qa) J_n (q \rho) - B_c (qa) Y_n (q \rho) \) with \( n = 0, 1 \). To ensure \( A_\phi (z, b) = 0 \), at the truncation surface, we define \( q_i \) as the positive roots of \( F_1 (q_i b) = 0 \). From the continuity of \( B_\rho \) and \( H_z \) at the ferrite core surface where \( \rho = a \), it is found that

\[ A_c (q_i a) = \frac{\pi q_i a}{2} \left [ \mu_r J_1 (q_i a) Y_0 (q_i a) - J_0 (q_i a) Y_1 (q_i a) \right ] \]  

(2)

and

\[ B_c (q_i a) = \frac{\pi q_i a}{2} (\mu_r - 1) J_0 (q_i a) J_1 (q_i a) . \]  

(3)
The diagonal matrix, $D$, is determined by the discontinuity of $H_\rho$ at the plane $z = z_0$.

To find the field of a finite length cylindrical ferrite core encircled by a filamentary coil above a layered conducting half-space, Figure 3, the problem space is divided into five regions at the end planes of the ferrite core and at the interfaces of the layered conductor. The vector potential in the region (1) outside of the core can be written

$$A^{(1)}(z, \rho) = \mu_0 I \left\langle F_1(q\rho) \left| \frac{1}{2} \left[ e^{-q|z-z_0|} + e^{-qz} \right] \right| D^{-1} \left| \rho_0 F_1(q\rho_0) \right. \right\rangle. \quad (4)$$

Here $\alpha$ and $\beta$ are the reflection matrices due to the end effects of the ferrite core, which have the relationship:

$$\alpha = R \left( e^{-qz_0} + e^{-ql} \beta \right) \quad (5)$$

and

$$\beta = R_0 \left[ e^{-q(l-z_0)} + e^{-ql} \alpha \right]. \quad (6)$$

For a coil with finite width $r_2 - r_1$, finite length $z_2 - z_1$ and having $N$ turns, Figure 1, the factor $\nu = N / [(r_2 - r_1)(z_2 - z_1)]$ is the coil turns density. The magnetic vector potential $A(z, \rho)$ in the rectangular region of the coil cross-section is found by integrating the filament potential over the same region. Thus by integrating equation (4) we get

$$A(z, \rho) = \nu \int_{z_1}^{z_2} \int_{r_1}^{r_2} A^{(1)}(z, \rho) d\rho_0 dz_0$$

$$= \frac{\mu_0 I \nu}{2} \left\langle F_1(q\rho) \left| q^{-1} \left[ 2I - e^{q(z-z_1)} - e^{q(z-z_2)} \right] + e^{q(z-l)} G_1^{-1} \left[ e^{qz_1} - e^{qz_2} \right] + R_0 e^{-q\ell} \left( e^{qz_2} - e^{qz_1} \right) \right| q^{-3} D^{-1} \left| \chi_F(qr_1, qr_2) \right. \right\rangle, \quad (7)$$

where

$$\chi_F(s_1, s_2) = \int_{s_1}^{s_2} x F_1(x) dx, \quad (8)$$

which can be calculated in terms of Struve and Bessel functions [6]. Then, the expression of the coil impedance can be found by integrating the filament potential over the same region.

---

**FIGURE 3.** Filamentary current loop coil encircling a finite length cylindrical ferrite core above a layered conductive half-space.
section to give:

\[
Z = \frac{2\pi \mu \nu}{I} \int_{z_1}^{z_2} \int_{r_1}^{r_2} A(z, \rho) \rho d\rho dz \\
= j\omega \mu_0 \nu^2 \left\{ 2 (z_2 - z_1) q + 2 I - 2 e^{q(z_2 - z_1)} \right\} \\
+ (e^{q_2} - e^{q_1}) e^{-q} G_1^{-1} \left[ e^{q_1} (e^{-q_2} - e^{-q_2} z_1) + R_0 e^{-q} (e^{q_2} - e^{q_1}) \right] \\
+ (e^{-q_1} - e^{-q_2}) G_2^{-1} \left[ R_0 (e^{-q_1} - e^{-q_2} z_1) + (e^{q_2} - e^{q_1}) \right] q^{-3} D^{-1} \frac{dF}{dz} (q_1, q_2). \tag{9}
\]

The matrices appearing above are defined through:

\[
G_1 = I - R_0 e^{-q} Re^{-q}, \tag{10}
\]

\[
G_2 = I - Re^{-q} R_0 e^{-q}, \tag{11}
\]

\[
R = [(U + V) + \Gamma (U - V)]^{-1} [(U - V) + \Gamma (U + V)], \tag{12}
\]

\[
D = [d_{ij}], d_{ij} = \mu_r \int_0^a J_1 (q_i \rho) J_1 (q_j \rho) \rho d\rho + \int_a^b F_1 (q_i \rho) F_1 (q_j \rho) \rho d\rho, \tag{14}
\]

\[
U = [u_{ij}], u_{ij} = \int_0^a J_1 (\kappa_i \rho) J_1 (q_j \rho) \rho d\rho + \int_a^b J_1 (\kappa_i \rho) F_1 (q_j \rho) \rho d\rho, \tag{15}
\]

\[
V = [v_{ij}], v_{ij} = \mu_r \int_0^a J_0 (\kappa_i \rho) J_0 (q_j \rho) \rho d\rho + \int_a^b J_0 (\kappa_i \rho) F_0 (q_j \rho) \rho d\rho, \tag{16}
\]

where \( \kappa_i, i = 1, 2 \ldots \infty \) denotes the positive zeros of \( J_1(\kappa_i b) = 0 \). The reflection coefficient, a diagonal matrix \( \Gamma \), due to the layered conducting half-space is defined by considering the field at its top surface, in this case at the plane \( z = -d_1 \). At this point one is free to determine the reflection coefficient for any multi-layered conductor whose material properties are independent of \( \rho \) [7].

**EXPERIMENT**

To verify the analytical results in the paper, experiments have been designed and completed with different configurations of layered test-pieces and an eddy current coil with a ferrite core. Their dimensions are shown in Tables 1 and 2 respectively. The experimental results are measured by an Agilent 4294A Precision Impedance Analyzer.

**TABLE 1.** Parameters of the coil and ferrite core. Unit of dimensions is mm and uncertainty is 0.01 mm.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Ferrite core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner diameter 2r₁</td>
<td>Diameter 2a</td>
</tr>
<tr>
<td>Outer diameter 2r₂</td>
<td>Length ( l )</td>
</tr>
<tr>
<td>Length ( z_2 - z_1 )</td>
<td>Liftoff ( d_1 )</td>
</tr>
<tr>
<td>Offset ( z_1 )</td>
<td>Rel. permeability ( \mu_r )</td>
</tr>
<tr>
<td>Number of turns ( N )</td>
<td></td>
</tr>
<tr>
<td>DC resistance</td>
<td></td>
</tr>
<tr>
<td>10.05</td>
<td>7.76</td>
</tr>
<tr>
<td>20.40</td>
<td>29.95</td>
</tr>
<tr>
<td>18.18</td>
<td>0.59</td>
</tr>
<tr>
<td>2.65</td>
<td>250</td>
</tr>
<tr>
<td>776</td>
<td>8.09 Ω</td>
</tr>
</tbody>
</table>
TABLE 2. Parameters of metal plates. Conductivity $\sigma$, thickness $T$ and lateral dimensions $w \times d$. Uncertainty in thickness is 0.01 mm and in lateral dimensions is 1 mm.

<table>
<thead>
<tr>
<th>Metal</th>
<th>$\sigma$ (MS/m)</th>
<th>$T$ (mm)</th>
<th>$w \times d$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass (C2600)</td>
<td>16.42±0.09</td>
<td>5.66</td>
<td>615×616</td>
</tr>
<tr>
<td>Ti (Ti-6Al-4V)</td>
<td>0.58±0.01</td>
<td>12.47</td>
<td>318×331</td>
</tr>
</tbody>
</table>

TABLE 3. Skin depths of Brass (C2600) and Ti (Ti-6Al-4V) at different frequencies, unit (mm).

<table>
<thead>
<tr>
<th>Metal</th>
<th>100 (Hz)</th>
<th>500 (Hz)</th>
<th>1 (kHz)</th>
<th>3 (kHz)</th>
<th>5 (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass (C2600)</td>
<td>12.42</td>
<td>5.55</td>
<td>3.93</td>
<td>2.27</td>
<td>1.76</td>
</tr>
<tr>
<td>Ti (Ti-6Al-4V)</td>
<td>66.09</td>
<td>29.55</td>
<td>20.90</td>
<td>12.07</td>
<td>9.35</td>
</tr>
</tbody>
</table>

Besides, the skin depths of the test pieces at different frequencies are given in Table 3. The measured and calculated DC inductances of the isolated coil are 4.149 mH and 4.155 mH, respectively. Those for the ferrite-cored coil are 19.27 mH and 19.32 mH, respectively.

Two-layer Case

Since the skin depth decreases greatly with the frequency increment, to verify the model for a two-layer case, a large thin brass C2600 plate and a thicker titanium alloy (Ti-6Al-4V) plate are used and in both cases air is as the bottom layer. The experimental data are compared with the theory curves in Figure 4 and 5. In these figures, the good agreement shows that the experiment results can be accurately predicted by the two-layer model, which can be used to determine the thickness or conductivity of a thin metal plate. Moreover, in Figure 4, there are obvious differences between the theory results based on the half-space and two-layer models at relative low frequencies. On the contrary, in Figure 5 the difference can be seen at relative high frequencies. These are due to the skin depths at different frequencies and thickness of the brass and titanium plates. For the brass plate, when the frequency is higher than 1kHz, the skin depth becomes much smaller compared with its thickness. Then it can be treated as a uniform half-space. As for the titanium plate, for the whole frequency range of the experiment, the skin depths are similar or even much bigger than the thickness of the titanium plate. However, since the signal at the low frequency region is too weak for the differences to be observed, it is only obvious at the high frequency region.

CONCLUSION

In this paper, the TREE method for a ferrite-cored coil above a multi-layered conductive half-space has been studied. Reflection coefficient matrices due to the end effects of the ferrite core have been introduced. Besides, the field and impedance of a rectangular cross-section coil with a finite length ferrite core above a multi-layered conducting plate are developed. Furthermore, this can be extended to other axisymmetric ferrite core shapes, such as the cup shape core, etc.
FIGURE 4. Impedance change as a function of frequency for an air-cored coil ($\mu_r=1$) and ferrite-cored coil ($\mu_r=250$) due to the brass C2600 plate. Circles are experiment data, solid lines are theory results of the two-layer model and dash lines for half-space model.

FIGURE 5. Impedance change as a function of frequency for an air-cored coil ($\mu_r=1$) and ferrite-cored coil ($\mu_r=250$) due to the titanium alloy(Ti-6Al-4V) plate. Circles are experiment data, solid lines are theory results of the two-layer model and dash lines for half-space model.

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