Resonances and Crack Roughness Effects in Surface Breaking Cracks

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This study deals with two different aspects of scattering from the cracks. The first one is the measurement of crack resonances by local probes and the second is the effect of the roughness of the crack forces on the scattering data.

Unlike the scattering measurement reported elsewhere, these measurements of crack resonances are to be performed by local probes. Two different types of probes are described and their problems are discussed.

The effect of crack roughness on the scattering data is investigated via the reciprocity relation. The crack roughness is analyzed using perturbation theory and the equivalent boundary condition concept developed by Brekhovskikh is used to express the fields in the presence of the roughness. The effect of roughness on the scattering data is then discussed in qualitative terms.

**INTRODUCTION**

In our last year's report [1], we proposed a model to explain the crack resonance phenomena, which regards the resonances as standing waves due to the modes that propagate on crack surfaces along the length and depth directions. For these resonances, we gave the following formulas which agreed well with the available experimental data available at that time [2,3]:

$$ f_L = \frac{V_L}{2L} (M + \frac{1}{2}) $$

$$ f_D = \frac{V_D}{2D} (N) $$

Here, $f_L$ and $f_D$ are length and depth resonance frequencies, $V_L$ and $V_D$ are the velocities of the guided modes along the length and the depth directions, $M$ and $N$ are integers that describe the mode indices. The $1/2$ factor in Eq. (1) arises from the boundary conditions of non-zero velocity fields at the crack tips. For large cracks $V_L$ and $V_D$ can be taken as the Rayleigh wave velocities.

**Measurement Scheme** - The measurement scheme is outlined in Fig. 1. We place input and output transducers near the crack and we cover the bottom of the plate by a damping material. Within our frequency range of operation (~ 50-1000 kHz) we found that both heavily tungsten loaded epoxy and Duxseal sealing putty work fairly well as dampers. We always repeat the measurements on a control plate (without the crack) to isolate the signature of the crack resonance, with the transducers placed on the same location on both cracked and uncracked plates to eliminate any false signal that might arise due to the lateral plate resonances.

We generally work on cw mode, scanning the frequency and monitoring the output signal through a gain-phase meter (HP 3575A) [Fig. 1(b)].

**Transducer Types** - We basically worked with large slots, since they are easier to manufacture.

Therefore our frequency of operation scaled down accordingly. Typically we worked with slots in the order of tens of millimeters in length and depth and therefore we needed to develop transducers that work in the hundred kilohertz range.

One transducer type is two side electrode length expander bars bonded together [Fig. 2(a)]. One electrode pair acts as the input port to the probe and the other pair is the output port. These transducers have been used successfully to determine the mechanical damping at frequencies of 30 to 200 kHz [4]. To reduce the electromagnetic pickup between the transducers, one of the bars is rotated 90° before bonding. For transducer material, we used Channel 5800 (PZT-8) poled along the direction between the electrodes.

Figure 2(b) shows the response of such transducer with both ends free. The transducer assembly
The valley between the first two resonance peaks is found to be the most appropriate region to detect the crack resonances due to its clean and fairly smooth variation. The second valley is not very appropriate, since the transducer is not thin enough for single mode operation in that region. When the transducer thickness satisfies the condition
d < \frac{\lambda}{5},

the motion of the bar is basically along the length direction while for thicker transducers higher order modes become more effective [4].

In our frequency range of operation, thickness and practicality considerations limit the transducer thickness to 2-3 mm. Therefore the contact area of the transducer is small and the alignment of the transducer becomes critical. Since our aim was to obtain a removable probe, we avoided permanent bonding of those transducers. Instead, we aligned the transducer by a three point holder and tried water, vacuum grease or medical gels for coupling material. We observed that the coupling depended critically to the coupling material, its amount and the way it was applied. In particular, water was not very useful since it failed to "wet" the contact area properly.

In the experiment we placed the transducer in the vicinity of the crack (about 1 mm near the crack edge) and near the center where the standing wave distribution for the first length resonance is at its peak. In one case we obtained the data given in Fig. 3 with a 27 mm transducer placed near an EDM notch of length 43 mm on aluminum [5]. Theoretical calculations predict a length resonance at \( f = 52 \text{ kHz} \) and there is an "S" type signature on the observed response between 47-58 kHz for the transducer on the cracked plate (Fig. 5). The same kind of signature was obtained from the far field scattering experiments by P. Khuri-Yakub, et al [2] at higher frequencies. Note that the Q for this resonance is around 5.

As mentioned previously, coupling is the basic drawback of these transducers, leading to poor repeatability of the measurements. The transducer used in obtaining the data given in Fig. 3 was eventually damaged, and we were unable to repeat the results. We will continue to use these transducers in the future, using permanent bonding to eliminate the coupling problem.

The difficulties of the length expander bars led us to look for different transducer type, and we decided to use interdigital transducers with a small number of elements. We used PZT-5H or PZT-8 slabs of thicknesses in the order of 0.3-1 mm and tested several geometries shown in Fig. 4. In Fig. 4(a) we formed the transducer by depositing electrodes on the surface of the slab. Figures 4(b) and 4(c) show counterpoled and unpoled pieces that are bonded to the surface separately. The pieces are epoxy bonded to the surface of the aluminum plate. In all cases we observed that the center frequency is not determined by the separation of fingers, \( L_s \), but rather by the width of each finger \( L_T \). Their efficiencies are surprisingly high, and for the case of a single finger transducer, we obtained an insertion loss as low as 13 dB (including the 6 dB loss due to bidirectionality) between two such transducers. Their bandwidth is narrower than the theoretical expectations. The 15 dB bandwidth is in the order of 35%.

To detect the depth resonances of a 2-D slot, we bonded two single finger transducers on an aluminum plate whose bottom is fully covered with non-uniform grooves of depth \( \approx 10 \text{ mm} \) filled with tungsten epoxy. The total plate thickness is \( \approx 30 \text{ mm} \), and the
separation between the transducers is 25 mm. We measured the transmission response between the transducers with no slot and then cut the slot, increasing the depth in 0.5 mm increments and measuring the response at each depth (Fig. 5). We were able to see the slot because of its effect on the measurements but were unable to recognize the signature of the depth resonances (if there were any). We have not tried placing transducers closer or further apart, but will do so in the future.

\[ \psi_{n}(R) = \frac{1}{4(P_1 P_2)^{1/2}} \int_{S_F} (V_1 \cdot T_2 - V_2 \cdot T_1) \cdot \hat{n} \, ds \]  

(4)

where \( V_1, T_1 \) are the solutions in the presence of the smooth crack when power \( P_1 \) is applied to the first transducer; \( V_2, T_2 \) are the solutions in the presence of the rough crack when power \( P_2 \) is applied to the second transducer; \( \hat{n} \) is the inward normal (towards the roughness) on the surface \( S_F \); and \( \psi_{n}(R) \) is the contribution of the surface roughness to the scattering coefficient.

Consider the scattering geometry given in Fig. 6 where our aim is to calculate the contribution of the surface roughness to the scattering coefficient from transducer 1 to transducer 2. We assume that we can enclose the rough crack with a smooth open crack which is characterized by the surface \( S_F \).

We start with the reciprocity relation [6]

\[ \psi_{n}(R) = \frac{1}{4(P_1 P_2)^{1/2}} \int_{S_F} (V_1 \cdot T_2 - V_2 \cdot T_1) \cdot \hat{n} \, ds \]  

(4)

In Eq. (5) \( V_1 \) is the velocity field in the presence of the smooth crack, and \( T_2 \cdot \hat{n} \) is the normal stress in the presence of the rough crack evaluated on the smooth crack surfaces. To evaluate \( T_2 \cdot \hat{n} \) we follow the analysis of Brekhovskikh [7,8] which has been successfully used to analyze the attenuation of Rayleigh waves on rough surfaces [7,9].

Consider a rough surface, \( S_R \) as shown in Fig. 7. We define a flat surface \( S_F \) under the roughness, defined by the coordinate variables \( r_1 \) and \( r_2 \), and denote the normal vectors to the flat and rough surfaces by \( \hat{n} \) and \( \hat{n}(r_1, r_2) \) respectively. The roughness is then defined as a function of the coordinate variables \( r_1, r_2 \),

\[ r_n = f_R(r_1, r_2) \]  

(6)

where \( r_n \) is the coordinate variable in the direction of the normal vector \( \hat{n} \). The rough surface can then be expressed by the equation

\[ \psi(r_1, r_2, r_n) = r_n - f_R(r_1, r_2) = 0. \]  

(7)

Then the normal vector to the rough surface is simply
the unit vector in the direction of the gradient of the function defining the rough surface,
\[ \hat{n}(r_1, r_2) = \nabla \psi / |\nabla \psi| \]  
(8)
where
\[ \nabla \psi = \hat{n} - \frac{\partial f_R}{\partial r_1} \hat{r}_1 - \frac{\partial f_R}{\partial r_2} \hat{r}_2 \]  
(9)
and
\[ |\nabla \psi| = [1 + (\partial f_R/\partial r_1)^2 + (\partial f_R/\partial r_2)^2]^{1/2}. \]  
(10)
If
\[ (\partial f_R/\partial r_i)^2 \ll 1 \quad i = 1, 2 \]  
(11)
then Eq. (8) can be approximated as
\[ \hat{n}(r_1, r_2) \approx \hat{n} - \sum_{i=1}^{2} (\partial f_R/\partial r_i) \hat{r}_i. \]  
(12)

Since the rough surface is stress free, we have
\[ T \cdot \hat{n}(r_1, r_2) = 0 \quad \text{on} \quad S_R \quad (r_n = f_R). \]  
(13)
The stress field can be expressed as the sum of the unperturbed stress field and a first order perturbation, i.e.
\[ T = T(0) + T(1) + \ldots \]  
(14)
where
\[ T(0) \cdot \hat{n} = 0 \quad \text{on} \quad S_F \quad (r_n = 0). \]  
(15)
Each term in Eq. (14) is then expanded in power series in \( r_n \). For example,
\[ T(0)(r_n = 0) = T(0)(r_n) + r_n \frac{\partial T(0)}{\partial r_n}(r_n = 0) + \ldots \]  
(16)
Substitution of Eq. (16) into Eq. (14) gives
\[ T = T(0)(r_n = 0) + T(1)(r_n = 0) \]  
\[ + r_n \frac{\partial T(0)}{\partial r_n}(r_n = 0) + \ldots \]  
(17)

Further substitution of Eqs. (17) and (12) into (15), keeping only the first order terms, yields
\[ T \cdot \hat{n}(r_1, r_2) = T(1) \cdot \hat{n} + f_R \frac{\partial T(0)}{\partial r_n} \cdot \hat{n} - \sum_{i=1}^{2} (\partial f_R/\partial r_i) \cdot \hat{r}_i \]  
(18)
where we have also used Eq. (15). This leads, within the accuracy of first order perturbation theory, to replacement of the actual boundary condition [Eq. (13)] on the rough surface by an equivalent boundary condition
\[ T(1) \cdot \hat{n} = -f_R \frac{\partial T(0)}{\partial r_n} \hat{n} \]  
(19)
on the flat surface \( S_F \). Note that the left-hand side of Eq. (19) is the quantity required to evaluate the reciprocity integral given in Eq. (5).

One implication of Eq. (19) and Eq. (5) is that the effects of the roughness on different faces of the crack are different. Noting that \( V_1 \) in Eq. (5) is the total unperturbed field, the fields on the shadowed face of the crack are less than those on the illuminated face. Since \( T_2 \) is related to the roughness function \( f_R \) via Eq. (19), the roughness on the illuminated face has more effect on the scattering coefficient than that on the shadowed face.

Example - We chose the simple example of scattering of SH waves from a rough surface (Fig. 8). We assume plane wave incidence at an angle \( \theta \) and receive the signal at an angle \( \phi \), using an infinitely wide transducer.

In free space the SH wave components are
\[ V_x = V_0 e^{ik_0 \sin \phi} \quad (20) \]
\[ T_{xx} = V_0 \frac{2P_0}{\cos \alpha} \quad (21) \]
\[ T_{xy} = \frac{2P_0}{\sin \alpha} \quad (22) \]
where \( \alpha \) is the angle of propagation and \( P_0 \) is the power per unit length along the angle \( \alpha \).

The unperturbed fields for the problem are the total fields after reflection from the smooth surface \( S_F \), i.e.
\[ V_x(0) = 2V_0 e^{ik_0 \sin \phi} \cos (k_0 \cos \alpha) \]  
(23)
\[ T_{xx}(0) = \frac{4P_0}{\cos \alpha} e^{ik_0 \sin \phi} \sin (k_0 \cos \alpha) \]  
(24)
\[ T_{xy}(0) = \frac{4P_0}{\sin \alpha} e^{ik_0 \sin \phi} \cos (k_0 \cos \alpha) \]  
(25)
where $\alpha = \theta$ or $\phi$ for the two waves shown in the figure. Assuming a one dimensional roughness function $f_R(y)$ and applying Brekhovskikh formula

\[
P_1 = \int P_0 \, d\xi \quad \text{and} \quad P_2 = \int P_0 \, d\eta \quad \text{(28)}
\]
or

\[
P_1 = P_0 \cos \theta \int dy
\]
and

\[
P_2 = P_0 \cos \phi \int dy
\]

Equation (27) therefore reduces to

\[
\delta \Gamma(R) = - \frac{2}{(\cos \theta \cos \phi)^{1/2}} \lim_{L \to \infty} \frac{1}{2L} \int_{-L}^{L} \left\{ -ik_0 \cos^2 \phi_R + \sin \phi \frac{df_R}{dy} \right\} \\
\times \left\{ -ik_0 (\sin \theta + \sin \phi)y \right\} dy.
\]

In Eq. (30) the limit arises because excitation and reception are with infinitely wide transducers and the roughness of the surface extends to infinity. We identify the exponential term inside the integral as the Fourier transform kernel. Neglecting the difficulties associated with $f_R$ (the total "power" of $f_R$ is infinite), one obtains

\[
\delta \Gamma(R) = -2ik_0 \left(1 + \sin \theta \sin \phi \right)
\]

where the Fourier transform variable is

\[
k = k_0 \left(\sin \theta + \sin \phi \right)
\]
and we have used the relation

\[
\Im \left\{ \frac{df}{dy} \right\} = ik\Im(f).
\]

As a check, consider the case when $f_R = f_0$ (constant). This corresponds simply to a shift on coordinate axis $x$ by an amount $f_0$. Then Eq. (31) says that the scattered field is in the $-\theta$ direction, and its value is

\[
\delta \Gamma(R) = -2ik_0 f_0 \cos \theta.
\]

This is the additional phase shift introduced by the path length difference $2f_0 \cos \theta$ due to the layer of thickness $f_0$.

If $f_R(y)$ is periodic, i.e.

\[
f_R(y) = f_0 (1 + \cos k_R y)
\]
than in addition to the specular reflection component, there are two other components at angles $\phi$ satisfying the relation

\[
\sin \phi = -\sin \theta \pm \frac{k_R}{k_0}
\]

In general, the roughness function $f_R(y)$ is unknown. Therefore it is preferable to take it as a stochastic process and examine the statistics of the scattering coefficient. The first moment (expected
value or mean value) of the scattering coefficient gives simply
\[ E\{\delta\Gamma(R)\} = -2k_o A(\theta, \phi) \Omega(E\{f_R\}) \]  \( (37) \)
where we have lumped the angle dependence into an angle factor, i.e.,
\[ A(\theta, \phi) = \frac{1 + \sin \theta \sin \phi}{(\cos \theta \cos \phi)^{1/2}} \]  \( (38) \)

If \( f_R \) is an ergodic process (i.e., statistics of the whole ensemble of roughness functions can be determined by examining a single member), then
\[ E\{f_R\} = \text{spatial average of } f_R = f_0 \]  \( (39) \)
and the expected value of \( \delta\Gamma(R) \) is simply a phase shift due to the mean value of \( f_R \). For the second moment we have
\[ E\{\delta\Gamma(R)\delta\Gamma(R)\} = 4k_o^2 A^2(\theta, \phi) S_f \]  \( (40) \)
where \( S_f \) is the power spectrum (Fourier transform of the autocorrelation function) of the roughness function \( f_R \). The variance of the scattering coefficient is then
\[ \text{Var} (\delta\Gamma) = E\{\delta\Gamma(R)\delta\Gamma(R)^*\} - \mathcal{E}^2 \delta\Gamma(R) \]  \( (41) \)

The implications of Eqs. (37) and (41) are the following:

When a scattering measurement is made, the results are obscured by the effect of roughness. If one tests \( N \) different rough samples for which the statistics can be determined by examining a single member (ergodicity), the average of the measurements is simply a phase shift due to the mean value of the roughness functions. For any measurement, the measure of the deviation from the ideal value (no roughness case) is related to the variance of the "noise" (contribution of the roughness) which is expressed in Eq. (41). In other words, any measurement value contains an uncertainty for which the measure is the variance.

For instance, if one is making angular scattering measurements in the high \( k_a \) regime, one obtains a scattering pattern which can be approximated as \([3, 10]\)
\[ \eta(0) \approx \frac{\sin \theta}{x}, \quad x = 2m \frac{L}{x} \sin \theta \]
and the nulls can be used to determine the length \( L \) of the crack. In the presence of the roughness
\[ \eta = \eta(0) + \eta(R) \]
and the uncertainty imposed by the roughness will introduce an uncertainty in the position of the nulls.

In the case of resonance measurements, the effect of roughness is attenuating the crack modes that travel along the depth and the length of the crack. Although the resonance frequencies do not change, the \( Q \) of the resonances decreases due to the attenuation caused by the roughness. Since those resonances are low \( Q \) in nature, the effect of the roughness will either be increasing the uncertainty in determining the center frequency or wiping them off at all.

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References

SUMMARY DISCUSSION

Bernie Tittmann, Chairman (Rockwell Science Center): We have time for one or two questions.

Gerald Quentin (University of Paris): I think it is quite interesting to work with reciprocity because we can carry out calculations, but there is a problem left with this sphere as it applies only for small slots on the surface. You give the expression that the roughness cannot vary very much with the distance on the surface. And the problem is that you just apply the theory. For example, when we made the experiments with Laszlo Adler on fracture cracks, even at the opening of the crack the slope is quite large.

Sevig Ayter (Stanford University): The theory applies so long as the roughness function - this is the roughness function - so long as the roughness function is small.

Laszlo Adler (Ohio State University): That's what he was saying.

Sevig Ayter: As a perturbation, the roughness function must be small, and also the --

Gerald Quentin: Which is the slope? It almost has to be equal to the angle with the normal.

Sevig Ayter: I don't quite get the implication of your remark.

J.D. Achenbach (Northwestern University): In what you have drawn there, you need a much more gradual slope. Even at this small slope, you would need a small roughness function as well.

Bert Auld (Stanford University): I wanted to make a comment there. I think the approximation there is not the derivative estimate. But the square is small, and I think if you go to second order you can push that up further and you can actually modify the Brekhovskikh theory.

Bernie Tittmann, Chairman: Thank you. We will now have to go on.