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Realistically Modeled Transcranial Magnetic Stimulation Coils for Lorentz Force and Stress Calculations During MRI

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Transcranial magnetic stimulation (TMS) uses transient magnetic field to activate brain regions by inducing an electric field across and an electric current through neurons. Functional magnetic resonance imaging (fMRI) allows the measurement of brain activity making it a potentially useful tool in combination with TMS to analyze the sites of stimulation within the brain. TMS typically utilizes a high current pulse up to 8 kA at approximately 2.5 kHz to induce an electric field inside the brain sufficient for neural stimulation. Lorentz forces are created on the TMS coil and are increased in the presence of a high external magnetic field from the MRI magnet. This study implements a realistic coil model developed from X-ray images of a commercial figure-of-eight TMS coil. The Lorentz forces and stress profile inside the current carrying material of this realistically modeled coil are presented. Results show that the maximum Lorentz force density is significantly larger than previously calculated, by a factor of more than 3.

Index Terms—Coil, transcranial magnetic stimulation (TMS), functional magnetic resonance imaging (fMRI), Lorentz force, stress.

I. INTRODUCTION

TRANSCRANIAL MAGNETIC STIMULATION is a noninvasive method to excite nerves by magnetic induction produced by time varying electric current in an excitation coil [1], [2]. Induced electric fields in neural tissue can change transmembrane potentials leading to action potentials [3]. The main benefits of TMS lie in its noninvasive nature – other neuromodulation methods require surgery or lack the relative localization of stimulation provided by TMS. As a result, this method of stimulation has been employed for cognitive neuroscience studies [4] and investigated for therapeutic applications for neurological diseases such as Parkinson’s disease [5]. Recently safety guidelines have been established to minimize the likelihood of encountering the potential adverse effects of TMS [6].

Combining TMS with a neuroimaging technique such as fMRI enables changes in brain activity during TMS to be monitored by measuring blood flow in the brain and promises to enhance our understanding of the effects of TMS [7]-[9]. However, performing such a procedure introduces new challenges; artifacts in fMRI images and large forces on the TMS coils.

The forces experienced by a TMS coil during normal operation can be very high due to large magnetic fields in excess of 1.5 T that can be generated when current is pulsed through the coil. When operated in the presence of a large static external field, such as is produced by an MRI magnet, this force can be significantly increased. The forces experienced by TMS coils under such conditions have previously been investigated [10] using simplified coil geometry. In this study we implement a realistic coil to accurately determine the magnitude and location of the Lorentz forces and stresses produced. The results of this study can be used to make further inferences about safe operation and construction of TMS coils.

II. DETAILS OF THEORETICAL ANALYSIS

The Lorentz force density \( f \) [N/m³] inside the coil can be computed by:

\[
\mathbf{f} = \mathbf{J} \times \mathbf{B}
\]

Where \( \mathbf{J} \) [A/m²] is the current density and \( \mathbf{B} \) [T] is the magnetic flux density. In order to compute the von Mises stresses, the components of the Maxwell stress tensor \( T_{i,j} \) were computed by:

\[
T_{i,j} = \varepsilon_0 \left( E_i E_j - \frac{1}{2} \delta_{i,j} E^2 \right) + \frac{1}{\mu_0} \left( B_i B_j - \frac{1}{2} \delta_{i,j} B^2 \right)
\]

Where \( \varepsilon_0 \) and \( \mu_0 \) are the relative permittivity and permeability respectively, \( \delta_{i,j} \) is the Kronecker delta, which is 1 if \( i = j \) and 0 if \( i \neq j \) and \( E \) is the electric field. In this case the stress tensor depends only on a part of the electromagnetic field quantities. The mechanical part can be assumed to be zero as no external mechanical stress or deformation is applied. Based on the stress tensor, the von Mises stress [Pa] was calculated by:

\[
\sigma_v = \sqrt{T_{xx}^2 + T_{yy}^2 + T_{zz}^2 - T_{xx} T_{yy} - T_{xx} T_{zz} - T_{yy} T_{zz} + 3(T_{xy}^2 + T_{xz}^2 + T_{yz}^2)}
\]

An initial 2D axisymmetric simulation was performed to investigate the influence of skin effect and determine if a change in the overall magnetic flux density was observed when considering the problem in an AC case as opposed to the DC case. Both calculations were performed with a current of \( I_0 = 5 \text{ kA} \) in a simulated copper medium with \( \sigma = 5.998 \times 10^7 \text{ S/m} \) and a surrounding air region with \( \sigma = 0 \text{ S/m} \). In the AC case a frequency of 2.5 kHz was assumed. The coil consisted of nine windings with inner and outer radii of 32 mm and 48 mm respectively, with a 1 mm separation between windings to account for an air gap. The dimensions of the windings in this simulation were 1 mm x 5 mm. Due to the rotational symmetry of this model a very fine mesh can be applied to the
The skin effect was found to have some effect on the overall current distribution inside the windings but in the AC case the majority of the coil cross section carries an instantaneous current density of approximately $1 \times 10^9$ A/m$^2$, corresponding to the current density in the DC case. Fig. 1 shows the underlying geometry.

From these results it was determined the calculation could be considered in the DC case for the 3D simulation of a realistic coil as this greatly reduces the computational requirements of the simulation.

The TMS coil was modeled with SolidWorks 2011 (SolidWorks Corp., Waltham, MA, USA) based on x-rays of a commercially available figure-of-eight type [11] coil (Magstim 2nd Generation 70 mm Double Coil). Simulations were performed with COMSOL Multiphysics 4.2a (COMSOL, Inc., Burlington, MA, USA) with current $I_0 = 5$ kA in copper wire with electrical conductivity $\sigma = 5.998 \times 10^7$ S/m.

III. RESULTS AND DISCUSSION

The effect of an applied external field has been analyzed in a number of cases. Initially the forces and stresses generated with no external field are described. Application of the external field in different orientations is then investigated.

A. With no External Field

The distribution of magnetic flux density in the coil plane, due only to the current in the coil is shown in Fig. 2 (a). The maximum field intensity in the coil plane is 2.313 T. The maximum magnitude of the Lorentz force density on the coil when no external field was present was found to be $2.8 \times 10^9$ Nm$^{-3}$. The directions in which the forces act are shown in Fig. 2 (b). The maximum von Mises stress was found to be $4.3175 \times 10^6$ Pa as shown in Fig. 2 (c). The largest stresses were found to be on the inner-most turn of the coil windings.

B. Coil Oriented Perpendicular to External Field

We have investigated the effect of orienting the realistically modeled TMS coil perpendicular to the direction of a 3 T external field. The external field here is aligned with the z-axis of the coil model. The distribution of magnetic flux density in

Fig. 1. (a) Axisymmetric simulation of magnetic flux density $B$ (T) produced by modeled TMS coil in DC and (b) AC case.

Fig. 2. (a) Magnetic flux density $B$ (T) inside modeled figure-of-eight coil with no external magnetic field applied, (b) resultant Lorentz force density $f$ (Nm$^{-3}$) and (c) von Mises stresses $\sigma$, (Pa).
the coil plane is shown in Fig. 3 (a). The maximum field intensity calculated in this case has increased to 5.3356 T. The maximum amplitude of the Lorentz force density on the coil when oriented perpendicular to the external field was found to be 6.4248 \times 10^9 \text{ Nm}^{-3}, indicating the force density has more than doubled due to the external field. The directions in which these forces act are shown in Fig. 3 (b). In this case the forces (a) Total magnetic flux density (T) - $B_z$

(b) Lorentz force density (N/m$^3$) - $B_z$

(c) von Mises stress (Pa) - $B_z$

![Fig. 3. (a) Magnetic flux density B (T) inside modeled figure-of-eight coil with 3 T external field applied along the model z-axis, perpendicular to the coil plane and (b) resultant Lorentz force density f (Nm$^{-3}$) producing radial forces in the coil plane and (c) von Mises stresses $\sigma_v$ (Pa).](image)

C. Coil Oriented Parallel to External Field

The TMS coil can be oriented parallel to the external field in two distinct ways. First we examine the effect of the external field operating parallel to the line joining the centers of the two windings, meaning that the field is oriented along the x-axis of the coil model shown in Fig. 4. The distribution of the flux density in the coil plane in this instance is shown in Fig. 4 (a). The peak magnetic flux density in the coil plane was 4.4106 T. This is significantly less than in the case of the perpendicular orientation where a peak magnetic flux density of approximately 5.3 T was calculated. The maximum Lorentz force density in this case was calculated to be $4 \times 10^9 \text{ Nm}^{-3}$. The directions in which the forces act are shown in Fig. 4 (b). (a) Total magnetic flux density (T) - $B_x$

(b) Lorentz force density (N/m$^3$) - $B_x$

(c) von Mises stress (Pa) - $B_x$

![Fig. 4. (a) Magnetic flux density B (T) inside modeled figure-of-eight coil with 3 T external field applied along the model x-axis, parallel to the coil plane and (b) resultant Lorentz force density f (Nm$^{-3}$) producing a flexural force about the coil center and (c) von Mises stresses $\sigma_v$ (Pa).](image)

The forces result in a flexural force about the central point of the coil and with the largest forces acting on the coil element that connects the two windings. The largest von Mises stress was found to be 1.6242 \times 10^7 \text{ Pa} with stresses generally occurring on the top or bottom surface of the coil, depending on the direction of the current flow, relative to the model x-axis as shown in in Fig. 4 (c).

The external field can also be applied parallel to the width of the TMS coil, such that the external field is oriented along the y-axis of the coil model as shown in Fig. 5. The distribution of magnetic flux density in the coil plane in this instance in shown in Fig. 5 (a). The peak magnetic flux density in the coil plane is 3.8906 T. The maximum Lorentz
force calculated in this case was $3.9797 \times 10^9\ \text{Nm}^{-3}$. The directions in which the forces act are demonstrated in Fig. 5 (b). In this case, the forces are again found to act perpendicular to the coil plane with the largest forces acting on the coil element that connects the two windings, but in this case the remaining forces result in a torsional force about the center of the coil. The largest von Mises stress calculated for this case was $1.5453 \times 10^7\ \text{Pa}$, with high stresses occurring on the top or bottom surface of the coil, depending on the direction of the current flow, relative to the model x-axis as shown in Fig. 5 (c).

The magnitude of the calculated forces in all instances are below the yield strength of copper (70 MPa) so we do not anticipate the coils will plastically deform. However, the plastic casing should be selected such that it should have yield strength higher than 16.25MPa. Since the coil undergoes large elastic deformation, one should also consider low cycle fatigue which has the potential to lead to coil failure.

IV. COMPARISON OF REALISTIC AND SIMPLIFIED MODEL

Values of the maximum calculated magnetic flux density and Lorentz force density found in this study and from a previous study where simplified coil geometry was used are given in Table I. Values of von Mises stress were not calculated in the earlier study.

V. CONCLUSION

The magnetic flux density, Lorentz force density and von Mises stress experienced by a modeled TMS coil have been calculated during normal operation and when in the presence of a 3 T external field such as can be found in an fMRI scanner. The results show that under normal conditions the maximum Lorentz force density can exceed $2.8 \times 10^9\ \text{Nm}^{-3}$ but a 3 T external field can increase this force density to $6.4 \times 10^9\ \text{Nm}^{-3}$. This is a factor of more than 3 larger than had been calculated previously where simplified coil geometry indicated the maximum Lorentz force density in the presence of a 3 T field was $1.85 \times 10^9\ \text{Nm}^{-3}$, much less than this new study reveals. It has also been demonstrated that the maximum von Mises stress encountered under an applied 3 T field is $2.2575 \times 10^7\ \text{Pa}$. The results of this study can be utilized in the implementation of combined TMS/fMRI systems and in the development of TMS coils to ensure mechanical failure does not occur.

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<th>Table I</th>
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<th>MAXIMUM CALCULATED MAGNETIC FLUX DENSITY AND LORENTZ FORCE FOR ORIENTATIONS OF EXTERNAL FIELD</th>
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<tbody>
<tr>
<td>Simplified Geometry [8]</td>
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<tr>
<td>Max Magnetic Flux Density (T)</td>
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<tr>
<td>B$_0$</td>
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REFERENCES


