Magnetic contribution to dilepton production in heavy-ion collisions

Kirill Tuchin
Iowa State University, tuchin@iastate.edu

Follow this and additional works at: http://lib.dr.iastate.edu/physastro_pubs
Part of the Astrophysics and Astronomy Commons, and the Physics Commons

The complete bibliographic information for this item can be found at http://lib.dr.iastate.edu/physastro_pubs/129. For information on how to cite this item, please visit http://lib.dr.iastate.edu/howtocite.html.

This Article is brought to you for free and open access by the Physics and Astronomy at Digital Repository @ Iowa State University. It has been accepted for inclusion in Physics and Astronomy Publications by an authorized administrator of Digital Repository @ Iowa State University. For more information, please contact digirep@iastate.edu.
We calculate a novel “magnetic contribution” to the dilepton spectrum in heavy-ion collisions arising from interaction of relativistic quarks with intense magnetic field. Synchrotron radiation by quarks, which can be approximated by the equivalent photon flux, is followed by dilepton decay of photons in an intense magnetic field. We argue that the “magnetic contribution” dominates the dilepton spectrum at low lepton energies, whereas a conventional photon dilepton decay dominates at higher lepton energies.

DOI: 10.1103/PhysRevC.88.024910

PACS number(s): 25.75.Cj

I. INTRODUCTION

Electromagnetic radiation that accompanies any relativistic heavy-ion collision weakly interacts with hot nuclear matter. However, it strongly interacts with a highly intense magnetic field that is frozen into the nuclear matter [1–4]. Therefore, it bears witness to the magnetic field existence and provides a rare opportunity to experimentally study its properties. In practice, electromagnetic radiation caused by a magnetic field is masked by the electromagnetic radiation of quark–gluon plasma (QGP). It is not an easy problem to disentangle the two contributions. Good theoretical control over both is required to achieve this goal.

In the present article we address the problem of dilepton production in heavy-ion collisions due to a magnetic field. Our work is partly motivated by the recent experimental results on dileptons and photons produced in heavy-ion collisions, which challenge the prevailing theories of electromagnetic processes in high-energy nuclear physics that neglect strong electromagnetic interactions. It was observed in Refs. [5,6] that there is a significant understimation of electromagnetic spectra by theoretical models in the low-momentum region. This indicates that there are additional contributions that have not been taken into account. We dub the additional lepton production due to a magnetic field as the “magnetic contribution” as opposed to the “conventional contributions.”

Dilepton production in a magnetic field proceeds in two stages. First, quark (or antiquark) is produced in a heavy-ion collision. We will refer to the quark (or antiquark) distribution at this stage as the initial quark distribution. Second, it radiates the lepton–antilepton pair in a processes mediated by a virtual photon as depicted in Figs. 1 and 2. Double lines indicate quark and lepton propagators in a magnetic field. In Fig. 1 both quark and lepton move in a magnetic field, whereas in Fig. 2 only one of them does. The final result is sum over all these processes. However, in this article we will focus on the square of the amplitude shown in Fig. 1, because we anticipate that it is most sensitive to a magnetic field. Other contributions will be considered elsewhere.

The initial quark distribution is made up of soft and hard quarks and antiquarks. Soft quarks have typical energies $\epsilon \sim T$, where $T$ is the plasma temperature and are a part of QGP. The exact mechanism of their production and equilibration is not fully understood and is actually not essential for our arguments. What is important is an observation that, according to the state-of-the-art phenomenology, the QGP equilibration happens over a very short time on the order of $1/Q_s$, where $Q_s$ is the saturation momentum, see, e.g., Ref. [7]. Hard quarks in the central rapidity region $y = 0$ have typical energies $\epsilon \sim Q_s T > 1$ and their production mechanism has been thoroughly investigated [8–14]. In this article we focus on lepton energies $\epsilon_+ \in (T, Q_s T)$ in the interval $T < \epsilon_+ < Q_s T$. In this region, the spectrum of soft quarks falls off exponentially, whereas the spectrum of hard quarks is only logarithmic. Moreover, the number of soft quarks in plasma of volume $V$ is on the order of $T^3 V$. At early times $V \sim S/Q_s$, where $S$ is the cross-sectional area of the ion overlap region. On the other hand, the number of hard quarks is on the order of $S Q_s^2$ (see Sec. IV), which is much larger than the number of soft quarks. Therefore, the contribution of hard quarks to dilepton production is dominant. It is this contribution that we discuss in this article.

The paper is organized as follows. In Sec. II we employ the Weizsäcker-Williams method to write the dilepton production rate by a hard quark as a convolution of the real photon decay rate with the flux of equivalent photons emitted by a fast quark. In Sec. III we compute the rate of photon dissociation into dilepton as a function of lepton transverse momentum and rapidity. Initial quark distribution produced in heavy-ion collisions is computed in Sec. IV using the quasiclassical approximation. In Sec. V we apply the developed formalism to calculate the magnetic contribution to $e^+ e^-$ pair production at midrapidity at the Relativistic Heavy Ion Collider (RHIC). We compare dilepton production in a magnetic field and in vacuum in Sec. VI and argue that the magnetic field contribution dominates at photon energies $\omega < 200$ MeV. Finally, we summarize in Sec. VII.

Throughout the article, we assume that the time dependence of the magnetic field is adiabatic. This is a reasonable approximation if the center-of-mass energy per nucleon satisfies the condition $\sqrt{s_{NN}} \gtrsim m_N / (\sigma R_A)$, where $m_N$ is nucleon mass, $\sigma$ is electrical conductivity of nuclear matter, and $R_A$ is nuclear radius [15]. It is easy to check that this condition is satisfied at RHIC and the Large Hadron Collider.1

1See, however, Ref. [16] for a dissenting opinion.
is exponentially suppressed [18] and, hence, can be neglected. Schwinger’s critical field, and
mediated by a virtual photon. Double lines denote fermion propagator because light quarks are ultra-relativistic
\(z_q \gg m_q\) in the center-of-mass frame. This allows us to employ the Weizsäcker-Williams method to calculate the dilepton production with logarithmic accuracy. According to this method, we can relate the cross section of the \(q \to q e^+ e^-\) process to the cross section of photoproduction \(\gamma \to e^+ e^-\). The logarithmically enhanced contribution arises from the kinematic region where photon virtuality has a negligible effect on photon emission and on dilepton photoproduction. The dilepton production rate can be written as (see notations in Fig. 1)

\[
\frac{dN_{q \to e^+ e^-}}{dt d\Omega_+ d\epsilon_+} = \int n(\omega) \frac{dN_{\gamma \to e^+ e^-}}{dt d\Omega_+ d\epsilon_+} d\omega, \tag{1}
\]

where \(n(\omega)\) is the flux of equivalent real photons replacing the virtual photon. It is given by [17]

\[
n(\omega) = \frac{2 \varepsilon_q^2 \alpha}{\pi} \frac{1}{\omega} \ln \frac{\omega}{[1 + (\chi \omega / \epsilon)^1/3]}, \tag{2}
\]

where \(\varepsilon_q\) is the quark’s charge and

\[
\chi = \frac{\varepsilon_q}{m_q^3} \sqrt{(F_{\mu \nu} p^\nu)^2} = \frac{\varepsilon_q}{m_q^3} \frac{p \times B}{m_{\gamma}^2} \tag{3}
\]

is a boost-invariant parameter. Here \(p_i\) is quark’s momentum component transverse to \(B\) (not to be confused with \(p_\perp\), which is transverse to the heavy-ion collision axis). Equation (1) takes into account that the spectrum of photons is enhanced at \(\omega \ll \epsilon\). Angular distribution of photons strongly peaks at small angles \(\sim m_{\gamma}^2/\epsilon^2\). Therefore, the equivalent photon momentum \(k\) is approximately collinear with the quark’s momentum \(p\).

It is convenient to introduce dimensionless parameters \(b = \alpha^{1/2} B / m_q^2\), which is the value of magnetic field in units of the Schwinger’s critical field, and \(\gamma = \epsilon / m_q\). In terms of these parameters \(\chi = b \sin \theta\), where \(\theta\) is the angle between \(p\) and \(B\). In heavy-ion collisions \(\gamma \gg 1\) and \(b \gtrsim 1\), implying that \(\chi \gg 1\). This estimate breaks down only at very small angles \(\theta \approx (b \gamma)^{-1}\), where \(\chi\) becomes small and the photon decay rate is exponentially suppressed [18] and, hence, can be neglected.

Integration in (1) runs over frequencies \(2m_\ell \leq \omega \leq \omega_m\) with \(\omega_m\) satisfying the equation

\[
\omega_m[1 + (\chi \omega / \epsilon)^{1/3}] = \epsilon. \tag{4}
\]

Since \(\omega \ll \epsilon\) and \(\chi \gg 1\) it follows that \(\epsilon \chi / \omega \gg 1\). Using this in (4) we get

\[
\omega_m \approx m_\ell \sqrt{\frac{\gamma}{\beta}}. \tag{5}
\]

To determine the region of applicability of the equivalent photon approximation, note that the logarithmic contribution to the equivalent photon flux comes about only if \(\omega_m \gg m_\ell\). In view of (5) we have

\[
\frac{\gamma}{\beta} \gg \frac{m_\ell^2}{m_q^2}. \tag{6}
\]

This condition is satisfied for not too strong fields and for light leptons. In particular, we will apply the equivalent photon approximation to calculate the magnetic contribution to electron-positron pair production at RHIC in Sec. V.

\[\text{FIG. 1. Dilepton production by quark in external magnetic field, mediated by a virtual photon. Double lines denote fermion propagator.}
\]

\[\text{FIG. 2. Magnetic contributions that are not considered in this article.}
\]

III. PHOTON DISSOCIATION RATE

Now we turn to the photon dissociation rate which is convenient to perform in a frame where photon’s momentum is perpendicular to the magnetic field. We denote such frame as \(K’\) and all quantities in it bear the prime. The corresponding dilepton rate is given in Sec. III A. In Sec. III B the rate is transformed to frame \(K\), where photon moves at an arbitrary angle with respect to magnetic field.

A. \(K’\) frame: \(k' \cdot B' = 0\)

In reference frame \(K’\) photon moves in a plane perpendicular to the magnetic field: \(k' \cdot B' = 0\). The rate of dilepton photoproduction in \(K’\) frame reads [18]

\[
\frac{dN_{\gamma \to e^+ e^-}}{dt d\Omega_+ p^+} = \frac{\alpha}{(2\pi)^3 \omega} \int_0^{\infty} d\tau \left[ \frac{m_\ell^2}{\epsilon_+ \epsilon_-} - (\epsilon_+^2 + \epsilon_-^2) \omega_0^4 \frac{1}{4\epsilon_+^2} \right] \times \exp \left\{ i \frac{\epsilon_+ - \epsilon_-}{\omega_0 \tau} \left( 1 - k' \cdot \nu_+ \frac{\omega_0^2 \nu_+^2}{24} \right) \right\}, \tag{7}
\]

where \(\epsilon_+, \epsilon_- = \omega_0'\) and the synchrotron frequency is

\[
\omega_0 = \frac{e B'}{\epsilon_+}. \tag{8}
\]
Integration over the time parameter $\tau$ can be done explicitly using the following formulas:

$$\int_{-\infty}^{\infty} \cos[\beta \tau + a \tau^3] d\tau = \frac{2\pi}{(3\alpha)^{1/3}} \cos \left( \frac{b}{(3\alpha)^{1/3}} \right), \quad (9)$$

$$\int_{-\infty}^{\infty} \tau^2 \cos[\beta \tau + a \tau^3] d\tau = -\frac{b}{3\alpha} \int_{-\infty}^{\infty} \cos[\beta \tau + a \tau^3] d\tau, \quad (10)$$

where $\text{Ai}(x)$ is the Airy function. Using (9) and (10) into (7) we obtain

$$dN_{\gamma \rightarrow e^-e^+} \frac{d^3p_e}{d^3p_{\gamma}'} = \frac{\alpha m_e^2}{\omega' L} \left[ \text{Ai}(\xi) d\xi + \left( \frac{2}{\chi'} - \frac{\chi^{1/2}}{\chi'} \right) \text{Ai}'(\chi') \right], \quad (12)$$

where $\text{Ai}'(\chi)$ is the derivative of the Airy function and

$$\chi' = \left( \frac{m_e^2 \omega'}{eB' e'_{\gamma} e'_\beta} \right)^{2/3}, \quad \chi = \frac{eB' \omega'}{m_e^2}. \quad (13)$$

Parameter $\chi$ is boost invariant. In an arbitrary frame it reads as follows:

$$\chi = \frac{e}{m_e} \sqrt{-(k \times B)^2} = \frac{e}{m_e} |k \times B|. \quad (14)$$

The total rate is

$$\frac{d^2N_{\gamma \rightarrow e^-e^+}}{d\Omega d\epsilon} = -\frac{\alpha eB'}{m_e \chi} \int_{(4\pi)^{1/3}}^{\infty} 2x^{3/2} + 1/\chi \text{Ai}'(x') x^{1/4} x^{3/2} - 4x^{1/2} dx'. \quad (15)$$

**B. K frame: $k \cdot B \neq 0$**

Now we need to transform equations (1), (2), and (7) into an arbitrary frame $K$. This is done by making a boost in the magnetic field direction. Let $z$ be the collision axis and $y$ be the magnetic field direction. We will use the following notations: $\alpha$ and $\beta$ are the polar and azimuthal angles of photon and quark with respect to the $\hat{z}$ axis (collision axis) and $\theta$ and $\phi$ are the polar and azimuthal angles of the photon and quark with respect to the $\hat{y}$ axis (magnetic field direction). The same symbols with the “+” subscript refer to lepton $\ell^+$. For example, quark momentum reads

$$p_\ell' = \epsilon (\hat{x} \sin \alpha \cos \beta + \hat{y} \sin \alpha \sin \beta + \hat{z} \cos \alpha), \quad (16)$$

$$= \epsilon (\hat{x} \sin \theta \cos \phi + \hat{y} \cos \theta + \hat{z} \sin \theta \sin \phi), \quad (17)$$

and similarly for other vectors.

Suppose that $K$ moves with velocity $V = V\hat{y}$ with respect to $K'$. A boost along the magnetic field direction does not change the field, i.e., $B' = B$. The Lorentz transformation formulas for the quark momentum read

$$p_\ell' = p_\ell, \quad p_y' = \gamma_V (p_y + V \epsilon), \quad \gamma_V = (1 - V^2)^{-1/2} = \frac{1}{\sin \theta}. \quad (19)$$

Transformation of quark and photon energies is given by

$$\epsilon' = \epsilon \sin \theta = \frac{\epsilon}{\gamma_V}, \quad \omega' = \frac{\omega}{\gamma_V}. \quad (20)$$

Analogously to (18), we can write the transformation of the positively charged lepton's $\ell^+$ momentum,

$$p_{\ell^+}' = p_{\ell^+}, \quad p_{\ell^+ y} = \gamma_V (p_{\ell^+ y} + V \epsilon'), \quad \gamma_V = (1 + V \cos \beta) \gamma_{\ell^+}(1 + V \sin \alpha \sin \beta_+). \quad (21)$$

Thus, the lepton's energy $\epsilon_+$ transforms as

$$\epsilon_+ = \gamma_{\ell^+} \epsilon_+ (1 + V \cos \theta_+) = \gamma_{\ell^+} \epsilon_+ (1 + V \sin \alpha \sin \beta_+). \quad (22)$$

Consider a relativistic invariant,

$$k \cdot p_+ = \omega \epsilon_+ (1 - \hat{k} \cdot \hat{v}_+) = \omega' \epsilon'_+ (1 - \hat{k}' \cdot \hat{v}'_+). \quad (23)$$

Using (20) and (22) we find

$$1 - \hat{k} \cdot \hat{v}_+ = \frac{1}{1 + V \cos \theta_+} \frac{1 - \hat{k}' \cdot \hat{v}'_+}{1 - \cos \theta \cos \theta_+ - \sin \theta \sin \theta_+ \cos (\phi - \phi_+)}, \quad (24)$$

Finally, employing transformation of the time interval and the solid angle

$$\frac{d^2N_{\gamma \rightarrow e^-e^+}}{d\Omega d\epsilon} = \frac{1}{\gamma_V^2 (1 + V \cos \theta_+)} \frac{dN}{dtd\Omega_+ d\epsilon_+}. \quad (26)$$

we obtain that in the $K'$ frame

$$\frac{dN}{dtd\Omega_+ d\epsilon_+} = \frac{1}{\gamma_V^2 (1 + V \cos \theta_+)} \frac{dN}{dtd\Omega_+ d\epsilon_+}. \quad (27)$$
where $\epsilon'$, $\epsilon_+$, and $\omega'$ on the right-hand side should be replaced with the corresponding expressions in the $K$ frame using (19), (20), (22), and (25).

Experimental data on dilepton production is usually represented in terms of rapidity $y$, and transverse momentum $p_{+\perp}$, which are convenient if there is no magnetic field, in place of energy $\epsilon_+$ and polar angle $\alpha_+$. In the ultrarelativistic limit

$$p_{+\perp} = \sqrt{p_{+x}^2 + p_{+y}^2} = \epsilon_+ \sin \alpha_+, \quad y_+ = -\ln \tan \frac{\alpha_+}{2}.$$  

(28)

Inverting equations (28) yields

$$\epsilon_+ = p_{+\perp} \cosh y_+., \quad \sin \alpha_+ = \frac{1}{\cosh y_+}.$$  

(29)

Using (29) in (19), (20), (22), and (25) we can transform parameters appearing in (11) to the $K$ frame as follows:

$$\cos \theta_+ = \frac{\sin \beta_+}{\cosh y_+}, \quad V = -\frac{\sin \beta}{\cosh y},$$  

(30)

$$\gamma' = \sqrt{\cosh^2 y - \sin^2 \beta},$$  

$$\epsilon'_+ = \sqrt{\cosh^2 y - \sin^2 \beta} \times (\cosh y \sin y_+ - \sin \beta \sin \beta_+),$$  

(31)

$$\omega' = \frac{\omega \sqrt{\cosh^2 y - \sin^2 \beta}}{\cosh y}.$$  

(32)

$$1 - k' \cdot v_+ = \frac{\cosh(y - y_+) - \cos(\beta - \beta_+)}{\cosh y \sin \beta_+}.$$  

(33)

In terms of the lepton’s transverse momentum with respect to the collision axis $p_{+\perp}$, its rapidity $y_+$ and azimuthal angle $\beta_+$, the dilepton spectrum reads

$$\frac{dN_{q\rightarrow \ell^+\ell^-}}{dt d\epsilon_+} = \frac{dN_{q\rightarrow \ell^+\ell^-}}{d\Omega_{+\perp} d\epsilon_+} \times \frac{\gamma' dN_{q\rightarrow \ell^+\ell^-}}{\gamma' \pi m \epsilon},$$  

(34)

where (27) and (22) are used. The rate on the right-hand side of (34) is a function of $\epsilon'_+, \omega', \ldots$, etc., which should be expressed through $p_{+\perp}, \beta_+, y_+$ using (30)–(33).

Integration over the lepton direction can be performed if we recall that in the $K$ frame the main contribution stems from the collinear configuration $\vec{k} \cdot v'_+ \approx 1$. Because $V \cos \theta_+ \ll 1$ and $1 - k \cdot v_+ \ll 1$, it follows from (24) that $1 - k \cdot v_+ \ll 1$.

Therefore, the integral over the lepton directions in the $K$ frame is still dominated by the collinear configuration $\theta_+ \approx \theta$ and $\phi_+ \approx \phi$. Using (22) and (19) we get $\epsilon'_+ = \gamma \epsilon_+ (1 - \cos^2 \theta) = \epsilon_+ \sin \theta$, in agreement with (20). To the same approximation $d\ell' d\epsilon'_+$ is boost invariant and, hence, we get in $K$ a formula similar to (12)

$$\frac{dN_{q\rightarrow \ell^+\ell^-}}{dt d\epsilon_+} = \frac{a m^2}{\omega^2} \left\{ \int_\infty^{\epsilon_+} A(\xi) d\xi + \left( \frac{2}{x} - \frac{x}{x^{1/2}} \right) A'(x) \right\},$$  

(35)

where now

$$x = \left( \frac{m^3}{eB \epsilon_+ \sin \theta} \right)^{2/3}, \quad \epsilon = \frac{eB \epsilon_+}{m^2} \sin \theta.$$  

(36)

To calculate the dilepton spectrum produced by quarks, (34) and (35) must be integrated with the equivalent photon flux $n(\omega) d\omega$ given by (2). It is helpful to note that the equivalent photon spectrum $n(\omega) d\omega$ (2) is boost invariant in our approximation. The differential rate per unit $\epsilon_+$, $y_+$, and $\beta_+$ is rather bulky and we will not write it down explicitly. The rate of lepton production with energy $\epsilon_+$ reads

$$\frac{dN_{q\rightarrow \ell^+\ell^-}}{dt d\epsilon_+} = \frac{2e^2}{\pi} \left( \int_{\epsilon_+}^{\infty} \frac{d\omega}{\omega^2} \ln \frac{\epsilon}{\omega(1 + (\chi \epsilon/\omega)^{1/3})} \right)$$  

$$\times \left\{ \int_{x}^{\infty} A(\xi) d\xi + \left( \frac{2}{x} - \frac{x}{x^{1/2}} \right) A'(x) \right\},$$  

(37)

where, the invariant parameter $\chi$ defined in (3) is

$$\chi = \frac{eB \epsilon_+ \sin \theta}{m_q^3}$$  

(38)

and $\omega_\min$ is a solution of (4). Integration over $\epsilon_+$ as in (15) yields the total lepton rate

$$\frac{dN_{q\rightarrow \ell^+\ell^-}}{dt d\Gamma_+} = \sum_q \int dN_{A_1 A_2 \rightarrow qX} \frac{dN_{q\rightarrow \ell^+\ell^-}}{dy d^2 p_{\perp} \frac{dt}{d\Gamma_+}} d^2 p_{\perp} dy.$$  

(39)

IV. INITIAL QUARK DISTRIBUTION

To calculate the magnetic contribution to the dilepton spectrum produced in a collision of two heavy ions with atomic weights $A_1$ and $A_2$ we need to convolute the initial quark distribution with the dilepton spectrum in (37) or (39). For the differential rate per lepton’s phase space $d\Gamma_+$ we have

$$\frac{dN_{A_1 A_2 \rightarrow qX}}{dy d^2 p_{\perp}} = \sum_q \int dN_{A_1 A_2 \rightarrow qX} \frac{dN_{q\rightarrow \ell^+\ell^-}}{dy d^2 p_{\perp} \frac{dt}{d\Gamma_+}} d^2 p_{\perp} dy.$$  

(40)

The sum runs over all light quarks and antiquarks. In this section we focus on the initial quark distribution.

First, consider the dependence of the initial quark spectrum on the transverse momentum $p_{\perp}$. The key parameters here are two saturation momenta $Q_{s1}$ and $Q_{s2}$ of the two colliding nuclei. They depend only on the quark’s rapidity $y$ and total collision energy. At a small quark’s transverse momentum $p_{\perp} \ll \min\{Q_{s1}, Q_{s2}\}$, the spectrum increases as

$$\frac{dN_{q \rightarrow \ell^+\ell^-}}{dy d^2 p_{\perp}} \propto 1/p_{\perp}^3$$  

and at $p_{\perp} \gg \max\{Q_{s1}, Q_{s2}\}$ it falls off as

$$\frac{dN_{q \rightarrow \ell^+\ell^-}}{dy d^2 p_{\perp}} \propto 1/p_{\perp}.$$  

(41)

The maximum is at $p_{\perp} \approx \min\{Q_{s1}, Q_{s2}\}$. It is a reasonable approximation to write

$$\frac{dN_{q \rightarrow \ell^+\ell^-}}{dy d^2 p_{\perp}} \approx \frac{1}{2\pi p_{\perp}} \frac{dN_{q \rightarrow \ell^+\ell^-}}{dy}.$$  

(41)
Using this equation in (40) we obtain
\[
\frac{dN_{A_1A_2+\ell^{+}\ell^{-}}}{dt d\Gamma_+} = \sum_q \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{-Y/2}^{Y/2} dy \frac{dN_{A_1A_2-q\chi}}{d\Gamma_+} \bigg|_{p_\perp = \min|Q_{\perp 1}, Q_{\perp 2}|} 
\]
where we assumed for definiteness that $Q_{\perp 1} > Q_{\perp 2}$, when $y > 0$. $Y$ is the rapidity interval between the two ions.

Initial differential cross section for quark production in heavy-ion collisions in the chiral limit reads \[8,14,19\]
\[
\frac{d\sigma_{A_1A_2+q\chi}}{d^2l dy dz} = \frac{4\pi e^2}{(2\pi)^2} \int d^2b_1 \int d^2b_2 \int d^2r \int d^2r' e^{-i(r-r') \cdot \cdot} \frac{1}{r'^2 (r-r')^2} \times \left\{ [S_1((1-z)(r-r'))S_2(z(r-r')) - 1] [S_2((1-z)(r-r'))S_1(z(r-r')) - 1] \right\}
\]
\[
\times \frac{1}{r^2} \left\{ [S_1((1-z)r)S_1(zr) - 1] [S_2((1-z)z)S_2(zr) - 1] \right\}
\]
\[
\times \frac{1}{r^2} \left\{ [S_1((1-z)r')S_1(zr') - 1] [S_2((1-z)r')S_2(zr') - 1] \right\}.
\]

where $\ell$ is the relative transverse momentum of the $q\bar{q}$ pair,
\[
S_\alpha(r) = \exp \left\{ -\frac{i}{\pi} r^2 Q_\alpha^2 \left(y_a, b_a \right) \right\}
\]
is the scattering matrix element, and $Q_\alpha(r, b_a)$ is the saturation momentum of nucleus $a = 1, 2$. $b_a$ is the impact parameter. Integrating (44) over $\ell$ we obtain the $\delta$ function $(2\pi)^2 \delta(r-r')$. Subsequent integration over $0 \leq r < \infty$ and $0 \leq z \leq 1$ yields
\[
\frac{d\sigma_{A_1A_2+q\chi}}{dy} = \frac{N_c}{24\pi^3} \int d^2b_1 \int d^2b_2 \left[ Q_{11}^2 \ln \left( 1 + \frac{Q_{11}^2}{Q_{12}^2} \right) + Q_{22}^2 \ln \left( 1 + \frac{Q_{21}^2}{Q_{22}^2} \right) \right].
\]

It is phenomenologically reasonable approximation to treat the impact parameter dependence of $Q_\perp$ as a step function. Denoting by $S$ the overlap area of the two ions, we obtain for the quark yield
\[
\frac{dN_{A_1A_2+q\chi}}{dy} = \frac{N_c S}{24\pi^3} \left[ Q_{11}^2 \ln \left( 1 + \frac{Q_{11}^2}{Q_{12}^2} \right) + Q_{22}^2 \ln \left( 1 + \frac{Q_{21}^2}{Q_{22}^2} \right) \right],
\]
where now $Q_{11}$’s are taken at $b_a = 0$. Rapidity dependence of the saturation momentum in the center-of-mass frame is $Q_{11}(y) = Q_{11} e^{2\lambda y/2}$, where $\lambda$ is known phenomenological parameter \[20\] and $Q_{11}$ depends only on the collision energy \[21\]. With this notation we cast (47) in the following form:
\[
\frac{dN_{A_1A_2+q\chi}}{dy} = \frac{N_c S Q_{11}^2}{24\pi^3} \left[ e^{2\lambda y} \ln(1 + e^{-2\lambda y}) + e^{-2\lambda y} \ln(1 + e^{2\lambda y}) \right].
\]

Equation (48) is valid for quark rapidities not too close to the kinematic boundary at $y = \pm Y/2$ (fragmentation regions of the nuclei). Behavior near the kinematic boundary can be inferred from the dependence of the valence quark distribution function $q_v$: When Bjorken’s $x$ is close to one, $q_v(x) \propto (1-x)^3$. Using $x = (p_\perp / \sqrt{s}) e^{y}$ with $Y = \ln(s/\mu^2)$, where $s$ is the center-of-mass energy squared and $\mu \approx 1$ GeV, we obtain
\[
\frac{dN_{A_1A_2+q\chi}}{dy} = \frac{N_c S Q_{11}^2}{24\pi^3} f(y),
\]
where we introduced a shorthand notation,
\[
f(y) = \left[ e^{\lambda y} \ln(1 + e^{-2\lambda y}) + e^{-\lambda y} \ln(1 + e^{2\lambda y}) \right] \times \left( 1 - \frac{Q_{11}}{\mu} e^{-y/2} \right)^3 \right\}.
\]

V. ELECTRON ENERGY SPECTRUM

To illustrate the derived results, we numerically compute the magnetic contribution to the electron energy spectrum produced in heavy-ion collisions at RHIC at the central rapidity $y_+ = 0$. Magnetic field strength in this case is such that $eB \gg m_e^2$. Taking also into account that $\omega \gg m_e$, we infer from (36) that $x \gg 1$. (This is not the case only at very small angles $\theta$ that are beyond the experimental resolution). At large $x$, the first term in the curly brackets of (35) and (37) is negligible compared to the second one. Bearing this in mind and substituting (37) and (48) into (43), we derive
\[
\frac{dN_{A_1A_2+\ell^{+}\ell^{-}}}{dt d\Gamma_+} = \frac{N_c S Q_{11}^2}{3\pi^5} \frac{m_e^2}{d^2} \int_0^{\pi/2} d\phi \int_0^{Y/2} dy \int_0^{\omega_m} d\omega \ln \frac{1}{\omega/\omega_m} e^{2\lambda y/2} \left\{ \frac{2}{x(1-x)} \right\} A'(x),
\]
where $x, \omega$, and $\chi$ are given by (36) and (38) with the following substitutions:
\[
\varepsilon = Q_{11} \cos y e^{\lambda y/2}, \quad \sin \theta = \sqrt{1 - (\sin \beta / \cosh y)^2}.
\]
Our calculation is valid when \( \varepsilon_+ \ll \varepsilon \), i.e., in the kinematic region \( \varepsilon_+ \ll Q_0 \sim 1-1.5 \) GeV. For electrons produced at \( y_+ = 0 \) at RHIC this translates into a condition \( p_{+\perp} = \varepsilon_+ \lesssim 0.5 \) GeV. In fact, as we argue in the next section, at larger \( \varepsilon_+ \) the magnetic contribution to the lepton spectrum is negligible.

Total lepton production rate also simplifies at large \( \varepsilon \). Taking integral over \( x \) in (15) yields [18]

\[
\int_{2\langle 1/2 \rangle}^{\infty} 2 (x^{3/2} + 1/\varepsilon) A^2(x) x^{1/4} (x^{3/2} - 4/\varepsilon)^{1/2} dx = -0.38 \varepsilon^{2/3}, \quad \varepsilon \gg 1.
\]  

(53)

Thus, in place of (39) we have

\[
\frac{dN_{q-\ell^+\ell^-}}{dt} = 0.38 \frac{2\alpha^2 e B z^2_{q} }{\pi m_e} \int_{2\langle 1/2 \rangle}^{\infty} d\omega \frac{\ln \frac{\varepsilon}{\omega}}{\varepsilon [1 + (\chi \varepsilon/\omega)^{1/3}]^2}.
\]  

(54)

Convoluting (54) with the hard quark spectrum (49) yields the total electron production rate as a function of magnetic field \( B \),

\[
\frac{dN_{A_1A_2-\ell^+\ell^-}}{dt} = 0.38 \frac{N_c S Q_{0\varepsilon}^2}{3\pi^2} \frac{\alpha^2 e B}{m_e} \sum_q z^2_{q} \int_{0}^{\pi/2} d\beta \int_{0}^{Y/2} dy f(y) 
\]

\[
\times \int_{2\langle 1/2 \rangle}^{\infty} d\omega \frac{\ln \frac{\varepsilon}{\omega}}{\varepsilon [1 + (\chi \varepsilon/\omega)^{1/3}]^2}.
\]  

(55)

All equations that we derived so far pertain to lepton production in a static magnetic field. In fact, magnetic field does change with time, albeit adiabatically. Its time dependence at the central rapidity \( y = 0 \) is approximately given by [22,23]

\[
eB(t) = \frac{\gamma Z R_A \sigma}{t^2} \exp \left\{ -\frac{R^2_{\perp} \sigma}{4t^2} \right\},
\]  

(56)

where \( \sigma \) is QGP electrical conductivity and \( Z \) and \( R_A \) are nuclear charge and radius correspondingly. Derivation of (56) assumes that QGP conductivity is constant. In expanding medium \( \sigma \) is a function of time, e.g., in Bjorken scenario \( \sigma \sim t^{-1/3} \). However, it has only a mild effect on the time dependence of magnetic field given by (56). Taking for the Gold nucleus \( Z = 79, R_A = 6.5 \) fm, we obtain that at \( t = 0.2 \) fm (which is \( \sim 1/Q_0 \)) \( eB = 1.3 m^2_e \), in agreement with earlier estimates [25]. However, time dependence of (56) significantly differs from the one found in Ref. [25] because it takes into account the electromagnetic response of QGP. The time dependence of magnetic field in conducting medium is shown by a solid line in Fig. 3. It is seen that even at \( t = 10 \) fm the magnetic field is about two orders of magnitude larger than the Schwinger field for electrons: \( eB/m_e^2 = (m_e/m_{\gamma^*})^2 = 1.3 \times 10^{-3} \). For comparison, we also calculated the dilepton spectrum without the medium effect on magnetic field. In this case, time dependence of the magnetic field can be modeled by boosted Coulomb field as

\[
eB = \frac{2\alpha Z R_A^2}{\gamma^2 t^2},
\]  

(57)

VI. COMPARISON OF MAGNETIC AND CONVENTIONAL PHOTON DECAY MECHANISMS

In the absence of a magnetic field, dilepton production by a fast quark is related to the dilepton production by a virtual...
FIG. 5. (Color online) Ratio of dilepton production yields: via photon splitting in magnetic field $N_{g\to\ell^+\ell^-}$ and via decay of virtual photon without magnetic field $P_{\gamma\to\ell^+\ell^-}$. Solid and dashed lines correspond to the time dependence of magnetic field according to (56) and (57). Photon’s rapidity $y = 0$.

photon as follows:

$$\frac{dN_{g\to\ell^+\ell^-}}{d\Gamma_+} = \frac{2\alpha}{3\pi} \int \frac{dM}{M} \frac{dN_{\gamma^*}}{d\Gamma_+},$$  \hspace{1cm} (58)

provided that the invariant mass $M$ satisfies $M \gg m_\ell$. If $\omega \gg M$, then the photon spectrum is $M$ independent and can be approximated by that of real photons. In that case, we obtain

$$\frac{dN_{g\to\ell^+\ell^-}}{d\Gamma_+} = \frac{2\alpha}{3\pi} \ln \frac{\omega}{m_\ell} \frac{dN_{\gamma^*}}{d\Gamma_+} = P_{\gamma^*\to\ell^+\ell^-} \frac{dN_{\gamma^*}}{d\Gamma_+},$$  \hspace{1cm} (59)

where $P_{\gamma^*\to\ell^+\ell^-}$ describes the probability to produce dilepton via photon decay. This quantity should be compared with $N_{g\to\ell^+\ell^-}$, which describes such a probability in a magnetic field. If one tries to reconstruct the photon spectrum using (58), then the result will be incorrect as it misses an important $B$-dependent contribution. To demonstrate how different these contributions are, we plotted their ratio in Fig. 5 for azimuthal angle $\beta = 0$, i.e., perpendicular to $B$. (We neglect small variations with $\beta$.) Magnetic contribution dominates at low frequencies. At midrapidity $\omega = k_\perp$ and we can infer from Fig. 5 that at $k_\perp = \omega < 0.2$ GeV the contribution to the dilepton yield induced by magnetic field dominates over the conventional virtual photon splitting. Even at $k_\perp \sim 0.6$ GeV it contributes as much as 20% to the yield. Dilepton production in a magnetic field certainly contributes to resolution of the puzzle of enhancement of dilepton production reported in Ref. [6]. However, as has been already mentioned, the exact value of magnetic field contribution is hard to pin down without accurate knowledge of the magnetic field dynamics.

VII. SUMMARY

It is hardly surprising that a magnetic field generated in heavy-ion collisions has a profound impact on dilepton production. Indeed, magnetic field strength by far exceeds the critical Schwinger’s value during the entire QGP lifetime. In the present work we calculated a contribution to the dilepton spectrum in the region $T < \varepsilon_+ < Q_s$ due to a magnetic field as a convolution of three factors: (i) the initial hard (i.e., nonthermal) quark distribution, (ii) equivalent photon flux, and (iii) photon decay rate. The last two factors exhibit a very strong dependence on a magnetic field. Because momentum is conserved only in the direction of magnetic field, the notion of invariant mass applies only to the part of the spectrum independent of magnetic field. Consequently, we plotted the electron spectra in Fig. 3 as a function of electron’s energy. We also derived formulas for fully differential distribution of leptons. We argued that the magnetic contribution is important at electron energies below ~0.5 GeV at midrapidity at RHIC. In fact, it becomes the dominant source of dileptons at lower electron energies (see Fig. 5).

In the region $\varepsilon_+ < T$, the contribution of soft quarks, i.e., those quarks that are part of the QGP, cannot be neglected and must be added to the magnetic contribution of hard quarks computed in this paper. Its calculation, however, is much more complicated because the equivalent photon approximation is no longer applicable. We plan to discuss this contribution elsewhere.

ACKNOWLEDGMENTS

I am grateful to Yoshimasa Hidaka and Kazunori Itakura for interesting discussions that initiated this work and to Thomas Hemmick for useful correspondence. This work was supported in part by the US Department of Energy under Grant No. DE-FG02-87ER40371.