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J/Psi dissociation in parity-odd bubbles

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We calculate the quarkonium dissociation rate in the $P$ and $CP$-odd domains (bubbles) that were possibly created in heavy-ion collisions. In the presence of the magnetic field produced by the valence quarks of colliding ions, parity-odd domains generate electric field. Quarkonium dissociation is the result of quantum tunneling of quark or antiquark through the potential barrier in this electric field. The strength of the electric field in the quarkonium comoving frame depends on the quarkonium velocity with respect to the background magnetic field. We investigate momentum, electric field strength and azimuthal dependence of the dissociation rate. Azimuthal distribution of quarkonia surviving in the electromagnetic field is strongly anisotropic; the form of anisotropy depends on the relation between the electric and magnetic fields and quarkonium momentum $P_\perp$. These features can be used to explore the properties of the electromagnetic field created in heavy ion collisions.

1. Introduction

Solid theoretical arguments [1] and numerical calculations [2, 3] indicate a possible existence of very strong magnetic fields in heavy-ion collisions. Electromagnetic fields of such enormous intensity have never been experimentally studied and therefore present a great interest, which extends far beyond applications in the nuclear physics. What are the possible manifestations of such magnetic field? An effect that has recently attracted a lot of attention is the Chiral Magnetic Effect (CME) [1, 4–7]. If a metastable $P$ and $CP$-odd bubble is induced by axial anomaly in the hot nuclear matter, then in the presence of external magnetic field $B_0$ the bubble generates an electric field which is parallel to the magnetic one. According to [5] the value of the electric field $E_0$ in the bubble is

$$E_0 = -N_c \sum_f \frac{e_f^2}{4\pi^2} \frac{\alpha}{N_f} B_0 = -\frac{2}{3} \frac{\alpha}{\pi} B_0$$

(1)

where the sum runs over quark flavors $f$ and we assumed that only three lightest flavors contribute. The value of the $\Theta$-angle fluctuates from event to event. CME refers to the macroscopic manifestation of this effect – separation of electric charges with respect to the reaction plane. This effect is a possible explanation of experimentally observed charge asymmetry fluctuations [8–10].

Other effects of the magnetic field that do not require existence of the parity-odd bubbles and have been recently discussed are: synchrotron radiation by fast fermions [11], polarization of the fermion spectra [11] (also known as the Sokolov–Ternov effect [12]), enhanced dilepton production [13] and azimuthal anisotropy of the quark–gluon plasma (QGP) [14, 15]. These possible effects await their experimental study. Thus, strong magnetic field must have a powerful impact on the behavior of the quark–gluon plasma.

Since CME cannot be the only effect induced by the magnetic field we are motivated to look for magnetic field manifestations in other observables. We have recently pointed out in Ref. [16] that the Lorentz ionization, i.e. dissociation of a moving bound state in external magnetic field, is phenomenologically significant in relativistic heavy-ion collisions in the presence of strong [1, 2] quasi-static [11] magnetic field generated by colliding ions. The ionization or, equivalently, dissociation happens because quarkonium constituents have finite probability to tunnel through the potential barrier in the presence of electric field, which appears upon boosting to the comoving frame. The dissociation rate depends on the magnetic field strength, bound state velocity and its binding energy. The dissociation rate indirectly depends on the properties of the nuclear matter by the way of dependence of the binding energy on temperature. Since at higher temperatures the binding energy is smaller, the dissociation rate is higher. Still, the Lorentz ionization can happen even if no matter is formed, provided that the magnetic field is strong enough and/or the bound state is fast enough.
In this Letter we address a different type of ionization which is possible only if a metastable $P$ and CP-odd bubbles are formed and trigger emergence of electric field $E_0$ given by (1). Due to this field, there is a finite dissociation rate of quarkonium at rest in the laboratory frame. When boosted to the comoving frame quarkonium dissociation is a combined effect of the boosted electric $E_0$ and magnetic $B_0$ fields. In the comoving frame, electric and magnetic fields are directed at some angle with respect to each other depending on the quarkonium kinematics. The main goal of this Letter is to investigate the dissociation rate in this case.

2. Dissociation rate

Ionization probability of quarkonium equals its tunneling probability through the potential barrier. In the WKB approximation the later is given by the transmission coefficient and was calculated in [16]. In this method contribution of the quark spin can be easily taken into account. Another method of calculating the ionization probability, the imaginary time method [17,18], was employed in [19–21] by Popov, Karnakov and Mur. In particular, they derived in the quasi-classical approximation. Such a calculation requires matching quark wave function inside and outside the potential barrier [23]. Extension of this approach to the relativistic case is challenging due to analytical difficulties of the relativistic two-body problem. Fortunately, it was argued in [16,21] that the non-relativistic approximation provides a very good accuracy in the $\epsilon$-relative to the relativistic case. In particular, they derived in the non-relativistic approximation the pre-exponential factor that appears due to the deviation of the quark wave function from the non-relativistic approximation. Such a calculation requires matching the wave function from the quasi-classical approximation, which provides a very good accuracy to the dissociation rate [16,21]. The quarkonium dissociation rate in the comoving frame in the non-relativistic approximation is given by [19]

$$W = \frac{8E_b}{\epsilon} P(\gamma, \theta) C^2(\gamma, \theta) e^{-\frac{2}{3} \frac{\epsilon}{\gamma}}$$

where function $g$ reads

$$g = \frac{3\tau_0}{2\rho} \left[ \frac{1}{1 + \frac{\rho_0^2}{\gamma^2} - 1} \right]^{1/2} \sin \theta - \frac{2}{3\gamma} \cos^2 \theta$$

and functions $P$ and $C$ are given in Appendix A. The contribution of quark spin is taken into account by replacing $\epsilon_0 \rightarrow \epsilon_0' = \epsilon_0 - \frac{g}{e} \vec{e} \cdot \vec{B}$ [16]. Function $g$ represents the leading quasi-classical exponent. $P$ is the pre-factor to the $S$-wave state of quarkonium and $C$ accounts for the Coulomb interaction between the valence quarks. Parameter $\tau_0$ satisfies the following equation

$$\tau_0^2 - \sin^2 \theta (\rho_0 \coth \rho_0 - 1)^2 = \gamma^2$$

which establishes its dependence on $\theta$ and $\gamma$. Note, that in the limit $E \rightarrow 0$ the dissociation rate (6) exponentially vanishes. This is because pure magnetic field cannot force a charge to tunnel through a potential barrier.

Eq. (6) gives the quarkonium dissociation rate in a bubble with a given value of $\theta$. Its derivation assumes that the dissociation process happens entirely inside a bubble and that $\theta$ is constant inside the bubble. Since in a relativistic heavy ion collision many bubbles can be produced with a certain distribution of $\theta$'s (with average $\langle \theta \rangle = 0$) more than one bubble can affect the dissociation process. This will result in a destructive interference leading to reduction of the CP-odd effect on quarkonium dissociation. However, if a typical bubble size $R_0$ is much larger than the size of quarkonium $R$, the dissociation is affected by one bubble at a time independently of others, and hence the interference effect can be neglected. In this case (6) provides, upon a proper average, a reasonable estimate of quarkonium dissociation in a heavy ion collision. We can estimate the bubble size as the size of the sphaleron, which is of the order of the chromo-magnetic screening length $\sim 1/g^2T$, whereas the quarkonium size is of the order $a_s/\epsilon_0$. Consequently, at small coupling and below the zero-field dissociation temperature (i.e. when $\epsilon_0$ is not too small) $R_0$ is parametrically much larger than $R$. A more quantitative estimate of the sphaleron size is $R_0 \sim 1.2/a_sN_cT \sim 0.4$ fm [22], whereas for $J/\psi$ $R_1 \approx 0.1-0.2$ fm. Thus, based on this estimate bubble interference can be neglected in the first approximation. However, since the ratio $R_1/R_0$ is actually not so small this effect nevertheless warrants further investigation.

To obtain the experimentally observed $J/\psi$ dissociation rate we need to average (6) over the bubbles produced in a given event and then over all events. To this end it is important to note that

1 Calculations in Refs. [1–3] yield very short relaxation time of magnetic field because they neglect the electromagnetic response of the quark-gluon medium, see [11] for details.
Fig. 1. Dissociation rate of $J/\psi$ at $eB_0 = 15m_b^2$, $\phi = \pi/2$ (in the reaction plane), $\eta = 0$ (midrapidity) as a function of (a) $p_\perp$ at $\varepsilon_b = 0.16$ GeV and (b) $\varepsilon_b$ at $p_\perp = 1$ GeV.

because the dissociation rate depends only on $\rho_0^2$ it is insensitive to the sign of the $E_0$ field or, in other words, it depends only on absolute value of $\Theta$ but not on its sign. Therefore, it stands to reason that although the precise distribution of $\Theta$’s is not known, (6) gives an approximate event average with parameter $\Theta$ representing a characteristic absolute value of the theta-angle.

3. Limiting cases

Before we proceed with the numerical calculations, let us consider for illustration several limiting cases. If quarkonium moves with non-relativistic velocity, then in the comoving frame electric and magnetic fields are approximately parallel $\theta \approx 0$, whereas in the ultra-relativistic case they are orthogonal $\theta \approx \pi/2$, see (4).

In the later case the electromagnetic field in the comoving frame does not depend on $E_0$ as seen in (3) and therefore the dissociation rate becomes insensitive to the CME. In our estimates we will assume that $\rho_0 < 1$ which is the relevant phenomenological situation. Indeed, it was proposed in [5] that $\rho_0 \sim \alpha < 1$ produces charge fluctuations with respect to the reaction plane of the magnitude consistent with experimental data.

1) $\theta \geq 0$, i.e. electric and magnetic fields are approximately parallel. This situation is realized in the following two cases. (i) Non-relativistic quarkonium velocities: $V \ll \rho_0$ or (ii) motion of quarkonium at small angle $\phi$ to the direction of the magnetic field $b_0$: $\phi \ll \rho_0 / \gamma_1 V$. In both cases $E \approx E_0$ and $B \approx B_0$. This is precisely the case where the dissociation rate exhibits its strongest sensitivity to the strength of the electric field $E_0$ generated by the local parity violating QCD effects. Depending on the value of the $\gamma$ parameter defined in (5) we can distinguish the case of strong electric field $\gamma \gg 1$ and weak electric field $\gamma \ll 1$ [20]. In the former case, $g = (3/8)\gamma$, $P = (8e) \gamma e^{-\gamma^2/2}$, $C = e^{\pi\gamma/2}/\gamma$. Substituting into (6) the dissociation rate reads

$$w = \frac{8e_b}{e\gamma} \sqrt{\frac{3}{e}} e^{-\gamma^2/2} e^{-\frac{2}{3} \rho_0} \frac{16e_b^2 m}{eB_0} \sqrt{\frac{3}{e}} e^{-\frac{2}{3} \rho_0} \gamma^2, \quad \gamma \gg 1$$

(9)

In the later case, $g = P = C = 1$ and

$$w = \frac{8e_b}{e} e^{-\frac{2}{3} \rho_0} = \frac{8e_b m^2}{eE_0} \left( \frac{2e_b}{m} \right)^{3/2} e^{-\frac{2\rho_0}{3}} \frac{2\rho_0}{3}, \quad \gamma \ll 1$$

(10)

where the electromagnetic field in the comoving frame equals one in the laboratory frame as was mentioned before.

2) $\theta \sim \pi/2$, i.e. electric and magnetic fields are approximately orthogonal. This occurs for an ultra-relativistic motion of quarkonium $V \to 1$. In this case

$$B = E = B_0 \gamma M \frac{1}{\sqrt{1 + \rho_0^2}}$$

(11)

This case was discussed in detail in our previous paper [16]. In particular for $\gamma \ll 1$ we get

$$w = \frac{8e_b m^2}{eE} \left( \frac{2e_b}{m} \right)^{3/2} e^{-\frac{2\rho_0}{3}} \frac{2\rho_0}{3}$$

(12)

Due to (9) and (12) dependence of $w$ on $E_0$ is weak unless $\rho_0 \gg 1$.

4. $J/\psi$ dissociation rate

One of the most interesting applications of the formalism described in the previous sections is calculation of the dissociation rate of $J/\psi$ which is considered a litmus test of the quark-gluon plasma [25].

Let $z$ be the heavy ions collision axis; heavy-ion collision geometry implies that $b_0 \cdot \hat{z} = 0$. The plane containing $z$-axis and perpendicular to the magnetic field direction is the reaction plane. We have

$$b_0 \times \mathbf{V}^2 = V_z^2 + V_\perp^2 \sin^2 \phi$$

(13)

where $\phi$ is the angle between the directions of $b_0$ and $\mathbf{V}$ and we denoted vector components in the $xy$-plane by the subscript $\perp$. We can express the components of the quarkonium velocity $\mathbf{V}$ in terms of the rapidity $\eta$ as $V_z = \tanh \eta$, $V_\perp = P_\perp/(M_\perp \cosh \eta)$, where $P$ and $M$ are the quarkonium momentum and mass and $M_\perp^2 = M^2 - P_\perp^2$.

Results of numerical calculations are exhibited in Figs. 1–3. In Fig. 1 we show the dissociation rate of $J/\psi$ for several values of the electric field $E_0$ induced by the Chiral Magnetic Effect. Note, that the typical size of the medium traversed by a quarkonium is a few fm. Therefore, $w \sim 0.3–0.5$ fm$^{-3}$ corresponds to complete destruction of $J/\psi$’s. This means that in the magnetic field of strength $eB_0 \sim 15m_b^2$ all $J/\psi$’s with $P_\perp \geq 0.5$ GeV are destroyed independently of the strength of $E_0$. This $P_\perp$ is lower than what we estimated previously in [16] neglecting the pre-factors in the dissociation rate. Since magnetic field strength decreases towards the QGP periphery, most of $J/\psi$ surviving at later times originate from

\footnote{Note, that the limit $\gamma \gg 1$ is different in $\theta = \pi/2$ and $\theta < \pi/2$ cases [19].}
that region. Effect of the electric field $E_0$ of the parity-odd bubble is strongest at low $P_\perp$, which is consistent with our discussion in the previous section. The dissociation rate at low $P_\perp$ exponentially decreases with decrease of $E_0$. Probability of quarkonium ionization by the fields below $E_0 \lesssim 0.1B_0$ (i.e. $\rho_0 \lesssim 0.1$) is exponentially small. This is an order of magnitude higher than the estimate $\rho_0 \sim \alpha$ proposed in [5].

As the plasma temperature varies, so is the binding energy of quarkonium although the precise form of the function $\varepsilon_b(T)$ is model-dependent. The dissociation rate picks at some $\varepsilon_b^0$ (see Fig. 1(b)), where $\varepsilon_b^{ac}$ is the binding energy in vacuum, indicating that $J/\psi$ breaks down even before $\varepsilon_b$ drops to zero, which is the case at $B_0 = 0$. This $\varepsilon_b^0$ is a strong function of $E_0$ as can be seen in Fig. 2. It satisfies the equation $d\varepsilon_b/d\varepsilon_b = 0$. In the case $\gamma \ll 1$ (10) and (12) imply that

$$
\varepsilon_b^0 = m \left( \frac{5eE}{2m^2} \right)^{2/3}, \quad \gamma \ll 1
$$

(14)

At $\gamma \gg 1$ and $\theta = \pi/2$ we employ (9) to derive the condition $(\varepsilon_b^0)^2 + eB\varepsilon_b^0/2m - e^2/B = 0$. In view of (11) $E \approx B$ and we obtain

$$
\varepsilon_b^0 = \frac{eB}{4m} \left( \sqrt{\frac{16m^2}{eB} + 1} - 1 \right) \approx \sqrt{eB}, \quad \gamma \gg 1
$$

(15)

where in the last step we used that $eB \ll m^2$. For a given function $\varepsilon_b(T)$ one can convert $\varepsilon_b^0$ into the dissociation temperature, which is an important phenomenological parameter.

In the absence of the CME the dissociation probability peaks in the direction perpendicular to the direction of magnetic field $b_0$, i.e. in the reaction plane. Dissociation rate vanishes in the $b_0$ direction. Indeed, for $V \cdot b_0 = 0$ (3) implies that $E = 0$. This feature is seen in the left panel of Fig. 3. At finite $E_0$ the dissociation probability is finite in the $b_0$ direction making the azimuthal distribution more symmetric. The shape of the azimuthal distribution strongly depends on quarkonium velocity: while at low $V$ the strongest dissociation is in the direction of the reaction plane, at higher $V$ the maximum shifts towards small angles around the $b_0$ direction. Extrema of the azimuthal distribution are roots of the equation $d\varepsilon_b/d\phi = 0$. At $\gamma \ll 1$ it yields minimum at $\phi_0 = 0$, maximum at $\phi_0 = \pi/2$ and another maximum that satisfies the condition (neglecting the spin-dependence of $\varepsilon_b$)

$$
eE_0\sqrt{1 + \gamma_1^2(V_2^2 + V_2^2 \sin^2 \phi_0)(1 + \rho_0^{-2})} = \frac{2m^2}{3} \left( \frac{2\varepsilon_b}{m} \right)^{3/2}
$$

(16)

In order to satisfy (16) $\phi_0$ must decrease when $V$ increases and visa versa. This features are seen in the left panel of Fig. 3.

Spectrum of quarkonia surviving in the electromagnetic field is proportional to the survival probability $P = 1 - \rho_0$, where $t$ is the time spent by the quarkonium in the field. Consider $P$ as a function of the angle $\chi$ between the quarkonium velocity and the reaction plane $\chi = \pi/2 - \phi$. Fourier expansion of $P$ in $\chi$ reads

$$
P(\chi) = \frac{1}{2} P_0 + \sum_{n=1}^{\infty} P_n \cos(n\chi), \quad P_n = \frac{1}{\pi} \int_0^\pi P(\chi) \cos(n\chi) d\chi.
$$

(17)

Ellipticity of the distribution is characterized by the “elliptic flow” coefficient $v_2$ defined as

$$
v_2 = \frac{P_2}{P_0} = \frac{\int_0^\pi (1 - \omega t) \cos(2\chi) d\chi}{\pi \langle P \rangle} = -\frac{\langle \chi \rangle}{\pi \langle P \rangle} \int_0^\pi w \cos(2\chi) d\chi
$$

(18)

where $\langle P \rangle$ denotes average of $P$ over the azimuthal angle. These formulas are applicable only as long as $\omega t < 1$ because otherwise there are no surviving quarkonia. In the right panel of Fig. 3 we show $v_2(P)/t$, which is independent of $t$, as a function of $P_\perp$. As expected, in the absence of the CME, $v_2$ is negative at low $P_\perp$ and positive at high $P_\perp$. $v_2$ changes sign at $P_\perp$ that depends on the strength of the electric field. It decreases as $E_0$ increases until at $E_0 \approx B_0$ it becomes positive at all $P_\perp$. Fig. 3(b) provides the low bound for $v_2$ because $P \ll 1$ and $t \gtrsim 1$ fm. We thus expect that magnetic field strongly modifies the azimuthal distribution of the produced $J/\psi$. Role of the magnetic field in generation of azimuthal anisotropies in heavy-ion collisions has been pointed out before in [13,14].

5. Summary

In this Letter we studied the effect of the parallel electric and magnetic fields on the dissociation rate of quarkonia, and particularly of $J/\psi$. Our main observation is that the CME effect on the dissociation rate is significantly different than the effect of the pure magnetic field if $E_0 \gtrsim 0.1B_0$ that implies an estimate of the $\Theta$-parameter: $|\Theta|/\pi \simeq 0.1/\alpha$. This is about an order of magnitude larger than is required for the charge separation [5]. Due to the electric field $E_0$ of the parity-odd bubble quarkonium dissociation rate is finite at low $P_\perp$, as indicated in Fig. 1. Fortunately, $J/\psi$'s can be measured down to very low $P_\perp$ [26,27]. The effect of the electric field is most clear along the direction perpendicular to the reaction plane, because magnetic component of the Lorentz force vanishes in this direction, see Fig. 3(a).

Azimuthal distribution of dissociation rate is strongly asymmetric in external magnetic field. The second harmonic $v_2$ of the azimuthal distribution of survival probability is large and negative at low $P_\perp$, while at high $P_\perp$ it is positive; zero of $v_2$ depends on the relation between the electric and magnetic fields. According to the preliminary experimental data $J/\psi$'s $v_2$ is either small, about a few per cent, or zero [28,29]. Absence of such asymmetry in the experimental data may have two reasons. (i) Magnetic field is significantly weaker and short-lived than suggested in [1,11], which
is however at odds with the charge separation observations [8–10].
(ii) Almost none of $J/\psi$’s produced in the center of QGP survive. Rather they originate from the peripheral regions. The later scenario is realized if time $t$ spent by quarkonium in the field is large because the dissociation rate only linearly increases with $t$ but exponentially decreases with the decrease of the field strength toward the QGP periphery. Finally, if the bubble interference effects due to finite bubble size discussed at the end of Section 2 are important they can significantly reduce the CP-odd effect on $J/\psi$ dissociation rate.

Abundance of possible effects associated with strong magnetic field calls for a detailed experimental investigation.

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Appendix A

Here we list functions $C$ and $P$ that appear in (6):

$$P = \frac{\gamma^2}{\tau_0} \left( \frac{\tau_0 \coth \tau_0 + \frac{\sinh \tau_0 \cosh \tau_0}{\tau_0} - 2}{\tau_0} \right) \sin^2 \theta$$

$$+ \sinh^2 \tau_0 \cos^2 \theta \right)^{-1/2} \right) \left( \frac{1}{2} \frac{\tau_0}{\tau_0 - \tau} \right)$$

$$C = \exp \left[ \ln \frac{\tau_0}{2\gamma} + \int_0^{\tau_0} d\tau \left( \frac{\gamma}{\xi(\tau)} - \frac{1}{\tau_0 - \tau} \right) \right]$$

$$\xi(\tau) = \left\{ \frac{1}{4} \left( \frac{\tau_0^2 - \tau^2}{\sinh \tau_0} \right)^2 + \frac{\tau_0^2}{\sinh \tau_0} \right\} \left( \frac{\cosh \tau_0 - \cos \tau}{\sin \tau_0} \right)^2$$

$$- \left( \frac{\sinh \tau}{\sinh \tau_0} - \frac{\tau}{\tau_0} \right)^2 \sin^2 \theta \right\}^{1/2} \left( \frac{1}{\tau_0 \tau_0} \right)$$

References