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Quarkonium dissociation in quark-gluon plasma via ionization in a magnetic field

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We study the impact of a magnetic field, generated in collisions of relativistic heavy ions, on the decay probability of a quarkonium produced in the central rapidity region. The quark and antiquark components are subject to mutually orthogonal electric and magnetic fields in the quarkonium comoving frame. In the presence of an electric field, the quarkonium has a finite dissociation probability. We use the WKB approximation to derive the dissociation probability. We find that the quarkonium dissociation energy, i.e., the binding energy at which the dissociation probability is of order unity, increases with the magnetic field strength. It also increases with quarkonium momentum in the laboratory frame owing to the Lorentz boost of the electric field in the comoving frame. We argue that $J/\psi$'s produced in heavy-ion collisions at the Large Hadron Collider with $P_L > 9$ GeV would dissociate even in vacuum. In plasma, the $J/\psi$ dissociation in a magnetic field is much stronger because of the decrease of its binding energy with temperature. We discuss phenomenological implications of our results.

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I. INTRODUCTION

In their seminal 1986 paper [1], Matsui and Satz suggested that quarkonium production in high-energy nucleus-nucleus collisions can be used as an indicator for production of quark-gluon plasma (QGP). They argued that a quarkonium of radius $r$ will dissociate in a QGP when the Debye screening radius $r_D(T)$ becomes smaller then $r$. The observed effect would be an "anomalous" suppression of quarkonium yield. In practice, determination of the screening radius $r_D(T)$ turned out to be a remarkably difficult problem even in a static medium. The existing approaches to solve this problem include lattice QCD calculations of quarkonium correlators [2–7], construction of potential models of quarkonium spectral functions [8–14], and use of effective field theory [15–17]. It is remarkable that in spite of much progress there still exists substantial uncertainty in the value of the $J/\psi$ dissociation temperature and in the functional form of $r_D(T)$; see, e.g., [18,19]. Another complication arises due to "cold nuclear matter effects," i.e., nuclear effects that are independent of temperature and that would occur even if no plasma were formed. There is ongoing controversy as to the nature of these effects, although there is agreement that they lead to suppression of quarkonium yield [20–29].

It has been recently realized [30] that colliding heavy ions produce a very strong magnetic field in the direction perpendicular to the reaction plane (defined as a plane containing the momenta of the ions and the impact parameter). This has a number of interesting phenomenological consequences [30–33]. The strength of the magnetic field at the Relativistic Heavy Ion Collider (RHIC) is estimated as $eB_0 \approx m_r^2$, and at the Large Hadron Collider (LHC) as $eB_0 \approx 15 m_r^2$ [30,34]. The relaxation time of the magnetic field, neglecting the back reaction, is of the order of the width of the two Lorentz-contracted nuclei divided by $c$, which is $\sim 0.1$ fm at the RHIC and $\sim 0.01$ fm at the LHC. However, we argued recently [31] that the relaxation time of this field is actually much larger if the back reaction is taken into account.\(^{1}\) In fact, the magnetic field can be treated as static if the distance over which it significantly varies is much larger than the quarkonium radius. We will refer to this approximation as "quasistatic." For a quarkonium with binding energy $\epsilon_b$ and radius $\alpha_s/\epsilon_b$, the quasistatic approximation applies when $\epsilon_b\tau/\alpha_s \gg 1$. For $\tau = 2$ fm we get $\epsilon_b\tau/\alpha_s \approx 23$, which is comfortably large enough to justify the quasistatic approximation, where we assumed that $\epsilon_b$ is given by its vacuum value. As $T$ increases, $\epsilon_b$ drops, while $\tau$ increases. The temperature dependence of $\epsilon_b$ is model dependent, but it is certain that eventually $\epsilon_b$ vanishes at some finite temperature $T_0$. Therefore, only in the close vicinity of $T_0$, i.e., at very small binding energies, is the quasistatic approximation not applicable. We thus rely on the quasistatic approximation throughout the paper.

\(^{1}\)This can be seen as follows. Upon collision, the magnetic field $B_0$ rapidly decreases with time, inducing an electric field of strength $E \sim B_0 R/\tau$ circulating around the direction of $B_0$; here $R$ is the nuclear radius and $\tau$ the relaxation time. This electric field generates circular currents that, by Lenz's law, support the original magnetic field. There are two types of generated current: (i) Foucault currents in the QGP; (ii) a current of charged fermions produced via the Schwinger mechanism [35]. Note that lepton Schwinger pairs are certainly not in equilibrium with the QGP and therefore do not contribute to the plasma electrical conductivity. In the first case, for a medium with electrical conductivity $\sigma$ we estimate $B_0 \sim j R \sim \sigma E R \sim \sigma B_0 R^2/\tau$, which implies

$$\tau \sim \sigma R^2. \quad (1)$$

Using the values of the electrical conductivities obtained in Ref. [36] and in Refs. [37,38] we get $\tau \sim 8$ fm and $\tau \sim 0.4$ fm, respectively. In the second case, recalling that the density of the Schwinger pairs is $n \sim (eE)^2$, we have $B_0 \sim env R \sim e^2 B_0^2 R^3/\tau$, which implies

$$\tau \sim e^2(eB_0) R^3. \quad (2)$$

We took into account that the Schwinger pairs are relativistic, $v \sim 1$, since their longitudinal momentum is of order $eE \sim eB_0 R \gg m_r$ for $R = 5$ fm. We estimate that at the RHIC $\tau \sim 0.5$ fm, while at the LHC $\tau \sim 8$ fm. A more accurate estimate of $\tau$ requires not only better knowledge of electrical conductivity in expanding plasma, but also numerical solution of the magnetohydrodynamic equations.
A magnetic field has a threefold effect on a quarkonium:

(1) Lorentz ionization. Suppose the quarkonium travels with constant velocity in a magnetic field in the laboratory frame. Boosting to the quarkonium comoving frame, we find mutually orthogonal electric and magnetic fields given by Eqs. (3) and (4). In the presence of an electric field the quark and antiquark have a finite probability to tunnel through the potential barrier, thereby causing quarkonium dissociation. In atomic physics such a process is referred to as Lorentz ionization. In the nonrelativistic approximation, the tunneling probability is of order unity when the electric field $E$ in the comoving frame satisfies $eE \gtrsim m^{1/2} v_b^{3/2}$ (for weakly bound states), where $\varepsilon_b$ is the binding energy and $m$ is the quark mass; see Eq. (26).

This effect causes a significant increase in the quarkonium dissociation energy $\varepsilon_d$. The corresponding results for $J/\psi$ are exhibited in Fig. 4. They suggest that a $J/\psi$ in plasma dissociates at a much lower temperature than it would in the absence of a magnetic field.

(2) Zeeman effect. The energy of a quarkonium state depends on spin $S$, orbital angular momentum $L$, and total angular momentum $J$. In a magnetic field these states split; the splitting energy in a weak field is $\Delta M = \frac{eB}{2m} g_J S$, where $J_z = -J, -J + 1, \ldots, J$ is the projection of the total angular momentum on the direction of the magnetic field, $m$ is the quark mass, and $g$ is the Landé factor depending on $J$, $L$, and $S$ in a well-known way; see, e.g., [39]. For example, a $J/\psi$ with $S = 1, L = 0$, and $J = 1$ ($g \approx 2$) splits into three states with $J_z = \pm 1, 0$, and with mass difference $\Delta M = 0.15$ GeV, where we used $eB_0 = 15m_\psi^2$ (the value of magnetic field expected at the LHC [30,34]). Thus, the Zeeman effect leads to the emergence of new quarkonium states in plasma.

(3) Distortion of the quarkonium potential in a magnetic field. This effect arises in higher-order perturbation theory and becomes important at field strengths of order $B \sim 3\pi a^2 / e^3$ [40]. This is $3\pi/a$ times stronger than the critical Schwinger field. Therefore, this effect can be neglected at the present collider energies.

The purpose of this paper is the analysis of the Lorentz ionization of quarkonia. Our paper is organized as follows. In Sec. II we give a fully relativistic derivation of the Lorentz ionization probability $w$ using the WKB approximation. We treat the force binding $q$ and $\bar{q}$ into a quarkonium as a short-range one, i.e., $(M\varepsilon_b)^{1/2} R \ll 1$, where $\varepsilon_b$ and $M$ are the binding energy and mass of the quarkonium, respectively, and $R$ is the radius of the nuclear force given by $R \approx \alpha_s / \sigma^{1/2}$, where $\sigma = 1$ GeV/fm is the string tension. For example, the binding energy of $e$ and $\gamma$ in a $J/\psi$ in vacuum is $\varepsilon_b = 0.64$ GeV $\ll M / R^2 = M\sigma / \alpha_s \approx 3$ GeV. This approximation is even better at finite temperature on account of the decrease in $\varepsilon_b$. Regarding $J/\psi$ as being bound by a short-range force enables us to calculate the dissociation probability $w$ with exponential accuracy, $w \approx e^{-f}$, independently of the precise form of the quarkonium wave function. This is especially important since solutions of the relativistic two-body problem for quarkonium are not readily available. We will argue that exponential accuracy is adequate for determining the quarkonium ionization energy due to the rapid variation of $f$ with binding energy. Our result for $w$ is in agreement with previous calculations [41].

In Sec. III we study the nonrelativistic approximation of the ionization probability $w$. We argue that it provides a remarkably good estimate of relativistic formulas; see Fig. 2. This is an important observation as it allows us to include the contribution of the quark spin interaction with the magnetic field. A fully relativistic calculation that accounts for the spin contribution is not yet available. This is discussed in Sec. III D. In Sec. III we also show that the relativistic formulas for $w$ derived in Sec. II reduce to nonrelativistic formulas found in the literature.

In Sec. IV we calculate the dissociation energy of the $J/\psi$ in a magnetic field and claim that it strongly depends on the magnetic field and the $J/\psi$’s velocity $V$ in the laboratory frame. Our results are summarized in Fig. 4. In a strong magnetic field such as the one expected to be produced at the LHC, a $J/\psi$ moving with $P_z > 9$ GeV in the reaction plane is expected to dissociate because of the magnetic field even in vacuum. At finite temperature, when the binding energy decreases, dissociation becomes prominent at lower transverse momenta. The magnetic field has no influence on the $J/\psi$ dissociation when it moves perpendicularly to the reaction plane (i.e., parallel to the field) because the corresponding electric field vanishes in the comoving frame. The nontrivial azimuthal angle dependence of $w$ may be an important source of azimuthal anisotropy of $J/\psi$ production in heavy-ion collisions.

In Sec. V we discuss the phenomenological significance of our results. Before continuing with the main part of the paper, we list here some of our notational definitions in order to avoid confusion: $V$ and $P$ are the velocity and momentum of the quarkonium in the laboratory frame; $M$ is its mass; $p$ is the momentum of the quark or antiquark in the comoving frame; $m$ is its mass; $B_0$ is the magnetic field in the laboratory frame; $E$ and $B$ are the electric and magnetic fields in the comoving frame; $\gamma_L$ is the quarkonium Lorentz factor; and $\gamma$ is a parameter defined in Eq. (21). We use Gauss units throughout the paper; note that the expressions $eB$, $eE$, and $eB_0$ are the same in Gauss and Lorentz-Heaviside units.

II. QUARKONIUM IONIZATION RATE

A. Comoving frame

Consider a quarkonium traveling with velocity $V$ in a constant magnetic field $B_0$. Let $B$ and $E$ be magnetic and electric fields in the comoving frame, and let the subscripts $\parallel$ and $\perp$ denote field components parallel and perpendicular to $V$, respectively. Then,

$$E_\parallel = 0, \quad E_\perp = \gamma_L V \times B_0,$$

$$B_\parallel = \frac{B_0 \cdot V}{V}, \quad B_\perp = \gamma_L \frac{(V \times B_0) \times V}{V^2},$$

where $\gamma_L = (1 - V^2)^{-1/2}$. Clearly, in the comoving frame $B \cdot E = 0$. If the quarkonium travels at angle $\phi$ with respect to the
magnetic field in the laboratory frame, then
\[ B = B_0 \sqrt{\cos^2 \phi (1 - \gamma_L^2) + \gamma_L^2}, \quad E = B_0 \gamma_L V \sin \phi. \quad (4) \]
We choose the \( z \) and \( y \) axes of the comoving frame such that \( B = B z \) and \( E = E \hat{y} \). A convenient gauge choice is \( A = -B y \hat{x} \) and \( \varphi = -E y \). For future reference we also define a useful dimensionless parameter \( \rho \) [42]
\[ \rho = \frac{E}{B} = \frac{\gamma_L V \sin \phi}{\sqrt{\cos^2 \phi (1 - \gamma_L^2) + \gamma_L^2}}. \quad (5) \]
Note that (i) \( 0 \leq \rho \leq 1 \) because \( B^2 - E^2 = B_0^2 \geq 0 \) and (ii) when the quarkonium moves perpendicularly to the magnetic field \( B_0, \rho = V \).

**B. WKB method**

It is natural to study quarkonium ionization in the comoving frame. As explained in the Introduction, ionization is quantum tunneling through the potential barrier caused by the electric field \( E \). In this section we employ the quasi-classical, or WKB, approximation to calculate the quarkonium decay probability \( w \). For the gauge choice specified in Sec. II A the quark energy \( \epsilon_0 (\epsilon_0 < m) \) in an electromagnetic field can be written as
\[ \epsilon_0 = \sqrt{m^2 + (p - eA)^2 + e \varphi} = \sqrt{m^2 + (p_x + eB y)^2 + p_z^2 + e E y}. \quad (6) \]
In terms of \( \epsilon_0 \), the quarkonium binding energy is \( \epsilon_b = m - \epsilon_0 \). To simplify notation, we set \( p_z = 0 \), because the quark moves with constant momentum along the direction of the magnetic field.

The effective potential \( U(y) = \epsilon_0(y) - \sqrt{m^2 + p^2} \) corresponding to Eq. (6) is plotted in Fig. 1. We can see that the tunneling probability is finite only if \( E > 0 \). It is largest when \( B = 0 \). It has already been noted in Refs. [41–43] that the effect of the magnetic field is to stabilize the bound state. In spite of the linearly rising potential (at \( B > E \)), the tunneling probability is finite as the result of rearrangement of the QED vacuum in the electric field.

The ionization probability of quarkonium equals its tunneling probability through the potential barrier. The latter is given by the transmission coefficient
\[ w = e^{-f} \int_0^\infty \sqrt{-p dy} = e^{-f}. \quad (7) \]
In the nonrelativistic approximation one can also calculate the preexponential factor, which appears due to the deviation of the quark wave function from the quasiclassical approximation. Such a calculation requires matching quark wave functions inside and outside the barrier [44]. To determine the preexponential factor in the relativistic case, one needs to solve the relativistic two-body problem, which is analytically challenging [45]. Fortunately, as we argue later, one does not need to know the prefactor to make reliable estimates of the quarkonium dissociation energy.

We now proceed with the calculation of the function \( f \). Since \( B > E \), Eq. (6) can be written as
\[ p_y^2 = -e^2(B^2 - E^2)(y - y_1)(y - y_2), \quad (8) \]
where
\[ y_{1,2} = \frac{\epsilon_0 E - p_x B \pm \sqrt{\epsilon_0^2 E^2 - p_x^2 B^2 - (B^2 - E^2)(-\epsilon_0^2 + m^2 + p_z^2)}}{e(B^2 - E^2)}. \quad (9) \]
Define dimensionless variables \( \epsilon_0 = \epsilon_0/m \) and \( q = p_x/m \). Integration in Eq. (7) gives
\[ \frac{f}{m^2} = \frac{\sqrt{-\epsilon_0^2 + 1 + q^2(\epsilon_0 E - q B)}}{e(B^2 - E^2)} \frac{(\epsilon_0 E - q B)^2 - (B^2 - E^2)(-\epsilon_0^2 + 1 + q^2)}{e(B^2 - E^2)^{3/2}} \times \ln \left\{ \frac{\epsilon_0 E - q B + \sqrt{(B^2 - E^2)(-\epsilon_0^2 + 1 + q^2)}}{\sqrt{\epsilon_0^2 E^2 - q B^2 - (B^2 - E^2)(\epsilon_0^2 + 1 + q^2)}} \right\}. \quad (10) \]
For different \( q \)'s, \( w = e^{f} \) gives the corresponding ionization probabilities. The largest probability corresponds to the smallest \( f \), which occurs at momentum \( q_m \) determined by the equation [41]
\[ \frac{\partial f(q_m)}{\partial q_m} = 0. \quad (11) \]
Using (10) and the parameter \( \rho \) defined in Eq. (5), we find
\[ \rho(\epsilon_0 - \rho q_m) \ln \left\{ \frac{\epsilon_0^2 \rho - q_m \sqrt{1 - \rho^2} - \epsilon_0^2 + 1 + q_m^2}{\sqrt{(\epsilon_0^2 - \rho q_m^2)^3 - 1 + \rho^2}} \right\} = \sqrt{-\epsilon_0^2 + 1 + q_m^2} \sqrt{1 - \rho^2}. \quad (12) \]
This is an implicit equation for the extremal momentum \( q_m = q_m(\epsilon_0, \rho) \). Substituting \( q_m \) into Eq. (10), one obtains \( f = f(\epsilon_0, \rho) \), which by means of Eq. (7) yields the ionization probability. The quasiclassical approximation that we employed in this section is valid inasmuch as \( f(q_m) \gg 1 \).
In order to compare with the results obtained in Ref. [41] using the imaginary time method, we can rewrite Eq. (12) in terms of an auxiliary parameter \( \tau_0 \) as

\[
\tau_0 = \frac{\sqrt{1 - \rho^2 - \epsilon_0^2} \sqrt{1 + \epsilon_0^2 + q_m^2}}{\rho (\epsilon_0 - \rho q_m)}, \quad (13a)
\]

\[
\frac{\tanh \tau_0}{\tau_0} = \frac{\rho}{\epsilon_0 \rho - q_m}. \quad (13b)
\]

Taking advantage of these equations, Eq. (10) can be cast into a more compact form:

\[
f_m = \frac{m^2 \tau_0 \rho}{eE \sqrt{1 - \rho^2}} [1 - \epsilon_0 (\epsilon_0 - q_m \rho)], \quad (14)
\]

where we denoted \( f_m = f(q_m) \). This agrees with the results of Ref. [41]. Numerical solution of Eqs. (13a), (13b), and (14) is shown in Fig. 2.

C. Special case: Crossed fields

An important limiting case is that of crossed fields \( E = B \). Since also \( E \perp B \) (see Sec. II A), both field invariants vanish. Nevertheless, the quarkonium ionization probability is finite [41]. This limit is obtained by taking \( \rho \to 1 \) in the equations from the previous section. Employing (13a) and (13b), we get the following condition for the extremum:

\[
\epsilon_0^2 - 1 - 2q_m^2 - 3\epsilon_0 q_m = 0, \quad (15)
\]

with the solution

\[
q_m = \frac{1}{4} (3\epsilon_0 \pm \sqrt{\epsilon_0^2 + 8}). \quad (16)
\]

Substitution into (14) produces

\[
f_m = \frac{2 m^2 (\epsilon_0^2 + 1 + q_m^2)^{3/2}}{3 eE (\epsilon_0 - q_m)}. \quad (17)
\]

III. NONRELATIVISTIC APPROXIMATION

A very useful approximation of the relativistic formulas derived in the previous section is the nonrelativistic limit because (i) it provides a very good numerical estimate (see Fig. 2), (ii) it allows us to eliminate the parametric dependence in Eqs. (10) and (12) and write \( f(q_m) \) explicitly in terms of \( \rho \) and \( \epsilon_0 \), and (iii) spin effects can be taken into account.

A. Arbitrary binding

The motion of a particle can be treated nonrelativistically if its momentum is much less than its mass. In such a case \( \epsilon_0 \approx m \) or \( \epsilon_b = m - \epsilon_0 \ll m \). Additionally, the motion of a charged particle in an electromagnetic field is nonrelativistic if \( E \ll B \). Indeed, the average velocity of a nonrelativistic particle is of order \( v \sim E/B = \rho \). Thus, the nonrelativistic limit is obtained by taking the limits \( \epsilon_b = \epsilon_b/m \ll 1 \) and \( \rho \ll 1 \). In these limits the extremum conditions (13a),(13b) reduce to

\[
\tau_0 = \frac{\sqrt{2\epsilon_b + q_m^2}}{\rho}, \quad (18a)
\]

\[
\frac{\tanh \tau_0}{\tau_0} = \frac{\rho}{\rho - q_m}. \quad (18b)
\]

Of two solutions to Eq. (18a) we pick the following one:

\[
q_m = -\sqrt{\tau_0^2 \rho^2 - 2\epsilon_b}. \quad (19)
\]

The sign of \( q_m \) is fixed using (18b) by noticing that \( \tanh \tau_0/\tau_0 < 1 \). Elimination of \( q_m \) gives

\[
\tau_0^2 - (\tau_0 \coth \tau_0 - 1)^2 = \gamma^2, \quad (20)
\]

where

\[
\gamma = \frac{\sqrt{2\epsilon_b}}{\rho}. \quad (21)
\]

\( \gamma \) is analogous to the adiabaticity parameter of Keldysh [46]. Taking the nonrelativistic limit of Eq. (14) and using (19) we...
obtain
\[ f_m = \frac{2m^2(2\epsilon_b)^{3/2}}{3eE} g(\gamma), \quad (22) \]
where \( g(\gamma) \) is the Keldysh function [46]
\[ g(\gamma) = \frac{3\tau_0}{2\gamma} \left[ 1 - \frac{1}{\gamma} \left( \frac{\gamma_0^2}{\gamma^2} - 1 \right)^{1/2} \right]. \quad (23) \]

In Fig. 2 we show the dimensionless ratio \( f_m eE/m^2 \) as a function of the binding energy \( \epsilon_b \) (in units of \( m \)) for several values of \( \rho \). The vacuum binding energy of \( J/\psi \) corresponds to \( \epsilon_b = 0.68 \). We observe an excellent agreement between the full relativistic calculation and the nonrelativistic approximation. At \( \rho = 0.9 \) and \( \epsilon_b = 0.68 \), the difference between the two lines is 10% and it can be further improved by considering higher-order corrections to \( f_m \) [43].

\[ \text{B. Weak binding} \]
Of special interest is the limit of weak binding, \( \gamma \ll 1 \), i.e., \( \sqrt{2\epsilon_b} \ll \rho \). Expansion of Eq. (20) at small \( \gamma \) and \( \tau_0 \) yields
\[ \tau_0 = \gamma \left( 1 + \frac{1}{18} \gamma^2 \right) \quad (24) \]
and substitution into Eq. (23) and subsequently into Eq. (22) yields
\[ f_m = \frac{2m^2}{3eE}(2\epsilon_b)^{3/2}. \quad (25) \]
Hence, the quarkonium dissociation probability reads [44]
\[ w = \exp \left\{ -\frac{2(2\epsilon_b m)^{3/2}}{3meE} \right\}. \quad (26) \]
Since the quasiclassical approximation employed in this paper is valid if \( f(q_m) \gg 1 \), it follows that the binding energy must satisfy
\[ \frac{(eE)^{2/3}}{m^{1/3}} \ll \epsilon_b \ll \rho^2 m. \quad (27) \]
Note also that we work in the approximation of the short-range binding potential, meaning that \( \sqrt{2\epsilon_b} \ll 1/(mR) \) (see Sec. I).

\[ \text{C. Strong binding} \]
In the limit \( \gamma \gg 1 \), (20) and (23) imply that
\[ \tau_0 = \frac{\gamma^2}{2}, \quad g(\gamma) = \frac{3\gamma}{8}. \quad (28) \]
Substituting (28) into (22), we derive
\[ f_m = \frac{\epsilon_b^2 B}{eE E}. \quad (29) \]
Thus, the quarkonium dissociation probability in the case of strong binding is
\[ w = \exp \left\{ -\frac{\epsilon_b^2 B}{eE E} \right\}. \quad (30) \]
This formula is valid when
\[ \rho^2 m, \quad \sqrt{eE \rho} \ll \epsilon_b \ll 1/R. \quad (31) \]

\[ \text{D. Contribution of quark spin} \]
So far we have neglected the contribution of spin interaction with the external field. We can use the squared Dirac equation for a bispinor \( \psi \):
\[ [(\epsilon - e\varphi)^2 - (p - eA)^2 - m^2 + e\Sigma \cdot B - i e\alpha \cdot E] \psi = 0, \quad (32) \]
where
\[ \Sigma = \begin{pmatrix} 0 & 0 \\ 0 & \sigma \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 & 0 \\ 0 & -\sigma \end{pmatrix}. \quad (33) \]
The operators \( \Sigma_e \) and \( \sigma_e \) do not commute. Therefore, in order to apply the WKB method for calculation of the ionization probability one actually needs to square (32), which leads to a differential equation of the fourth order in derivatives. The problem becomes more tractable in the nonrelativistic case and for crossed fields. Spin effects in crossed fields were discussed in Ref. [47].

With quark spin taken into account, the nonrelativistic version of Eq. (6) becomes
\[ \frac{1}{2m} \left( [p_x + eB y]^2 + p_y^2 \right) - eE y - \frac{\mu s \cdot B}{s} = -\epsilon_b, \quad (34) \]
and hence
\[ p_y^2 = 2m \left( -\epsilon_b + \frac{\mu s \cdot B + eE y}{s} \right) - (p_x + eB y)^2, \quad (35) \]
where \( \mu \) is the quark magnetic moment and \( s \) is the projection of the spin in the direction of the magnetic field. For a point quark, \( \mu = \mu_B = \frac{eB}{mc} \). The effect of quark spin on the quarkonium dissociation probability can be taken into account by replacing \( \epsilon_b \rightarrow \epsilon_b' = \epsilon_b - \frac{e}{s} s \cdot B \) in formulas for \( f_m \). With this replacement, all results of this section apply to a particle with spin. Note that the effective binding energy \( \epsilon_b' \) decreases if the spin is parallel to the magnetic field and increases if it is antiparallel. In particular, in the case of weak binding,
\[ w = \sum_{s=\pm 1/2} \exp \left\{ -\frac{2(2\epsilon_b m + 2seB)^{3/2}}{3meE} \right\}. \quad (36) \]
Since the nonrelativistic limit provides a good approximation of the full relativistic formulas, we will implement the quark spin dependence using the nonrelativistic prescription.

\[ \text{IV. DISSOCIATION ENERGY OF J/ψ} \]
Now, we would like to use the results of the preceding sections to analyze the dissociation probability \( w \) of the quarkonium as a function of its binding energy \( \epsilon_b \). We suppose that the quarkonium moves with velocity \( V \) at angle \( \phi \) with respect to the external magnetic field \( B_0 \). Different binding energies may correspond to either different quarkonium states or the same quarkonium state at different temperatures.
The dissociation probability is given by Eq. (7) as \( \frac{eB}{2m^2} \). At \( f_m(\varepsilon_b) \approx 1 \), the quarkonium is stable with respect to Lorentz ionization because \( w \ll 1 \). The dissociation becomes probable only at values of binding energy \( \varepsilon_b = \varepsilon_d \) such that \( f_m(\varepsilon_d) \approx 1 \). In view of the steep dependence of \( f_m(\varepsilon_b) \) on \( \varepsilon_b \), the probability \( w(\varepsilon_b) \) is in fact close to a step function; it becomes 1 when \( \varepsilon_b < \varepsilon_d \) and is zero otherwise. Consequently, we introduce the quarkonium dissociation energy in a magnetic field, \( \varepsilon_d = m\varepsilon_d \), as the solution to the equation

\[
\frac{f}{\varepsilon_d - \frac{eB}{2m^2}} = 1, \quad \text{(37)}
\]

where we took into account the contribution of quark spin (see Sec. III D). The negative sign in front of the spin term in Eq. (37) is chosen to give the highest dissociation energy. (Which of two particles \( c \) or \( \bar{c} \) has the higher probability to tunnel depends on the direction of the magnetic field.) The solution of Eq. (37) depends on the electric and magnetic fields in the comoving frame, which in turn depend on the magnetic field in the laboratory frame, \( B_0 \), and the quarkonium kinematics through the Lorentz transformation formulas (4).

In Fig. 3 we show the function \( f_m(\varepsilon_b) \) for different velocities \( V \) of \( J/\psi \), assuming it moves perpendicularly to the magnetic field. In vacuum, \( M = 3.1 \) GeV and \( m = 1.87 \) GeV, so that \( \varepsilon_b = 2m - M = 0.64 \) GeV. The corresponding dimensionless parameter \( \varepsilon_b \) varies in the interval \( 0 \leq \varepsilon_b \leq 0.68 \).

The numerical solution to Eq. (37) is plotted in Fig. 4. We see that the dissociation energy of the \( J/\psi \) increases with its velocity and is larger in a strong magnetic field. At \( eB_0 = 15 m^2 \), which according to estimates may be reached in heavy-ion collisions at the LHC [30,34], the dissociation energy equals the vacuum binding energy at \( V = 0.94 \). This implies that most of the \( J/\psi \)'s moving in the reaction plane with \( P_\perp > 9 \) GeV will dissociate in a magnetic field even in vacuum.

In a thermal medium, \( \varepsilon_b \) exhibits strong dependence on temperature \( T \). At \( T = 0 \), \( \varepsilon_b \) equals its vacuum value. As \( T \) increases \( \varepsilon_b \) drops until it vanishes at some \( T_0 \) near \( T_r \). The precise value of \( T_0 \) as well as the functional form of \( \varepsilon_b(T) \) are model dependent; see, e.g., [18,19]. To illustrate the temperature effect, we indicated in Fig. 4 one-half and one-quarter of the \( J/\psi \) binding energy, \( \varepsilon_b/2 \) and \( \varepsilon_b/4 \). Suppose that the plasma temperature is such that the binding energy is one-half of that in vacuum. Then, at \( eB_0 = 15 m^2 \), all \( J/\psi \)'s with \( V > 0.71 \) (\( P_\perp > 3.1 \) GeV) will dissociate, while at \( eB_0 = 7 m^2 \) this occurs at \( V > 0.92 \) (\( P_\perp > 7.3 \) GeV).

So far in this section we have discussed the case of a quarkonium moving in the reaction plane, i.e., at \( \phi = \pi/2 \). Since the electric field in the comoving frame is proportional to \( \sin\phi \) [see (4)], the dissociation probability decreases at oblique angles. \( J/\psi \)'s moving parallel to the magnetic field \( B_0 \), i.e., at \( \phi = 0 \), are not affected by electromagnetic field at all. This nontrivial azimuthal angle dependence translates into azimuthal anisotropy of the produced \( J/\psi \)'s, which we plan to address in a separate presentation.

The size of the spin contribution is determined by the dimensionless ratio \( \xi = eB/2\varepsilon_b m^2 \) [see (36)]. Its magnitude for a \( J/\psi \) moving with a nonrelativistic velocity perpendicular to the magnetic field \( B_0 \) is \( \xi \approx (eB_0/24m^2)(0.68/e_b) \). Therefore, even in the strongest field expected at the LHC, \( eB_0 = 15 m^2 \), the effect of spin is quite small at \( T = 0 \). \( \xi \) increases as the temperature of the plasma and/or the velocity of the \( J/\psi \) increases, owing to decrease of the binding energy from its vacuum value \( e_b = 0.68 \) down to zero. When \( \xi \approx 1 \) nonrelativistic treatment of spin breaks down.
V. DISCUSSION AND CONCLUSIONS

In this paper we chose the quarkonium proper frame to calculate the dissociation probability. In this frame the potential energy of, say, an antiquark (with \( e < 0 \)) is the sum of its potential energy in the binding potential and its energy in the electric field \(-eEy\), where \( y \) is the electric field direction. Since \(|e|Ey\) becomes large and negative at large and negative \( y \) (far away from the bound state) and because the quarkonium potential has finite radius, this region opens up for the motion of the antiquark. Thus there is a quantum mechanical probability to tunnel through the potential barrier formed on one side by the vanishing quarkonium potential and on the other by the increasing absolute value of the antiquark energy in an electric field. Of course the total energy of the antiquark (not counting its mass) is negative after tunneling. However, its kinetic energy grows proportionally to \( eEy \) as it goes away. By picking up a light quark out of vacuum it can hadronize into a \( D \) meson.

If we now go to the reference frame where \( E = 0 \) and there is only a magnetic field \( B \) (we can always do so since \( E < B \)), then the entire process looks quite different. An energetic quarkonium travels in the external magnetic field and decays into a quark-antiquark pair that can late-hadronize into \( D \) mesons. This happens in spite of the fact that the \( J/\psi \) mass is smaller than the masses of two \( D \) mesons because of the additional momentum \( eA \) supplied by the magnetic field. Similarly, a photon can decay into an electron-positron pair in an external magnetic field.

In the preceding section, we observed that the \( J/\psi \) dissociation energy increases with increasing magnetic field strength and quarkonium momentum. As a consequence, quarkonia dissociate at lower temperature than one would have expected based on calculations neglecting the magnetic field. Figure 4 indicates that in heavy-ion collisions at the LHC, \( J/\psi \)'s moving with \( P_\perp > 9 \) GeV in the reaction plane dissociate with probability of order unity even in vacuum, where we rely on the magnetic field estimates of Ref. [30]. This effect is more pronounced at finite temperature owing to the decrease in binding energy and, as seen in Fig. 4, it occurs in weaker fields.

Although magnetic fields in \( pp \) and \( pA \) collisions are much weaker than in \( AA \) collisions, they are still strong enough to cause \( J/\psi \) dissociation at sufficiently high momenta \( P_\perp \). A truly spectacular feature of such a process would be \( J/\psi \) decay into two heavier \( D \) mesons. The validity of the quasistatic approximation can perhaps be advocated for a prehadronic \( c\bar{c} \) state owing to its short production time. This intriguing possibility will be addressed elsewhere.

The effect of \( J/\psi \) dissociation in a magnetic field vanishes in the direction parallel to the magnetic field, i.e., perpendicular to the reaction plane. Therefore, \( J/\psi \) dissociation gives a negative contribution to the total azimuthal asymmetry coefficient \( v_2 \). The size of this contribution is a subject of our ongoing investigation.

Finally, we would like to mention the possibility of \( J/\psi \) dissociation directly by electric fields induced in a plasma by the chiral magnetic effect [30]. We plan to address this interesting possibility in a forthcoming presentation.

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\[2\] This observation is due to D. Kharzeev.

[44] L. D. Landau and L. E. Lifshitz, Quantum Mechanics Non-Relativistic Theory (Ref. [39], Sec. 77).