MODELING ULTRASONIC WAVES USING FINITE DIFFERENCE METHODS

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INTRODUCTION

Models based on explicit finite difference methods are used to study pulsed elastic wave propagation and scattering. These models were first developed in seismology and have now been developed and applied to SAW electronics and ultrasonic NDT.

The region under consideration is divided into a 2-D grid of points. The initial conditions require that the displacements at two initial time levels are specified. For each point the time development of the displacement field is then calculated in sequence. The models employ difference forms of the continuum equations which provide nodal formulations for each class of point, e.g., body node, 90 degree corner node, etc. The nodal formulations are then the building blocks from which models are constructed.

Table 1 lists the references to initial conditions and nodal formulations used in modelling systems which introduce slots into the stress-free surface of an elastic solid, and on the interface of an elastic solid with a viscous fluid.

Table 1 - References to various algorithms used in the models

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Reference</th>
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<tbody>
<tr>
<td>Solid interior points</td>
<td>Alterman and Rotenberg [1]</td>
</tr>
<tr>
<td>Horizontal and vertical free surfaces</td>
<td>Ilan and Loewenthal [2]</td>
</tr>
<tr>
<td>90° corner</td>
<td>Ilan [3]</td>
</tr>
<tr>
<td>270° corner</td>
<td>Ilan et al [4]</td>
</tr>
<tr>
<td>Absorbing boundaries</td>
<td>Reynolds [5]</td>
</tr>
<tr>
<td>Input pulse: C-pulse</td>
<td>Ilan et al [6]</td>
</tr>
<tr>
<td>SV-pulse</td>
<td>Punjani [7]</td>
</tr>
<tr>
<td>Solid-fluid interface</td>
<td>Saffari [8]</td>
</tr>
<tr>
<td>Solid-solid interface with partial closure</td>
<td>Punjani [7]</td>
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</table>
A classical problem is the interaction of a line impulse of compression waves with a 270° wedge.

A series of vector plots are shown for the case of a 60° incident pulse as figure 1. Alternatively, the surface displacements were recorded at all points along the horizontal and vertical surfaces at one time step, figure 2. The magnitude of the mode-converted Rayleigh wave component was then considered. This problem was studied as the angle of incidence was varied from +90° to -90°. The conversion coefficients thus obtained were compared with experimental data due to Gangi and Wesson [9], figure 3. They are seen to be in good agreement.

Figure 1. Vector plots of the interaction of a compression impulse with a 270° wedge, incident at 60°.
Figure 2. Displacements along the vertical (a) and horizontal (b) surfaces of the wedge at one time step.

Figure 3. Numerical C-to-R conversion coefficients (solid line) compared to the experimental results (dots) of [9].

COMPRESSION WAVE INTERACTION WITH A VERTICAL STEP

The vertical step is the simplest feature which provides frequency dependent scattering in the mid-frequency regime.

A line impulse of compression waves was normally incident from below on a vertical up step. A series of vector plots are shown in figure 4. At a receiver point located on the free surface to the right hand of the step, the vertical component of displacement was recorded with time, figure 5. The receiver was sufficiently distant from the step to allow the various scattered components in the wave field to separate. The Rayleigh wave component was gated and its spectrum deconvolved with that of the input pulse, figure 6. The position of the peak was measured for a number of models with different step depths and an average value of 0.74 found. The position of this peak is found to be in good agreement with that predicted by an interference model proposed in [8].
Figure 4. Vector plots for the interaction of a compression impulse with a step.

Figure 5. Vertical displacements at a receiver point located on the free surface.

Figure 6. Deconvolved spectrum of the mode-converted Rayleigh wave on the free surface.
In a previous paper [10] the study of the scattering of transient plane compressional and shear vertical waves by a broken crack with regions of partial closure and vacuum, was reported. Here, the previous work is extended to consider the scattering by a crack with continuous partial closure (a partially closed perfect crack). In contrast to the complicated broken crack, the geometry of the partially closed perfect crack is simple. The boundary condition for partial closure is, physically, continuity of traction but not of displacement. The constants $k \text{ Nm}^{-3}$ of this condition are stiffness (spring) coefficients.

Figures 7a and 7b, displacement vector and magnitude plots, respectively, show the diffraction of a plane compression wave by a crack with $k = 0$. In this figure, the well-known diffraction pattern of scattered body and surface waves is clearly observed. Figures 8a and 8b show the diffraction of a plane compression wave by a crack with $k_1 = k_2 = 0.5 \times 10^7 \text{ Nm}^{-3}$. It is evident in this figure that the crack becomes almost transparent to the incident wave. The scattering, which is seen to be weak, still consists of outgoing circular body waves plus scattered waves due to the continuous partial closure.
A line compression wave was normally incident on a vertical slit from below. The depth of the slit below the free surface was a fraction of the incident wavelength (0.1 to 0.2), and the length being up to about a wavelength. A series of vector plots are shown in figure 9. When the displacements are plotted at selected points along the free surface for models with a range of depths and lengths, figure 10, the mode-conversion phenomena are found to be related to the void dimensions. It is seen that the amplitude of the Rayleigh wave component, $R_T$, is a function of void depth and the surface-skimming shear wave, $S_{HS}$, is a function of both depth and length.

Figure 9. Vector plots for the interaction of a compression impulse with a vertical near-surface slit.
Figure 10. Displacement versus time plots at different receiver points along the free surface for three different slits: (a) length 10, depth 4; (b) length 30, depth 4; (c) length 30, depth 8 - the slit is at J = 80.
A line compression wave was incident on a solid-fluid interface, from the fluid side, at the Rayleigh critical angle. On the interface, and decaying into the solid, a leaky Rayleigh wave was thus excited which is shown interacting with a near-surface perpendicular slit, figure 11. Part of the energy is reflected which travels back along the the interface as a leaky Rayleigh wave. The vertical displacement due to this reflection wave is sampled on the curve for the frequency dependent reflection coefficient obtained, figure 12. The reflection coefficients for two slits with different depth (d) to length (l) ratios are shown These are compared with those for Rayleigh wave interactions with slots on free surfaces calculated in [11]. They are seen to be similar.

Figure 11. Vector plots for the interaction of a leaky Rayleigh wave pulse with a near-surface slit.
SCATTERING OF A SHEAR-VERTICAL WAVE ON A HALF SPACE WITH A SLOT

A new source function, which consists of a Ricker-type amplitude spectral distribution, has been developed to model a plane shear-vertical wave at a half space for wave incidence from 0 to 90°. The details of the analytic synthesis of this shear-vertical Ricker-type wavelet for a numerical scattering model have been presented in [7].

Figure 13 shows a sequence of displacement fields which arise when a transient plane shear-vertical wave interacts with a slot perpendicular to the surface of a half space. In this figure, the angle of wave incidence is 45° relative to the half space, and the incident wave is traversing the half space from left to right.

Figure 12. Leaky Rayleigh wave reflection coefficients for near-surface slits of different d/l ratios (solid lines) compared with the results of [11] (dots).

Figure 13. Scattering of a transient plane shear-vertical wave by a slot perpendicular to a half space; displacement field and contour plot of magnitude of displacement field.
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REFERENCES