A COMPUTATIONAL MODEL FOR ELECTROMAGNETIC INTERACTIONS WITH ADVANCED COMPOSITES

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INTRODUCTION

Composite materials in the form of fiber-reinforced matrix materials as, for example, graphite-epoxy, are being increasingly used in critical structures and structural components because of their high strength-to-weight ratio. In order to assess the integrity of these structures, it is necessary to employ suitable methods for quantitative nondestructive evaluation (NDE). One such method uses electromagnetics (eddy-currents), but the problem is that composite materials are inherently anisotropic, which means that much of the classical eddy-current technology and design procedures are not applicable. In this paper we compute fields by applying a rigorous model of electromagnetic interactions with graphite-epoxy composites, which is based on a continuum approach. In this approach the graphite fibers produce a macroscopic conductivity tensor that has different conductivities in the directions parallel and transverse to the fibers.

In [1] we described a method for computing electromagnetic fields within composite media, which is based on a matrix form of Maxwell's equations in Fourier space. By using this approach we computed the tensor Green's function for a plane-parallel slab of graphite-epoxy. With the Green's tensor in hand, we showed how one can compute the fields interior to the slab, due to arbitrary sources of excitation; we gave as an example the field due to an infinite current sheet parallel to the slab. In [2] we extended the computations to include the fields due to planar, circular current-loops that are also parallel to the slab. These results simulate some of the effects one might encounter when dealing with circular coils in the presence of a composite slab.

The fundamental relationship between the field at level \( z \), due to an impressed current source at \( z' \), is, in Fourier-transform space, \((k_x, k_y)\):

\[
\tilde{\mathbf{e}}(k_x, k_y, z) = \int \tilde{\overline{G}}(k_x, k_y, z; z') \cdot \tilde{\mathbf{j}}^{(i)}(k_x, k_y, z') dz',
\]

where \( \tilde{\mathbf{e}}(k_x, k_y, z) \) is the transform of the transverse field vector at \( z \), \( \tilde{\overline{G}}(k_x, k_y, z; z') \) is the transform of the Green's tensor, with source point at \( z' \) and field point at \( z \), and \( \tilde{\mathbf{j}}^{(i)}(k_x, k_y, z') \) is the transform of the transverse components of the impressed current source at \( z' \). The computation of \( \tilde{\mathbf{j}}^{(i)}(k_x, k_y, z') \) for circular current sheets is discussed in Appendix C of [2].
When $\tilde{G}$ and $\tilde{j}$ are substituted into (1), we obtain the transverse Fourier transform of the transverse field vector at any level, $z$, within the slab. Then, upon taking the inverse Fourier transform, say by using the Fast Fourier Transform (FFT) algorithm, we get the fields in physical space, at level $z$. We have done this for a number of circular current distributions, including a filamentary loop, and have found that the results are qualitatively similar in all cases; hence, we will display results of the filamentary loop, only. The frequency of excitation is $10^6$ Hz, and the parallel and transverse conductivities are, respectively, $\sigma_p = 2 \times 10^4$ S/m, $\sigma_t = 100$ S/m. Hence, the anisotropy ratio is 200. The slab thickness is 1.27 cm (0.5 inches), and the current loop is 2.54 mm (0.1 inch) above the slab. The radius of the current loop is 0.5 inch (1.27 cm).

Before going into the anisotropic problem, we illustrate in Figure 1 the field induced into an isotropic medium (with a conductivity of $2 \times 10^4$ S/m) at a depth of 0.05 inch (0.127 cm). The isotropic nature of the response is clearly apparent; if we were to look vertically downward we would see a circular response region. Each pixel is a square, whose side is 0.05 inch. Thus, the response region has a diameter of about 1.0 inch, which is the diameter of the current loop. Therefore, the result agrees with our intuition.
The situation in an anisotropic material is changed dramatically, however. In this case the fibers will 'guide' the field, so that it will die out much less rapidly in the $x$-direction (along the fibers) than in the $y$-direction. This is illustrated quite clearly in Figure 2, where the complex values of the $x$- and $y$-components of the electric field at a depth of 0.05 inch are shown. The response region in this figure is highly elongated in the $x$-direction, when viewed from directly above. The $x$-component of the induced electric current field is obtained by multiplying the $x$-component of the electric field by $\sigma_p$, which is equal to $2 \times 10^4 \text{ S/m}$, and the $y$-component of the current is given by the product of the $y$-component of electric field with $\sigma_t$ (100 S/m). Therefore, the eddy-currents do not flow in the usual circular paths of an unbounded isotropic medium, as suggested by Figure 1, but, rather, flow in highly elongated quasi-elliptical paths. The degree of eccentricity of the paths depends upon the degree of anisotropy, as measured by the ratio, $\sigma_p/\sigma_t$.

![Fig. 2 Field induced into composite (anisotropic) slab by a circular current loop. Depth = 0.05 inch.](image-url)
In many applications it is important to know how rapidly the induced field dies out with depth into the slab. In an anisotropic medium there is no unique skin-depth, because the conductivity varies with direction of the electric field. Therefore, the problem must be handled numerically in most cases. We present in Figure 3 model calculations of the induced electric field at a depth of 0.4 inch within the slab, under the same conditions as above. Upon comparing this figure with Figure 2, we draw the following conclusions: the field magnitude is reduced by about an order-of-magnitude, the field is much more spread out in the \( y \)-direction and is very uniform in the \( x \)-direction. These results are consistent with the notion of diffusion in isotropic media; they are an obvious manifestation of the filtering-out of the higher spatial frequencies, \( k_x, k_y \), with depth, and the crowding of the spatial-frequency spectrum toward the origin. In addition, we note that the \( x \)-component of the electric field dies out much more rapidly with depth than does the \( y \)-component. This is due to the fact that in the principal axis coordinate system the \( x \)-components 'sees' a much larger conductivity, \( \sigma_p \), than does the \( y \)-component. This supports our statement that there is no unique skin-depth in a single layer of graphite-epoxy.

Model computations of the type presented here can be very useful in setting up eddy-current experiments in graphite-epoxy and interpreting the results.

Fig. 3 Field induced into composite (anisotropic) slab by a circular current loop. Depth = 0.40 inch.
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REFERENCES
