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A Semantic Importing Approach to Reusing Knowledge from Multiple Autonomous Ontology Modules

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Abstract. Semantic web aims to provide seamless access to a large number of inter-connected, autonomous resources. Hence, it is inevitable that ontologies on the semantic web are distributed and context-specific. There is an urgent need for ontology languages and reasoning services that allow knowledge from multiple ontologies to be combined and reused from specific points of view. We present the syntax and semantics of a modular ontology language $\mathcal{SHOIQP}$ to accomplish this goal. Specifically, a $\mathcal{SHOIQP}$ ontology consists of multiple ontology modules (each of which can be viewed as a $\mathcal{SHOIQ}$ ontology) and concept, role and nominal names can be shared by "importing" relations among modules. The proposed language supports contextualized interpretation, i.e., interpretation from the point of view of a specific package. We establish the necessary and sufficient constraints on domain relations (i.e., the relations between individuals in different local domains) to preserve the satisfiability of concept formulae, monotonicity of inference, and transitive reuse of knowledge. We also explore the relationship between $\mathcal{SHOIQP}$ and existing modular ontology languages such as distributed description logics (DDL) and $\mathcal{E}$-connections.

1 Introduction

The success of the world wide web can be attributed the *network effect*: The absence of central control on content and organization of the web allows thousands of independent actors to contribute resources (web pages) that are interlinked to constitute the web. Current efforts to extend the current web into a *semantic web*, are aimed at enriching the web with machine interpretable content and interoperable resources and services [4]. Realizing the full potential of the Semantic web requires the large-scale adoption and use of ontology-based approaches to sharing of information and resources. Constructing large ontologies typically requires collaboration among multiple individuals or groups with expertise in specific areas, with each participant contributing only a part of the ontology. Therefore, instead of a single, centralized ontology, in most application domains, it is natural to have multiple distributed ontologies covering parts of the domain. Such ontologies represent the *local* knowledge of the ontology designers, that is, knowledge that is applicable in a *context*. Because no single ontology can meet the needs of all users under every conceivable scenario, there is an urgent need for theoretically sound yet practical approaches that allow knowledge from multiple autonomously developed ontologies to be adapted and reused in a user, context, or application-specific scenarios.

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As noted by Bouquet et al. [7], ontologies on the semantic web need to satisfy two apparently conflicting objectives: *Sharing* or *r*euse of knowledge across autonomously developed ontologies; and accommodation of the *local points of view* or *contextuality* of knowledge, as illustrated by the following examples.

Example 1. Consider two independently developed ontologies: a *People* ontology and a *Work* ontology. The *People* ontology asserts: "every individual that is not a male is a female", "man is male" and "woman is female". Attempts to reuse knowledge from the people ontology without regard to the context in which it is applicable can lead to unintended and undesirable consequences. For example, an attempt to reuse some of the knowledge from the *People* ontology in the context of the*Work* ontology, such as to say "an equal opportunity enterprise employs both men and women" (without regard to the contexts in which the respective assertions were made), we end up with the absurd conclusion: "every enterprise that is not a male is a female" (when in fact, we ought to be able to recognize that an enterprise is neither male nor female).

Example 2. Suppose we will query two ontologies about people in two different departments. In the first ontology, the universe of people is explicitly enumerated by their names, which can be modelled using nominals in description logics, e.g.,

$$\mathsf{People} = \{\mathsf{Alice}, \mathsf{Bob}\}$$

In the second ontology, the universe of people is also explicitly enumerated but with a different set of names (which is disjoint from the ones in the first ontology), e.g.,

$$\mathsf{People} = \{\mathsf{Carol}, \mathsf{Dave}, \mathsf{Eve}\}$$

Hence, the two ontologies disagree on the members of the People concept such that it has a different semantics in the two contexts. In order to be able to reuse knowledge from the two ontologies, the differences in the corresponding contexts need to be reconciled.

Against this background, there have been several efforts aimed at developing formalisms that allow reuse of knowledge from multiple ontologies via **contextualized interpretations** in multiple local domains instead of a single shared global interpretation domain. Contextualized reuse of knowledge requires the interactions between local interpretations to be controlled. Examples of such modular ontology languages include: Distributed Description Logics (DDL) [6], $\mathcal{E}$-Connections [13], Package-based Description Logics (P-DL) [2] and Semantic Importing [18].

An alternative approach to knowledge reuse is based on the notion of **conservative extension** [9–12] which allows ontology modules to be interpreted using standard semantics by requiring that they share the same global interpretation domain. To avoid undesired combination of ontology modules as in Example 1 and 2, this approach requires the combination of ontology modules to be a conservative extension of component modules. More precisely, if O is the union of a set of ontology modules $\{O_1, ..., O_n\}$, then we say O is a conservative extension of O_i if $O \models \alpha_i \Leftrightarrow O_i \models \alpha_i$ for any α_i of the form $C_1 \sqsubseteq C_2$, where C_1, C_2 are concepts in the language of O_i. This guarantees that the combination of ontology modules will not alter the knowledge in any component module. Thus, combining ontology modules cannot induce a new concept inclusion relation between existing concepts in any module. This requirement is enforced through a syntactical restriction over ontology modules which forbids the use of any axiom that is not "local" (e.g., $\top \sqsubseteq C$).

The existing approaches to knowledge reuse are limited in several ways. To preserve contextuality, existing modular ontology languages offer only limited ways to connect ontology modules (and hence limited ability to reuse knowledge across modules). For instance, DDL does not allow concept construction using foreign roles or concepts; $\mathcal{E}$-Connections does not allow concept inclusion between ontology modules or the use of foreign roles; P-DL and Semantic Importing in their current forms only allow each component module to be in $\mathcal{ALC}$. None of the existing approaches support knowledge reuse in a setting where each ontology module uses a representation language that is as expressive as OWL-DL, i.e. $\mathcal{SHOIN}(D)$.

Furthermore, some of the existing modular ontology languages suffer from reasoning difficulties that can be traced to an absence of natural ways to restrict the relations between individuals in different local domains. For example, DDL does not support the transitivity of inter-module concept subsumptions (called bridge rules in DDL) in general, and a concept that is declared as being more specific than two disjoint concepts in another module may still be satisfiable (the inter-module satisfiability problem) [2, 13]. Undisciplined use of generalized links in $\mathcal{E}$-Connections has also been shown to lead to reasoning difficulties [1].

Conservative extensions [10–12], in their current form, by forcing a single global interpretation domain, prevent the different modules from interpreting axioms within their own local contexts. Thus, the designers of different ontology modules have to play it safe by accounting for all possible contexts in which knowledge from a specific module might be reused. Locality of knowledge is achieved by precluding several otherwise useful modelling scenarios, such as the refining of relations between existing concepts in an ontology module and the reuse of nominals [17].

Against this background, this paper explores a formalism that can support *context-aware reuse* from multiple ontology modules. The resulting modular ontology language:

• Allows each ontology module to use subset of $\mathcal{SHOIQ}$, i.e., $\mathcal{ALC}$ augmented with transitive roles, role inclusion, role inversion, qualified number restriction and nominal concepts, hence covers a significant fragment of OWL-DL.

• Supports more flexible modelling scenarios than those supported by existing approaches, using a mechanism of *semantic import* of names (including concept, role and nominal names) across ontology modules.

• Contextualizes the interpretation of reused knowledge. Locality of axioms in ontology modules is obtained "for free" by its *contextualized semantics*, thereby freeing ontology engineers from the burden of ensuring the reusability of an ontology module in contexts that are hard to foresee at the time of construction of the module in question. A natural consequence of contextualized interpretation is that inferences that are drawn are always *from the point of view* of a witness package. Thus, different modules might infer different consequences, based on the knowledge that they import from other modules.

• Ensures that the result of reasoning is always the same as that obtained from a standard reasoner over an integrated ontology resulting from combining the relevant knowledge in a context-specific manner. This ensures the *monotonicity* of inference in the distributed setting.

• Avoids many of the known reasoning difficulties of the existing approaches.

2 Semantic Importing

This section will introduce the syntax and semantics of the proposed language.

2.1 Syntax

Definition 1 *A distributed TBox contains a set of modules called* packages, *each is a TBox of a subset of $\mathcal{SHOIQ}$. Each package P has associated with it, a local signature: a subset of its symbols (the set of concept, role and individual names)* $\mathsf{Loc}(P) \subseteq \mathsf{Sig}(P)$*; for any symbol $s \in \mathsf{Loc}(P)$, P is the* home package *of s, denoted by $P = \mathsf{Home}(s)$. The set of symbols in $\mathsf{Imp}(P) = \mathsf{Sig}(P) \backslash \mathsf{Loc}(P)$ is called P's* imported signature.

If a symbol $s \in \mathsf{Loc}(Q)$ also appears in the imported signature of a different package P (i.e., $s \in \mathsf{Imp}(P)$), we say that P imports $Q : s$ *and denote it as $Q \xrightarrow{s} P$. If any local symbol of Q is imported into P, we say that P imports Q and denote it as $Q \mapsto P$.*

A concept, role, nominal or general concept inclusion (GCI) axiom that contains only the names in the local signature of P_i is called a pure i-formula. For any package P_i, we refer to a formula that appears in P_i that contains names from more than one packages an hybrid i-formula. An i-formula can be either a pure i-formula or a hybrid i-formula. We denote an i-formula X by $i : X$, and we drop the prefix $i :$ when there is no possibility of confusion.

Each package P_i has associated with it, a context which constrains the scope of knowledge in it. In particular, for each package P_i, instead of the universal top ($\top$), bottom ($\bot$) concepts, we have their contextualized counterparts: top $\top_i$, contextualized bottom $\bot_i$; and (global) negation ($\neg$) is replaced by its contextualized counterpart negation $\neg_i$. Without the loss of generality, in the syntax we assume $\neg_i$ is only used before an i-concept[1].

The importing transitive closure *$I(P_i)$ of a package P_i contains packages that are directly or indirectly imported by P_i. That is,*

– $\forall j \neq i, P_j \mapsto P_i \Rightarrow P_j \in I(P_i)$
– $\forall k \neq j \neq i, (P_k \mapsto P_j) \wedge (P_j \in I(P_i)) \rightarrow P_k \in I(P_i)$

[1] For an i-concept C, we can always transform $\neg_j C$ appearing in package k to $\neg_j C'$ where C' is a new j-concept and add an axiom $C' = C$ in package k, k and j may or may not be the same.

We use P_i^* to denote the union of a package P_i and its importing transitive closure $I(P_i)$.

A distributed TBox $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$ has acyclic importing relation *if for any $i \neq j$, $P_j \in I(P_i) \rightarrow P_i \notin I(P_j)$, otherwise it has* cyclic importing relation. *Σ is said* closed *if every symbol used in Σ is defined in one of its component packages, i.e., for $\forall s, P_k, s \in \mathsf{Imp}(P_K) \rightarrow \mathsf{Home}(s) \in \{P_i\}$.*

We denote a package-based Description Logics (DL) by adding the letter $\mathcal{P}$ to the notation for the corresponding DL. Thus, $\mathcal{ALCP}$ is the package-based DL $\mathcal{ALC}$. In this paper, we focus on $\mathcal{SHOIQP}$, thereby extending some of the results of [3] which studied $\mathcal{ALCP}_\mathcal{C}$, a restricted type of $\mathcal{ALCP}$ that only allows import of concept names.

Decidability [15] requires that the reuse of role names be restricted such that a *locally simple role* (i.e. a role that is not transitive nor has any transitive sub-role in its home package) that is used in number restriction will not be declared as a super-role of a transitive role in a different ontology module. In practice, it is usually hard to check if imported role names and all their super-roles in the importing transitive closure are not used in number restrictions. A stronger condition may be used such that *a locally non-simple role can not be declared as a sub-role of an imported role or its inverse*.

2.2 Semantics

Definition 2 *A $\mathcal{SHOIQP}$ KB has* localized semantics *in that each package has its own local interpretation domain. Formally, for a $\mathcal{SHOIQP}$ KB $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$, a* distributed interpretation *is $\mathcal{I} = \langle \{\mathcal{I}_i\}, \{r_{ij}\}_{i \neq j} \rangle$, where $\mathcal{I}_i = \langle \Delta^{\mathcal{I}_i}, (.)^{\mathcal{I}_i} \rangle$ is the* local interpretation *of package P_i; $r_{ij} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$ is the* image domain relation *from P_i to P_j. For convenience, we may also denote $r_{ii} = \{(x,x) | \forall x \in \Delta^{\mathcal{I}_i}\}$ as the identity mapping in local domain $\Delta^{\mathcal{I}_i}$. For a subset S of $\Delta^{\mathcal{I}_i}$, $r_{ij}(S) = \{y | \forall x \in S, \langle x, y \rangle \in r_{ij}\}$*

Each local interpretation $\mathcal{I}_i$ has a non-empty domain $\Delta^{\mathcal{I}_i}$, and the interpretation function $(.)^{\mathcal{I}_i}$ which maps every concept name to a subset of $\Delta^{\mathcal{I}_i}$, every role name to a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$, and every individual name to an element in $\Delta^{\mathcal{I}_i}$, such that equations in Table 1 are satisfied.

A local interpretation $\mathcal{I}_i$ satisfies *a role inclusion axiom $R_1 \sqsubseteq R_2$ iff $R_1^{\mathcal{I}_i} \subseteq R_2^{\mathcal{I}_i}$, and it satisfies a GCI $C \sqsubseteq D$ iff $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$. An interpretation $\mathcal{I}_i$ is said to be a* model *of P_i (denoted by $\mathcal{I}_i \vDash P_i$), if it satisfies all of the axioms in P_i.*

The proposed semantics of $\mathcal{SHOIQP}$ is motivated by the need to overcome some of the limitations of existing approaches that can be traced to the arbitrary construction of domain relations and the lack of support for contextualized interpretation. Specifically, we seek a semantics that satisfies the following desiderata:

- **The preservation of concept unsatisfiability**. The intuition is that an unsatisfiable concept formula can never be reused so as to be interpreted as a satisfiable concept. DDL, in its current form, does not preserve concept unsatisfiability due to the fact that a domain relation r_{ij} can map two disjoint non-empty subsets S_1, S_2 of $\Delta^{\mathcal{I}_i}$ to a non-empty set $r_{ij}(S_1) \cap r_{ij}(S_2)$. Formally, we say a domain relation r_{ij} *preserves*

$$
\begin{aligned}
R^{\mathcal{I}_i} &= (R^{\mathcal{I}_i})^+, \text{for transitive role role } R \\
(R^-)^{\mathcal{I}_i} &= \{\langle x, y\rangle | \langle y, x\rangle \in R^{\mathcal{I}_i}\} \\
(C \sqcap D)^{\mathcal{I}_i} &= C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}, \\
(C \sqcup D)^{\mathcal{I}_i} &= C^{\mathcal{I}_i} \cup D^{\mathcal{I}_i} \\
(\neg_i C)^{\mathcal{I}_i} &= \Delta^{\mathcal{I}_i} \backslash C^{\mathcal{I}_i} \\
(\neg_j C)^{\mathcal{I}_i} &= r_{ji}((\neg_j C)^{\mathcal{I}_j}), \text{for } i \neq j \\
(\exists R.C)^{\mathcal{I}_i} &= \{x \in \Delta^{\mathcal{I}_i} | \exists y, \langle x, y\rangle \in R^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\} \\
(\forall R.C)^{\mathcal{I}_i} &= \{x \in \Delta^{\mathcal{I}_i} | \forall y, \langle x, y\rangle \in R^{\mathcal{I}_i} \rightarrow y \in C^{\mathcal{I}_i}\} \\
(\geqslant R.C)^{\mathcal{I}_i} &= \{x \in \Delta^{\mathcal{I}_i} | \#\{y | \langle x, y\rangle \in R^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\} \geqslant n\} \\
(\leqslant R.C)^{\mathcal{I}_i} &= \{x \in \Delta^{\mathcal{I}_i} | \#\{y | \langle x, y\rangle \in R^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\} \leqslant n\}
\end{aligned}
$$

Table 1. Local Interpretations

the unsatisfiability of an i-concept C if it is the case that whenever $C^{\mathcal{I}_i} = \emptyset$, it is necessarily the case that $C^{\mathcal{I}_j} = \emptyset$.

- **The transitive reusability of knowledge**. The intuition is that the consequences of some of the axioms defined in one module can be propagated in a transitive fashion to other ontology modules. For example, if a package P_i asserts that $C \sqsubseteq D$, and P_j (directly or indirectly) imports that axiom from P_i, then it should be the case that $C \sqsubseteq D$ from the *point of view* of P_j.
- **Contextualized interpretation of knowledge**. The intuition is that the interpretation of assertions in each ontology module are constrained by its context; When knowledge (e.g., axioms) in that module are reused by other modules, then the interpretation of the reused knowledge should be constrained by the context in which the knowledge is being reused.
- **Improved expressivity**. In particular, the language should support 1) both inter-module concept inclusion, role inclusion (supported by DDL but not $\mathcal{E}$-Connections) and concept construction using foreign concepts (supported by $\mathcal{E}$-Connection but not DDL); 2) more general reuse of roles and of nominals than allowed by existing approaches.

A major goal of this paper is to explore the constraints that need to be placed on local interpretations so the resulting semantics for $\mathcal{SHOIQP}$ satisfies the desiderata enumerated above. We start by defining the domain relation restrictions of $\mathcal{SHOIQP}$:

Definition 3 *An interpretation of* $\mathcal{I} = \langle\{\mathcal{I}_i\}, \{r_{ij}\}_{i\neq j}\rangle$ *is a* model *of a* $\mathcal{SHOIQP}$ *KB* $\Sigma = \langle\{P_i\}, \{P_i \mapsto P_j\}_{i\neq j}\rangle$ *if the following conditions are satisfied.*

1. *For any* i, j, r_{ij} *is one-to-one, i.e., it is an injective partial function.*
2. *For any* i, j, k, r_{ij} *is compositionally consistent, i.e.* $r_{kj} \circ r_{ik} = r_{ij}$.
3. *For every atomic pure* i*-concept* C *that appears in* P_j *(*$i \neq j$*), we have* $r_{ij}(C^{\mathcal{I}_i}) = C^{\mathcal{I}_j}$.

4. *For every atomic i-role p that appears in P_j ($i \neq j$), for every $(x, y) \in p^{\mathcal{I}_j}$, we have $(r^-_{ij}(x), r^-_{ij}(y)) \in p^{\mathcal{I}_i}$, and for every $z \in \Delta^{\mathcal{I}_i}$, we have:*
   - *forward closure: $(r^-_{ij}(x), z) \in p^{\mathcal{I}_i}$ iff $(x, r_{ij}(z)) \in p^{\mathcal{I}_j}$*
   - *backward closure: $(z, r^-_{ij}(y)) \in p^{\mathcal{I}_i}$ iff $(r_{ij}(z), y) \in p^{\mathcal{I}_j}$*
5. *For every i-nominal o that appears in P_j ($i \neq j$), $(o^{\mathcal{I}_i}, o^{\mathcal{I}_j}) \in r_{ij}$.*
6. *$\mathcal{I}_i \vDash P_i$ (for all i).*

The proposed semantics for $\mathcal{SHOIQP}$ is an extension of the semantics of $\mathcal{ALCP_C}$ [3] (which used conditions 1,2,3,6 above) and Semantic Importing [18] (which introduced condition 4 above). In section 4, we will show that the conditions 1-6 are necessary and sufficient to ensure that the desiderata we outlined above for the semantics of $\mathcal{SHOIQP}$ are indeed satisfied.

In what follows, we will use $r_{ij}(f^{\mathcal{I}_i}) = f^{\mathcal{I}_j}$ to denote the relation between the local interpretations $f^{\mathcal{I}_i}$ and $f^{\mathcal{I}_j}$ of an atomic i-formula f.

Definition 4 *A closed Σ is* consistent *as witnessed by a package P_i of Σ if P_i^* has a model. A concept C is* satisfiable *as witnessed by a package P_i if there is a model of P_i^* such that $C^{\mathcal{I}_i} \neq \emptyset$. P_i witnesses a concept subsumption $C \sqsubseteq D$ (denoted by $C \sqsubseteq_i D$), if for every model of P_i^*, $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$, i.e., $C^{\mathcal{I}_i} \backslash D^{\mathcal{I}_i} = \emptyset$.*

Hence, in $\mathcal{SHOIQP}$, consistency, satisfiability and subsumption problems are always answered from the point of view of the *witness package*, and it is possible that different packages draw different conclusions from their own points of view.

2.3 $\mathcal{SHOIQP}$ Examples

The semantic importing approach described here can model a broad range of scenarios that can be modeled using existing approaches.

Example 1: *Inter-module concept and role inclusions*. Suppose we have a *people* ontology P_1:

$$
\begin{aligned}
\top_1 &\sqsubseteq 1:\mathsf{Man} \sqcup 1:\mathsf{Woman} \\
1:\mathsf{Boy} \sqcup 1:\mathsf{Girl} &\sqsubseteq 1:\mathsf{Child} \\
1:\mathsf{Husband} &\sqsubseteq 1:\mathsf{Man} \sqcap \exists 1:\mathsf{marriedTo}.1:\mathsf{Woman}
\end{aligned}
$$

Suppose the *Work* ontology P_2 imports some of the knowledge from the *people* ontology:

$$1:\mathsf{marriedTo} \sqsubseteq 2:\mathsf{knows} \tag{1}$$
$$2:\mathsf{FemaleEmployee} \sqsubseteq 2:\mathsf{Employee} \tag{2}$$
$$2:\mathsf{MaleEmployee} \sqsubseteq 2:\mathsf{Employee} \tag{3}$$
$$2:\mathsf{MaleEmployee} \sqsubseteq 1:\mathsf{Man} \tag{4}$$
$$2:\mathsf{FemaleEmployee} \sqsubseteq 1:\mathsf{Woman} \tag{5}$$
$$1:\mathsf{Child} \sqsubseteq \neg_2 2:\mathsf{Employee} \tag{6}$$

Axiom (1) models inter-module role inclusion and (4-5) models inter-module concept inclusions. It also shows the semantic importing approach can realize concept specialization (4-5) and generalization (6).

Example 2: *Use of foreign roles or foreign concepts to construct local concepts*. Suppose a *marriage* ontology P_3 reuses the *people* ontology:

$$(= 1\ (1 : \mathsf{marriedTo}).(1 : \mathsf{Woman})) \sqsubseteq 3 : \mathsf{Monogamist} \tag{7}$$

$$3 : \mathsf{MarriedPerson} \sqsubseteq \forall(1 : \mathsf{marriedTo}).(3 : \mathsf{MarriedPerson}) \tag{8}$$

$$3 : \mathsf{NuclearFamily} \sqsubseteq \forall(\mathsf{hasMember}).(1 : \mathsf{Child}) \tag{9}$$

A complex concept in P_3 may be constructed using an imported role (8), an imported concept (9), or both an imported role and an imported concept (7).

Example 3: *The use of nominals*. Suppose the *work* ontology P_2 defined above is augmented with additional knowledge from a *calendar* ontology P_4, to obtain an augmented *work* ontology. Suppose P_4 contains the following axiom:

$$\mathsf{4{:}WeekDay} = \{\mathsf{4{:}}Mon, \mathsf{4{:}}Tue, \mathsf{4{:}}Wed, \mathsf{4{:}}Thu, \mathsf{4{:}}Fri\}$$

where the nominals are shown in *italic* font. Suppose the new version of P_2 contains the following additional axioms:

$$4 : Fri \sqsubseteq \exists 2 : \mathsf{hasDressingCode}.2 : \mathsf{CasualDress}$$

$$\top_2 \sqsubseteq 2 : \mathsf{hasDressingCode}^-.(4 : \mathsf{WeekDay})$$

3 Reduction to Ordinary DL

A reduction $\Re$ from a closed $\mathcal{SHOIQP}$ KB $\Sigma_d = \langle\{P_i\}, \{P_i \mapsto P_j\}_{i\neq j}\rangle$ to a $\mathcal{SHOIQ}$ KB Σ can be obtained as follows:

• The signature of Σ is the union of the local signatures of the component packages, i.e. $\bigcup_i \mathsf{Loc}(P_i)$

• Σ is constructed such that: $\forall i$, the concepts $\top_i$ and $\bot_i \in \Sigma$; $\top, \bot \in \Sigma$. $\bot_i \sqsubseteq \bot \in \Sigma$.

• $\forall i, j, k$ such that $P_i \in I(P_j)$, $P_i \in I(P_k)$ and $P_k \in I(P_j)$, add $\top_i \sqcap \top_j \sqsubseteq \top_k$ to Σ.

• Copy each GCI or role inclusion $X \sqsubseteq Y$ in P_i as $\#(X) \sqsubseteq \#(Y)$. The mapping #() is defined below.

• For each local atomic concept or nominal C in P_i, add $i : C \sqsubseteq \top_i$ to Σ.

• For each local atomic role P in P_i, add $\top_i$ as its domain and range, i.e. add $\top \sqsubseteq \forall P^-.\top_i$ and $\top \sqsubseteq \forall P.\top_i$ to Σ.

• For each imported atomic role P in P_i, add the following axioms to Σ:
- $\exists P.\top_i \sqsubseteq \forall P.\top_i$ (forward closure)
- $\exists P^-.\top_i \sqsubseteq \forall P^-.\top_i$ (backward closure)
- $\#(P) \sqsubseteq P$
- $\exists \#(P).\top_i \sqsubseteq \top_i$ (local domain)
- $\exists \#(P)^-.\top_i \sqsubseteq \top_i$ (local range)
- $Trans(\#(P))$ if P is transitive

The mapping #() is adapted from a similar one for DDL [6] with modifications to allow name importing. For a formula X used in P_j, $\#(X)$ is:

- X, for an atomic j-term X.
- $X \sqcap \top_j$, for an atomic pure i-concept or i-nominal X ($i \neq j$).
- $\neg\#(X) \sqcap \top_j$, for $\neg_i X$ where X is a pure i-concept ($i \neq j$).
- $X^{i \to j}$, for an atomic i-role X ($i \neq j$), where $X^{i \to j}$ is a new "image" role name.
- $\top_j \sqcap \top_i \sqcap \rho(\#(X_1), ..., \#(X_k))$, for an i-concept $X = \rho(X_1, ..., X_k)$, where ρ is a concept constructor with k arguments.

For example:

$$
\begin{aligned}
\#(j : (\neg_i i : C)) &= \top_j \sqcap \top_i \sqcap \neg C \\
\#(j : (j : D \sqcup i : C)) &= \top_j \sqcap ((\top_j \sqcap D) \sqcup (\top_j \sqcap C)) \\
\#j : (\forall(j : P).(i : C)) &= \top_j \sqcap \forall P.(\top_j \sqcap C) \\
\#j : (\exists(i : P).(i : C)) &= \top_j \sqcap \top_i \sqcap \exists P^{i \to j}.(\top_j \sqcap C)
\end{aligned}
$$

It should be noted that #() is *contextualized*, so as to allow a formula with the same syntax to have different translations when it appears in different packages.

4 Properties of Semantic Importing

In this section, we further justify the proposed semantics for $\mathcal{SHOIQP}$. Specifically, we summarize the main results that we have proved that show that $\mathcal{SHOIQP}$ satisfies the desiderata summarized in section 2.

Formally, we have:

Lemma 1 *A $\mathcal{SHOIQP}$ KB Σ is consistent as witnessed by a package P_i iff $\Re(P_i^*)$ is consistent.*

Theorem 1 (Reasoning Exactness) *For a $\mathcal{SHOIQP}$ KB $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$, $C \sqsubseteq_i D$ iff $\Re(P_i^*) \models \#(C) \sqsubseteq \#(D)$.*

Corollary 1 (The Preservation of Unsatisfiability) *For a $\mathcal{SHOIQP}$ KB $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$, $P_i \in I(P_j)$, if $C \sqsubseteq_i \bot_i$ then $C \sqsubseteq_j \bot_j$.*

Theorem 2 (Monotonicity) *For a $\mathcal{SHOIQP}$ KB $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$, if $P_i \in I(P_j)$ and $C \sqsubseteq_i D$, then $C \sqsubseteq_j D$, where $\mathsf{Sig}(C)$ and $\mathsf{Sig}(D)$ are subsets of $\mathsf{Sig}(P_i) \cap \mathsf{Sig}(P_j)$.*

Theorem 2 ensures that when some part of an ontology module is reused, the restrictions asserted by it (e.g. domain restrictions of roles) will not be relaxed to prohibit the reuse of imported knowledge. Theorem 2 also ensures that consequences of imported knowledge can be transitively propagated across importing chains.

From the proof of Lemma 1 and Theorem 2, we also have:

Lemma 2 *For every concept C such that $\mathsf{Sig}(C) \subseteq \mathsf{Sig}(P_i) \cap \mathsf{Sig}(P_j)$ where P_i, P_j are two packages and $P_i \in I(P_j)$., we have $r_{ij}(C^{\mathcal{I}_i}) = C^{\mathcal{I}_j}$.*

Finally, the semantics of $\mathcal{SHOIQP}$ ensures that the interpretation of axioms in an ontology module is constrained by their *contexts*, as seen from the reduction to a corresponding integrated ontology: $\mathsf{C} \sqsubseteq \mathsf{D}$ in P_i is mapped to $\top_i \sqcap \#(\mathsf{C}) \sqsubseteq \top_i \sqcap \#(\mathsf{D})$.

When an i-GCI is propagated to module P_j, it will only affect the overlapped domain $r_{ij}(\Delta^{\mathcal{I}_i}) \cap \Delta^{\mathcal{I}_j}$, and not the entire domain $\Delta^{\mathcal{I}_j}$. Suppose package P_i contains an axiom $\neg_i \mathsf{Male} \sqsubseteq \mathsf{Female}$ and package P_j imports P_i, then it is not required in P_i such that $\top_j \sqsubseteq \mathsf{Male} \sqcup \mathsf{Female}$, since $r_{ij}(\Delta^{\mathcal{I}_i}) \subseteq \Delta^{\mathcal{I}_j}$, $\Delta^{\mathcal{I}_i} \backslash \mathsf{Male}^{\mathcal{I}_i} \subseteq \mathsf{Female}^{\mathcal{I}_i}$, i.e., $\Delta^{\mathcal{I}_i} = \mathsf{Male}^{\mathcal{I}_i} \cup \mathsf{Female}^{\mathcal{I}_i}$ does not necessarily mean $\Delta^{\mathcal{I}_j} = \mathsf{Male}^{\mathcal{I}_j} \cup \mathsf{Female}^{\mathcal{I}_j}$.

Hence, the effect of an axiom is always contextualized within its original designated context. Therefore, it is not necessary to explicitly restrict the use of ontology language to ensure locality of axioms as required by conservative extension [10]. The locality of axioms follows from the semantics of $\mathcal{SHOIQP}$.

The constraints on domain relations in the semantics of $\mathcal{SHOIQP}$ is minimal in the sense that if we drop any of them, we can no longer satisfy the desiderata summarized in section 2.2. In the absence of restriction 3 in Definition 3, the reuse of concept names will be only syntactical, i.e., the local interpretations of shared concept names can be independently determined hence have no shared meaning. Requirement 5 is needed to ensure that nominals can only have one instance (which may be "copied" by multiple local interpretations associated by domain relations). Requirement 6 is rather natural.

Dropping requirement 1 (one-to-one domain relation) leads to difficulties in preservation of concept unsatisfiability. For example, if domain relations are not injective, then $C_1 \sqsubseteq_i \neg_i C_2$, $D \sqsubseteq_j C_1$ and $D \sqsubseteq_j C_2$ does not ensures $D \sqsubseteq_j \bot_i$. If domain relations are not partial functions, multiple individuals in $\Delta^{\mathcal{I}_i}$ may get mapped to the same individual in $\Delta^{\mathcal{I}_j}$ via r_{ij}, then unsatisfiability of a complex concept can no longer be guaranteed when both number restriction and role importing is allowed.

Dropping requirement 2 (compositional consistency of domain relations) would result in violation of transitive reusability requirement in particular, and monotonicity of inference based on imported knowledge in general. In the absence of compositional consistency of domain relations, the importing relation will be like bridge rules in DDL in that it is localized w.r.t. the connected pairs of modules without support for connected composition [19].

With the dropping of requirement 4 (forward and backward closure of role instances), it will not be possible to ensure the consistency of local interpretations of complex concepts that use number restrictions. Such a requirement ensures the counting of numbers of p-successors and p-predecessors of an individual is always kept the same as that in the interpretation of role p's home package.

5 Relation to Other Formalisms

Several other modular ontology formalisms can be simulated using the semantic importing approach we adopted in this paper.

5.1 Reduction of DDL

First, a **Distributed Description Logics (DDL)** knowledge base $\langle\{\mathcal{T}_i\}, \{\mathcal{B}_{ij}\}_{i\neq j}\rangle$ contains a set of component TBoxes $\mathcal{T}_i$ each is in a subset of $\mathcal{SHIQ}$, and a set of bridge rules $\mathcal{B}_{ij}$ between pairs of component TBoxes. In the paper we only study *homogenous bridge rules* in the form of:

INTO rule: $i : \phi \xrightarrow{\sqsubseteq} j : \psi$ (semantics: $r_{ij}(\phi^{\mathcal{I}_i}) \subseteq \psi^{\mathcal{I}_j}$)

ONTO rule: $i : \phi \xrightarrow{\sqsupseteq} j : \psi$ (semantics: $r_{ij}(\phi^{\mathcal{I}_i}) \supseteq \psi^{\mathcal{I}_j}$)

where ϕ is an i-concept and ψ is a j-concept (defined in [5]), or ϕ is an i-role and ψ is a j-role (defined in [8]). $r_{ij} \subseteq \Delta_i \times \Delta_j$ is the interpretation of $\mathcal{B}_{ij}$. We use $r_{ij}(x)$ to denote $\{y \in \Delta_j | (x, y) \in r_{ij}\}$; $r_{ij}(\phi^{\mathcal{I}_i})$ is defined as

- $\bigcup_{x\in\phi^{\mathcal{I}_i}} r_{ij}(x)$, when ϕ is a concept
- $\bigcup_{(x,y)\in\phi^{\mathcal{I}_i}} r_{ij}(x) \times r_{ij}(y)$, when ϕ is a role name

Lemma 3 *DDL homogenous bridge rules between concepts and between roles can be reduced to $\mathcal{SHOIQP}$ axioms.*

However, the other way around, i.e., reduction from $\mathcal{SHOIQP}$ to DDL, does not hold. Since DDL allows arbitrary domain relations, compositionally consistent domain relations as required in $\mathcal{SHOIQP}$ may not be realized in DDL. Hence, there is no reduction from a $\mathcal{SHIQP}$ knowledge Σ to a DDL knowledge base $\mathcal{T}$ such that it ensures every model of $\mathcal{T}$ can be mapped to a model of $\mathcal{SHIQP}$. Syntactically, it is represented by the bridge rule propagation problem in DDL [2], e.g., $i : C \xrightarrow{\sqsubseteq} j : D$ and $j : D \xrightarrow{\sqsubseteq} k : E$ does not entail $i : C \xrightarrow{\sqsubseteq} k : E$. On the contrast, Theorem 2 ensures transitive reuse of subsumptions in $\mathcal{SHOIQP}$ hence can avoid such inference difficulties.

5.2 Reduction of $\mathcal{E}$-Connections

$\mathcal{E}$**-Connections** between DLs [16, 13] focus on offering inter-module role connections and constructing concepts using foreign concepts. Roles are divided into disjoint sets of *local roles* (connecting concepts in one module) and *links* (connecting concepts in different modules). Links can be used to construct local concepts like roles, which is summarized in Table 2.

In what follows, we will show one-way $\mathcal{E}$-Connections KBs as given in [16, 13] can be reduced to $\mathcal{SHOIQP}$ KBs.

Lemma 4 *One-way $\mathcal{E}$-Connections $C^{\mathcal{E}}_{\mathcal{IHQ}}(\mathcal{SHOIN})$ knowledge bases can be reduced to $\mathcal{SHOIQP}$ knowledge bases.*

It is easy in $\mathcal{SHOIQP}$ to reuse imported transitive roles or symmetric roles without the need of any specifically designed mechanism. $\mathcal{SHOIQP}$ also provides some modelling ability that is not covered by $\mathcal{E}$-Connections in its current form. For example, it is possible in $\mathcal{SHOIQP}$ to use foreign roles to define local concepts, or to define role inclusion between a foreign roles and a local role.

The expressivity of DDL, $\mathcal{E}$-Connections and $\mathcal{SHOIQP}$ are summarized in Table 3.

Table 2. $\mathcal{E}$-Connections Syntax and Semantics

	Syntax	**Semantics**
Existential Link Restriction	$i : (\exists E.(j : D))$	$r_E^-(D^{\mathcal{I}_j})$
Universal Link Restriction	$i : (\forall E.(j : D))$	$\Delta_i \backslash r_E^-(\Delta_j \backslash D^{\mathcal{I}_j})$
Number Link Restriction ($_\mathcal{Q}$)	$i : (\geqslant nE.D)$	$\lvert r_E(x) \cap D^{\mathcal{I}_j} \rvert_{\neq} \geqslant n$, for $\forall x \in (\geqslant nE.D)^{\mathcal{I}_i}$
	$i : (\leqslant nE.D)$	$\lvert r_E(x) \cap D^{\mathcal{I}_j} \rvert_{\neq} \leqslant n$, for $\forall x \in (\leqslant nE.D)^{\mathcal{I}_i}$
Link Inverse ($_\mathcal{I}$)	$E = F^-$	$r_E = r_F^-$
Link Inclusion ($_\mathcal{H}$)	$E_1 \sqsubseteq E_2$	$r_{E_1} \subseteq r_{E_2}$

E, E_1, E_2 is an $\mathcal{E}$-connection from i to j, F is an $\mathcal{E}$-connection from j to i; D is a j-concept; r_E is the domain relation for E, r_E^- is the inverse of r_E; $|S|_{\neq}$ stands for all-different cardinality of a set S, i.e. the number of elements in S if equivalent elements only counted as one element.

5.3 Relation to P-DL and Semantic Importing of Pan et al.

The proposed $\mathcal{SHOIQP}$ improves the P-DL $\mathcal{ALCP_C}$ by [3] and a related proposal for semantic importing introduced by [18] in several significant aspects:

- **Increased expressivity**: provided by support for the use of $\mathcal{SHOIQ}$ instead of the much more restricted $\mathcal{ALC}$ by individual modules and by support for concept, role and nominal importing (unlike in the case of P-DL $\mathcal{ALCP_C}$ which only allows concept importing).
- **Contextualized negation**: a necessary condition for preservation of unsatisfiability
- **Monotonicity**: a property not guaranteed by the semantic importing approach of Pan et al. [18].

Hence, $\mathcal{SHOIQP}$ is appealing in its expressivity power to model many representative scenarios supported by existing approaches. In addition, it also supports the general reuse of role and nominals which is not supported by any of the existing approach.

6 Summary and Discussion

In this paper, we have introduced a modular ontology language $\mathcal{SHOIQP}$ to reuse knowledge from multiple autonomous ontology modules. A $\mathcal{SHOIQP}$ ontology consists of multiple ontology modules (each of which can be viewed as a $\mathcal{SHOIQ}$ ontology) and concept, role and nominal names can be shared by "importing" relations among modules.

The proposed language supports contextualized interpretation, i.e., interpretation from the point of view of a specific package. We establish the necessary and sufficient constraints on domain relations (i.e., the relations between individuals in different local domains) to preserve the satisfiability of concept formulae, monotonicity of inference, and transitive reuse of knowledge.

Ongoing work is aimed at developing a distributed reasoning algorithm for $\mathcal{SHOIQP}$ by extending the results of [3] and [18]. Instead of a single reasoner, the goal is to use

Table 3. Comparison of Expressivity

Modelling Scenario	**Syntax**	**DDL**	**$\mathcal{E}$-Connections**	$\mathcal{SHOIQP}$
Concept Subsumption	$C \sqsubseteq D$	$\surd$	$\times$	$\surd$
	$C \sqsupseteq D$	$\surd$	$\times$	$\surd$
Concept Negation	$\neg C$	$\surd$	$\times$	$\surd$
Concept Conjunction	$C \sqcap D$	$\surd$	$\times$	$\surd$
Concept Disjunction	$C \sqcup D$	$\surd$	$\times$	$\surd$
Universal Restriction	$\forall R.C$	$\times$	$\surd$	$\surd$
	$\forall P.D$	$\times$	$\times$	$\surd$
	$\forall P.E$	$\times$	$\times$	$\surd$
Existential Restriction	$\exists R.C$	$\times$	$\surd$	$\surd$
	$\exists P.D$	$\times$	$\times$	$\surd$
	$\exists P.E$	$\times$	$\times$	$\surd$
Number Restriction[1]	$\leq nR.C$	$\times$	$\surd$	$\surd$
	$\leq nP.D$	$\times$	$\times$	$\surd$
	$\leq nP.E$	$\times$	$\times$	$\surd$
Role Inclusion	$P \sqsubseteq R$	$\surd$	$\times$	$\surd$[3]
	$P \sqsupseteq R$	$\surd$	$\times$	$\surd$[3]
Role Inverse	P^-	$\times$	$\times$	$\surd$[3]
Transitive Role	$trans(P)$	$\times$	$\surd$[2]	$\surd$
Nominal	$\{x\} \rightarrow \{y\}$	$\surd$	$\times$	$\surd$

[1] $\geq$ case is similar. [2] only with generalized links. [3] a locally non-simple role can not be declared as a sub-role of an imported role or its inverse.
C is an i-concept, D is a j-concept, E is a k-concept; P is an i-role, R is a j-role, Q is a k-role; x is a i-individual, y is a j-individual; $i \neq j$, $j \neq k$, i may be or may not be k. All formulas represent module j's point of view and constructed concepts (roles) are j-terms. Local domains of modules may be partially overlapping.

a federation of multiple communicating reasoners local reasoners to realize distributed reasoning. Each local reasoner creates and maintains a local tableau, which may share some nodes with other tableaux (indicated by domain relations). This approach which has proven successful in the setting where component logics are from $\mathcal{ALC}$ [3], and offers a promising avenue for extending the reasoning algorithm for $\mathcal{SHOIQ}$ [14] to obtain a reasoning algorithm for $\mathcal{SHOIQP}$.

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A Appendix

Some useful properties of image domain relations are listed in the below:

Lemma 5 *Suppose C, D are concepts. A model $\mathcal{I} = \langle \{\mathcal{I}_i\}, \{r_{ij}\}_{i \neq j} \rangle$ of a $\mathcal{SHOIQP}$ KB has the following properties:*

1. *If $P_i \in I(P_j)$ and $P_j \in I(P_i)$, then $r_{ij} = r_{ji}^-$.*
2. $r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) = r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i})$
3. $r_{ij}(C^{\mathcal{I}_i} \cup D^{\mathcal{I}_i}) = r_{ij}(C^{\mathcal{I}_i}) \cup r_{ij}(D^{\mathcal{I}_i})$

4. $r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}((\neg_i C)^{\mathcal{I}_i}) = \emptyset$
5. $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i} \rightarrow r_{ij}(C^{\mathcal{I}_i}) \subseteq r_{ij}(D^{\mathcal{I}_i})$
6. $r_{ij}(C^{\mathcal{I}_i} \setminus D^{\mathcal{I}_i}) = C^{\mathcal{I}_j} \setminus D^{\mathcal{I}_j}$

Proof: 1): We must have $r_{ij} \circ r_{ji} = r_{ii}$. If $(x,y) \in r_{ij}$ and $(y,z) \in r_{ji}$, we must have $x = z$, i.e. $(x,y) \in r_{ji}^{-}$. The other direction is similar.

2) and 3): They are true since domain relations are one-to-one.

4): A corollary of 2).

5): $x \in r_{ij}(C^{\mathcal{I}_i}) \rightarrow r_{ij}^{-}(x) \in C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i} \rightarrow x \in r_{ij}(D^{\mathcal{I}_i})$.

6): From $C^{\mathcal{I}_i} \setminus D^{\mathcal{I}_i} = C^{\mathcal{I}_i} \cap (\neg_i D)^{\mathcal{I}_i}$ and 2)

Proof of Lemma 1

First we prove the "if" direction. If $\Re(P_i^*)$ is consistent, it has at least one model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, (.)^{\mathcal{I}} \rangle$. We will try to construct a model of P_i^* from $\mathcal{I}$. For each package P_i, we will create a projection of $\mathcal{I}$ as its local interpretation $\mathcal{I}_i$ in the following way:

- $\Delta^{\mathcal{I}_i} = (\top_i)^{\mathcal{I}}$
- For every atomic concept name C that appears in P_i, $C^{\mathcal{I}_i} = C^{\mathcal{I}} \cap \top_i^{\mathcal{I}}$
- For every j-role name R that appears in P_i, $R^{\mathcal{I}_i} = (R^{j \rightarrow i})^{\mathcal{I}} = R^{\mathcal{I}} \cap (\top_i^{\mathcal{I}} \times \top_i^{\mathcal{I}})$
- For every nominal name o that appears in P_i, $o^{\mathcal{I}_i} = o^{\mathcal{I}}$

For every pair of P_i and P_j, $P_i \in I(P_j)$, we define the domain relation $r_{ij} = \{\langle x, x \rangle | x \in \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}\}$. Now we prove it is a model of the modular ontology P_i^*.

Condition 1. Domain Relation is One-to-One

It is true from the definition of domain relation.

Condition 2. Domain Relation is Compositionally Consistent

We show that domain relations is also compositionally consistent.

- If $\langle x, x \rangle \in r_{ij}$, it must be that $x \in \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$ (from the definition), hence $x \in \Delta^{\mathcal{I}_k}$ for all k such that $P_i \in I(P_k)$ and $P_k \in I(P_j)$ (because of $\top_i \sqcap \top_j \sqsubseteq \top_k$), therefore $\langle x, x \rangle \in r_{ik}$ and $\langle x, x \rangle \in r_{kj}$, therefore $\langle x, x \rangle \in r_{kj} \circ r_{ik}$.
- If $\langle x, x \rangle \in r_{kj} \circ r_{ik}$, we must have that $x \in \Delta^{\mathcal{I}_i}$ and $x \in \Delta^{\mathcal{I}_j}$, hence by definition $\langle x, x \rangle \in r_{ij}$.

Together, we have that $r_{kj} \circ r_{ik} = r_{ij}$.

Condition 3,4,5. $r_{ij}(t^{\mathcal{I}_i}) = t^{\mathcal{I}_j}$ **for every concept, role or nominal name** t**.**

If X is an atomic concept, role or nominal name, $X^{\mathcal{I}_j} \subseteq X^{\mathcal{I}_i} = X^{\mathcal{I}}$ must be true, i.e., every name has a "official" interpretation in $\Delta^{\mathcal{I}_i}$ and other local domains may have copies of its subsets.

Next we prove $r_{ij}(X^{\mathcal{I}_i}) = X^{\mathcal{I}_j}$ for every i-formula X that appears in P_j. Suppose C and D are i-concepts, E, F are concepts appearing in P_j, R/o is a i-role/nominal that appears in P_j, if $C^{\mathcal{I}_j} \subseteq C^{\mathcal{I}_i}$ and $D^{\mathcal{I}_j} \subseteq D^{\mathcal{I}_i}$, we must have

- If R is imported into P_j, then for every $x, y \in \Delta_j$ such that $(x,y) \in P^{\mathcal{I}_j}$, we must have $\forall z, (x,z) \in P^{\mathcal{I}} \rightarrow z \in \Delta_j$ (since $\exists R.\top_j \sqsubseteq \forall P.\top_j$), therefore $(x,z) \in P^{\mathcal{I}_j}$; similarly, for $\forall z, (z,y) \in P^{\mathcal{I}} \rightarrow z \in \Delta_j$, therefore $(z,y) \in P^{\mathcal{I}_j}$. Hence the forward and backward closure requirements are satisfied.

- $(\neg_i C)^{\mathcal{I}_j} = \top_i^{\mathcal{I}} \cap \top_j^{\mathcal{I}} \backslash C^{\mathcal{I}} \subseteq \top_i^{\mathcal{I}} \backslash C^{\mathcal{I}} = (\neg_i C)^{\mathcal{I}_i}$.
- $(C \sqcap D)^{\mathcal{I}_j} = C^{\mathcal{I}_j} \cap D^{\mathcal{I}_j} \subseteq C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i} = (C \sqcap D)^{\mathcal{I}_i}$
- $(C \sqcup D)^{\mathcal{I}_j} = C^{\mathcal{I}_j} \cup D^{\mathcal{I}_j} \subseteq C^{\mathcal{I}_i} \cup D^{\mathcal{I}_i} = (C \sqcup D)^{\mathcal{I}_i}$
- For every $x \in (\exists R.C)^{\mathcal{I}_j}$, there must be a $y \in \Delta^{\mathcal{I}_j}$ such that $(x, y) \in R^{\mathcal{I}_j}$ and $y \in C^{\mathcal{I}_j}$, hence $(x, y) \in R^{\mathcal{I}_i} = R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}_i}$, therefore $x \in (\exists R.C)^{\mathcal{I}_i}$. $\therefore$ $(\exists R.C)^{\mathcal{I}_j} \subseteq (\exists R.C)^{\mathcal{I}_i}$
- For every $x \in (\forall R.C)^{\mathcal{I}_j}$, for all $y \in \Delta^{\mathcal{I}_j}$, we have $(x, y) \in R^{\mathcal{I}_j}$ implies $y \in C^{\mathcal{I}_j}$; for all $z \in \Delta^{\mathcal{I}_i}$, if $(x, z) \in R^{\mathcal{I}_i}$, we must have $(x, z) \in R^{\mathcal{I}_j}$ (forward closure), hence $z \in C^{\mathcal{I}_j} \subseteq C^{\mathcal{I}_i}$. $\therefore$ $(\forall R.C)^{\mathcal{I}_j} \subseteq (\forall R.C)^{\mathcal{I}_i}$
- For every $x \in (\leq nR.C)^{\mathcal{I}_j}$ we have $|\{y|(x, y) \in R^{\mathcal{I}_j} \wedge y \in C^{\mathcal{I}_j}\}|_{\neq} \leq n$ [2]. For every $z \in \Delta^{\mathcal{I}_i}$ such that $(x, z) \in R^{\mathcal{I}_i} \wedge z \in C^{\mathcal{I}_i}$, according to the forward closure property, $(x, z) \in R^{\mathcal{I}_j}$ and $z \in C^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} = C^{\mathcal{I}_j}$, therefore $|\{z\}|_{\neq} \leq n$. $\therefore (\leq nR.C)^{\mathcal{I}_j} \subseteq (\leq nR.C)^{\mathcal{I}_i}$. Similarly, $(\geq nR.C)^{\mathcal{I}_j} \subseteq (\geq nR.C)^{\mathcal{I}_i}$.
- For an i-nominal o in P_j, we have $o^{\mathcal{I}_i} = o^{\mathcal{I}} = o^{\mathcal{I}_j}$, hence $(o^{\mathcal{I}_i}, o^{\mathcal{I}_j}) \in r_{ij}$.

Hence, we have $X^{\mathcal{I}_j} \subseteq (X^{\mathcal{I}_i})$ for every i-formula X that appears in P_j. We now prove $X^{\mathcal{I}_j} = (X^{\mathcal{I}_i}) \cap \Delta^{\mathcal{I}_i}$ for every i-concept X. It is true when X is an atomic concept. By induction, suppose $C^{\mathcal{I}_j} = (C^{\mathcal{I}_i}) \cap \Delta^{\mathcal{I}_i}$ and $D^{\mathcal{I}_j} = (D^{\mathcal{I}_i}) \cap \Delta^{\mathcal{I}_i}$, we have:

- It is straightforward to verify when X is of the form of $\neg_i C$, $C \sqcap D$ or $C \sqcup D$
- For every $x \in (\exists R.C)^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$, there must be a $y \in \Delta^{\mathcal{I}_i}$ such that $(x, y) \in R^{\mathcal{I}_i}$ and $y \in C^{\mathcal{I}_i}$; since $x \in \Delta^{\mathcal{I}_j}$, $y \in (\exists R^{-}.\top_j)^{\mathcal{I}}$, hence $y \in \top_j^{\mathcal{I}} = \Delta^{\mathcal{I}_j}$. $\therefore x \in (\exists R.C)^{\mathcal{I}_j}$.
- For every $x \in (\forall R.C)^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$, we have $\forall y, (x, y) \in R^{\mathcal{I}_i}$ implies $y \in C^{\mathcal{I}_i}$; similar to the $\exists R.C$ case, since $x \in \Delta^{\mathcal{I}_j}$ implies $y \in \Delta^{\mathcal{I}_j}$, hence $(x, y) \in R^{\mathcal{I}_j} \rightarrow (x, y) \in R^{\mathcal{I}_i} \rightarrow y \in C^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} = C^{\mathcal{I}_j}$. $\therefore x \in (\forall R.C)^{\mathcal{I}_j}$.
- For every $x \in (\leq nR.C)^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$, we have $|\{y|(x, y) \in R^{\mathcal{I}_i} \wedge y \in C^{\mathcal{I}_i}\}|_{\neq} \leq n$ and such a $y \in \Delta^{\mathcal{I}_j}$. $\therefore$, $x \in (\leq nR.C)^{\mathcal{I}_j}$. It is similar for the $\geq$ case.

Condition 6. $\mathcal{I}_j$ is a model of P_j

Next we prove $\mathcal{I}_j$ is a model of P_j. For every GCI of the form $E \sqsubseteq F$ in P_j, suppose E is an i-concept and F is a k-concept, i, j, k may be or not be the same, from the above discussion we must have $E^{\mathcal{I}_j} = E^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} = E^{\mathcal{I}} \cap \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$ and $F^{\mathcal{I}_j} = F^{\mathcal{I}_k} \cap \Delta^{\mathcal{I}_j} = F^{\mathcal{I}} \cap \Delta^{\mathcal{I}_k} \cap \Delta^{\mathcal{I}_j}$. Since $\#(E) \sqsubseteq \#(F)$ must be satisfied in $\mathcal{I}$, we have $\Delta^{\mathcal{I}_j} \cap \Delta^{\mathcal{I}_i} \cap E^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}_j} \cap \Delta^{\mathcal{I}_k} \cap F^{\mathcal{I}}$, i.e. $E^{\mathcal{I}_j} \subseteq F^{\mathcal{I}_j}$.

For every role inclusion of the form $R \sqsubseteq S$ in P_j, we must have $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$, hence $R^{\mathcal{I}_j} = P^{\mathcal{I}} \cap (\Delta^{\mathcal{I}_j} \times \Delta^{\mathcal{I}_j}) \subseteq S^{\mathcal{I}} \cap (\Delta^{\mathcal{I}_j} \times \Delta^{\mathcal{I}_j}) = S^{\mathcal{I}_j}$.

Therefore, $\langle\{\mathcal{I}_i\}, \{r_{ij}\}_{i \neq j}\rangle$ constructed by copying some individuals among local domains indeed is a model of P_i^*. Hence, If $\Re(P_i^*)$ is consistent, then P_i^* is consistent.

Second, we prove the “only if” direction. Suppose P_i^* is consistent, it has a model $\langle\{\mathcal{I}_i\}, \{r_{ij}\}_{i \neq j}\rangle$, we may construct a model $\mathcal{I}$ of $\Re(P_i^*)$ by merging individuals that are connected by image domain relations. $\mathcal{I} = \langle \Delta^{\mathcal{I}}, (.)^{\mathcal{I}} \rangle$ is defined as following:

[2] $|S|_{\neq}$ stands for all-different cardinality of a set S, i.e. the number of element in S if we treat every group of equivalent elements as a single element.

- $\Delta^{\mathcal{I}}$ is the union of all $\Delta^{\mathcal{I}_i}$: $\Delta^{\mathcal{I}} = \cup_i \Delta^{\mathcal{I}_i}$.
- For every named concept, role, or nominal X, $X^{\mathcal{I}} = X^{\mathcal{I}_i}$.
- For every "image" role[3] $X^{i \to j}$, $(X^{i \to j})^{\mathcal{I}} = X^{\mathcal{I}_j}$
- Remove duplicated individuals: For every $(x, y) \in r_{ij}$, remove y from $\Delta^{\mathcal{I}}$, and replace all occurrences of y in $(.)^{\mathcal{I}}$ with x, until no such individuals can be replaced. If both $(x, y) \in r_{ij}$ and $(y, x) \in r_{ji}$ are true, arbitrarily choose one of x or y to be replaced.

We now prove $\mathcal{I}$ is a model of $\Re(P_i^*)$.

- For every i, $\bot_i \sqsubseteq \bot$ is true, since $\bot_i^{\mathcal{I}} = \bot_i^{\mathcal{I}_i} = \bot^{\mathcal{I}} = \emptyset$.
- For every i, j, k, if $P_i \in I(P_j)$, $P_i \in I(P_k)$ and $P_k \in I(P_j)$, we have $r_{kj} \circ r_{ik} = r_{ij}$ in the distributed model $\langle \{\mathcal{I}_i\}, \{r_{ij}\}_{i \neq j} \rangle$. Hence, for every $\langle x, x'' \rangle \in r_{ij}$, there must be a $x' \in \Delta^{\mathcal{I}_k}$ such that $\langle x, x' \rangle \in r_{ik}$ and $\langle x', x'' \rangle \in r_{kj}$. According to the duplicated individual removal rules, x' and x'' are replaced by x in $\mathcal{I}$. For every $x \in \Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j}$, we must have $x \in \Delta^{\mathcal{I}_k}$ (since $x' \in \Delta^{\mathcal{I}_k}$), hence $\top_i \sqcap \top_j \sqsubseteq \top_k$.
- For every atomic i-concept or i-nominal C, $i : C \sqsubseteq \top_i$ is true, since $C^{\mathcal{I}} = C^{\mathcal{I}_i} \subseteq \Delta^{\mathcal{I}_i}$.
- For every atomic i-role P, $P^{\mathcal{I}} = P^{\mathcal{I}_i} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_i}$, hence its domain and range are both restricted to $\top_i$.
- For every atomic i-role P,
  - and every pair $(x, y) \in (P^{i \to j})^{\mathcal{I}} = P^{\mathcal{I}_j}$, we must have $(x, y) \in P^{\mathcal{I}_i} = P^{\mathcal{I}}$, hence $\#(P) \sqsubseteq P$ is true;
  - since $P^{\mathcal{I}_j} \subseteq \Delta^{\mathcal{I}_j} \times \Delta^{\mathcal{I}_j}$, domain and range restriction (both in $\top_j$) of $\#(P) = P^{i \to j}$ is satisfied.
  - according to the forward and backward closure properties, $\exists P.\top_i \sqsubseteq \forall P.\top_i$ and $\exists P^-.\top_i \sqsubseteq \forall P^-.\top_i$ are true.
  - If P is transitive, then $(P^{i \to j})^{\mathcal{I}} = ((P^{i \to j})^{\mathcal{I}})^+$, since if $(x, y) \in (P^{i \to j})^{\mathcal{I}}$ and $(y, z) \in (P^{i \to j})^{\mathcal{I}}$, we have $(x, y) \in P^{\mathcal{I}_j}$, $(y, z) \in P^{\mathcal{I}_j}$, hence $(x, y) \in P^{\mathcal{I}_i}$, $(y, z) \in P^{\mathcal{I}_i}$ (by condition 4 in the semantics of domain relation and the removal of "duplicated individuals"). Since $P^{\mathcal{I}_i} = P^{\mathcal{I}}$, there must be $(x, z) \in P^{\mathcal{I}_i}$, thus $(x, z) \in P^{\mathcal{I}_i} \cap (\Delta^{\mathcal{I}_j} \times \Delta^{\mathcal{I}_j}) = (P^{i \to j})^{\mathcal{I}}$. Therefore $Trans(\#(P))$ is true if P is transitive.
- For every j-GCI of the form $E \sqsubseteq F$, suppose E is an i-concept and F is a k-concept, i, j, k may be or not be the same, we have $E^{\mathcal{I}_j} \subseteq F^{\mathcal{I}_j}$. It is easy to verify $E^{\mathcal{I}_j} = E^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} = (\#(j : E))^{\mathcal{I}}$ (similar to the induction technique we used in the "if" direction proof) and $F^{\mathcal{I}_j} = F^{\mathcal{I}_k} \cap \Delta^{\mathcal{I}_j} = (\#(j : F))^{\mathcal{I}}$, hence $j : \#(E) \sqsubseteq \#(F)$ is true.
- For every role inclusion of the form $R \sqsubseteq S$ in P_j, since $(R^{i \to j})^{\mathcal{I}} = R^{\mathcal{I}_j}$ and $(S^{i \to j})^{\mathcal{I}} = S^{\mathcal{I}_j}$, $\#(R) \sqsubseteq \#(S)$ must be true.

Summarize up, $\Re(P_i^*)$ has a (standard) model if and only if P_i^* has a (distributed) model, hence the lemma is true. Q.E.D.

[3] For convenience, we may also denote $X^{i \to i}$ as the same of X.

Proof of Theorem 1

As usual, we reduce subsumption to (un)satisfiability. Immediately from Lemma 1, we can see $P_i \cup I(P_i)$ and $C \sqcap \neg D$ has a common model if and only if $\Re(P_i^*)$ and $\#(C) \sqcap \neg\#(D)$ has a common model. Hence the lemma is true.

Proof of Theorem 2

Since $C \sqsubseteq_i D$, then $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$ is true for every model of P_i^*, since $I(P_i) \subseteq I(P_j)$ by definition of importing transitive closure, $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$ is also true for every model of $P_j \cup I(P_j)$. Now we prove $r_{ij}(C^{\mathcal{I}_i}) = C^{\mathcal{I}_j}$:

- It is true from the definition of semantics and the proof of lemma 1 if C is an i-concept.
- If C is a k-concept ($k \neq i$), then $C^{\mathcal{I}_i} = r_{ki}(C^{\mathcal{I}_k})$, hence $r_{ij}(C^{\mathcal{I}_i}) = r_{ij}(r_{ki}(C^{\mathcal{I}_k})) = r_{kj}(C^{\mathcal{I}_k}) = C^{\mathcal{I}_j}$.

Similarly, we have $r_{ij}(D^{\mathcal{I}_i}) = D^{\mathcal{I}_j}$. Therefore, $C^{\mathcal{I}_j} = r_{ij}(C^{\mathcal{I}_i}) \subseteq r_{ij}(D^{\mathcal{I}_i}) = D^{\mathcal{I}_j}$ for every model of $P_j \cup I(P_j)$, i.e. $C \sqsubseteq_j D$. Q.E.D.

Proof of Lemma 3

A reduction from a DDL knowledge base $\mathcal{T} = \langle \{\mathcal{T}_i\}, \{\mathcal{B}_{ij}\}_{i \neq j} \rangle$ to a $\mathcal{SHOIQP}$ knowledge base $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$ can be defined in the following way:

- First, for every component TBoxes $\mathcal{T}_i$, define a package P_i with the same set of signature and axioms
- For every into bridge rules between concepts (roles) of the form $i : \phi \xrightarrow{\sqsubseteq} j : \psi$, add the importing axiom $P_i \xrightarrow{i:\phi} P_j$ and a GCI (role inclusion) $(i : \phi) \sqsubseteq (j : \psi)$ to P_j.
- For every onto bridge rules between concepts (roles) of the form $i : \phi \xrightarrow{\sqsupseteq} j : \psi$, add the importing axiom $P_i \xrightarrow{i:C} P_j$ and a GCI (role inclusion) $(j : \psi) \sqsubseteq (i : \phi)$ to P_j.

For every concept $i : C$ and every model $\mathcal{I}$ of Σ, we have $C^{\mathcal{I}_j} = r_{ij}(C^{\mathcal{I}_i})$, hence if $\mathcal{I}$ satisfies $i : C \sqsubseteq j : D$, it must also satisfy the into bridge rule $i : C \xrightarrow{\sqsubseteq} j : D$; similarly, if $\mathcal{I}$ satisfies $j : D \sqsubseteq i : C$, it must also satisfy the onto bridge rule $i : C \xrightarrow{\sqsupseteq} j : D$.

For every concept $i : R$ and every model $\mathcal{I}$ of Σ, for any $(x, y) \in R^{\mathcal{I}_i}$, if $(x, x') \in r_{ij}$ or $(y, y') \in r_{ij}$, then we must have $(x', y') \in R^{\mathcal{I}_j}$, and $(x, y) \in R^{\mathcal{I}_j}$ only if $(r_{ij}^-(x), r_{ij}^-(y)) \in R^{\mathcal{I}_i}$; hence $R^{\mathcal{I}_j} = \bigcup_{(x,y) \in R^{\mathcal{I}_i}} r_{ij}(x) \times r_{ij}(y) = r_{ij}(R^{\mathcal{I}_i})$. Similar to the GCI case, $\mathcal{I}$ satisfies a bridge rule between roles if it satisfies the corresponding role inclusion axiom in Σ.

Therefore, every model of Σ is also a model of $\mathcal{T}$. Hence, DDL with bridge rules between concepts and between roles can be reduced to $\mathcal{SHOIQP}$. Q.E.D.

Proof of Lemma 4:

A $C^{\mathcal{E}}_{\mathcal{IHQ}}(\mathcal{SHOIN})$ knowledge base $\mathcal{T} = \langle \{\mathcal{T}_i\}, \{\mathcal{E}_{ij}\}_{i \neq j} \rangle$ contains a set of local TBoxes $\mathcal{T}_i$, each is from a subset of $\mathcal{SHOIN}$, and a set of one-way $\mathcal{E}$-connections $\mathcal{E}_{ij}$.

A reduction from $\mathcal{T}$ to a $\mathcal{SHOIQP}$ knowledge base $\Sigma = \langle \{P_i\}, \{P_i \mapsto P_j\}_{i \neq j} \rangle$ can be defined in the following way:

- First, for every component TBoxes $\mathcal{T}_i$, define a package P_i with the same set of signature and axioms.
- For every role name $i : R$, add both domain and range restriction of $i : R$ to be in $\top_i$ at P_i, where $\top_i$ is the short of $C \sqcup \neg_i C$ for any local concept name C of P_i.
- For every $E \in \mathcal{E}_{ij}$, add a role name $i : E$ to the local signature of P_i, with domain in $\top_i$ and range in $\top_j$.
- For every concepts constructed by existential link restriction, universal link restriction or number link restriction in the forms given Table 2, add importing axiom $P_j \xrightarrow{j:D} P_i$.
- For every link inverse axiom $E = F^-$, add importing axiom $P_j \xrightarrow{j:F} P_i$ and a local role inverse axiom $i : E = j : F^-$ in P_i.
- For every link inclusion $E_1 \sqsubseteq E_2$ in $\mathcal{T}_i$, add a local role inclusion axiom $E_1 \sqsubseteq E_2$ in P_i.

It is easy to verify that for every $E \in \mathcal{E}_{ij}$, its interpretation in a model of Σ is a subset of $\Delta_i \times (\Delta_i \cap \Delta_j)$. Every model of Σ can be uniquely mapped to a model of $\mathcal{T}$. Hence, Σ can answer any query that might be answered by $\mathcal{T}$. Q.E.D.