2-D ELASTODYNAMIC SCATTERING FROM A FINITE CLOSED CRACK

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INTRODUCTION

In the present paper the problem of 2-D elastodynamic scattering of horizontally polarized transverse waves from a finite planar or nonplanar closed crack is studied. The boundary conditions on the flaw are of a type which incorporate restoring forces (as well as energy dissipation), and this enables the modelling of a crack which is partly closed under a static background pressure. Given an incident plane wave and the crack geometry we calculate the backscattered far field in the time-harmonic case. In this study there are also a numerical comparison between two well known theoretical methods for 2-D scattering of ultrasonic sound by flaws in elastic solids. The methods are the GTD (Geometrical Theory of Diffraction) method that gives an asymptotic solution for high frequencies and the nullfield approach that yields an "exact" numerical solution. The boundary conditions for the partly closed crack are proposed by Boström and Wickham [1]. For a thorough description of the details of the GTD method as applied to scattering problems in elastodynamics, the reader is referred to the book by Achenbach, Gautesen, and McMaken [2]. The nullfield approach has previously been used for treating 3-D planar and nonplanar cracks with similar boundary conditions [3], the same ideas will here be used in the treatment of the 2-D case. The backscattered far field amplitude is numerically calculated and compared between the two methods.

BOUNDARY CONDITIONS

The boundary conditions on the flaw are taken as those of a spring contact model (expressed in scalar form):

$$u_+^{tot} - u_-^{tot} = \frac{g}{k_T}\left[\frac{\partial u^{tot}}{\partial n}\right]_+ \tag{1}$$

$$\left[\frac{\partial u^{tot}}{\partial n}\right]_+ = \left[\frac{\partial u^{tot}}{\partial n}\right]_- \tag{2}$$

where eq. (2) is continuity in traction. Here k_T is the wavenumber and $\hat{n}$ the unit normal of the flaw surface. Subscripts + and – denote limits as we approach the crack from the two different sides. g is a dimensionless parameter which in general may be frequency dependent and complex. These boundary conditions include a number of cases of physical interest and some of them are discussed in Ref.[4]. For example, letting $f \to \infty$, $g \to \infty$ we obtain the open crack and $f = 0$, $g \to \infty$ yields the perfectly lubricated crack. The case of a crack, which is partly closed under static background pressure is in Ref.[1] proposed as

$$g = \frac{\pi\, k_T\, \tilde{r}\, (1 - \nu)}{2C} \tag{3}$$

where the fractional area of contact C is the static background pressure divided by the "flow pressure", ν is Poisson´s ration and $\tilde{r}$ (<< the wavelenght) is the mean radius of the areas of contact.

THE GTD SOLUTION

The version of GTD used here is nonuniform and includes no correction terms. The quantities entering in the GTD solution for the spring contact model are more complicated than the corresponding quantities for the open crack. Here we will only have use of the diffraction coefficient $D(\alpha,\beta)$ which is a function of the incident angle β, the outgoing angle α and the parameter g. The diffraction coefficient is stated in Ref. [4] and will not be repeated here. Figure 1 shows the planar crack where the two crack tips are labeled 1 and 2, the angle of the incident plane wave is $\theta + \pi$ and we will calculate the back-scattered far field in the direction θ. Including multiple diffraction due to interaction between the two crack tips we get the diffracted far field as

$$u^d = u^{pd} + u^{md1} + u^{md2} \tag{4}$$

Here u^{pd} is the primary diffracted field

$$u^{pd} = \frac{e^{ik_T r}}{(k_T r)^{1/2}} \{ D(2\pi - \theta, \pi - \theta)\, e^{i2k_T a\cos\theta} + D(\pi + \theta, \theta)\, e^{-i2k_T a\cos\theta} \} \tag{5}$$

and u^{md1} is the multiply diffracted field emanating from cracktip 1

$$u^{md1} = \frac{e^{ik_T r}}{(k_T r)^{1/2}} \frac{1}{2} e^{ik_T a\cos\theta} D(\theta, \pi) \{ u_{11} + u_{21} \} \tag{6}$$

where u_{11} is the total multiply diffracted field incident on cracktip 1 due to the ray incident on cracktip 1 and u_{21} is the total multiply diffracted field incident on cracktip 1 due to the

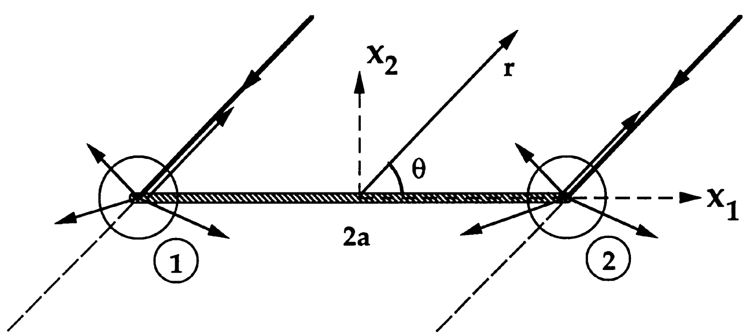

Fig. 1. The planar crack. Fig. also shows the two rays striking the two cracktips.

ray incident on cracktip 2. The multiply diffracted rays form a geometrical series and we have

$$u_{11} = 2\, e^{ik_T a\cos\theta}\, D(0, \pi - \theta)\, \frac{e^{ik_T 4a}}{2k_T a}\, D(2\pi, \pi) \sum_{n=0}^{\infty} (S_1)^n \qquad (7)$$

$$u_{21} = 2\, e^{-ik_T a\cos\theta}\, D(0, \pi - \theta)\, \frac{e^{ik_T 2a}}{(2k_T a)^{1/2}} \sum_{n=0}^{\infty} (S_1)^n \qquad (8)$$

where

$$\sum_{n=0}^{\infty} (S_1)^n = \left[\, 1 - \frac{e^{ik_T 4a}}{2k_T a}\, D(2\pi, \pi)^2 \,\right]^{-1} \qquad (9)$$

The multiply diffracted field emanating from cracktip 2 is then

$$u^{md2} = \frac{e^{ik_T r}}{(k_T r)^{1/2}}\, \frac{1}{2}\, e^{-ik_T a\cos\theta}\, D(\pi - \theta, \pi)\, \{\, u_{22} + u_{12} \,\} \qquad (10)$$

where the expressions for u_{22} and u_{12} correspond to the ones for u_{11} and u_{21}.

The nonplanar crack is assumed to be part of a circle and the opening angle is v, see Fig. 2. The primary diffracted field is written in the same manner as for the planar crack, the only difference is that the crack tips have the angle v to the x_1 axis. That is

$$u^{pd} = \frac{e^{ik_T r}}{(k_T r)^{1/2}} \{\, D(2\pi - \theta - v, \pi - \theta + v)\, e^{i2k_T a\cos\theta} + D(\pi + \theta + v, \theta + v)\, e^{-i2k_T a\cos\theta} \,\} \qquad (11)$$

In this solution we are restricted to $v < \theta < \pi/2 - v$. The lower limit is to secure for no diffraction due to transmitted rays and no tangential incidence on the crackface. The upper limit is to secure for no rays reflected on the crackface. The multiply diffracted field is obtained in the same way as for the planar crack. The difference is that there is an interaction between the crack tips on only one side of the crack. Here we have the expressions for the

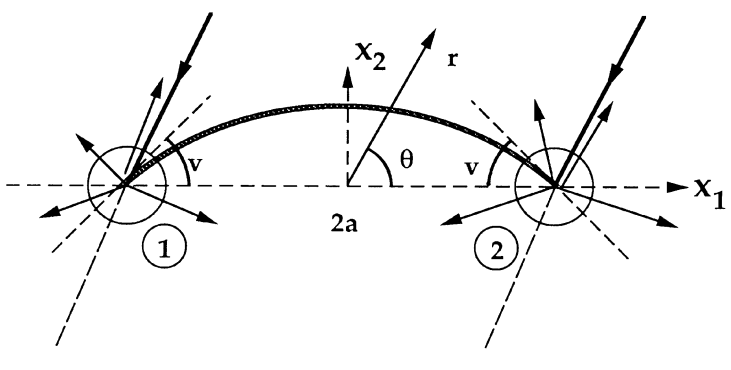

Fig. 2. The nonplanar crack. Fig. also shows the two rays striking the two cracktips.

total multiply diffracted field, incident on cracktip 1 and 2 due to the ray incident on cracktip 1

$$u_{11} = e^{ik_T a\cos\theta}\, D(v, \pi - \theta + v)\, \frac{e^{ik_T 4a}}{2k_T a}\, D(2\pi - v, \pi - v) \sum_{n=0}^{\infty} (S_2)^n \tag{12}$$

$$u_{12} = e^{ik_T a\cos\theta}\, D(v, \pi - \theta + v)\, \frac{e^{ik_T 2a}}{(2k_T a)^{1/2}} \sum_{n=0}^{\infty} (S_2)^n \tag{13}$$

where

$$\sum_{n=0}^{\infty} (S_2)^n = [\, 1 - \frac{e^{ik_T 4a}}{2k_T a}\, D(2\pi - v, \pi - v)^2 \,]^{-1} \tag{14}$$

The total contribution to the backscattered far field is obtained as for the planar crack.

THE NULLFIELD APPROACH

In the nullfield approach we consider a cracklike flaw on an open surface S. The surface may be planar or nonplanar. The boundary conditions on S are the spring boundary conditions. To the surface S we append another surface S_0, thus forming the closed surface $S + S_0$, see Fig. 3.

On S_0 the boundary conditions are the ones of welded contact (g = 0). For a given incoming field we want to compute the scattered field, doing this we will need the 2-D cylindrical wave functions χ_n and $Re\chi_n$. These functions is stated in Ref.[5] and will not be repeated here. Starting from the inner and the outer integral representation

$$u^{\alpha}(r') + \int_{S+S_0} [\, u^{tot}_{+}(r) \frac{\partial}{\partial n} G(r,r') - G(r,r') \frac{\partial}{\partial n} u^{tot}_{+}(r) \,]\, dS = \begin{cases} u^{tot}(r') & , r' \text{ outside } S + S_0 \\ 0 & , r' \text{ inside } S + S_0 \end{cases} \tag{15}$$

$$- \int_{S+S_0} [\, u^{tot}_{-}(r) \frac{\partial}{\partial n} G(r,r') - G(r,r') \frac{\partial}{\partial n} u^{tot}_{-}(r) \,]\, dS = \begin{cases} u^{tot}(r') & , r' \text{ outside } S + S_0 \\ 0 & , r' \text{ inside } S + S_0 \end{cases} \tag{16}$$

we expand Greens function G, the incident field u^a, the scattered field u, the total field u^{tot} and it its normal derivative in cylindrical wave functions according to

$$G(r,r') = i \sum Re\chi_n(r_<)\, \chi_n(r_>) \tag{17}$$

$$u^{\alpha}(r) = \sum a_n\, Re\chi_n(r) \tag{18}$$

$$u(r) = u^{tot} - u^{\alpha}(r) = \sum f_n\, \chi_n(r) \tag{19}$$

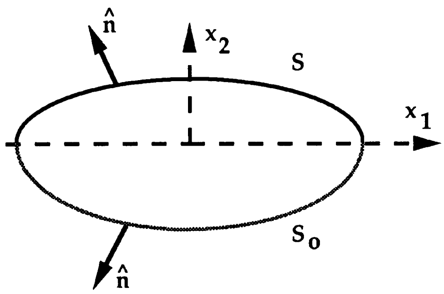

Fig. 3. The closed surface $S + S_0$

$$u_-^{tot}(r) = \sum \beta_{n'} \, Re\chi_{n'}(r) \tag{20}$$

$$\frac{\partial}{\partial n} u_-^{tot}(r) = \sum \gamma_{n'} \frac{\partial}{\partial n} Re\chi_{n'}(r) \tag{21}$$

The expansion coefficients a_n are known and we want to find the coefficients f_n. Using the expansions (17)-(20) in eq.(16) we see that $\beta_n = \gamma_n$. Using this together with the expansions above, the boundary conditions and eq.(15) we will obtain the Q and the ReQ matrices

$$Q_{nn} = i\,\delta_{nn} + \int_S [\, \frac{\partial}{\partial n} \chi_{n'}(r) \frac{\partial}{\partial n} \chi_n(r)\,]\, dS \tag{22}$$

$$ReQ_{nn} = \int_S [\, \frac{\partial}{\partial n} Re\chi_{n'}(r) \frac{\partial}{\partial n} Re\chi_n(r)\,]\, dS \tag{23}$$

We then know the transition matrix

$$T = -\, ReQ\, Q^{-1} \tag{24}$$

and it is possible to calculate the expansion coefficients f_n according to

$$f = T\,a \tag{25}$$

For the case of the nonplanar crack where the crack is part of a circle it is possible to calculate the integrals in the Q and ReQ matrices analytically.

NUMERICAL EXAMPLES AND CONCLUDING REMARKS

The numerical examples will be given without any discussion of the numerical methods or accuracy and will be restricted to the following. Figure 4 shows the backscattered far field vs the wave number for the planar crack. The parameter g = 2. The plane TH-wave strikes the crack at an angle of 45°. The solid line is the GTD solution where the multiply diffracted field is included and the dotted line is only the primary diffracted field. If we compare these two solutions we see that the multiply diffracted field strongly affects the GTD solution for low frequencies. We can also see that the GTD solution shows a very good agreement with the nullfield solution even though the frequency is quite low. Figure 5 shows the same as Fig. 4 but for the boundary conditions of a closed crack. Here $\tilde{r} = 0.008a$, $\nu = 0.29$ and $C = 1/15$. Compared to Fig.4 we see that the frequency dependence of the boundary conditions strongly affects the behaviour of the solution.

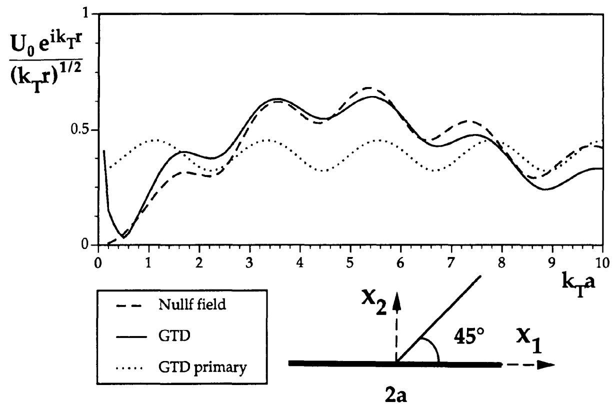

Fig. 4. The backscattered far field vs the wave number for the planar crack. The parameter g = 2.

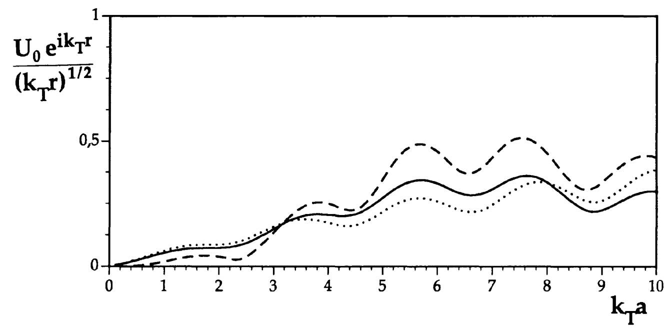

Fig. 5. Same as Fig. 4., but the parameter g is frequency dependent.

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