AN EXPERIMENTAL STUDY OF TOMOGRAPHIC IMAGING IN LAYERED MEDIA

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INTRODUCTION

Ultrasonic tomography has found its applications in material evaluation since the later 70's. However, the techniques in this field are far less developed compared to their x-ray counterparts, which have been widely used in the medical community. One of the practical problems in acoustic tomography is that acoustic waves will not necessarily propagate along straight paths in a nonhomogeneous medium. The situation will be more complicated when material inhomogeneities are coupled with anisotropy as the approach is applied to composite media. In order to resolve the situation, one has either to tolerate the consequence of using straight line ray paths or to seek a way to correct the errors due to ray bending. Indeed, most of the previous work in this area has been based on the straight path assumption. As pointed out by Dines and Lytle[1], if the material inhomogeneity is not serious, the errors caused by straight path assumption can be safely neglected. However, in practice, situations may arise where serious inhomogeneities exist. Even with small inhomogeneities correction is highly desirable when accuracy is of particular concern.

Although the literature on raybending corrections is relatively limited, some work has been done in this area. In addition to several ray tracing algorithms suggested by a number of geologists and seismologists[2][4], Lytle and Dines[5] used an iterative approach based on the differential form of Snell's law to find the curved ray paths and these curved ray paths were then used in the iterative tomographic reconstruction process. Results were obtained for several simulated cases. Berryman[6] also used an iterative technique based on Fermat's principle but with a more involved optimization technique to the same problem. These algorithms may be useful for solving ray bending problems in isotropic media, but the formulation is not readily applicable to anisotropic media. In the current research, we seek to use an iterative approach to find the curved ray paths...
and incorporate it in a tomographic reconstruction algorithm. The key step in this approach is to find the curved ray paths for the current image result. The ray tracing algorithm we presently use is a direct variational formulation of Fermat’s principle. One of the advantages of this approach is its conceptual and algebraic simplicity when applied to anisotropic media such as fiber reinforced composites. As an extension part of our previous work in the area\textsuperscript{[17]}, the primary interest here is to experimentally study the effectiveness of the ray-bending-correction procedure when this algorithm is applied to layered media with each layer being isotropic and homogeneous. Some discussions and observations will be made based on the results obtained.

ACOUSTIC RAY TRACING

According to Fermat’s principle (principle of least time), when a wave propagates from point A to point B in a known velocity field, the wave will propagate along the ray path which takes least time to propagate from A to B. As shown in Fig. 1, the actual wave path will not in general be the straight line connecting A and B in a nonhomogeneous medium. However, the governing equation for the actual ray path can be readily obtained using a variational calculus. Here, a two dimensional problem is considered. Using the slowness $m(x,y)$ (inverse of the velocity), the total time for the wave to propagate from point A to point B can be expressed as

$$T(y) = \int_{x_A}^{x_B} m(x,y) \sqrt{1+y'^2} \, dx$$

where $\sqrt{1+y'^2} \, dx$ represents the differential arc length and $x_A$ and $x_B$ are the x coordinates of point A and point B respectively. A more general form of Eq.(1) is

$$T(y) = \int_{x_A}^{x_B} F(x,y,y') \, dx$$

According to variational calculus, the path minimizing $T(y)$ is given by the following Euler equation.

$$F_y y'' + F_{yy} y' + F_{yx} - F_y = 0$$

For acoustic ray tracing, we have

$$F(x,y,y') = m(x,y) \sqrt{1+y'^2}$$

and, therefore, the various coefficients can be obtained by differentiating Eq.(4) when the slowness field is known. The formal governing equation is true for both isotropic
and anisotropic media. However, other than the algebraic complexity caused by the directional dependence of acoustic properties, there is a conceptual difference when the formulation is applied to anisotropic media. In anisotropic materials, acoustic rays are defined by speed of energy propagation, or group velocity, which is different from phase velocity in both magnitude and direction. Therefore, the extension can be made by replacing \( m(x,y) \) with the inverse of group velocity. With boundary conditions determined by the positions of the transmitter and receiver, the path with least time delay can be obtained by solving a second order ordinary differential equation. However, due to the variable coefficients and nonlinearity of the equation, care needs to be exercised in the process of the solution.

**MODIFIED ART IMAGING ALGORITHM**

In a two dimensional imaging problem, the domain of interested is first divided into \( N \times N \) pixels as shown in Fig. 2. The reconstruction process amounts to solving a system of \( N \times N \) linear equations

\[
T_{kl} = \sum_i \sum_j a_{kij} s_{ij}
\]

where \( a_{kij} \) denotes the length of the \( k \)th ray in the \( ij \)th pixel, \( s_{ij} \) represents slowness to be constructed and \( T_{kl} \) is the measured time delay for the \( k \)th ray path.

With straight path assumption many existing algorithms can be used to solve the system of equations. The most widely used one is the so called algebraic reconstruction technique (ART). This reconstruction algorithm has the advantage of simplicity but with reasonable accuracy even in a relatively noisy measurement environment. Since it deals with one equation at each time it is not necessary to store the large and sparse coefficient matrix. In addition, the algorithm is equally applicable to tomography in anisotropic media with a linearization technique.
Algebraic reconstruction technique is an iterative process. One begins with an arbitrary or available initial guess and modify the solution continually until a satisfactory image is obtained. To illustrate the process for the \( m \)th iteration and \( k \)th ray path, the estimate time delay is computed from the current values of the slowness \( s_{ij} \) by the following equation

\[
T_{kl}^m = \sum_{i} \sum_{j} a_{kl} s_{ij}^m
\]

Then, the current values of \( s_{ij} \) (only those appeared in this equation) are updated so that the estimated time delay matches the measured time delay. This is accomplished by adding a correction factor

\[
\Delta s_{ij}^m = \frac{( T_{kl}^m - T_{kl}^m ) a_{kl}^j}{\sum_{i} \sum_{j} (a_{kl})^2}
\]

to the current slowness of each cell. When the process is repeated for all the paths one completes one single iteration cycle.

If time delay measurements are carried out in a noisy environment, a damping factor is usually employed to improve the convergence behavior. In this modified version,

\[
s_{ij}^{m+1} = s_{ij}^m + \lambda \Delta s_{ij}^m
\]

where \( \lambda \) is a damping factor with a value between 0 and 1.

To consider the ray bending effects, we incorporate the ray tracer discussed earlier in the above solution process. In the case of curved ray paths, the coefficients \( a_{kl}ij \) in Eq.(6) are also unknowns. One possible solution to this problem is to find the approximate values of \( a_{kl}ij \) using the ray tracing technique for the current values of the
image profile. Then, the $a_{klj}$ values obtained are used in the next iteration of the standard ART. However, due to the iterative characteristics of the process, it is often desirable to make several number of additional iterations using the same modified $a_{klj}$ for the image to catch up with the modified curved paths. In practice, this process is applied after some number of iterations using straight paths and therefore an approximate image profile can be obtained very quickly. As one would expect, being able to obtain at least a qualitatively close enough image to the real one is a prerequisite for the approach to work properly. It was also found that it is better to use the slowness image of the last iteration cycle in tracing the ray paths instead of using the most updated one before the iteration cycle is completed.

In solving the differential equation in Eq. (3), the various coefficients must be evaluated. These coefficients include the partial derivatives of the slowness with respect to $x$, $y$ and $y'$. Besides, the original grid division is usually too coarse to numerically integrate the differential equation. Therefore, in tracing the ray path the mesh must be further discretized. This requires a proper interpolation method to give the values of the slowness and its derivatives at any point (possibly out of the domain to be reconstructed). In simple cases where velocity variation is relatively smooth, the shooting approach with a modified metric interpolation method developed by Shepard is employed to accomplish this task. In this method, the interpolated (or extrapolated) value at any location is given by

$$f(x,y) = \frac{\sum_{i=1}^{N} f_i / r_i^\alpha}{\sum_{i=1}^{N} 1 / r_i^\alpha}$$

(10)

where $f_i$ is the value of function $f$ at the $i^{th}$ known point and $r_i$ is the Euclidean distance from point $i$ to the point to be interpolated (or extrapolated), that is,

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$$

(11)

The behavior of the function given by Eq. (10) is strongly dependent upon the value of $\alpha$. In order to reasonably reflect the boundaries between regions with different values of slowness. A value from 10 to 20 for $\alpha$ is preferred. The advantage of this interpolation scheme is that it is simple to use, but the smooth of the interpolant is often not good enough to get convergent results for relatively complex velocity fields. In these situations, it was found that a more sophisticated interpolation scheme such as B-spline combined with a finite difference method to solve the differential equation is necessary to achieve convergence. This approach has been used to reconstruct the image of the aluminum-resin-aluminum specimen where sharp velocity discontinuities exist.

**EXPERIMENTAL PROCEDURE**

To experimentally study the modified ART approach and possible effects of curved ray paths on image reconstruction, we apply this algorithm to two specimens and
compare the results from straight wave paths. One of the specimens is a layered square sample made of 30 layers. In each layer the acoustic velocity is approximately the same. Variation of the velocity among the layers was achieved by filling high polymer resin with a different percentage of aluminum powder in the layers. This sample was made to simulate continuous velocity fields with relatively simple variation. The velocity difference between two edges is about 17%. The specimen has an essentially linear variation of velocity across its width. In the other case, the specimen is made of two pieces of aluminum between which is a piece of cured resin. The geometry of this specimen is shown in Fig. 3. Here the velocity difference is more than 100% relative to the velocity of the resin. This sample is used to test the algorithm's capability of handling sharp velocity discontinuities. To collect the travel time information, two small transducers (known as pinducers) were used to approximate the point source and receiver. Acoustic time delays are determined by identifying the first arrivals in the digitized waveforms.

Fig. 3. Aluminum-resin-aluminum sample geometry.

![Aluminum-resin-aluminum sample geometry.](image)

Fig. 4. Typical ray paths from experimental samples.

(a) Layered  
(b) Aluminum-resin-aluminum

![Typical ray paths from experimental samples.](image)
RESULTS AND DISCUSSIONS

In this section we present the results from the experiments. Fig. 4a shows some typical ray paths from the layered specimen. We observe that wave paths are no longer straight lines even though the velocity difference is small. Also, we note that along the curved ray paths the travel time is shorter than the travel time along the corresponding straight path. However, the difference is small compared with the errors from experimental measuring process due to serious scattering of aluminum powder and reflection and refraction at layer boundaries. Therefore, errors caused by ray bending can be simply neglected. This agrees with the conclusion made by Dines and Lytl.

For the aluminum-resin-aluminum specimen, ray paths are plotted in Fig. 4b, and the images reconstructed from straight paths and the modified algorithm are shown in Figs. 5a and 5b respectively. Due to the large velocity differences, Curved wave path phenomenon is readily observable. Now, the travel time differences between the curved paths and straight paths are so large that the quality of reconstructed image will be degraded if straight ray model is used. Comparison of Figs. 5a and 5b shows that the modified algorithm does improve the quality of the reconstructed image qualitatively. A quantitative comparison reveals a 30% reduction in error. The result is satisfactory considering the following facts: Although a theoretical wave path can be predicted, the energy magnitude along this wave path does not guarantee it can be experimentally detected by the receiver. Furthermore, due to ray bending, fewer rays will pass through the lower velocity regions and therefore those regions may not benefit from the correction process. To overcome this problem, a preferred sampling scheme has to be used for uniform improvement in image reconstruction. Considerations for these factors could become future subjects in tomographic image reconstruction.
CONCLUSIONS

A modified imaging technique with acoustic ray tracing was described. When the algorithm was applied to experimental specimens, it was found that the ray tracer correctly traces the wave paths. For reconstruction with large inhomogeneities the modified algorithm improves the quality of image reconstruction. Due to the limitation discussed above this approach is more suitable for the situations where moderate material inhomogeneities exist and high accuracy reconstruction is required.

REFERENCES