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ACTIVE INTERMEDIATION IN OVERLAPPING GENERATIONS ECONOMIES WITH PRODUCTION AND UNSECURED DEBT

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It is well known that the first welfare theorem can fail for overlapping generations economies with private production and unsecured debt. This paper demonstrates that the reason for this failure is that intermediation is modeled as a purely passive coordination activity implemented by a Walrasian Auctioneer. When intermediation is modeled instead as a contestable activity carried out by a corporate intermediary owned by consumer-shareholders and operated in their interest, every equilibrium is Pareto efficient. In broader terms, these findings caution that the inefficiency observed in standard modelings of overlapping generations economies may not be the reflection of an intrinsic market failure. Rather, the observed inefficiency could instead be due to a fundamental incompleteness in the model specification—the presumed inability of private agents to exploit the earnings opportunities associated with incurring and forever rolling over debt.

Keywords: Intermediation, Overlapping Generations, Pareto Efficiency, Production, Unsecured Debt, Bubbles

1. INTRODUCTION

Tirole (1985) studies an overlapping generations economy in which agents can issue consumption loans, as in Samuelson (1958), and also can engage in private goods production as in Diamond (1965, pp. 1130–1135). In particular, the savings of each generation can be used in part to finance the consumption by agents whose consumption demands are in excess of their endowments and in part to finance the capital investment of firms. Tirole shows that the resulting economy fails to satisfy the first welfare theorem. Specifically, as reviewed in Section 2, below, two stationary competitive equilibria exist for this economy: a Pareto-inefficient equilibrium with no consumption loans, and a Pareto-efficient "golden rule" equilibrium in which consumption loans are made.

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This paper suggests that the failure of the Tirole economy to satisfy the first welfare theorem can be attributed to the passive role assigned to intermediation.[1] The only intermediary in the Tirole economy is an implicitly present Walrasian Auctioneer concerned with coordination but not with optimization. As detailed in Section 3, below, private agents in the Tirole economy do not exploit viable and profitable debt-issue opportunities.

To explore this point further, the Tirole economy is generalized in Section 4 to include the explicit presence of a corporate intermediary owned by consumer-shareholders.[2] The efficiency properties exhibited by the resulting brokered economy then depend on the exact modeling of the intermediary's objective. One possibility is that the brokered-economy intermediary behaves as a Walrasian Auctioneer, i.e., as a price-setting agent concerned only with trade and credit coordination. In Section 5.1, it is shown that the brokered economy reduces to the Tirole economy in this case.

Another possibility, however, is that the intermediary moves beyond mere coordination and acts in the interests of its shareholders. In Section 5.2, we retain the assumption that the intermediary is a price setter, but we also assume that the intermediation market is contestable. That is, we assume that entry occurs when an entrant can feasibly assume the liabilities of the existing corporate intermediary and can strictly increase the dividends paid to its current shareholders. We assume that the existing corporate intermediary sets its stock share prices so as to prevent the entry of any competitors, and we extend the standard definition of an equilibrium to include this corporate intermediary objective. Under these assumptions, we show that *every* brokered-economy equilibrium is Pareto efficient.

The crucial fact used to establish this brokered-economy first welfare theorem is that entry into the intermediation market is infeasible only if a certain price condition holds. This price condition is analogous to the well-known Cass–Balasko–Shell transversality condition shown by Balasko and Shell (1980) to be necessary for Pareto efficiency in the context of a pure-exchange overlapping generations economy. Our findings thus suggest an economic interpretation for this transversality condition as a constraint on the price-setting behavior of an optimizing corporate intermediary in a contestable intermediation market.

Overall, our findings indicate that the Pareto inefficiency exhibited by commonly used models of dynamic open-ended economies with finite-lived agents can, to a large extent, be accounted for by the fact that intermediation is modeled as a purely passive coordination activity. As demonstrated here for the Tirole economy, this inefficiency may be ameliorated or even eliminated if intermediation is more realistically modeled as an activity oriented toward the exploitation of available profit opportunities.

2. TIROLE ECONOMY

2.1. Basic Model Structure

As depicted in Figure 1, the model of an economy developed by Tirole (1985)—hereafter referred to as the Tirole economy—is an overlapping generations

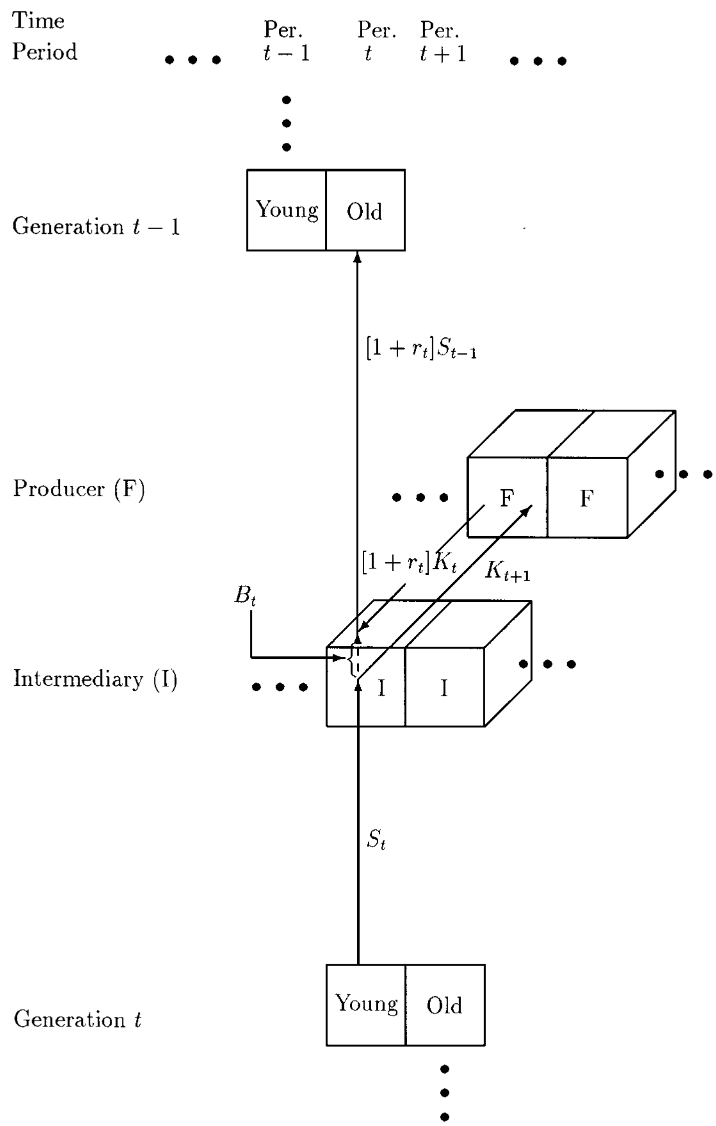

FIGURE 1. Tirole economy.

economy in which each consumer lives for just two periods: youth and old age. The generation of consumers born at the beginning of period t consists of L_t consumers and is referred to as generation t. Population grows at the rate $n \geq 0$, so that $L_{t+1} = [1+n]L_t$. The economy begins in period 1 with $L_0 > 0$ old consumers of generation 0 and $L_1 = [1+n]L_0$ young consumers of generation 1.

The Tirole economy has a single physical resource that may be either consumed or used in production as capital. Adopting the conventional time dating of goods, the resource during period t is referred to as good t. Each young consumer in each generation $t \geq 1$ inelastically supplies one unit of labor in return for a real (resource) wage w_t. Wage income is used by young consumers to provide young-age consumption c_t^1 and savings s_t. Old-age consumption c_{t+1}^2 is provided entirely from savings and accumulated interest; old consumers do not work.

Consumer saving is held in two possible forms: units of capital, and units of a bubble asset that pays no dividends.[3] The bubble asset can be thought of as vouchers representing real purchasing power—for example, as real money balances, or as a credit or debit held on account with an intermediary. If capital and the bubble asset are both to be held in competitive equilibrium, the bubble asset must bear the same yield as capital. Thus, it is assumed that saving in either form has a common rate of return r_{t+1} over each period $t \geq 1$.

Consumers have identical preferences. The objective of each generation-t young consumer is to maximize his lifetime utility $U(c_t^1, c_{t+1}^2)$, where $U(\cdot)$ has the usual curvature properties.[4] Given any $w_t > 0$ and $r_{t+1} > -1$, the problem faced by this young consumer takes the form

$$\max U\left(c_t^1, c_{t+1}^2\right) \tag{1}$$

with respect to (s_t, c_t^1, c_{t+1}^2) subject to the budget and nonnegativity constraints

$$\begin{aligned} c_t^1 &= w_t - s_t; \\ c_{t+1}^2 &= [1 + r_{t+1}]s_t; \\ c_t^1 \geq 0, &\qquad c_{t+1}^2 \geq 0. \end{aligned}$$

Let the solution to this problem be denoted by

$$x(w_t, r_{t+1}) = \left(s(w_t, r_{t+1}), c^1(w_t, r_{t+1}), c^2(w_t, r_{t+1})\right). \tag{2}$$

Each old consumer in the initial period 1 receives a payment $[1 + r_1]s_0$ of principal plus interest, where the savings level s_0 satisfies $s_0 = s(w_0, r_1)$ and the savings function $s(\cdot)$ coincides with the savings function determined for each generation $t \geq 1$ agent in (2). The consumption by each old consumer in period 1 is therefore $c_1^2 = [1 + r_1]s(w_0, r_1)$.

Output in the Tirole economy is produced at the beginning of each period by a firm that uses capital and labor inputs in accordance with the production relation $Y = F(K, L)$. The production function $F(\cdot)$ is assumed to exhibit constant returns to scale, and to satisfy the usual continuity and curvature restrictions.[5] Letting $k \equiv K/L$ and $y \equiv Y/L$ denote the capital/labor ratio and the output/labor ratio, respectively, the production relation can be expressed in per-capita (i.e., per-worker) form as $y = F(k, 1) \equiv f(k)$.

In each period $t \geq 1$ the firm must pay the wage rate w_t to each unit of employed labor as well as a rental rate for each unit of employed capital. The rental rate on capital in period t is assumed to be the same as the interest rate r_t on savings.

The price-taking firm selects levels of capital and labor inputs to maximize profits. Formally, the firm's problem may be stated as

$$\max_{K\geq 0, L\geq 0} [F(K, L) - r_t K - w_t L]. \tag{3}$$

For a positive pair (K_t, L_t) of capital and labor inputs to solve problem (3), it is both necessary and sufficient that the capital/labor ratio $k_t \equiv K_t/L_t$ satisfy

$$r_t = f'(k_t); \tag{4}$$

$$w_t = f(k_t) - f'(k_t)k_t. \tag{5}$$

Conditions (4) and (5) generate the well-known factor-price frontier relationship between the wage rate w_t and the interest rate r_t. For any given interest rate $r_t > 0$, let $k(r_t)$ denote the capital/labor ratio k_t that uniquely satisfies condition (4). Substituting $k(r_t)$ into condition (5), the wage rate w_t that satisfies condition (5) then is uniquely determined as a strictly decreasing function $w(r_t)$ of the interest rate r_t.

In each period $t \geq 1$ the supply of capital consists of aggregate savings $S_{t-1} \equiv L_{t-1}s_{t-1}$ less that part of savings held in the form of the bubble asset. Let $B_{t-1} \equiv L_{t-1}b_{t-1}$ denote aggregate bubble asset holdings and K_t denote the firm's aggregate demand for capital. Then, in per-capita terms, supply equals demand[6] in the capital market when

$$[s_{t-1} - b_{t-1}]/[1 + n] = k_t. \tag{6}$$

As in Tirole (1985, p. 1503), the following restriction is imposed on the growth of the aggregate bubble asset holdings B_t:

$$B_t = [1 + r_t]B_{t-1}, \tag{7}$$

or, in per-capita terms,

$$b_t = [(1 + r_t)/(1 + n)]b_{t-1}. \tag{8}$$

The implications of restriction (7) are examined in Section 3.

Young consumers in each period $t \geq 1$ supply labor inelastically, in total amount L_t. Supply equals demand in the labor market when the firm chooses to employ this labor supply. As in Tirole (1985), it is assumed that supply equals demand in the labor market in each period t, so that L_t denotes the period-t workforce as well as the period-t population of young consumers.

Because capital in the Tirole economy does not depreciate, the total supply of product available in each period t is $Y_t + K_t$. The total demand for product includes capital demand for the following period, K_{t+1}, and aggregate consumption for the current period, $L_t c_t^1 + L_{t-1}c_t^2$. In per-capita terms, supply equals demand in the product market for period t when

$$y_t + k_t = [1 + n]k_{t+1} + c_t^1 + c_t^2/[1 + n]. \tag{9}$$

Finally, following Tirole (1985), the economy is initialized by assuming that the capital/labor ratio $k_0 > 0$ and the per-capita bubble asset holdings $b_0 \geq 0$ are exogenously given. The capital/labor ratio k_0 in turn determines the initial interest rate r_0 and the initial wage rate w_0 in accordance with the marginal productivity conditions

$$r_0 = f'(k_0); \tag{10}$$

$$w_0 = f(k_0) - f'(k_0)k_0. \tag{11}$$

The Tirole economy then can be reduced to a pair of difference equations in the state variables k_t and b_t over times $t \geq 1$,

$$[1+n]k_t = s(f(k_{t-1}) - f'(k_{t-1})k_{t-1},\ f'(k_t)) - b_{t-1}; \tag{12}$$

$$b_t = ([1 + f'(k_t)]/[1+n])b_{t-1}, \tag{13}$$

starting from the exogenously determined initial values k_0 and b_0.

2.2. Competitive Equilibrium and Efficiency

As in Tirole (1985), a competitive equilibrium is defined for the Tirole economy in terms of optimality conditions for the consumers and the firm, the capital market-clearing condition, and the growth restriction on bubble asset holdings, all in per-capita form.[7]

DEFINITION 1 (Tirole Equilibrium). *Given initial values $k_0 > 0$ and $b_0 \geq 0$ for capital and bubble asset holdings, a sequence $(s_t, c_t^1, c_t^2, k_t, b_t, r_t, w_t : t \geq 1)$ of savings levels s_t, consumption levels c_t^1 and c_t^2, positive capital/labor ratios k_t, per-capita bubble asset levels b_t, interest rates $r_t > 0$, and wage rates $w_t > 0$ is a* competitive equilibrium $\boldsymbol{e}(k_0, b_0)$ *for the per-capita Tirole economy if and only if it satisfies the following four conditions*:

(i) Firm optimization. *Conditions* (4) *and* (5) *hold for each $t \geq 0$.*
(ii) Consumer optimization. *In each period $t \geq 1$, the young consumer's choice vector $\boldsymbol{x}_t = (s_t, c_t^1, c_{t+1}^2)$ solves the lifetime utility maximization problem* (1) *conditional on w_t and r_{t+1}, i.e., $\boldsymbol{x}_t = x(w_t, r_{t+1})$; and each old consumer in period* 1 *consumes $c_1^2 = [1+r_1]s_0$ with $s_0 = s(f(k_0) - f'(k_0)k_0, r_1)$.*
(iii) Capital market clearing. *In each period $t \geq 1$, condition* (6) *holds.*
(iv) Bubble asset growth restriction. *In each period $t \geq 1$, per-capita bubble asset holdings grow in accordance with condition* (8).

A competitive equilibrium $\boldsymbol{e}(k_0, b_0)$ for the Tirole economy is called stationary *if $k_t = k_0$ and $b_t = b_0$ for each period $t \geq 1$.*

Tirole (1985) assumes that there exists a unique $\bar{r}$ satisfying $s(w(\bar{r}), \bar{r}) = [1+n]k(\bar{r})$, with $0 < \bar{r} < n$. Also, as detailed in Section A.1 of the Appendix, several additional technical regularity conditions are imposed on the savings function $s(\cdot)$ and the production function $f(\cdot)$. Given these regularity conditions, the Tirole economy has two distinct stationary competitive equilibria: a Pareto-inefficient equilibrium characterized by the interest rate $r = \bar{r}$ and *zero* bubble asset holdings, and a Pareto-efficient golden-rule equilibrium characterized by the interest rate $r = n$ and *positive* bubble asset holdings.

Given the stationary interest rate $r = \bar{r}$, the optimal consumer and firm choice variables and the wage rate are stationary and are given by $\bar{\boldsymbol{x}} = x(w(\bar{r}), \bar{r})$, $\bar{k} = k(\bar{r})$, and $\bar{w} = w(\bar{r})$. It is easily established that the sequence $\boldsymbol{e}(\bar{k}, 0) = (\bar{\boldsymbol{x}}, \bar{k}, 0, \bar{r}, \bar{w} : t \geq 1)$ satisfies the conditions in Definition 1 characterizing a stationary competitive equilibrium. Hereafter, this bubbleless stationary competitive equilibrium is abbreviated by $\bar{\boldsymbol{e}}$.

The allocation achieved under the bubbleless stationary equilibrium $\bar{\boldsymbol{e}}$ is not Pareto efficient. Because $\bar{r} < n$, a suitably small reduction in the equilibrium capital/labor ratio $\bar{k}$ in any given period t', offset by a corresponding increase in bubble asset holdings, permits an increased stationary level of net output $y - nk$ in all periods $t \geq t'$. This in turn implies that young- and old-age per-capita consumptions also can be increased in all periods $t \geq t'$; see condition (9).

Given the stationary interest rate $r = n$, the optimal consumer and firm choice variables and the wage rate take on the stationary values $\boldsymbol{x}^n = x(w(n), n)$, $k^n = k(n)$, and $w^n = w(n)$. Bubble asset holdings then are given by $b^n = s(w(n), n) - [1 + n]k(n)$. Although the sign of b^n is not explicitly determined by Tirole (1985), it can be shown that b^n is positive.[8] It then is verified easily that the sequence $\boldsymbol{e}^n(k^n, b^n) = (x^n, k^n, b^n, n, w^n : t \geq 1)$ satisfies all conditions in Definition 1 characterizing a stationary competitive equilibrium. The allocation generated under $\boldsymbol{e}(k^n, b^n)$ is the Pareto-efficient allocation yielding maximum net output $y - nk$ in each period $t \geq 1$. Hereafter, the equilibrium $\boldsymbol{e}^n(k^n, b^n)$ is abbreviated as $\boldsymbol{e}^n$.

Tirole (1985, Prop. 1, p. 1504, and Prop. 2, p. 1507) provides a complete characterization of the stability and efficiency properties of the Tirole economy under the assumption that $\bar{r} < n$. Specifically, given any initial capital/labor ratio $k_0 > 0$, he shows that there exists a maximum feasible value $b(k_0)$ for the initial bubble asset holdings b_0 such that the following two results are true. First, given the initial conditions $(k_0, b(k_0))$, there exists a unique competitive equilibrium $\boldsymbol{e}(k_0, b(k_0))$; this equilibrium is Pareto efficient, and it converges to the Pareto-efficient golden-rule stationary equilibrium $\boldsymbol{e}^n$. Second, given the initial conditions (k_0, b_0) for any b_0 satisfying $0 \leq b_0 < b(k_0)$, there exists a unique competitive equilibrium $\boldsymbol{e}(k_0, b_0)$; this equilibrium is Pareto inefficient, and it converges to the Pareto-inefficient bubbleless stationary equilibrium $\bar{\boldsymbol{e}}$. These properties are schematically depicted in Figure 2, a modified version of Figure 1 in Tirole (1985, p. 1505).

In general, of course, Pareto inefficiency of a stationary equilibrium does not necessarily imply that any equilibrium converging to this stationary equilibrium must itself be Pareto inefficient. Consider, for example, the Ramsey–Cass–Koopmans one-sector neoclassical optimal growth model with a positive social discount rate and an infinite time horizon as depicted by Blanchard and Fischer (1989, Fig. 2.2, p. 46). To each positive initial capital/labor ratio there corresponds a unique optimal (hence Pareto-efficient) growth path; but this optimal growth path converges to a stationary equilibrium that is strictly Pareto-dominated by the golden-rule

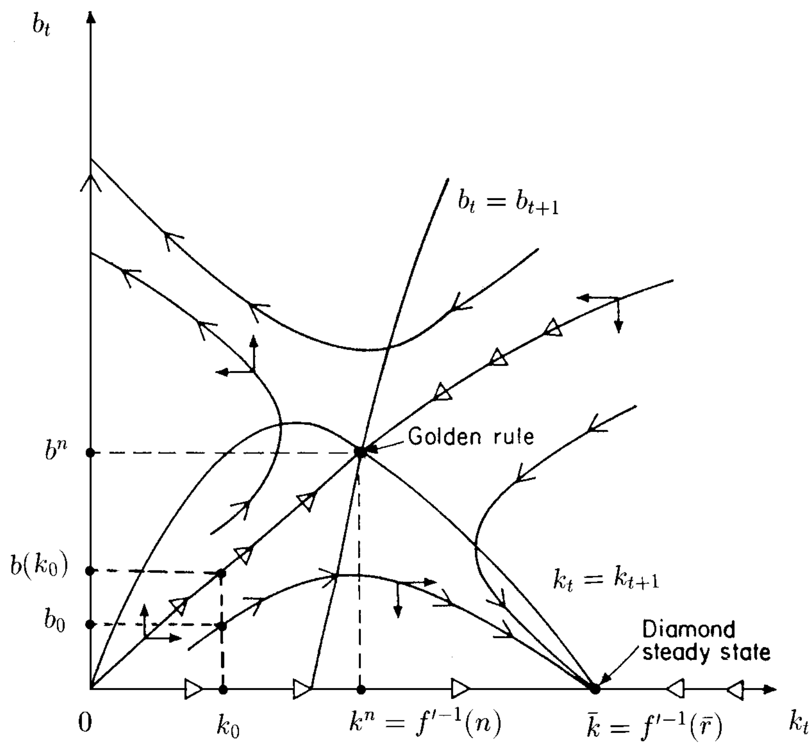

FIGURE 2. Phase diagram for Tirole economy.

stationary equilibrium. For the Tirole economy, however, each equilibrium $\boldsymbol{e}(k_0, b_0)$ with $0 \leq b_0 < b(k_0)$ is Pareto inefficient because it is characterized by capital over-accumulation in every period. A suitably small increase in the bubble asset holdings b_{t-1} offset by a decrease in the capital/labor ratio k_t in any period t permits an increase in the period-t net output $y_t - nk_{t+1}$ and hence also in the per-capita consumptions for the young and old consumers in all periods $t' \geq t$.

3. PASSIVE MEDIATION IN TIROLE ECONOMY

The Tirole economy does not include an explicit intermediating institution. Nevertheless, it is still possible to consider production and intermediation as distinct functions. Production is the transformation of capital and labor inputs into output. Intermediation matches agents who have demands with suppliers who can meet these demands. Here, attention is focused on the intermediation of demands and supplies for savings, and the term "intermediary" is used as a shorthand for the implicit agency that mediates savings activities in the Tirole economy.

The Tirole economy differs from the economy modeled by Diamond (1965) in one crucial respect: The market-clearing condition for capital is relaxed to allow for consumption loans, i.e., intergenerational transfers of goods that permit some agents to consume more than their current endowments. Specifically, as depicted in Figure 1, the Diamond market-clearing condition $S_t = K_{t+1}$ is replaced by the condition $S_t = B_t + K_{t+1}$. From the viewpoint of period-t young consumers, B_t represents the portion of their savings held in the form of a bubble asset. From the viewpoint of the intermediary, B_t represents the portion of the intermediary's deposits *not* invested in capital. The intermediary is thus potentially able to use B_t to enhance the consumption of generation $t-1$ old consumers.

More precisely, the incoming receipts of the Tirole intermediary in period t consist of the savings deposits S_t received from young consumers and the repayment $[1+r_t]K_t$ of principal plus interest received from the firm for capital borrowed in period $t-1$. On the other hand, the intermediary is obliged to pay $[1+r_t]S_{t-1}$ of principal plus interest to generation $t-1$ old consumers and to provide the firm's demand for capital, K_{t+1}. The *net receipts* received by the intermediary in period t therefore are given by

$$\begin{aligned} \Pi_t &= S_t + [1+r_t]K_t - [1+r_t]S_{t-1} - K_{t+1} \\ &= [S_t - K_{t+1}] - [1+r_t][S_{t-1} - K_t] \\ &= B_t - [1+r_t]B_{t-1}. \end{aligned} \tag{14}$$

Comparing (14) with the restriction (7) imposed by Tirole on the growth of the bubble asset holdings B_t, the latter restriction appears in a new light. Specifically, the Tirole restriction (7) forces the intermediary's net receipts (14) to be zero in each period $t \geq 1$. In imposing condition (7), Tirole (1985, p. 1503) correctly notes that physical capital and the bubble asset must earn the same yield in order for both assets to be held in equilibrium. The equality of these yields, however, does not imply that restriction (7) must hold. Rather, restriction (7) holds if and only if an additional special-case restriction is invoked: Namely, the bubble asset remains in fixed supply.

To see this, let Θ_{t-1} denote the quantity of the bubble asset supplied by the intermediary in period $t-1$, and let p^{θ}_{t-1} denote the price of the bubble asset during period $t-1$ measured in units of good $t-1$. Because the bubble asset is used exclusively to finance consumption loans, it follows that $B_{t-1} = p^{\theta}_{t-1}\Theta_{t-1}$. For the bubble asset to be held at the same time that capital loans are being made, the bubble asset must earn the same rate of return as capital, meaning that $p^{\theta}_t / p^{\theta}_{t-1} = [1+r_t]$. Consequently, the following sequence of equalities must hold:

$$\begin{aligned} \Pi_t &= B_t - [1+r_t]B_{t-1} \\ &= p^{\theta}_t\Theta_t - [1+r_t]p^{\theta}_{t-1}\Theta_{t-1} \\ &= p^{\theta}_t[\Theta_t - \Theta_{t-1}]\,. \end{aligned} \tag{15}$$

As condition (15) makes clear, whether or not the Tirole restriction (7) holds, and hence whether or not the intermediary has nonzero net receipts, depends upon whether or not the supply of the bubble asset is fixed.

As shown in note 7, the Tirole restriction (7) is equivalent to the restriction that supply exactly equals demand in the product market. Thus, if restriction (7) need not hold, product market-clearing in this strong form need not hold either (cf. note 6). In fact, the intermediary's ability to achieve positive net receipts depends upon his ability to generate an excess supply in the product market. When there is an excess *demand* in the product market, B_t is strictly less than $[1 + r_t]B_{t-1}$ and the intermediary's net receipts Π_t are negative. This situation cannot arise in competitive equilibrium because the intermediary is unable to fulfill all contractual obligations. In contrast, when there is an excess *supply* in the product market, i.e., when B_t is strictly greater than $[1 + r_t]B_{t-1}$, the intermediary's net receipts are positive. In this case, the intermediary can meet all contractual obligations and then keep the remaining excess supply.

In short, once the assumption of a fixed bubble asset supply is relaxed, it is possible for the intermediary to obtain ownership over a positive quantity of the economy's resources by increasing the supply of the bubble asset. With this opportunity present, is it reasonable to suppose that the coordination of trade and credit transactions remains the intermediary's sole objective? If not, the intermediary's stance toward accumulating net receipts must be clarified in order for the model of the economy to be complete.

The next section sets out a generalization of the Tirole economy that explicitly includes a corporate intermediary with an objective that goes beyond passive coordination. In particular, the intermediary is not required to keep the bubble asset in fixed supply. The efficiency implications of assuming this more active form of intermediation are explored in Section 5.

4. BROKERED ECONOMY

Consider an extension of the Tirole economy, hereafter referred to as the brokered economy, in which all savings activities are explicitly mediated by a single intermediary. Consumers have the opportunity to purchase two assets from the intermediary: namely, bonds, and shares of stock in the intermediary. There are no risk differences between bonds and stock shares, so that consumers base their asset choices only upon expected rate of return.

Each young consumer in generation $t \geq 1$ demands λ_t^d bonds and θ_t^d shares of stock, to be sold in the subsequent period. The period-t prices of bonds and stock shares in terms of good t are p_t^λ and p_t^θ, $t \geq 1$, and are taken as given by consumers. The intermediary distributes dividends to its consumer-shareholders in accordance with the following dividend policy: Each share of stock purchased during period t entitles the owner to a per-share dividend in period $t + 1$.

Let d_{t+1} denote the per-share dividend expected by each young consumer in generation $t \geq 1$. The utility maximization problem of this price-taking young consumer then takes the form

$$\max U\left(c_t^1, c_{t+1}^2\right) \tag{16}$$

with respect to $(c_t^1, c_{t+1}^2, \lambda_t^d, \theta_t^d)$ subject to the budget and nonnegativity constraints

$$c_t^1 = w_t - p_t^\lambda \lambda_t^d - p_t^\theta \theta_t^d;$$
$$c_{t+1}^2 = p_{t+1}^\lambda \lambda_t^d + p_{t+1}^\theta \theta_t^d + d_{t+1}\theta_t^d;$$
$$c_t^1 \geq 0, \qquad c_{t+1}^2 \geq 0.$$

The utility function appearing in (16) is assumed to be the same as described in Section 2 for the Tirole economy.

No sign restrictions are placed on λ_t^d or θ_t^d, implying that short sales are allowed for both bonds and stock shares. Consequently, assuming that all prices are positive, no finite solution exists for problem (16) unless bonds and stock shares yield a common positive gross rate of return, i.e., unless

$$p_{t+1}^\lambda / p_t^\lambda = \left[p_{t+1}^\theta + d_{t+1}\right] / p_t^\theta = [1 + r_{t+1}] \tag{17}$$

for some $r_{t+1} > -1$.

Given positive prices, together with the viability condition (17), it is straightforward to show that the utility maximization problem (16) faced by each generation-t young consumer in the brokered economy can equivalently be expressed in form (1), i.e., in the same form as the utility maximization problem faced by each generation-t young consumer in the Tirole economy. Consequently, as for the Tirole economy, an optimal solution exists for the consumer's planned savings level $s_t \equiv [c_t^1 - w_t]$ and consumption levels (c_t^1, c_{t+1}^2) as a function of the wage w_t and the common rate of return r_{t+1}. Hereafter, this optimal solution is denoted by[9]

$$\big(s(w_t, r_{t+1}), c^1(w_t, r_{t+1}), c^2(w_t, r_{t+1})\big). \tag{18}$$

On the other hand, the individual bond and stock-share demands of each brokered-economy consumer are indeterminate because the consumer is indifferent among all bond and stock-share combinations $(\lambda_t^d, \theta_t^d)$ that yield his optimal planned savings level.

The problem facing the firm in the brokered economy is identical to that facing the firm in the Tirole economy. In each period $t \geq 1$, the profit-maximizing firm has the opportunity to rent capital from the intermediary to be used as an input to production along with the labor L_t supplied inelastically by young consumers. The rental capital K_t^d that the firm plans to employ during period t is demanded from the intermediary during period $t-1$. As for the Tirole economy, it is assumed that the interest rate charged to the firm for the rental of this capital is the same as the interest rate r_t on savings. Consequently, the firm plans to make a payment $[1 + r_t]K_t^d$ of principal plus interest to the intermediary in period t.

The intermediary's net receipts Π_t in period $t \geq 1$ are equal to the quantity of good t remaining in the intermediary's possession after all of his contractual obligations are fulfilled, apart from dividend payments. These net receipts are determined by a consideration of the intermediary's bond, stock-share, and capital transactions.

Let $\Lambda_t^s \equiv L_t \lambda_t^s$ and $\Theta_t^s \equiv L_t \theta_t^s$ denote the total amounts of bonds and stock shares that the intermediary plans to supply to consumers in period t, and let K_t^s denote the amount of rental capital that the intermediary plans to supply to the firm for use in period t. As a result of bond sales and purchases, the intermediary in period t plans to receive $p_t^\lambda \Lambda_t^s$ units of good from generation-t young consumers and to deliver $p_t^\lambda \Lambda_{t-1}^s$ units of good t to generation-($t-1$) old consumers. Moreover, as a result of stock-share sales and purchases, the intermediary in period t plans to receive $p_t^\theta \Theta_t^s$ units of good from generation-t young consumers and to deliver $p_t^\theta \Theta_{t-1}^s$ units of good t to generation-($t-1$) old consumers. Finally, as a result of capital rental transactions, the intermediary in period t plans to receive a principal-plus-interest payment $[1+r_t]K_t^s$ from the firm and deliver capital K_{t+1}^s to the firm to be employed in the subsequent period $t+1$. Consequently, the intermediary's planned period-t net receipts take the form

$$\Pi_t = p_t^\lambda \left[\Lambda_t^s - \Lambda_{t-1}^s\right] + p_t^\theta \left[\Theta_t^s - \Theta_{t-1}^s\right] - \left[K_{t+1}^s - (1+r_t)K_t^s\right], \qquad t \geq 1. \quad \textbf{(19)}$$

In the Tirole economy, only one asset—the bubble asset—is used to finance consumption loans. In the brokered economy, there are two assets that the intermediary could use to finance consumption loans: namely, bonds and stock shares. To achieve a more direct comparison with the Tirole economy, it is useful to place the following financing restriction on the intermediary, which ensures that stock shares in the brokered economy correspond to units of bubble asset in the Tirole economy:

$$p_{t-1}^\lambda \Lambda_{t-1}^s = K_t^s \quad \text{and} \quad p_{t-1}^\theta \Theta_{t-1}^s = B_{t-1}, \qquad t \geq 1. \qquad \textbf{(20)}$$

Specifically, restriction (20) guarantees that the intermediary finances his capital loans K_t^s solely by means of bond transactions and his consumption loans B_{t-1} solely by means of stock-share transactions. Letting $b_t \equiv B_t/L_t$ denote per-capita consumption loans, this financing restriction in per-capita terms reduces to

$$p_{t-1}^\lambda \lambda_{t-1}^s = [1+n]k_t^s \quad \text{and} \quad p_{t-1}^\theta \theta_{t-1}^s = b_{t-1}, \qquad t \geq 1. \qquad \textbf{(21)}$$

Given the viability condition (17) and the financing restriction (20), the intermediary's period-t net receipts (19) can be expressed as

$$\begin{aligned} \Pi_t &= p_t^\theta \left[\Theta_t^s - \Theta_{t-1}^s\right] \\ &= B_t - \left[p_t^\theta / p_{t-1}^\theta\right] B_{t-1} \\ &= d_t \Theta_{t-1}^s + B_t - [1+r_t]B_{t-1}. \end{aligned} \qquad \textbf{(22)}$$

By assumption, the intermediary uses these net receipts to fulfill his dividend obligations, $d_t \Theta_t^s$, where d_t is the period-t per-share dividend expected by generation-($t-1$) old shareholders.

If the intermediary's dividend obligations exhaust his net receipts, i.e., if $\Pi_t = d_t \Theta_{t-1}^s$, then it follows from (22) that Tirole's bubble asset growth restriction (7) holds for the brokered economy. In particular, (7) holds with net receipts and dividend payments both equal to zero if the intermediary issues no new stock shares in period t. Note, however, that (7) is not an ex ante restriction on intermediary behavior. For example, as is explored in Section 5.2, it may be that a competing intermediary is able to successfully enter against an existing intermediary in some period t by offering a windfall dividend to shareholders in excess of d_t, which the competing intermediary finances by new stock-share issue. In this case, $\Pi_t > d_t \Theta^s_{t-1}$ and Tirole's restriction (7) fails to hold.

To close the model, we now impose initial conditions. As for the Tirole economy, it is assumed that initial values $k_0 > 0$ and $b_0 \geq 0$ for per-capita capital and per-capita consumption loans are exogenously given, and that the initial interest rate r_0 and wage rate w_0 are determined by conditions (10) and (11).

Extending Tirole's initial conditions to recognize financial assets, it is also assumed that each old consumer in the initial period 1 owns a positive number λ_0 of bonds and a nonnegative number θ_0 of stock shares, where λ_0 and θ_0 are exogenously given. Thus, the aggregate quantities of bonds and stock shares in existence in period 1 are $\Lambda_0 \equiv L_0\lambda_0$ and $\Theta_0 \equiv L_0\theta_0$. It is assumed throughout the remainder of the paper that $\Lambda^s_0 \equiv \Lambda_0$ and $\Theta^s_0 \equiv \Theta_0$. Moreover, for reasons clarified below, it is assumed that $\theta_0 = 0$ if and only if $b_0 = 0$.

The expected income of each old consumer in the initial period 1 is determined by the sale value of his bond and stock shareholdings as well as by his dividend payment $d_1\theta_0$. Thus, each old consumer in period 1 plans to consume

$$c^2_1 = p^\lambda_1\lambda_0 + p^\theta_1\theta_0 + d_1\theta_0. \tag{23}$$

This planned consumption is assumed to derive from an unmodeled time-0 choice problem of form (16).

More precisely, it is assumed that the viability condition (17) holds for $t = 0$ for some $r_1 > 0$, where the initial prices p^λ_0 and p^θ_0 are taken to be any positive values satisfying the financing restriction (21). In particular, then, $p^\lambda_0\lambda_0 = [1 + n]k(r_1)$ and $p^\theta_0\theta_0 = b_0$. Consequently, the planned consumption of old agents in period 1 can be expressed equivalently in the form

$$c^2_1 = [1 + r_1]s(w_0, r_1), \tag{24}$$

where the savings function $s(\cdot)$ coincides with the savings function determined for each generation $t \geq 1$ agent in (18). Moreover, it follows from condition (11) and from the preceding discussion that this savings function satisfies

$$s(f(k_0) - f'(k_0)k_0, r_1) = [1 + n]k(r_1) + b_0. \tag{25}$$

Given the regularity conditions imposed by Tirole (1985, p. 1502) on the curvature of the savings function $s(\cdot)$ and the production function $f(\cdot)$ and assumed also in the present paper (see Section 2 and Appendix A), relation (25) uniquely determines $r_1 > 0$ as a function of k_0 and b_0.

The brokered economy is not yet complete. An objective for the intermediary needs to be specified, along with market-clearing conditions. In standard overlapping generations models, these two specifications traditionally have been equated; the intermediary is assumed to be a passive Walrasian Auctioneer concerned only with trade and credit coordination. As will now be clarified, changing the specification of the intermediary from a passive coordinator to an active agent motivated by net receipts has immediate and dramatic implications for the efficient operation of the economy.

5. MODELING THE INTERMEDIARY: EFFICIENCY IMPLICATIONS

5.1. Intermediary as a Passive Coordinator

Suppose the brokered-economy intermediary passively mediates savings activities in concert with the Walrasian Auctioneer who passively mediates trades in goods and services. That is, the intermediary and the Walrasian Auctioneer together set prices and per-share dividends and then stand ready to mediate trades for bonds, stock shares, goods, and services and to channel savings into consumption loans and capital investment. In particular, the intermediary does not engage in new stock-share issue. Rather, the sole objective of the intermediary is to coordinate market transactions.

The labor, capital, and product markets for the brokered economy then operate in the same manner as for the Tirole economy. In particular, labor is supplied inelastically to the firm by young consumers and the firm is assumed to employ this labor. Also, in accordance with the financing restriction (20), the intermediary engages in bond transactions with consumers to supply the capital demanded by the firm. Finally, the intermediary distributes all net receipts back to consumer-shareholders as dividends in each period t.

Given the viability condition (17), it follows from (22) that the intermediary's per-capita net receipts $\pi_t \equiv \Pi_t/L_t$ in each period $t \geq 1$ are given by

$$\begin{aligned}\pi_t &= p_t^\theta\left[\theta_t^s - \theta_{t-1}^s\big/(1+n)\right]\\ &= b_t - \left[p_t^\theta\big/(1+n)\,p_{t-1}^\theta\right]b_{t-1}\\ &= d_t\left[\theta_{t-1}^s\big/(1+n)\right] + b_t - [(1+r_t)/(1+n)]b_{t-1}.\end{aligned} \quad \mathbf{(26)}$$

The stock shares Θ_t^s supplied by the intermediary in period t are given by the stock shares Θ_{t-1}^s that generation-$(t-1)$ old agents sell back to the intermediary in period t. That is, in per-capita terms,

$$\theta_t^s = \theta_{t-1}^s\big/(1+n). \quad \mathbf{(27)}$$

Given relation (27), it follows from (26) that the net receipts of the intermediary are zero in each period $t \geq 1$.

Given zero net receipts in each period $t \geq 1$, the intermediary can feasibly make the per-share dividend payments d_t expected by shareholders if and only if these expected dividend payments are zero. It then follows from (26) with $d_t = 0$ that Tirole's state equation (13) must hold. As previously shown in note 7, this state equation is equivalent to product market clearing in a strong supply-equal-demand sense.

Finally, given the viability condition (17), the optimization problems faced by consumers and the firm in the brokered economy coincide with the optimization problems faced by consumers and the firm in the Tirole economy once the identification $s_t \equiv [c_t^1 - w_t]$ is made for the brokered economy. The specific mix of bonds and stock shares purchased by each consumer in the brokered economy is determined by the specific mix of bonds and stock shares supplied to young consumers by the intermediary in accordance with his capital supply objective, subject to the restriction that the portfolio purchased by each consumer must be equal in value to the consumer's optimally chosen savings level. In particular, for each $t \geq 1$, the intermediary's stock-share supply $b_{t-1} = p_{t-1}^{\theta}\theta_{t-1}$ must satisfy Tirole's state equation (12). The only difference from the Tirole economy is that the instruments for achieving the consumers' optimal savings levels are now explicitly identified as bonds and stock shares.

For later reference, the brokered-economy market-clearing conditions for bonds, stock shares, capital, and product are listed below in per-capita form[10]:

$$p_t^{\lambda}\lambda_t^s = p_t^{\lambda}\lambda_t^d, \qquad t \geq 1; \tag{28}$$

$$p_t^{\theta}\theta_t^s = p_t^{\theta}\theta_t^d, \qquad t \geq 1; \tag{29}$$

$$k_t^s = k_t^d, \qquad t \geq 1; \tag{30}$$

$$y_t + k_t^s = [1+n]k_{t+1}^d + c_t^1 + c_t^2\big/[1+n], \qquad t \geq 1. \tag{31}$$

In summary, when the market coordination conditions (27) through (31) are imposed on the brokered economy, the resulting Walrasian brokered economy is essentially equivalent to the Tirole economy.[11] In particular, every Walrasian brokered economy satisfies the two Tirole state equations (12) and (13), implying that the same paths are generated for per-capita capital k_t and per-capita consumption loans b_t for the brokered economy and the Tirole economy if the same initial values $k_0 > 0$ and $b_0 \geq 0$ are given.

One implication of this equivalence is that the Walrasian brokered economy supports the Pareto-inefficient outcome $\bar{\boldsymbol{e}}$ as a competitive equilibrium. Another implication is that the intermediary implicitly present in the Tirole economy is a peculiarly passive agent who does not seek out opportunities to increase the welfare of his consumer-shareholders. In particular, as indicated by (26), the intermediary ignores the possibility that his net receipts—paid out as dividends to his consumer-shareholders—could potentially be increased by issuing additional shares of stock. In contrast, real-world intermediaries generally take advantage of the opportunity to issue additional stock shares up to the point where stock dilution is perceived as having adverse consequences.

It is now shown that the passive intermediary in period 0 is susceptible to entry in period 1 unless b_0 happens by chance to be equal to $b(k_0)$ (cf. Figure 2).

Specifically, if $b_0 < b(k_0)$, a competitor can feasibly assume the liabilities of the existing passive intermediary while offering current shareholders a windfall dividend secured by means of new stock-share issue.

5.2. Potential Entry by Active Intermediaries

Suppose the existing intermediary in period 1 is currently subject to government regulations that protect his monopoly position against potential entrants. Some measure of protection against unrestricted entry is certainly necessary, for an entering intermediary with no liabilities can always attract potential shareholders away from an existing intermediary who has liabilities resulting from the previous issue of unsecured debt. In addition, suppose the existing intermediary is subject to a no-profit regulation and hence is currently playing a purely passive coordination role as described in Section 5.1.

Suddenly, government in period 1 decides to open the intermediation market to permit entry in any period $t \geq 1$ by active profit-seeking intermediaries in accordance with the following chartering restriction: Entrance is permitted by any competitor who can assume the liabilities of the existing intermediary and offer a feasible intermediation plan under which shareholder dividends in the entry period are increased.[12] In particular, both the existing intermediary and potential entrants now are permitted to be active intermediaries who can issue bonds and stock shares (bubble asset) in the pursuit of higher dividends for shareholders as long as all contractual obligations are met. If he wishes to retain his monopoly position, the existing intermediary then must seek a feasible intermediation plan that will prevent entry, and the question arises whether this preventive behavior is socially beneficial. As shown below, social welfare is unambiguously increased by this preventive behavior unless the initially given level b_0 for per-capita consumption loans happens by chance to be set at just the right level.

The first task is to describe more concretely our modeling of an active intermediary. Two basic possibilities can be considered: The active intermediary is a price-taker who sets quantities, or the active intermediary is a price-setter who takes quantities as given.

The first possibility is attractive in that the consumers and the firm are already assumed to be price-takers. Nevertheless, the original motivation for explicitly introducing the intermediary was to put a corporate business suit on the passive Walrasian Auctioneer in the Tirole economy, and this auctioneer is most certainly a price-setter rather than a price-taker. We thus choose to model the active intermediary as a price-setting agent who posts prices for bonds and stock shares. Moreover, to restrict attention to the most interesting case in which bonds and stock shares are both held in equilibrium, we further assume that these posted prices are positive.

Specifically, following the decision by government in period 1 to open up the intermediation market, the existing now-active intermediary publicly posts a positive bond price sequence $\boldsymbol{p}^{\lambda} = (p_1^{\lambda}, p_2^{\lambda}, \ldots)$, a positive stock share price sequence $\boldsymbol{p}^{\theta} = (p_1^{\theta}, p_2^{\theta}, \ldots)$, and a nonnegative per-share dividend sequence $\boldsymbol{d} = (d_1, d_2, \ldots)$.

The collection of sequences $\boldsymbol{I} = (\boldsymbol{p}^{\lambda}, \boldsymbol{p}^{\theta}, \boldsymbol{d})$ is referred to as the intermediary's *prospectus*.

Assuming entry does not occur, consumers and the firm then solve their individual optimization problems conditional on the intermediary's posted prospectus. Although the price and dividend expectations of the consumers and the firm can differ in principle from the planned prices and dividends posted by the intermediary in his prospectus, the definition of an equilibrium for the brokered economy presented below includes the requirement that the intermediary fulfills all of his contractual obligations. Without loss of generality, then, we simplify the exposition by assuming from the start that the price and dividend expectations of the consumers and the firm coincide with the planned prices and dividends appearing in the intermediary's prospectus.

Roughly stated, a prospectus $\boldsymbol{I}$ is feasible if it enables the intermediary to fulfill all contractual obligations. An analytical characterization of feasibility for a prospectus $\boldsymbol{I}$ will now be developed.

First, we assume that the intermediary understands that it is not feasible in the brokered economy for the per-capita consumer shareholdings, $b_t = p_t^{\theta}\theta_t^s$, to become arbitrarily large over time. This fact, established in Section A.2 of the Appendix, follows from the assumption of diminishing marginal returns to capital and is independent of the intermediary's behavior.

Second, the intermediary presumably would recognize or discover that bonds and stock shares must earn a common positive gross rate of return if consumer optimization is to result in finite demands for bonds and stock shares. Moreover, given the financing restriction (21), it follows from the regularity conditions imposed on the production function in Section 2 that interest rates on bondholding must be positive. We therefore assume that the intermediary requires his prospectus to satisfy the viability condition (17), where the implied common interest rate r_{t+1} in each period $t \geq 1$ is strictly positive.

Third, once the intermediary has posted his prospectus $\boldsymbol{I}$, he must fulfill certain contractual obligations to young consumers and the firm in each period $t \geq 1$. Specifically, the intermediary must be able to sell bonds and stock shares to period-t young consumers at the posted bond and stock-share prices p_t^{λ} and p_t^{θ} in order to realize their savings-level objectives, and he must be able to supply the capital demanded by the period-t firm at the implied interest rate r_t. Because bonds and stock shares earn the same interest rate, however, only the overall level of savings matters to consumers. The specific mix of bonds and stock shares demanded by consumers thus is determined by the specific mix of bonds and stock shares supplied by the intermediary. In particular, given the financing restriction (21), the per-capita level of consumption loans (stock shareholding) is determined by the extent to which the optimal consumer savings level exceeds the firm's optimal capital demand. Consequently, for each $t \geq 0$, the intermediary's period-t per-capita consumption loans $b_t = p_t^{\theta}\theta_t^s$ must satisfy[13]

$$b_t = b(r_t, r_{t+1}) \equiv s(w(r_t), r_{t+1}) - [1+n]k(r_{t+1}). \tag{32}$$

Fourth, the intermediary must fulfill his contractual obligations to old consumers in each period $t \geq 1$. Specifically, he must redeem their bond and stock shareholdings and make per-share dividend payments to them in accordance with the prices and per-share dividends posted in his prospectus. The intermediary can ensure redemption of bond and shock shareholdings at his posted prospectus prices by ensuring that (32) holds for each $t \geq 0$. As now shown, however, the intermediary also needs to abide by a further restriction in order to ensure that he can fulfill his dividend obligations.

Given the viability condition (17), the financing restriction (21), and relation (32) for consumption loans, it follows from (22) that the per-capita net receipts of the intermediary take the form

$$\pi_t = \frac{d_t \theta^s_{t-1}}{1+n} + b(r_t, r_{t+1}) - \frac{1+r_t}{1+n} b(r_{t-1}, r_t), \qquad t \geq 1, \tag{33}$$

where d_t denotes the per-share dividend promised to period-t consumer-shareholders in the intermediary's prospectus. Consequently, for the intermediary to be able to make these dividend payments, it must hold that

$$\pi_t \geq \frac{d_t \theta^s_{t-1}}{1+n}, \qquad t \geq 1, \tag{34}$$

or equivalently, that

$$b(r_t, r_{t+1}) - \frac{1+r_t}{1+n} b(r_{t-1}, r_t) \geq 0\,, \qquad t \geq 1. \tag{35}$$

Finally, in total-value terms, note that the intermediary can meet all of his obligations to agents in each period $t \geq 1$, apart from dividend obligations, if and only if the intermediary's per-capita net receipts π_t are nonnegative in each period $t \geq 1$. Relation (34), together with the assumption that the intermediary posts positive stock-share prices and nonnegative dividend payments, shows that this nonnegativity holds if $b_{t-1} = p^{\theta}_{t-1}\theta^s_{t-1}$ is nonnegative for each $t \geq 1$. The latter condition is ensured by conditions (32) and (35), however, given the assumed nonnegativity of $b_0 = b(r_0, r_1)$ and the positivity of r_1.

In summary, a collection of sequences $\boldsymbol{I} = (\boldsymbol{p}^{\lambda}, \boldsymbol{p}^{\theta}, \boldsymbol{d})$ are said to constitute a *feasible prospectus* for the intermediary if it satisfies the following three conditions: (i) The bond and stock-share prices posted in $\boldsymbol{I}$ are positive and the dividend payments posted in $\boldsymbol{I}$ are nonnegative; (ii) $\boldsymbol{I}$ satisfies the viability condition (17) with $r_{t+1} > 0$ for each $t \geq 1$; and (iii) the per-capita consumption loans supplied to consumers by the intermediary, given by (32), are uniformly bounded above and satisfy condition (35).

Given a feasible prospectus $\boldsymbol{I}$, it follows from the general structure of the brokered economy set out in Section 4 that the only aspect of the prospectus that affects real outcomes in the economy is the associated interest-rate sequence $(r_1, r_2, \ldots)$.

Hereafter, this interest-rate sequence is referred to as the *intermediation plan* associated with $\boldsymbol{I}$, denoted by $\boldsymbol{r}(\boldsymbol{I})$. Conversely, any interest-rate sequence $\boldsymbol{r} = (r_1, r_2, \ldots)$ with $r_t > 0$ for all $t \geq 1$ that satisfies condition (iii) in the definition of a feasible prospectus is said to be a *feasible intermediation plan* for the intermediary, because any such interest-rate sequence constitutes an intermediation plan $\boldsymbol{r}(\boldsymbol{I})$ for at least one feasible prospectus $\boldsymbol{I}$.[14]

Conditions have now been set out that define what it means for the existing intermediary to view an interest-rate sequence as a feasible intermediation plan. Still to be specified, however, is the objective of the intermediary that determines how he will choose a particular intermediation plan from this feasible set in order to protect himself against potential entrants able to attract away his prospective shareholders.

It is shown first that, relative to an existing intermediary with a feasible intermediation plan $\boldsymbol{r}$, a potential entrant in period t that assumes the liabilities of the existing intermediary can feasibly increase the dividends paid out in period t if and only if he can feasibly offer a higher interest rate r_{t+1}.

To see this, suppose that a potential entrant offers an intermediation plan $\boldsymbol{r}'$ in some period $t \geq 1$ as a feasible alternative to an existing intermediation plan $\boldsymbol{r}$. As explained above, feasibility of an intermediation plan requires the period-1 interest rate r_1 to satisfy relation (32) for $t = 0$. As discussed in Section 4, this requirement determines a unique positive value for r_1, so potential entrants have no feasible way to change r_1.

Suppose, then, that $r'_s = r_s$ for all $s \leq t$ and $r'_{t+1} > r_{t+1}$. Given that the potential entrant assumes the liabilities of the existing intermediary, his net receipts in period t are determined by (33) with r'_{t+1} in place of r_{t+1}. It follows that the increase in r_{t+1} will increase period-t net receipts relative to the net receipts of the existing intermediary—and thus provide for the possibility of a windfall per-share dividend payment to period-t shareholders in excess of their currently expected per-share dividend payment d_t—if and only if the consumption loan level $b\,(r_t, r_{t+1})$ of each period-t young consumer increases with increases in r_{t+1}, that is, if and only if

$$\frac{\partial s(w(r_t), r_{t+1})}{\partial r_{t+1}} > [1+n]k'(r_{t+1}). \qquad \textbf{(36)}$$

Condition (36) is equivalent to a condition imposed by Tirole (1985, p. 1502) on the slope of the savings supply curve relative to the capital demand curve. As noted in Section 2 and detailed in Section A.1 of the Appendix, condition (36) is one of the regularity conditions assumed to hold throughout the present paper. Consequently, the increase in r_{t+1} will increase feasible dividend payments in the entry period.

Conversely, suppose $r'_s = r_s$ for all $s \leq t$ and $r'_{t+1} < r_{t+1}$. Given (36), it is immediately seen that dividend payments in the entry period then must decrease.

It follows as a corollary to these arguments that a potential entrant in period t who assumes the liabilities of the existing intermediary can always offer an alternative intermediation plan $\boldsymbol{r}'$ that is feasible through period t and that increases period-t dividends: Simply let $\boldsymbol{r}'$ coincide with the existing intermediation plan $\boldsymbol{r}$ through period t and satisfy $r'_{t+1} > r_{t+1}$. The possibility of successful entry against the existing intermediary in period t thus hinges upon whether or not there exists an intermediation plan of this form that is also feasible for all periods $s > t$. If so, the existing intermediation plan $\boldsymbol{r}$ will be said to be *susceptible to entry in period t*.

An intermediation plan $\boldsymbol{r}$ will be said to be *optimal* if (a) it is feasible, and (b) it is not susceptible to entry in any period t. In principle, the set of optimal intermediation plans can be determined through the use of conditions (32) and (35) once values are given for the exogenous parameters k_0 and b_0. Letting $R\,(k_0, b_0)$ denote this set of optimal intermediation plans, it is assumed that the existing intermediary in period 1 selects a new intermediation plan from the set of optimal plans in $R\,(k_0, b_0)$ after the government in period 1 opens the intermediation market to potential entrants.

A definition of an equilibrium is now given for the brokered economy in per-capita form that includes the no-entry objective of the intermediary as well as the objectives of the consumers and the firm.

DEFINITION 2 (Brokered-Economy Equilibrium). *Let initial values* $(k_0, \lambda_0) > 0$ *and* $(b_0, \theta_0) \geq 0$ *for per-capita capital, bonds, consumption loans, and stock shares be given, where* $\theta_0 = 0$ *if and only if* $b_0 = 0$. *Then a sequence* $(\boldsymbol{v_t}, k_t^d, \boldsymbol{m_t}, w_t : t \geq 1)$ *consisting of consumer-choice vectors* $\boldsymbol{v_t} = (s_t, c_t^1, c_t^2, \lambda_t^d, \theta_t^d)$, *firm per-capita capital demands* $k_t^d > 0$, *intermediary choice vectors* $\boldsymbol{m_t} = (r_t, p_t^\lambda, p_t^\theta, d_t, k_t^s, \lambda_t^s, b_t, \theta_t^s)$, *and wage rates* $w_t > 0$ *is a* brokered-economy equilibrium $\boldsymbol{e}(k_0, \lambda_0, b_0, \theta_0)$ *if the following five conditions are met*:

- (i) Firm optimization. *In each period* $t \geq 0$, $r_t = f'(k_t^d)$ *and* $w_t = f(k_t^d) - f'(k_t^d)k_t^d$, *where* $k_0^d \equiv k_0$.
- (ii) Consumer optimization. *In each period* $t \geq 1$, *the young consumer's choice vector* $\boldsymbol{x_t} = (s_t, c_t^1, c_{t+1}^2)$ *solves the lifetime utility maximization problem* (1), *conditional on* w_t *and* r_{t+1}, *i.e.,* $\boldsymbol{x_t} = x(w_t, r_{t+1}) = (s(w_t, r_{t+1}), c^1(w_t, r_{t+1}), c^2(w_t, r_{t+1}))$; *and each old consumer in period* 1 *consumes* $c_1^2 = [1 + r_1]s(w_0, r_1)$.
- (iii) Financing restriction. *In each period* $t \geq 1$, *capital loans* $(1+n)k_t^s$ *are financed solely by means of bond transactions* $p_{t-1}^\lambda \lambda_{t-1}^s$ *and consumption loans* b_{t-1} *are financed solely by means of stock-share transactions* $p_{t-1}^\theta \theta_{t-1}^s$, *where* $\lambda_0^s \equiv \lambda_0$, $\theta_0^s \equiv \theta_0$, *and* p_0^λ *and* p_0^θ *are any positive values satisfying* $(1+n)k_1^s = p_0^\lambda \lambda_0$ *and* $b_0 = p_0^\theta \theta_0$.
- (iv) Intermediary optimization. *For each* $t \geq 0$, *the consumption loan level* $b_t = p_t^\lambda \lambda_t^s$ *equals* $[s(w(r_t), r_{t+1}) - (1+n)k(r_{t+1})]$ *and the total portfolio value* $p_t^\lambda \lambda_t^s + p_t^\theta \theta_t^s$ *supplied to each young consumer equals* $s(w(r_t), r_{t+1})$, *where* $w(r)$ *and* $k(r)$ *are the factor-price frontier functions derived from* $r = f'(k)$ *and* $w = f(k) - f'(k)k$. *Moreover, the interest-rate sequence* $\boldsymbol{r} = (r_1, r_2, \ldots)$ *is an element of the set* $R(k_0, b_0)$ *of optimal intermediation plans, and the collection of sequences* $\boldsymbol{I} = (\mathbf{p}^\lambda, \boldsymbol{p}^\theta, \boldsymbol{d})$ *constitutes a feasible prospectus that supports this optimal intermediation plan.*
- (v) Market-clearing conditions. *In each period* $t \geq 1$, *the bond, stock-share, and capital market-clearing conditions* (28) *through* (30) *hold, and the product market clears in the sense that excess supply is nonnegative.*[15]

A brokered-economy equilibrium $\boldsymbol{e}(k_0, \lambda_0, b_0, \theta_0)$ *will be called* stationary *if* $k_t = k_0$, $\lambda_t = \lambda_0$, $b_t = b_0$, *and* $\theta_t = \theta_0$ *for all* $t \geq 1$.

An immediate and important implication of this equilibrium definition with active intermediation is that the Pareto-inefficient stationary equilibrium $\bar{\boldsymbol{e}}$ for the Tirole economy cannot be supported as an equilibrium real outcome for the brokered economy because it is susceptible to entry. More generally, as shown in Theorem 1, below, the optimizing behavior of the active intermediary rules out any path for the brokered economy that converges to $\bar{\boldsymbol{e}}$ in real terms because all such equilibria are susceptible to entry.

One key observation used in the proof of Theorem 1 is that the active intermediary in the brokered economy, able to issue new stock shares, functions as a selection mechanism for picking out which Tirole equilibrium path the brokered economy will follow in real terms.

To see this, note that the two basic state equations (12) and (13) characterizing a Tirole economy equilibrium also are satisfied by any brokered-economy equilibrium once the intermediary stops issuing new stock shares, in which case all actual dividend payments are zero. More generally, these two state equations hold whenever the per-share dividends promised in the intermediary's prospectus absorb all of his net receipts, whether or not these net receipts are zero and whether or not new stock shares are issued. These claims follow from a consideration of form (33) for the intermediary's per-capita net receipts.

Another key observation used in the proof of Theorem 1 is that the possibility of successful entry by a competing intermediary through additional stock-share issue permits the brokered economy to traverse from one Tirole equilibrium path to another if the existing brokered-economy intermediary is not currently pursuing an optimal intermediation plan.

THEOREM 1. *Given any value* $k > 0$ *for per-capita capital, let* $b(k) \geq 0$ *denote the maximum feasible value for per-capita consumption loans as determined for the Tirole economy by Tirole* (1985, *Prop*. 1, *p*. 1504).[16] *Then the following holds. Given* (k_0, b_0) *such that* $0 \leq b_0 \leq b(k_0)$, *there exists a unique optimal intermediation plan for the brokered economy. Moreover, the interest rates* r_t *that constitute this optimal intermediation plan satisfy* $r_t \to n$ *as* $t \to \infty$.

Proof. The proof of Theorem 1 follows directly from simple modifications in the proof of Proposition 1 by Tirole (1985, p. 1504).

Suppose, first, that b_0 satisfies $0 \leq b_0 < b(k_0)$. This initially places the brokered economy at a point (k_0, b_0) in the (k, b) plane that is strictly below the locus of points (k, b) with $b = b(k)$, hereafter referred to as the *Tirole saddle path*. One such point is illustrated in Figure 2.

Given k_0 and b_0, feasibility requires the existing intermediary, as well as any potential entrant in period 1, to set the period-1 interest rate in accordance with relation (32) for $t = 0$. As discussed in Section 4, the latter relation determines a unique positive value $\hat{r}_1$ for r_1 as a function of k_0 and b_0, which in turn determines a unique positive value $\hat{k}_1$ for the period-1 level k_1 of per-capita capital through the firm profit maximization condition $\hat{r}_1 = f'(\hat{k}_1)$.

However, as detailed earlier in this section, a potential entrant in period 1 could deliver a greater dividend to shareholders in period 1 than the existing intermediary by increasing the period-2 interest rate r_2. Condition (36) ensures that this increase in r_2 moves the economy vertically upward from the point $(\hat{k}_1, b_1)$ where the economy would otherwise have been in period 1 if the economy had simply pursued the Tirole equilibrium resulting under passive intermediation, starting from the initial conditions (k_0, b_0).

If the increase in r_2 results in an increase in the consumption loan level b_1 that is less than $b(\hat{k}_1)$—implying that the economy is below the Tirole saddle path—this places the economy on another feasible Tirole equilibrium path; but further entry in period 1 is still feasible. If the increase in r_2 results in an increase in b_1 to the value $b(\hat{k}_1)$, this places the economy on a feasible Tirole equilibrium path—the Tirole saddle path—for which further entry is not feasible. The reason for this is that, once on the saddle path, any attempt to offer a higher interest rate in some period t than is currently being offered would move the economy above the Tirole saddle path and hence onto an infeasible path where the consumption loan levels eventually explode. Finally, if the increase in r_2 results in an increase in b_1 to a value that lies above $b(\hat{k}_1)$, implying the economy is above the Tirole saddle path, the economy immediately moves onto an infeasible path where the consumption loan levels eventually explode, implying that the entrant's proposed intermediation plan is not feasible. These arguments directly follow from Proposition 1 in Tirole (1985, p. 1504) and can be understood intuitively by referring to Figure 2.

It follows from these arguments that, to protect against entry in period 1, the optimizing intermediary should set the period-2 interest rate so that the period-1 consumption loan level equals $b(\hat{k}_1)$. Note by condition (36) that $b(\hat{r}_1, r_2)$ is a strictly increasing function of r_2; hence the particular interest rate $\hat{r}_2$ that achieves $b(\hat{k}_1)$ is a uniquely determined value. Moreover, $\hat{r}_2$ must be positive, because, by construction, all interest rates that support Tirole equilibria must be positive.

Similar arguments apply for protection against entry in all subsequent periods $t \geq 2$. Consequently, to protect against entry in all periods $t \geq 1$, the optimizing intermediary must set the period-1 interest rate to the unique value $\hat{r}_1 > 0$ that satisfies relation (32) for $t = 0$ for the given initial values k_0 and b_0 and then set the uniquely determined interest rates $\hat{r}_t > 0$ for subsequent periods $t \geq 2$ that ensure that the brokered economy attains the Tirole saddle path, i.e., that ensure a consumption loan level equal to $b(\hat{k}_t)$ for all $t \geq 1$. By construction, the consumption loan levels $b(\hat{k}_t)$ for $t \geq 1$ then lie along a Tirole equilibrium path—namely, the Tirole saddle path—and hence satisfy the Tirole state equation (13) for all $t \geq 2$. Consequently, relation (35) holds for all $t \geq 2$. Moreover, b_1 and b_0 lie along a (different) Tirole equilibrium path and hence satisfy (13) for $t = 1$ with $r_1 = \hat{r}_1 = f'(\hat{k}_1)$. Because $b(\hat{k}_1) \geq b_1$ by construction, it follows that condition (35) holds for $t = 1$. The uniquely determined positive interest-rate sequence $(\hat{r}_1, \hat{r}_2, \hat{r}_3, \ldots)$ thus constitutes a unique optimal intermediation plan in the sense defined earlier in this section.

Finally, as established by Tirole (1985, Prop. 1, p. 1505) and indicated in Figure 2, along the Tirole saddle path the level $\hat{k}_t$ for per-capita capital converges to $k^n = k(n)$ as $t \to \infty$, implying that the interest rate $\hat{r}_t$ converges to the golden-rule interest rate n as $t \to \infty$.

If $b_0 = b(k_0)$, it is clear from the above arguments that the optimizing intermediary would again set $r_t = \hat{r}_t$ in all periods $t \geq 2$ to ensure that the brokered economy moves along the Tirole saddle path in periods $t \geq 1$. The only change from the preceding case is that the value for the period-1 interest rate r_1 now supports the per-capita consumption loan level $b(k_0)$ on the Tirole saddle path, implying that the brokered-economy equilibrium path coincides with the Tirole saddle path for all $t \geq 0$. ■

The corollary, below, demonstrates that the no-entry objective of the active brokered-economy intermediary is consistent with the goal of market-value maximization, where market value is measured by the value of the intermediary's outstanding stock shares. The proof of this corollary follows immediately from the proof of Theorem 1.

COROLLARY 1. *Given any brokered-economy equilibrium, the market value* $B_t = p_t^{\theta}\Theta_t^s$ *of the active intermediary in each period* $t \geq 1$ *is positive and takes on its maximum feasible value* $L_t b(k_t)$.

For use in the next theorem, define

$$q_t \equiv \frac{[1 + r_t]}{[1 + n]}, \qquad t \geq 1, \tag{37}$$

and

$$Q_t \equiv [q_t q_{t-1} \ldots q_1] b_0, \qquad t \geq 1. \tag{38}$$

Also, for each t and k satisfying $t \geq k \geq 1$, define

$$s_{t,k} \equiv [q_t q_{t-1} \ldots q_{k+1} q_k] \tag{39}$$

and

$$S_t \equiv \sum_{k=1}^{t} s_{t,k}. \tag{40}$$

THEOREM 2. *If* $\mathbf{r}$ *is a feasible intermediation plan for the brokered economy that is not susceptible to entry, then*

$$\lim_{t \to +\infty} S_t = +\infty, \tag{41}$$

and the sequence (Q_t) *is uniformly bounded above. Conversely, if* $\mathbf{r}$ *is a feasible intermediation plan that is susceptible to entry under which stock shares are in fixed supply, then the sequence* (S_t) *must converge and the sequence* (Q_t) *must converge to* 0.

Proof. As established in the proof for Theorem 1, the unique intermediation plan for the brokered economy that is not susceptible to entry is the plan $\hat{\boldsymbol{r}}$ that follows the Tirole saddle path for $t \geq 1$ with r_t converging to n. Given $r_t \rightarrow n$ as $t \rightarrow \infty$, however, the tail sum $S_t - S_{t'}$ can be made arbitrarily large by making t sufficiently larger than t' for some sufficiently large t', because in this way, the terms $s_{t,k}$ appearing in this tail sum can be made arbitrarily close to 1. Because $S_{t'}$ is positive for all t', it follows that (41) must hold.

Also, if $\boldsymbol{r}$ is an optimal (hence feasible) intermediation plan for the brokered economy, it follows by the definition of feasibility that the associated sequence (b_t) of per-capita consumption loan levels must be uniformly bounded above and must satisfy $b_t \geq Q_t$ for each $t \geq 1$. The sequence (Q_t) therefore must be uniformly bounded above.

Conversely, suppose that $\boldsymbol{r}$ is an intermediation plan susceptible to entry under which stock shares are in fixed supply. Any feasible intermediation path for the brokered economy that involves a fixed stock-share supply must satisfy the two state equations (12) and (13) characterizing Tirole equilibria and hence must move the brokered economy in real terms along a Tirole equilibrium path. If $\boldsymbol{r}$ is susceptible to entry, however, it follows from the proof of Theorem 1 that this Tirole equilibrium path cannot be the Tirole saddle path. As established by Tirole (1985, Prop. 1, p. 1504), all Tirole equilibrium paths apart from the Tirole saddle path are characterized by interest rates that converge to $\overline{r}$ as $t \rightarrow \infty$, where $\overline{r} < n$ (cf. Figure 2). Consequently, any such brokered-economy equilibrium must be characterized by interest rates r_t that ultimately converge to $\overline{r}$, which implies that q_t defined as in (37) converges to $\overline{q} = [1 + \overline{r}]/[1 + n] < 1$. By construction, the terms S_t defined in (40) are positive terms that satisfy $S_t = q_{t-1}[1 + S_{t-1}]$. Given $q_t \rightarrow \overline{q} < 1$, it follows by backward recursion that (S_t) is uniformly bounded, hence S_t must converge to $\overline{q}/[1 - \overline{q}]$.

Finally, if $r_t \rightarrow \overline{r}$ as $t \rightarrow \infty$, then $Q_t \rightarrow 0$ as $t \rightarrow \infty$. ■

The price condition (41) appearing in Theorem 2 is the brokered-economy analog of the well-known Cass–Balasko–Shell transversality condition shown by Balasko and Shell (1980, Prop. 5.6, pp. 296–297) to be necessary for Pareto efficiency in a pure-exchange overlapping generations context.[17] Balasko and Shell do not provide an economic motivation for their transversality condition. Theorem 2 demonstrates that, for the brokered economy, condition (41) constitutes a necessary condition for the intermediary's selected intermediation plan to be optimal. Moreover, in the absence of new stock-share issue, it is also sufficient for optimality.

Interestingly, given a brokered economy with $b_0 < b(k_0)$, it is feasible (but not optimal) for an active intermediary to continually traverse across Tirole-economy equilibria with successive new stock-share issues so that $b_t < b(k_t)$ for each $t \geq 0$ but $r_t \rightarrow n$ as $t \rightarrow \infty$. Consequently, condition (41) holds despite the fact that the intermediation plan is susceptible to entry in each period $t \geq 1$. This is why the qualifier concerning stock-share issue is needed in the sufficiency direction for Theorem 2.

THEOREM 3. *There exists at least one stationary equilibrium for the brokered economy. Every stationary equilibrium generates the same real outcome* $(s, c^1, c^2, k, b, r, w)$, *where* $s \equiv [c^1 - w]$; *namely, the Pareto-efficient real outcome* $\boldsymbol{e}^n = \boldsymbol{e}(k^n, b^n)$ *constituting the golden-rule equilibrium for the Tirole economy. In particular, the unique stationary equilibrium interest rate* r *for the brokered economy is given by* $r = n$.

Proof. Theorem 1 implies that the unique optimal stationary intermediation plan $\boldsymbol{r}$ for the brokered economy is given by $r_t = n$ for all $t \geq 1$. Consequently, the corresponding stationary equilibrium values for per-capita capital and per-capita consumption loans are given by the Tirole golden-rule solution values (k^n, b^n), where $f'(k^n) = n$ and $b^n = b(k^n)$ (see Section 2). The remaining assertions of Theorem 3 are then easily verified. ■

Theorem 3 demonstrates that the Pareto-efficient golden-rule equilibrium for the Tirole economy is the unique real outcome possible for the brokered economy in any stationary equilibrium. In particular, the Pareto-inefficient stationary equilibrium $\bar{\boldsymbol{e}}$ for the Tirole economy is not supported as a stationary-equilibrium real outcome for the brokered economy. As noted in Section 2.2, all Pareto-inefficient equilibria for the Tirole economy converge to $\bar{\boldsymbol{e}}$; hence one might surmise that the elimination of this Pareto-inefficient outcome would lead to the restoration of a first welfare theorem. The next theorem establishes that this is indeed the case.

THEOREM 4. *All brokered-economy equilibria are Pareto efficient.*

Proof. As established by Tirole (1985, Prop. 1, p. 1504), the set of equilibria for the Tirole economy can be partitioned into two subsets: the Pareto-efficient equilibria characterized by interest rates that converge to n; and the Pareto-inefficient equilibria characterized by interest rates that converge to $\bar{r} < n$. As shown in the proof of Theorem 1, above, the unique optimal intermediation plan for the brokered economy is characterized by interest rates converging to n, and the equilibrium paths for k_t and b_t that are supported by this intermediation plan coincide for all $t \geq 1$ with the Pareto-efficient Tirole-economy equilibrium given by the Tirole saddle path. Moreover, given k_0 and b_0, there is no way to achieve a Pareto improvement of these k_t and b_t paths without violating the feasibility condition (32) at $t = 0$. ■

To summarize, the key result of this section is that the inefficiency observed in the Tirole economy is eliminated completely when intermediation is modeled as a contestable activity carried out by a corporate intermediary owned by consumer-shareholders and operated in their interest. Assuming $b_0 < b(k_0)$, this elimination requires the active intermediary to issue new stock shares in period 1, which results in an increase in his net receipts in excess of his current contractual obligations. The intermediary's excess net receipts constitute a windfall dividend payment for his current shareholders, the initial generation-0 old consumers.

These findings caution that the inefficiency observed in standard modelings of overlapping generations economies with production and unsecured debt may not be the reflection of an intrinsic market failure. Rather, the observed inefficiency could instead be due to a fundamental incompleteness in the model specification—the presumed inability of private agents to exploit the earnings opportunities associated with incurring and forever rolling over debt.

NOTES

1. In Pingle and Tesfatsion (1991a), it is shown that passive intermediation is the root cause of the Pareto inefficiency problems for Samuelson's (1958) pure exchange economy, a special case of the Tirole economy.

2. Bernanke and Gertler (1987) also introduce intermediaries (banks or insider investment coalitions) into an overlapping generations model. However, they focus on the potential role of these intermediaries in reducing deadweight losses due to principal-agent problems in the loan market for investment projects. They do not permit intermediaries to issue consumption loans. The closest forerunner to the present paper appears to be Thompson (1967). Thompson argues that genuinely perfect competition requires the introduction of a market for private debt instruments (e.g., a corporate pension fund), a market omitted from models such as Samuelson (1958).

3. An asset is said to exhibit a bubble at time t if its price at time t differs from its fundamental value, determined as the present value of its current and future dividends. A bubble asset is any asset on which a bubble may form. When an asset pays no dividends, it necessarily exhibits a bubble whenever its price is positive. Tirole (1985) does not incorporate a market for stock shares into his model; hence bubbles on capital holdings are not considered.

4. Specifically, it is assumed that $U(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly quasi concave, with $U(0, c^2_{t+1}) = U(c^1_t, 0) = U(0, 0)$.

5. More precisely, the following restrictions are imposed on $F(\cdot)$: twice-continuous differentiability and strict concavity over the positive orthant R^2_{++} with $F_K > 0$, $F_L > 0$, and $F_{KK} < 0$; continuity over the nonnegative orthant R^2_+ with $F(0, 0) = F(0, L) = F(K, 0) = 0$; and, for each $L > 0$, $F_K(K, L) \to 0$ as $K \to +\infty$ and $F_K(K, L) \to +\infty$ as $K \to 0$.

6. In the standard Arrow–Debreu general equilibrium model with nonsatiated consumers, market-clearing conditions typically only require that supply be at least as great as demand in quantity terms. Walras' law then implies that any good in excess supply must have a zero price. However, as detailed by Pingle and Tesfatsion (1994), Walras' law need not hold in overlapping generations models without unsecured debt. Consequently, market-clearing conditions traditionally have been stated in a stronger form requiring directly that supply equal demand in each market in real-value terms. See, for example, Diamond (1965), Gale (1973), Balasko and Shell (1980), Tirole (1985), and Azariadis (1993, Ch. 13).

7. Given consumer and firm optimization and the capital market-clearing condition (6), the bubble asset growth restriction (8) is equivalent to the product market-clearing condition (9). To see this, consider the following sequence of implications in both forward and reverse directions:

$$(1+n)k_{t+1} + c^1_t + c^2_t/[1+n] = y_t + k_t;$$

$$(1+n)k_{t+1} + [w_t - s_t] + [1+r_t]s_{t-1}/[1+n] = y_t + k_t;$$

$$(1+n)k_{t+1} + [w_t - s_t] + [1+r_t]s_{t-1}/[1+n] = w_t + [1+r_t]k_t;$$

$$(1+n)k_{t+1} - s_t + [1+r_t]s_{t-1}/[1+n] = [1+r_t]k_t;$$

$$(1+n)k_{t+1} - [b_t + [1+n]k_{t+1}] + [1+r_t][b_{t-1} + [1+n]k_t]/[1+n] = [1+r_t]k_t;$$

$$-b_t + [1+r_t]b_{t-1}/[1+n] = 0$$

8. Because the proof is not entirely straightforward, it is given in Section A.1 of the Appendix.

9. Recalling the definitions for the factor-price frontier functions $w(r)$ and $k(r)$ given in Section 2.1, it is assumed here as for the Tirole economy that there exists a unique $\bar{r}$ satisfying $s(w(\bar{r}), \bar{r}) =$

$[1+n]k(\bar{r})$, with $\bar{r} < n$. Also, as noted in Section 2.2, Tirole (1985) imposes several additional technical regularity conditions on the savings function $s(\cdot)$ and the production function $f(\cdot)$. These regularity conditions also are assumed to hold for the brokered economy. A detailed statement of these regularity conditions can be found in Section A.1 of the Appendix.

10. As noted in Section 3, excess supply in the product market could feasibly occur in the Tirole economy because the intermediary could accumulate positive net receipts (positive amounts of good) that he does not distribute back to consumers. Consequently, the supply-equal-demand equilibrium condition for the Tirole product market is overly restrictive. In the brokered economy with passive intermediation, however, it is assumed that all of the intermediary's net receipts are distributed back to consumers in the period in which they are accumulated. Thus, all goods produced in any period t are acquired by consumers through wages, dividends, and net receipts from bond and stock transactions. The supply-equal-demand condition (31) is therefore an appropriate product market-clearing condition for the brokered economy with passive intermediation.

11. Specifically, any equilibrium $(s_t, c_t^1, c_t^2, k_t, b_t, r_t, w_t : t \geq 1)$ for a Tirole economy with initial conditions of the form $k_0 > 0$ and $b_0 \geq 0$ is an equilibrium sequence of real outcomes for a Walrasian brokered economy with initial conditions of the form $(k_0, \lambda_0) > 0$ and $(b_0, \theta_0) \geq 0$, and vice versa, where $s_t \equiv [c_t^1 - w_t]$ for the brokered economy.

12. Empirical findings suggest that chartering and other government-imposed restrictions currently in force do restrict entry into U.S. financial markets [Amel and Liang (1992)]. Indeed, as noted by Kaufman (1992, p. 296), among the requirements typically included for obtaining a commercial bank charter in the U.S. is a demonstration that the services proposed by the applicant are needed and will not endanger the solvency of other similar financial institutions. Our assumption that any entering intermediary must assume the liabilities of the existing intermediary can be interpreted as a buy-out condition that prevents the insolvency of the existing intermediary.

13. In reality, the intermediary would have to discover the form of the function $b(r_t, r_{t+1})$ in the process of implementing his prospectus. For this initial study, however, we make the simplifying assumption that the intermediary knows this function.

14. To see this, first note that no intermediary in period 1 has any leeway in the setting of r_1, or any nontrivial leeway in the setting of the associated initial prices p_0^λ and p_0^θ if existing contractual obligations are to be fulfilled. Specifically, it follows from condition (iii) in the definition of a feasible prospectus that $r_1 > 0$ is uniquely determined as a function of the exogenously given values k_0 and b_0 by relation (32) for $t = 0$, which then determines p_0^λ and p_0^θ as well (see Section 4). To construct a feasible prospectus that supports the interest-rate sequence $\boldsymbol{r}$, it then suffices to let the dividend payment d_t be zero for each $t \geq 1$ and to set the bond and stock-share prices p_t^λ and p_t^θ for $t \geq 1$ to positive values that satisfy the viability condition (17) in each period $t \geq 1$ for the given interest-rate sequence.

15. These market-clearing conditions are actually implied by previous conditions of the definition and are stated only for clarity. As the intermediary acts to mediate savings into investment, he also acts to ensure the market-clearing conditions for bonds, stock shares, and capital. Moreover, nonnegative excess supply in the product market is ensured by condition (35) (cf. note 7). Positive excess product supply can occur only if the intermediary is able to generate net receipts in excess of his dividend obligations without permitting successful entry by a competitor.

16. See Section 2.

17. In further analogy to Theorem 2, Balasko and Shell (1980) also show the sufficiency of this transversality condition for Pareto efficiency if certain additional restrictions hold.

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APPENDIX

A.1. PROOF THAT $b^n > 0$

Tirole [1985, eq. (8), p. 1502] assumes that the equation $0 = z(r)$ has a unique solution $\bar{r}$, where

$$z(r) \equiv [L_{t-1}s(w(r), r) - L_t k(r)]/L_t = s(w(r), r)/[1+n] - k(r) \qquad \textbf{(A.1)}$$

denotes the excess supply of savings per young consumer when each optimizing consumer and producer is faced with the interest rate r. In addition, much of Tirole's analysis focuses on the case in which inefficiency is possible, which requires $\bar{r} < n$. As noted in Section 2.2, these regularity conditions also are assumed to hold in the present paper.

It is first shown that, under these assumptions, the excess supply of savings $z(r)$ is an increasing function of r at $r = \bar{r}$; i.e., $0 < z'(\bar{r})$. Using this result, it then is shown that

$$0 < z(r) \quad \text{for all} \quad r \quad \text{satisfying} \quad \bar{r} < r. \qquad \textbf{(A.2)}$$

Hence, in particular, $0 < z(n) = b^n/[1+n]$.

Given the stationary interest rate $\bar{r}$, producer optimization in the Tirole economy implies that the capital/labor ratio k and the wage rate w satisfy $\bar{r} = f'(k)$ and $w = f(k) - f'(k)k$. Consequently, it follows from Section 2.2 that $k = k(\bar{r})$ and $w = w(\bar{r})$ with

$$k'(\bar{r}) = 1/f''(k(\bar{r})); \qquad \textbf{(A.3)}$$

$$w'(\bar{r}) = -k(\bar{r}) \cdot f''(k(\bar{r})) \cdot k'(\bar{r}) = -k(\bar{r}). \qquad \textbf{(A.4)}$$

Using conditions (44) and (45), one obtains

$$\begin{aligned} z'(\bar{r}) &= [s_w(w(\bar{r}), \bar{r})w'(\bar{r}) + s_r(w(\bar{r}), \bar{r})]/[1+n] - k'(\bar{r}) \\ &= [s_r(w(\bar{r}), \bar{r}) - s_w(w(\bar{r}), \bar{r})k(\bar{r}) f''(k(\bar{r}))k'(\bar{r})]/[1+n] - k'(\bar{r}) \\ &= [s_r(w(\bar{r}), \bar{r}) - s_w(w(\bar{r}), \bar{r})k(\bar{r})]/[1+n] - k'(\bar{r}). \end{aligned} \quad \textbf{(A.5)}$$

Diamond [1965, eq. (11), p. 1133] places a condition on the relative slopes of the capital market supply and demand curves along any competitive equilibrium path. In particular, for the unique stationary competitive equilibrium $\bar{\boldsymbol{e}} = \boldsymbol{e}(k(\bar{r}), 0)$ associated with $\bar{r}$, this condition reduces to

$$[f''(k(\bar{r}))s_w(w(\bar{r}), \bar{r})]/[1+n - f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})] < 0. \quad \textbf{(A.6)}$$

Moreover, to guarantee that consumption is a normal good in each period of life for each agent, Diamond (1965, pp. 1131–1132 n. 6) assumes that $0 < s_w < 1$. The numerator in condition (A.6) is therefore negative. Thus, for condition (A.6) to hold, the denominator must be positive, i.e.,

$$[1+n - f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})] > 0. \quad \textbf{(A.7)}$$

Diamond [1965, eq. (12), p. 1134] also assumes that $\bar{\boldsymbol{e}}$ is locally stable, and he gives a condition that is necessary for this to be the case, namely,

$$|[-k(\bar{r}) f''(k(\bar{r}))s_w(w(\bar{r}), \bar{r})]/[1+n - f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})]| \leq 1. \quad \textbf{(A.8)}$$

These same conditions are imposed by Tirole (1985, p. 1502) and are assumed to hold in the present paper as well.

A *sufficient* condition for the local stability of $\bar{\boldsymbol{e}}$ is that condition (A.8) holds with strict inequality. This slightly stronger condition is assumed by Tirole (1985, p. 1502) and also is assumed to hold in the present paper. Using conditions (A.3), (A.5), (A.7), and (A.8) with strict inequality, one then obtains the following series of implications:

$$\begin{gathered} [-k(\bar{r}) f''(k(\bar{r}))s_w(w(\bar{r}), \bar{r})] < [1+n - f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})]; \\ 0 < [k(\bar{r}) f''(k(\bar{r}))s_w(w(\bar{r}), \bar{r}) + 1 + n - f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})]; \\ 0 < [-k'(\bar{r})k(\bar{r}) f''(k(\bar{r}))s_w(\bar{r}, w(\bar{r})) - k'(\bar{r})[1+n] + k'(\bar{r}) f''(k(\bar{r}))s_r(w(\bar{r}), \bar{r})]; \\ 0 < [-k(\bar{r})s_w(w(\bar{r}), \bar{r}) - k'(\bar{r})[1+n] + s_r(w(\bar{r}), \bar{r})]; \\ 0 < [s_r(w(\bar{r}), \bar{r}) - s_w(w(\bar{r}), \bar{r})k(\bar{r})]/[1+n] - k'(\bar{r}); \\ 0 < z'(\bar{r}). \end{gathered} \quad \textbf{(A.9)}$$

As previously noted, $\bar{r}$ satisfies $0 = z(\bar{r})$. It follows from (A.9) that, for some positive ϵ, $0 < z(r)$ for all r in the interval $(\bar{r}, \bar{r} + \epsilon)$. Suppose that $z(r*) \leq 0$ for some $r*$ satisfying $\bar{r} + \epsilon < r*$. By continuity of $z(\cdot)$, this would imply the existence of some r^o lying between $\bar{r} + \epsilon$ and $r*$ that satisfies $0 = z(r^o)$, a contradiction of the assumption that there exists a unique solution to the equation $0 = z(r)$.

It follows that condition (A.2) must hold for the Tirole economy. ■

A.2. PROOF THAT b_t CANNOT DIVERGE IN ANY BROKERED-ECONOMY EQUILIBRIUM

Along any equilibrium path for the brokered economy, the per-capita consumption loan levels b_t supplied by the intermediary are given by relation (32), implying that $b_t = [s_t - (1+n)k_{t+1}]$ for each $t \geq 0$. In addition, they must satisfy condition (35) in order to guarantee that dividend obligations can be met. Suppose the sequence (b_t) diverges to infinity. Because the historically given value b_0 is nonnegative by assumption, and all equilibrium interest rates are positive, it follows from (35) that the sequence (b_t) is nonnegative. Consequently, b_t must become infinitely large as t approaches infinity.

The equilibrium savings s_t of each generation-t young consumer must be nonnegative and less than his endowment w_t for the consumer to consume a nonnegative amount of good when young and when old. Because all equilibrium interest rates are positive, the producer optimization condition (4) implies that the equilibrium capital/labor ratios k_t also must be positive. Consequently, it follows from (32) that b_t can only become infinitely large if s_t and thus w_t both become infinitely large.

Using these observations, together with the producer optimization condition (5), it must hold for each $t \geq 1$ that

$$0 \leq b_t/k_t \leq s_t/k_t \leq w_t/k_t = f(k_t)/k_t - f'(k_t) < f(k_t)/k_t. \qquad \textbf{(A.10)}$$

By conditions (4) and (5), and the production-function restrictions given in note 5, for w_t to become infinitely large as t approaches infinity, the interest rate r_t must approach zero and the capital/labor ratio k_t must become infinitely large as t approaches infinity. Consequently, by strict concavity of the production function $f(\cdot)$, and the assumption that $f'(k)$ approaches zero as k approaches infinity, the average product of capital $f(k_t)/k_t$ must approach zero as t approaches infinity, implying from relation (A.10) that the ratio s_t/k_t also approaches zero as t approaches infinity.

Finally, it follows from (32) and previous observations that

$$s_{t-1}/k_{t-1} - [1+n]k_t/k_{t-1} = b_{t-1}/k_{t-1} \geq 0 \qquad \textbf{(A.11)}$$

for each $t \geq 1$. Because s_{t-1}/k_{t-1} approaches zero as t approaches infinity, the (positive) term $[1+n]k_t/k_{t-1}$ also must approach zero as t approaches infinity in order to have the right term remain nonnegative for all t. Consequently, $k_t < k_{t-1}$ for all sufficiently large t, but this contradicts the fact, established above, that k_t becomes infinitely large as t approaches infinity.

It follows that no equilibrium consumption loan sequence (b_t) for the brokered economy can diverge to infinity. ■