A COMPUTER MODEL OF EDDY-CURRENT PROBE-Crack INTERACTION

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INTRODUCTION

A general three-dimensional eddy-current probe model, developed by Sabbagh Associates and reported in [1], [2] and [3], has been adapted for the calculation of probe-flaw interactions. The theoretical model, [4] and [5], uses integral equations with dyadic Green's function kernels, and is applicable to both probe and flaw calculations at arbitrary skin depths and frequencies. Discrete approximations of the integral equations are solved using a highly efficient algorithm based on recent developments in numerical techniques and their application to the solution of large problems in electromagnetic field-theory.

The model was validated internally through self-consistency tests and externally by comparing predictions with experimental data.

NUMERICAL EXPERIMENTS

The results presented here are in three sections which correspond to the following tasks:

1. Generate impedance plane curves (lift-off amplitude and phase) for 1, 2, 4, 8, 16 mils at frequencies 1, 2, 4, 8, 16, 32 MHz, using Probe 1 and Probe 2 that are shown below in Figure 1.

2. Obtain theoretical EC impedance responses from scanning six (6) simulated EDM notches in a conductor for Probe 1 and Probe 2 at a frequency of 2MHz, and a scanning index of 10 mils. The six slots and their dimensions are shown below.

<table>
<thead>
<tr>
<th>NAME</th>
<th>LENGTH (mils)</th>
<th>WIDTH (mils)</th>
<th>DEPTH (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot.1</td>
<td>30</td>
<td>3</td>
<td>5.0</td>
</tr>
<tr>
<td>Slot.2</td>
<td>30</td>
<td>3</td>
<td>6.0</td>
</tr>
<tr>
<td>Slot.3</td>
<td>30</td>
<td>3</td>
<td>7.5</td>
</tr>
<tr>
<td>Slot.4</td>
<td>30</td>
<td>3</td>
<td>10.0</td>
</tr>
<tr>
<td>Slot.5</td>
<td>30</td>
<td>3</td>
<td>15.0</td>
</tr>
<tr>
<td>Slot.6</td>
<td>500</td>
<td>3</td>
<td>15.0</td>
</tr>
</tbody>
</table>
3. Make comparisons when the probe response is at a maximum for Probe 1 and Probe 2 at frequencies 1, 2, 4, 8, 16, and 32 MHz.

For Tasks 1, 2 and 3, the conductivity is to be 1.20% IACS, and for Tasks 2 and 3, the probe lift-off is 3.5 mils.

**IMPEDEANCE PLANE RESULTS**

The complex normalized impedance is defined as $Z_N = Z/X_0$, where $Z$ is the probe impedance measured on the workpiece (or at the prescribed lift-off), and $X_0$ is the free-space probe reactance. We have assumed that the intrinsic probe losses are negligible; if not, the equivalent resistance must be subtracted from $Z$ in computing $Z_N$. The actual probe will have losses of various types (such as core losses and copper losses), so these losses will have to be determined experimentally, and then subtracted before comparing the measured results with our computations.

To generate the data for this task, we used our probe model code. The input data to this model includes the probe description, lift-off, and frequency. Thus, by varying the frequency and lift-off values, we can compute the probe impedance, which is then normalized. The normalized impedance at infinite lift-off is the complex value $(0.0,1.0)$, or, $X_N = 1.0$, $R_N = 0.0$. If we consider $Z_N = Z_N(\omega)$, then $Z_N(0) = (0.0,1.0)$ and $Z_N(\infty) = (0.0,x)$, and the coupling coefficient is $1.0 - x$.

The results for Probe 1 and 2 are presented in Figure 2. We see from the figures that Probe 1, the larger probe, has a larger coupling coefficient, which is what the theory predicts.

![Fig. 1 Preprocessor graphics for the eddy-current probe model showing the subdivision of the core. 'Resolution' refers to the cell dimensions, here given in mil. The left probe has a 50 mil diameter coil and the right probe, a 30 mil diameter coil.](image-url)
Fig. 2 Normalized impedance diagrams of Probe 1 (left) and Probe 2 (right) for liftoff = 1, 2, 4, 8, 16 mils, and frequencies = 1, 2, 4, 8, 16, 32 mHz.

SCAN RESULTS

We used our crack model to scan six (6) slots with Probe 1 and five slots with Probe 2. The section headed ‘NUMERICAL EXPERIMENTS’ gives the names and dimensions of the slots. We used 5 mils for the scanning index.

In predicting the response of a ferrite core eddy-current probe, it is necessary first to determine the induced magnetization of the ferrite. This makes it possible to calculate the incident field at the flaw as a sum of contributions from the coil current and the magnetic dipole moment of the core. The incident field is then typically two or three times what it would be if the core were absent.

Having computed the magnetic source strength of the core elements, the field anywhere in air or the conductor can be found. For the flaw calculation, the induced field in the conductor is needed only at the location of the flaw. If values of ΔZ were required at a number of different probe positions, it would not be necessary to recompute the core magnetization each time, nor the matrix elements used for finding the anomalous flaw current. This makes the algorithm particularly efficient for simulating or predicting scan data.

In order to calibrate (scale) our results we scanned Slot 5, (the calibration slot), in 0.5 mil increments to find the maximum response. This value of ΔZ was then rotated to the $\hat{z}$-axis, which gave us a rotation angle, and scaled to 2000, which gave a scaling factor. All impedances, ΔZ, were then rotated by this angle and scaled by this factor. For Probe 1, this maximum response occurred when the probe was positioned at 18 mils; for Probe 2, the maximum occurred at position 15.5 mils.

The scans were longitudinal (along the length of the flaw) and each started at $y = 0$. This corresponds to the probe being centered over the slot. The
induced field was computed once per slot; a simple interpolation of this field over the slot simulated the action of moving the probe. Since the probes were axisymmetric, only the azimuthal component of the field was nonzero, and since the width of the slot was small compared to the length, we made a thin slot approximation by using the computed value of the field along the axis of symmetry of the slot (y-axis).

The results for Probes 1 and 2 are shown in Figures 3 and 4, respectively.

**Fig. 3** Response of ferrite probe P1 (50 mil diameter coil). Scaled $|\Delta Z|$ (left) and Rotated $\arg(\Delta Z)$ (right) vs probe position for Slot_1 through Slot_5 and scanning increment of 5 mils.

**Fig. 4** Response of ferrite probe P2 (30 mil diameter coil). Scaled $|\Delta Z|$ (left) and Rotated $\arg(\Delta Z)$ (right) vs probe position for Slot_1 through Slot_5 and scanning increment of 5 mils.
FREQUENCY RESPONSE

Having found the position of maximum probe response for Slot_5 at 2 MHz, we were able to compare the impedance responses using Probe 1 on slots 1-6, and Probe 2 on slots 1-5, as a function of frequency. Since we want these computations to be efficient, careful planning was needed.

As mentioned previously, the coil current and magnetic dipole moment of the core are functions of frequency, but independent of the slot. Thus, for a fixed frequency, we used the Probe model to compute these magnetic sources, and then used the Crack model to compute the impedances for each slot.

```
foreach frequency do
    use Probe model to compute magnetic sources
    foreach slot do
        use Crack model to compute impedance for slot
    end
end
```

The codes for these models are written so that their use, as illustrated above, is quite easy.

These results are again scaled and rotated in the manner described in the previous section, using the response to Slot_5 at 2 MHz as the standard. Figure 5 shows the frequency response for Probe 1, and Figure 6 for Probe 2. We see that for a fixed frequency, comparing Slots 1-5, $|\Delta Z|$ increases with depth, while $\arg(\Delta Z)$ decreases with depth. For a fixed slot, $|\Delta Z|$ increases with frequency, while $\arg(\Delta Z)$ decreases with frequency.

![Fig. 5 Scaled $|\Delta Z|$ (left) and Rotated $\arg(\Delta Z)$ (right) vs frequency at frequencies from 1 to 32 MHz for Slots 1-6 using Probe 1 at position 18 mils.](image-url)
Fig. 6 Scaled $|\Delta Z|$ (left) and Rotated $\arg(\Delta Z)$ (right) vs frequency at frequencies from 1 to 32 MHz for Slots 1-5 using Probe 2 at position 15.5 mils.

REFERENCES


