A 3-D FEM-BEM MODEL FOR ELECTROMAGNETIC WAVE INTERACTIONS WITH MATERIAL

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INTRODUCTION

This paper continues the authors effort in numerical modeling of electromagnetic forward scattering for high frequency nondestructive testing (NDT) applications. Started in [1], a two-dimensional problem in the time domain was solved by finite difference method in conjunction with a local absorbing boundary condition located some distance away from the scatterer. In the present work, more complicated, three-dimensional models in the frequency domain are considered. A typical electromagnetic NDT situation involves interaction of a given incident wave field with a material sample. Numerical modeling of such an NDT situation usually involves a large solution domain or inverting a very large matrix, as a direct consequence of the following two features. One is the unboundedness of the problem, because it is assumed that the incident wave field is emitted from and the measurement is performed in an infinite background medium. In finite difference or finite element modeling, an artificial boundary condition (more or less, frequency dependent) is located far away from the scatterer in order to simulate the nonreflecting nature of the scattered wave field. Another feature is the vector nature of the electromagnetic waves. Thus, the field variable at each node in the solution domain has three components. To get rid of the interface condition, finite element modeling usually reformulates the problem by introducing scalar and vector potentials, which means more unknowns per node and even larger matrices. The purpose here is to review and propose a class of edge-based methods with minimal solution domains, fewer independent unknowns and higher accuracy. These methods use a new class of elements having unknowns on the edges (tangential component of field along the edge) instead of the nodes. By using vector basis functions, each edge has only one degree of freedom (first order model)[2].

This paper is organized as follows. First, the physical model and governing equations are introduced. Then, the solution of surface integral equations based on vector boundary elements (BEM) is considerd. Coupled finite element method - boundary element method (FEM-BEM) using scalar and vector finite elements are given next. And finally, some representative results and conclusions are provided.

PHYSICAL MODEL AND BASIC EQUATIONS

Consider the problem of obtaining a numerical solution to the time - harmonic Maxwell's equations in a source - free domain. This physical configuration is shown

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in Fig. 1. Let the space $\Re^3$ be divided into two parts by a closed surface S (see figure 1). The exterior domain Ω_1 is characterized by $(\mu_1, \epsilon_1, \sigma_1 = 0)$, and the interior domain Ω_b by $(\mu_2, \epsilon_2(\vec{\mathbf{r}}), \sigma_2(\vec{\mathbf{r}}))$, where the $\vec{\mathbf{r}}$ dependence of conductivity and permittivity reflects possible defects and inclusions in the material. Let the source $(\vec{\mathbf{E}}^{inc}, \vec{\mathbf{H}}^{inc})$ be emitted from domain Ω_1 and excite the object Ω_2 externally. The purpose here is to compute the scattered wave fields $(\vec{\mathbf{E}}_1^s, \vec{\mathbf{H}}_1^s)$ and the penetrating wave fields $(\vec{\mathbf{E}}_2^s, \vec{\mathbf{H}}_2^s)$. Then the total wave fields are governed by the following basic equations

$$\begin{cases} \nabla \times \vec{\mathbf{E}}_1 = -j\omega\mu_1\vec{\mathbf{H}}_1, & \nabla \times \vec{\mathbf{H}}_1 = j\omega\epsilon_1'\vec{\mathbf{E}}_1 \quad \text{in } \Omega_1 \\ \nabla \times \vec{\mathbf{E}}_2 = -j\omega\mu_2\vec{\mathbf{H}}_2, & \nabla \times \vec{\mathbf{H}}_2 = j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}}_2 \quad \text{in } \Omega_2 \\ (\vec{n} \times \vec{\mathbf{E}})^+ = (\vec{n} \times \vec{\mathbf{E}})^-, & (\vec{n} \times \vec{\mathbf{H}})^+ = (\vec{n} \times \vec{\mathbf{H}})^- \quad \text{on } S \end{cases} \tag{1}$$

where $\epsilon_1' = \epsilon_1 + \sigma_1/j\omega$, $\epsilon_2' = \epsilon_2 + \sigma_2/j\omega$. It is possible to eliminate one field variable and obtain a *curlcurl* equation in terms of the other field variable, for example, in Ω_2,

$$\nabla \times \frac{1}{j\omega\mu_2}\nabla \times \vec{\mathbf{E}} + j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} = \vec{\mathbf{0}}. \tag{2}$$

BEM MODEL USING VECTOR ELEMENTS

In this section, some edge-based boundary element models are outlined for electromagnetic wave interation with homogeneous inclusions, i.e, parameters in Ω_2 are constants. The Stratton-Chu [3] formula plays a central role in deriving the surface integral equations and in coupled methods:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}^{inc} - \frac{1}{4\pi}\iint_S \left[j\omega\mu(\hat{\mathbf{n}} \times \vec{\mathbf{H}})G + (\hat{\mathbf{n}} \times \vec{\mathbf{E}}) \times \nabla G + (\hat{\mathbf{n}} \cdot \vec{\mathbf{E}})\nabla G\right] dS, \tag{3}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}^{inc} + \frac{1}{4\pi}\iint_S \left[j\omega\epsilon'(\hat{\mathbf{n}} \times \vec{\mathbf{E}})G - (\hat{\mathbf{n}} \times \vec{\mathbf{H}}) \times \nabla G - (\hat{\mathbf{n}} \cdot \vec{\mathbf{H}})\nabla G\right] dS. \tag{4}$$

By the equivalence principle, the scattered electric and magnetic fields in Ω_1 and Ω_2 can be expressed as [4,5]

$$\vec{\mathbf{E}}_1^s(\vec{\mathbf{r}}) = -j\omega\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) - \nabla \mathbf{V}_1(\vec{\mathbf{r}}) - \left(1/\epsilon_1'\right)\nabla \times \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \tag{5}$$

$$\vec{\mathbf{H}}_1^s(\vec{\mathbf{r}}) = -j\omega\vec{\mathbf{F}}_1(\vec{\mathbf{r}}) - \nabla \mathbf{U}_1(\vec{\mathbf{r}}) + (1/\mu_1)\nabla \times \vec{\mathbf{A}}_1(\vec{\mathbf{r}}) \tag{6}$$

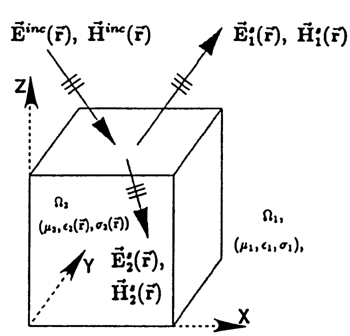

Fig. 1 Physical Configuration.

for $\vec{\mathbf{r}}$ on or outside S and

$$\vec{\mathbf{E}}_2^s(\vec{\mathbf{r}}) = j\omega\vec{\mathbf{A}}_2(\vec{\mathbf{r}}) + \nabla\mathbf{V}_2(\vec{\mathbf{r}}) + \left(1/\epsilon_2'\right)\nabla\times\vec{\mathbf{F}}_2(\vec{\mathbf{r}}) \tag{7}$$

$$\vec{\mathbf{H}}_2^s(\vec{\mathbf{r}}) = j\omega\vec{\mathbf{F}}_2(\vec{\mathbf{r}}) + \nabla\mathbf{U}_2(\vec{\mathbf{r}}) - (1/\mu_2)\nabla\times\vec{\mathbf{A}}_2(\vec{\mathbf{r}}) \tag{8}$$

for $\vec{\mathbf{r}}$ on or inside S. The various potential functions are

$$\vec{\mathbf{A}}_i(\vec{\mathbf{r}}) = \frac{\mu_i}{4\pi}\iint_S \vec{\mathbf{J}}(\vec{\mathbf{r}}')\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \quad \mathbf{V}_i(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_i'}\iint_S \rho^e(\vec{\mathbf{r}}')\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}')$$

$$\vec{\mathbf{F}}_i(\vec{\mathbf{r}}) = \frac{\epsilon_i'}{4\pi}\iint_S \vec{\mathbf{M}}(\vec{\mathbf{r}}')\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \quad \mathbf{U}_i(\vec{\mathbf{r}}) = \frac{1}{4\pi\mu_i}\iint_S \rho^m(\vec{\mathbf{r}}')\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}')$$

where the Green's functions are defined by $\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}') = e^{-jk_iR}/R$, $R = |\vec{\mathbf{r}} - \vec{\mathbf{r}}'|$. The currents are related to the fields by $\vec{\mathbf{J}} = \hat{\mathbf{n}}\times\vec{\mathbf{H}}$, $\vec{\mathbf{M}} = \hat{\mathbf{E}}\times\vec{\mathbf{n}}$ by the equivalence principle. The electric and magnetic charges are related to the currents by the continuity relations

$$\rho^e(\vec{\mathbf{r}}') = \frac{-1}{j\omega}\nabla_S'\bullet\vec{\mathbf{J}}(\vec{\mathbf{r}}') \quad \rho^m(\vec{\mathbf{r}}') = \frac{-1}{j\omega}\nabla_S'\bullet\vec{\mathbf{M}}(\vec{\mathbf{r}}') \tag{9}$$

The combined field integral equation (CFIE) formualtion is obtained by requiring the continuity of tangential components of the electric and magnetic fields:

$$\vec{\mathbf{E}}^i(\vec{\mathbf{r}})\,|_{\tan} = \Big\{j\omega[\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) + \vec{\mathbf{A}}_2(\vec{\mathbf{r}})] + [\nabla\mathbf{V}_1(\vec{\mathbf{r}}) + \nabla\mathbf{V}_2(\vec{\mathbf{r}})] + [\frac{1}{\epsilon_1'}\nabla\times\vec{\mathbf{F}}_1(\vec{\mathbf{r}}) + \frac{1}{\epsilon_2'}\nabla\times\vec{\mathbf{F}}_2(\vec{\mathbf{r}})]\Big\}\,|_{\tan} \quad \vec{\mathbf{r}}\in S \tag{10}$$

$$\vec{\mathbf{H}}^i(\vec{\mathbf{r}})\,|_{\tan} = \Big\{j\omega[\vec{\mathbf{F}}_1(\vec{\mathbf{r}}) + \vec{\mathbf{F}}_2(\vec{\mathbf{r}})] + [\nabla\mathbf{U}_1(\vec{\mathbf{r}}) + \nabla\mathbf{U}_2(\vec{\mathbf{r}})] - [\frac{1}{\mu_1}\nabla\times\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) + \frac{1}{\mu_2}\nabla\times\vec{\mathbf{A}}_2(\vec{\mathbf{r}})]\Big\}\,|_{\tan} \quad \vec{\mathbf{r}}\in S \tag{11}$$

A rigorous solution to the CFIE is achieved by dividing the surface S into triangular elements. A vector basis function and its divergence are given by

$$\vec{\mathbf{f}}_n(\vec{\mathbf{r}}) = \begin{cases} \frac{l_n}{2A_n^+}\vec{\rho}_n^+, & \vec{\mathbf{r}} \text{ in } T_n^+ \\ \frac{l_n}{2A_n^-}\vec{\rho}_n^-, & \vec{\mathbf{r}} \text{ in } T_n^- \\ \vec{0}, & \text{otherwise} \end{cases} \quad \nabla_S\bullet\vec{\mathbf{f}}_n(\vec{\mathbf{r}}) = \begin{cases} \frac{l_n}{A_n^+}, & \vec{\mathbf{r}} \text{ in } T_n^+ \\ -\frac{l_n}{A_n^-}, & \vec{\mathbf{r}} \text{ in } T_n^- \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

where T_n^+ and T_n^- are two adjacent triangles with the nth edge common, l_n is the length of the nth common edge, $A_n^{\pm}$ the area of triangle $T_n^{\pm}$, and $\vec{\rho}_n^{\pm}$ are the approriate position vector in $T_n^{\pm}$. Let there be N common edges in the surface triangulation. Now the surface electric and magnetic currents are expanded in terms of the basis function,

$$\vec{\mathbf{J}}(\vec{\mathbf{r}}') = \sum_{n=1}^{N} I_n\vec{\mathbf{f}}_n(\vec{\mathbf{r}}'), \quad \vec{\mathbf{M}}(\vec{\mathbf{r}}') = \sum_{n=1}^{N} M_n\vec{\mathbf{f}}_n(\vec{\mathbf{r}}') \tag{13}$$

where the coefficients I_n and M_n stand for the perpendicular components of the electric and magnetic current flowing past the nth common edge. To determine these coefficients, the CFIE is tested against the basis function in the sense of the following symmetrical product

$$< \vec{\mathbf{f}}, \vec{\mathbf{g}} > = \iint_S \vec{\mathbf{f}} \bullet \vec{\mathbf{g}} dS. \tag{14}$$

Careful calculation yields a partitioned matrix equation

$$\begin{bmatrix} \{Z_{mn}^{JJ}\} & \{Z_{mn}^{JM}\} \\ \{Z_{mn}^{MJ}\} & \{Z_{mn}^{MM}\} \end{bmatrix} \begin{bmatrix} \{I_n\} \\ \{M_n\} \end{bmatrix} = \begin{bmatrix} \{E_n^i\} \\ \{H_n^i\} \end{bmatrix} \tag{15}$$

FEM-BEM MODEL USING SCALAR ELEMENTS

Now consider the situation in which Ω_2 is a nonhomogeneous medium. Since the parameter ϵ_2' depends on position vector $\vec{\mathbf{r}}$, no Green function can be found for this case. Surface type integral equations discussed in the previous section cannot be applied, and instead, a volume type integral equation or a finite element procedure has to be considered. A volume type integral formulation uses the concept of volume polarization current. A detailed account of the method is not contained here. We focus on implementation of coupled FEM-BEM approaches. The traditional node-based method is discussed first. For brevity, only the solution procedure using $\vec{\mathbf{E}}$ as state variable is discussed. The dual procedure, using $\vec{\mathbf{H}}$ as state variable, can be obtained analogously.

The weighted integral form of (2) is

$$\int_{\Omega_2} \nabla \times \frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}} \phi_m d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}}) \vec{\mathbf{E}} \phi_m d\Omega = \vec{\mathbf{0}}, \tag{16}$$

where ϕ_m are any set of real scalar weighting functions. Applying the divergence theorem to each component yields the weak form

$$\int_{\Omega_2} \left(\frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}} \right) \times \nabla \phi_m d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}}) \vec{\mathbf{E}} \phi_m dS = \oint_S \hat{\mathbf{n}} \times \vec{\mathbf{H}} \phi_m dS. \tag{17}$$

Obviously, in the $\vec{\mathbf{E}}$ formulation, the surface electric current $\vec{n} \times \vec{\mathbf{H}} = \vec{\mathbf{J}}$ is used as a nonlocal boundary condition, while in the $\vec{\mathbf{H}}$ formulation, the surface magnetic current $\vec{E} \times \vec{\mathbf{n}} = \vec{\mathbf{M}}$ is the nonlocal boundary condition. This feature provides a natural way to couple the interior FEM solution with the exterior BEM solution [7]. Now consider the Galerkin procedure for discretizing the weak form. $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ are expanded by

$$\vec{\mathbf{E}} = \sum_{n=1}^{M_v} \vec{\mathbf{E}}_n \phi_n, \quad \hat{\mathbf{n}} \times \vec{\mathbf{H}} = \sum_{n=1}^{M_v} (\hat{\mathbf{n}} \times \vec{\mathbf{H}})_n \phi_n = \sum_{n=1}^{M_v} \vec{\mathbf{F}}_n \phi_n \tag{18}$$

where M_v is the total number of nodes in the finite element mesh (including the boundary nodes N_v), and ϕ_n is the scalar basis function. Then the discretized version of (17) is a matrix equation relating $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$

$$[A]\{\mathcal{E}\} = [B]\{\mathcal{F}\}. \tag{19}$$

Through the interface between two elements with different properties, the following relations have to be satisfied:

$$\hat{\mathbf{n}} \cdot (\epsilon_1' \vec{\mathbf{E}}_1 - \epsilon_2' \vec{\mathbf{E}}_2) = 0, \quad \hat{\mathbf{n}} \times (\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2) = 0, \tag{20}$$

$$\hat{\mathbf{n}} \cdot (\mu_1 \vec{\mathbf{H}}_1 - \mu_2 \vec{\mathbf{H}}_2) = 0, \quad \hat{\mathbf{n}} \times (\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2) = 0. \tag{21}$$

Let $\hat{\mathbf{n}} = n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}} + n_z \hat{\mathbf{z}}$. By careful manipulations, the above become [7]

$$\left\{\begin{matrix} E_{x_2} \\ E_{x_2} \\ E_{y_2} \end{matrix}\right\} = \begin{bmatrix} n_x^2\frac{\epsilon_1'}{\epsilon_2'} + n_y^2 + n_z^2 & n_x n_y\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) & n_x n_z\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) \\ n_y n_x\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) & n_x^2 + n_y^2\frac{\epsilon_1'}{\epsilon_2'} + n_z^2 & n_y n_z\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) \\ n_z n_x\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) & n_z n_y\left(\frac{\epsilon_1'}{\epsilon_2'} - 1\right) & n_x^2 + n_y^2 + n_z^2\frac{\epsilon_1'}{\epsilon_2'} \end{bmatrix} \left\{\begin{matrix} E_{x_1} \\ E_{x_1} \\ E_{y_1} \end{matrix}\right\}. \tag{22}$$

Similar expression holds for $\vec{\mathbf{H}}$ with ϵ' replaced by μ. The condition (22) has to be applied during the assembly of the global stiffness equation (19).

The successful implementation of the relation (22) needs a specification of a nodal normal $\hat{\mathbf{n}}$ at each node. However, the normal direction along the boundary of the finite element grid may not be unique. In the three-dimensional finite element calculation, a unique normal for node i, based on the conservation principle [8], is defined by

$$n_x = \frac{1}{n}\int_\Omega \frac{\partial \phi_i}{\partial x} d\Omega, \quad n_y = \frac{1}{n}\int_\Omega \frac{\partial \phi_i}{\partial y} d\Omega, \quad n_z = \frac{1}{n}\int_\Omega \frac{\partial \phi_i}{\partial z} \tag{23}$$

where the normalizing factor is

$$n = \left[\left(\int_\Omega \frac{\partial \phi_i}{\partial x} d\Omega\right)^2 + \left(\int_\Omega \frac{\partial \phi_i}{\partial y} d\Omega\right)^2 + \left(\int_\Omega \frac{\partial \phi_i}{\partial z} d\Omega\right)^2\right]^{1/2}. \tag{24}$$

Equation (19) cannot be solved because one does not know $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ on S *a priori*. This information is obtained by considering the stratton-chu formula (3). Let the position vector be on the surface S. One obtains

$$\frac{1}{2}\vec{\mathbf{E}}_m = \vec{\mathbf{E}}_m^i - \frac{1}{4\pi}\iint_S \left[j\omega\mu_2(\hat{\mathbf{n}} \times \vec{\mathbf{H}})G_m + (\hat{\mathbf{n}} \times \vec{\mathbf{E}}) \times \nabla G_m + (\hat{\mathbf{n}} \cdot \vec{\mathbf{E}})\nabla G_m\right] dS, \tag{25}$$

where G_m has its singularity at node m. Similar to the finite element formulation, the surface S is modeled by triangular elements (consistent with a tetrahedral FEM model) or quadrlateral elements (consistent with a hexahedral FEM model). Let $\vec{\mathbf{E}}$ and $\vec{n} \times \vec{\mathbf{H}}$ on S be expanded by

$$\vec{\mathbf{E}} = \sum_{n=1}^{N_v} \vec{\mathbf{E}}_n\psi_n, \quad \hat{\mathbf{n}} \times \vec{\mathbf{H}} = \sum_{n=1}^{N_v}(\hat{\mathbf{n}} \times \vec{\mathbf{H}})_n\psi_n = \sum_{n=1}^{N_v} \vec{\mathbf{F}}_n\psi_n \tag{26}$$

where ψ_n are scalar basis functions and N_v is the total number of nodes in the boundary element mesh. Substituting (26) into (25) and letting m vary from 1 to N_v yields the matrix equation relating $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ to $\vec{\mathbf{E}}$ and incident wave field

$$[C]\{\mathcal{F}\} = -\{\mathcal{E}^i\} + [D]\{\mathcal{E}\}. \tag{27}$$

The nodal normal on surface S is computed by [7,8]

$$n_{x_m} = \frac{1}{n}\oint_S n_x\psi_m dS, \quad n_{y_m} = \frac{1}{n}\oint_S n_y\psi_m dS, \quad n_{z_m} = \frac{1}{n}\oint_S n_z\psi_m dS \tag{28}$$

where the normalizing factor is

$$n = \left[\left(\oint_S n_x\psi_m dS\right)^2 + \left(\oint_S n_x\psi_m dS\right)^2 + \left(\oint_S n_x\psi_m dS\right)^2\right]^{1/2}. \tag{29}$$

Equation (19) and (27) can be easily coupled to get a solution. Assume that the interior nodes are numbered first and the boundary nodes second. Equation (19) may be split into a partitioned matrix equation

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}. \tag{30}$$

While the boundary element formulation is

$$[C]\{F_2\} = -\{E_2^i\} + [D]\{E_2\}. \tag{31}$$

The two equation may be combined into

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} - BC^{-1}D \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -BC^{-1}E_2^i \end{Bmatrix}. \tag{32}$$

FEM-BEM USING VECTOR ELEMENTS

The previous section establishes an elegant way to couple interior-exterior wave fields. This node-based method has a disadvantage, in that the inter-element conditions and the surface conditions are rather tedious. The method proposed next uses edge-based elements with vector basis functions. The resulting matrix equation has fewer unknowns, and is more sparse.

The weighted integral form of (2) is [9]

$$\int_{\Omega_2} \nabla \times \frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega = 0, \tag{33}$$

where $\vec{\mathbf{w}}_m$ are any set of real vector weighting functions. Application of some vector algebra yields the weak form

$$\int_{\Omega_2} \left(\frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}}\right) \cdot (\nabla \times \vec{\mathbf{w}}_m) d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega = \oint_S \left(\hat{\mathbf{n}} \times \vec{\mathbf{H}}\right) \cdot \vec{\mathbf{w}}_m dS. \tag{34}$$

Next consider the Galerkin procedure for discretizing the weak form. $\vec{\mathbf{E}}$ and $\vec{n} \times \vec{\mathbf{H}}$ are expanded by

$$\vec{\mathbf{E}} = \sum_{n=1}^{M_e} E_n \vec{\mathbf{w}}_n(\vec{\mathbf{r}}), \quad \hat{\mathbf{n}} \times \vec{\mathbf{H}} = \sum_{n=1}^{M_e} F_n \vec{\mathbf{w}}_n(\vec{\mathbf{r}}) \tag{35}$$

where M_e is the total number of edges in the finite element mesh, and $\vec{\mathbf{w}}_n$ is the vector basis function associated with the nth edge. These vector basis functions are constructed such that the expansion coefficients in (35) are tangential components of corresponding field variables along that edge. Because the tangentially continuous finite elements are used, the inter-element conditions are automatically guaranteed. The discretized version of (34) is

$$[A]\,\{\mathcal{E}\} = [B]\,\{\mathcal{F}\}, \tag{36}$$

where the coefficients are

$$a_{mn} = \int_{\Omega_2} \left[\frac{1}{j\omega\mu_2} (\nabla \times \vec{\mathbf{w}}_n) \cdot (\nabla \times \vec{\mathbf{w}}_m) + j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{w}}_n \cdot \vec{\mathbf{w}}_m\right] d\Omega, \tag{37}$$

$$b_{mn} = \oint_S \vec{\mathbf{w}}_n \cdot \vec{\mathbf{w}}_m dS, \qquad m,\, n = 1, 2, \cdots, M_e. \tag{38}$$

In an edge-based BEM, field variables are expanded by a set of vector basis functions $\vec{\mathbf{f}}_n$, for example,

$$\vec{\mathbf{E}} = \sum_{n=1}^{N_e} E_n \vec{\mathbf{f}}_n(\vec{\mathbf{r}}), \quad \hat{\mathbf{n}} \times \vec{\mathbf{H}} = \sum_{n=1}^{N_e} F_n \vec{\mathbf{f}}_n(\vec{\mathbf{r}}) \tag{39}$$

where N_e is the total number of edges in the boundary element mesh. In a conformal way, $\vec{\mathbf{f}}_n$ should be $\vec{\mathbf{w}}_n$ acting only on S. However, in a nonconformal case, $\vec{\mathbf{f}}_n$ can be any set of vector functions as long as the coefficents in the expansion have the same physical meaning as that in (35). Since both $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ are tangentially continous, the surface conditions are automatically guaranteed. Testing (3) in the sense of symmetric product (14) yields a matrix equation of the form

$$[C]\,\{\mathcal{F}\} \;=\; -\{\mathcal{E}^i\} \;+\; [D]\,\{\mathcal{E}\}, \tag{40}$$

where the coefficients are

$$c_{mn} \;=\; \iint_S \left[-\frac{1}{4\pi} \iint_S j\omega\mu_2 \vec{\mathbf{f}}_n \mathbf{G} dS \right] \bullet \vec{\mathbf{f}}_m dS \tag{41}$$

$$d_{mn} \;=\; \frac{1}{2} \iint_S \vec{\mathbf{f}}_n \bullet \vec{\mathbf{f}}_m dS + \iint_S \left[\frac{1}{4\pi} \iint_S (\hat{\mathbf{n}} \bullet \vec{\mathbf{f}}_n) \nabla \mathbf{G} dS \right] \bullet \vec{\mathbf{f}}_m dS$$

$$+ \iint_S \left[\frac{1}{4\pi} \iint_S (\hat{\mathbf{n}} \times \vec{\mathbf{f}}_n) \times \nabla \mathbf{G} dS \right] \bullet \vec{\mathbf{f}}_m dS \tag{42}$$

$$e_m^i \;=\; \iint_S \vec{\mathbf{E}}^i \bullet \vec{\mathbf{f}}_m dS, \qquad m, n = 1, 2, \cdots, N_e. \tag{43}$$

A complete solution is obtained by solving the two coupled equations, (36) and (40).

Computationally speaking, in the edge-based method, each edge is shared by 4 hexahedra, resulting in 21 nonzero coefficients for each edge. Thus, the edge-based FEM is more economical than the node-based method. The present edge-based BEM is more involved as it involves double surface integrals when evaluating matrix coefficients, compared with simple surface integrals in the node-based formulation.

REPRESENTATIVE RESULTS AND ON-GOING WORK

Several methods have been considered for efficiently modeling electromagnetic wave interaction with materials. In all three methods, CFIE, node-based FEM-BEM, and the edge-based FEM-BEM, discretizing the boundary integral plays an important role. The computer implementation of these methods starts with the numerical implementation of edge-based integral procedures. As an example, consider a conducting cube under plane wave illumination. The cube has the electrical size $ks = 2$, where s is the cube side width. The incident plane wave has its electric field polarized along the $+z$ direction, and propagates in the $+y$ direction normal to the front face of the cube. Each cube face is spanned by 32 triangular elements. This model requires filling and inverting a 288×288 matrix. Fig. 2 shows the magnitude of the "looping" surface current along the locus $\overline{abcd}$ (in the E-plane of the incident wave, slightly shifted to the right of the center cut). The results coincide with that in Figs. 11(a) of [5].

Substantial work is under way to build a computer code using a coupled finite element - boundary element method in conjunction with edge type elements, proposed in the previous section. Two kinds of vector elements are reported in the literature. One is a tetrahedral element with six degrees of freedom, another is a hexahedral element with twelve degrees of freedom [10]. The latter is prefer-

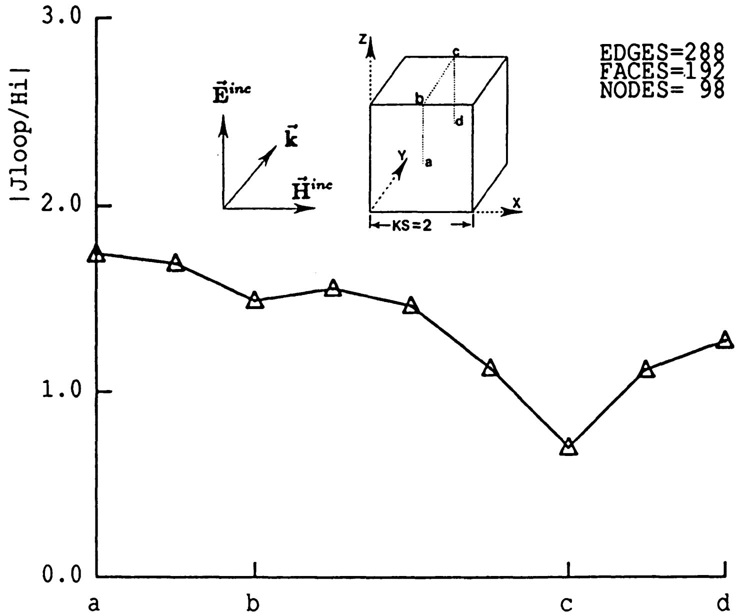

Fig. 2 The Magnitude of the "Looping" Current.

able due to the fact that hexahedral elements are easy to generate and visualize. On-going work also includes extending the method to multiple interface cases, for example, a nonhomogeneous inclusion buried inside a homogeneous material under wave illumination from free space, and modeling any transducer - receiver effects.

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