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Keywords
Excited states, Excitation energies, Solvents, Polarization, Absorption spectra

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Excited-state quantum mechanics/molecular mechanics molecular dynamics simulations are performed, to examine the solvent effects on the fluorescence spectra of aqueous formaldehyde. For that purpose, the analytical energy gradient has been derived and implemented for the linear-response time-dependent density functional theory (TDDFT) combined with the effective fragment potential (EFP) method. The EFP method is an efficient ab initio based polarizable model that describes the explicit solvent effects on electronic excitations, in the present work within a hybrid TDDFT/EFP scheme. The new method is applied to the excited-state MD of aqueous formaldehyde in the n-π* state. The calculated π*→n transition energy and solvatochromic shift are in good agreement with other theoretical results. © 2011 American Institute of Physics. [doi:10.1063/1.3523578]

I. INTRODUCTION

The development of quantum mechanical (QM) methods to describe the properties of electronically excited states of solvated molecules is crucial in the study of photochemical and photobiological processes in solution. The understanding of photochemical and photophysical phenomena relies on the accurate description of excited-state potential energy surfaces. In general, the relaxation of molecules in their excited states involves a dramatic change in both the electronic and geometric structures. Therefore, it is highly desirable to develop and apply accurate QM methods in the simulation of the excited electronic states of such molecules.

The linear-response time-dependent density functional theory (TDDFT) (Refs. 2–11) is a convenient and efficient tool for calculating excitation energies with reasonable computational cost. The TDDFT method has been successfully applied to the study of excited states in a broad class of large molecular systems. The ability to perform excited-state geometry optimizations is essential for elucidating the mechanism of a photochemical process. The prediction of fluorescence spectra, for example, requires the geometries at the excited-state potential energy minima. Furche and Ahlrichs have proposed a variational formulation of TDDFT and on this basis derived analytic excited-state gradients.

The application of TDDFT to solvated molecules is an important step in understanding solvent effects on the excited-state dynamics and properties of large molecules, because a large number of experiments have been conducted in solution. Although TDDFT is employed routinely for predicting the absorption spectra of solvated molecules, TDDFT excited-state molecular dynamics in solution is still limited, because a direct application of TDDFT to the whole solute-solvent system is computationally demanding. Furthermore, for many functionals, TDDFT is not generally reliable for describing excited states with a strong charge-transfer (CT) character. Bernasconi et al. have applied a plane wave-based TDDFT approach to the acetone-water system and observed spurious low-lying solute to solvent CT excitations. Therefore, solvent effects are often taken into account by appropriate classical models. The dielectric continuum model is a popular approach for describing the electronic structure of solvated molecules. In this approach, solvent effects are directly incorporated into the QM Hamiltonian by using surface charges on the cavity surrounding the QM molecule. The TDDFT method combined with the polarizable continuum model (PCM) has been developed and applied to predict the absorption and fluorescence spectra of solvated molecules. Scalmani et al. have reported analytic TDDFT gradients for molecules in solution within the framework of the PCM method and evaluated solvent effects on the fluorescence peak shift. Very recently, Wang and Li have applied the conductor-like PCM/TDDFT analytic gradient to examine the excited-state potential energy surface of the solvated photoactive yellow protein chromophore.

Although the TDDFT/PCM method has achieved some success in reproducing absorption and fluorescence spectra in solution, the method cannot correctly describe local solute-solvent interactions such as hydrogen bonding, as the PCM solvent is characterized by a homogeneous macroscopic medium. Therefore, it is important to take the molecular and electronic aspects of the solvent-solute interactions, and often solvent-solvent interactions, into account.

Hybrid QM-molecular mechanics (QM/MM) is an alternative method for incorporating solvent effects by introducing an explicit solvent model. The effective fragment potential (EFP) method provides a polarizable QM-based force field to describe intermolecular interactions. The EFP
method has been applied successfully to QM/MM studies of molecules in clusters and in solution. The interface of the EFP model with the TDDFT method has recently been developed for describing electronically excited states of solvated molecules. Yoo et al.\textsuperscript{28} have combined the linear-response TDDFT (LR-TDDFT) method with the original EFP model (EFP1) and applied the hybrid method successfully to simulate the absorption spectrum of the $n\rightarrow\pi^*$ vertical transition of acetone in aqueous solution. Very recently, Si and Li\textsuperscript{134} have derived the analytic energy gradient for combined LR-TDDFT and polarizable force field methods.

In this work, the analytic energy gradient is implemented for the combined TDDFT/EFP1 method to describe the excited state dynamics of solvated molecules. First, the TDDFT/EFP1 energy formulation is extended on the basis of the matrix formulation for the EFP1 polarization energy and derivative as given by Li et al.\textsuperscript{30} This is a more general approach than that presented in Ref. 28, and is closely related to the configuration interaction formulation presented by Arora et al.\textsuperscript{31} and by DeFusco et al.\textsuperscript{32} Second, the formulation of TDDFT/EFP1 analytic energy gradients is presented. The EFP1 contribution to the gas-phase TDDFT analytic gradient that was derived by Furché and Ahlrichs\textsuperscript{8} is examined based on the new TDDFT/EFP1 excitation energy formula. Finally, the proposed method is applied to the excited-state molecular dynamics (MD) simulation of aqueous formaldehyde in its $n-\pi^*$ state. Although there are no experimental fluorescence spectra available for aqueous formaldehyde, this solute-solvent system has been the subject of several theoretical studies.\textsuperscript{20,21,33–45} The ultimate goal is to apply the solute-solvent interaction terms is an initial step toward applications to more complex systems. In the present QM/EFP1 MD simulations, the DFT-based EFP1 water model\textsuperscript{27} is adopted.

The organization of this paper is as follows. Section II describes the formulation of the TDDFT/EFP1 energy and gradient on the basis of the matrix equation for the EFP polarization energy. In Sec. III, the proposed method is applied to the excited-state MD simulation of aqueous formaldehyde in the $n-\pi^*$ state. Concluding remarks are summarized in Sec. IV.

II. THEORETICAL METHOD

A. QM/EFP1 method

The EFP1 method\textsuperscript{25–27} contains three terms: electrostatic (Coulomb), polarization (induction), and a remainder term that largely represents repulsive interactions. The Coulomb interaction is modeled with static multipoles located at atoms and bond midpoints. The polarization/induction effects are described by anisotropic dipole polarizability tensors located at the centroids of localized bonds and lone-pair orbitals. The repulsive term represents the exchange repulsion and charge-transfer interactions, as well as short-range electron correlation effects, and is determined empirically by fitting to a large number of points on the DFT/B3LYP water dimer potential energy surface.\textsuperscript{27} In order to describe the QM-EFP1 interaction, it is necessary to define an effective QM Hamiltonian $\hat{H}^{\text{eff}}$ by adding the solute-solvent interaction terms,

$$\hat{H}^{\text{eff}} = \hat{H}_{\text{gas}} + \hat{V}^{\text{es}} + \hat{V}^{\text{pol}} + \hat{V}^{\text{rep}}, \tag{1}$$

where $\hat{H}_{\text{gas}}$ is the Hamiltonian of the isolated QM molecule, and the remaining three terms describe the QM-EFP1 interaction terms. $\hat{V}^{\text{es}}$ represents the electrostatic (Coulomb) potential generated by EFP permanent multipoles: monopoles, dipoles, quadrupoles, and octopoles. The last term $\hat{V}^{\text{rep}}$ is the repulsive potential. Since $\hat{V}^{\text{es}}$ and $\hat{V}^{\text{rep}}$ are independent of the QM electron density, these two terms can be treated in a similar manner to the electron-nucleus attractive potential. In contrast, care must be taken to evaluate the polarization term $\hat{V}^{\text{pol}}$, because the EFP induced dipoles depend on the QM electron density.\textsuperscript{28}

In the remainder of this work, the notation DFT/EFP1 refers to the interface between DFT for a solute and EFP1 for the solvent. The DFT version of EFP1 is used throughout. Within the DFT/EFP1 framework, an element of the Fock matrix in the Kohn-Sham (KS) equation is given by

$$F_{pq\sigma} = h_{pq\sigma} + V^{\text{es}}_{pq\sigma} + V^{\text{pol}}_{pq\sigma} + V^{\text{rep}}_{pq\sigma} + V^{\text{xc}}_{pq\sigma} + \sum_{i\tau}[(pq\sigma | i\tau) - c_\tau \delta_{\sigma\tau}(pi\sigma | iq\sigma)], \tag{2}$$

where

$$(pq\sigma | rst) = \int \int dr_1 dr_2 \psi_{po\sigma}(r_1) \psi_{q\sigma}(r_1) \frac{1}{r_{12}} \psi_{r\tau}(r_2) \psi_{s\tau}(r_2).$$

As usual, indices $i, j, \cdots$ label occupied, $a, b, \cdots$ virtual, and $p, q, \cdots$ general molecular orbitals (MO), and orbitals are assumed to be real throughout the paper. Greek letters $\sigma$ and $\tau$ are spin labels. $c_\tau$ is the mixing weight of the Hartree-Fock exchange contribution. $h_{pq\sigma}$ is a one-electron Hamiltonian matrix element that consists of the kinetic energy and nuclear attraction, and $V^{\text{xc}}_{pq\sigma}$ is a matrix element of the exchange-correlation potential derived by the functional derivative with respect to the KS electron density $n_\sigma(r)$, in the MO basis,

$$V^{\text{xc}}_{pq\sigma} = \int \int dr \psi_{po\sigma}(r) \delta E^{\text{xc}}[n] \frac{\delta}{\delta n_\sigma(r)} \psi_{q\sigma}(r). \tag{3}$$

To obtain a simplified expression for the polarization potential $\hat{V}^{\text{pol}}$, a matrix formulation for the EFP induced dipoles\textsuperscript{30} is introduced. The induced dipoles at polarizable points are determined by solving the equation,

$$\mu = \alpha(E^{\text{nucl}} + E^{\text{el}} + E^{\text{fp}} + U\mu). \tag{4}$$

Here the collective variables are introduced: $(\mu)_k = \mu_k$, $(\alpha)_k = \alpha_k$, $(\sigma)_k = \sigma_k$, and $(U)_k = U_k(1 - \delta_{kl})$, where $\delta_{kl}$ is the Kronecker delta. The vector $\mu$ and matrices $\alpha$ and $U$ have the dimension three times the number of polarizable points, while $\mu_k$, $\alpha_k$, and $U_{kl}$ are the corresponding elements defined as a three-dimensional vector (or matrix). $\mu_k$ and $\alpha_k$ are the induced dipole and dipole polarizability tensor at polarizable point $k$, respectively. Note that the dipole polarizabilities in the EFP method are modeled with the asymmetric anisotropic tensors located at the centroids of the localized MOs. The
first three terms in parentheses in Eq. (4) are the electric fields generated by the QM nuclei, the QM electrons, and the EFP permanent multipoles, respectively, and the last term describes the contribution of the induced dipoles in other EFP molecules. \( U_{kl} \) is a \( 3 \times 3 \) symmetric tensor that describes the inter-fragment dipole-dipole interaction between points \( k \) and \( l \). To obtain the induced dipoles, Eq. (4) is solved self-consistently by a common iterative method or directly by applying the inverse matrix

\[
\mu = M^{-1}(E^{\text{nuc}} + E^{\text{el}} + E^{\text{efp}}) \equiv M^{-1}E, \quad \tilde{\mu} = (M^T)^{-1}E,
\]

where \( M \equiv \alpha^{-1} - U \) and the superscript \( T \) means the transpose. Note that the matrix \( M \) is not symmetric because of the intra-fragment matrix \( \alpha \).

The polarization energy of the EFP induced dipoles is given by

\[
E^{\text{pol}}[n] = -\frac{1}{2} E^T \mu
\]

\[
= -\frac{1}{2} \sum_{ki} \left( E^{\text{nuc}} + E^{\text{el}} + E^{\text{efp}} \right)^T \left( M^{-1} \right)_k \left( E^{\text{nuc}} + E^{\text{el}} + E^{\text{efp}} \right)_i
\]

\[
= -\frac{1}{2} E^T M^{-1} E
\]

(6)

where the summations over \( k \) and \( l \) in the second line are over the EFP polarization points. \( E^{\text{pol}}[n] \) has a quadratic functional dependence on the electron density due to the electric field \( E^{\text{el}} \). The polarization potential is defined in a similar way to the exchange-correlation potential by the functional derivative of the polarization energy functional, Eq. (6),

\[
V^{\text{pol}}_{\rho\sigma} = \int dr \psi_{\rho\sigma}(r) \frac{\delta E^{\text{pol}}[n]}{\delta n_{\sigma}(r)} \psi_{\rho\sigma}(r) = -\frac{1}{2} (\mu + \tilde{\mu})^T E^{\text{el}}_{\rho\sigma},
\]

(7)

where the definitions of induced dipoles \( \mu \) and \( \tilde{\mu} \) [Eq. (5)] are employed.

**B. TDDFT/EFP1 excitation energies**

Within the linear-response TDDFT method, \( 4 \) the excitation energy \( \Omega \) is obtained by solving the matrix equation,

\[
\left( \begin{array}{cc} A & B \\ B & A \end{array} \right) \left( \begin{array}{c} X \\ Y \end{array} \right) = \Omega \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \left( \begin{array}{c} X \\ Y \end{array} \right),
\]

(8)

where \( A \) and \( B \) are components of the coupling matrix and \( X \) and \( Y \) are components of the transition amplitude. Note that only spin-conserving blocks, i.e., \( \alpha \alpha \) and \( \beta \beta \), of \( X \) and \( Y \) are allowed to be nonzero. The TDDFT-EFP1 interaction terms modify the matrices \( A \pm B \) as follows:

\[
(A + B)_{\rho\sigma, j\tau} = (\epsilon_{\rho\sigma} - \epsilon_{\tau\sigma}) \delta_{ij} \delta_{\rho\sigma} + 2 (i \rho \sigma | \eta \tau)
\]

\[
+ f_{\rho\sigma, j\tau}^{\text{xc}} + c_{\rho\sigma} \delta_{\rho\sigma} \rho | i \eta \sigma | a \beta \sigma
\]

\[
+ (i \beta \sigma | j \alpha \sigma) + f_{\eta \sigma, j\tau}^{\text{pol}}
\]

(9)

and

\[
(A - B)_{\rho\sigma, j\tau} = (\epsilon_{\rho\sigma} - \epsilon_{\tau\sigma}) \delta_{ij} \delta_{\rho\sigma} - c_{\rho\sigma} \delta_{\rho\sigma} \times [(i \eta \sigma | a \beta \sigma) - (i \beta \sigma | j \alpha \sigma)],
\]

(10)

where \( \epsilon_{\rho\sigma} \) is the orbital energy obtained by solving the ground-state KS equation with the Fock operator defined in Eq. (2). \( f_{\rho\sigma, j\tau}^{\text{xc}} \) is the exchange-correlation kernel defined by the second derivative of the exchange-correlation energy functional,

\[
f_{\rho\sigma, j\tau}^{\text{xc}} = \int \int dr' \psi_{\rho}(r) \psi_{\tau}(r') \times \frac{\delta^2 E^{\text{xc}}[n]}{\delta n_\rho(r) \delta n_\tau(r')} \psi_{\rho}(r) \psi_{\tau}(r'),
\]

(11)

Similarly, the polarization kernel \( f_{\rho\sigma, j\tau}^{\text{pol}} \) is given by the second derivative of the polarization energy, Eq. (6),

\[
f_{\rho\sigma, j\tau}^{\text{pol}} = \int \int dr' \psi_{\rho}(r) \psi_{\tau}(r') \times \frac{\delta^2 E^{\text{pol}}[n]}{\delta n_\rho(r) \delta n_\tau(r')} \psi_{\rho}(r) \psi_{\tau}(r') \times \frac{1}{2} \frac{E^{\text{pol}}_{\rho\sigma}}{(M^{-1} + (M^T)^{-1}) F^{\text{pol}}_{j\tau}}
\]

(12)

where the operator \( F^{\text{pol}}_{j\tau} \) generates the electrostatic field at point \( k \) due to the QM electrons, and the summations over \( k \) and \( l \) in the second line are taken with respect to the EFP polarizable points. Note that the inter-fragment contribution appears through the off-diagonal elements of \( M^{-1} + (M^T)^{-1} \). In Ref. 28, the polarization kernel is represented by the polarizability tensor \( \alpha + \alpha^T \), and thus the inter-fragment contribution is neglected. Equations (5) and (12) indicate that this approximation becomes exact only in the single-fragment case.

A modified Davidson algorithm \( 5,46 \) is used to solve Eq. (8). Therefore, it is not necessary to perform the matrix multiplication for each \( j\tau-bt \) pair in Eq. (12). During the computation, the entire matrix \( 2 \) is not stored in memory. Rather, the array is constructed by multiplying the polarization kernel by any of the \( N \) trial vectors \( \{ b^{(n)} \}_{n=1}^N \) in the Davidson procedure,

\[
\sum_{j\tau} 2 f_{\rho\sigma, j\tau}^{\text{pol}} b^{(n)}_{j\tau} = - \left( F^{\text{pol}}_{\rho\sigma} \right) [(M^{-1} + (M^T)^{-1}) \text{Tr}(E^{\text{el}} b^{(n)})].
\]

(13)

Although the computation in Eq. (13) involves a relatively small number of matrix multiplications, the inversion of the \( M \) and \( M^T \) matrices is required. The inversion of these matrices is potentially difficult as the number of polarizable points can be very large. In the calculation of a QM solute with 100 EFP water molecules, for example, the dimension of \( M \) is 1500. Since it is desirable to avoid the computation of inverse matrices with large dimensions, the present implementation is based on the iterative method instead of the
matrix formulation. It is easy to derive an equation similar to Eq. (4) by rewriting Eq. (13) with the definition of $M$ and $M^T$ in mind,

$$
\mu^{(n)} = \alpha[\text{Tr}[E_b^{[n]}] + U \mu^{(n)}] \quad \text{for} \quad n = 1, \ldots, N.
$$

The dipole moments $\{\mu^{(n)}\}$ are determined self-consistently by the iterative method for each trial vector $b^{(n)}$. Using these dipole moments, Eq. (13) can be simplified to

$$
\sum f_{iaa, jbt}^{\mu} b_{jbt}^{(n)} = - (E_{iaa}^{\mu})^T [\mu^{(n)} + \tilde{\mu}^{(n)}].
$$

In this manner, the matrix inversions are eliminated.

### C. TDDFT/EFP1 analytical energy gradient

To derive the TDDFT analytic energy gradient, Furche and Ahlrichs$^8$ have introduced the following Lagrangian,

$$
L[X, Y, \Omega, C, Z, W] = G[X, Y, \Omega] + \sum_{iaa} Z_{iaa} F_{iaa} - \sum_{pq, p \leq q} W_{pq} (S_{pq} - \delta_{pq}),
$$

where the vector $C$ consists of MO coefficients $C_{\mu\sigma}$ ($\mu$ indexes the atomic basis function) and $S_{pq}$ is an element of the overlap matrix. As described in Ref. 7, the TDDFT excitation energy $\Omega$ is a stationary point of the excitation energy functional $G[X, Y, \Omega]$. The vector $Z$ enforces the condition that the occupied-virtual block of the Fock matrix is zero ($F_{iaa} = 0$). The Lagrange multipliers $Z$ and $W$ are determined from the stationary condition of the Lagrangian: $\partial L/\partial C_{\mu\sigma} = 0$.

To obtain the TDDFT/EFP1 gradient, it is necessary to solve the so-called $Z$-vector equation$^7$

$$
\sum_{iaa, jbt} (A + B)_{iaa, jbt} Z_{jbt} = - \sum_{\mu} \left( \frac{\partial G}{\partial C_{\mu\sigma}} C_{\mu\sigma} - \frac{\partial G}{\partial C_{\mu\sigma}} C_{\mu\sigma} \right) \equiv -R_{iaa}.
$$

The vector $R$ is determined by the occupied-virtual and virtual-occupied blocks of the excitation energy functional $G[X, Y, \Omega]$. As may be seen from the definition of $(A + B)_{iaa, jbt}$ in Eq. (9), the left-hand side of Eq. (17) contains the EFP polarization kernel $f_{iaa, jbt}^{\mu}$. To evaluate the EFP contribution to the right-hand side of Eq. (17), $-R_{iaa}$, the derivation by Furche and Ahlrichs$^8$ is followed. The unrelaxed difference density matrix $T$ is defined by

$$
T_{aba} = \frac{1}{2} \sum_i ((X + Y)_{iaa} (X + Y)_{iba} + (X - Y)_{iaa} (X - Y)_{iba}),
$$

$$
T_{ijs} = - \frac{1}{2} \sum_a ((X + Y)_{iaa} (X + Y)_{jas} + (X - Y)_{iaa} (X - Y)_{jas}),
$$

where $a, b$ are virtual orbitals and $i, j$ are occupied orbitals. $X$ and $Y$ are defined in Eq. (8). The addition of the EFP polarization kernel contribution modifies the linear transformations for arbitrary vectors $V$, as follows:

$$
H_{pq}^+ [V] = \sum_{r \neq s} [2(pq | rs) + 2 f_{pq, rrs} + 2 f_{pol, rrs}] - c_{r} \delta_{rs} [(ps | rq + (pr | sq))] V_{rst}
$$

When $V$ is the unit vector, i.e., 1 for the element $(r \neq s)$ and 0 otherwise, $H_{pq}^{rs} [V]$ and $H_{pq}^+ [V]$ lead to the integral-only part of $(A + B)_{pq, rrs}$ [terms 2–6 in Eq. (9)] and similarly that of $(A - B)_{pq, rrs}$ [the second and third terms in Eq. (10)]. Note that only $H_{pq}^+ [V]$ has an EFP1 contribution. Using these definitions, $\tilde{R}_{iaa}$ is evaluated as

$$
\tilde{R}_{iaa} = \sum_b ((X + Y)_{iba} H_{a}^{+} [X + Y]
$$

$$
+ (X - Y)_{iba} H_{a}^{-} [X - Y])
$$

$$
- \sum_j ((X + Y)_{jas} H_{a}^{+} [X + Y]
$$

$$
+ (X - Y)_{jas} H_{a}^{-} [X - Y]) + H_{iaa}^+ [T]
$$

$$
+ 2 \sum_{jba', kca'} \delta_{iab', kca'} ((X + Y)_{jab'} (X + Y)_{kca'} - (X - Y)_{jab'} (X - Y)_{kca'}).
$$

The EFP polarization contributes to the first, second, and third terms of Eq. (20) through the linear transformation [Eq. (19)] of $X + Y$ and $T$: $H_{a}^{+} [X + Y]$, $H_{a}^{-} [X + Y]$, and $H_{iaa}^+ [T]$. The last term in Eq. (20) involves the matrix elements of the third-order functional derivative of the exchange-correlation energy,

$$
\delta_{iaa, jbt', kca'} g_{xc}^{\mu}(x^{\mu}, y^{\mu}, z^{\mu}) = \int \int d\mathbf{r} d\mathbf{r}' \psi_{iaa}(\mathbf{r}) \psi_{jbt'}(\mathbf{r}') \psi_{kca'}(\mathbf{r}').
$$

Note that the third-order derivative of the polarization energy vanishes because the polarization kernel, Eq. (12),

$$
\frac{\delta^3 W}{\delta n_{\mu}(\mathbf{r}) \delta n_{\sigma}(\mathbf{r}') \delta n_{\tau}(\mathbf{r}'')}
$$

$$
\times \psi_{iaa}(\mathbf{r}) \psi_{jbt'}(\mathbf{r}') \psi_{kca'}(\mathbf{r}'').
$$
is independent of the electron density. To summarize, the original TDDFT Z-vector equation can be extended to a TDDFT/EFP1 Z-vector equation by adding the computation of $2f_{\text{pol}}$ multiplied by vectors on both sides of Eq. (17). This has already been implemented in the TDDFT/EFP1 energy calculations.\textsuperscript{28}

Furche and Ahlrichs\textsuperscript{8} derived the analytic energy gradient of the excitation energy with respect to a QM nuclear coordinate $\xi$, expressed in the atomic orbital (AO) basis as

$$
\Omega^\xi = \sum_{\mu\nu\sigma} (h_{\mu\nu}^{\xi} + V_{\mu\nu}^{\xi} + V_{\mu\nu}^{\text{rep},\xi}) P_{\mu\nu\sigma} - \sum_{\mu\nu\sigma} S_{\mu\nu}^{\xi} W_{\mu\nu\sigma} + \sum_{\mu\nu\sigma,\kappa\lambda\tau} (\mu\nu|\kappa\lambda)^{2} \Gamma_{\mu\nu\sigma,\kappa\lambda\tau}^{\text{}\xi} + \Omega^{\text{xc},\xi} + \Omega^{\text{pol},\xi}, \quad (22)
$$

where $\mu$, $\nu$, $\kappa$, and $\lambda$ label the AO basis sets. The matrix element $P_{\mu\nu\sigma}$ is calculated from the corresponding quantity in the MO representation using $P_{\mu\nu\sigma} = \sum_{pq} C_{\mu\rho\sigma} P_{pq\sigma} C_{\nu\sigma\rho}$, and a similar transformation is applied to obtain $(X \pm Y)_{\mu\nu\sigma}$ and $W_{\mu\nu\sigma}$. In the present work, $\xi$ can also represent an EFP translation or EFP rotation. $P_{\mu\nu\sigma} = T_{\mu\nu\sigma} + z_{\mu\nu\sigma}$ is the relaxed density matrix, which is the difference between the excited and ground-state density matrices, and $\Gamma_{\mu\nu\sigma,\kappa\lambda\tau}^{\text{}\xi}$ is an element of the effective two-particle difference density matrix [see Eq. (27) of Ref. 8]. $h_{\mu\nu}^{\xi}$, $S_{\mu\nu}^{\xi}$, and $(\mu\nu|\kappa\lambda)^{2}$ are the derivative of the one-electron gas-phase Hamiltonian, the overlap matrix, and the two-electron integrals, respectively. Equation (22) includes the gradients of the EFP1 electrostatic and repulsive terms as well as those of the polarization interaction. The energy-weighted density matrix $W$ is given in terms of the MO basis as follows:

$$
(1 + \delta_{ij})W_{ij\sigma} = \sum_{a} \Omega \left[(X + Y)_{i\alpha\sigma}(X - Y)_{j\alpha\sigma} + (X - Y)_{i\alpha\sigma}(X + Y)_{j\alpha\sigma}\right] \\
- \sum_{a} \epsilon_{i\sigma}(X + Y)_{i\alpha\sigma}(X + Y)_{j\alpha\sigma} + (X - Y)_{i\alpha\sigma}(X - Y)_{j\alpha\sigma} \\
+ H_{ij\sigma}^{\text{}\xi}(P) + 2 \sum_{k\alpha\sigma,\lambda\delta\sigma^\prime} g_{ij\sigma,\kappa\lambda\sigma^\prime,\lambda\delta\sigma^\prime}^{\text{}\xi}(X + Y)_{k\alpha\sigma}(X + Y)_{\lambda\delta\sigma^\prime} \\
(1 + \delta_{ab})W_{a\beta\sigma} = \sum_{i} \Omega \left[(X + Y)_{i\alpha\sigma}(X - Y)_{i\beta\sigma} + (X - Y)_{i\alpha\sigma}(X + Y)_{i\beta\sigma}\right] \\
+ \sum_{i} \epsilon_{i\sigma}(X + Y)_{i\alpha\sigma}(X + Y)_{i\beta\sigma} + (X - Y)_{i\alpha\sigma}(X - Y)_{i\beta\sigma} \\
W_{i\alpha\sigma} = \sum_{f} ((X + Y)_{j\alpha\sigma} H_{j\sigma}^{\text{}\xi}(X + Y) + (X - Y)_{j\alpha\sigma} H_{j\sigma}^{\text{}\xi}(X - Y)) + \epsilon_{i\sigma} Z_{i\alpha\sigma}.
$$

In Eq. (23), the bold term in square brackets (e.g., $P$, $X+Y$) indicates arguments of the respective functions. The EFP polarization kernel contributes to the occupied-occupied and occupied-virtual blocks of $W$ through the linear transformation [Eq. (19)] of $P$ and $X+Y$: $H_{ij\sigma}^{\text{}\xi}(P)$ and $H_{ij\sigma}^{\text{}\xi}(X + Y)$. The gradient of the exchange-correlation part is given by

$$
\Omega^{\text{xc},\xi} = \sum_{\mu\nu\sigma} V_{\mu\nu\sigma}^{\text{}\xi}(P) P_{\mu\nu\sigma} + \sum_{\mu\nu\sigma,\kappa\lambda\tau} f_{\mu\nu\sigma,\kappa\lambda\tau}^{\text{}\xi}(X + Y)_{\mu\nu\sigma}(X + Y)_{\kappa\lambda\tau}. \quad (24)
$$

Here, the superscript in parentheses ($\xi$) on the right-hand side indicates that the derivatives are evaluated with respect to $\xi$ but keeping the MO coefficients constant at their zeroth-order values. The details of the evaluation of Eq. (24) are presented in Ref. 18.

The gradient of the EFP polarization term is

$$
\Omega^{\text{pol},\xi} = \sum_{\mu\nu\sigma} V_{\mu\nu\sigma}^{\text{pol},\xi}(P) P_{\mu\nu\sigma} + \sum_{\mu\nu\sigma,\kappa\lambda\tau} f_{\mu\nu\sigma,\kappa\lambda\tau}^{\text{pol},\xi}(X + Y)_{\mu\nu\sigma}(X + Y)_{\kappa\lambda\tau}. \quad (25)
$$

The first term includes the change in the one-electron density matrix

$$
\sum_{\mu\nu\sigma} V_{\mu\nu\sigma}^{\text{pol},\xi}(P) P_{\mu\nu\sigma} = -\frac{1}{2} (\mathbf{\mu} + \bar{\mathbf{\mu}})^{T} \mathbf{E}^{\xi}(\mathbf{\xi}) - \frac{1}{2} (\mathbf{\mu}^\Delta + \bar{\mathbf{\mu}})^{T} \mathbf{E}^{\xi}(\mathbf{\xi}) \\
+ \frac{1}{2} (\bar{\mathbf{\mu}})^{T} \frac{\partial \mathbf{M}}{\partial \mathbf{\xi}} \mathbf{\mu} + \frac{1}{2} \mathbf{\mu}^{T} \frac{\partial \mathbf{M}}{\partial \mathbf{\xi}} \mathbf{\mu}^\Delta. \quad (26)
$$

Here $\mathbf{E}^{\Delta} = \text{Tr}(\mathbf{PE}^{\Delta})$ is the solute electric field due to the difference electron density matrix, and $\mathbf{\mu}^\Delta = \mathbf{M}^{-1} \mathbf{E}^{\Delta}$. The second term in Eq. (25) is specific to the linear-response theory.
and results from the derivative of the polarization kernel in the matrix \((A + B)\),

\[
\sum_{\mu\nu,\kappa\lambda\tau} f^{\text{pol.}(\xi)}_{\mu\nu,\kappa\lambda\tau}(X + Y)_{\mu\nu\sigma}(X + Y)_{\kappa\lambda\tau} = -(\mu X + Y + \tilde{\mu} X + Y)^T E^{x + Y,\xi} + (\mu X + Y) \frac{\partial M}{\partial \xi} \mu X + Y.
\]

(27)

In Eq. (27), \(E^{x + Y} = \text{Tr}[(X + Y)E^{\text{pol}}]\) is the electric field contribution related to the transition density, and \(\mu X + Y = M^{-1} E^{x + Y}\).

Finally, the gradient of the excitation energy obtained above is added to that of the ground-state energy,

\[
E^\xi = \sum_{\mu\nu\sigma} (h^\xi_{\mu\nu} + V^{\text{es},\xi}_{\mu\nu} + V^{\text{rep},\xi}_{\mu\nu}) D_{\mu\nu\sigma} - \sum_{\mu\nu\sigma} S^\xi_{\mu\nu} W'_{\mu\nu\sigma}
+ \sum_{\mu\nu,\kappa\lambda\tau} (\mu \nu | \kappa \lambda)^T \Gamma'_{\mu\nu,\kappa\lambda\tau} + E^{\text{exc},\xi} + E^{\text{pol},\xi},
\]

where \(D_{\mu\nu\sigma}\) is a ground-state density matrix element and \(\Gamma'_{\mu\nu,\kappa\lambda\tau}\) is the ground-state two-particle density matrix,

\[
\Gamma'_{\mu\nu,\kappa\lambda\tau} = \frac{1}{2} \left( D_{\mu\nu\sigma} D_{\kappa\lambda\tau} - c_{\delta\sigma\tau} D_{\mu\delta\sigma} D_{\nu\lambda\sigma} \right).
\]

Note that the superscript in parentheses \((\xi)\) appears in the last two terms, and the MO coefficients are held constant in the gradient evaluation. It is not necessary to compute the derivatives of MO coefficients because these terms are absorbed in the energy-weighted density matrix (the second term).25 The last term in Eq. (28), the gradient of the ground-state polarization energy, is easily derived by differentiating Eq. (6) with respect to the coordinate \(\xi\),

\[
E^{\text{pol}}_{\mu\nu\sigma}(\xi) = -\frac{1}{2}(\mu + \tilde{\mu})^T [E^{\text{nuc},\xi} + E^{\text{pol},\xi}] + \frac{1}{2} \tilde{\mu}^T \frac{\partial M}{\partial \xi} \mu.
\]

(29)

In summary, the TDDFT/EFP1 gradient can be computed easily by modifying the ground-state DFT/EFP1 gradient as follows: (a) electrostatic interaction between QM electrons and EFP permanent multipoles:

\[
\text{Tr}[D V^{\text{ex},\xi}] \rightarrow \text{Tr}[(D + P) V^{\text{ex},\xi}],
\]

(30)

(b) repulsive interaction between QM electrons and EFP molecules:

\[
\text{Tr}[D V^{\text{rep},\xi}] \rightarrow \text{Tr}[(D + P) V^{\text{rep},\xi}],
\]

(31)

and polarization contribution due to (c) QM nuclei-EFP induced dipoles:

\[
-\frac{1}{2}(\mu + \tilde{\mu})^T E^{\text{nuc},\xi} \rightarrow -\frac{1}{2}(\mu + \tilde{\mu} + \mu^\Delta + \tilde{\mu}^\Delta)^T E^{\text{nuc},\xi},
\]

(32)

(d) QM electrons-EFP induced dipoles:

\[
-\frac{1}{2}(\mu + \tilde{\mu})^T E^{\text{pol},\xi} \rightarrow -\frac{1}{2}(\mu + \tilde{\mu})^T [E^{\text{pol},\xi} + E^{\Delta,\xi}],
\]

\[
-\frac{1}{2}(\mu^\Delta + \tilde{\mu}^\Delta)^T E^{\text{pol},\xi} \rightarrow -(\mu^{x + Y} + \mu^{x + Y})^T E^{x + Y,\xi},
\]

(33)

(e) EFP permanent multipoles-EFP induced dipoles

\[
-\frac{1}{2}(\mu + \tilde{\mu})^T E^{\text{pol},\xi} \rightarrow -\frac{1}{2}(\mu + \tilde{\mu} + \mu^\Delta + \tilde{\mu}^\Delta)^T E^{\text{pol},\xi},
\]

(34)

and (f) EFP induced dipoles-EFP induced dipoles

\[
-\frac{1}{2}(\mu^\Delta + \tilde{\mu}^\Delta)^T E^{\text{pol},\xi} \rightarrow -(\mu^{x + Y} + \mu^{x + Y})^T E^{\text{pol},\xi}.
\]

(35)

Equations (34) and (35) are evaluated only when multiple fragments are present. All these terms can be computed with the existing code by modifying the arguments for the density matrix and the induced dipoles.

D. Corrected linear response method

The EFP induced dipoles optimized for a given excited state are not obtained by the linear-response TDDFT/EFP1 method, as it is the ground-state density that is used for the evaluation of the terms in Eq. (8). In addition, there is no density dependence of \(f^{\text{pol}}_{\text{far-fer}}\) as given in Eq. (12), assuming the ground-state Kohn-Sham orbitals are fixed. Thus, the electronic relaxation of EFP molecules interacting with the excited state density is not taken into account. Here, the solvent electronic relaxation effect is evaluated using the corrected linear response (cLR) scheme originally proposed in Ref. 19 for the TDDFT/PCM method as an estimate of the correction to the linear-response excitation energy. A similar correction scheme has been proposed in the interface of the EFP solvent model with the equation-of-motion coupled cluster with single and double excitations.26 Note that it has been demonstrated recently31,32 that solvent effects on electronic excitations are largely determined by the modification of the ground state density by the solvent. Therefore, the impact of the modification of the excited state density by the solvent is expected to be small, and the use of an estimate for this small correction is expected to be reasonable.

The ground-state equilibrium energy is obtained as:

\[
G^0_0 = \langle \Psi_0 | \hat{H}_{\text{gas}} + \hat{V}^{\text{es}} + \hat{V}^{\text{rep}} - \frac{1}{2}(\mu_0 + \tilde{\mu}_0)^T E^{\text{exc}} | \Psi_0 \rangle
- \frac{1}{2}(\mu_0 + \tilde{\mu}_0)^T (E^{\text{nuc}} + E^{\text{pol}}) + \frac{1}{2}(\tilde{\mu}_0)^T E_0,
\]

\[
\equiv G_0 + \frac{1}{2}(\tilde{\mu}_0)^T E_0
\]

(36)

where \(E_0 = E^{\text{nuc}} + E^{\text{pol}} + E^{\text{exc}}\) and \(E^{\text{exc}} = \langle \Psi_0 | \hat{E}^{\text{exc}} | \Psi_0 \rangle\). Here, the induced dipoles \(\{\mu_0, \tilde{\mu}_0\}\) are determined to be
self-consistent among themselves and the QM electron density in the ground state.

If the induced dipoles are kept frozen upon excitation, the non-equilibrium energy of excited state $I$ would be given as:

$$G_{\text{eq}}^{\text{neq}} = \langle \Psi_I | \hat{H}_{\text{gas}} + \hat{V}^{\text{ex}} + \hat{V}^{\text{rep}} - \frac{1}{2} (\mu_0 + \mu_0) (\mathbf{E}^{\text{rez}} + \mathbf{E}^{\text{efp}} \rangle$$

When the induced dipoles are relaxed in the excited state, the excited-state energy becomes

$$G_{\text{eq}}^{\text{eq}} = \langle \Psi_I | \hat{H}_{\text{gas}} + \hat{V}^{\text{ex}} + \hat{V}^{\text{rep}} - \frac{1}{2} (\mu_I + \mu_I) (\mathbf{E}^{\text{rez}} + \mathbf{E}^{\text{efp}}) + \frac{1}{2} (\mu_I + \mu_I)^2 \mathbf{E}_{I},$$

$$\equiv G_I + \frac{1}{2} (\mu_I)^2 \mathbf{E}_{I} \quad (38)$$

where $\mathbf{E}_{I} = \mathbf{E}^{\text{rez}} + \mathbf{E}^{\text{efp}}$ and $\mathbf{E}_{I}^{\text{eq}} = \langle \Psi_I | \hat{\mathbf{E}}^{\text{eq}} \rangle$. Note the difference between Eqs. (37) and (38). In the former, the induced dipoles are treated as an external perturbation while in the latter the set of $\{\mu_I, \mu_I\}$ is determined to be self-consistent among themselves and the QM electrons in the excited state $I$. These correspond to method 2 and the fully self-consistent method, respectively, in Ref. 31. The relaxation of the solvent electronic polarization is estimated by

$$G_{\text{eq}}^{\text{eq}} - G_{\text{eq}}^{\text{neq}} = -\frac{1}{2} (\mu_0 - \mu_0^2) (\mathbf{E}^{\text{rez}} + \mathbf{E}^{\text{efp}} + \mathbf{E}^{\text{rez}}).$$

In deriving Eq. (39), $\mathbf{E}_{I} - \mathbf{E}_{0}$ and $\mu_I - \mu_0$ are replaced by $\mathbf{E}^{\text{rez}}$ and $\mu_0^2$ [see the definitions given below Eq. (26)], respectively, because the difference in the density matrices between the two states may be thought of as that of the difference density matrix within the LR-TDDFT framework. Using the relation $(\mu_0^2)^T \mathbf{E}_0 = (\mathbf{E}^{\text{rez}})^T \mathbf{E}_0$, the excited-state energy is obtained as follows:

$$G_I^{\text{eq}} = G_{\text{eq}}^{\text{eq}} + (G_{\text{eq}}^{\text{neq}} - G_0) - \frac{1}{2} (\mu_0^2)^T \mathbf{E}^{\text{rez}}. \quad (40)$$

The linear-response TDDFT/EFP1 method evaluates the excitation energy in the presence of frozen induced dipoles, i.e., the ground-state KS Fock operator using $\{\mu_0, \mu_0\}$. Therefore, the second term of Eq. (40) may be thought of as the TDDFT/EFP1 excitation energy and the last term as the energy correction. The correction scheme is applied after the relaxed density matrix is obtained.

### III. EXCITED-STATE MD SIMULATION ON AQUEOUS FORMALDEHYDE

In this Section, the TDDFT/EFP1 method is applied to the excited-state MD simulation of aqueous formaldehyde in the $n$-$\pi^*$ state. As is well known, the $n\rightarrow\pi^*$ transition of formaldehyde is electric dipole forbidden in C$_2v$ symmetry and has a very small oscillator strength due to vibronic coupling. Since there are no experimental data available for aqueous formaldehyde in the $n$-$\pi^*$ state, the calculated results are compared with those reported for other theoretical studies.

#### A. MD simulation protocol

The TDDFT/EFP1 gradient code was implemented in the GAMESS (General Atomic and Molecular Electronic Structure System) package. The B3LYP hybrid functional and a double zeta plus polarization (DZP) quality basis set were employed in all of the QM calculations. The solute (formaldehyde) was placed in 100 water molecules, and the entire system was treated as a cluster. The DFT-based EFP1 water model was used for the solvent molecules. All simulations were performed at a constant temperature of 300 K by the Nosé-Hoover thermostat, and a time step of 1 fs. The TDDFT/EFP1 MD simulations were performed for both the ground and $n$-$\pi^*$ states. For the ground-state MD of solvated (isolated) formaldehyde, the system was equilibrated for 5 (5) ps and the production run was performed for 334 (400) ps. To examine the ground-state properties and absorption spectra, 667 (800) different snapshots were taken from the MD simulations. For the $n$-$\pi^*$ excited-state MD, the system was equilibrated for 5 (5) ps, and a 354 (396) ps production run was performed. To analyze the fluorescence spectra, 886 (990) snapshots were taken from the gas-phase TDDFT/EFP1 MD simulation in the $n$-$\pi^*$ state.

#### B. Accuracy of TDDFT/EFP1 analytic gradient

Before describing the MD simulation results, consider the accuracy of the present implementation of TDDFT/EFP1 analytic energy gradients. The analytic gradient method was applied to the H$_2$CO-2(H$_2$O) cluster. For comparison, numerical gradient calculations were also performed using a five-point numerical differentiation formula. A QM atom was displaced along a Cartesian coordinate, while the molecular translation and rotation was examined for the EFP1 water due to the frozen internal geometry. The numerical gradients were evaluated with a translational step size of 0.001 bohr and a rotational step size of 0.001 radian. The comparison was made for several configurations. In all cases, the analytic gradients are in very good agreement with the numerical gradients: the difference is $\sim 10^{-6}$ a.u. for the EFP translation and rotation coordinates and $\sim 10^{-3}$ a.u. for the QM atomic coordinates. Note that the default convergence tolerance employed in the geometry optimization is $1 \times 10^{-4}$ hartree/bohr. The small difference between the analytic and numerical gradients indicates that the present implementation of analytic energy gradients is correct.

Further test calculations were performed on acetamide (CH$_3$CONH$_2$) and the CH$_3$CONH$_2$-2(H$_2$O) cluster (see Fig. 1). The geometries were optimized for the $n$-$\pi^*$ state at the TD-B3LYP/DZP level of theory. In the cluster, the two water molecules are treated as QM or EFP1. The TDDFT/EFP1 (full TDDFT) calculation on the acetamide cluster provides an $\pi^*$ emission energy of 3.20 (3.21) eV.
which is slightly blue-shifted with respect to that of the isolated acetamide, 3.23 eV. Notably, the TDDFT/EFP1 value is comparable to the full TDDFT estimate. Table I summarizes the n-π* state geometric parameters and vibrational frequencies of acetamide and the acetamide cluster. The vibrational frequencies were obtained by the numerical differentiation of the analytic energy gradients, and no imaginary frequency was observed. The TDDFT/EFP1 results are in good agreement with those obtained by the full TDDFT. Therefore, the present implementation of TDDFT/EFP1 analytic energy gradients is correct for both geometry optimizations and force constant matrix calculations.

C. Absorption spectra of formaldehyde in gas phase and in solution

Table II summarizes the average ground state geometric parameters of formaldehyde. The computed ground-state geometry shows good agreement with experimental results.\(^{54}\) To examine the solvent effects on the solute electronic structure, the average geometric parameters are determined using the snapshots taken from the simulations of isolated and solvated formaldehyde. As clearly seen in the geometry and dipole moment, the presence of water molecules significantly affects the solute electronic structure, as embodied in the dipole moment.

<table>
<thead>
<tr>
<th>n-π* state</th>
<th>Gas (0 K)</th>
<th>Gas (300 K)</th>
<th>Experimental</th>
<th>Solution (300 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>r(π)</td>
<td>1.213</td>
<td>1.21 (0.02)</td>
<td>1.20 (0.03)</td>
<td>1.22 (0.02)</td>
</tr>
<tr>
<td>r(CH)</td>
<td>1.111</td>
<td>1.12 (0.03)</td>
<td>1.099 (0.009)</td>
<td>1.10 (0.03)</td>
</tr>
<tr>
<td>r(HCO)</td>
<td>115.9</td>
<td>116 (4)</td>
<td>116.5 (1.2)</td>
<td>117 (4)</td>
</tr>
<tr>
<td>r(O-CCH)</td>
<td>0.0</td>
<td>4 (3)</td>
<td></td>
<td>4 (3)</td>
</tr>
<tr>
<td>Dipole</td>
<td>2.35</td>
<td>2.34 (0.09)</td>
<td></td>
<td>3.15 (0.26)</td>
</tr>
</tbody>
</table>

\(^{a}\)Experimental estimation based on the infrared measurement (Ref. 54).

but has very little effect on the geometric structure. The average dipole moment is increased by 0.81 D with respect to the gas-phase average value of 2.34 D. In addition, the distribution of dipole moments becomes much broader; the standard deviation is increased by 0.17 D.

For the optimized gas phase ground-state geometry, the TDDFT method predicts a vertical transition energy of 3.94 eV, in good agreement with the experimental value of 4.07 eV.\(^{35}\) In order to take into account the thermal motion, the gas-phase absorption spectrum was constructed using MD snapshots. The resulting distribution of excitation energies is well represented by a Gaussian curve. Figure 2 shows the simulated absorption spectra based on the procedure above, and the average peak values are summarized in Table III. The average excitation energy in the gas phase at 300 K is estimated to be 3.91 ± 0.12 eV.

Next, consider the aqueous formaldehyde system. It is not the main purpose of the present paper to simulate the absorption spectrum. However, the TDDFT/EFP1 polarization kernel derived in the present work is more rigorous than the simplified formula in Ref. 28, and it is interesting to examine the error introduced by the approximation to the polarization kernel. In addition, the electronic relaxation of EFP induced dipoles in the n-π* excited state is estimated by the cLR approach.

The vertical excitation energies are calculated using the snapshots taken from the 300K QM(B3LYP/DZP)/EFP1 simulation for the ground state. The average excitation energy in water is calculated to be 4.25 ± 0.19 eV, and the solution-phase absorption spectrum has a slightly broader Gaussian distribution compared to the gas-phase spectrum; the standard deviation is increased by 0.07 eV. First, the approximation to the polarization kernel is examined. For that purpose, the TDDFT/EFP1 energy is calculated by setting M = \(\alpha^{-1}\) in Eq. (12). This simplified formula is identical with that in
Ref. 28 and provides an excitation energy of 4.27 ± 0.18 eV, which is increased only by 0.02 eV with respect to that obtained by the present more rigorous TDDFT/EFP1 excitation energy formula (4.25 ± 0.19 eV). The small energy difference indicates a weak dependence of the excitation energy on the expression of polarization kernel. Yoo et al. 28 have discussed the direct and indirect contributions of EFP1 solvent molecules to the TDDFT excitation energy. The former comes from the polarization kernel $f_{\text{pol}}$ in the coupling matrix given in Eq. (9) and the latter results from the other components in the coupling matrix such as orbital energies and the exchange-correlation kernel. The present results imply that the indirect component is the dominant factor in determining the TDDFT/EFP1 excitation energies. Although the computed excitation energies are slightly different, the approximation employed in Ref. 28 is very useful in that the simplification can reduce the computational cost dramatically. The TDDFT/EFP1 energy computation based on the present formula requires a number of iterations for each Davidson trial vector to obtain the self-consistent induced dipole moments [see Eq. (14)]. In a serial run using a 2.66 GHz workstation, for example, it takes 31 (16) seconds to obtain the TDDFT excitation energy and the response density by the present (previous approximate) method. Interestingly, the corresponding gas-phase computation requires 15 seconds.

Now, consider the cLR method applied to estimate the relaxation of the EFP induced dipoles. Table III shows the TDDFT/EFP1 excitation energies obtained with and without the cLR approach. Evidently, the cLR method makes a negligible contribution (~0.01 eV) to the excitation energies. This may be attributed to the use of the explicit EFP solvent model. The EFP1 water has permanent multipoles for describing the electrostatic interaction. The excitation energy in solution reflects not only the difference in the solvent electronic polarization between the ground and excited states, but also that arising from the solute-solvent electrostatic interaction. In the following discussion, the transition energies in the fourth column in Table III are employed. The excitation energy difference between the gas and solution phases, the solvatochromic shift, is calculated to be 0.34 eV, which is comparable to the experimental value of 0.21 eV for acetone 26 as well as to previous theoretical results. 20, 34–42

D. Fluorescence spectra of formaldehyde in gas phase and in solution

The average geometrical parameters of formaldehyde in the $n$-$\pi^*$ excited state are tabulated in Table II. Compared with the ground-state optimized structure, the most important change is the pyramidalization of the carbonyl group; the out-of-plane angle from the O=C bond to the CH$^+$ plane ($\angle$O-CHH) is about 30°. The change in out-of-plane angle is due to the electron migration from the oxygen lone pair to the $\pi^*$ orbital, and the $sp^2$ carbonyl carbon atom gains some additional $p$ character. Furthermore, the C=O bond is stretched by 0.1 Å and the $\angle$HCO angle decreases by 5°. The solvent effects hardly modify the $n$-$\pi^*$ state solute geometry; the difference in geometric parameters between the gas and aqueous solution is less than 0.01 Å and 1°. Both in the gas and solution phases, the dipole moment is much smaller in the excited state than in the ground state; the average dipole moment decreases by 0.66 and 0.94 D for the isolated and solvated formaldehyde, respectively.

The gas-phase fluorescence spectrum of formaldehyde is calculated using the snapshots from the TDDFT MD simulation in the $n$-$\pi^*$ state at 300 K. The gas-phase average emission energy is estimated to be 3.00 ± 0.25 eV, which gives a Stokes shift (the difference between the absorption and emission energies) of 0.91 eV. As shown in Fig. 2, the fluorescence spectrum deviates strongly from a Gaussian distribution. To estimate the asymmetry of the distribution of excitation energies and geometric parameters, skewness is introduced for the distribution of variable $x$ as follows,

$$\sum_{i=1}^{N_{\text{data}}} \frac{(x_i - \bar{x})^3}{N_{\text{data}} \sigma_x^3}, \quad (41)$$

where, $x_i$ is the value of $x$ for sample $i$ and $N_{\text{data}}$ is the total number of data points. $\bar{x}$ and $\sigma_x$ are the average and

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TABLE III. Average vertical transition energies (eV) of formaldehyde in vacuum and aqueous solution obtained by the (TD)DFT (B3LYP/DZP) method. The standard deviations are given in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Gas (0 K)</th>
<th>Gas (300 K)</th>
<th>Solution (300 K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$-$\pi^*$</td>
<td>3.94</td>
<td>3.91 (0.12)</td>
<td>4.25 (0.19)</td>
</tr>
<tr>
<td>$\pi^*$-$\pi$</td>
<td>2.96</td>
<td>3.00 (0.25)</td>
<td>3.03 (0.27)</td>
</tr>
</tbody>
</table>

*Corrected linear response method, Eq. (40).
standard deviation of the distribution \( x \), respectively. A positive (negative) skewness means that the right (left) tail is longer. In the gas phase, the fluorescence spectrum has a skewness of \(-0.43\), which is in sharp contrast to the corresponding absorption spectrum \((\sim 0.06)\). The deviation from a Gaussian distribution is attributed to the intramolecular contribution of formaldehyde: the emission energy has a strong negative correlation with the out-of-plane motion of formaldehyde (Fig. 3). As the molecule is distorted from the planar conformation, the ground state is destabilized, and the ground-state energy increase leads to the decrease in the emission energy.

In aqueous solution, the vertical emission energies are computed by taking the configurations from the 300 K TDDFT/EFP1 simulation for the \( n-\pi^* \) state. As in the case of absorption energies, the cLR approach in the TDDFT/EFP1 method gives a negligible contribution \((\sim 0.01\ \text{eV})\), see Table III) to the emission energy. Therefore, the values in the fourth column in Table III are employed in the following discussion. The average emission energy in aqueous solution is calculated to be \( 3.03 \pm 0.27\ \text{eV} \), and the resultant Stokes shift is \( 1.22\ \text{eV} \), which is increased by \( 0.31\ \text{eV} \) compared to the gas phase value \((0.91\ \text{eV})\). A very small solvatochromic shift \((0.03\ \text{eV})\) is obtained by subtracting the gas-phase average emission energy at 300 K \((3.00\ \text{eV})\). The solvatochromic shift of the emission spectrum \((0.03\ \text{eV})\) is much smaller than that of the absorption spectrum \((0.34\ \text{eV})\).

The shape of the emission spectrum is not altered by the solvent effects. The standard deviation of the emission spectrum in water is \(0.27\ \text{eV} \), which is comparable to that in the gas phase \((0.25\ \text{eV})\), and the spectral shape remains asymmetric though the degree of asymmetry is decreased (skewness of \(-0.15\)). To evaluate the solvent fluctuation effects on the absorption and emission energies, a configuration-dependent solvatochromic shift is computed for each snapshot; the emission (absorption) energy is calculated with and without the EFP molecules for each snapshot taken from the TDDFT/EFP1 \( n-\pi^* \) state (DFT/EFP1 ground state) simulation in water. Figure 4 plots the correlation between the transition energies in water and the configuration-dependent solvatochromic shift obtained for each snapshot. The transition energy, or the potential energy gap in solution, is often employed as an effective solvation coordinate in electron-transfer theory, in which solvent fluctuations play a vital role. A strong correlation is clearly seen in the absorption spectrum: the positive (negative) side with respect to the average absorption energy has a tendency to enhance (diminish) the blueshift. As discussed in the previous study on the acetone-water system, the solvation coordinate (absorption energy) reflects the strength of hydrogen bonds formed mainly between the carbonyl oxygen and water hydrogen. In contrast, the solvatochromic shift is nearly independent of the solvation coordinate in the emission spectrum; the distribution is nearly uniform, and the absolute solvatochromic shift values are smaller than those of the absorption spectrum. Thus, solvent fluctuations have a marginal effect on the distribution of emission energies, and this weak solvent perturbation accounts for the similarity between the gas and solution spectra. Rather, the emission energy distribution in solution is determined mainly by the geometry fluctuations of the solute formaldehyde as shown in Fig. 3: there exists much stronger correlation between the out-of-plane motion of formaldehyde and the emission energy.

The solvatochromic shift of the \( \pi^* \rightarrow n \) emission for formaldehyde has been studied using various solvation models. These works estimate the solvatochromic shifts by using the emission energy of the isolated formaldehyde at 0 K. Therefore, the corresponding value obtained by the TDDFT/EFP1 method, 0.07 eV, is adopted to compare with the previous studies. Some continuum model studies on the aqueous formaldehyde system have been performed with a variety of QM methods. Sánchez et al. obtained a blueshift of 0.082 and 0.033 eV using the configuration interaction singles (CIS) and CI singles and doubles (CISD) methods with the nonequilibrium continuum model. Improta et al. have reported a blueshift of 0.16 and 0.09 eV using the TDDFT/PCM method with the state-specific and linear-response approaches. The present TDDFT/EFP1 shift is very close to these values. The good agreement is partly due to the weak solute-solvent interaction in the \( n-\pi^* \) state, and the absence of local solvent structure, such as hydrogen bonds, may
improve the accuracy of dielectric continuum approaches. This rationale is corroborated by comparing the \( n \rightarrow \pi^* \) excitation energies; the TDDFT(PBE0)/6-31G(d)/PCM method gives a solvatochromic shift of 0.07 eV\(^2\) while the present TDDFT(B3LYP)/DZP/EFP1 method provides 0.34 eV.

Discrete solvent models have been employed as well for evaluating the solvatochromatic shift. Coutinho and Canuto\(^4\) have provided a shift of 0.20 eV using cluster calculations based on the semi-empirical intermediate neglect of differential overlap (INDO) with the CIS method. Öhrn and Karlström\(^4\) have obtained a very small blue shift of 0.003 eV using a QM (the complete active space state interaction method, CASSI)/MM Monte Carlo simulation. Another approach is to treat the whole solute-solvent system quantum mechanically by dividing into small fragments. Recently, Chiba et al.\(^5\) have developed the analytical energy gradient for the fragment molecular orbital (FMO) based TDDFT method. Using this FMO-TDDFT method, the authors studied the de-excitation energy of formaldehyde with 25 water molecules, and a slight blueshift of 0.04 eV was obtained.\(^5\) The solvatochromatic shift calculated by the present TDDFT/EFP1 method is located between the values of these studies. In general, the computation of solvatochromatic shifts depends on the QM methods and solvent models that are employed, and ultimately the computed results should be compared with experiments conducted in solution. Although no reliable experimental data are available for the formaldehyde-water system, the present results are encouraging. The next step is to apply the present TDDFT/EFP1 method to larger solute-solvent molecular systems, and such work is in progress.

IV. CONCLUDING REMARKS

In the present paper, excited-state QM/MM MD simulations have been performed to examine solvent effects on the fluorescence spectra. For this purpose, the analytic energy gradient has been implemented for the linear-response TDDFT method combined with the EFP1 solvent model. The TDDFT/EFP1 excitation energy formula has been derived by redefining the polarization energy, potential, and second derivative on the basis of the matrix formulation for the EFP induced dipoles. The derived formula reveals the existence of an inter-fragment component in the coupling matrix, which is missing in the previous study. In addition, an energy correction scheme has been introduced to take account of the electronic relaxation of EFP induced dipoles in the excited states. The TDDFT/EFP1 gradient calculation has been applied to the excited-state MD simulation of aqueous formaldehyde in the \( n \rightarrow \pi^* \) state. The calculated \( \pi^* \rightarrow n \) transition energy and solvatochromatic shift are in quantitatively good agreement with other theoretical results.

The present method has great possibilities in several directions. First, the present method enables a large-scale solution-phase QM/MM MD simulation for electronically excited states, although some algorithmic developments are required for efficient computation of the TDDFT/EFP1 energy and gradient. In particular, it is time-consuming to evaluate the integrals required to compute the QM-EFP interaction, and the numerical integration of the exchange-correlation functional. Another attractive future direction is the prediction of nonadiabatic dynamics of solvated molecules. Several authors have applied the TDDFT method combined with the trajectory surface-hopping approach to investigate the relaxation pathways of photoexcited molecules in the gas phase.\(^5\) The TDDFT/EFP1 excited-state simulations proposed in the present study may be applicable to examine the solvent effects on such surface crossing problems. Finally, the present TDDFT/EFP1 analytic energy gradient formula is also applicable to the interface of the spin-flip TDDFT (SF-DFT)\(^5\) method and the EFP1 solvent model by treating the excitation from the occupied \( \alpha \) to virtual \( \beta \) orbitals in the coupling matrix and transition amplitude. The resultant formula is much simpler than conventional linear response TDDFT due to the spin-orthogonality. Recently, the SF-DFT method has been shown to be useful for describing conical intersections,\(^5, 6\) and the extension to the SF-DFT/EFP1 is promising way to describe the conical intersections of solvated molecules. Work is in progress along these lines.

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