Estimation and analysis of beef gain roughage-concentrate production functions

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Estimation and Analysis of Beef Gain Roughage-Concentrate Production Functions

Shashanka Bhide, Francis Epplin, Earl O. Heady, Bryan E. Melton, and M. Peter Hoffman

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CONTENTS

Introduction ................................................. 947
Previous studies ............................................ 947
Objectives .................................................... 947
Feeding experiment ....................................... 948
    Constant energy and ad libitum .................. 948
    Ration characteristics ................................. 948
Theoretical model for economic analysis ........... 949
    Review of basic concepts ............................. 949
    Optimization with gain production functions .... 949
    Analysis of derived isoquants ..................... 950
Econometric problems and procedures
    in estimating physical relationships .............. 951
        Estimation of the gain production functions 952
        Estimation of the days functions ............... 953
Results of econometric analysis ...................... 954
    The gain production function ..................... 954
    The days functions .................................. 957
Economic analysis ......................................... 958
    Analysis of derived isoquants ..................... 958
    Comparison of decision models .................... 961
Summary ..................................................... 963
References .................................................. 964

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Estimation and Analysis of Beef Gain Roughage-Concentrate Production Functions

by Shashanka Bhide, Francis Epplin, Earl O. Heady, Bryan E. Melton, and M. Peter Hoffman

Through the 1960s and early 1970s, many cattle feeders formulated feedlot rations composed primarily of grains. Rations high in grain were relatively inexpensive and economical. For example, Scott and Broadbent [20] constructed a programming model in 1972 that utilized the California net energy system as developed by Lofgreen and Garrett [16] and adopted by the National Academy of Sciences (NAS) to estimate economical rations. They concluded, "In most feedlot operations, it appears that the maximum possible rate of gain will be most profitable under usual price relationships" [20, p. 24]. Although maximizing rate of gain is a biological objective, it was congruent with the economic objective of maximizing profits. Therefore, there were several reasons for little interest in investigating the trade-off or substitution rates between roughages and concentrates in the beef feeding ration. First, concentrates were relatively inexpensive. Second, addition of roughages to rations generally reduces rate of gain. A longer time on feed thus increases the nonfeed costs, such as labor, yardage fees, and carrying charges, and reduces the annual volume of a lot. Third, roughages generally are bulkier and more difficult to handle than concentrates. They may require more expensive equipment and large long-term capital investments. Fourth, many feedlots were designed and constructed to provide high-concentrate rations. Hence, little effort was exerted toward investigating the rate of substitution between roughages and concentrates.

In 1973, world crop shortfalls and U. S. currency devaluation were in part responsible for an increase in U. S. grain exports. United States grain prices rose substantially. These changes in the relative price of concentrates to roughages led to increased concern over the rate of substitution between roughages and concentrates.

The NRC recommendations assume additivity of nutrient contributions, such as energy, among feed inputs and, hence, linear roughage-concentrate isoquants. To gain more information about the marginal rate of substitution between concentrates and roughages in the beef feeding ration, a 3-year experiment was conducted. The experiment was initiated in 1975. This report uses the data obtained from that 3-year feeding experiment to estimate and analyze the beef gain roughage-concentrate production function.

PREVIOUS STUDIES

In a previous manuscript [6], we discussed difficulties involved in estimating the shape of the roughage-concentrate beef gain isoquant. Some evidence suggests that, over a range of roughage-concentrate mixtures, the isoquant may not be convex to the origin. For example, Brokken et al. [3] have presented a strong case for a concave region. Brokken's study [2] of alfalfa hay-corn grain substitution yields S-shaped or sigmoid isoquants. They are convex in the high-roughage region in which gut fill may restrict energy intake and concave over the range in which gut fill is not expected to limit energy intake.

Brokken refers to a study of corn grain-corn silage substitution by Goodrich et al. [10]. A plot of their estimates indicates a very slightly concave isoquant.

Byers et al. [4] also compared alternative levels of corn silage and corn grain. They concluded that the NRC recommendations, which assume additivity of energy contributions among feed inputs, are incorrect. The inputs of corn silage and corn grain yield less energy when combined than when fed separately to steers. This interaction suggests a concave isoquant.

We have analyzed the data from a study of substitution rates between soilage (green-chopped alfalfa-brome grass) and corn grain and also concluded that, over a range, the isoquant may not be convex [6].

OBJECTIVES

Although estimation procedures such as quadratic production functions do not force convex isoquants, they do not allow them to have inflection points. Thus, estimation of production functions, which may not yield strictly convex or strictly concave isoquants, requires adjustments of the traditional methods of analysis.

This study has two primary objectives. One objective is to develop and present a methodology that can be used to statistically estimate and analyze...
production functions that do not yield strictly convex or strictly concave isoquants. The second objective is to apply the method to investigate the shape of the roughage-concentrate beef gain isoquant with whole-plant corn silage as the roughage source and corn grain and alfalfa pellets as the concentrate source.

This report is divided into several sections. The first section provides a brief description of the feeding experiment from which data for the study were obtained, and the next section includes a theoretical framework for analyzing the data. A later section is devoted to the econometric problems encountered and procedures adopted to handle these problems. Econometric results and economic analysis of estimated functions are presented in subsequent sections.

**FEEDING EXPERIMENT**

The feeding trial was conducted over a 3-year period at the Western Iowa Outlying Research Center near Castana. Yearling steers were placed on feed in the fall, fattened, and slaughtered in the following spring. The first group of steers was placed on feed in the fall of 1975. The experiment was completed in the spring of 1978. The steers were of unknown origin composed primarily of Angus and Hereford breeding. A total of 278 cattle was fed: 96 in the first and third years and 86 in the second year.

The steers were randomly assigned to one of six alternative rations. Rations were composed of whole-plant corn silage, whole shelled corn grain, dehydrated alfalfa pellets, and a soybean meal base supplement fortified with vitamins and minerals. All six rations were formulated to meet the protein, vitamin, and mineral requirements as established by the NRC [18]. The six rations in terms of dry-matter (DM) ratio of corn silage to concentrate, ignoring the supplement, were approximately: 100:0, 82:18, 63:37, 44:56, 23:77, and 0:100. The 100:0 ration was composed entirely of corn silage and supplement. Corn grain, alfalfa pellets, and supplement made up the 0:100 ration. All the animals were implanted with 30 mg of diethylstilbestrol immediately preceding the feeding trial. The supplement was fed at a constant rate of about 0.8 kg per animal daily. The proportion of corn silage in each of the six rations was held constant throughout the feeding trial. For example, the steers receiving the 44:56 ration received the same mixture at each feeding.

Corn grain and alfalfa pellets were fed in a constant ratio of two parts corn grain to one part alfalfa pellets on an as-fed basis. This fixed-proportion mixture is referred to as concentrate.

**Constant Energy and Ad Libitum**

Twelve pens of cattle were fed in each year. Six pens were fed a constant quantity of metabolizable energy (ME) per day. The other six pens were fed *ad libitum*. One of the objectives of the trial, which is reported elsewhere [14], was to determine the efficiency of energy conversion. The six "constant energy" pens were fed under the constraint that each pen received the same quantity of ME per day. The rations were formulated so that the efficiency of energy (as measured by ME) from corn silage could be compared with energy from other sources. The 2-to-1 mixture of corn grain to alfalfa pellets was selected to be the other energy source. The addition of alfalfa pellets to the corn grain minimized the problem of feeding a constant level of ME to the six pens per day.

Analysis of the data indicates that there were no statistically significant differences among rates of gain and feed intake between the steers fed the constant-energy levels as compared with those fed *ad libitum*. Also, there were no statistically significant differences in carcass quality grades and yield grades among the rations. Steers from all pens consistently graded low choice to high good across years, rations, and treatments. For these reasons, the data for steers fed a constant level of energy are combined with data for those fed *ad libitum*.

**Ration Characteristics**

The metabolizable energy (ME), net energy for maintenance (NE<sub>m</sub>), net energy for gain (NE<sub>g</sub>), total digestible nutrients (TDN), and crude fiber of alfalfa pellets, corn silage, corn grain, and concentrate are reported in Table 1. Characteristics of the concentrate, which is composed of corn grain and alfalfa pellets in a 2-to-1 ratio as-fed, were calculated by assuming that the concentrate was composed of 65.7 percent corn grain and 34.3 percent alfalfa pellets on a dry-matter basis. Alfalfa pellets have a lower energy content per unit dry matter and a higher fiber percentage than does corn silage.

<table>
<thead>
<tr>
<th>Item</th>
<th>NRC Ref.</th>
<th>ME (Mcal/kg)</th>
<th>NE&lt;sub&gt;m&lt;/sub&gt; (Percent)</th>
<th>NE&lt;sub&gt;g&lt;/sub&gt; (Percent)</th>
<th>TDN (Percent)</th>
<th>Fiber (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alfalfa pellets</td>
<td>1-00-023</td>
<td>2.24</td>
<td>1.33</td>
<td>0.73</td>
<td>62.0</td>
<td>26.1</td>
</tr>
<tr>
<td>Corn silage</td>
<td>3-08-153</td>
<td>2.53</td>
<td>1.56</td>
<td>0.99</td>
<td>70.0</td>
<td>24.4</td>
</tr>
<tr>
<td>Corn grain</td>
<td>4-02-931</td>
<td>3.29</td>
<td>2.28</td>
<td>1.48</td>
<td>91.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Concentrate&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.93</td>
<td>1.95</td>
<td>1.72</td>
<td>81.1</td>
<td>10.4</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Source: NRC [18].

<sup>b</sup>Concentrate is composed of 65.7 percent corn grain and 34.3 percent alfalfa pellets.

The NE<sub>g</sub>, NE<sub>m</sub>, ME, and crude fiber percentages for the six rations are reported in Table 2. The energy concentration in terms of megacalories per kilogram (Mcal/kg) of the rations as measured by NE<sub>g</sub> ranged from 0.99 for the full corn silage ration to 1.22 for the full concentrate ration. The energy concentration of the full concentrate ration corresponds to that of a ration composed of 53 percent corn silage and 47 percent corn grain on a dry-matter basis. The fiber content of the 0:100 ration is similar to that which would result from a ration composed of 37 percent corn silage and 63 percent corn grain. Addition of alfalfa pellets to the five rations containing concentrate reduced the energy
concentration and increased the fiber content over the level it would have been if corn grain had been used as the sole component of the concentrate source.

In recent years, the price per unit of dry matter of alfalfa pellets has been very close to or higher than that of corn grain. Because corn grain contains more energy and less fiber, it is unlikely that producers would include alfalfa pellets as an energy source in a corn silage, corn grain ration. It is not possible to differentiate between the impacts of alfalfa pellets and corn grain on steer performance in this feeding trial.

### Review of Basic Concepts

In this section, a brief review of basic concepts used in the economic analysis of the estimated production function is presented. For a detailed presentation of these concepts, refer to Heady and Dillon [12].

A production function is the mathematical relationship between outputs and inputs in a production process. Algebraically, it may be represented by an equation, with output expressed as a function of inputs. Graphically, its presentation is limited to the case of one or two inputs. When only one input is involved in the production of an output, graphical presentation of the production function is accomplished by plotting output levels on the vertical axis and input levels on the horizontal axis. When two inputs are used to produce an output, the production function can be represented as a production surface in three-dimensional space. Graphical presentation is difficult for production processes involving more than two inputs, unless it is assumed that only one or two of the inputs are variable and that all other inputs are held at fixed levels.

A production function describes the quantitative nature of the production process. To gain more information on the specific aspects of production, various quantities can be derived from the production function. One such quantity is the isoquant. The isoquant is defined as a set of combinations of inputs that yields a constant level of output. Isoquants also may be represented algebraically or graphically. The concept of an isoquant is useful only when more than one input is used in production. Inputs may be substitutes or complements in production. Complementarity of inputs implies that production of a given output level is possible only when the inputs are used in a fixed proportion. When inputs are substitutes, isoquants reflect the nature and extent of substitutability among them.

An isocline is a set of input combinations for which slopes of all isoquants corresponding to various output levels are equal. Slopes of isoquants define the rates at which one input substitutes for another to produce a given output level while all other inputs are held at some fixed levels. Economic optimum, defined by the lowest cost of inputs required to produce a given level of output, is obtained when the slope of the isoquant equals the ratio of input prices, when the shape of the isoquant is convex. The isocline corresponding to the given price ratio is called the expansion path.

The two inputs involved in this experiment are concentrate and silage. The output is beef gain. Thus, the production function is a gain function. The gain isoquants derived from it show all combinations of the two feeds that will produce a given level of gain. The isoclines connect points on the gain isoquants where the marginal rate of substitution between concentrate and silage is fixed.

### Optimization With Gain Production Functions

The economic decisions to be analyzed in this study are from a microeconomic point of view. The analysis is relevant to a single decision maker. The analysis is in terms of average results on a per-steer basis.

Weight gain in a steer is considered as the "output," and concentrate and corn silage are the variable inputs. Another important input that should be considered is "time." Time required to achieve a given level of weight gain depends upon the particular combination of concentrate and silage levels with which the gain level is achieved. Along with time, several other inputs vary monotonically. For example, the feed supplement fed to steers to supply the essential levels of vitamins, protein, and minerals was fed at a constant amount per day. Amount of labor required depends upon the number.

### Table 1. Energy and fiber characteristics of the rations

<table>
<thead>
<tr>
<th>Item</th>
<th>Ration a</th>
<th>100:0</th>
<th>82:18</th>
<th>63:37</th>
<th>44:56</th>
<th>23:77</th>
<th>0:100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (Mcal/kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NE</td>
<td>0.99</td>
<td>1.02</td>
<td>1.07</td>
<td>1.12</td>
<td>1.17</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>NE n</td>
<td>1.56</td>
<td>1.63</td>
<td>1.70</td>
<td>1.78</td>
<td>1.86</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>ME</td>
<td>2.53</td>
<td>2.60</td>
<td>2.68</td>
<td>2.76</td>
<td>2.84</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td>Fiber (percent)</td>
<td></td>
<td>24.4</td>
<td>21.9</td>
<td>19.2</td>
<td>16.6</td>
<td>13.6</td>
<td>10.4</td>
</tr>
</tbody>
</table>

a Ration is the dry matter ratio of corn silage to concentrate. The supplement, which was fed at a rate of 0.8 kg per animal daily to all animals, is not included.
of days the steers are kept on feed. For credit purposes, the borrowing period depends on the length of the feeding period. Hence, these inputs vary monotonically with time.

**Homogeneous output and inputs**

To analyze various techniques of production, the output produced from these techniques should be the same. No significant differences were detected in the quality of beef produced by using different proportions of silage and concentrate in the feed [14]. Therefore, it is assumed that the beef gain output is homogeneous.

In the case of inputs, there can be a considerable degree of variation in quality. For example, the moisture content of silage may vary widely across farms and years. To overcome some of these differences, all feed inputs are converted into a dry-matter (DM) basis. The dry-matter content of feeds was evaluated and recorded weekly during the feeding experiment.

With these considerations, the gain production function can be represented in implicit form as

\[ G = g(C, S|\Omega, M) \]  

where:
- \( G \) = gain in kilograms per steer;
- \( g \) = functional form of the gain production function;
- \( C \) = concentrate consumption in kilograms of dry matter per steer;
- \( S \) = silage consumption in kilograms of dry matter per steer;
- \( \Omega \) = parameters of the gain production function; and
- \( M \) = factors other than concentrate and silage.

**Economic decision models**

Several economic objectives could be formulated and analyzed with the experimental data available. Examples include: minimizing cost per head daily, maximizing return per head, or minimizing feed cost. However, we analyze only two objectives. One is to minimize the cost of feed inputs ignoring time costs, and the second is to minimize feed costs including time costs. These objectives constitute decision models I and II, respectively. Each of the two objectives is analyzed by using isoquants derived from the overall gain production function.

**Analysis of Derived Isoquants**

A producer may decide to feed steers only up to a given weight level. Also, he may feed different rations over different weight ranges. In these cases, analysis of the isoquant relating to the chosen weight gain level is appropriate. Also, if the interactions resulting from changes in rations during a feeding program are so strong as not to be reflected

in a single gain function, then results of derived isoquant analysis are more in line with design of the experiment from which data for this study were obtained.

Consider a gain isoquant derived from the gain production function, where concentrate is a function of silage:

\[ C_i = f_i(S|\Omega, G_i) \]  

where:
- \( C_i \) = concentrate consumption (in DM);
- \( S \) = silage consumption (in DM);
- \( f_i \) = functional form of the \( i \)-th isoquant;
- \( G_i \) = level of weight gain; and
- \( \Omega \) = parameters of the gain production function.

In most economic applications, isoquants as in equation (2) are downward sloping and convex from the origin. However, some special models have been suggested recently for beef gain (concentrate-roughage) production functions.

**Sigmoid isoquants**

It has been suggested that beef gain concentrate-roughage isoquants may not be strictly convex [1,17]. Nevertheless, convex shape for the isoquants over some range is not ruled out. If we consider concentrates such as corn grain and roughages such as an alfalfa-brome grass mixture for beef grain production, the isoquant may be convex at both ends (near the concentrate and roughage axes) and concave over the range in between. Inasmuch as the ration contains a high proportion of roughage, the gut fill limits DM intake by the steers, and less and less energy becomes available for gain production. The isoquant becomes convex over the high-roughage region in the isoquant. Because the ration contains a high proportion of grain, lack of sufficient amount of fiber in the ration may result in less efficient use of grain. Hence, the isoquant may be convex over the high-grain region. The region in between the high-grain and high-roughage regions is hypothesized to be concave.

In the present experiment, corn silage and concentrate (made up of corn grain and alfalfa pellets) were the feeds used. Because corn silage contains a high proportion of corn grain, it is not expected that gut fill would limit DM intake of corn silage. Hence, we propose a sigmoid shape for the beef-gain, corn-silage-concentrate isoquant. The isoquant is convex over the high-concentrate region and concave elsewhere. One such isoquant is graphed in Figure 1. Isoquants of this shape also were obtained by Melton [17].

**Decision Model I: Minimizing cost of concentrate and corn silage**

Mathematically, the objective function corresponding to this decision model can be written as:

\[ \text{Minimize } TC_i = p_c \cdot f_i(S|\Omega, G_i) + p_s \cdot S \]  

where:
- \( p_c \) = concentrate cost;
- \( p_s \) = silage cost; and
- \( f_i \) = functional form of the \( i \)-th isoquant.
where:

\[ TC_j = \text{total cost to be minimized for } j^{\text{th}} \text{ isoquant}; \]
\[ P_0, P_8 = \text{prices of concentrate and silage, respectively; and } f_j, S_j, G_j \text{ are as previously defined.} \]

For sigmoid isoquants, such as the one as shown in Figure 1, equality of slope of the isoquant and ratio of silage to concentrate prices does not always mean that total cost \((TC_j)\) is lowest of these points. Over the concave region of the isoquant, only the two end points—the point on the full-silage axis and the inflection point on the isoquant—can result in minimum total cost.

Therefore, the minimum cost of achieving a given gain level may occur at a point satisfying the first-order condition for minimum where the slope of the isoquant equals the price ratio of feeds over the convex range of the isoquant, or one of the two end points of the concave segment of the isoquant may give minimum cost of the isoquant. If the point satisfying the first-order condition for minimum cost of the isoquant indicates negative levels of silage, the full-concentrate ration may be the least expensive ration under Decision Model I.

All these considerations are incorporated in a solution procedure. The procedure is used for each of the selected isoquants to obtain optimal quantities of concentrate and silage for a given price ratio. The price ratio can then be parameterized to obtain optimal solutions over a wide range of prices of concentrate and silage.

**Decision Model II: Minimizing cost of concentrate, silage, and time**

As the proportions of concentrate and silage in the feed change, the number of days required for steers to achieve given levels of weight gain also is expected to vary. Because costs other than feeds also are involved, the total costs of fattening a steer to market weight also may vary with rations. Decision Model II considers these costs, which are affected by the length of the feeding period.

The costs associated with daily expenses in feeding the steers can be represented as \(P_d\) (price of day). Substituting the isoquant equation for \(C_i\) (concentrate) and using the days functions for \(D_i\) (number of days), the cost equation becomes

\[ TC_i = P_0 \cdot f_i(S_i|\Omega, G_i) + P_8 \cdot S_i + P_d \cdot d_i(R_i) \]  

where:

\[ TC_i = \text{total cost of achieving the gain level corresponding to the } i^{\text{th}} \text{ isoquant}; \]
\[ C_i, S_i = \text{concentrate and silage consumption (DM) in the } i^{\text{th}} \text{ isoquant}; \]
\[ P_0, P_8, \text{ and } P_d = \text{prices of concentrate, silage, and day;} \]
\[ R_i = \frac{\text{ME from silage } (S_i)}{\text{ME from silage } (S_i) \text{ and concentrate } (C_i)} \]
\[ d_i = \text{functional form of the relationship between } R_i \text{ and number of days; and other expressions are as previously defined.} \]

It is rather tedious to obtain numerical solutions for the first and second-order conditions, which define the minimum total cost, especially if the days function is complicated. Hence, we follow a more direct approach. For a large number of points on a given isoquant, the cost of associated silage, concentrate, and days is calculated. Then, the point yielding lowest cost is selected as optimal from a total cost standpoint. By checking a large number of points covering the entire range of a given isoquant, a good approximation to the best possible solution is obtained. As under Decision Model I, we parameterize the ratio of concentrate to silage prices and obtain solutions under a wide range of price situations.

**ECONOMETRIC PROBLEMS AND PROCEDURES IN ESTIMATING PHYSICAL RELATIONSHIPS**

The problems associated with estimating relationships representing livestock production have been well documented in various studies [5, 12]. Two major estimation problems are: First, repeated observations on the same experimental units (viz., animal) may result in serially dependent error terms in regression analysis. Second, levels of feed consumption by animals are not controlled in the experiment, thus resulting in random independent variables in a regression model.

The first problem has fairly simple remedies once the order of autocorrelation is determined or assumed. The second problem, that of stochastic or random regressors, requires the use of special variables, "the instrumental variables." These variables are themselves fixed or nonrandom, but are correlated with the model's independent
variables. In recent studies, researchers also have resorted to "direct estimation" of isoquants and "weight interval functions" [13, 17]. Use of the "direct estimation" procedure eliminates the problem of autocorrelated errors. Isoquants derived from the gain function are used in this study. Accordingly we do not "directly estimate" them.

Data for the present analysis were obtained from a 3-year experiment. Each year's data could be analyzed separately and a comparison of variability among years could be made, but the analysis in this report is based on data pooled for the 3 years.

The following two physical relationships are estimated: (a) the gain production function and (b) the days functions corresponding to selected weight intervals. In this section, we discuss different problems associated with the estimation procedures used.

**Estimation of the Gain Production Functions**

The first problem in estimating a gain production function is to determine the "best" explicit functional form. Past studies indicate the need to consider a wide variety of functional forms in estimating the roughage-concentrate gain production function [1, 2, 6, 17, 19]. A procedure for determining the best functional form would be to try several functional forms and then select the one that meets certain criteria. For this purpose, regressions are fit in polar coordinates in various degrees of power. Once the shape of the function is determined, it is approximated to an arbitrary degree of accuracy by splicing or grafting two quadratic functions.

**Regression using polar coordinates**

Figure 2 represents an input plane for a gain production surface. Let A1 and A2 be two points on the input plane. Distances of the points A1 and A2 from the origin "O" are d_A1 and d_A2, respectively. Thus, any point, A_i, on the input plane can be located by the distance d_Ai and angle Θ_Ai. The gain production function can be written as

\[ G = g(d, Θ) \] (5)

where:

\[ d = (C^2 + S^2)^{1/2} \]

\[ Θ = \tan^{-1}(\frac{C}{S}), \quad \text{since} \quad \frac{C}{S} = \tan Θ \]

Isoquants of various shapes are possible from gain production functions estimated in polar coordinates. The following three functional forms were utilized.

1. \[ G = b_1d + b_2d^2 + b_3dΘ + b_4dΘ^2 + b_5d^3Θ + b_6d^2Θ^2 \]

2. \[ G = b_1d + b_2d^2 + b_3dΘ + b_4dΘ^2 + b_5d^3Θ + b_6d^2Θ^2 + b_7d^4Θ^2 \]

3. \[ G = b_1d + b_2d^2 + b_3dΘ + b_4dΘ^2 + b_5d^3Θ + b_6d^2Θ^2 + b_7d^4Θ^2 + b_8d^6Θ^4 \]

The criterion of choice among the three forms is the mean square error (MSE). The model with the lowest MSE is selected as the "best" functional form.

**Autocorrelation.** Because weight gain data are taken at fixed intervals of time by using the same group of steers repeatedly, correlation between observations from one period to the next can prevail. If the autocorrelation problem is severe, OLS estimators for the regression coefficients are unbiased and consistent, but they are not efficient. Moreover, the estimates of standard errors of OLS estimators are biased. Therefore, under such a situation the generalized least-squares (GLS) procedure is preferred to OLS [15]. Comparison of various functional forms is required in selecting the "best" form for representing the gain production surface. Thus, an estimate of the autocorrelation coefficient is obtained independent of the functional form of the regression model. The procedure used is explained in Heady et al. [11] and Roehrkasse [19].

**Stochastic inputs.** Although concentrate and silage consumption levels are subject to random fluctuations, only the "distance" variable in polar coordinate regression is subject to these random fluctuations. The "angles" are fixed variables. In past studies, the effect of "errors" in input variables on estimated regression coefficients in livestock production function analysis was reported to be minimal [17]. Hence, this problem is ignored in the present context.

---

*We wish to acknowledge the help of Wayne A. Fuller and Ray Brokken who suggested this approach to determine the nature of the gain surface.*
Grafted polynomials

Isoquants can be derived from the polar coordinates production functions. The shapes of these isoquants provide insight as to an appropriate functional form in rectangular coordinates. The best fitting functional form in polar coordinates can be used to derive the implied shapes for the isoquants. Depending on shapes of the derived isoquants, the input plane is divided into different segments. For example, if the derived isoquants are of the form as in Figure 3, the input plane can be divided into three regions:

Region 1: over the range \( S < k_1 C \) isoquants are convex;
Region 2: over the range \( k_1 C < S < k_2 C \) isoquants are concave; and
Region 3: over the range \( S > k_2 C \) isoquants are convex.

Where \( k_1 \) and \( k_2 \) are positive constants.

If a sufficient number of observations is available, the three functions can be separately estimated. It is more reasonable, however, to consider these functions as parts of a function representing the overall gain production surface. By imposing certain restrictions on the three functions involved, Fuller [71] has provided an efficient method to estimate parameters of these functions. The restrictions ensure that the overall function is smooth and continuous on the lines where component functions join.

![Figure 3. Hypothetical isoquants.](image)

Autocorrelation. For estimating the "grafted" or the "spliced" gain production function, the GLS procedure is used for the same reasons as explained in the case of polar coordinate regression. The estimate of the autocorrelation coefficient is obtained in the usual manner [15]. Consistent estimates of error terms \( (\hat{U}_t) \) are obtained from OLS regression, and then the estimate of autocorrelation coefficient is obtained by regressing \( \hat{U}_t \) on \( \hat{U}_{t-1} \) without an intercept.

Stochastic inputs. Econometric estimation of production functions in grafted quadratic functional form should take into account the random nature of the independent variables. Statistical tests have been developed that test the severity of bias of OLS estimators when the independent variables are stochastic. For example, see Fuller [9] and Wu [21]. Because of the convenience in computations, Fuller's F-test is used in this study.

Pooling data for estimating gain production function

In pooling data from different pens and different trials of the experiment, care must be taken to consider effects of factors, other than those of interest, on the gain response. Two such factors in this experiment are: (a) The nature of the relationship between gain and inputs may differ over years because of differences in starting weights of the steers, differences in climate, and other differences in steers; and (b) the nature of the relationship between gain and inputs may vary across pens because of different feeding methods used.

For differences from trial to trial, dummy variables are used to account for these differences in regression. For polar coordinate regression, dummy variables are defined in terms of the "distance" variable. Because feed inputs, concentrate and silage, were mixed in fixed proportions for each of the six rations, the angle variable does not change from trial to trial for the same rations.

For grafted quadratic functions, the procedure is the same as used by Melton [17]. The basic emphasis is on accounting for the differences in starting weight of the steers.

Estimation of the Days Functions

A general relationship between the metabolizable energy contribution (ME) of silage in the feed and the number of days required for steers to achieve a given level of gain is supposed. Time on feed required to achieve a given level of gain is estimated as a function of the ration fed.

The general form of the days functions is,

\[
D = d_i(R)
\]

where:

\[
D = \text{number of days;}
\]

\[
R = \frac{\text{ME from silage}}{\text{ME from (concentrate + silage)}}; \text{ and}
\]

\[
d_i = \text{the functional form for the days function corresponding to ith weight interval.}
\]

A functional form will be chosen from linear, quadratic, and cubic equations in \( R \). The criterion for selection is the MSE and consistency in the form chosen. The OLS estimates of the parameters of these days functions are best linear unbiased estimates.

Differences due to methods of feeding——ad libitum

953
and constant energy—and differences from trial to trial should be considered in estimating the days functions. After choice of a functional form, the functions are fit separately for ad libitum and constant energy data sets first. Then, overall functions also are estimated. If the methods of feeding cause significant difference in days functions, then we have two separate sets of days functions. If the difference is not statistically significant, as indicated by the F-test, then the overall days functions are used.

The F-test used is:

\[ F = \frac{(\text{SSE}_{\text{ov}} - \text{SSE}_{\text{AD}} - \text{SSE}_{\text{CE}})}{(\text{SSE}_{\text{AD}} + \text{SSE}_{\text{CE}})} \times \frac{\text{df}_{\text{ov}}}{\text{df}_{\text{AD}} + \text{df}_{\text{CE}}} \]

where:

- \( \text{SSE} \) = the sum of squares due to error in regression;
- \( \text{df} \) = degrees of freedom for error in regression; and
- \( \text{OV}, \text{AD}, \) and \( \text{CE} \) = subscripts indicating overall days functions, days functions using data from pens fed ad libitum, and days functions using data from pens fed on constant energy.

RESULTS OF ECONOMETRIC ANALYSIS

The procedures used in obtaining empirical estimates of various relationships important to the economic analysis of the beef production process have been described. Results of application of these estimation procedures are presented in this section.

The Gain Production Function

Shape of the gain production surface

Three alternative forms of gain production functions in polar coordinates were estimated. Dummy variables were used to account for the difference between trials in the "distance" from the origin of the point on the input plane. Because rations, in terms of proportions of silage and concentrate in the feed, were fixed for all trials, the measure of "angles" does not vary among trials. The following example illustrates the type of dummy variables and the form of the function used in estimation by a second-degree function.

\[ G = \beta_0 + \beta_1 d + \beta_2 d^2 + \beta_3 d \Theta + \beta_4 d \Theta^2 + \beta_5 d^2 \Theta^2 + \beta_6 (d \cdot D1) + \beta_7 (d \cdot D2) \]

where:

- \( G \) = weight gain;
- \( d = (C^2 + S^2)^{1/2} \);
- \( \Theta = \tan^{-1} \frac{C}{S} \);
- \( D1 = 1 \) if second trial of the experiment, 0 otherwise; and
- \( D2 = 1 \) if third trial of the experiment, 0 otherwise.

Autocorrelation

The first-order autocorrelation coefficient for the estimated equations was 0.9389, with a standard error of 0.0502. This estimate of autocorrelation was used to transform the data. The estimated generalized least-squares equations follow. Statistical significance of estimated regression coefficients at 0.10, 0.05, or 0.01 level of probability is indicated by one, two, or three asterisks, respectively. Calculated t-values (absolute value) for the regression coefficients are given below the coefficients.

1. Second-degree equation in angle \( \Theta \):

\[ G = 0.153507d - 0.000019d^2 + 0.128842d \Theta \]

\[ 17.08*** \quad 3.08*** \quad 4.21*** \]

\[ -0.077704d \Theta^2 - 0.000025d \Theta^3 \]

\[ 4.06*** \quad 1.04 \]

\[ +0.000013d \Theta^3 - 0.011928(d \cdot D1) \]

\[ 0.82 \quad 2.88*** \]

\[ -0.031756(d \cdot D2) \]

\[ 8.03*** \]

MSE = 24.71 n = 204 \( R^2 = 0.7836 \)

2. Third-degree equation in angle \( \Theta \):

\[ G = 0.158118d - 0.000020d^2 + 0.084447d \Theta \]

\[ 16.36*** \quad 3.16*** \quad 1.24 \]

\[ -0.003077d \Theta^2 - 0.000024d \Theta^3 \]

\[ 0.03 \quad 0.04 \]

\[ +0.000012d \Theta^3 - 0.031669d \Theta^3 \]

\[ 0.14 \quad 0.69 \]

\[ +0.000001d \Theta^3 - 0.012155(d \cdot D1) \]

\[ 0.004 \quad 2.94*** \]

\[ -0.031972(d \cdot D2) \]

\[ 8.11*** \]

MSE = 24.53 n = 204 \( R^2 = 0.7873 \)

3. Fourth-degree equation in angle \( \Theta \):

\[ G = 0.157021d - 0.000020d^2 + 0.125028d \Theta \]

\[ 15.68*** \quad 2.98*** \quad 1.00 \]

\[ -0.141886d \Theta^2 - 0.000046d \Theta^3 \]

\[ 0.37 \quad 0.45 \]

\[ +0.000088d \Theta^3 + 0.112489d \Theta^4 \]

\[ 0.27 \quad 0.29 \]

\[ -0.000078d \Theta^4 - 0.045974d \Theta^4 \]

\[ 0.24 \quad 0.38 \]

\[ +0.000025d \Theta^4 - 0.012152(d \cdot D1) \]

\[ 0.24 \quad 2.92*** \]

\[ -0.031961(d \cdot D2) \]

\[ 8.05*** \]

MSE = 24.76 n = 204 \( R^2 = 0.7876 \)

\*This convention in notation is followed throughout this report.
In terms of MSE, the third-degree equation is slightly the best fit. The second-degree equation gives almost the same MSE. In terms of $R^2$, all models do equally well. Plots of isoquants from the reference point of first year’s group of steers derived from these estimates are presented in Figures 4 through 6. We do not try to find an explicit expression for isoquants (i.e., $C$ as a function of $S$, or $S$ as a function of $C$). The following procedure is used to obtain coordinates for the plot. Consider the second-degree equation (dummy variables deleted):

$$G = \beta_1d + \beta_2d^2 + \beta_3d^3 + \beta_4d^4 + \beta_5d^5 + \beta_6d^6$$

For a given gain level $G_j$, we can solve for $d$ in terms of $\Theta$ and $G$.

$$d = \frac{-(\beta_1 + \beta_3\Theta + \beta_5\Theta^3)}{2(\beta_2 + \beta_4\Theta + \beta_6\Theta^2)}$$

$$+ \left[ \frac{(\beta_1 + \beta_3\Theta + \beta_5\Theta^3)^2 + 4(\beta_2 + \beta_4\Theta + \beta_6\Theta^2)G_j^{0.5}}{2(\beta_2 + \beta_4\Theta + \beta_6\Theta^2)} \right]^{0.5}$$

For a given level of $\Theta$ and $G_j$, we select the smallest value of $d$.

For the other functional forms, the same procedure is used in solving for $d$ in terms of $\Theta$ and $G$. Once the angle and distance are known, the point on the isoquant can be located. The plots are drawn accordingly.

Figures 4 through 6 suggest sigmoid-shaped isoquants. Not all the coefficients of any of the three models used are statistically significantly different from zero. The reason for retaining all the coefficients was to maintain the symmetrical nature of the function.

The regression for the gain production function in polar coordinates suggests sigmoid-shaped isoquants. This result conforms with the findings of Melton [17]. Furthermore, plots also suggest that the ray from the origin at the 60° angle may serve as the join line, which divides the gain surface into two regions: (a) one in which silage substitutes for concentrate at increasing rates and (b) another in which silage substitutes for concentrate at decreasing rates. Therefore, in further analysis, the 60° line from the origin in the concentrate-silage input plane is chosen to be the join line for splicing two quadratic surfaces.

**The grafted or spliced gain production function**

The join line where two quadratic surfaces join over the input plane was determined to be the ray
from origin at an angle of 60°. This translates into the regions \( S \leq 0.5774C \) and \( S \geq 0.5774C \). The overall gain surface can be represented by two grafted quadratics.

\[
G = a_0 + a_1S + a_2C + a_3S^2 + a_4C^2 + a_5CS \\
\text{for } S \leq 0.5774C \tag{12} \\
G = b_0 + b_1S + b_2C + b_3S^2 + b_4C^2 + b_5CS \\
\text{for } S \geq 0.5774C
\]

For estimation purposes, we put restrictions on the functions as described by Fuller [7]. The function to be estimated is

\[
G = a_0 + a_1S + a_2C + a_3S^2 + a_4C^2 + a_5CS + (b_5 - a_5)Z \
\text{for } S \leq 0.5774C \tag{13} \\
G = b_0 + b_1S + b_2C + b_3S^2 + b_4C^2 + b_5CS \
\text{for } S \geq 0.5774C
\]

where:

\[
Z = 0 \text{ if } S \leq 0.5774C; \text{ or} \\
(S - 0.5774C)^2 \times 0.866 \text{ otherwise.}
\]

The remaining parameters of the model are estimated as follows:

\[
b_0 = a_0 \\
b_1 = a_1 \\
b_2 = a_2 \\
b_3 = a_3 \cdot \frac{(b_5 - a_5)}{2 \times 0.5774} \\
b_4 = a_4 \cdot \frac{0.5774 \times (b_5 - a_5)}{2} \\
b_5 = [(b_5 - a_5) + a_5]
\]

Dummy variables are used to account for the starting weight differences. All the variables in the regression model are corrected for autocorrelation with the coefficient calculated from OLS residuals. The equation to be estimated is,

\[
G = a_0 + a_1S + a_2C + a_3S^2 + a_4C^2 + a_5CS + (b_5 - a_5)Z + a_6(D0 \cdot S) + a_7(D1 \cdot S) + a_8(D2 \cdot C) + a_9(D3 + a_9(D4 \cdot C) + a_{10}(D5 \cdot C) \tag{14}
\]

where:

\[
G \text{ = weight gain in kilograms;} \\
C \text{ = concentrate consumption;} \\
S \text{ = silage consumption;} \\
Z = -(S - 0.5774C)^2 \times 0.866 \text{ if } S \geq 0.5774C, 0 \text{ otherwise;} \\
D0, D1, D2 = 1 \text{ if second trial, 0 otherwise; and} \\
D3, D4, D5 = 1 \text{ if third trial, 0 otherwise.}
\]

Two alternative models are estimated: One includes the interaction terms in both segments of the overall function, and the other does not have the interaction term in the segment over the region \( S \leq 0.5774C \). The estimated equations follow.

1. \( G = 1.423010 + 0.154490S + 0.149640C \)
   \( 0.74 \) \( 24.73*** \) \( 23.88*** \)
   \(-0.000077S^2 - 0.000023C^2 + 0.000010SC \)
   \( 4.92*** \) \( 4.77*** \) \( 0.75 \)
   \( + 7.140937D0 - 0.021881D1 \cdot S \)
   \( 2.62*** \) \( 5.40*** \)
   \(-0.0025407D4 \cdot S - 0.027592D5 \cdot C \)
   \( 6.76*** \) \( 7.04*** \)
   \( \text{for } S \leq 0.5774C \) \( (15a) \)

\[
G = 1.423010 + 0.154490S + 0.149640C \\
0.74 \ 24.73*** \ 23.88*** \\
-0.000018S^2 - 0.00003C^2 - 0.000058SC \\
1.96** \ 0.50 \ 3.79*** \\
+ 7.140937D0 - 0.021881D1 \cdot S \\
2.62*** \ 5.40*** \\
-0.0025407D4 \cdot S - 0.027592D5 \cdot C \\
6.76*** \ 0.65 \ 0.97 \\
\text{for } S \geq 0.5774C \\
\text{(15b)}
\]

The two sets of equations (15a and 15b vs. 16a and 16b) have similar statistical results. The MSE for equation (16) is slightly smaller than for equation (15).
Since the shapes of the isoquants are the same for both models and since all estimated coefficients are significantly different from zero, except for the intercept term and two dummy variable coefficients in equation (16), it is chosen as the function to represent the gain production process. Note that the coefficients for $S^2$ and $C^2$ in equation (16b) are derived from the estimated coefficients as described before. The isoquants derived from these two production functions are sigmoid, convex from the origin in the high-concentrate region and concave in the high-silage region. The isoquants derived from equation (16) are shown in Figure 7.

![Figure 7. Isoquants derived from grafted polynomial gain production function.](image)

**Test for OLS bias**

An F-test does not lead us to reject the null hypothesis of no bias in the OLS estimates. The calculated value of $F_{10}^{18}$ is 0.52. The table value of $F_{10}^{18}$, ($\alpha = 0.05$) is 1.83. Hence, we do not reject the null hypothesis that OLS estimates of regression coefficients in the gain production function are nearly unbiased.

**The Days Functions**

Corresponding to each of the five weight intervals, three alternative forms of relationships are estimated between the proportion of energy supplied by silage in the feed and the number of days cattle are on feed. The estimated equations are:

**I. 320-370 kg weight interval**

1. Linear form
   \[
   D = 60.31667 - 8.55000R \quad (17a)
   \]
   \[
   MSE = 13.73 \quad n = 12 \quad R^2 = 0.4270
   \]

2. Quadratic form
   \[
   D = 61.27500 - 15.73750R + 7.18750R^2 \quad (17b)
   \]
   \[
   25.05*** \quad 1.37 \quad 0.65
   \]
   \[
   MSE = 14.57 \quad n = 12 \quad R^2 = 0.4528
   \]

3. Cubic form
   \[
   D = 62.96806 - 54.39560R + 113.00347R^2 - 70.54398R^3 \quad (17c)
   \]
   \[
   2.41** \quad 2.02
   \]
   \[
   MSE = 11.23 \quad n = 12 \quad R^2 = 0.6250
   \]

**II. 370-400 kg weight interval**

1. Linear form
   \[
   D = 28.28726 + 0.12214R - 8.00677D1 \quad (18a)
   \]
   \[
   20.69*** \quad 0.06 \quad 5.96***
   \]
   \[
   MSE = 10.83 \quad n = 24 \quad R^2 = 0.6284
   \]

2. Quadratic form
   \[
   D = 29.88949 - 11.89460R + 12.01674R^2 - 8.00667D1 \quad (18b)
   \]
   \[
   19.36*** \quad 1.80 \quad 1.89*
   \]
   \[
   MSE = 9.65 \quad n = 24 \quad R^2 = 0.6847
   \]

3. Cubic form
   \[
   D = 31.25568 - 43.08905R + 97.40303R^2 - 56.92419R^3 - 8.00667D1 \quad (18c)
   \]
   \[
   21.75*** \quad 3.35*** \quad 3.04***
   \]
   \[
   MSE = 7.33 \quad n = 24 \quad R^2 = 0.7725
   \]

**III. 400-425 kg weight interval**

1. Linear form
   \[
   D = 27.37381 + 0.45571R \quad (19a)
   \]
   \[
   26.49*** \quad 0.34
   \]
   \[
   - 7.27917D1 - 4.50917D2
   \]
   \[
   6.52*** \quad 4.04***
   \]
   \[
   MSE = 7.48 \quad n = 36 \quad R^2 = 0.5759
   \]

2. Quadratic form
   \[
   D = 28.16617 - 5.68699R + 5.94271R^2 - 7.27917D1 - 4.50917D2 \quad (19b)
   \]
   \[
   23.75*** \quad 3.35*** \quad 3.04***
   \]
   \[
   6.60*** \quad 4.09***
   \]
   \[
   MSE = 7.31 \quad n = 36 \quad R^2 = 0.5983
   \]

3. Cubic form
   \[
   D = 28.31159 - 8.80734R + 15.03125R^2 - 6.05903R^3 - 7.27817D1 - 4.50917D2 \quad (19c)
   \]
   \[
   22.23*** \quad 0.83
   \]
   \[
   + 15.03125R^2 - 6.05903R^2 \quad 0.57 \quad 0.35
   \]
   \[
   - 7.27817D1 - 4.50917D2
   \]
   \[
   6.50** \quad 4.03**
   \]
   \[
   MSE = 7.52 \quad n = 36 \quad R^2 = 0.5999
   \]
IV. 425-455 kg weight interval

1. Linear form
   \[ D = 29.15905 \cdot 2.52654R - 2.07750D^1 + 0.05417D^2 \]
   \[ + 0.03 \]
   MSE = 22.70  n = 36  \( R^2 = 0.0789 \)

2. Quadratic form
   \[ D = 30.38236 - 11.70128R + 9.17485R^2 - 2.07750D^1 + 0.0541667D^2 \]
   MSE = 22.46  n = 36  \( R^2 = 0.1172 \)

3. Cubic form
   \[ D = 31.70417 - 41.88251R + 91.78770R^2 - 55.07523R^3 - 2.07750D^1 + 0.05417D^2 \]
   MSE = 20.69  n = 36  \( R^2 = 0.2129 \)

V. 455-480 kg weight interval

1. Linear form
   \[ D = 26.03798 - 0.01238R + 11.21*** 0.004 \]
   MSE = 37.79  n = 36  \( R^2 = 0.3739 \)

2. Quadratic form
   \[ D = 28.27935 - 16.79789R + 16.81027R^2 - 3.29000D^1 + 7.41750D^2 \]
   MSE = 35.74  n = 36  \( R^2 = 0.4263 \)

3. Cubic form
   \[ D = 29.87254 - 53.17583R + 11.06*** 2.39** + 116.38492R^2 - 66.38310R^3 \]
   MSE = 33.27  n = 36  \( R^2 = 0.4831 \)

Except in one weight interval, 400-425 kg, the cubic form fits consistently better than the linear and quadratic forms of days function, in terms of both MSE and \( R^2 \). Therefore, we choose the cubic form for the days functions for use in the economic analysis. The functions estimated suggest that a greater number of days is required to complete the selected weight intervals for cattle on full-concentrate ration than on full-silage ration. Although the difference in days required is not very large, the smaller number of days for the full-silage rations may seem inconsistent with the ration composition. However, the addition of the alfalfa pellets to the concentrate portion of the ration reduced the energy concentration and evidently reduced rate of gain from what might be expected. Thus, the cubic form is chosen because it does provide a consistently better fit among alternative forms of days function.

After selection of the cubic form for days functions, F-tests were made to check the validity of pooling data over ad libitum and constant-energy fed pens. These tests are explained in the section on econometric problems and procedures. For all five weight intervals, the null hypothesis of no difference between the two groups of pens cannot be rejected at the 5-percent level of significance. Following are the results of the F-tests. The superscript and subscript for F refer to the numerator and denominator degrees of freedom, respectively. Numbers in parentheses indicate the level of confidence.

320-370 kg weight interval:
   Calculated F = 0.8457
   Table value F\(_{4,4}(0.95)\) = 6.39

370-400 kg weight interval:
   Calculated F = 1.8154
   Table value F\(_{5,14}(0.95)\) = 2.96

400-425 kg weight interval:
   Calculated F = 1.8644
   Table value F\(_{6,24}(0.95)\) = 2.51

425-455 kg weight interval:
   Calculated F = 0.3967
   Table value F\(_{2,62}(0.95)\) = 2.51

455-480 kg weight interval:
   Calculated F = 1.8281
   Table value F\(_{2,62}(0.95)\) = 2.51

ECONOMIC ANALYSIS

Results of application of the two decision rules to the estimated gain production function are presented in this section.

Analysis of Derived Isoquants

Isoquants can be derived for any desired gain level. Isoquants of 50 kg, 80 kg, 105 kg, 135 kg, and 160 kg were derived for this analysis because they correspond with the estimated days functions. Input prices are required to determine least-cost rations, and product prices are necessary to determine the optimal levels of gain. Since the concentrate is composed of 2:1 mixture of corn grain and alfalfa pellets, the price of concentrate is calculated as a weighted
average of the prices of corn grain and alfalfa pellets.

The isoquants derived from the gain function estimated in equation (16) are as follows for a 320-kg starting weight:

\[
C = \begin{cases} 
0.150137 \pm \sqrt{(0.150137)^2 + 4 \times 0.000022 \times (1.219687 + 0.155839S - 0.000067SP - G)} & \text{for } S < 0.5774C \\
-(0.150137 - 0.000048S) \pm \sqrt{(0.150137 - 0.000056S)^2 - 4 \times 0.000006 \times (1.219688 + 0.155839S - 0.000019S^2 - G)} & \text{for } S \geq 0.5774C
\end{cases}
\]

The average relative prices of alfalfa pellets and corn grain were obtained as follows. Monthly prices of alfalfa pellets and corn grain were obtained for a period of 3 years, 1976-1978. The price of alfalfa pellets was regressed on corn grain prices without an intercept. The resulting regression coefficient was approximately 40. Hence, when the price of corn is at $2.50 per bushel, the price of alfalfa pellets is expected to be approximately $100 per ton. The weighted average price of concentrate on a dry-matter basis, based on $2.50 per bushel of corn grain and $100 per ton of alfalfa pellets, is $0.1132 per kilogram.

**Minimizing cost of concentrate and silage (Decision Model I)**

Concentrate price is fixed at a level of $0.1132 per kilogram of dry matter. Least-cost levels of concentrate and silage are obtained for the five isoquants corresponding to different gain levels under various corn silage price situations. These results are summarized in Table 3. The number of days required to

<table>
<thead>
<tr>
<th>Table 3. Optimal least-cost per animal rations ignoring the cost of time for five isoquants at selected price ratios$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price per ton of corn silage</strong></td>
</tr>
<tr>
<td>Ratio of concentrate to corn silage price (DM basis)</td>
</tr>
<tr>
<td>$$30$</td>
</tr>
<tr>
<td>1.28</td>
</tr>
</tbody>
</table>

Isoquant I (320 to 370kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>1.00</th>
<th>0.25</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Silage (kg DM)</td>
<td>325</td>
<td>84</td>
<td>0</td>
</tr>
<tr>
<td>Concentrate (kg DM)$^c$</td>
<td>0</td>
<td>251</td>
<td>342</td>
</tr>
<tr>
<td>Days</td>
<td>51</td>
<td>56</td>
<td>63</td>
</tr>
</tbody>
</table>

Isoquant II (320 to 400kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>1.00</th>
<th>0.25</th>
<th>0.09</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Silage (kg DM)</td>
<td>341</td>
<td>138</td>
<td>51</td>
<td>0</td>
</tr>
<tr>
<td>Concentrate (kg DM)</td>
<td>0</td>
<td>415</td>
<td>512</td>
<td>574</td>
</tr>
<tr>
<td>Days</td>
<td>90</td>
<td>82</td>
<td>88</td>
<td>94</td>
</tr>
</tbody>
</table>

Isoquant III (320 to 425kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>1.00</th>
<th>0.25</th>
<th>0.14</th>
<th>0.04</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Silage (kg DM)</td>
<td>370</td>
<td>187</td>
<td>106</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>Concentrate (kg DM)</td>
<td>0</td>
<td>560</td>
<td>649</td>
<td>741</td>
<td>782</td>
</tr>
<tr>
<td>Days</td>
<td>108</td>
<td>109</td>
<td>113</td>
<td>119</td>
<td>123</td>
</tr>
</tbody>
</table>

Isoquant IV (320 to 455kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>1.00</th>
<th>0.25</th>
<th>0.17</th>
<th>0.10</th>
<th>0.04</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Silage (kg DM)</td>
<td>401</td>
<td>248</td>
<td>170</td>
<td>102</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>Concentrate (kg DM)</td>
<td>0</td>
<td>744</td>
<td>810</td>
<td>914</td>
<td>996</td>
<td>1,057</td>
</tr>
<tr>
<td>Days</td>
<td>135</td>
<td>135</td>
<td>139</td>
<td>144</td>
<td>150</td>
<td>154</td>
</tr>
</tbody>
</table>

Isoquant V (320 to 480kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>1.00</th>
<th>0.25</th>
<th>0.19</th>
<th>0.14</th>
<th>0.09</th>
<th>0.04</th>
<th>0.00</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Silage (kg DM)</td>
<td>432</td>
<td>302</td>
<td>231</td>
<td>172</td>
<td>113</td>
<td>51</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Concentrate (kg DM)</td>
<td>0</td>
<td>907</td>
<td>986</td>
<td>1,058</td>
<td>1,139</td>
<td>1,230</td>
<td>1,306</td>
<td>1,314</td>
</tr>
<tr>
<td>Days</td>
<td>161</td>
<td>158</td>
<td>161</td>
<td>166</td>
<td>172</td>
<td>178</td>
<td>184</td>
<td>184</td>
</tr>
</tbody>
</table>

$^a$The objective is to minimize the sum of the corn silage and concentrate costs. Price of concentrate fixed at $0.1132 per kilogram of dry matter. Alfalfa pellets and corn grain prices fixed at $100 per ton and $2.50 per bushel, respectively.

$^b$Ration is the dry matter proportion of corn silage in the ration.

$^c$Concentrate is composed of two parts corn grain and one part alfalfa pellets by weight.
achieve given levels of weight gains are obtained for rations that result in minimum cost of concentrate and silage.

Isoquants derived from the gain production function are sigmoid. They are convex to the origin in the high-concentrate region and concave in the high-silage region. Only rations in the convex region of the isoquant or the full corn silage ration result in lowest feed costs under Decision Model I. These results are reported in Table 3. When the price of concentrate is more than or equal to 1.10 times the price of corn silage (on DM basis), the full-silage ration results in lowest feed cost for all five selected isoquants. For Decision Model I, price ratios, rather than absolute values of prices are important. We note that as-fed prices of $2.50 per bushel of corn grain, $35 per ton of corn silage, and $100 per ton of alfalfa pellets translate into a dry-matter price ratio of 1.10.

As the concentrate-to-silage price ratio declines from 1.10 to 0.96, the optimal ration under Decision Model I switches from full silage to the convex region of the isoquant for all five selected gain levels. At the price ratio of 0.96, a ration composed of 25 percent corn silage results in lowest feed cost for all gain levels. For price ratios lower than 0.96, optimal rations are obtained in the convex region of the isoquant.

At a concentrate-to-silage price ratio of 0.86, the full-concentrate ration results in lowest feed cost for the first 50 kg gain. For higher gain levels, the proportion of silage slightly increases in the ration. For price ratios lower than 0.86, the full-concentrate ration results in lowest feed cost for gain levels higher than 50 kg. For example, at a concentrate-to-silage price ratio of 0.55, the full-concentrate ration is optimal for all five selected gain levels.

These results also can be illustrated graphically. Optimal rations under Decision Model I, for two alternative price ratios of feed inputs, are shown in Figure 8. For a price ratio of 1.10, the full-silage ration is optimal for all five selected gain levels. R1, R2, R3, R4, and R5 are the optimal rations for price ratio of 1.10 in Figure 8. For a price ratio of 0.86, optimal rations are R1, R2, R3, R4, and R5 for 50-kg, 80-kg, 105-kg, 135-kg, and 160-kg gain levels, respectively.

Minimizing cost of concentrate, silage, and other per-day expenses (Decision Model II)

Minimizing the cost of concentrate and silage may be appropriate for farmers who feed one lot per year, but, farmers who feed continuously must also consider other variable expenses. Typically, these expenses depend on how long the cattle are on feed, and a cost for time must be included.

These expenses other than concentrate and silage costs are calculated to be $0.30 per head per day. These costs include cost of labor but not the cost

\[ \text{Pc} = \text{Price of concentrate (\$/kg DM)} \]
\[ \text{Ps} = \text{Price of corn silage (\$/kg DM)} \]
\[ \text{Pd} = \text{Cost of time (\$/day)} \]
\[ R_i = \text{Optimal ration for } i\text{th gain level} \]
\[ (i=1, 2, \ldots, 5) \text{ under } \frac{\text{Pc}}{\text{Ps}} \geq 0.86, \text{ Pd}=0 \]
\[ R_i = \text{Optimal ration for } i\text{th gain level} \]
\[ (i=1, 2, \ldots, 5) \text{ under } \frac{\text{Pc}}{\text{Ps}} \leq 1.10, \text{ Pd}=0 \]

Figure 8. Optimal rations under isoquant analysis for Decision Model I.

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\[ ^4 \text{Per-day costs include $0.172 for supplement and minerals, } \$0.078 \text{ for interest, and } \$0.050 \text{ for power, fuel, and miscellaneous variable costs. Cost estimates based on "Suggested farm budgeting costs and returns" Iowa Cooperative Extension Service, (PUBL.) FM 1186 (revised), January 1978.} \]
Table 4. Optimal least-cost rations per animal including the cost of time for five isoquants at selected price ratios

<table>
<thead>
<tr>
<th>Price per ton of corn silage</th>
<th>$30</th>
<th>$35</th>
<th>$40</th>
<th>$45</th>
<th>$50</th>
<th>$55</th>
<th>$60</th>
<th>$65</th>
<th>$70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of concentrate to corn silage price (DM basis)</td>
<td>1.28</td>
<td>1.10</td>
<td>0.96</td>
<td>0.86</td>
<td>0.77</td>
<td>0.70</td>
<td>0.64</td>
<td>0.59</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Isoquant I (320 to 370 kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>Corn Silage (kg DM)</th>
<th>Concentrate (kg DM)</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>326</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td>1.00</td>
<td>326</td>
<td>0</td>
<td>51</td>
</tr>
</tbody>
</table>

Isoquant II (320 to 400 kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>Corn Silage (kg DM)</th>
<th>Concentrate (kg DM)</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>541</td>
<td>541</td>
<td>80</td>
</tr>
<tr>
<td>1.00</td>
<td>541</td>
<td>541</td>
<td>80</td>
</tr>
</tbody>
</table>

Isoquant III (320 to 425 kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>Corn Silage (kg DM)</th>
<th>Concentrate (kg DM)</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>730</td>
<td>730</td>
<td>108</td>
</tr>
<tr>
<td>1.00</td>
<td>730</td>
<td>730</td>
<td>108</td>
</tr>
</tbody>
</table>

Isoquant IV (320 to 455 kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>Corn Silage (kg DM)</th>
<th>Concentrate (kg DM)</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>971</td>
<td>971</td>
<td>135</td>
</tr>
<tr>
<td>1.00</td>
<td>971</td>
<td>971</td>
<td>135</td>
</tr>
</tbody>
</table>

Isoquant V (320 to 480 kg)

<table>
<thead>
<tr>
<th>Ration</th>
<th>Corn Silage (kg DM)</th>
<th>Concentrate (kg DM)</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1,188</td>
<td>1,188</td>
<td>161</td>
</tr>
<tr>
<td>1.00</td>
<td>1,188</td>
<td>1,188</td>
<td>161</td>
</tr>
</tbody>
</table>

---

The objective is to minimize the sum of silage, concentrate and days cost. Price of concentrate fixed at $0.1132 per kilogram of dry matter. Alfalfa pellets and corn grain prices fixed at $100 per ton and $2.50 per bushel, respectively. Cost per day fixed at $0.30.

Ration is the dry matter proportion of corn silage in the ration.

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of steer. With these expenses fixed at $0.30 per day and concentrate price fixed at $0.1132 per kilogram of dry matter, the objective function for the decision model representing variable costs is minimized for the five different gain levels under a range of silage price situations.

Results of this analysis are summarized in Table 4. Even with per-day expenses included in the objective function, the full-silage ration is optimal for all levels of isoquants up to a silage price of $35 per ton. Only when the price of silage is increased to $40 per ton does the proportion of concentrate increase in the optimal ration. These results are not surprising considering the nature of the days functions used for the analysis. The cubic days functions estimated imply that fewer days are required on the full-silage ration, as compared with the full-concentrate ration, for a given isoquant. But for some isoquants, the minimum number of days is obtained on rations high in concentrate content. However, the lower cost of silage more than offsets the advantage that other rations may have because of faster rates of gain.

In Figure 9, optimal rations for two price situations under Decision Model II are graphed. At concentrate-to-silage price ratios of 0.96 and 1.10, the full-silage ration (R₁, R₉) is optimal for the 50-kg isoquant. Under the price ratio of 1.10, the full-silage rations (R₂, R₃, R₄, and R₅) also are optimal for the next four isoquants.

Under the concentrate-to-silage price ratio of 0.96, the full-silage ration is optimal for the first two isoquants. For higher isoquants, optimal rations (R₆, R₇, and R₈) fall in the convex region of the isoquant.

### Comparison of Decision Models

Rations that are optimal for the two decision models under various concentrate to corn silage price ratios have been obtained. As pointed out earlier, rates of gain do not differ greatly among ra-
Optimal factor usages under different price situations are compared.

Optimal silage levels under the two decision models are graphed in Figure 10 for various prices of silage with concentrate and time costs fixed at $0.1132 per kilogram dry matter and $0.30 per day, respectively. The graph corresponds to a normative factor demand curve for a fixed output level. Optimal silage use under the objective of minimizing total variable costs, Decision Model II, denotes greater silage as compared with minimizing feed costs or Decision Model I at a given price ratio. The days functions, which denote faster gains with the full-silage ration, bring about this result when time is included in costs.

Optimal concentrate levels under various concentrate prices are graphed in Figure 11. Silage price and costs per day are fixed at $30 per ton and $0.30, respectively. Again, less concentrate is used when cost of time is considered than when the objective function ignores cost of time.

Figure 9. Optim al rations under isoquant analysis for Decision Model II.

Figure 10. Optimal use of silage under derived isoquant analysis for 160 kg weight gain per steer.
SUMMARY

This study was initiated to gain more information about the marginal rate of substitution between concentrates and roughages in the beef feeding ration. Data were obtained from a 3-year experiment in which yearling steers were randomly assigned to one of six rations. The rations varied from one composed entirely of whole-plant corn silage and supplement to one containing whole shelled corn grain, alfalfa pellets, and supplement. There were no significant differences in live and carcass grades across rations. Thus, the product was assumed to be homogeneous.

Previous studies have indicated that the beef gain roughage-concentrate isoquant may not be strictly convex to the origin. Therefore, a methodology was developed that can be used to estimate production or gain functions that may yield isoquants of various slopes and configurations.

An estimate of the production function was obtained in polar coordinates. Three alternative functional forms were estimated. One was selected as best fitting the data. Isoquants derived from the selected polar coordinates production or gain function indicate sigmoid isoquants, with a concave region extending from the corn silage axis to rations composed of approximately 30 percent corn silage and 70 percent concentrate. Because polar coordinates production functions are difficult to solve, graphs of isoquants were utilized to derive a join line for grafted polynomial production function estimation. Two quadratic functions were grafted at the estimated join line. The grafted functional form permits, but does not force, sigmoid isoquants. The grafted quadratic production function used for economic analysis is:

\[
G = 1.219687 + 0.155840S + 0.150137C - 0.000067S^2 - 0.000022C^2 \quad \text{for } S < 0.5774C
\]
\[
G = 1.219687 + 0.155840S + 0.150137C - 0.000119S^2 - 0.000006C^2 - 0.000056SC \quad \text{for } S \geq 0.5774C
\]

To obtain estimates of the time (number of days) required to achieve weight gain on alternative rations, days functions were estimated for each of five selected weight intervals. Information contained in the estimated grafted quadratic production function and the estimated days functions was utilized in the economic analysis.

One objective in the economic analysis was to minimize the cost of corn silage, corn grain, and alfalfa pellets. A second objective was to minimize the cost of corn silage, corn grain, alfalfa pellets, and time. In general, the rate of gain as estimated by the days functions did not differ greatly across rations. Hence, results from the two objectives were very similar.

Economic analysis was conducted on isoquants derived from the grafted quadratic production function. Because of the concave region in the isoquants, either full corn silage rations or rations high in concentrates, with a minimal amount of roughage, are generally optimal. If the price per unit dry matter of concentrate is more than 1.10 times the price per unit dry matter of corn silage, the full corn silage ration is optimal for both objectives.
REFERENCES


