AN IMPROVED ULTRASONIC TECHNIQUE FOR THREE DIMENSIONAL INCLUSIONS

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INTRODUCTION

Conventional C-scan ultrasonic techniques are excellent for imaging two dimensional defects such as delaminations, cracks, and plate type inclusions [1]. This technique is most effective when these defects are oriented parallel to the scanning plane. However, for three-dimensional inclusions difficulties are encountered when the conventional scanning technique is adopted. If one wants to reconstruct the three dimensional inclusion from generated C-scan images at different depths a significant amount of error is introduced in its size and shape. Even for a simple 3D object such as a sphere it is often difficult to accurately estimate its size and location from its C-scan generated image.

The V(z) curve method, used in acoustic microscopy, is an effective quantitative method for investigating planar objects [2] and not fully developed for nonplanar objects. The aim of this paper is to extend the V(z) curve method to spherical objects. It is shown that from the positions of different peaks in the V(z) curves one can determine the size and acoustical parameters of solid spherical particles in a liquid or a solid.

THEORY

First we consider the spherical particle in an immersion liquid.
The output signal generated by the focused beam scanning of this object has been given in [3]. It was shown in [3] that for small particles (ka < 1, where k is the wave number in liquid and a is the sphere radius) the output signal is proportional to the intensity of incident acoustical field at the point where the sphere is located.

The axial (V(z) curve) and off-axial (V(r) curve or image curve) output signals for large rigid spheres (ka(1-cosα) > 1, where α is the half aperture angle of the focused beam) are shown in Fig.1 a,b.

One can see in Fig.1a that for two positions of z the V(z) values become large - when the focus coincides with the top surface of the sphere (maximum at the point z=-a in fig.1a) and with the sphere center (maximum at the point z=0 in fig.1a). We will denote these two maxima as Top Maximum (TM) and Center Maximum (CM). The distance between these two maxima is exactly equal to the sphere its radius a [3].

The image curves or V(r) curves of the rigid sphere for these two positions (z=-a and z=0) are shown in Fig.1b. The image sizes for both z positions do not coincide with the actual size of the sphere. The size of the central image (z=0) is equal to Δc=0.3λ/sinα (λ is the wavelength in liquid) that is the half of the Airy spot radius [4,5]. This image does not depend on the sphere size and is determined only by the parameters of the focused beam. The size of the top image (z=-a) is approximately equal to Δt=a sinα [5]. But one can see from fig.1b that for z=-a the V(r) curve slowly decreases with r, hence it is difficult to determine exactly the sphere size from this image curve. On the other hand the distance between two peaks of the V(z) curve is exactly equal

![Diagram](image)

Figure 1. (a) - V(z) curve for a large rigid sphere, (b) - V(r) curves of this sphere when focus coincides with its top (z=-a, solid line) and center (z=0, dashed line). ka = 100, α = π/6.
to the sphere radius $a$ (fig.1a). Thus this simple example shows that for spherical object characterization the $V(z)$ curve method is preferable than the C-scan technique.

The $V(z)$ curve for a solid spherical particle has four well-defined maxima (Fig.2). Besides Top Maximum and Center Maximum two additional maxima arise at the positions for which the focal points of longitudinal and shear waves into the solid sphere coincide with its back surface. The distances from Central Maxima ($z=0$) and these two maxima depend on the size of the sphere and its acoustical properties and are equal to

$$Z_i = \frac{a}{2C_{1}^{i}C - 1}$$

(1)

Here $Z_i$ ($i$ is $l$ or $s$) are the positions of the Longitudinal Maximum (LM) or the Shear Maximum (SM), $C_{1}^{i}$ is the longitudinal or shear wave speed, and $C$ is the longitudinal wave speed in the immersion liquid. Thus from the positions of the maxima of the $V(z)$ curve for a solid spherical particle one can determine its size as well as elastic properties.

This method may also be applied for characterization of spherical inclusions in solid. Let the spherical inclusion center coincides with the spherical focused transducer axis and be placed at a distance $z_s$ from its center of curvature. The radius of the inclusion is $a$. The distance between the solid-liquid interface and the sphere center is $d$.

The total output signal is calculated as the sum of the signal reflected from the solid-liquid interface and the signal scattered by the inclusion. Besides, it is convenient to write the output signal expressions corresponding to different types of waves which arrive on
the transducer surface at different times. The first signal that is received by the transducer is the wave reflected from the liquid-solid interface. This signal has the standard form of \( V(z) \) curve \([2]\) and we will denote it as \( V_R \) (R for reflection). The first wave from the inclusion that is received by the transducer is the signal that is transmitted as the longitudinal wave into the solid and scattered by the inclusion again as the longitudinal wave. We will denote this wave as \( \text{ll-wave} \) and corresponding output signal voltage will be denoted as \( V_{ll} \). After \( \text{ll-wave} \) two types of waves reach the transducer simultaneously. These two waves are \( \text{lt} \) (transmitted into solid as the transverse wave and scattered by the inclusion as the longitudinal wave) and \( t_l \) (this is reverse of \( \text{lt-wave} \)). The output signal corresponding to these two waves is \( (V_{lt} + V_{tl}) \). The final wave received by the transducer is \( \text{tt-wave} \). This wave is transmitted into the solid as the transverse wave and scattered by the inclusion as the transverse wave also. The voltage corresponding to this signal is \( V_{tt} \).

Then the total voltage \( V \) is equal to

\[
V(z) = V_R + V_{ll} + (V_{lt} + V_{tl}) + V_{tt},
\]

(2)

Where

\[
V_{ll} = \frac{V_0}{1 - \cos \alpha} \frac{\rho_s c_l}{\rho_c} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{\tau_{ll}^n}{L_n^2},
\]

(3)

\[
(V_{lt} + V_{tl}) = \frac{V_0}{1 - \cos \alpha} \frac{\rho_s c_t}{\rho_c} \sum_{n=0}^{\infty} (-1)^n (2n+1) (c_{lt} \tau_{lt}^n + c_{tl} \tau_{tl}^n) L_n T_{ll}^{SV},
\]

(4)

\[
V_{tt} = \frac{V_0}{1 - \cos \alpha} \frac{\rho_s c_t}{\rho_c} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{\tau_{tt}^n}{(T_{ll}^{SV})^2}
\]

(5)

Here \( V_0 \) is the output signal of the transducer when a perfect reflector is placed at the focal plane, \( \rho \) and \( \rho_s \) are the densities of the liquid and the solid respectively, \( c_l \) and \( c_t \) are the longitudinal and the transverse wave speeds in the solid, \( c \) is the wave speed in immersion liquid, and \( \tau_{ij} \) are the scattering coefficients for the spherical inclusion \([6]\). The integral coefficients \( L_n \), \( T_{ll}^{SV} \) depend on the inclusion position and elastic properties of the solid and can be expressed as

\[
L_n = \int_{-\alpha}^{\alpha} L(\theta) \exp(i\Phi_x) P_n(\cos \theta) \sin \theta \, d\theta,
\]

(6)

\[
T_{ll}^{SV} = \int_{-\alpha}^{\alpha} T(\theta) \exp(i\Phi_x) \frac{\partial P_n(\cos \theta)}{\partial \theta} \sin \theta \, d\theta,
\]

(7)
where $P_n(\cos \theta)$ is the Legendre polynomials, $k$ is the wave number in the immersion liquid, $k_1$ and $k_t$ are the wave numbers of the longitudinal and the transverse wave in the solid, $\phi_1 = -k(z_1 + d)\cos \theta_1 + k_1 d \cos \theta_1$, $\phi_t = -k(z_t + d)\cos \theta_t + k_t d \cos \theta_t$, and the angles $\theta_1, \theta_t$ follow Snell's law, $k \sin \theta_1 = k_1 \sin \theta_1 = k_t \sin \theta_t$.

The $V(z)$ curves for large spherical cavities in plexiglas and aluminum are shown in Fig. 3a, b. One can see that for plexiglas the $V$ due to $11$ signal is larger than that due to $1t$ or $tt$ signal, which almost coincide with the horizontal axis. This is because the transverse wave speed in plexiglas is less than the wave speed in water and the incident focused beam of the transverse wave is diverged in plexiglas. The $V(z)$ curve for aluminum from $tt$ wave is larger than those from other components scattered by the cavity. This is because in aluminum the transverse wave speed is about two times but the longitudinal wave speed is four times the wave speed in water.

The $V_{11}(z)$ and $V_{tt}(z)$ curves for a large cavity have two maxima. These two maxima arise when the focus of the incident beam coincides with the top surface of the cavity (position $z_t$) and with its center (position $z_c$) respectively. For a spherical object in the immersion liquid the distance between these maxima is equal to its radius, but for a spherical cavity in a solid this distance is different because of the focused beam aberrations. From equations (3) and (5) one can show that for small aberrations $z_t$ and $z_c$ for the longitudinal and the transverse waves are equal to

$$-100 \quad -50 \quad 0 \quad 50 \quad 100 \quad 150$$

$$0.0 \quad 0.2 \quad 0.4 \quad 0.6$$

Figure 3. $V(z)$ curves for spherical cavities, (a) in plexiglas, $ka=30$, $kd=50$, (b) in aluminum, $ka=50$, $kd=100$, 1 - $11$ wave, 2 - $1t$ wave, 3 - $tt$ wave. $\alpha = 30^\circ$. 

127
\[ z_{l,t} = -d + \frac{(d - a) \cos \theta_{0}^{l,t}}{\sqrt{\cos^{2} \theta_{0}^{l,t} - 1 + (c/c_{l,t})^{2}}} , \quad (8) \]

\[ z_{c} = -d + \frac{d \cos \theta_{0}^{l,t}}{\sqrt{\cos^{2} \theta_{0}^{l,t} - 1 + (c/c_{l,t})^{2}}} , \quad (9) \]

where \( \cos \theta_{0}^{l,t} = (1 + \cos \theta_{m}^{l,t})/2 \), and \( \theta_{m}^{l,t} = \alpha \) if \( (c/c_{l,t}) \sin \alpha < 1 \) and \( \theta_{m}^{l,t} = \arcsin(c/c_{l,t}) \) for opposite case. Hence one can determine the true radius of the cavity from \( z_{l} \) and \( z_{c} \).

It should be noted that the \( V_{l}^{t}(z) \) curve has only one peak because in the case of focusing at the center of the cavity there is no transformation from longitudinal to transverse wave for a normally incident wave on the cavity surface. When aberrations increase (due to larger difference in wave speeds in solid and liquid or due to greater depth of the cavity location) the \( V(z) \) curve becomes flatter making it difficult to determine the peak positions. One can see this situation for \( V_{l}^{t}(z) \) curve for a cavity in aluminum (fig. 3b). But in this situation it is possible to obtain the cavity size and its depth using the positions of the maxima of \( V_{l}^{t}(z) \) and \( V_{t}^{t}(z) \) curves and waves reflected from the solid-liquid interface, that is \( V_{R}^{t}(z) \) curve.

EXPERIMENT

The digitally controlled acoustic microscope used in this work has been described in reference [7]. It is based on the mechanical scanner with lateral resolution 10 \( \mu \)m and axial resolution 0.3 \( \mu \)m. RF pulse burst of several periods length at a frequency of 25 MHz were used for ultrasound excitation.

For investigating the \( V(z) \) curve of a spherical particle in an immersion liquid the aluminum ball was glued to 20 \( \mu \)m maylar film or was placed into gel with almost the same acoustical properties as the immersion liquid (water). To create the spherical inclusion in solid the bearing ball was encapsulated into epoxy.

Experimental \( V(z) \) curves for aluminum sphere 3.38 mm radius are shown in Fig.4. The number of periods in the impulse excitation is adjusted such as we can receive separately the signals reflected from the top and the bottom surfaces of the sphere. The first reflected impulse in Fig.4 is the z-dependence of the output signal the specular reflected wave. The second and the third reflected impulse are the signals from the longitudinal and the shear wave into sphere reflected from its back surface. The parameters of the particle obtained from \( V(z) \)
curves (radius \(a = 3.35\) mm, longitudinal and shear speed \(c_l = 6360\) km/sec and \(c_t = 3110\) km/sec) are in good agreement with real values (\(a = 3.38\) mm, \(c_l = 6420\) km/sec, and \(c_t = 3040\) km/sec).

The experimental curves for the bearing ball of 1mm diameter with its center located at 2.5mm depth from water-epoxy interface are shown in Fig.5a,b. The theoretical curves \(V_R(z)\) from water-epoxy interface and \(V_{11}(z)\) from the ball are also presented in Fig.5a for rectangular pupil function \((P(\theta) = 1, \text{ if } \theta < \alpha, \text{ and } P(\theta) = 0 \text{ if } \theta > \alpha)\). One can see that the distance between the peaks of the theoretical \(V_{11}(z)\) curve is greater than that of the experimental curve. According to the equations (8) and (9) this distance depends on the ball radius, the relation between the wave speeds in water and epoxy, and the aperture angle of the focused beam. At the same time the experimental \(V_R(z)\) curve from the epoxy-water interface is wider than that of the theoretical curve, and experimental \(V_R(z)\) curve has almost nonoscillating form. It means that the wave amplitude on the lens surface is not constant and it decreases from the center of the lens to its edge. So the effective angle of the focused beam is less than its geometrical one.

To overcome this shortcoming we have used Sasaki pupil function [8] to describe approximately the amplitude distribution on the lens surface

\[
P(\theta) = \begin{cases} 
1 + \frac{\cos(\pi \frac{1-cos\theta}{1-cos\alpha})}{2}, & \text{if } \theta < \alpha \\
0, & \text{if } \theta > \alpha
\end{cases}
\]

(10)

The theoretical \(V_R(z)\) and \(V_{11}(z)\) curves with Sasaki pupil function are presented in Fig.5b. One can see good agreement for both curves between theory and experiment.
Figure 5a,b. The experimental $V(z)$ curves for the bearing ball in epoxy ($a = 1\, \text{mm}, d = 2.5\, \text{mm}$). Frequency 25 MHz, $\alpha = 22^\circ$. 1 - experimental $V_R(z)$ curve, 2 - experimental $V_{11}(z)$ curve, 3 - theoretical $V_R(z)$ curve, 4 - theoretical $V_{11}(z)$ curve. (a) - rectangular pupil function, (b) - Sasaki pupil function [8].

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