Design and provisioning of WDM networks for traffic grooming

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Design and provisioning of WDM networks for traffic grooming

by

Raza Ul-Mustafa

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This is to certify that the doctoral dissertation of
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has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

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Signature was redacted for privacy.

For the Major Program
DEDICATION

I would like to dedicate this thesis to my parents, for their love and encouragement, and to my wife Nadia and to my son Affaan, for their love and support.
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ABSTRACT

Wavelength Division Multiplexing (WDM) is the most viable technique for utilizing the enormous amounts of bandwidth inherently available in optical fibers. However, the bandwidth offered by a single wavelength in WDM networks is on the order of tens of Gigabits per second, while most of the applications' bandwidth requirements are still subwavelength. Therefore, cost-effective design and provisioning of WDM networks require that traffic from different sessions share bandwidth of a single wavelength by employing electronic multiplexing at higher layers. This is known as traffic grooming. Optical networks supporting traffic grooming are usually designed in a way such that the cost of the higher layer equipment used to support a given traffic matrix is reduced. In this thesis, we propose a number of optimal and heuristic solutions for the design and provisioning of optical networks for traffic grooming with an objective of network cost reduction. In doing so, we address several practical issues. Specifically, we address the design and provisioning of WDM networks on unidirectional and bidirectional rings for arbitrary unicast traffic grooming, and on mesh topologies for arbitrary multipoint traffic grooming. In multipoint traffic grooming, we address both multicast and many-to-one traffic grooming problems. We provide a unified framework for optimal and approximate network dimensioning and channel provisioning for the generic multicast traffic grooming problem, as well as some variants of the problem. For many-to-one traffic grooming we propose optimal as well as heuristic solutions. Optimal formulations which are inherently non-linear are mapped to an optimal linear formulation. In the heuristic solutions, we employ different problem specific search strategies to explore the solution space. We provide a number of experimental results to show the efficacy of our proposed techniques for the traffic grooming problem in WDM networks.
CHAPTER 1 INTRODUCTION

In this chapter we provide a brief overview of optical networks and their different generations. We also discuss the research challenges in these networks, including the traffic grooming problem and the challenges it poses. Finally, we discuss the contribution of this dissertation, and its organization.

1.1 Optical Networks: Background

Optical fiber communication systems offer a huge bandwidth as compared to copper cables. Also, they are less susceptible to electromagnetic interferences. The first transatlantic optical communication system, TAT-8, was installed in 1988, operating at 140 Mbps. Since then, in almost 15 years, advances in optical communication technology have facilitated transmission speeds exceeding 1 Tbps [1]. The advances in optical devices, transmission systems, and network theory, combined have enabled this to happen. Currently, two spectral regions, centered at 1300 nm and 1550 nm, of an optical fiber are used for transmission. Using single mode optical fibers, the effective transmission windows available at these spectral regions correspond to 14 THz and 15 THz of potential frequency space for transmission.

To utilize such a vast amount of bandwidth, one requires high-speed electronics. The current electronics, however, cannot handle switching beyond a few gigabits per second. In order to utilize this bandwidth the transmission spectrum of an optical fiber is divided into multiple independent channels, which can be used to support multiple transmissions. This technique is called Wavelength Division Multiplexing (WDM). Since, WDM channels operate at rates in the tens of gigabits per second range, they can be supported by state-of-the-art electronics, thus achieving aggregate rates in the terabits per second range. Hence, the huge
bandwidth reserve offered by the optical fibers, is efficiently utilized by WDM. Figure 1.1 shows how \( N \) signals, each on a different wavelength, are multiplexed together using a WDM multiplexer.

![WDM multiplexer diagram](image)

Figure 1.1 Wavelength division multiplexing.

A popular style in the literature to categorize optical networks is to divide their evolution into two phases: first generation optical networks in which optical fiber was used as a mode of communication while all the processing happens at the electronic level, and second generation optical networks in which some of the decisions take place in the optical domain. We too follow the same categorization here.

### 1.1.1 First Generation Optical Networks

First generation optical networks employ fiber only as a transmission medium. These networks essentially replace copper cables with optical fibers. The key feature of first generation optical networks is that all processing is carried out in the electronic domain. The electronics at a node must handle all data intended for that node as well as all data passing through the node and destined to other nodes in the network. This essentially puts a limitation on the transmission speed of such networks due to the electronic processing bottleneck. Examples of first generation optical networks are Synchronous Optical Networks (SONET) and Synchronous Digital Hierarchy (SDH) networks, computer interconnects, such as Enterprise Serial
Connection (ESCON), Fiber Channel, and High-Performance Parallel Interface (HIPPI), and metropolitan area networks, such as Fiber Distributed Data Interface (FDDI) and gigabit Ethernet.

SONET networks probably are the most popular among first generation networks. They incorporate a variety of functions. For example, they provide point-to-point connections between different node pairs in the network. Also, they provide add/drop functionalities, such that only a part of the streams is dropped at a node, and the rest can pass through. Also, SONET networks consist of cross-connects, which can switch multiple traffic streams. Moreover, one of the attractive features of SONET networks is their fault tolerant capabilities, where they handle node and link failures without disrupting the services.

1.1.2 Second Generation Optical Networks

Since the first generation optical networks are in place, an increasing realization that optical networks are capable of providing more functions than just point-to-point transmission led to the emergence of second generation optical networks. Second generation optical networks use WDM technology to split the huge bandwidth provided by a fiber into multiple wavelength channels, that can be used to support multiple transmissions simultaneously. Also, some of the switching and routing functions that are performed by the electronics in first generation optical network can be carried out in the optical domain in second generation optical networks. Second generation optical networks offer different types of services to the higher network layers. The most commonly used service is the lightpath service. A lightpath is a dedicated connection on a wavelength between two nodes in the network, such that no electronic conversion takes place on the path between these two nodes. Moreover, second generation optical networks offer transparency, i.e., they are insensitive to the nature of the coding or modulation techniques used over the lightpaths.

Since, second generation optical networks employ WDM technology, they are also known as WDM networks. A typical WDM link is shown in Figure 1.2. It consists of a set of transmitters, optical amplifiers and receivers. The transmitters are indeed lasers, such that one
laser per wavelength is used. The outgoing signals from different transmitters is multiplexed together using a multiplexer. The power amplifier immediately after the multiplexer amplifies the combined signal. The signal after travelling some distance on fiber may need amplification again due to attenuation; this task is carried out by a line amplifier. Finally, at the destination, the combined signal is amplified again and demultiplexed. Due to demultiplexing, the signal is split into different wavelengths which are converted to the electronic domain using photodetectors, where each photodetector is tuned to a specific wavelength.

WDM network architectures can be classified broadly into two categories: broadcast and select networks, and wavelength routing networks. In a broadcast and select WDM network, the nodes can normally transmit at different wavelengths. The signals are broadcast by a passive device, which is either an optical star coupler or a bus. In case of a coupler, the signals are combined from all nodes by the coupler and the combined signal is transmitted on each of its output ports using a fraction of the power of each signal. When using a bus topology, each node employs a set of couplers to transmit and receive from the bus. The number of the couplers and their usage is different in both topologies. In a star coupler topology, there is a single \( n \times n \) star coupler for a total of \( n \) nodes in the network, which can also be implemented using \( n/2 \log_2 n \) 2 × 2 couplers. A corresponding bus topology requires 2\( n \) 2 × 2 couplers. In a broadcast and select WDM network, routing is provided to all nodes by default, and hence,
no routing function is provided by the network. This is one of the features that differentiates broadcast and select networks from the wavelength routed networks, in which, as the name implies, the network provides the routing functionality. A broadcast and select WDM network, using a star coupler, is shown in figure 1.3.

![Figure 1.3 A broadcast and select network with a star coupler.](image)

Broadcast and select WDM networks are suitable for local-area or metropolitan-area networks. However, these networks are limited by the number of nodes they can support because wavelengths cannot be reused in the network (at a time, a specific wavelength can be used by only one node in the network,) and also because the transmitted power from a node must be split among all the receivers in the network.

In case of wavelength routing networks, the nodes in the network are capable of routing different wavelengths at an input port to different output ports. This feature enables many nodes in the network to transmit simultaneously at the same wavelength using spatial diversity. For example, a simple wavelength routed network is shown in Figure 1.4. Nodes A and D have established a lightpath on wavelength $\lambda_1$, while nodes A and E have established a lightpath on wavelength $\lambda_2$. The lightpath between nodes B and E can also be established on wavelength $\lambda_1$ because this wavelength has not been occupied on this link earlier. Also note that the lightpath between node A and node D is using the same wavelength on both links between nodes A and C, and between nodes C and D. This restriction to maintain the same wavelength on all the
physical links traversed by a lightpath is called *wavelength-continuity* constraint. However, with the use of optical wavelength converters at some nodes this constraint can be relaxed. We will illustrate this point with the help of an example. Figure 1.5 is similar to Figure 1.4, except that node A is equipped with a wavelength converter. Say we need to establish a new lightpath between nodes B and C. For the network shown in Figure 1.4, we need a new wavelength, as $\lambda_2$ is occupied on a link between nodes B and A, and $\lambda_1$ is occupied on a link between nodes A and C. However, in the network shown in Figure 1.5, we do not need a new wavelength, as we can use wavelength $\lambda_1$ on the link between nodes B and A, convert $\lambda_1$ into $\lambda_2$ optically using the wavelength converter at node A, and then use wavelength $\lambda_2$ on the link between nodes A and C. Thus, wavelength converters in a network can save on the total number of wavelengths in a network while supporting a given traffic matrix. However, the trade-offs between the cost of wavelengths and wavelength converters need to be taken into consideration.

An inside view of a typical wavelength routed network is shown in the Figure 1.6. Each node is equipped with an Optical Cross Connect (OXC) that can route wavelengths from one input port to another output port without converting them into the electronic domain. The wavelengths that are dropped at a node are handed over to the higher layer. The higher layer employs Light Terminals (LTs) to terminate the optical signal and convert it into electronic form. In the figure we assumed a SONET layer operating on the top of the optical layer and
Figure 1.5 A wavelength routing network with a wavelength converter at node A.

hence label the LT as an ADM. One ADM is needed for each dropped wavelength. The ADMs themselves may or may not be connected to a Digital Cross Connect (DCS).

Figure 1.6 A wavelength routed network with a SONET layer operating over it.
Most wide area networks employ wavelength routing networks. In wide area networks nodes are interconnected by point-to-point links and hence, unlike broadcast and select networks, the signals are transmitted over only a subset of the links.

The networks are generally viewed as consisting of many layers, each layer performing specific functions. For example, Internet Protocol (IP) can be thought of as a network layer that uses several underlying layers, such as an Ethernet or a token ring local area network, to provide services to the layers residing above it in the layered hierarchy. WDM networks introduce another layer to the layering hierarchy, the Optical layer. As most of the services provided by the WDM networks are due to the presence of the optical layer, we will discuss it below.

1.1.3 The Optical Layer

In first generation optical networks the physical layer is merely an optical fiber providing a single wavelength. In second generation optical networks, the physical layer is much more intelligent and is capable of providing many services. The components of a typical optical layer are shown in Figure 1.7. The layering structure is somewhat similar to the SONET layer. It consists of three sub-layers. The top most sub-layer is the Optical Channel (OC) layer. This layer provides end-to-end routing of lightpaths. Each lightpath may traverse several physical links, and may use one or more wavelengths depending on the availability of the optical wavelength converters in the network. The middle sub-layer is the Optical Multiplex Section (OMS) layer, and takes care of the issues related to each point-to-point link along the route of a lightpath. Each point-to-point link basically consists of many link segments, each between a pair of optical amplifier stages. Each link segment between a pair of optical amplifiers constitutes an Optical Amplifier Section (OAS) layer. The optical layer resides at the bottom of the network layer hierarchy. The upper layers could be SONET, Asynchronous Transfer Mode (ATM) or IP. However, most of the current networks employ SONET on top of the optical layer [2], while IP-over-WDM [3] is also gaining grounds. In case of a SONET network, the lightpaths are a replacement of the hard-wired fiber connections between SONET
The optical layer provides the following functionalities:

- It provides transparency, which means that the lightpaths can carry data at different data rates, different formats, follow different protocols etc. This functionality indeed helps an optical layer to support many different layers operating on top of it concurrently. This basically means that some lightpaths can be carrying SONET data while others are carrying ATM or IP data.

- It provides wavelength re-useability. As different lightpaths may be traversing different physical links, the non-overlapping lightpaths at the physical level can use the same wavelength simultaneously.

- It provides reliability at the optical level. The network can be configured in a way that in the case of failures some automated process re-routes the connections on the failed link(s) or node(s) onto alternative paths.

However, although the optical layer is efficient at processing traffic at a wavelength granularity, it is not very good at processing traffic at subwavelength granularities.
1.2 WDM Network Challenges

In this section we will discuss major challenges regarding the design and operation of WDM networks. Under the heading of WDM network, we will cover only wavelength routed networks and not the broadcast and select networks.

In general the design of the WDM networks is an off-line activity, where a designer is supplied with a projected static traffic matrix and is required to design and provision the corresponding WDM network that satisfies the traffic matrix while meeting an objective, such as minimizing the cost of the network. Alternatively, he may be given the network architecture and his task is to either fully accommodate the given traffic matrix using the least number of network resources, or to maximally accommodate the given matrix using available resources. The network design problem involves solving many problems most of which are NP-Complete.

On the other hand, operation of WDM networks is an on-line activity in which the network has already been designed and most probably supporting some traffic. The task here is to either accommodate as many new traffic requests while optimizing the network resource usage, or to accommodate all the new traffic with a minimum number of additional resources. The distinct classification of each problem into off-line and on-line, however, is sometimes blurred. The design and operation of the WDM networks basically address the following issues:

- For a given number of resources, how much traffic can be accommodated?
- How many resources are needed to fully accommodate the given traffic matrix?
- How much does each component help in improving the capacity of the network? For example, how much benefit one can get by deploying wavelength converters in the network?
- How to route and assign wavelengths to traffic requests in a way that maximizes some objective function?
- How to provide protection to either all or critical traffic requests, while achieving some objective, such as optimization of network resources?
• How to take into account the cost of the higher layers while making decisions at the optical layer?

The last issue will be explored in detail in this dissertation. The design and operation of the WDM networks depends to a great extent on the traffic patterns that are to be accommodated. There could exist many possibilities in this regard. Here we present the most commonly used traffic models.

• Fixed Traffic Model: In this case all the traffic requests are know a priori. Mostly, a traffic matrix $C = [c(i, j)]$ represents the set of traffic demands, where $c(i, j)$ represents the amount of traffic units from node $i$ to node $j$. Such a matrix is used in a typical off-line problem. The task is to accommodate the whole matrix using the least number of resources, or to accommodate the maximum amount of this traffic using given resources.

• Maximum Load: In this case, a parameter load, defined as the maximum number of lightpaths on any link in the network, is given. The problem comes in two flavors. In case of offline requests, a fixed traffic matrix is given, and the maximum load model reduces to the fixed traffic model except with the additional restriction of not exceeding the load. In case of online requests, the lightpaths arrive one at a time and need to be accommodated without any knowledge of the future requests. However, the lightpaths need to be accommodated without exceeding the load parameter. This model is helpful in determining the efficiency of a provisioned network with no, limited or full wavelength conversion. If a network with full wavelength conversion is provisioned with load $L$, then the additional number of wavelengths required to accommodate the same traffic matrix with no wavelength conversion available in the network, can be used to quantify the benefit of wavelength conversion.

• Statistical Model: In this case, the nature of arrivals and departures of the traffic is known, or assumed, to follow some stochastic process. Analytical methods can then be used to either drive bounds, or exact or approximate solutions for the dimensioning of the network resources to support the traffic. Alternatively, given network resources,
blocking probabilities can be determined for the traffic pattern. The blocking probability is defined as the number of requests that are blocked over a long time period, divided by the total number of the requests in that time [4].

We will now briefly discuss a few challenges that arise in the design and operation of the WDM Networks.

1.2.1 Routing and Wavelength Assignment

The Routing and Wavelength Assignment (RWA) problem is defined as: given a network topology, a set of end-to-end lightpath requests, determine how to route those requests, and which wavelengths they should be assigned using the minimum possible number of wavelengths [4]. This is a very well studied problem [6, 7, 8, 9, 10, 11, 12, 13, 14] and is known to be NP-Complete\(^1\) on arbitrary topologies[4]. Sometimes routing is either given or is straightforward, e.g., in a unidirectional ring. In such cases, the RWA problem reduces to solving the wavelength assignment problem only. The wavelength assignment for a network must satisfy two constraints, namely, no two lightpaths on the same physical link be assigned the same wavelength, and if wavelength conversion is not available, then wavelength continuity constraints be observed on all the links that a lightpath traverse. Interestingly, even the wavelength assignment problem is NP-Complete [15].

1.2.2 Virtual Topology Design

The virtual topology design problem studies the issue of how a higher layer views the lightpaths provided to it by the optical layer. In most cases, for a higher layer the lightpaths provided by the optical layer are like physical links between its node pairs, thus called virtual links. The network topology consisting of virtual links is called virtual topology or logical topology. The virtual topology design problem basically consists of two design levels. The levels are interdependent, however, in most approaches they are handled as two independent problems. At the bottom level is the design of the physical topology itself, while at the top

\(^{1}\)For a detailed discussion of NP-Completeness please see [5].
level is the design of the virtual topology which should be supported by the physical level. One of the advantages of virtual topology design is that as its (virtual) links are lightpaths which can be re-configured to meet changes in the traffic, the very same physical topology can be used without tearing apart the physical links of the network. The virtual topology design problem too is a well studied problem in optical network and a large body of literature exists in this area, e.g., [16, 17, 18, 19]. The virtual topology design problem is also NP-Complete [19].

1.2.3 Survivability

Survivability of a network refers to a network's capability to provide continuous service in the presence of failures [20]. In a WDM network, as a single channel may be carrying tens of gigabits of data per second, a single failure would cause service disruption to a large number of users. Design of survivable WDM networks has therefore attracted the attention of the research community. The basic types of failures in the network can be categorized as either link or node failures. Link failure usually occurs because of cable cuts, while nodes failures occurs because of equipment failure at network nodes. Beside link and node failures, channel failure is also possible in WDM networks. A channel failure is usually caused by the failure of transmitting and/or receiving equipment operating on that channel (wavelength).

Survivability approaches can be broadly classified into two main categories: protection and restoration. Protection is a pro-active approach that uses a pre-assigned capacity between nodes in order to replace the failed entities in the network. Restoration, on the other hand, is a reactive approach that uses any capacity and/or alternative equipment available between nodes in order to replace the failed entity in the network. Restoration schemes are based on re-routing algorithms to find a new path to recover failed network entities, once the failure occurs. Compared to restoration techniques, protection techniques are much faster but the utilization of the network resources by protection techniques is less efficient. They are classified as $1+1$, $1:1$, and $1:N$ to represent the number of the entities reserved for protection. For example, in a $1+1$ protection scheme, the signal is sent simultaneously over two disjoint lightpaths.
The receiver receives the signal from both lightpaths and selects the better one. In a 1 : 1 protection scheme each primary lightpath has a dedicated backup lightpath, however, the backup lightpath is used only in a case of failure. In 1 : N protection scheme, N primary lightpaths share a common backup lightpath. These schemes are quite similar to those of SONET networks [20]. For example, in point-to-point optical WDM systems, 1 + 1, 1 : 1, and 1 : N optical protections are used in a way similar to Automatic Protection Switching (APS) in SONET systems, except that switching is done in the optical domain. However, in WDM systems, due to the availability of multiple wavelengths on a single fiber, protection methods can be more flexible. This, however, makes the problem more complicated.

WDM networks provide survivability by providing fault-tolerance at the optical layer. However, this has its own merits and demerits. Here, we will provide a brief account of such aspects:

• Optical layer protection and restoration may be used to provide an additional level of resilience in the network. For example, many transport networks are designed to handle a single failure at a time, but not multiple failures. Optical restoration can be used to provide resilience against multiple failures.

• Some of the layers operating above the optical layer may not be fully able to provide all the protection functions needed in the network, hence optical layer survivability techniques can fill this deficiency.

• Optical layer protection can be more efficient at handling certain types of failures, such as fiber cuts. Handling such failures at a higher layer, say SONET, will overwhelm the network as a large number of SONET connections need to be restored.

• Optical layer protection cannot handle all types of faults in the network. For example, it cannot handle the failure of an IP router or a SONET ADM attached to the optical network.

• The optical layer protects traffic in units of lightpaths. Thus owing to its transparency, the optical layer cannot provide different levels of protection to different parts of the traffic.
being carried on a lightpath (part of the traffic may be high-priority, while another may be low-priority).

Many approaches have appeared in the literature in the field of survivability in optical network. For a brief list, interested readers are referred to [20, 21, 22, 23, 25, 24, 26].

For a detailed study of WDM networks we recommend the books in references [4, 27, 28].

1.3 Design and Provisioning of WDM Networks for Traffic Grooming

WDM networks provide a vast amount of bandwidth per wavelength. While 10 gigabits per second is the most common transmission rate that is commercially available, the technology for 40 gigabits rates is already available. Most of the applications currently supported by optical networks, however, may not even require the entire amount of bandwidth provided by a full wavelength. Hence, to exploit the bandwidth available on one wavelength, multiplexing different sets of traffic demands per wavelength is the natural approach. Traffic grooming is defined as an intelligent allocation of subwavelength tributaries onto a set of full wavelengths in such a way that leads to efficient utilization of network resources. Earlier, the emphasis was to reduce the number of required wavelength channels while grooming the traffic. However, with the realization that the dominant cost factor in traffic grooming is not the number of the wavelengths, but rather the higher layer line terminating electronic components [57], most of the research now is focused on reducing the total number of the electronic components in the network while accommodating a given set of traffic. Depending on the layer on the top of the optical layer, the line terminals (LTs) can be of any type, for example, SONET Add/Drop multiplexers (ADMs), or IP or MPLS router ports. Without loss of generality, in this dissertation, we will use the terms LT and ADM interchangeably.

To understand how traffic grooming can reduce the network cost, let us consider a 5-node linear topology network, shown in Figure 1.9. Let each node be equipped with an Optical cross connect (OXC). Each OXC is in turn connected to a number of LTs. Having an OXC on each node will help only drop those wavelengths that carry the traffic to, or from that specific node. A wavelength can bypass a node if it carries no traffic that is terminated at
that node. This will result in the saving of an LT. Hence, our objective is to minimize the total number of LTs used in the network to support all the traffic by intelligently assigning traffic to wavelengths. Let \( g \) denote the total number of basic units of traffic supported by each wavelength. For example, if a wavelength supports an OC-12 connection, and the basic unit of traffic is an OC-3, then \( g = 4 \). For illustrative purpose, we assume that \( g = 2 \) in Figure 1.9. Traffic demands or connections are shown by the line segments between the source and destination nodes. Also, black circles are used on the edges of the segments to indicate that an LT is used at the corresponding node for a specific wavelength. To minimize the total number of required LTs, out of many possibilities, we could have the following two allocations of traffic to wavelengths.

(a) \( \lambda_1: 1\leftrightarrow2, \lambda_2: 1\leftrightarrow5; \lambda_3: 2\leftrightarrow4; \lambda_4: 2\leftrightarrow5; \)

(b) \( \lambda_1: 1\leftrightarrow2, \lambda_2: 1\leftrightarrow5; \lambda_3: 3\leftrightarrow4; \lambda_4: 2\leftrightarrow5; \)

Assignments (a) and (b) are shown in Figure 1.9(a) and 1.9(b), respectively. For the first assignment the total number of required LTs is 10, while for the second assignment the total number of LTs is 9. Note that in the second assignment the traffic between nodes 1 and 2, and
between 2 and 4 are sharing the same LT. This example shows that by assigning the traffic to appropriate wavelengths, one can reduce the number of required LTs. Also, given very high cost of LTs\(^2\), even saving a few LTs translates into savings of hundreds of thousands of dollars.

Traffic grooming in WDM networks involves both design and provisioning of the network, as one needs to determine both the lightpaths carrying the traffic demands, and the amount of capacity claimed by each traffic demand on these lightpaths. However, the above problem is different from the traditional network provisioning and dimensioning problem, in which the total number of wavelengths in the network is minimized [29, 30]. The latter usually assumes that the basic traffic requirement is a full wavelength. Network providers can then use multiplexing to share a wavelength among many users, thus reducing the total number of required wavelengths. In this dissertation, the network design goal is not just to minimize the total number of the wavelengths, but is rather to minimize the overall network cost by minimizing the electronic cost at network edges and by making efficient use of wavelengths.

1.4 Research Challenges in Traffic Grooming

The general problem of traffic grooming with arbitrary traffic has been proven to be NP-Complete [44]. The authors in reference [44] considered a very simple example where all the traffic on a unidirectional ring is destined to a single node, and proved that even this instance of the problem is NP-Complete. In fact, the complexity of the traffic grooming problem is far more than that of the traditional Routing and Wavelength Assignment (RWA) problem [58]. The RWA problem is solvable within polynomial time, for the path and star topologies. The authors in reference [58], however, showed that the decision version of the traffic grooming problem in unidirectional paths is also NP-Complete, thus leading to the conclusion that the general traffic grooming optimization problem is NP-hard. Hence, designing efficient schemes to minimize the network cost, given the high complexity of the problem is a major challenge to the research community.

Currently, WDM networks are extending their presence from merely in backbone networks\(^2\)a single port SONET ADM costs approximately $40,000.
to metro-area and access networks. This poses many other research challenges regarding the current and future trends in the field of traffic grooming in WDM networks. Most of these challenges arise due to the following dimensions:

1. Although most of the current WDM networks are configured as a regular topology, such as rings, such settings are very restrictive and may lead to wastage of resources. In many cases the deployed networks indeed are forming a mesh topology. However, due to the lack of algorithms for the design of WDM networks for traffic grooming on mesh topologies, in such cases network operators superimpose a regular topology, such as network of rings, on top of the given topology. Hence to utilize the network resources efficiently, while designing the networks for traffic grooming, arbitrary mesh topologies need to be considered.

2. Assumptions like, all nodes transmitting to only one node, or through one node, or having the same amount of traffic between all nodes in the networks are not practical cases for metro-area and access networks, and even for backbone networks. Therefore, an arbitrary amount of traffic between any node pair needs to be considered.

3. In spite of the high complexity of the problem, more realistic models need to be developed to reflect the practical networks. For example, designing a virtual topology without the knowledge of its corresponding physical topology, or assuming a very large number of wavelength channels between each node pair, or splitting the traffic from a source into multiple streams to ease the analysis, may not reflect real world networks.

The interest in traffic grooming problem in WDM networks started a few years ago. The general trend at the beginning, naturally, was to solve the problem for simple topologies and with restricted traffic patterns. Researchers focused on the path, unidirectional, and bidirectional rings, and devised heuristics, computed bounds, and developed Integer Linear Programs (ILPs) for the design and/or operation of WDM networks while performing traffic grooming. Recently a few researchers are considering arbitrary topologies too. Also, to make the problem somewhat less difficult, many relaxations have been considered in the literature. For example,
most of the studies allow the traffic between each source-destination set to be (vertically) split over multiple wavelengths - a condition known as bifurcation. Due to bifurcation different components of the same traffic demand may traverse different links. This provision provides flexibility in traffic allocation, which may lead to a reduction in the number of wavelengths as well as the number of ADMs. Although allowing bifurcation simplifies the problem, it is unrealistic since it increases the complexity and the cost of traffic reassembly, and may also introduce jitter at the application layer. Many applications, especially real-time applications, require that their traffic be kept intact, i.e., without demultiplexing at the source, independent switching at intermediate nodes, and multiplexing at the destination. Therefore, there is a need for considering more realistic models while developing the solutions for traffic grooming in WDM networks.

Furthermore, the major portion of current research activity is the area in centered around the unicast traffic only. In the foreseeable future, a sizable portion of the traffic is expected to be multipoint [31]. Multipoint traffic can take different forms, e.g., one-to-many and many-to-one. One-to-many, or multicast applications, include multi-party conferencing, video distribution, network news distribution, collaborative processing, and web content distribution to proxies [31]. On the other hand, the many-to-one service model is used by applications such as resource discovery, data collection, auctions, polling and audience to speaker communication. There is a critical need to develop solutions for the design and provisioning of WDM networks that can handle multipoint traffic, while performing cost-effective traffic grooming.

Currently, most optical networks are provisioned every few months [4], and as such the traffic can be considered static. However, the entry of WDM networks in access networks necessitates consideration of dynamic traffic too. However, in this dissertation we consider only the static traffic case.

A significant amount of research has been carried out in providing fault-tolerance in WDM networks. However, most of the schemes work with an inherent assumption that the applications' bandwidth requirements are integer multiple of a single wavelength. In case of traffic grooming, many applications could be sharing a single wavelength. Hence, schemes need to be
developed that provide fault tolerance while exploiting the dynamics of traffic grooming.

1.5 Contributions of this Dissertation

In this dissertation we address a few of the challenges discussed in Section 1.4. Our intention is to start with simple models and evolve into more complex but realistic models for the design and provisioning of WDM network considering static traffic grooming. We start with special topologies, but consider arbitrary traffic and allow traffic bifurcation. We propose approximate schemes for non-uniform traffic grooming on unidirectional as well as bidirectional rings. We then include the arbitrary topologies and ensure non-bifurcation. We develop models to handle multicast traffic, and propose optimal as well as approximate solutions. We explore a few practical variants of the multicast traffic grooming problem and propose solutions for them. We then develop models for many-to-one traffic grooming, also proposing optimal as well as approximate solutions. The main thrust of this work is to develop more realistic and generic models and provide corresponding solutions. The generic models reduce to many special practical cases. We summarize the contribution of the work in this dissertation as follows.

- It provides solutions for the design and provisioning of WDM networks for non-uniform traffic grooming on unidirectional and bidirectional rings such that the traffic between a node pair could be asymmetric.

- It provides a unified framework for optimal network dimensioning and channel provisioning for multicast traffic grooming. The framework encompasses the generic multicast grooming problem and a few of its variants.

- It provides a generic model to dimension and provision the network while grooming many-to-one traffic demands, and minimizing the network cost. The grooming model provides the flexibility in aggregating the traffic streams with arbitrary ratios, such that the ratios are a function of the number of the traffic streams participating in the aggregation.
• It provides optimal as well as approximate solutions to the unicast, multicast or many-to-one traffic grooming problems.

• In case of optimal solutions routing, wavelength assignment and traffic grooming is solved in an integrated way, to achieve global minima.

• For optimal solutions Mixed Integer Linear Programs (MILPs) are developed. Few problems that are inherently non-linear are solved in a linear fashion by exploiting the specifics of the problems. In particular, in case of many-to-one traffic grooming, the non-linearity which arises due to the dependency of the aggregation factor on the number of the traffic streams participating in the aggregation, is removed by decoupling the routing and aggregation sub-problems, while still maintaining the optimality of the solution. The decoupling strategy can be used in many other problems.

• The heuristic approaches use a myriad of techniques. In some cases the problems are solved by first making innovative approximations and mapping the problem into other well-known problems, and then using the knowledge of the well-known mapped domain. In other cases, some specifics of the problems are exploited, for example, cost reduction by following non-shortest paths, or cost reduction by exploring different wavelength assignments. Also, a hybrid of general-purpose and problem specific techniques are developed.

• The solution approaches maintain non-bifurcation of the traffic demands. Most of the techniques proposed in the literature bifurcate the traffic demands while provisioning the network. However, as we discussed in Section 1.4 it is more realistic not to bifurcate traffic demands. One proposed technique [34] solves the non-bifurcation problem for special cases only, but does not solve the general non-bifurcation problem as in Appendix C. This problem will be elaborated on further in Chapter 4.

• Network cost is minimized by minimizing multiple resources instead of a single resource. The introduced models minimize the total number of required LTs, and simultaneously reduce the total number of the required wavelengths in the network.
The developed models, and the corresponding solutions, are generic and can be reduced to many interesting cases of the traffic grooming problem.

1.6 Dissertation Organization

The rest of the dissertation is organized as follows. In Chapter 2, we provide an overview of the related work in the literature on traffic grooming in WDM networks. In Chapter 3, we consider unidirectional and bidirectional rings and propose approximate solutions. In Chapter 4, we address the multicast traffic grooming problem, by developing a unified framework. We discuss the generic multicast grooming problem and few of its variants under this framework. Also, optimal as well as approximate solutions are proposed in this chapter. In Chapter 5, we address the many-to-one traffic grooming problem and provide the optimal as well as approximate solutions. In Chapter 6, we conclude this dissertation and outline future research directions in the field of traffic grooming in WDM networks.
CHAPTER 2 LITERATURE REVIEW

In this chapter we provide an overview of the literature available in the field of traffic grooming in WDM networks.

Traffic grooming in WDM networks is a comparatively newly developed field. To the best of our knowledge the earliest papers that explicitly dealt with the topic appeared in 1998 [38, 40, 35, 41]. There are currently several survey papers addressing different issues in the design and operation of WDM networks with traffic grooming, e.g., [32, 33, 34].

Research on traffic grooming can be categorized according to: (1) traffic types, e.g., static traffic and dynamic traffic, (2) traffic patterns, e.g., uniform traffic and arbitrary traffic, (3) network topologies, e.g., unidirectional rings, bidirectional rings, and random topologies, (4) solution approaches to the traffic grooming problem, e.g., exact, heuristics, and bounds, (5) objective functions, e.g., minimization of network resources, network resource utilization, blocking probability etc.

In traffic grooming the subwavelength tributaries are groomed on a set of available wavelength while achieving some objective. For the cases in which the objective function is the minimization of the network cost, most researchers are now convinced that the cost of the higher layer components is the dominant cost factor and not the total number of the wavelengths. Advocacy for the minimization of the number of the higher layer components, instead of wavelengths, actually has started even in case of the wavelength routed networks that does not employ traffic grooming [35, 36, 37].

It should be mentioned that there is a significant amount of literature on the virtual topology design domain, as mentioned in Chapter 1. Many researchers find the logical/virtual topology design problem helpful in solving, or at least conceptualizing, the traffic grooming
problem. However, both problems are significantly different, and the traffic grooming problem is considered harder than the virtual topology design problem [57, 58].

To review the literature in the field of traffic grooming on WDM networks, we will first review works that employ static traffic, and sub-categorize it based on topology, i.e., ring and mesh topologies. We will then review some studies made in the area of dynamic traffic grooming. Finally, we will present, in separate sections, works related to survival traffic grooming and multipoint traffic grooming problems. Also, a list of the references, related to research work in static and dynamic traffic grooming, is provided in Table 2.1.

### 2.1 Static Traffic

In this section the literature pertaining to traffic grooming on WDM networks under static traffic patterns will be reviewed.

In case of static traffic, a fixed traffic matrix is given, i.e., all the demands are known a priori. Given a physical topology, the task is to design and provision the network such that the network cost is minimized. The traffic matrix itself further can be classified into uniform traffic demands, distance dependent traffic demands, and non-uniform traffic demands. In the case of uniform traffic demands, all the nodes in the network have the same number of traffic

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**Table 2.1** References related to work in static and dynamic traffic grooming.

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units between each other. In case of distance dependent traffic, the number of traffic units between a node pair depends on the distance between them; a node shares more traffic with the neighboring nodes then the nodes farther from it. We will call both uniform and distance dependent traffic as fixed traffic pattern. In case of non-uniform traffic demands, the number of traffic units between node pairs is arbitrary, which will be called arbitrary traffic patterns.

2.1.1 Ring Topology

Some of the earliest studies in the area of traffic grooming in WDM networks were produced by Simmon, Goldstein, and Saleh [38, 42, 59], Gerstel, Lin and Sasaki [39, 43], Chiu and Modiano [40, 44], and Zhang and Qiao [41, 53]. All such studies advocated that to reduce the network cost more emphasis should be placed on minimizing the number of higher layer components than the number of wavelengths. However, this problem is a hard problem, and it was proven to be NP-Complete by Chiu and Modiano [44], and by Li et al. [49].

2.1.1.1 Fixed Traffic Patterns

In this subsection we will review the literature related to the design of WDM networks for traffic grooming on ring topologies using fixed traffic patterns.

Simmon, Goldstein, and Saleh provided some initial work in [38] and later refined it in [42]. In both references they considered ring topology with fixed traffic pattern. Specifically, in [42], the authors considered uniform all-to-all traffic and distance dependent traffic between nodes connected as a ring topology. They provided upper bounds on the number of ADMs and also provided procedures to obtain these bounds. They consider the cases in which there is no traffic grooming, i.e, each traffic demand occupies the whole wavelength, and more involved cases when traffic demands are 1/2, 1/4, 1/8 and 1/16 of a wavelength, and computed the required number of ADMs and wavelengths. They also introduced an interesting concept of super-node where nodes were grouped under different super nodes such that the amount of traffic between super nodes occupied the full wavelengths. The traffic between super nodes then can be routed using procedures developed for full wavelength traffic. The nodes, however,
were grouped into super nodes using approximations, and hence the overall solution would be an approximate one. They also mentioned that the number of ADMs may further be reduced by grooming within each super node, however, they did not elaborate on how this can be done. They mentioned that to reduce the number of the ADMs, more connections need to be routed optically at each node, and thus quantified the benefit of using wavelength add-drops by measuring the *through-to-total* ratio of connections at each node in the network. Another important contribution of this work was the realization that minimizing the number of ADMs will not only reduce the cost of the network, but also some cases like, IP over WDM, such saving may imply a substantial simplification in packet routing.

Reference [43] focused on obtaining bounds on the number of SONET ADMs when nodes are placed on Unidirectional Path-Switched Ring (UPSR) and Bidirectional Line-Switched Ring (BLSR). They, however, restricted their analysis to uniform traffic between the nodes. Their emphasis was to reduce the cost of the entire system by minimizing the total number of SONET ADMs in the network. They considered single-hub UPSRs, and assumed that low speed traffic can be cross-connected at nodes. They computed upper bounds on the number of the SONET ADMS and showed that using the bounds for cross-communication on a single hub UPSR, the number of the SONET ADMs in the network can be reduced over that of non-hub architecture. However, the down side of using single-hub architecture is the increase in the number of wavelengths. They also demonstrated that although spatial reuse of the bandwidth in BLSR/2 ring networks makes it less expensive to implement than UPSR, in a few cases UPSR does a little better than BLSR/2 in terms of the number of ADMs required to accommodate the traffic. Finally, they considered a network architecture that has different line speeds, and computed the required number of wavelengths and ADMs. Specifically, they considered a network that has a mix of OC-48 and OC-12 SONET rings, and has a single-hub architecture. They computed the number of wavelengths and ADMs required for such an architecture under uniform traffic, and concluded that the cost per node, in terms of number of ADMs required at that node, in most cases is lesser when mixed line-speeds were used.

In [40], Chiu and Modiano considered all-to-all uniform traffic and computed the upper
bounds when the nodes in the network are connected through a unidirectional SONET/WDM ring. They proposed a heuristic approach whose performance was close to their computed upper bounds. In [44], the same authors extended the work in reference [40]. They proved that the traffic grooming problem is NP-Complete by considering an example of a unidirectional ring such that arbitrary amounts of subwavelength traffic from all nodes in the network are destined to a single node. They proved that even this simple instance of the problem is NP-Complete. After that they considered all-to-all uniform and distance dependent traffic on WDM unidirectional ring networks. They propose an optimal solution for a special case, namely, when the traffic from all nodes is destined to a single node, and all traffic rates are the same. The solution was polynomial-time, because this special case is not NP-Complete. For general all-to-all uniform and distance dependent traffic they proposed some heuristic approaches. They also provided two results that are sometimes misunderstood by the research community. They showed that there exists an optimal solution such that no traffic from a node is split onto two rings. Few researchers interpret this result to hold for all topologies and traffic types. However, they showed this result to hold only when all the traffic was heading towards a single node, i.e., a hub node. Otherwise, it can be shown with the help of a simple example that if all the nodes are equipped with switching capabilities, traffic splitting can reduce the number of ADMs as well as the number of wavelengths. In Appendix B, we provide such an example. The other often misunderstood result mentioned in [44] is that the number of wavelengths and ADMs cannot be minimized simultaneously. Again, few researchers interpret this result as if it means that while minimizing the ADMs, the number of wavelengths cannot be reduced. However, in fact the authors in [44] also stated that it is possible to have a solution that has the minimum number of wavelengths as well as the ADMs. Their conclusion only suggests that while minimizing the number of ADMs, the wavelength minimization could occur at the cost of an ADM, and vice versa. Hence, when the ADM is an expensive resource, then it is feasible to minimize the number of ADMs while also reducing the number of wavelengths, but not at the cost of an ADM.

In reference [45], the authors computed lower bounds on the number of wavelengths in
a BLSR network under static all-to-all uniform traffic. Also, they computed lower bounds on the number of ADMs in unswitched UPSR rings for uniform all-to-all traffic. They made the observation that properly balancing the utilization of ADMs across the nodes tends to minimize the total number of ADMs in the network. For UPSR they also presented a heuristic approach that first constructs full circles, and later grooms these circles onto wavelengths.

The authors in reference [46], used tools from graph theory and design theory to address the traffic grooming problem in unidirectional WDM ring networks. They considered uniform all-to-all traffic demands among the nodes of the networks, i.e., there is exactly one request of a given size between each node pair in the network. They showed that if $N$ be the number of nodes in the network then the problem of minimizing the number of ADMs for uniform all-to-all traffic demands can be expressed by partitioning the edges of a complete graph on $N$ vertices into $W$ subgraphs, where $W$ is the number of wavelengths, such that the total number of vertices in each subgraph is minimized. Each subgraph in this case represents a wavelength, while the edges in each subgraph correspond to ADMs. They then used design theory to compute the number of ADMs required for many possible combinations of grooming factor and the number of nodes in the network.

Some researchers used meta-heuristics to solve the traffic grooming problem on WDM ring networks. In reference [47], the authors used Simulated Annealing as a part of the solution approach. They considered WDM ring networks with uniform traffic and presented two different approaches. In one approach the traffic needs to be routed through a hub node in the network. They defined this approach as a multihop approach, because each traffic demand delivery needed to employ more than one lightpath hop. Also, they consider the case in which traffic can be routed without going through any hub node. They defined such an approach as single-hop approach. For the multihop approach they first place ADMs on the nodes for each request and then try to re-assign the traffic such that the maximum number of ADMs and wavelengths can be saved. For the single-hop approach they employed a two-phase approach. In the first phase they constructed the virtual circles, while in the second phase they used Simulated Annealing to group the circles into different wavelengths. Based on simulation re-
suits, they argued that it was beneficial in terms of the number of ADMs to use the single-hop approach while employing simulated annealing for small grooming ratios. However, for large grooming ratios and large networks the multihop approach could lead to better savings in terms of the number of ADMs.

The authors in reference [48] explore an interesting variant of the ring topology. They considered a topology in which many rings are connected to each other forming a WDM multi-ring network. They classify the traffic connections as either intra-connection or as inter-connection. Using intra-connections they constructed intra-connection rings. Similarly, inter-connections were used to construct inter-connection rings. Considering only uniform traffic, they explored four different ways of combining intra-connection rings and inter-connection rings to reduce the total number of the SONET ADMs in the network.

2.1.1.2 Arbitrary Traffic Patterns

In this subsection we will review the literature related to design of WDM networks for traffic grooming on ring topologies that used arbitrary traffic between the pair of nodes.

Reference [49] extended the work done in [43] by focusing only on single-hub SONET/WDM rings, but considering non-uniform traffic too. First, they proved that given a set of traffic demands, the single-hub BLSR/2 costs no more than the single-hub UPSR under any traffic pattern\(^1\). They then showed that optimal traffic grooming can be confined to a narrow subset of valid grooming strategies, and referred to these as canonical groomings. One of the contributions of their work is to prove that the traffic grooming problem is NP-Complete. For this purpose, they reduced the bin-packing problem instance when the size of the objects are fractional, to a single-hub ring architecture with non-uniform traffic demands. The problem setting and corresponding proof bear many resemblances to that laid out in reference [44]. Furthermore, they showed that grooming on single-hub BLSR/2 can be solved using bin-packing approaches, and proved that the First-Fit-Decreasing (FFD) algorithm [50] produces a 10/9

\(^1\)Compare this with [43] in which they showed that in few cases UPSR does a little better than (non-hub) BLSR/2. Both results are not contradicting, as the results in [49] are for single-hub ring only, while results in [43] are for non-hub ring.
approximation ratio. Moreover, for integer traffic demands and a grooming factors of 2, 4, and 8, they also presented optimal algorithms using a strategy similar to that of FFD.

In reference [52], the authors considered SONET/WDM BLSRs, while taking into account non-uniform traffic. They defined each traffic stream to consist of unitary traffic demand. As each node pair was allowed to establish a variable number of traffic streams, they conclude that such a setting permits non-uniform (arbitrary) traffic. They extended the work presented in reference [51] by considering two versions of the minimum ADM cost problem. In the first version each traffic stream followed the shortest path routing, while in the second version the routing of each traffic stream is not known in advance. For the second version, however, they assumed that each traffic stream is full duplex with symmetric and unitary demands, which must be routed along the same path but in the opposite direction. In terms of solution approaches, they presented some lower bounds, and also presented a set of heuristics. The basic idea is to first construct primitive rings from the unitary traffic demands and then partition them into a number of groups such that each groups consists of a set of primitive rings whose number does not exceed the grooming factor. The paper did not include any performance study of the heuristics.

Probably, one of the most cited work in the literature about traffic grooming is [53]. In this reference, the authors extended their earlier work in [41] and proposed heuristic approaches to solve the traffic grooming problem on unidirectional and bidirectional rings, while considering both uniform and non-uniform traffic. Also, they presented bounds on the number of wavelengths and ADMs for the uniform traffic case. The main idea of the heuristics is to follow a two-phase approach. In the first phase circles are constructed using traffic streams, and in the second phase those circles are groomed onto the set of the available wavelengths. Three heuristics were presented to construct circles for uniform traffic, and one heuristic for the construction of circles for non-uniform traffic was also introduced. In addition, a heuristic for circle grooming was provided.

A Genetic Algorithm based approach was used by the authors in reference [54] to solve the traffic grooming problem in unidirectional SONET/WDM rings with arbitrary and asymmetric
traffic. They chose an interesting representation of the chromosome by making use of order-based approach, which essentially determines the order in which each traffic stream will be considered for grooming. Given a chromosome, they decoded the chromosome using a greedy heuristic, which worked like a first-fit bin packing algorithm. New offsprings were generated using a \((\mu + \lambda)\) strategy, in which \(\mu\) parents are used to generate \(\lambda\) children and all \((\mu + \lambda)\) solutions compete for the next generation.

Some researchers employed linear programming tools to determine the optimal solution for the traffic grooming problem. In reference [55], the authors developed an ILP for bidirectional WDM rings allowing arbitrary traffic between node pairs. They also presented approximate solutions using Simulated Annealing and a greedy approach. In reference [56], the author also developed an ILP to minimize the total number of ADMs on a WDM ring network. He then reduced the complexity of the problem by relaxing a few integer variables to real variables but without compromising the solution quality for uniform traffic only. Extension to incorporate non-uniform traffic, using integer variables, is also provided. Finally, he also illustrated how his formulation can be extended to include dynamic traffic. Results were provided using fixed traffic only. In [57] the authors formulated an ILP for the traffic grooming problem on unidirectional rings. The objective of the optimization problem was to minimize electronic routing. An interesting contribution of this work was the formulation of a sequence of bounds, where each successive bound in the sequence was tighter, or at least as good as, the previous one. Each successive bound, however, involved more computation. The underlying principle in obtaining bounds was to decompose the ring into many sets of nodes such that in each set nodes were arranged in a path, and then adopting a locally optimal topology within that set.

### 2.1.2 Mesh Topology

The ubiquity of SONET rings was the impetus behind the surge of studies of traffic grooming in ring networks. However, deployment of optical mesh networks, and the use of SONET ADMs, and IP and MPLS routers, for traffic grooming in such networks motivated researchers to consider mesh topologies.
In reference [59] the first study of traffic grooming on mesh topologies was carried out. The authors applied the concept of clustering nodes into super nodes, developed earlier in reference [42] by the same authors, to mesh networks. However, the architecture of the mesh network was not arbitrary, and was rather a grid. The objective in this case, however, was not to minimize the number of LTs or the number of wavelengths. The authors argued that the value of optical bypass in an optical network can be measured by the fractional decrease in required router size. The size of the router was defined as the capacity required to handle the sum of the router traffic. So if the router receives $T_i$ amount of traffic from all other routers, and $T_L$ amount of traffic from directly connected LTs, then the capacity of the router should be $T_i + T_L$. As an optical bypass reduces the amount of traffic handed over to higher layers for electronic processing, increasing the amount of optical bypass leads to decreasing the size of the router. By plotting their computed upper bounds, the authors showed that optical bypass significantly reduced the size of the router. The curve in the plots showed that the fractional saving increases with the increase in the size of the network. They thus claim that such a technique would scale well in future networks.

Reference [60] considered arbitrary mesh networks while making a few simplifying assumptions. It defined **opaque nodes** as nodes that can perform O-E-O conversion and **optical nodes** as nodes that do not perform O-E-O conversion. The paper, therefore, considered a special type of the network consisting of optical and opaque nodes such that each optical node had only opaque neighbors. The set of optical nodes thus formed an independent set. Minimizing the number of LTs in such a network is now equivalent to finding a maximum weighted independent set, which is an NP-Complete problem. The authors then presented a heuristic to approximately determine a maximum weighted independent set. Note that the authors did not consider static traffic, but rather incremental traffic, i.e., traffic that can come at any time, but does not leave the network.

A very interesting linear formulation for traffic grooming problem on mesh topologies is given in reference [61]. The authors made some simplifying assumptions that may not reflect realistic networks. There focus was to design a virtual topology that supports the given traffic
matrix, i.e., to determine the set of lightpaths between node pairs to support the traffic matrix, while minimizing the number of transceivers in the network. They assumed that a large number of fibers existed between each node pair and hence any virtual topology can essentially be supported by the underlying physical topology\textsuperscript{2}. Once such a simplification was made, the task reduced to the determination of a virtual topology only, for which they provided an ILP. Also, they presented a heuristic approach to solve the virtual topology problem using minimum cost flow problem. Note that their formulation allowed bifurcation of the traffic streams. They provided some limited experimental results.

In [62], the authors presented an iterative greedy algorithm that aimed at reducing the total number of lightpaths while accommodating the given traffic set. The basic idea of the algorithm is to allocate the residual capacity of already routed lightpaths to a traffic stream, while creating a new lightpath only if residual capacities are not enough. To allow the earlier assignments of traffic streams to take benefit of lightpaths created after their assignment, they implemented an iterative approach. The heuristic approach, however, bifurcated each traffic demand into multiple basic units, and each basic unit then could be routed along different paths in the network.

Reference [63] used an abstraction model, namely, Blocking Islands, to solve the traffic grooming problem on mesh topologies. The paper also presented an ILP for the design of WDM networks with mesh topologies in order to support traffic grooming such that the number of transceivers is minimized. The ILP is an improvement over the one presented in reference [61] because, unlike [61], during the design of the virtual topology the authors also considered the physical topology restrictions. Although the number of transceivers was minimized by minimizing the number of the lightpaths, in reality it is also the maximization of the sharing of end points of the lightpaths, which lead to the minimization of the total number of LTs in the network. Finally, the study showed that, given a fixed number of transceivers at each node, the Blocking Islands based heuristic approach can accommodate more connections than other approaches reported in the literature.

\textsuperscript{2}In general, not all virtual topologies are supported by a given physical topology.
An interesting variation of the traditional traffic grooming problem on mesh topologies is presented in reference [64]. The authors considered a multi-layer network model, consisting of optical layer, electronic layer and OEO junction. They developed an ILP with a multi-objective function. Requests were allowed to be routed either entirely through the electronic layer, or optical layer or through both using OEO junctions. However, the size of the ILP was formidable and the authors did not present any results from the ILP. They also proposed a heuristic and presented some results. A part of their objective was to also minimize the cost of the allocated capacity to cover the 1:1 protection or 1:N protection routes, and hence this work can be categorized as belonging to the survivable traffic grooming problem.

One of the most cited works in the literature, regarding traffic grooming on WDM mesh networks, is in reference [65]. The authors presented an ILP that maximizes the network throughput on a mesh topology. They view the network at three levels: (i) The lowest level consists of physical topology, (ii) the second level consists of virtual topology made up by lightpaths such that each lightpath may consist of multiple physical links, and (iii) the highest level consists of connections such that each connection may use multiple lightpaths to deliver the traffic from source to destination. They also presented two heuristics that try to maximize the one hop traffic and the utilization of the lightpaths, respectively. Although, the authors claimed that their formulation ensured non-bifurcation, in Appendix C we show that this is not the case. Later in Chapter 4 we will show that how non-bifurcation can be achieved.

2.2 Dynamic Traffic Grooming

In case of dynamic traffic, all the traffic demands are not known a priori. Requests arrive one at a time, and need to be accommodated without waiting for the future requests. Given a physical topology, the task is to design and provision the network such that either the blocking probability can be minimized or the network resource utilization can be maximized.

Some early work on dynamic traffic grooming was reported in references [39, 66, 67, 68]. Few analytical models, using Markov processes, have also been proposed to study the performance of different grooming strategies under dynamic traffic conditions. Many researchers,
however, took heuristic approaches to maximize the number of admitted calls, or to minimize the blocking probabilities, given a specific resource set.

In reference [66], the authors considered three different traffic models and analyzed the performance of six different OADM ring networks while supporting one or more of the traffic models. They considered dynamic non-uniform traffic with pre-computed routes, dynamic non-uniform traffic with restrictions on the amount of traffic that terminates at the nodes, and uniform static traffic. The OADM ring network architectures they considered are fully optical ring, single-hub ring, double-hub ring, point-to-point WDM ring, hierarchical ring, and incremental ring. All architectures supported uniform and one of the other traffic type. For each architecture they computed bounds on the number of wavelengths, average number of transceivers per node, and maximum number of hops of lightpaths. Using static uniform traffic they compared all six architectures, and concluded that on one extreme the single-hub ring required a large amount of bandwidth but incurred a small transceiver cost, and on other extreme the point-to-point WDM ring required minimal bandwidth but was the most expensive architecture in terms of the number of the transceivers. However, the hierarchical, double-hub and the incremental ring architectures, provided a trade-off between the number of the wavelengths and the number of the transceivers.

In reference [67] to study the performance of different types of grooming nodes in a mesh network, an analytical model was developed using discrete-time markov chains. Two types of nodes were considered: wavelength-selective crossconnect nodes (WSXC), which cannot groom at subwavelength level and wavelength-grooming crossconnect nodes (WGXC), which can groom at the subwavelength level by switching the traffic streams within a wavelength to another wavelength. The other assumptions made were: routing is fixed-path, i.e., each request uses a pre-specified path, wavelength assignment is random, and link and wavelength independence exists, i.e., events are independent on different links of the network, and wavelength usage is independent of utilization of other wavelengths on the same link or on other links. Performance of the networks that have only WSXC and networks in which few of the nodes are of type WGXC were studied, to evaluate the efficiency of grooming at the subwavelength
level. From the analytical and simulation results, it was concluded that the networks with WGXC nodes had lesser blocking probabilities than the networks with WSXC nodes, thus re-assuring that networks perform better by allowing grooming at the subwavelength level. Also, it was reported that the performance of networks carrying WGXC nodes was more profound on mesh-torus networks as compared to that of ring networks.

In reference [68], the authors defined a set of traffic matrices that reflects the changes in the traffic over a specified time period. The task was then to minimize the number of electronic ADMs needed to support any traffic matrix in this set; hence essentially supporting the traffic which dynamically changes within the given set of matrices. They considered a unidirectional ring. They also assumed that the traffic matrices are t-allowable, i.e., no node is sourcing more than t units of traffic at any time, where t is a specified constant. They then started out by placing an ADM on each wavelength at every node, and provided heuristics that remove the maximum possible number of ADMs, while supporting every t-allowable traffic matrix. They mentioned that their heuristic approaches can save up to 27% of the number of ADMs.

An analytical model was developed in reference [69] for single-hop traffic grooming in WDM mesh networks. A single-wavelength link-blocking model was considered and was modelled using a Markov process. Using bulk arrivals, the arrival of a call was converted into multiple fictitious micro calls. If the number of idle servers in the system exceeded the number of micro calls, the corresponding call was accepted; otherwise the call was blocked. Moreover, no wavelength conversion was considered, and alternate routing and sequential wavelength assignment was employed. The analytical model was found to be accurate when compared to simulation results.

A novel graph model was introduced in reference [70]. Considering a heterogeneous WDM mesh network environment, the authors captured the different abstract layers in the network with the help of a single auxiliary graph. The presence of certain equipment or functionality at a node was represented by the edges to (from) that node. The novel feature of the auxiliary graph was that different objective functions under different grooming policies could be represented by drawing the edges as per the corresponding physical constraints and assigning
appropriate weights to the edges. They presented two procedures that together can solve many grooming problems by running shortest-path algorithms on the auxiliary graph. The model works for both static and dynamic traffic grooming. The work presented by the same authors in reference [71] also uses the same concept of auxiliary graph, however, they considered different grooming OXC architectures. They presented four such architectures: (1) single-hop grooming OXC, which can switch only at wavelength granularity and also supports low-data-rate interfaces (2) multi-hop partial-grooming OXC, which can switch at wavelength granularity using a wavelength-switch fabric, and can also switch few, but not all, of the wavelength channels at subwavelength granularity (3) multi-hop full-grooming OXC, which can switch all the wavelengths channels at subwavelength granularity, and (4) light-tree-based source-grooming OXC, which has the capability to duplicate the traffic from one input port to multiple output ports, using opaque (O-E-O conversion) or transparent (no O-E-O conversion) technology. They also presented two algorithms to perform grooming using auxiliary graph, and compared the network performance of each type of OXC.

In reference [72], the authors presented a generic framework to collect the link information and then employed it effectively in order to carry out dynamic traffic grooming in a WDM network with heterogeneous grooming capabilities. The framework, named MICRON, collects the path information, interprets it, selects a path, and then carry out sub-trunk (subwavelength) assignment. The abstraction of groups of channels as trunks allow the framework to hold for different levels of grooming, e.g., wavelength level grooming, time slot level grooming, etc.

Reference [73] explored different routing strategies for dynamic traffic grooming. The authors considered Widest-Shortest path routing, Shortest-Widest path routing, Shortest-MaxSum path routing and MaxSum-Shortest path routing. Also, they considered a mesh network. The objective was to evaluate the performance of each routing strategy in terms of network utilization. They defined network utilization as the ratio of effective-used-capacity to the maximum capacity of the network, where effective-used-capacity is the minimum amount of the capacity required by a request, irrespective of the employed routing strategy. They noted the variations in the network utilization for each of the routing strategy by keeping the link
capacity constant but changing the total number of wavelengths and the grooming capacity on that link. It was found that Shortest-Widest path routing offered better network utilization with increased grooming capability. However, Widest-Shortest path routing offered degraded network utilization with increased grooming capability. They conjectured that this could be due to the reason that increased grooming capabilities aids in finding wider paths at the cost of increased path length.

Meta-heuristics have also been used for dynamic traffic grooming problem. Reference [74] employed tabu search to solve the dynamic traffic grooming problem on unidirectional rings. However, the traffic follows an incremental model rather than a dynamic model. The authors assumed that in the beginning a static traffic matrix is given and the network has been provisioned for that traffic matrix. At some later time, another traffic matrix is introduced which is different than the initial one. The objective is to accommodate the new traffic requests by disrupting as few current connection as possible. Two flavors of the problem are considered: best fit, in which they try to place as much new traffic as possible without adding any additional SONET ADMs, and full fit, in which they try to accommodate all the new traffic by adding minimum number of additional SONET ADMs. They presented an ILP, greedy heuristic and tabu search approach to solve the best fit case. They also presented an ILP and a tabu search based technique for full fit case.

2.3 Survivable Traffic Grooming

Survivability of a network refers to a network’s capability to provide continuous service in the presence of failures [20]. Survivable WDM network design is a very active research area. However, most of the earlier work was done with the assumption of full wavelength traffic requirements. Currently, providing survivability with traffic grooming consideration has started to catch the researchers’ attention.

Reference [75] provided some earliest work in this field, in which dependable low-rate traffic streams were established under a dynamic setting. To groom primary and backup streams onto wavelengths, two options were explored. In the first option, the primary and the backup
traffic streams share the same wavelength, while in the second option they occupied different wavelengths. They found that in terms of the blocking probabilities the former option is better for rings, while the latter option is better for mesh-torus networks.

In reference [76], the authors presented a dynamic path-protection traffic-grooming algorithm. In reference [77] and [78], the authors studied the survivable traffic-grooming problem considering both shared protection and dedicated protection. Reference [79] considered a unidirectional ring with all-to-all uniform traffic, such that a specific number of streams need protection. Under such settings, they derived bounds on the number of the ADMs and the number of the wavelengths for different grooming factors and traffic units. Finally, reference [80] developed optimal solutions for survivable WDM network design taking into account the subwavelength traffic demands. Two ILPs were developed, one for backup multiplexing and another for dedicated backup. In one of the ILPs the objective was to minimize the total number of wavelength-links, while in the other ILP the objective was to minimize both the number of wavelength-links as well as link-primary-sharing. They defined link-primary-sharing of a link \((i,j)\) as the total number of primary paths that utilize this link \((i,j)\).

2.4 Multipoint Traffic Grooming

Multipoint traffic consists of multicast (one-to-many) and many-to-one traffic. Multicasting is the ability of a communication network to accept a single message from an application and to deliver copies of the message to multiple recipients at different locations [82]. Thus in a typical multicast routing tree, a single stream of data coming from a source is duplicated onto multiple streams at branching points. This could result in substantial savings in terms of the number of wavelengths and transmitting equipment needed at the nodes in the network. Multicast applications, include multi-party conferencing, video distribution, network news distribution, collaborative processing, and web content distribution to proxies [31].

In a many-to-one service model, a set of sources transmit data to a common destination. The data transmitted by the sources could be correlated or uncorrelated. In case the data transmitted by the sources is correlated, networks resources, e.g., bandwidth and transceivers
at the nodes, can be saved by aggregating and removing the redundant data from different sources on its way to the destination, thus making the bandwidth requirements of a many-to-one routing tree very different than that of a typical multicast tree. The many-to-one service model is used by applications such as resource discovery, data collection, auctions, polling and audience to speaker communication.

A significant amount of literature on multicast in non-optical networks and all-optical networks exists. Please refer to references [81, 82] and [83] for some works on multicast in non-optical networks and all-optical networks, respectively. Research on multipoint traffic grooming has just started to emerge. To the best of our knowledge there exists no earlier work on many-to-one traffic grooming. One of the contributions of this dissertation is that it provides the first work in the design and provisioning of WDM networks with many-to-one traffic grooming. Also our work on the design of WDM networks for multicast traffic grooming, included in this dissertation, pioneered research in this area [84, 85]. Since the publication of this work, two other papers on the topic of multicast traffic grooming have appeared in the literature.

Reference [86], presented a heuristic approach for routing and wavelength assignment of multicast sessions with subwavelength traffic demands on a WDM ring network. The authors assumed that traffic demand of each multicast session is one unit. Also, they allowed duplication of traffic streams at the optical as well as electronic level. However, their objective was to minimize the electronic duplication. For that purpose, they tried different routing strategies, and constructed circles by grouping non-overlapping arcs and combining those circles into different wavelengths. They showed through simulations that their approach leads to lesser equipment cost than that obtained by routing each multicast session along its minimum spanning tree and then using a best known heuristic for circle construction.

Reference [87] solved the multicast traffic grooming in mesh network with sparse nodal light splitting capabilities, with an objective of minimizing the total number of the wavelengths. The authors provided a heuristic solution by providing an ILP to compute the minimum number of the wavelengths on a link, and then estimating the total number of wavelengths for the
entire network. Their model assumed that the multicast routing trees are given. Also, they provide a heuristic approach for constructing multicast routing trees and a first-fit wavelength assignment algorithm to perform traffic grooming.

2.5 Chapter Summary

In this chapter we provided a review of the literature in the area of traffic grooming in WDM networks. Many flavors of the traffic grooming problem in WDM networks have been explored and are still being explored. A significant amount of work has been done on ring topologies using uniform traffic. Some work has also appeared on non-uniform traffic grooming in WDM rings, non-uniform traffic grooming in mesh networks, dynamic traffic grooming, survivable traffic grooming and multipoint traffic grooming. Most of the current research seems to focus more on accommodating arbitrary traffic on ring networks. Also, there is an increasing interest in exploring mesh topologies for traffic grooming. Static traffic seems to remain important, while interest in accommodating dynamic traffic is clearly increasing.
CHAPTER 3 NON-UNIFORM TRAFFIC GROOMING ONTO UNIDIRECTIONAL AND BIDIRECTIONAL RINGS

In this chapter we discuss non-uniform traffic grooming onto unidirectional and bidirectional rings. We present heuristic approaches with the objective of minimizing the number of higher layer equipment, like SONET Add/Drop Multiplexers (ADMs), or IP or MPLS routers. What sets this work apart from other work on non-uniform traffic grooming on rings, is the innovative mapping scheme we employed to map the unidirectional and bidirectional ring onto a linear topology. We then develop a generalized two-step approach to solve the grooming problem on the mapped topology. In the first step, we allocate the traffic while minimizing the possible number of strings (each string being a collection of non-overlapping traffic streams) in a manner that yields the optimal number of strings in the linear topology case. We also prove the optimality of this step in the number of strings (wavelengths). In the second step we employ a grouping technique to efficiently combine $g$ strings onto a wavelength while minimizing the total number of LTs. Another contribution of the work presented in this chapter is the observation that shortest path routing does not always lead to minimum number of LTs. We, therefore, propose an approach to route the traffic in such a way that reduces the total number of required wavelengths and LTs. The time complexity of our technique is at least an order of $n$ less than other proposed approaches, where $n$ is the total number of nodes in the network. We also demonstrate the efficacy of the proposed techniques through a large number of experiments.

The contribution of the work in this chapter is summarized as follows:

- We develop a generic model that can accommodate general non-uniform and asymmetric traffic with an arbitrary number of nodes and arbitrary grooming factor for the design
and provisioning of WDM networks.

- We devise algorithms to solve the traffic grooming problem for our generic model. The proposed algorithms scale well with the problem size, and are efficient in terms of the solution quality, and run time.

- In case of bidirectional rings, we demonstrate that the shortest-path routing does not necessarily lead to minimizing the number of the wavelengths and the LTs, and propose an approach to route the traffic in a way that reduces the total number of the required wavelengths and LTs.

### 3.1 Traffic Grooming on Unidirectional Rings

In this section we define the terms used in the rest of the chapter. Also, in this section we map the unidirectional ring onto a linear topology and develop a two-step approach that handles all types of traffic, uniform and non-uniform (including both symmetric and asymmetric), on the mapped topology. We also show that the first step of our approach is optimal in the number of the wavelengths for a linear topology.

Let the total number of nodes be $N$, and let the traffic matrix $C$ be defined as, $C = [c_{ij}]$ such that $1 \leq i \leq N$ and $1 \leq j \leq N$. Each $c_{ij}$ entry is a set that represents the traffic units between nodes $i$ and $j$ and consists of $|c_{ij}| = n_{ij}$ number of basic units of data, namely, $c_{ij} = \{c_{ij}^{(1)}, c_{ij}^{(2)}, \ldots, c_{ij}^{(n_{ij})}\}$, with $|c_{ii}| = 0$ and $c_{ij} \geq 0$ for $j \neq i$. We call each such basic unit of data, $c_{ij}^{(k)}$, a stream or a connection. Let $r$ represent the rate of the basic stream, e.g., an OC-3 connection. Our objective is to accommodate the demands in the traffic matrix $C$ with the least possible number of wavelengths ($W$), and number of LTs ($U$). In general, by reducing $W$, we will be able to reduce $U$, since for each originating and terminating wavelength at a node (in the same direction) we need an LT. This, however, also suggests that to reduce $U$, besides reducing $W$, we need to allocate most of the traffic to and from a node on as few wavelengths as possible.

To determine the lower bound on the number of wavelengths we will define the term *density*
(θ) as the maximum number of streams on any of the N links. Let link l be the link between nodes l and (l + 1) mod N. Then density θ_l at link l can be defined as:

\[ \theta_l = \sum_{i \leq l, j > l} n_{ij} \]  

(3.1)

Similarly, the density can be defined at a node. Let \( \theta_i \) stand for the density at node i, and \( T_i, S_i, \) and \( P_i \) stand for the number of terminating streams at node i, the number of starting streams from node i, and the number of streams that are passing through node i, respectively. Then:

\[ \theta_i = \max(T_i, S_i) + P_i \]  

(3.2)

\[ \theta = \max_i \theta_i \ 1 \leq i \leq N \]  

(3.3)

The density at a node or link shows that to accommodate the traffic, we need at least this much bandwidth (in terms of the number of streams). Therefore, the minimum number of wavelengths can be determined as:

\[ W_{LB} = \left\lceil \frac{\theta}{g} \right\rceil \]  

(3.4)

To determine a tight lower bound on the number of LTs for non-uniform traffic is, however, quite difficult. In [52], the authors generalized the work done by [35], by incorporating the grooming factor, as follows.

\[ U_{LB} = \sum_{i=1}^{N} \left\lceil \frac{\max(T_i, S_i)}{g} \right\rceil \]  

(3.5)

In [36], the authors proposed an even tighter bound for circular rings as follows.

\[ W_{LBring} = \sum_{i=1}^{N} (T_i + S_i - b_i) \]  

(3.6)

where \( b_i \) is the size of the maximal matching of the bipartite graph, constructed at each node, of starting and terminating streams as vertices, and edges correspond to non-overlapping streams.

### 3.1.1 Mapping of a Unidirectional Ring onto a Linear Topology

In this subsection we devise a method to map the unidirectional ring onto the linear topology.
In a unidirectional ring of $N$ nodes, let the nodes be numbered from 1 to $N$, starting from any node, in the direction of communication. Then, the traffic from node $i$, where $1 < i \leq N$, destined to node $j$, where $j < i$, must be crossing the links between nodes $N$ and $j$. By adding $N - 1$ dummy nodes to an $N$ node linear topology, we can emulate the behavior of an $N$ node unidirectional ring. All the traffic sourced by $i$ and destined to $j$, for $j < i$ and $i > 1$, on a unidirectional link, will now be terminating at node $N + j$.

In Figure 3.1, a simple 5 node unidirectional ring is shown. Also the unfolding of the ring into a linear topology is depicted. Nodes 6, 7, 8, and 9 (also represented as 1', 2', 3', and 4', respectively) are the added dummy nodes, which correspond to nodes 1, 2, 3, and 4, respectively. Traffic sourcing from a node and destined to a lower indexed node will now terminate at the corresponding added dummy node. For example, traffic originating at node 4 and terminating at destination 1, will now terminate at node 6, as shown in Figure 3.1.

Once all the traffic is mapped from a unidirectional ring onto a linear topology, the solutions developed for the linear topology below will be applicable to the unidirectional ring too.

![Diagram](image)

Figure 3.1 Mapping a unidirectional Ring into linear topology. Nodes 6 to 9 are the added dummy nodes corresponding to nodes 1 to 4.
3.1.2 A Two Step Approach

In this sub section, we present a two step approach to solve the grooming problem on a linear topology. In the first step, we devise an algorithm MIN-STRINGS that arranges a number of non-overlapping streams into a string, thus making strings of streams in such a way that minimizes the total number of strings. Each string will be formed such that the space between streams in the same string is minimized. This algorithm, optimally minimizes the number of strings, which is equivalent to the lower bound on wavelengths. In the second step, we use a grouping heuristic that groups $g$ (or fewer) strings onto each wavelengths. During grouping, our objective is to group strings together in a manner that results in reducing the number of LTs.

3.1.2.1 Minimizing the Number of Strings

In this subsection we present a novel technique that accommodates the general non-uniform traffic and produces the least number of strings, equal to the density $\theta$, thus leading to the least number of wavelengths, $W$ for a linear topology.

To approach the problem in hand, a visualization of the problem will be helpful. Considering Figure 1.9 in Chapter 1, we notice that to compactly pack the traffic streams we can combine them horizontally as strings such that no two traffic streams in a string overlap. Visualizing each stream as an interval we can then form an interval graph consisting of all streams such that, each stream is represented by a vertex and there is an edge between each pair of vertices (intervals) that overlaps. The problem of finding the minimum number of strings is then equivalent to finding a vertex coloring of the corresponding interval graph with a minimal number of colors [88]. The graph coloring problem for general graphs is NP-Complete [89]. However, for interval graphs we will present a polynomial time algorithm that is optimal in the number of colors (in our case each color corresponds to a string).

Let us represent each stream $c_{kl}$ with a pair of coordinates $(X_{min}(c_{kl}), X_{max}(c_{kl}))$ such that $X_{min}(c_{kl}) = k$ and $X_{max}(c_{kl}) = l$. An $i^{th}$ string, $R_i$, can then be defined as a set of non-overlapping streams such that its member streams are not present in $R_j$, where $1 \leq j \leq |C|$. 

Let $H$ represent a sorted list of all the streams $c_{ij} \in C$, and $H(i)$ represent the $i^{th}$ element in the list $H$. The sorting criterion is explained in the algorithm itself. Also let us define an operation $REMOVE(H(i))$ on list $H$ that removes $H(i)$ from the list $H$ and hence decrements its size, $|H|$, by one (for $i \geq 0$). The algorithm MIN-STRINGS is given in Figure 3.2.

The algorithm MIN-STRINGS starts by sorting the segments in list $H$ in ascending order with respect to their $X_{\min}$ coordinate. For segments having the same $X_{\min}$ coordinates, longer segments are selected first (for segments with the same $X_{\min}$ and length, the tie can
be broken randomly). For each string we repeat the following (lines 3-18). We first initialize each string by the first element in the sorted list and remove that element from the list (lines 5-8). We then fill the string by searching the whole list, and accommodating the very first non-overlapping segments (lines 9-15). This way, in a single scan of the list we will be able to fill a string with available non-overlapping segments.

The algorithm MIN-STRINGS is simple and elegant and works for any general non-uniform traffic. Also, it always finds the optimal (minimum) number of strings, equal to the density $\theta$, as proven by Theorem 1 below. Moreover, simulation results verify that the algorithm compactly packs the segments in each string, which means that most of the segments are connected to each other. This reduces the number of LTs since two connected segments use the same LT at their connecting point.

We can determine the complexity of MIN-STRINGS as follows. For each string, MIN-STRINGS scans the whole list. As the number of strings produced by MIN-STRINGS is equal to the density $\theta$, the complexity of the algorithm is $O(\theta \times |C|)$. If the traffic between different node pairs is uniformly distributed between zero and some number $h$, then on average the total number of segments between all nodes pairs is $(N^2 \times \frac{h}{2})$, and the complexity of the MIN-STRINGS can be given as $O(\theta \times N^2 \times h)$.

Before proving the optimality of the MIN-STRING algorithm, we will define a few terms, followed by lemmas that are required for the proof. Let nodes be numbered $1, 2, ..., N$, and the links be numbered $1, 2, ..., N - 1$, such that link $i$ is between nodes $i$ and $i+1$. Let $\Omega$ be the set of all given segments, i.e., $\Omega = \{a_1, a_2, \ldots, a_{|C|}\}$. Let $\psi_j$ be a set that consists of segment $a_j$ and all segments that overlap with $a_j$ and starts earlier than or at the same node as $a_j$, i.e., $\psi_j = \{x | x \in \Omega \land (Xmin(x) \leq Xmin(a_j)) \land (Xmax(x) > Xmin(a_j))\}$. Let $\pi_i$ be a set defined as, $\pi_i = \{\psi_m | \sup_{Xmin(a_m) \leq i < Xmax(a_m)} m\}$. Basically, if a segment $a_m$ starts at node $i$, then $\pi_i = \psi_m$. If no segment starts at node $i$ then we will select such a segment $a_m$, which passes through node (link) $i$, and its starting point is closest to node $i$ among all the segments passing through node (link) $i$. $\pi_i$ will then consist of segment $a_m$ and all the segments that overlap with $a_m$. Note that if no segment passes through link $i$, then $\pi_i$ will be empty. Finally,
let \( \Pi \) be a sequence of sets \( \pi_i \), i.e., \( \Pi = (\pi_1, \pi_2, \ldots, \pi_{N-1}) \). The following lemmas are needed for the proof of the theorem.

**Lemma 1:** \( |\psi_j| \leq \theta \) for \( 1 \leq j \leq |\Gamma| \)

**Corollary:** \( |\pi_i| \leq \theta \) for \( 1 \leq i \leq N - 1 \)

**Lemma 2:** In each iteration, \( k \), of the MIN-STRINGS algorithm (lines 3-18, Figure 3.2), exactly one member of each nonempty \( \pi_i, 1 \leq i \leq N - 1 \), will be selected for inclusion in string \( k \).

The following theorem is the main result.

**Theorem 1:** MIN-STRINGS algorithm is optimal in the number of strings.

The proof of the lemmas and the theorem is given in Appendix A.

Figure 3.3 shows the output of MIN-STRINGS for a sample input. The traffic demands are shown in Figure 3.3(a). Each segment corresponds to a single traffic unit. Figure 3.3(b) shows the demands after sorting. Notice that longer segments precede shorter segments when their \( X_{min} \) coordinates are same. Figure 3.3(c) shows the output of MIN-STRINGS. Notice that the number of strings is exactly equal to the density, which is 8 in this case.

Please note that on a unidirectional ring, the segments between original nodes and the added dummy nodes could be overlapping. Any segment crossing the link between nodes \( i \) and \( i + 1 \), for \( 1 \leq i \leq N \), overlaps with any segment crossing the link between nodes \( N + i \) and \( N + i + 1 \). For example, from Figure 3.1(b) it appears that segments A and C can be combined in a string. However, they are overlapping segments. The links between nodes 6 and 8 are the same as the links between nodes 1 and 3. Therefore, we need to modify a few expressions for the unidirectional ring. Let \( \theta_{i,i+1} \) be the density of the link between nodes \( i \) and \( i + 1 \). For a unidirectional ring, \( \theta_{i,i+1} \) can be defined as:

\[
\theta_{i,i+1} = \sum_{k=1}^{i} \sum_{l=i+1}^{N} n_{kl} + \sum_{k=i+2}^{N} \sum_{l=i+1}^{k-1} n_{kl}
\]  

The first term includes all the sources before and including node \( i \), and the destinations after and including node \( i + 1 \). The second term considers all the sources \( k \) between \( i + 2 \) and \( N \), that are transmitting the traffic either to node \( i + 1 \), or to nodes after \( i + 1 \) but before node \( k \).
Similarly, we need to make few minor modifications in the algorithm MIN-STRINGS. While packing each string we need to select only non-overlapping segments. Therefore, besides checking the condition, \( X_{\text{max}}(A) \leq X_{\text{min}}(B) \) (line 11 in MIN-STRINGS), we also need to check \( X_{\text{min}}(A) \geq X_{\text{max}}'(B) \), where \( X_{\text{max}}'(B) \) is defined as:

\[
X_{\text{max}}'(B) = \begin{cases} 
0 & 1 \leq X_{\text{max}}(B) \leq N \\
X_{\text{max}}(B) \mod N & X_{\text{max}}(B) > N 
\end{cases}
\]

The number of strings determined by MIN-STRINGS in this case, will not necessarily be equal to the density. In fact, the problem of finding the minimum number of strings in rings can be reduced to a circular-arc coloring problem, which is an NP-Complete problem [36, 89].
experimental results, however, show that the number of strings determined by MIN-STRINGS even in this case is quite close to the density. Note that we are opening up the ring at a particular node only. Opening the ring at different locations can affect the results of MIN-STRINGS. The distribution of the traffic between actual node and its dummy node could also affect the total number of strings determined by MIN-STRINGS. For example, in Figure 3.1, out of a total of $h$ streams, say, between node 2 and 4, a fraction of traffic streams can be allocated between node 7 and 9, thus influencing the output of MIN-STRINGS. One of the possibility is to open the ring at each node, each time extending the ring as explained above, and selecting the best solution. However, this will increase the complexity of the technique by an order of $N$. In Section 3.3, we will examine the trade-off between number of LTs and the corresponding time complexity of the heuristic, for the unidirectional ring.

3.1.2.2 Grouping Algorithm

By now we have the set of strings, $R$, of size $|R|$. In this subsection we will combine the $|R|$ strings to obtain $W$ wavelengths, as given by equation (3.4). Also while combining the strings, our objective will be to minimize the total number of LTs, i.e., $U$.

Before presenting the algorithm for grouping we define a few terms. Define $E_{R_i}$ as the set of node points for which string $i$ needs an LT. In general each segment needs an LT at its $X_{min}$ and $X_{max}$ positions. However, two connected segments on the same string can share a single LT, because $X_{max}$ of one of the segments is the same as $X_{min}$ of the other segment. Please also note that in the mapped linear topology having any $X_{min}$ or $X_{max}$ at node $i$ ($1 \leq i \leq N$) is equivalent to having it on node $N + i$, and vice versa. We also define a saving function between strings $i$ and $j$ as:

$$\text{Saving}(R_i, R_j) = |E_{R_i} \cap E_{R_j}|$$

(3.8)

So the saving of two strings depends on the number of node points, where both strings need an LT due to the overlap of their component segments' endpoints ($X_{min}$ and/or $X_{max}$). Maximizing the saving function increases the sharing of LTs, resulting in fewer LTs. Also we
define an operation \( \text{MERGE} \) on any two strings \( R_i \) and \( R_j \) as follows:

\[
\text{MERGE}(R_i, R_j) \equiv E_{R_i} \leftarrow E_{R_i} \cup E_{R_j}
\]  

(3.9)

Thus the \( \text{MERGE} \) operation basically superimposes those node points of a string onto another where an LT is needed. Let the list \( H \) now include all the elements of the set of strings \( R \) in the order of their creation by \( \text{MIN-STRINGS} \). The grouping algorithm given in Figure 3.4 is then executed.

![Algorithm GROUPING](image)

Figure 3.4 Algorithm to group the strings obtained by \( \text{MIN-STRINGS} \) into \( W \) wavelengths while minimizing the number of the required LTs.

The algorithm GROUPING starts with a list consisting of all the elements of the set \( R \). Each wavelength is initialized with the first string in the list \( H \) (line 6). The added string is removed from \( H \) (line 8). Next, such a string is selected from all the remaining strings that
has maximum common node points, thus maximizing the saving (line 11). The largest value of saving, corresponding to a pair of strings, indicates that grouping these strings together in a wavelength will lead to sharing the largest number of LTs. The selected string is then merged with the previously selected strings on the same wavelength (line 13). For each wavelength we group \( g \) strings. However, note that the number of strings selected for the \( W^{th} \) wavelength may be fewer than \( g \). The number of LTs required for each wavelength is then the number of \( X_{\min} \) and \( X_{\max} \) points over all the segments in the wavelength, while excluding the multiplicity of common node points. Note that we are allowing traffic between same pair of nodes to be allocated to different wavelengths.

The complexity of the algorithm GROUPING is \( O(|R|^2) \). For the linear topology, \( |R| = \theta \), and hence the complexity for linear topology can be expressed as \( O(\theta^2) \). As \( |R| \) is upper bounded by the total number of segments \( N^2 \times \frac{h}{2} \) (an extreme case when each string has just one segment), hence the dominant factor in the overall complexity of the two-step approach comes from MIN-STRINGS that is \( O(\theta \times N^2 \times h) \).

### 3.2 Traffic Grooming on Bidirectional Rings

In this section we show how to groom the non-uniform traffic on a bidirectional ring by mapping it onto unidirectional rings, which in turn will be mapped onto the linear topology. Also, we explore two segment routing options, namely, when the segments are routed using the shortest path, and when the route is not fixed and thus may or may not follow the shortest path route.

We first start with the shortest path approach. First, note that similar to the case of the unidirectional ring, the linear topology can also emulate a bidirectional ring by adding some dummy nodes. However, this time the number of added dummy nodes is just \( \lfloor \frac{N}{2} \rfloor \). The allocation of the traffic streams, between source \( s \) and destination \( d \), to the extended linear topology then can be explained as follows. When \( (d > s) \), we will schedule the traffic from \( s \) to \( d \) if \( ((d - s) < \frac{N}{2}) \) and from \( s + \frac{N}{2} \) to \( d \) if \( ((d - s) > \frac{N}{2}) \). In the case when \( ((d - s) = \frac{N}{2}) \), we will split the traffic into two halves, and assign each half from \( s \) to \( d \) and from \( s + \frac{N}{2} \)
to \( d \). Also in the case when \((d < s)\) and \(((d - s) > \frac{N}{2})\) we will route the traffic from \( s \) to \( d + \frac{N}{2} \). Note, however, that the above assignment on a single ring can lead to segments in both directions. Applying MIN-STRINGS may result in combining opposite direction segments into a string, thus implying that a bidirectional LT (on a single wavelength) exists. To avoid such an assumption and to consider LTs that operate only in one direction, we need to modify our model. One necessary update is to consider that each wavelength can contain streams flowing in one direction only. Groups of such \( g \) wavelengths then can be assigned to two different fibers (in case of unidirectional fiber) or to the same fiber (in case of bidirectional fiber). Let the total traffic on a bidirectional ring be split between two unidirectional rings \( A \) and \( B \). Also, let the corresponding extended linear topology, of rings \( A \) and \( B \) be represented as \( L(A) \) and \( L(B) \), respectively. The assignment of traffic to \( L(A) \) and \( L(B) \) can then be carried out as explained in algorithm ASSIGNMENT, given in Figure 3.5.

Algorithm ASSIGNMENT is straight-forward. For each traffic request between a node pair, if the index of the destination is greater than that of the source then the traffic will be routed on ring \( B \) from the extended source to the destination, except when the distance between the source and the destination is less than or equal to \( N/2 \). In the former case, the traffic will be routed on ring \( A \) from the source to the destination, while in latter case the traffic will be split into two halves and one half will be routed on ring \( B \) from the extended source to the destination, and the other half will be routed on ring \( A \), also from the source to the destination. Similarly, if the index of the destination is less than that of the source, then depending on the distance between the source and the destination, the traffic will be either routed from the source to the extended destination on ring \( A \), from the source to the destination on ring \( B \), or it will be split into two halves and will be routed on both rings. Figure 3.6 shows how the traffic can be split between two different rings. Also it depicts the assignment of the traffic on corresponding extended linear topologies. The generality of the above mentioned assignment strategy is evident by noting that we are able to handle both symmetric and asymmetric traffic, and are still able to use the same heuristics that we developed for the extended linear topology for unidirectional ring.
Algorithm ASSIGNMENT

input: traffic matrix \( C \)
output: An assignment of traffic streams on \( L(A) \) and \( L(B) \)

BEGIN WHILE( \( c_{sd} > 0 \) \( \forall s, d \) DO
BEGIN
IF(\( d > s \) THEN
IF(\( (d - s) > \frac{N}{2} \)) THEN
Route traffic from \( s + N \) to \( d \) on \( L(B) \)
ELSE IF(\( (d - s) = \frac{N}{2} \)) THEN
Split total number of streams \( |c_{ij}| \) into two halves
Route one half from \( s + N \) to \( d \) on \( L(B) \)
Route another half from \( s \) to \( d \) on \( L(A) \)
ELSE
Route traffic from \( s \) to \( d \) on \( L(A) \)
ELSE IF(\( d < s \) THEN
IF(\( (s - d) > \frac{N}{2} \)) THEN
Route traffic from \( s \) to \( d + N \) on \( L(A) \)
ELSE IF(\( (s - d) = \frac{N}{2} \)) THEN
Split total number of streams \( |c_{ij}| \) into two halves
Route one half from \( s \) to \( d + N \) on \( L(A) \)
Route another half from \( s \) to \( d \) on \( L(B) \)
ELSE
Route traffic from \( s \) to \( d \) on \( L(B) \)
c_{sd} \leftarrow c_{sd} - 1
END WHILE
END ASSIGNMENT

Figure 3.5 Algorithm for assignment of segments on two unidirectional rings to emulate a bidirectional ring.

3.2.1 Non-Shortest Path Routing

In this subsection we will explore non-shortest path routing to reduce the number of wavelengths and LTs. The above mentioned assignment algorithm uses the shortest path to route the traffic between different source destination pairs. However, shortest path may not always lead to the least number of the wavelengths and the LTs. This can be best illustrated with the help of an example. In Figure 3.7, two rings \( A \) and \( B \) are shown. Suppose \( g \) in this case is 3.
Figure 3.6  Conversion of a Bidirectional Ring into linear topology. \( \lceil \frac{N}{2} \rceil \) nodes are added as dummy nodes. In (a) a sample traffic set is shown, while (b) and (c) shows how traffic in (a) can be split between between two unidirectional rings. Mapping of rings in (b) and (c) is shown in (d) and (e), respectively.

The density \( \theta \) of ring \( A \) currently is 2 while that of ring \( B \) is 3. The number of LTs required on each ring is also 3. Suppose we need to route a traffic stream originating from node 2 and terminating at node 1. Using shortest path routing we will end up selecting ring \( B \). However, this will not only increment the number of required wavelengths for ring \( B \) but also needs two more LTs. On the other hand, if we choose the longer route on ring \( A \), the already present LTs will be sufficient to accommodate the stream. This example clearly illustrates that there could arise situations where using shortest path routing does not lead to the minimum number of the wavelengths and the LTs. Hence, to address this issue, in the following we will develop an algorithm, TRAFFIC-SHIFTING, that uses three different criteria to relax the shortest path routing restriction, and is given in Figure 3.8.
Algorithm TRAFFIC-SHIFTING
input: traffic matrices of ring A and B
output: An assignment of traffic streams on \( L(A) \) and \( L(B) \)

BEGIN
Step 1: Assign traffic on both rings using algorithm ASSIGNMENT
Step 2: Compute density \( d(A) \) and \( d(B) \) for \( L(A) \) and \( L(B) \), respectively.
Step 3: Determine rings \( P \) and \( A \), such that,
    IF \( d(A) > d(B) \) THEN \( P = A \) and \( Q = B \)
    ELSE \( P = B \) and \( Q = A \)
Step 4: For each, longest-first, non-locked segment of \( L(P) \) that passes through link(s) with density \( d \), DO
    Step 4.1: Apply CRITERION 3, or CRITERION 2, or CRITERION 1 to approve the segment
    Step 4.2: If a segment \( c_{ij}^{(k)} \) is approved
        Step 4.2.1: Assign \( c_{ij}^{(k)} \) to \( L(Q) \) from \( j \) to \( i \),
        Step 4.2.2: Lock \( c_{ij}^{(k)} \) on \( L(Q) \)
    Step 4.2.2: Go to step 2
Step 5: Terminate the program
END TRAFFIC-SHIFTING

Figure 3.7 Example to illustrate that shortest path routing does not necessarily gives the minimum number of the wavelengths and LTs.

Figure 3.8 Algorithm for traffic shifting from shortest path to longer path, to reduce the number of required LTs.

In general, we will first assign all the traffic streams on \( L(A) \) and \( L(B) \) using shortest path. After that we will repeatedly select the ring with larger density and check each of the non-locked segments passing through its maximum density link, selecting the longest segment first. The
selected segment is then checked for its eligibility to move to the other ring using CRITERION 1, 2 or 3. In case the shifting of segment is approved, the segment is shifted (now being routed over longer path) and is locked to prohibit any further shifting (to avoid cycles). Note that, to be able to predict exactly that a shifting of the segment from \( L(P) \) to \( L(Q) \) will lead to the reduced number of the LTs, we need to determine the corresponding wavelengths of all other segments. Due to our two step approach, however, we do not determine the corresponding wavelengths before routing all of the segments. Hence, at this stage our approach tries to reduce the total number of strings on each ring in a manner that will not lead to an increment in the number of LTs. The three criteria used in algorithm TRAFFIC-SHIFTING are explained below.

**CRITERION 1:** Approve the segment if \( d(Q) \) is not an integral multiple of \( g \), else try CRITERION 2.

**CRITERION 2:** Approve the segment if \( d(P) > d(Q) \), else try CRITERION 3.

**CRITERION 3:** Approve the segment if shifting the segment from \( L(P) \) to \( L(Q) \) does not increase \( d(Q) \).

The intuition behind CRITERION 1 is that if \( d(Q) \) is not an integral multiple of \( g \) then some space will be left in at least one of the wavelengths of ring \( Q \), and hence we can utilize it by placing a segment in it, while potentially decreasing the number of strings from the ring \( P \). CRITERION 2 tries to balance the difference of densities (and hence number of strings) between both rings. Finally CRITERION 3, shifts the segment from one ring to another only if such a move does not increase the density of either of the two rings, hence potentially decreasing the number of strings from both the rings, \( P \) and \( Q \), but not at the expense of ring \( P \). Note that using CRITERION 3 fewer segments will be allowed to shift as compared to CRITERION 2 and CRITERION 1. Similarly CRITERION 2 is more restrictive than CRITERION 1.

### 3.3 Experimental Results

In this section we will present the results of applying the techniques proposed in Sections 3.1 and 3.2 to various networks with different topologies and parameters. We are more interested
in conducting experiments with nonuniform and asymmetric traffic, because uniform traffic can be considered as a special case of general arbitrary traffic.

For all the experiments the number of nodes, \( N \), was varied from 5 to 25 with an increment of 5. The value of the grooming factor, \( g \), was assigned to 1, 4, 8, and 16 in each of the experiments. Assuming that our basic data stream is an OC-3, these values of \( g \) then correspond to OC-3, OC-12, OC-24, and OC-48, respectively. For each of the network topologies, we generated a set of traffic streams \( c_{ij} \), between different node pairs \((i, j)\), whose cardinality is taken from an integer uniform distribution in the closed interval \([0, g]\). Each reported result is an average value obtained by running 30 batches of 30 runs each. The 90% confidence intervals were computed and are very tight, but are not shown here.

We divide the experiments into two suites. Suite 1 consists of experiments conducted on unidirectional rings. We study the number of wavelengths and LTs required for different values of grooming factor. Also we study the effect of opening the unidirectional ring, at each of the \( N \) nodes, on the number of required LTs. Suite 2 consists of experiments conducted on bidirectional rings. We study the performance of the different criteria developed in Section 3.2 for traffic shifting, in terms of saving in the number of ADMs.

We first discuss the experiments in suite 1. Figure 3.9 shows the number of LTs required for a unidirectional ring, for different values of \( g \). Note that when \( g = 1 \), there will be no traffic grooming, because each basic stream will occupy the whole wavelength between source and destination nodes. Figure 3.10 compares the number of wavelengths determined by MIN-STRINGS for a unidirectional ring, and the corresponding lower bound on the number of the wavelengths, i.e., density given by equation (3.3), when \( g = 8 \). From Figure 3.10, it is evident that the number of the wavelengths determined by MIN-STRINGS is exactly equal to the lower bound when the problem size is small (for example, when the number of nodes are 5 and 10), and slightly exceeds the lower bound when the problem size increases.

In Section 3.1, we discussed the time-cost tradeoff in opening the ring at single or multiple nodes. Figure 3.11 shows the savings in the number of the LTs that can be obtained by opening the unidirectional ring at each of the \( N \) nodes and selecting the best solution. The saving in
the number of the LTs increases when the problem size increases (and so does the computing time). On average we are able to save 5 to 10 LTs by opening it at each of the \( N \) nodes and selecting the best solution. Given that a single port SONET ADM costs $40,000 or more, this saving could mean a total saving of $200,000 to $400,000. On the other hand by opening the
ring at each of the $N$ nodes, the time complexity increases by a factor of $N$. However, the run time of our technique allows one to afford this additional computation to save large amounts of money while designing WDM networks. For example, the real time taken by the program, for $N = 20$ and $g = 8$, was 0.62 seconds and 5.2 seconds when it was opened at node zero and when it was opened at each of the $N$ nodes, respectively. Similarly, the real time taken by the program for the above mentioned two options was 2.71 seconds and 21.7 seconds when $g = 16$.

Comparison of our results with other proposed techniques, e.g., [53] reveals that with far less complexity (at least an order of $N$) we have achieved either less or comparable costs in terms of the number of the LTs.

![Graph](image)

Figure 3.11 Number of the LTs saved by opening the unidirectional ring at each of the $N$ nodes.

For all of the above experiments the amount of traffic generated between each node pair is related to the grooming factor (the traffic generated for each node pair is an integer uniformly distributed in the closed interval $[0, g]$). We also conducted experiments to study the effect of different grooming factors while using the same input traffic matrices. For these experiments, the traffic generated for each node pair is an integer uniformly distributed in the closed interval $[0, 16]$. The results are collected for $g = 4$ and $g = 8$. Figures 3.12 and 3.13 show the number
of LTs and the number of wavelengths required to accommodate the input traffic, respectively. Note that the number of the wavelengths required for \( g = 8 \) are almost exactly half that required for \( g = 4 \), while the number of the LTs required for \( g = 8 \) are close to half of that required for \( g = 4 \). This shows that our two-step approach is scalable with the grooming factor.

![Graph showing the number of LTs for a unidirectional ring](image)

**Figure 3.12** Number of LTs for a unidirectional ring, when the same input matrices are used for experiments with \( g = 4 \) and \( g = 8 \).

In the case of rings even the first step of our two-step technique, i.e., minimizing the number of strings, is NP-Complete. Therefore, to compare the performance of MIN-STRINGS, in this case, we compared the results MIN-STRINGS with the results given in [36]. In [36] the authors solved the wavelength assignment problem in WDM rings while minimizing the number of LTs. They proposed three heuristics, namely, Modified Assign First (MAF), Iterative Matching (IMat) and Iterative Merging (IMer), and presented the LT savings over 200 experiments. Whenever two segments shared a common node this was counted as an LT saving. Table 3.1, compares the performance of MIN-STRINGS to the MAF, IMat, and IMer heuristics, for the same network setup (\( N = 16 \), and number of streams generated randomly between 16 and 256). On average, our proposed MIN-STRINGS algorithm performed 204\%, 145\%, and 117\% better than MAF, IMat, and IMer, respectively, which is shown by Improvement1 in Table...
3.1. We also experimented by opening the ring at each of the \( N \) nodes, and selecting the best solution. This introduced an improvement over our initial solution which is about 8\% (shown in Table 3.1 as Improvement2). This translates into a further improvement of 20\%, 16\%, and 14\%, over MAF, IMat, and IMer, respectively.

![Graph showing the number of wavelengths for a unidirectional ring](image)

Figure 3.13  Number of the wavelengths for a unidirectional ring, when the same input matrices are used for experiments with \( g = 4 \) and \( g = 8 \).

<table>
<thead>
<tr>
<th></th>
<th>MIN-STRINGS</th>
<th>MAF</th>
<th>IMat</th>
<th>IMer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. LT saving</td>
<td>76</td>
<td>25</td>
<td>31</td>
<td>35</td>
</tr>
<tr>
<td>Improvement1</td>
<td>-</td>
<td>204%</td>
<td>145%</td>
<td>117%</td>
</tr>
<tr>
<td>Improvement2</td>
<td>8%</td>
<td>224%</td>
<td>161%</td>
<td>131%</td>
</tr>
</tbody>
</table>

Suite 2 consists of experiments for bidirectional rings. Figure 3.14 shows the number of LTs required for a bidirectional ring, using shortest path and TRAFFIC-SHIFTING algorithm, when \( g = 16 \). Note that the results were collected for the TRAFFIC-SHIFTING algorithm with all three different criteria, namely, CRITERION 1 (C1), CRITERION 2 (C2), and CR-
TERION 3 (C3). From the figure, it is evident that we can improve on the shortest path approach by relaxing the shortest path constraint. Also both C1 and C2 perform better than C3, because they have more flexibility in shifting the traffic streams from shortest path routes to other routes. Table 3.2 shows the saving in the number of LTs obtained by using C1, C2, and C3 over the shortest path routing (C0) option, when $g = 8$. Using either C1 or C2, for large problem sizes we were able to save 74 LTs. On average, criteria C1, C2 and C3 saved 36.1, 35.6, and 6.2 LTs, respectively, over shortest path option. Using $40,000 as the price for a single port SONET ADM, this saving translates into 1.44 million, 1.42 million, and 0.24 million dollars, respectively.

![Graph showing the number of LTs for a bidirectional ring with $g = 16$.](image)

Figure 3.14 Number of LTs for a bidirectional ring; $g = 16$.

3.4 Chapter Summary

In this chapter, we addressed the grooming of non-uniform traffic on unidirectional and bidirectional rings. We mapped the unidirectional rings onto a linear topology, and then developed a two-step approach to solve the grooming problem, while minimizing the number of wavelengths and LTs, for the mapped topologies. For the first step, an algorithm MIN-STRINGS is developed that produced the optimal (minimum) number of strings on a linear
Table 3.2  Saving in terms of the number of LTs over shortest-path routing, when TRAFFIC-SHIFTING algorithm is used for bidirectional ring and with $g = 8$.

<table>
<thead>
<tr>
<th>N</th>
<th>CO - C1</th>
<th>CO - C2</th>
<th>CO - C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.7</td>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>15</td>
<td>18.4</td>
<td>17.2</td>
<td>1.4</td>
</tr>
<tr>
<td>20</td>
<td>46.5</td>
<td>45.3</td>
<td>9.4</td>
</tr>
<tr>
<td>25</td>
<td>74</td>
<td>74</td>
<td>13.8</td>
</tr>
</tbody>
</table>

topology, while compacting each string with traffic streams. Optimality of the algorithm for linear topologies is proved. For the second step, an effective heuristic is designed to group $g$ strings for each wavelength such that the number of LTs used per wavelength are minimized. Similarly, the bidirectional rings are also mapped onto unidirectional rings and the two-step approach is used with some modifications. Moreover, a study is conducted on routing strategies for bidirectional rings to minimize the number of required wavelengths and LTs. Few approaches are proposed that lead to considerable reduction in the number of the required wavelengths and LTs. Finally, the efficacy of the proposed techniques is demonstrated using a large set of experiments. Experimental results show that in most of the cases and under non-uniform traffic, the proposed techniques perform better than other schemes reported in the literature.
CHAPTER 4 MULTICAST TRAFFIC GROOMING IN WDM NETWORKS

In this chapter we consider the optimal design and provisioning of WDM networks when multicast subwavelength traffic is given. Since most of the multicast service applications require subwavelength capacity, it is therefore important to design and dimension networks while grooming subwavelength traffic demands. We develop a unified framework to solve different practical scenarios of multicast traffic grooming. We solve each scenario optimally. We also introduce heuristic solutions. Optimal solutions are designed by exploiting the specifics of the problems to formulate Mixed Integer Linear Programs (MILPs), of otherwise non-linear problems. Specifically, we solve the traditional multicast problem in which, given a set of multicast demands, all demands need to be accommodated such that each destination node of a multicast session requires the same amount of traffic. The objective is to minimize the network cost by minimizing the higher layer electronic equipment and, simultaneously, minimizing the total number of the wavelengths used. We also solve two interesting and practical variants of the traditional multicast problem, namely, multicasting with partial destination set reachability and multicasting with traffic thinning. For both variants too, we provide optimal as well as heuristic solutions. We also present a number of examples based on the exact and heuristic approaches.

The contribution of the work in this chapter is summarized as follows:

- It provides a unified framework for optimal network dimensioning and channel provisioning for multicast traffic grooming. The framework provides solutions for the generic multicast traffic grooming problem as well as variants of it.

- Routing, wavelength assignment and traffic grooming is solved in an integrated way in
order to achieve global minima, while maintaining non-bifurcation of traffic.

- Network cost is minimized by minimizing multiple resources (i.e., the number of the LTs and the wavelengths) instead of a single resource.

4.1 Introduction

Most studies in traffic grooming on WDM networks have exclusively dealt with unicast traffic. However, in the foreseeable future, it is expected that a sizable portion of the traffic in future high performance networks will be multipoint in nature. Multipoint traffic can take different forms, e.g., one-to-many and many-to-one. In this chapter we will deal with one-to-many, or multicast, traffic, while in the next chapter we will consider many-to-one traffic.

Multicast traffic applications include multi-party conferencing, video distribution, network news distribution, collaborative processing, and web content distribution to proxies [31]. Interestingly, most multicast service applications require only sub-wavelength capacity. For example, HDTV can work well with just 20 Mbps per channel, while a normal TV channel typically requires less than 2 Mbps per channel, when compressed using MPEG-2, as in digital television. Hence, many such connections can be groomed together onto a single wavelength. It is therefore important to design and dimension networks in order to be able to support traffic of the multicast type, while grooming subwavelength traffic demands. The multicast traffic grooming problem differs significantly from the multicast problem in an all-optical network, and this will be explained in detail with the help of an example in Section 4.3 of this chapter.

As mentioned earlier, most of the studies allow bifurcation of traffic, i.e., the multiple-unit traffic between a source-destination pair is split into a set of components, such that the number of traffic units in each component is lesser than that of the original traffic. Each component, then, is routed separately from source to destination. Although allowing bifurcation provides flexibility in traffic allocation and may reduce the number of wavelengths and LTs, it increases the complexity and the cost of traffic reassembly, and may also introduce delay jitter at the application layer. As our thrust in this work is to develop more realistic models, therefore, in the rest of the dissertation we have considered the cases in which traffic bifurcation is not
allowed. Note that a specific traffic demand, as a whole, can still traverse different lightpaths while making its way to the destination (which we refer to as horizontal splitting). The only restriction here is that the whole traffic demand needs to be intact on each wavelength it traverses.

In this chapter, we solve the traditional multicast problem as well as two of its interesting and practical variants. In the traditional or generic multicast traffic grooming problem, given a set of multicast demands, all demands need to be accommodated such that each destination node of a multicast session requires the same amount of traffic. To lower the cost of the network, dominant cost factors, e.g., the number of line terminals (LT) are minimized. The two variants of the generic multicast traffic grooming problem we consider in this chapter are: **multicasting with partial destination set reachability** and **multicasting with traffic thinning**. For both problems the destination set of each multicast session consists of two subsets. In multicasting with partial destination set reachability only one subset of each multicast session must be accommodated while the other subset can be accommodated only if this results in no additional cost. On the other hand, in traffic thinning both subsets of each multicast session must be accommodated. However, each subset has different bandwidth requirements. All three types of multicast traffic grooming will be further explained with the help of examples in Section 4.3 of this chapter. Multicasting with partial destination set reachability is very useful when design and provisioning need to be done under a tight budget, and destinations of the multicast sessions can be classified as critical or non-critical. Destinations that are part of the critical set need to be accommodated in all cases, while destinations that are a part of the non-critical set are accommodated only if this accommodation can be done without incurring any additional resources\(^1\). Traffic thinning, however, is helpful when a subset of the destinations of a session can be satisfied by a bandwidth less than that of other destinations of the session. In practice, this can happen when destinations of a multicast session have different Quality-of-Service (QoS) requirements, e.g., cases in which destinations are served with different levels of video coding. Provisioning all the destinations of the session with the

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\(^1\)Without the loss of generality, we here assume that resources are represented by the LTs and the wavelength channels.
maximum amount of bandwidth required by any of the destinations of the session could result in potential wastage of resources. The latter issue is also reconfirmed by our experiments in Section 4.6 of the chapter. Hence, entertaining the destinations with different bandwidths, i.e., traffic thinning, can be helpful to reach all the destinations at an overall reduced network cost.

4.2 The Model

In this section we will describe the network model that we consider for the multicast traffic grooming problems. The network model consists of three levels of abstraction: the physical, the lightpath and the connection levels, as shown in Figure 4.1. The physical level corresponds to the network topology consisting of physical links between nodes, and is an input to the MILP formulation. We assume that each physical link is composed of two fibers that are used to communicate in opposite directions and each fiber can support W wavelengths in only one direction. The lightpath level represents the virtual topology, made of all-optical lightpaths, and is an output from the MILP. Each lightpath can span several physical links. Also, more than one lightpath may exist between a pair of nodes. If a pair of such lightpaths uses the same wavelength, then they must follow link disjoint physical routes. We assume that no optical wavelength conversion is available, and hence we maintain the wavelength continuity constraint at the lightpath level. Connection level links between nodes represent the traffic demands. Each link at the connection level may span several lightpaths. In Figure 4.1 nodes $d_1, d_2$ and $d_3$ are the members of the destination set of the multicast group originating at source node $s$. We consider the case in which branching for multicast traffic is implemented in the electronic domain; therefore, a multicast tree consists of several connection level links. Note that due to the implementation of the branching at electronic level we do not require any special features at a node for duplication of the traffic except that required for a electronic-multicast-capable node.

Regarding notations, we will use $s$ and $d$ to represent source and destination of a connection, $i$ and $j$ to represent the source and destination nodes of a lightpath, $m$ and $n$ to represent the source and destination nodes of a physical link, respectively. Let $K$ be the total number
of sessions from all sources. Then, each connection $c_a$, where $1 \leq a \leq K$, corresponds to an ordered pair $(s, k)$, where $k$ represents the $k^{th}$ (unicast or multicast) session originating from source $s$. Let $D_{ca}$ represent the destination set of session $c_a$, and $d$ represent a destination in the destination set. For the two variants of generic multicast traffic grooming problem we assume that each destination set, $D_{ca}$, consists of two disjoint subsets, represented by $D'_ca$ and $D''ca$, such that $D_{ca} = D'_ca \cup D''ca$ and $D'_ca \cap D''ca = \phi$. Let $m'_ca$ represent the number of basic units of traffic required by the destination set $D'_ca$, and $m''ca$ represent the number of basic units of traffic required by the destination set $D''ca$. If $m'_ca = m''ca$, then let $mca$ represent the number of basic units of traffic.

4.3 Problem Description

In this section we will describe the multicast traffic grooming problem and will show how it differs significantly from the multicast problem in an all-optical network.

In an all-optical network, multicasting is supported by developing a Steiner Minimum Tree (SMT). With the help of an example, we show that in case of multicast traffic grooming, using an SMT in terms of the number of hops, will not necessarily give an optimal solution in terms of the number of required LTs. Consider a six-node bidirectional ring as shown in Figure 4.2.

Let us assume that the capacity of each wavelength is 2 units and there exists 3 traffic
Figure 4.2 Routing using Steiner Minimum Tree.

sessions as follows:

Session 1: Source = A; Destination = {B, C}; Traffic demand = 1 unit;
Session 2: Source = B; Destination = {C}; Traffic demand = 2 unit;
Session 3: Source = A; Destination = {F}; Traffic demand = 1 unit;

Routing the demands using an SMT requires 7 LTs and two wavelengths, as shown in Figure 4.2. However, using the routing shown in Figure 4.3 costs just 6 LTs and one wavelength, which proves our claim. Hence, we need to take a totally different approach for designing WDM network to support multicast traffic grooming. One of the simple techniques applied in the literature, to handle multicast traffic, is to treat every multicast demand as a set of unicast demands. However, it is obvious that such a policy will not lead to a minimum number of LTs in most cases.

A given traffic matrix typically consists of a number of unicast and multicast sessions\(^2\), from a source node to a set of destination nodes. Using the notations defined earlier, we state that in a generic multicast traffic grooming problem each destination in the destination set of some multicast session \(c_a\), namely, \(d \in D_{c_a}\), has same bandwidth requirement. Also, all destinations, \(d \in D_{c_a}, \forall c_a\) need to be served. Hence, no differential treatment is meted out

\(^2\)Here we make no distinction between unicast and multicast sessions. A unicast session is treated as a multicast session with one destination.
to the destinations in a destination set in terms of delivery of traffic and the amount of the delivered traffic. However, in the two variants of generic multicast traffic grooming problem, for each multicast session, \( c_a \), the destination set, \( D_{ca} \), consists of two subsets, \( D'_{ca} \) and \( D''_{ca} \).

In the partial destination set reachability problem, we are required to accommodate all the destinations in subset \( D'_{ca} \) only, while minimizing the network cost. The destinations in \( D''_{ca} \) would be accommodated only if this action does not require any additional LTs or wavelength channels. The objective is also to maximize such an accommodation of \( d \in D''_{ca}, \forall c_a \), which does not increase the network cost. In traffic thinning case, a differential treatment is meted out to the destinations of the two subsets in terms of bandwidth requirements, with one of the two subsets receiving lower quality signal, i.e., lower bandwidth. More specifically, for each \( c_a \), \( d \in D''_{ca} \) will be entertained with a bandwidth \( m''_{ca} \), while \( d \in D'_{ca} \) will be entertained with a bandwidth \( m'_{ca} \), such that \( m''_{ca} < m'_{ca} \).

We will explain both variants with the help of an example. A 4-node network is shown in Figure 4.4. Suppose that a single wavelength channel can accommodate two basic units of traffic, and there is a total of 4 sessions, one from each node. The details of the sessions are given below:

\[ c_0: s = 0; D'_{c0} = \{1\}; D''_{c0} = \{2\}; m'_{c0} = m''_{c0} = \{1\} \]
\[ c_1: s = 1; D'_{c1} = \{2\}; D''_{c1} = \{\}; m'_{c1} = \{2\} \]
Figure 4.4 Partial destination set reachability problem on a 4-node network.

\[ c_2 : s = 2; D'_c = \{3\}; D''_c = \{0\}; m'_c = m''_c = \{1\}; \]

\[ c_3 : s = 3; D'_c = \{0\}; D''_c = \{\}; m'_c = \{2\}; \]

Figure 4.4 also shows the solution for the partial destination reachability set problem. Each square box represents an LT. Thus, the number of LTs required are 5, while a single wavelength channel is enough to accommodate the requests for the destinations \( d \in D'_c \). Note that the solution accommodates the request to \( D''_c = \{0\} \) because this does not increase the network cost. However, accommodating the request to \( D''_c = \{2\} \) would have required an additional wavelength channel, and is therefore not served.

For the traffic thinning problem, both \( d \in D'_c \) and \( D''_c \) need to be accommodated. The above example is used again, however, to illustrate the point, the traffic demands are changed as follows.

\[ m'_c = \{2\}; m''_c = \{1\}; \]

\[ m'_c = \{1\}; \]

\[ m'_c = \{2\}; m''_c = \{1\}; \]

\[ m'_c = \{2\}; \]

The solution is shown in Figure 4.5, and needs 6 LTs and 2 wavelength channels. Note that if
$D_\omega^\prime = \{2\}$ would have also been served with 2 units of traffic, as was the case for $D_\omega^\prime = \{1\}$, the number of LTs required would be 8. This shows that traffic thinning potentially can reduce the cost of the network, while still serving all the requests.

![Figure 4.5 Traffic Thinning problem on a 4-node network.](image)

### 4.4 Problem Formulation

In this section we will present the MILP for the generic multicast traffic grooming problem and its two variants: partial destination set reachability and traffic thinning problems. First, however, we will define all the variables used in the MILP.

#### 4.4.1 Definitions

The parameters involved can be divided into two classes: input parameters to the MILP, and parameters that are determined by the MILP, and hence are an output from the MILP. The definition of such parameters is given below.

- **Input parameters:**
  
  $N$ : total number of nodes in the network
  
  $W$ : maximum number of wavelengths per fiber \(^3\)

\(^3\)Although we start with $W$ wavelengths, the MILP minimizes the number of required wavelengths.
$g$: capacity of a wavelength in terms of number of basic units of traffic (also called grooming factor)

$\alpha$: cost of an LT

$\beta$: cost of a wavelength

$\gamma$: a scalar factor, smaller than both $\alpha$ and $\beta$, and is used to control the accommodation of $d \in D_{ca}$

$Q$: a very large integer number, (it suffices to set $Q$ such that $Q \geq N^2 - N$)

$P_{mn}$: number of physical fiber links (1 or 0) connecting nodes $m$ and $n$.

$m_{ca}$: number of basic units of traffic required by each member of the destination set $D_{ca}$

$m'_{ca}$: number of basic units of traffic required by each member of the destination set $D'_{ca}$

$m''_{ca}$: number of basic units of traffic required by each member of the destination set $D''_{ca}$

- Variables of the MILP:

$LT_n$: number of LTs at node $n$

$\psi$: highest index of wavelengths used over all fiber links

$y_w$: a binary indicator; must be 1 if there is at least one lightpath in the network on wavelength $w$.

$L_{ijn}^w$: number of lightpaths between node pair $(i, j)$ routed through fiber $(m, n)$ on wavelength $w$

$L_{ij}^w$: number of lightpaths from node $i$ to node $j$ on wavelength $w$

$L_{ij}$: number of lightpaths from node $i$ to node $j$ on all the wavelengths, $L_{ij} = \sum_w L_{ij}^w$

$Z_{ij}^{ca, d}$: a real number between 0 and 1, which takes non-zero values if and only if connection $c_a$, destined to $d$, is employing a lightpath from $i$ to $j$ as an intermediate virtual link

$M_{ij}^{ca}$: a binary indicator; is 1 if and only if connection $c_a$ is using a lightpath between nodes $i$ and $j$ to reach at least one destination $d$, where $d \in D_{ca}$. This means that if for any $d \in D_{ca}$, $Z_{ij}^{ca, d}$ is greater than zero, then $M_{ij}^{ca} = 1$.

$G_{ij}^{ca}$: a binary indicator; is 1 if and only if connection $c_a$ is using lightpath between nodes
i and j to reach at least one d, where d ∈ D_{c_a}.

\( X_{ij}^{c_a} \): a real number; represents the capacity used by connection \( c_a \) on lightpath(s) between nodes \( i \) and \( j \).

\( J_{ij}^{c_a,c_b} \): a binary indicator; is 1 if and only if connections \( c_a \) and \( c_b \) are groomed on the same lightpath from \( i \) to \( j \).

\( Y_{ij}^{c_a,c_b} \): a real number and is a product of \( J_{ij}^{c_a,c_b} \) and \( X_{ij}^{c_a} \).

### 4.4.2 Common Constraints

The common set of constraints for all the above mentioned problems are as follows:

- **Number of LTs:**
  
  The following two constraints ensure that for each originating or terminating lightpath at a node an LT is present:

  \[
  LT_i \geq \sum_{w, j \neq i} L^w_{ij} \quad \forall i \tag{4.1}
  \]

  \[
  LT_i \geq \sum_{w, j \neq i} L^w_{ji} \quad \forall i \tag{4.2}
  \]

- **Number of wavelengths:**

  The following constraints ensure that \( \psi \) will be the index of the highest numbered wavelength used on any fiber link in the network. Notice that minimizing \( \psi \) will minimize the total number of wavelengths in the network and not just the total number of wavelengths on each link.

  \[
  \psi \geq w \cdot y_w \quad \forall w \tag{4.3}
  \]

  \[
  y_w \geq \sum_i \sum_{j \neq i} L^w_{ij} / Q \quad \forall w \tag{4.4}
  \]

  \[
  y_w \leq \sum_i \sum_{j \neq i} L^w_{ij} \quad \forall w \tag{4.5}
  \]

- **Lightpath level constraints:**

  The following constraint ensures that the origin node, \( i \), and the terminating node, \( j \), of lightpath(s) between node \( i \) and \( j \) have no incoming and outgoing traffic carried on such
lightpaths, respectively:

\[ \sum_{m, m_{ij} = 1}^{m_{ij}} p_{ij}^{w} = \sum_{n, n_{in} = 1}^{n_{in}} p_{ij}^{w} = 0 \quad \forall i, j, w \]  

(4.6)

The following constraint determines the total number of lightpaths on wavelength \( w \) between node \( i \) and \( j \), supported by the underlying physical topology:

\[ \sum_{m, m_{ij} = 1}^{m_{ij}} p_{ij}^{w} = \sum_{n, n_{in} = 1}^{n_{in}} p_{ij}^{w} = \lambda_{ij}^{w} \quad \forall i, j, w \]  

(4.7)

The following constraint preserves wavelength continuity of lightpaths over multiple physical links:

\[ \sum_{m, m_{ix} = 1}^{m_{ix}} p_{ij}^{w} = \sum_{n, n_{nx} = 1}^{n_{nx}} p_{ij}^{w} \quad \forall w, i, j, x ; x \neq i, j \]  

(4.8)

Equations (4.7) and (4.8) together ensure that for each lightpath there exists a corresponding physical path, while maintaining wavelength continuity over the physical path. Equation (4.9) then ensures that on a wavelength, \( w \), on fiber from node \( m \) to node \( n \), only one lightpath is present.

\[ \sum_{i, j, m_{ij} = 1}^{m_{ij}} F_{mn}^{ij}^{w} \leq 1 \quad \forall m, n, w \]  

(4.9)

We would like to mention that our constraints involving the \( F_{mn}^{ij}^{w} \) variables are similar to those in [65].

- **Multicast connection topology constraints:**

The following two constraints ensure that for the connection \( c_{a} \) destined for \( d \), no traffic is coming in (going out) the source (destination), respectively

\[ \sum_{i, i \neq s}^{s} Z_{is}^{c_{a}, d} = 0 \quad \forall c_{a}, d \in D_{c_{a}} \]  

(4.10)

\[ \sum_{j, j \neq d}^{d} Z_{dj}^{c_{a}, d} = 0 \quad \forall c_{a}, d \in D_{c_{a}} \]  

(4.11)

The following constraint preserves the continuity of the traffic of connection \( c_{a} \) on multiple lightpaths

\[ \sum_{i, i \neq x}^{i} Z_{ix}^{c_{a}, d} = \sum_{j, j \neq x}^{j} Z_{xj}^{c_{a}, d} \quad \forall c_{a}, d \in D_{c_{a}}, x, (x \neq s, d) \]  

(4.12)
For a multicast connection, delivery to two or more members of the connection’s destination set can be made by sending the traffic only once over a lightpath. Then, traffic duplication can take place after terminating the lightpath. The following constraints set the variable $M_{ij}^{ca}$ to 1, if and only if at least one destination, $d \in D_{ca}$, is reached through a lightpath between nodes $i$ and $j$. Later, this variable will also be used to guarantee non-bifurcation.

\[ M_{ij}^{ca} \geq \sum_{d} Z_{ij}^{ca,d}/Q \quad \forall d \in D_{ca}, c_{a}, i, j \quad (4.13) \]

\[ M_{ij}^{ca} \leq \sum_{d} Z_{ij}^{ca,d} \quad \forall d \in D_{ca}, c_{a}, i, j \quad (4.14) \]

The following constraint ensures that the capacity, represented by $X_{ij}^{ca}$, used by connection $c_{a}$ on lightpath(s) between nodes $i$ and $j$, does not exceed the physical capacity of the lightpaths on which the traffic is accommodated. The value of $X_{ij}^{ca}$ itself will be computed for each problem separately.

\[ \sum_{c_{a}=1}^{K} X_{ij}^{ca} \leq L_{ij} \cdot g \quad \forall i, j \quad (4.15) \]

- **Non-Bifurcation:**

Bifurcation (vertical split) of a traffic demand can happen at three levels: among lightpaths between different nodes, among lightpaths between the same pair of nodes but on different wavelengths, and among lightpaths between the same pair of nodes and on the same wavelength, but with each lightpath taking a different physical route. Equations (4.18), (4.19) and (4.12) together prevent only the first type of bifurcation. However, these constraints do not preclude bifurcation of the second and third type. Note that one of the objective of the formulation in [65] was to ensure non-bifurcation of traffic. However, for arbitrary integer values of $g$ that formulation too guarantees non-bifurcation of the first type only. This is shown by using a counter example in Appendix 3. Hence to obtain a complete non-bifurcated solution we propose the following constraints:

\[ J_{ij}^{ca,cb} \leq (M_{ij}^{ca} + M_{ij}^{cb})/2 \quad \forall c_{a}, c_{b}, i, j \quad (4.16) \]
The above equation will ensure that if both \( M_{ca}^i \) and \( M_{cb}^i \) are 1, then the variable \( J_{ij}^{ca,cb} \) must also be 1 (i.e., connections \( c_a \) and \( c_b \) share the same lightpath between nodes \( i \) and \( j \)); otherwise the variable \( J_{ij}^{ca,cb} \) could be either 1 or 0.

The following equation ensures that the total number of lightpaths between these two nodes must be equal to the total number of shared and unshared lightpaths between nodes \( i \) and \( j \)

\[
L_{ij} = J_{ij}^{C1,C1} + \sum_{a=2}^{K} (J_{ij}^{ca,ca} - \bigvee_{b=1}^{a-1} J_{ij}^{cb,cb}) \quad \forall i, j
\]

where \( \bigvee_{b=1}^{a-1} J_{ij}^{cb,cb} \) is the disjunction function among \( J_{ij}^{C1,ca}, \ldots, J_{ij}^{C_{a-1},ca} \). This function is implemented using linear constraints as follows. If \( V = \bigvee_{b=1}^{a-1} J_{ij}^{cb,cb} \), then it is computed as:

\[
V \leq \sum_{b=1}^{a-1} J_{ij}^{cb,cb} \quad \text{and} \quad V \geq \sum_{b=1}^{a-1} J_{ij}^{cb,cb} / (a - 1)
\]

Note that equation (4.17) counts all the lightpaths between nodes \((i, j)\) by counting all the connections sharing a lightpath only once. The second term in equation (4.17) counts all the connections on a single lightpath only once, since if \( \bigvee_{b=1}^{a-1} J_{ij}^{cb,cb} \) is 1, then this means that connection \( c_a \) is sharing a lightpath with a lower numbered connection, \( c_b \). Therefore, subtracting this value from \( J_{ij}^{ca,ca} \) will avoid multiple counting. As will be illustrated below, minimizing the total number of LTs in the objective function, will minimize \( \sum_i \sum_j L_{ij} \). Therefore, equation (4.17) tries to maximize the sharing of each lightpath by minimizing the second term in the equation. A check that sharing of each lightpath does not exceed its physical capacity, however, will be required and will be shown below for each of the problems separately.

### 4.4.3 The Generic Multicast Problem

In addition to the above constraints, the set of the constraints required for the generic multicast problem is provided in this subsection. For generic multicast problem all the destinations need to be reached such that the amount of the traffic delivered to each destination of a multicast session is same. Hence, we need the following two constraints to ensure the
delivery:
\[ \sum_{j \neq s} Z_{c, d}^{c_a, d} = 1 \quad \forall c_a, d \in D_{ca} \quad (4.18) \]
\[ \sum_{i, j \neq d} Z_{c, d}^{c_a, d} = 1 \quad \forall c_a, d \in D_{ca} \quad (4.19) \]

The capacity used by connection \( c_a \) on lightpath(s) between nodes \( i \) and \( j \) is computed as follows:
\[ X_{ij}^{c_a} = m_{ca} M_{ij}^{c_a} \quad (4.20) \]

Note that referring to the bandwidth requirements of connection \( c_a \) on lightpath(s) between nodes \( i \) and \( j \) using the \( M_{ij}^{c_a} \) variables will avoid multiple counting of the same bandwidth \( m_{ca} \). Furthermore, we need to ensure that the shared capacity of each unique lightpath between nodes \( i \) and \( j \) is not exceeded, which is carried out by using the following constraint:
\[ m_{ca} + \sum_{c_b, c_b \neq c_a} m_{ca} J_{ij}^{c_a,c_b} \leq g \quad \forall c_a, i, j \quad (4.21) \]

**Objective function for the generic multicast problem:**
\[ \text{Minimize} : \alpha \sum_n LT_n + \beta \psi \quad (4.22) \]

In the objective function, \( \alpha \) and \( \beta \) represent the relative cost of an LT and a wavelength channel, respectively. Since, an LT is much more expensive than a wavelength channel, the values of \( \alpha \) and \( \beta \) should be assigned such that saving of an LT has preference over saving of a wavelength channel. In other words, no wavelength channel should be saved at the cost of an LT saving.

### 4.4.4 Multicasting with Partial Destination Set Reachability

In addition to the common set of constraints, the set of the constraints required for the partial destination set reachability problem is provided in this subsection. In case of partial destination set reachability problem the amount of the traffic delivered to the two classes of destination set, \( D'_{ca} \) and \( D''_{ca} \), is the same, i.e., \( m'_{ca} = m''_{ca} = m_{ca} \). Since source traffic delivery to the destinations, \( d \in D''_{ca} \), need only be done if it entails no additional cost, the following
set of constraints ensures the delivery to the destinations \( d \in D_{c_a}' \), while only providing the possibility of delivery to the destinations \( d \in D_{c_a}'' \):

\[
\sum_{j:j \neq s} Z_{s_j}^{c_a,d} = 1 \quad \forall c_a, d \in D_{c_a}'
\]

\[
\sum_{j:j \neq s} Z_{s_j}^{c_a,d} \leq 1 \quad \forall c_a, d \in D_{c_a}''
\]

\[
\sum_{i,i \neq d} Z_{id}^{c_a,d} = 1 \quad \forall c_a, d \in D_{c_a}'
\]

\[
\sum_{i,i \neq d} Z_{id}^{c_a,d} \leq 1 \quad \forall c_a, d \in D_{c_a}''
\]

The value of \( X_{ij}^{c_o} \) is same as that of generic multicast. Also, the following constraints is the same as that of generic multicast and is reproduced here for clarity:

\[
m_{c_a} + \sum_{c_b,c_b \neq c_a} m_{c_b} J_{ij}^{c_a,c_b} \leq g \quad \forall c_a, i, j
\]

**Objective function for the partial destination set reachability problem:**

\[
\text{Minimize : } \alpha \sum_n LT_n + \beta \psi - \gamma \sum_{c_a} \sum_{d \in D_{c_a}''} \sum_{j:j \neq s} Z_{s_j}^{c_a,d}
\]

In addition to choosing \( \alpha \) and \( \beta \) similar to the generic multicast problem, \( \gamma \) is chosen such that the accommodation of the destination set \( D_{c_a}'' \) will not be done at the expense of any additional LT or wavelength. The following assignments captures these objectives:

\[
\gamma = 1
\]

\[
\beta = \gamma + \sum_{c_a} |D_{c_a}''| \times (N - 1)
\]

\[
\alpha = b \times \beta,
\]

where \( b \) is a positive scalar number and is the ratio of the cost of an LT to the cost of a wavelength.
4.4.5 Multicasting with Traffic Thinning

In this subsection, we will provide the constraints specifically needed for the multicasting with traffic thinning problem. In this case, \( m''_c \) < \( m'_c \). Also, as all destinations in \( D'_c \) and \( D''_c \) must be accommodated, we need equation (4.18) and equation (4.19) which are reproduced in the following for the sake of completeness:

\[
\sum_{j,j \neq i} Z^c_{ij} = 1 \quad \forall c_a, d \in D_c 
\]

\[
\sum_{i \neq d} Z^c_{id} = 1 \quad \forall c_a, d \in D_c 
\]

The following constraint ensures that \( G^c_{ij} \) is 1 if and only if connection \( c_a \) is using a lightpath between nodes \( i \) and \( j \) to reach at least one destination \( d \), where \( d \in D'_c \).

\[
G^c_{ij} \geq \sum_d Z^c_{ijd} / Q \quad \forall d \in D'_c, c_a, i, j
\]

\[
G^c_{ij} \leq \sum_d Z^c_{ijd} \quad \forall d \in D'_c, c_a, i, j
\]

Let \( X^c_{ij} \) in this case be computed as:

\[
X^c_{ij} = m''_c M^c_{ij} + (m'_c - m''_c) G^c_{ij}
\]

Note that the value of \( X^c_{ij} \) depends on both \( M^c_{ij} \) and \( G^c_{ij} \). Hence the value of \( X^c_{ij} \) depends on a total of four combinations of the these binary variables. However, the combination \( M^c_{ij} = 0 \) and \( G^c_{ij} = 1 \) cannot take place, and hence this reduce the possible number of combinations to three. Therefore, the above equation will result in values determined by the following cases:

- When \( M^c_{ij} = 1 \) and \( G^c_{ij} = 0 \), \( X^c_{ij} \) will be \( m''_c \).
- When \( M^c_{ij} = 1 \) and \( G^c_{ij} = 1 \), \( X^c_{ij} \) will be \( m'_c \).
- When \( M^c_{ij} = 0 \) and \( G^c_{ij} = 0 \), \( X^c_{ij} \) will be zero.

To ensure that the bandwidth of all the connections sharing the very same lightpath is within the capacity of that lightpath, we need the following constraint

\[
X^c_{ij} + \sum_{c_a \neq c_b} X^c_{ij} \ast J^c_{ij} \leq q \quad \forall c_a, i, j
\]
However, equation (4.34) is non-linear. We map the non-linear term to a linear representation by defining $Y_{ij}^{ca,cb} = X_{ij}^{ca} * J_{ij}^{ca,cb}$, and rewriting equation (4.34) as follows.

$$X_{ij}^{ca} + \sum_{c_b \neq c_a} Y_{ij}^{ca,cb} \leq g \quad \forall c_a, i, j$$  \hspace{1cm} (4.35)

The product term, $Y_{ij}^{ca,cb}$, can now be computed using the following set of linear equations.

$$Y_{ij}^{ca,cb} \geq Q * J_{ij}^{ca,cb} - Q + X_{ij}^{cb} \quad \forall c_a, c_b, i, j$$  \hspace{1cm} (4.36)

$$Y_{ij}^{ca,cb} \leq X_{ij}^{cb} \quad \forall c_a, c_b, i, j$$  \hspace{1cm} (4.37)

$$Y_{ij}^{ca,cb} \leq Q * J_{ij}^{ca,cb} \quad \forall c_a, c_b, i, j$$  \hspace{1cm} (4.38)

**Objective function for Traffic Thinning:** The objective function is this case is the same as that of the generic multicast problem, and simply corresponds to a weighted combination of the total number of LTs, and the total number of wavelength channels used in the network, as given below.

$$\text{Minimize} : \alpha \sum \text{LT} + \beta \psi$$  \hspace{1cm} (4.39)

The complexity of the MILP, for all formulations, in terms of the number of variables is $O(N^4W + N^2K^2)$ and in terms of the number of constraints is $O(N^3W + N^2K^2)$, where $K$ is the total number of multicast connections. If $K > NW$, then both the number of variables and the constraints are $O(N^2K^2)$. While if $K^2 < NW$, then the number of variables and the constraints are $O(N^4W)$ and $O(N^3W)$, respectively.

Note that the variable $Z_{ij}^{ca,d}$ can be defined as an integer. However, experiments show that removing the integer constraint, results in significant reduction in computation time. In general, by reducing the number of integer variables, the number of branching variables is reduced, which helps in reducing the complexity of the problem. We declared the $Z_{ij}^{ca,s}$ variables as real numbers, however, the above constraints will essentially force them to take either 0 or 1 value, thus in essence acting as integer variables. Finally, we will make few notes that help speed up the MILP, though the order of the complexity will remain the same. Note that for the $J$ variables, the following two equations hold, which help reduce the number of variables.

$$J_{ij}^{ca,cb} = M_{ij}^{ca} \quad \forall c_a, i, j$$  \hspace{1cm} (4.40)
Also, all those variables that sum to zero, e.g., in equation (4.10) and equation (4.11), can be simply removed while generating the constraints.

4.5 A Heuristic Approach

In this section we will present a heuristic approach to solve the multicast traffic grooming problem. The basic idea is to incorporate the observations made from the results of the MILP and devise a heuristic to first obtain an initial feasible solution, and then improve that initial solution iteratively. Moreover, instead of presenting a different algorithm for the generic problem and each of its variants, we devise a generic algorithm that handles all of the above mentioned multicast traffic grooming problems. Let $H_{ca} = D'_{ca}$ if the problem to be solved is multicasting with partial destination set reachability, else let $H_{ca} = D_{ca}$. The pseudo-code of the algorithm is shown in Figure 4.6.

The algorithm starts by constructing Shortest Path Tree (SPT) for each request $c_a$, which approximates an SMT. The SPT may consist of multiple hops, where each hop corresponds to a lightpath, which is routed over the shortest physical path. Each traffic request is then routed on the SPT using the first-fit wavelength (heuristic starts with a single wavelength; a new wavelength is added each time when no accommodating wavelength is found among the available wavelengths). The corresponding number of required LTs is computed. An iterative approach (lines 7-26) is then used to improve this initial solution. For each request, $c_a$, and for each of its destinations $d$, many alternative routes(paths) are explored and corresponding saving in terms of the number of LTs, had the request be routed on these alternative routes, is computed (lines 7-17). Among all the alternative routes, the route that provides the maximum saving ($BestSaving$) is selected as the best alternative route for that connection. If the computed maximum saving is above zero the traffic demand is re-routed on the best alternative path, and the path tree (which is no longer a shortest-path) of the request $c_a$ is updated. Finally, in case of multicasting with partial destination set reachability each request is revisited, since teh above makes sure that only $D'_{ca}$ has been accommodated. Until now destinations $d \in D'_{ca}$
have been accommodated only. Now for each request, each of its destinations, \( d \in D''_{ca} \), are accommodated if and only if the accommodation does not increase both the number of LTs, as well as the number of wavelengths. The accommodation of \( d \in D''_{ca} \) is done by determining its shortest path from the source, and routing the demand along its shortest path using first-fit wavelength. However, by now an SPT of connection \( ca \) is already present, and hence only the part of the shortest path not traversing the already-present SPT is routed.

The complexity of constructing the initial solution is \( O(N^2 K \log N + NKW) \), while the complexity of the iterative improvement is \( O(N^3 K \log N) \). Hence, if \( N^2 \log N > W \), then the complexity of the heuristic is \( O(N^3 K \log N) \).

### 4.6 Experimental Results

In this section we will present the results of the MILP model and the heuristic for the multicast traffic grooming problems. For the MILP, we conduct experiments on two different network topologies. One of them consists of 6 nodes, and is shown in Figure 4.7, while the other one is the 14-node NSF network shown in Figure 4.8. Also, for heuristics we randomly generate large-sized networks. The traffic demands in all cases consist of integer multiples of OC-1 connections. The capacity of a wavelength, i.e., the grooming factor, is equivalent to OC-48.

The MILP is solved using the Cplex linear programming package [90]. The values of \( \alpha \) and \( \beta \) are selected to be 100 and 1, respectively. A higher value for \( \alpha \) captures the fact that an LT is much more expensive than providing an additional wavelength in the network. A sample traffic that consists of a mix of multicast and unicast sessions is generated. For comparison purposes, we use a traffic matrix for generic multicasting, and then modify it for the partial destination set reachability and the traffic thinning problems. We divide each destination set, \( D_{ca} \), which is a part of the original traffic matrix, into two subsets, \( D'_{ca} \) and \( D''_{ca} \), such that \( D'_{ca} \cup D''_{ca} = D_{ca} \) and \( D'_{ca} \cap D''_{ca} = \emptyset \). The generated traffic demands are shown in Table 4.1. For the generic and partial destination set reachability problems, the amount of traffic of each multicast connection is mentioned under the column \( m'_{ca} \), while for the traffic thinning problem
Algorithm Multicast Traffic Grooming
BEGIN //initial solution
1. For each traffic session $c_a$
2. For all $d \in H_{c_a}$
3. Construct a Shortest Path Tree, $SPT_{c_a}$, consisting of shortest paths $p_{c_a,d}$;
4. Accommodate $c_a$ on $SPT_{c_a}$ using first-fit wavelength;
5. Compute $LT_{c_a}$, the number of LTs required to accommodate $c_a$;
6. $total.LTs \leftarrow \sum_{c_a} LT_{c_a}$; 
//iterative improvement
7. For each traffic session $c_a$
8. For each of the destination $d \in H_{c_a}$
9. For each of the node $i \in p_{c_a,d}$ of $SPT_{c_a}$
10. $SPT_{c_a} \leftarrow SPT_{c_a}$; //save original tree
11. Remove the traffic demand $(c_a, d)$ from all the links, say $l_i$, between node $i$ and node $d$ on path $p_{c_a,d}$;
12. Find alternative path from node $i$ to $d$, such that new path does not include any of the links in $l_i$;
13. Route the traffic demand $(c_a, d)$ between node $i$ and $d$ over the alternative path using first-fit wavelength;
14. Update $p_{c_a,d}$ with the alternative path; update $SPT_{c_a}$;
15. Compute the number of LTs required for $SPT_{c_a}$;
16. $saving_i \leftarrow$ number of LTs for $SPT_{c_a} -$ number of LT for $SPT_{c_a}$;
17. $SPT_{c_a} \leftarrow SPT_{c_a}$; //recover original tree
18. $i^* \leftarrow \arg\{\min_i(saving_i)\}$;
19. $BestSaving \leftarrow saving_{i^*}$;
20. IF ($BestSaving > 0$) THEN
21. Remove the traffic demand $(c_a, d)$ from all the links between node $i^*$ and node $d$, i.e., $l_{i^*}$;
22. Find alternative path from node $i^*$ to $d$, such that new path does not include any of links in $l_{i^*}$;
23. Route the traffic demand $(c_a, d)$ between node $i$ and $d$ over the alternative path using first-fit wavelength;
24. Update the $SPT_{c_a}$ by incorporating the alternative path;
25. $total.LTs \leftarrow total.LTs - BestSaving$;
26. Return the least number of total LTs obtained at any stage during the iterative improvement;
27. IF(partial destination set reachability problem) THEN
28. For each traffic session $c_a$
29. For each $d \in D_{c_a}$
30. Compute the number of LTs and wavelengths required to accommodate the demand along its shortest path and using available SPT of request $c_a$;
31. IF (the number of LTs and wavelengths required are zero) THEN
32. accommodate the demand;
END

Figure 4.6 Multicast traffic grooming heuristic.

the amount of traffic of each multicast connection is mentioned under the column $m^w_{c_a}$. The problem is run using four wavelengths. Table 4.2 summarizes the total number of LTs and the total number of wavelengths obtained for each problem. The last column of the Table 4.2
Figure 4.7 A six node network. 

Figure 4.8 NSF Network topology.

Table 4.1 Multicast traffic demands on the 6-node network.

<table>
<thead>
<tr>
<th>$c_0$</th>
<th>$D_{c_0}$</th>
<th>$D_{c_0}''$</th>
<th>$m_{c_0}$</th>
<th>$m_{c_0}''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>{5}</td>
<td>{}</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>(0,2)</td>
<td>{1,2}</td>
<td>{4}</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>(0,3)</td>
<td>{5}</td>
<td>{1,2}</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>(0,4)</td>
<td>{2,4}</td>
<td>{3,5}</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>(1,1)</td>
<td>{3}</td>
<td>{}</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>(1,2)</td>
<td>{3}</td>
<td>{}</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>(1,3)</td>
<td>{2,5}</td>
<td>{1}</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>(2,1)</td>
<td>{1}</td>
<td>{}</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>(2,2)</td>
<td>{5}</td>
<td>{}</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>(2,3)</td>
<td>{0}</td>
<td>{3,5}</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>(2,4)</td>
<td>{1,3,4}</td>
<td>{0,5}</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>(3,1)</td>
<td>{0,1}</td>
<td>{}</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>(3,2)</td>
<td>{4}</td>
<td>{5}</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td>(4,1)</td>
<td>{0}</td>
<td>{}</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>(4,2)</td>
<td>{0}</td>
<td>{}</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>(4,3)</td>
<td>{0,2}</td>
<td>{1}</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>(5,1)</td>
<td>{2}</td>
<td>{}</td>
<td>48</td>
<td>-</td>
</tr>
</tbody>
</table>

shows the number of LTs at each node.

As expected, the total number of LTs and wavelengths are lesser for both the partial destination set reachability and traffic thinning problems. The small difference between the number of required LTs for generic traffic grooming and traffic thinning is primarily due to
Table 4.2 The number of wavelengths, total number of LTs, and location of LTs in the network as required by MILP for the generic, partial destination set reachability, and the traffic thinning problems.

<table>
<thead>
<tr>
<th></th>
<th>Wavelengths</th>
<th>Total LTs</th>
<th>Location of LTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generic</td>
<td>3</td>
<td>21</td>
<td>(LT_0=4, LT_1=2, LT_2=4, LT_3=3, LT_4=3, LT_5=5)</td>
</tr>
<tr>
<td>Partial</td>
<td>2</td>
<td>13</td>
<td>(LT_0=3, LT_1=1, LT_2=4, LT_3=2, LT_4=2, LT_5=1)</td>
</tr>
<tr>
<td>Thinning</td>
<td>2</td>
<td>20</td>
<td>(LT_0=3, LT_1=2, LT_2=5, LT_3=3, LT_4=3, LT_5=4)</td>
</tr>
</tbody>
</table>

the facts that in the traffic thinning problem all the destinations need to be reached, and the difference in the amount of the traffic between generic and traffic thinning problem is small. The difference in the number of LTs in case of partial destination set reachability problem, however, is profound. One of the possible explanations is that the MILP, after accommodating the must-accommodate sets, i.e., \(D_{ca}'\), has the flexibility to accommodate only those destinations which will require no additional LTs to be reached. There is a total of 37 destinations in the original traffic matrix, and a total of 13 destinations in optionally-accommodate destinations sets, i.e., \(D_{ca}''\). Out of these 13 destinations, the MILP manages to accommodate 7 destinations. In other words, out of a total of 37 destinations, partial destination set reachability problem accommodated 31 destinations while using only 13 LTs as compared to 21 LTs used by the generic multicast traffic grooming problem. This translates into a saving of 38% of the LTs while accommodating 83% of the destinations. Thus, while working under a tight budget, dividing the destinations into critical and non-critical sets and following an approach similar to partial destination set reachability, can alleviate the financial issues.

In Figure 4.9, we are showing the routing of a single multicast tree at the lightpath level and are also showing the corresponding physical links traversed by the lightpaths. Moreover, all the LTs required by the optimal solution of the generic multicast problem for the traffic matrix given in Table 4.1, are also shown. Note that the lightpaths between nodes 0 and 1, and between nodes 2 and 5, are not using shortest path at the physical level. In Table 4.3, we are listing the lightpaths used by each unicast or multicast connection, as determined by the optimal solution of the generic multicast problem for the traffic matrix given in Table
Similarly, in Table 4.4, we are listing the physical paths of all the lightpaths required by the optimal solution of the generic multicast problem for the traffic matrix given in Table 4.1. Also, the corresponding wavelengths used by the lightpaths are mentioned. A detailed inspection of the solutions produced by the MILPs reveals the following information.

- At the lightpath level, the multicast traffic is delivered either directly, from the source to the destination, or through another destination in the multicast destination set.
- Instead of establishing multiple individual lightpaths between a source and each of its destinations, many lightpaths carry the (multicast) traffic to more than one destination in the same session, while simultaneously grooming the traffic to other sessions.
- An inspection of the physical paths corresponding to lightpaths revealed that not all the lightpaths are routed over shortest physical path. This shows that to obtain the minimum number of LTs, one needs to explore routes other than the shortest-path routes.

To accommodate the traffic matrices given in Table 4.1, the heuristic approaches for the generic multicast traffic grooming, partial destination set reachability, and traffic thinning
Table 4.3  Lightpaths generated by the MILP for the generic multicast traffic grooming problem for the traffic demands given in Table 4.1.

<table>
<thead>
<tr>
<th>$c_n$</th>
<th>Lightpaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>0-5</td>
</tr>
<tr>
<td>(0,2)</td>
<td>0-2, 2-1, 2-4</td>
</tr>
<tr>
<td>(0,3)</td>
<td>0-2, 2-1, 2-5</td>
</tr>
<tr>
<td>(0,4)</td>
<td>0-2, 0-5, 5-3, 5-4</td>
</tr>
<tr>
<td>(1,1)</td>
<td>1-3</td>
</tr>
<tr>
<td>(1,2)</td>
<td>1-3, 3-5</td>
</tr>
<tr>
<td>(1,3)</td>
<td>1-0, 0-2, 0-5</td>
</tr>
<tr>
<td>(2,1)</td>
<td>2-1</td>
</tr>
<tr>
<td>(2,2)</td>
<td>2-5</td>
</tr>
<tr>
<td>(2,3)</td>
<td>2-4, 4-3, 3-5, 5-0</td>
</tr>
<tr>
<td>(2,4)</td>
<td>2-5, 5-0, 5-3, 5-4, 4-1</td>
</tr>
<tr>
<td>(3,1)</td>
<td>3-0, 0-2, 2-1</td>
</tr>
<tr>
<td>(3,2)</td>
<td>3-5, 5-4</td>
</tr>
<tr>
<td>(4,1)</td>
<td>4-0</td>
</tr>
<tr>
<td>(4,2)</td>
<td>4-3, 3-0</td>
</tr>
<tr>
<td>(4,3)</td>
<td>4-1, 4-0, 0-2</td>
</tr>
<tr>
<td>(5,1)</td>
<td>5-2</td>
</tr>
</tbody>
</table>

problems requires 29, 19, and 27, LTs and 4, 3, and 3, wavelengths, respectively. The results are summarized in Table 4.5. Comparison of these results to the corresponding results obtained from the MILP, the heuristic solutions are within 38%, 46%, and 35% of the optimal number of LTs for the generic traffic grooming, partial destination set reachability, and traffic thinning problems, respectively. Moreover, the heuristic solution for the partial destination set reachability problem, accommodated 5 destinations from the destination set which were in the must accommodate set. Hence, the heuristic accommodated 29 destinations, out of a total of 37 destinations, while using 19 LTs and 3 wavelengths. When compared with the solution generated by the heuristic for generic multicast traffic grooming problem, this translates into a saving of 34% of the LTs while accommodating 78% of the destinations. Furthermore, the experimental results from other medium sized examples, show that the heuristic solutions are within 30-40% of the optimal values.
Table 4.4 Lightpaths and their corresponding physical paths generated by the MILP for the generic multicast traffic grooming problem for the traffic demands given in Table 4.1.

<table>
<thead>
<tr>
<th>Lightpaths</th>
<th>Physical links</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>0-3, 3-1, 1-2 (λ₁)</td>
</tr>
<tr>
<td></td>
<td>0-3, 3-1, 1-2 (λ₂)</td>
</tr>
<tr>
<td></td>
<td>0-1, 1-2 (λ₃)</td>
</tr>
<tr>
<td>0-5</td>
<td>0-3, 3-4, 4-2, 2-5 (λ₃)</td>
</tr>
<tr>
<td>1-0</td>
<td>1-3, 3-0 (λ₁)</td>
</tr>
<tr>
<td>1-3</td>
<td>1-3 (λ₃)</td>
</tr>
<tr>
<td>2-1</td>
<td>2-1 (λ₃)</td>
</tr>
<tr>
<td>2-4</td>
<td>2-4 (λ₁)</td>
</tr>
<tr>
<td>2-5</td>
<td>4-5, 2-4 (λ₃)</td>
</tr>
<tr>
<td>3-0</td>
<td>3-0 (λ₃)</td>
</tr>
<tr>
<td>3-5</td>
<td>3-4, 4-2, 2-5 (λ₁)</td>
</tr>
<tr>
<td></td>
<td>3-4, 4-5 (λ₂)</td>
</tr>
<tr>
<td>4-0</td>
<td>4-3, 3-1, 1-0 (λ₃)</td>
</tr>
<tr>
<td>4-1</td>
<td>4-2, 2-1 (λ₃)</td>
</tr>
<tr>
<td>4-3</td>
<td>4-3 (λ₁)</td>
</tr>
<tr>
<td>5-0</td>
<td>5-2, 2-1, 1-0 (λ₁)</td>
</tr>
<tr>
<td>5-2</td>
<td>5-2 (λ₂)</td>
</tr>
<tr>
<td>5-3</td>
<td>5-4, 4-3 (λ₂)</td>
</tr>
<tr>
<td>5-4</td>
<td>5-4 (λ₁)</td>
</tr>
<tr>
<td></td>
<td>5-4 (λ₃)</td>
</tr>
</tbody>
</table>

Table 4.5 Results of the heuristic approach for the generic, partial destination set reachability, and the traffic thinning problems, using the traffic matrix shown in Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>wavelengths</th>
<th>Total LTs</th>
<th>Number of destinations reached</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generic</td>
<td>4</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>Partial</td>
<td>3</td>
<td>19</td>
<td>29</td>
</tr>
<tr>
<td>Thinning</td>
<td>3</td>
<td>27</td>
<td>37</td>
</tr>
</tbody>
</table>

To run the optimal approach on some real network, we chose the 14-node, 21-edge NSF network shown in Figure 4.8. We selected a traffic matrix, shown in Table 4.6, such that every node is selected as destination at least once. Also, out of 14 nodes, 10 nodes are acting as
source nodes. However, each source node is establishing only a single unicast or multicast session. Moreover, out of a total of 10 sessions, 5 sessions are unicast sessions, while the rest of the sessions are multicast. Although, we run the MILP with 4 wavelengths, it reduces the number of wavelengths to 1. The corresponding number of required LTs to accommodate the whole matrix is 16. An inspection of the solution, in terms of routing of lightpaths and their corresponding physical links, leads to the same conclusions drawn from 6-node topology network. The proposed heuristic for this example produced a solution which required 23 LTs and 3 wavelengths. The heuristic solution, in terms of the number of LTs, is within 43% of the optimal value.

To study the performance of the heuristic with large input files, we chose NSF network topology and randomly generated the traffic, for the generic multicast problem, with the following parameters:

- A number of sessions from each node is generated uniformly between 0 and 14.
- 50% of the sessions carry multicast traffic, while the remaining sessions carry unicast traffic.
- For each multicast session, the destination set size is uniformly distributed between 2 and 8 for the first scenario, and between 7 and 13 for the second scenario.

Table 4.6  Multicast traffic demands on the NSF network.

<table>
<thead>
<tr>
<th>$c_a$</th>
<th>$D_{ca}$</th>
<th>$m_{ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>{2}</td>
<td>12</td>
</tr>
<tr>
<td>(1,1)</td>
<td>{5,9}</td>
<td>24</td>
</tr>
<tr>
<td>(3,1)</td>
<td>{8}</td>
<td>36</td>
</tr>
<tr>
<td>(4,1)</td>
<td>{0,6}</td>
<td>24</td>
</tr>
<tr>
<td>(6,1)</td>
<td>{1}</td>
<td>36</td>
</tr>
<tr>
<td>(7,1)</td>
<td>{2,11,12,13}</td>
<td>18</td>
</tr>
<tr>
<td>(9,1)</td>
<td>{10}</td>
<td>24</td>
</tr>
<tr>
<td>(10,1)</td>
<td>{8,11}</td>
<td>24</td>
</tr>
<tr>
<td>(12,1)</td>
<td>{4}</td>
<td>3</td>
</tr>
<tr>
<td>(13,1)</td>
<td>{3,7}</td>
<td>12</td>
</tr>
</tbody>
</table>
• The destinations are chosen randomly among all nodes, excluding the source, for both unicast and multicast sessions.

• The generated traffic, for both unicast and multicast sessions, is an integer multiple of OC-1, and is uniformly chosen from the set \{1,3,9,12,18,24,36,48\}. These values represent the recommended rates for OC streams.

Table 4.7 shows the results of the experiments. For comparison purposes, experiments are conducted for the original traffic load, generated as mentioned above, and also by doubling the load. To double the load, we just duplicate all the requests from each source. Moreover, the results from the heuristic are compared to the case when all multicast traffic is accommodated using multiple unicast connections. In Table 4.7 the two scenarios correspond to different destination set sizes. As we mentioned earlier, for the first scenario the destination set size is uniformly distributed between 2 and 8, and for the second scenario it is uniformly distributed between 7 and 13. Hence the destination size in the second scenario is almost double than that of the first scenario. The total amount of traffic generated for the first scenario is equivalent to 504 OC-1 streams, while the total amount of the traffic generated for the second scenario is equivalent to 924 OC-1 streams. From Table 4.7 it is evident that both the number of LTs and the number of wavelengths increases by almost 100% when the load is doubled (the doubled load for the first and second scenario is equivalent to 1008 OC-1 streams and 1848 OC-1 streams, respectively). This shows that our heuristic is grooming the traffic effectively. Moreover, Table 4.7 shows that when the multicast traffic is accommodated by employing multiple unicast connections, the number of required LTs is more than 30% than that of our heuristic. Similarly, using multiple unicast connections one ends up using more than twice the number of wavelengths of computed by our heuristic to accommodate the same traffic demands. Finally, the comparison between the first and the second scenarios shows that if the traffic is accommodated by constructing multicast trees and the size of the destination set is doubled, the number of required LTs increases by almost 60%, while the increment in the number of wavelengths is less than twice. However, if multiple unicast connections are employed to accommodate the traffic, then doubling the size of the destination set increases
Table 4.7 The number of LTs and wavelengths, obtained by the generic traffic grooming heuristic and by accommodating the traffic demands using multiple unicast connections.

<table>
<thead>
<tr>
<th></th>
<th>Original load</th>
<th>Doubled load</th>
<th>Multiple uncasts with original load</th>
<th>Multiple uncasts with doubled load</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Scenario</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTs</td>
<td>237</td>
<td>462</td>
<td>321</td>
<td>631</td>
</tr>
<tr>
<td>Wavelengths</td>
<td>15</td>
<td>30</td>
<td>34</td>
<td>68</td>
</tr>
<tr>
<td><strong>Second Scenario</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTs</td>
<td>381</td>
<td>741</td>
<td>630</td>
<td>1229</td>
</tr>
<tr>
<td>Wavelengths</td>
<td>19</td>
<td>37</td>
<td>59</td>
<td>117</td>
</tr>
</tbody>
</table>

the number of LTs by almost 100% while the number of wavelengths increases by almost 75%. Thus, our heuristic has a clear advantage over considering the multicast traffic as multiple unicast connections and then grooming them together.

We also conducted experiments on the NSF network topology for partial destination set reachability and traffic thinning. We randomly generated traffic with the above mentioned parameters and with the destination set size uniformly distributed between 2 and 8. From this traffic matrix, we then generated the traffic matrices for the partial destination set reachability problem and for the traffic thinning problem using the following guidelines:

- The destination set size is equally divided into two subsets, $D'_a$ and $D''_a$.

- For traffic thinning an immediate lower rate is selected from the set \{1,3,9,12,18,24,36,48\}, e.g., if $m'_{c_a}$ is 12 units of traffic, then $m''_{c_a}$ is selected to be 9 units of traffic.

The results obtained by running the MILP on the modified traffic matrices will essentially capture the reduction in the cost of the network due to either partial destination set reachability or traffic thinning. Traffic generation as outlined above generated a total of 486 connections. Out of these connections, 294 connections belong to sets $D'_c$ (all unicast connections belong to these sets too) and 192 connections belong to sets $D''_c$. As shown in Table 4.8, the proposed heuristic, for partial destination set reachability problem, is able to accommodate 79 connections that belong to sets $D''_c$ without an increase in the cost of the network. Hence, a total of
76% of the connections are accommodated while achieving a 31% lower network cost. In case of traffic thinning each connection is served while achieving a 10% lower network cost.

Table 4.8 The results of the heuristic for generic traffic grooming, partial destination set reachability, and traffic thinning problems on NSF network.

<table>
<thead>
<tr>
<th></th>
<th>wavelengths</th>
<th>Total LTs</th>
<th>Percentage accommodated</th>
<th>Percentage saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generic</td>
<td>28</td>
<td>386</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Partial</td>
<td>23</td>
<td>266</td>
<td>76</td>
<td>31</td>
</tr>
<tr>
<td>Thinning</td>
<td>24</td>
<td>344</td>
<td>100</td>
<td>11</td>
</tr>
</tbody>
</table>

4.7 Chapter Summary

In this chapter, we developed a unified framework to design and provision WDM networks, given sub-wavelength multicast traffic. We solved different multicast grooming problems, which arise as a part of the dimensioning of optical networks, and developed both optimal as well as heuristic solutions. For optimal solutions we exploited the specifics of the problem to have a linear formulation, of an otherwise non-linear problem. The formulation is generic and also ensures non-bifurcation of the traffic. For the approximate technique we first developed an initial feasible solution, and then improved it iteratively. A number of experiments have been conducted. Also, we showed that both partial destination set reachability and traffic thinning incurs lower network cost and hence can be used while dimensioning the network under a tight budget.
CHAPTER 5 MANY-TO-ONE TRAFFIC GROOMING IN WDM NETWORKS

In this chapter we address the design and provisioning of WDM networking for many-to-one traffic grooming on arbitrary topologies. Traffic streams from different sources in the same session can be aggregated using arbitrary, but application dependent, aggregation ratios. We provide optimal as well as heuristic solutions to the problem. The objective is to minimize the cost of the network, by minimizing the total number of the higher layer components and the total number of the wavelengths used in the network. One of the main contributions of this work is to provide a mixed integer linear formulation, to an otherwise non-linear problem, by exploiting the specifics of routing and aggregation sub-problems, while still maintaining the optimality of the solution. Moreover, the formulation can handle varying amounts of traffic from each source to a common destination, as well as arbitrary aggregation fractions of the data coming from the different sources. Interestingly, using linear relationships, we manage to make this fraction a function of the number of the streams participating in the aggregation.

For the heuristic solution we developed a Dynamic Programming style approach that builds the solution progressively, going through a number of stages, while choosing the best partial solutions among a number of possible partial solutions at each stage.

The contributions of the work in this chapter are summarized as follows.

- We provide optimal as well as heuristic solutions to the design of optical networks supporting many-to-one traffic grooming.

- The model provide the flexibility in aggregating the traffic streams with arbitrary ratios, such that the ratios are functions of the number of the traffic streams participating in the aggregation.
5.1 Introduction

WDM networks are extending their presence from merely in backbone networks to metro and access networks. A direct consequence of this phenomenon is that WDM networks are required to support diverse applications, each with its own traffic characteristics. To efficiently utilize the network resources it is imperative to develop customized approaches for each traffic type. In the previous chapter, we addressed the one-to-many or multicast traffic type. In this chapter we will focus on the many-to-one traffic type. The many-to-one service model is used by applications such as resource discovery, data collection, auctions, polling and audience to speaker communication. Like most of the multicast service applications, several many-to-one service applications too require only sub-wavelength capacity, and hence to efficiently utilize the network resources traffic grooming is a natural choice. However, to the best of our knowledge, no one has yet addressed the many-to-one traffic grooming problem.

Although both multicast and many-to-one traffic types have the multipoint factor in common, each type has different characteristics and hence demands different design and operational approaches. Many-to-one traffic propagates from a set of sources to a particular destination. Depending on the application, each source may generate a different amount of data. There could be some overlap or redundancy between the data streams heading towards the destination. Examples of aggregation include aggregation of voice calls due to ubiquitous single speaker mode, aggregation of data from sensors, or aggregation of low frame rate video stream feeding to control centers. To efficiently use the resources, such data streams might be aggregated on their way to the destination. Hence a many-to-one routing tree, sourced at different nodes and terminating at a common destination can be created such that data streams are aggregated at each merging point of the many-to-one tree. Compared to a typical multicast tree where a single stream of data coming from a source is duplicated onto multiple streams at branching points, the many-to-one tree merges and may aggregate the data coming from different sources at an aggregation point. This changes the bandwidth requirements and equipment functionalities for a many-to-one tree. Hence, the algorithms developed for multicast trees are not very useful for designing the many-to-one trees.
In this chapter, we present an optimal as well as heuristic solutions for the many-to-one traffic grooming problem such that given a traffic matrix all the traffic should be accommodated with the least number of required higher layer light terminating equipment, while also trying to minimize the total number of wavelengths in the network.

5.2 Problem Formulation

In this section we will explain the model used in the paper and formulate an MILP for the many-to-one traffic grooming problem on arbitrary network topologies.

5.2.1 The Model

The model used for the many-to-one traffic grooming has many similarities to that of multicast traffic grooming, with some notable differences. Like the multicast model, in the many-to-one model too we visualize the network at three different levels: the physical level, the lightpath level and the connection level. These three levels represent the same functionality as that of multicast.

The bandwidth requirements of a many-to-one routing tree, however, is very different than that of a typical multicast tree. In the many-to-one case, a set of sources transmit data to a common destination over a many-to-one routing tree, while aggregating the data streams, and may be removing the redundant data at each merging point on the tree. In our model, we consider the case in which aggregation is implemented at an LT, and hence in the electronic domain\(^1\) We will then minimize the number of the LTs, hence leading to a minimal network cost. We consider a generic model that allows, at each aggregation point\(^2\), the selection of an arbitrary fraction of the data of each stream. Moreover, this fraction is made a function of the number of the streams participating in the aggregation. For arbitrary values for the aggregation ratios, the problem becomes much more difficult than fixed values for the aggregation ratios.

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\(^1\)Once a traffic stream is available in the electronic domain, several operations can be performed on it. For example, its size can be reduced by removing some of its data. Also, it can be multiplexed with other traffic streams using standard multiplexing techniques. The implementation details of aggregation, however, are application dependent and are therefore out of the scope of this work.

\(^2\)Since aggregation is done at merging points, we will use aggregation and merging points interchangeably.
For each many-to-one connection, at each aggregation point, MILP has to determine the total capacity after aggregation. This involves selection of an appropriate aggregation ratio, which itself depends on the number of sources of the many-to-one session participating in the aggregation at that specific aggregation point. For example, if \( m_{Ca,1}, m_{Ca,2}, \ldots, m_{Ca,h} \), are the capacities of \( h \) streams to be aggregated, then \( \alpha/h \) will be taken from each stream, where \( 0 < \alpha \leq 1 \), and hence the total capacity after aggregation will be \( \sum_{s=1}^{h} m_{Ca,s} \cdot \alpha/h \). However, the number of streams participating in the aggregation, \( h \), is a variable and needs to be determined. This inherently makes the problem non-linear. However, by exploiting the specifics of the routing and aggregation sub-problems, we will formulate the problem in a linear fashion while still maintaining the optimality of the solution.

Regarding notation, we will use the same notations used for multicast traffic grooming, except that we will define a few more variables and will also update the definitions of a few variables. Let \( K \) be the total number of sessions terminating at all the destinations. Each connection \( c_a \), where \( 1 \leq a \leq K \), in the many-to-one case corresponds to an ordered pair \((d, k)\), where \( d \) is the destination, and \( k \) represents the \( k^{th} \) (unicast or many-to-one) session terminating at \( d \). The source set of session \( c_a \) is \( S_{Ca} \), and \( s \) represents a source in the source set i.e., \( s \in S_{Ca} \). Few additional notations are defined below.

- **Input parameters:**
  \( r^\alpha_f \): aggregation ratio; the fraction of the capacity of each stream from \( s \in S_{Ca} \) selected for aggregation, when the number of sources participating in aggregation is \( f \); \( r^\alpha_f = 1 \), \( \forall c_a \)
  \( m_{ca,s} \): number of basic units of traffic originating from source \( s \) of connection \( c_a \)
  \( Q \): a very large integer, in many-to-one case it suffices to set \( Q \) such that 
  \( Q \geq max_{Ca} \left( \sum_{s \in S_{Ca}} m_{ca,s} \right) \)

- **Variables of the MILP:**
  \( Z^\alpha_{ij,s} \): a real number between 0 and 1, which takes non zero values if and only if the traffic stream from source \( s \in S_{Ca} \) is using a lightpath from \( i \) to \( j \)
  \( M^\alpha_{ij} \): a binary indicator; is 1 if and only if at least one of the sources of connection \( c_a \)
is using a lightpath from node $i$ to $j$ to reach the destination, i.e., $\exists s \in S_{ca}$, such that

$$Z_{ij}^{ca,s} = 1$$

$I_{ij}^{ca,f}$: a binary indicator; is 1 if and only if the number of sources $s \in S_{ca}$, on a lightpath from node $i$ to $j$, are $\leq f$

$X_{ij}^{ca}$: a real number which represents the amount of traffic on lightpath from node $i$ to $j$ due to all sources in $S_{ca}$

### 5.2.2 The MILP Formulation

In this subsection we present the MILP for the many-to-one grooming problem. Our objective is to minimize the total cost of the higher layer equipment by minimizing the total number of LTs, as well the maximum number of different wavelengths on a fiber.

**Objective function:**

$$\text{Minimize : } \alpha \sum_{n} LT_n + \beta \psi$$

(5.1)

The above function is a multi-objective function, in which LTs are given higher priority for minimization than the wavelengths.

**Subject to:**

- **Number of LTs:**
  
  The constraints to determine the number of the LTs in the network are given in equations (4.1) and (4.2).

- **Number of wavelengths:**
  
  The constraints to determine the index of the highest numbered wavelength used on any fiber link in the network are given in equations (4.3), (4.4), and (4.5).

- **Lightpath level constraints:**
  
  The lightpath level constraints for many-to-one traffic grooming are the same as those of multicast traffic grooming, i.e, equations (4.6) to equation (4.9).
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• **Connection topology constraints:**

The following constraint ensures that for the traffic request sourced at $s$ and is part of the many-to-one connection $c_a$, no traffic is coming in (going out) the source (destination), respectively

$$\sum_{i,i \neq s} Z_{is}^{c_a,s} = \sum_{j,j \neq d} Z_{dj}^{c_a,s} = 0 \quad \forall c_a, s \in S_{ca}$$  \hspace{1cm} (5.2)

The following constraint ensures that the traffic request between $s$ and $d$, which is part of the many-to-one connection $c_a$, is originating (terminating) at $s$ ($d$), respectively

$$\sum_{j,j \neq s} Z_{sj}^{c_a,s} = \sum_{i,i \neq d} Z_{td}^{c_a,s} = 1 \quad \forall c_a, s \in S_{ca}$$  \hspace{1cm} (5.3)

The following constraint preserves the continuity of connection traffic on multiple light-paths

$$\sum_{i,i \neq x, i \neq d} Z_{ix}^{c_a,s} = \sum_{j,j \neq x, j \neq s} Z_{xj}^{c_a,s} \quad \forall c_a, s \in S_{ca}, x, (x \neq s, d)$$  \hspace{1cm} (5.4)

Once data streams, from more than one source, are aggregated at some node, these should remain intact for the rest of their travel to the destination, i.e., these should follow the very same physical path for the rest of the journey. The following set of constraints ensures that the aggregated streams will remain intact at the lightpath level. As we are ensuring non-bifurcation too (using a separate set of constraints to be described later), these data streams will remain intact at the physical level too. The following set of constraints essentially also determines the routing of the many-to-one connections.

$$M_{ij}^{c_a} \geq \sum_{s \in S_{ca}} Z_{ij}^{c_a,s} / Q \quad \forall c_a, i, j$$  \hspace{1cm} (5.5)

$$M_{ij}^{c_a} \leq \sum_{s \in S_{ca}} Z_{ij}^{c_a,s} \quad \forall c_a, i, j$$  \hspace{1cm} (5.6)

$$\sum_{j} M_{ij}^{c_a} \leq 1 \quad \forall c_a, i$$  \hspace{1cm} (5.7)

The next task is to determine the amount of traffic before and after aggregation on each of the lightpaths. For this purpose, we first use the following constraints to set the binary indicator $I_{ij}^{c_a,f}$ to one if and only if the number of sources $s \in S_{ca}$ are $\leq f$

$$\sum_{s \in S_{ca}} Z_{ij}^{c_a,s} = \sum_{f=1}^{\left| S_{ca} \right|} I_{ij}^{c_a,f} \quad \forall c_a, i, j$$  \hspace{1cm} (5.8)
Once the $I^{ca, j}_{ij}$ variables have been determined by the above constraints, they are used to determine the exact amount of traffic after aggregation on lightpath(s) from node $i$ to node $j$ due to the sources in $S_{Ca}$. This is done using the following set of constraints.

\[ X^{ca, j}_{ij} \geq I^{ca, j}_{ij} \sum_{s \in S_{Ca}} m_{ca, s} Z^{ca, s}_{ij} - Q \sum_{s \in S_{Ca}} Z^{ca, s}_{ij} + Q * f * I^{ca, j}_{ij} \quad 1 \leq f \leq |S_{Ca}|, \forall c, a, i, j \quad (5.10) \]

\[ X^{ca, j}_{ij} \leq I^{ca, j}_{ij} \sum_{s \in S_{Ca}} m_{ca, s} Z^{ca, s}_{ij} + Q \sum_{s \in S_{Ca}} Z^{ca, s}_{ij} - Q * f * I^{ca, j}_{ij} \quad 1 \leq f \leq |S_{Ca}|, \forall c, a, i, j \quad (5.11) \]

To understand how the above constraints will compute the exact amount of traffic after aggregation on lightpath(s) from node $i$ to node $j$ due to the sources in $S_{Ca}$, we divide the range of $f$ into three sub-ranges: (1) $f < \sum_{s \in S_{Ca}} Z^{ca, s}_{ij}$, (2) $f > \sum_{s \in S_{Ca}} Z^{ca, s}_{ij}$, and (3) $f = \sum_{s \in S_{Ca}} Z^{ca, s}_{ij}$. Note that the value $\sum_{s \in S_{Ca}} Z^{ca, s}_{ij}$ is the number of the sources ($s \in S_{Ca}$) arriving at the node $i$ and hence taking part in the aggregation. For sub-range (1), $X^{ca}_{ij}$ will be greater than a large negative number and smaller than a large positive number. For sub-range (2), the values of $I^{ca, j}_{ij}$ will be zero and again $X^{ca}_{ij}$ will be greater than a large negative number and smaller than a large positive number. However, for sub-range (3) the last two terms will cancel each other and we will obtain an exact value for the amount of traffic after aggregation on lightpath(s) from node $i$ to node $j$ due to sources in $S_{Ca}$. The whole range $1 \leq f \leq |S_{Ca}|$ needs to be considered because the number of the data streams on lightpath(s) from node $i$ to node $j$, due to the sources in $S_{Ca}$, itself is a variable determined by the MILP.

Finally, to bound the total capacity on lightpath(s) between node $i$ and $j$, we add the following constraint

\[ \sum_{c_a = 1}^{K} X^{ca}_{ij} \leq L_{ij} * g \quad \forall i, j \quad (5.12) \]

- Non-Bifurcation:

Again, the above constraints just ensured that each traffic request of a many-to-one connection does not bifurcate (vertically split) its traffic between lightpaths of different node pairs. But there is no assurance that traffic demands will not get bifurcated over...
multiple lightpaths between a node pair. The following set of constraints, together with the above constraints, will ensure a complete non-bifurcated solution.

\[ J_{ij}^{ca,cb} \leq \frac{(M_{ij}^{ca} + M_{ij}^{cb})}{2} \quad \forall c_a, c_b, i, j \]  
\[ L_{ij} = J_{ij}^{ca,ci} + \sum_{a=2}^{K} (J_{ij}^{ca,ca} - \sum_{b=1}^{a-1} J_{ij}^{ca,cb}) \quad \forall i, j \]  
\[ X_{ij}^{ca} + \sum_{c_b, c_b \neq c_a} Y_{ij}^{ca,cb} \leq g \quad \forall c_a, i, j \]

The above constraints are the same as those of multicast traffic thinning given by equations (4.16), (4.17) and (4.35), except that the definition of a few variables is updated. The physical significance of the constraints, however, remains the same. Note that the variable \( Y_{ij}^{ca,cb} \) contributes a non-linear term, as \( Y_{ij}^{ca,cb} = J_{ij}^{ca,cb} \times X_{ij}^{ca} \). However, using the set of linear constraints given by equations (4.36), (4.37), and (4.38) we can determine the variable \( Y_{ij}^{ca,cb} \) in a linear fashion.

Note that while ensuring non-bifurcation we are inherently assuming that the total capacity of all aggregated traffic streams is less than or equal to that of a single wavelength, i.e., \( \sum_{s \in S_{ca}} m_{ca,s} \leq g \). We will observe this restriction while generating the traffic for the MILP in Section 5.4. Regarding the complexity of the MILP, because we are still able to determine the aggregated capacity in a linear fashion, the dominant factors in the MILP for many-to-one traffic grooming or same as that of multicast. Hence, the complexity in terms of the number of variables is \( O(N^4W + N^2K^2) \) and in terms of the number of constraints is \( O(N^3W + N^2K^2) \), where \( K \) is the total number of many-to-one connections.

### 5.3 A Heuristic Technique

In this section we will present a heuristic technique for many-to-one traffic grooming on random topologies that also performs data aggregation and does not bifurcate the traffic originating at each source.

The search space for the many-to-one traffic grooming problem is huge. To design a heuristic using non-exhaustive search, one needs to make decisions, which may be greedy in nature,
at a number of points during the search process. For example, the sequence in which each connection is selected and accommodated will affect the cost associated with the accommodation of other connections. Also, the decisions taken regarding the routing and wavelength assignment of each connection will determine the many-to-one tree and thus the aggregation points. Moreover, data aggregation will take place on those physical links where two or more streams of the same many-to-one session share the same wavelength. Once data aggregation takes place, we need to ensure that all the aggregated streams follow the same physical route afterwards until they reach the destination.

To explore the search space we designed a Dynamic Programming (DP) style algorithm. Let each many-to-one session, \( c_a \), consist of \( |S_{ca}| \) streams, one from each source \( s \in S_{ca} \), and let \( \Phi \) be the total number of streams from all the many-to-one connections, i.e., \( \Phi = \sum_{c_a} |S_{ca}| \).

The graphical depiction of the DP-style algorithm is given in Figure 5.1. As shown in the figure, the algorithm consists of \( \Phi \) stages, while each stage further consists of \( \Phi \) steps. The sequence in which the streams are stage ordered, or are ordered within each stage, is taken arbitrarily. However, the algorithm checks several combinations during its execution. At each step of a stage, we accommodate a new stream after conducting a systematic search, such that the decisions at each step of a stage depends only on the previous stage. The outcome of each step is a selection of a set of accommodated streams; let us call such a set an assignment. At the end of each stage we have a collection of several best assignments. After the final stage, we simply select the assignment that offers the least number of LTs, while breaking the tie by selecting solutions that offer the least number of wavelengths.

For the systematic search at each stage, we examine all the assignments of previous stage and determine which assignment can accommodate the current stream with the least cost. In Figure 5.1, steps 2 and 4 of stages 2 and 3 are shown in detail. For example, at step 2 of stage 2, the stream with I.D. 2 is accommodated. However, in order to do that, all the assignments from stage 1 are considered and an assignment is selected such that when combined with stream 2, incurs the least network cost. In this case the outcome of step 2 in stage 2 is the assignment \( \{b^*, 2\} \), while the outcome of the step 4 of the same stage is the assignment \( \{d^*, 4\} \).
Figure 5.1 Graphical depiction of the many-to-one traffic grooming heuristic. It is assumed that $a^* \neq 1, b^* \neq 2, c^* \neq 3, d^* \neq 4$, and $z^* \neq \Phi$.

### Table 5.1 Variables used in the heuristic.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{s,k,d}$</td>
<td>a stream originating at $s$, destined to $d$, and belonging to session $k$</td>
</tr>
<tr>
<td>$\gamma_{k,d}$</td>
<td>a set of streams, $\phi_{s,k,d}$, corresponding to a many-to-one connection, destined to $d$, and belonging to session $k$, i.e., { $\phi_{s,k,d}$</td>
</tr>
<tr>
<td>$\Pi_{i,j}$</td>
<td>an assignment of accommodated streams at step $j$ of stage $i$</td>
</tr>
<tr>
<td>$P$</td>
<td>a set of physical links corresponding a path</td>
</tr>
<tr>
<td>$\Lambda_{s,k,d}(P)$</td>
<td>a set of wavelengths used by $\phi_{s,k,d}$ on the physical links of path $P$</td>
</tr>
<tr>
<td>$\Delta_{i,j}$</td>
<td>cost of accommodating a stream at stage $i$, given the set $\Pi_{i-1,j}$</td>
</tr>
</tbody>
</table>

To present the heuristic formally we define the additional terms given in Table 5.1. Also, we define a procedure $\text{Evaluate}(\phi_{s,k,d}, P, \Lambda_{s,k,d}(P))$ which computes the total number of the LTs in the network, if $\phi_{s,k,d}$ is routed along path $P$ using wavelength(s) specified by $\Lambda_{s,k,d}(P)$.

The pseudo-code of the heuristic is shown in Figure 5.2. In the first stage, at each step, only one stream is accommodated using shortest path (line 1). This creates $\Phi$ independent assignments, each consisting of a single stream. The algorithm then, in each following stage, iteratively adds other streams by selecting the best assignments (lines 2-9). The stream to be accommodated at a step is $\phi_{s,k,d}$. To select the best assignment for $\phi_{s,k,d}$, a search procedure
is carried out (lines 3-9). All those assignments, from the previous stage only, which do not include the $\phi_{s,k,d}$ are considered (lines 3-4). The assignment that contributes the least cost if $\phi_{s,k,d}$ is added to it is selected, by using the procedure `compute_cost()`, and $\phi_{s,k,d}$ is made part of that assignment (lines 7-9). In the final stage, the assignment that incurs the least number of LTs is selected.

<table>
<thead>
<tr>
<th>Algorithm Many-to-one Traffic Grooming</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> Many-to-one traffic matrix</td>
</tr>
<tr>
<td><strong>Output:</strong> Number of LTs and wavelengths required to accommodate the input traffic matrix, and corresponding dimensioning of the network</td>
</tr>
<tr>
<td><strong>BEGIN</strong></td>
</tr>
<tr>
<td>1. $\Pi_{1,j} = j; \quad 1 \leq j \leq \Phi$</td>
</tr>
<tr>
<td>2. FOR $i = 2$ to $\Phi$ \quad //for each stage</td>
</tr>
<tr>
<td>3. FOR each $\phi_{s,k,d}$ \quad //for each stream</td>
</tr>
<tr>
<td>4. FOR $j = 1$ to $\Phi$ \quad //from each of the steps in previous stage</td>
</tr>
<tr>
<td>5. IF $\phi_{s,k,d} \notin \Pi_{i-1,j}$ \quad //if current stream is not a part of the assignment</td>
</tr>
<tr>
<td>6. $(\Delta_{i,j}, P^{(j)}<em>{\Phi</em>{s,k,d}}(P)) \leftarrow compute_cost(\Pi_{i-1,j}, \phi_{s,k,d})$</td>
</tr>
<tr>
<td>7. $j^* \leftarrow arg(min_{j} \Delta_{i,j})$ \quad //select least cost assignment;</td>
</tr>
<tr>
<td>8. $\Pi_{i, \Phi_{s,k,d}} = \Pi_{i-1,j^*} \cup \phi_{s,k,d}$</td>
</tr>
<tr>
<td>9. Route $\phi_{s,k,d}$ using $P^{(j^<em>)}$ and $\Lambda^{(j^</em>)}<em>{\Phi</em>{s,k,d}}(P)$;</td>
</tr>
<tr>
<td>10. Choose the assignment with least number of LTs, breaking the tie with the least number of wavelengths.</td>
</tr>
<tr>
<td><strong>END</strong></td>
</tr>
</tbody>
</table>

Figure 5.2 Many-to-one traffic grooming heuristic.

To compute the cost (line 6) in Figure 5.2, we employ the procedure shown in Figure 5.3. In Figure 5.3 the shortest path for the stream $\phi_{s,k,d}$ is computed (line 1), and different wavelength assignments are explored (lines 2-9). The cost corresponding to first-fit wavelength assignment, on the shortest-path, is stored in the variable $\delta_0$ (line 3). Note that we select such a first-fit wavelength that does not share the wavelength with any of the other streams of the same many-to-one connection. Consequently, no data aggregation can take place on the selected first-fit wavelength. We then explore wavelength assignment of each stream $\phi_{s',k,d} \in \gamma_{k,d}$, provided $\phi_{s',k,d}$ is also part of the wavelength assignment under consideration (line 4-8). We divide the wavelength assignment of the physical links corresponding to the path of stream $\phi_{s,k,d}$ into two subsets, $P_1$ and $P_2$. An illustration is given in Figure 5.4. The subset $P_1$ consists of physical links that are common between stream $\phi_{s,k,d}$ and $\phi_{s',k,d}$, while the subset
$P_2$ consists of physical links that are part of the path of stream $\phi_{s,k,d}$ but are not part of the path of stream $\phi_{s',k,d'}$. We use first-fit wavelength assignment for the members of the subset $P_2$, while we follow the wavelength assignment of the other stream, $\phi_{s',k,d'}$, for the members of the subset $P_1$, using aggregated capacity. In case the wavelength used by the other stream $\phi_{s',k,d'}$ is not accommodating, an infinite cost is recorded. Note that as the wavelength assignment of the other stream, $\phi_{s',k,d'}$, could also be using the same wavelength as determined by the first-fit wavelength assignment, we eventually end up aggregating the streams even on the first-fit wavelength. The only restriction is that aggregation is allowed when streams follow the same wavelength from the point of their first merging until they reach the destination. This ensures non-bifurcation of streams once they are aggregated. Finally, we select the wavelength assignment that results in the minimum cost (line 9). This procedure returns: 1) the least cost for accommodation of stream $\phi_{s,k,d}$, 2) the corresponding physical path, and 3) the corresponding wavelength assignment. The heuristic thus explores, in a systematic way, the different ways all the streams can be accommodated and explores the ways of constructing the different many-to-one trees that employ aggregation. The complexity of the heuristic is $O(\Phi^3 N^2)$, as the procedure `compute_cost()` is called $\Phi^3$ times, and the complexity of the procedure itself is $O(N^2)$.

```
compute_cost(\Pi_{i-1,j}; \phi_{s,k,d})
1. Compute shortest path $P_{s,k,d}$ for $\phi_{s,k,d}$;
2. $t = 0$;
3. $\delta_t = Evaluate(\phi_{s,k,d}, P_{s,k,d}; FF)$;   //determine the cost using First-Fit wavelength on path $P_{s,k,d}$
4. FOR each $\phi_{s',k,d} \in (\gamma_{k,d} \cup \Pi_{i-1,j})$
5. $t \leftarrow t + 1$;
6. $P_1 = P_{s,k,d} \cap P_{s',k,d}$;
7. $P_2 = P_{s,k,d} - P_{s',k,d}$;    //use aggregated capacity on $\Lambda_{s',k,d}(P_1)$
8. $\delta_t = Evaluate(\phi_{s,k,d}, P_2; FF) + Evaluate(\phi_{s,k,d}, P_1, \Lambda_{s',k,d}(P_1))$;
9. $\lambda_{s,k,d}(P_{s,k,d}) \leftarrow min_t \delta_t$;  // wavelength assignment that results in minimum cost
return($\min_t \delta_t, P_{s,k,d}, \lambda_{s,k,d}(P_{s,k,d})$)
```

Figure 5.3 Compute the cost of accommodating a stream given an assignment of already accommodated streams.
5.4 Experimental Results

In this section we will present the results of the MILP as well as the heuristic approach for the many-to-one traffic grooming problem. To experiment with the MILP, we used the network shown in Figure 4.7, while for the heuristic we also considered the NSF network topology shown in Figure 4.8. The traffic demands consist of integer multiples of OC-3 connections. The capacity of a wavelength is OC-48, and therefore, the grooming factor, $g$, is 16.

5.4.1 Exact Solutions

The MILP problem is solved using the CPLEX linear programming package [90]. The values of $\alpha$ and $\beta$ are selected to be 100 and 1, respectively. We selected these values to give higher priority to the minimization of the number of LTs over minimization of the number of wavelengths in the network. A sample traffic that consists of a mix of many-to-one and one-to-one sessions is generated and is shown in Table 5.2. The last column shows the fractions, from each data stream, to be accepted for aggregation corresponding to the number of streams being aggregated. These values are generated randomly between 0 and 1 such that $r_{ij}^{cn} = 1$, and $r_{ij}^{cn} \leq r_{ij}^{cn-1}, 2 \leq f \leq |S_{cn}|$. Multiple fractions correspond to the number of the streams being aggregated. Hence, for example, for a many-to-one connection terminating at node 0 and belonging to session 1, the fractional value is 1 if a single stream is aggregated, while it is 0.69.
if two streams are aggregated and 0.484 if three streams are aggregated. For example, if the capacity of 2 streams are $m_1$ and $m_2$ and they are being aggregated, then the total capacity of the aggregated stream will be $0.69 \times (m_1 + m_2)$. Note that each many-to-one connection has an independent set of values. It is possible that different many-to-one connections correspond to different applications, and hence aggregate the traffic using different fractions. A higher value of the fraction represents less correlation between the data streams of a many-to-one session, and vice versa. For example, note that the streams destined to node 3 have a less degree of correlation. We run the problem using 4 wavelengths. However, the optimal solution reduced the total number of wavelengths to 2. The total number of LTs required for this example is 13. In Figure 5.5, a part of the solution is shown. Node 1 aggregates the traffic stream coming from node 4 with the traffic stream generated by itself for connection (0,1). Both, node 4 and node 1 are generating 6 units of traffic. As the aggregation ratio when two streams are aggregated for connection (0,1) is 0.69 (see Table 5.2), the aggregated traffic from node 1 and heading towards the destination, instead of being 12, is just 8.286 units. Note that although node 2 is also a part of the same many-to-one connection, it is not a part of the aggregation tree. Similarly, node 4 aggregates the traffic from node 1 with the traffic stream generated by itself for connection (5,2). For this session, nodes 1 and 4 are generating 3 and 8 units of traffic, respectively. Note that the traffic from node 1 is arriving at node 4 after traversing two lightpaths: between nodes 1 and 2, and between nodes 2 and 4. The aggregation ratio for connection (5,2) when two streams are aggregated is 0.666, hence the aggregated traffic sent from node 4 to the destination, is 7.335 units.

Figure 5.6, shows the complete solution generated by the MILP, for the traffic matrix shown in Table 5.2, in terms of lightpaths between the nodes and in terms of LTs required at each node. An inspection of the solution at the lightpaths and physical level leads to the same conclusion as reached in Section 4.6. A brief list is given below:

- At the lightpath level, the traffic stream of a many-to-one session is either delivered directly from the source to the destination, or after getting aggregated with the traffic streams of other sources.
Table 5.2 Many-to-one traffic demands on the 6-node network.

<table>
<thead>
<tr>
<th>Destination</th>
<th>Session</th>
<th>Source set</th>
<th>Traffic (multiples of OC-3)</th>
<th>Aggregation ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>{1,2,4}</td>
<td>{6,4,6}</td>
<td>{1, 0.690, 0.484}</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>{0}</td>
<td>{12}</td>
<td>{1}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>{3,5}</td>
<td>{8,6}</td>
<td>{1, 0.698}</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>{1}</td>
<td>{8}</td>
<td>{1}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>{0,3,5}</td>
<td>{3,8,4}</td>
<td>{1, 0.889, 0.863}</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>{0}</td>
<td>{16}</td>
<td>{1}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>{0,1}</td>
<td>{12,3}</td>
<td>{1, 0.997}</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>{3}</td>
<td>{8}</td>
<td>{1}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>{1,4}</td>
<td>{3,8}</td>
<td>{1, 0.666}</td>
</tr>
</tbody>
</table>

Figure 5.5 Two many-to-one aggregation trees generated by the MILP for the traffic matrix shown in Table 5.2.

- Many lightpaths are carrying the traffic streams of not just one many-to-one session, but rather from different many-to-one sessions, thus grooming the traffic of different sessions.

- The corresponding physical path of a lightpath is not necessarily the shortest path.
5.4.2 Results From the Heuristic Solution

Due to the high complexity of the problem, one cannot obtain optimal solutions within reasonable time, e.g., hours or a whole day of computation, for even 10-node topologies. Therefore, one needs to resort to the heuristic approach. However, the above MILP can act as a baseline to determine the performance of any heuristic designed for many-to-one traffic grooming problem. Our heuristic technique, for the topology shown in Figure 4.7 and the inputs shown in Table 5.2, produced a solution with 16 LTs and 2 wavelengths. Note that the number of the LTs are within 33% of the optimal value. For other small size examples we observed that the heuristic technique produces results, in terms of the number of LTs, that are within 30%-40% of the optimal values.

To collect the results on some real network topologies, we chose the NSF network topology shown in Figure 4.8. The grooming factor, $g$, used is 48. To study the effect of aggregation, traffic granularity, grooming factor, and source set size, we conducted many experiments and divided them into four scenarios. To assess the efficiency of the DP-style optimization step in the proposed heuristic and the degree of its success, we also implemented a simple heuristic, in which streams were routed using shortest path and first-fit wavelength assignment with
aggregation where possible. Note that our proposed heuristic can be reduced to this simple heuristic. For comparison purposes let us call our proposed heuristic as Many-to-one Traffic Grooming using Dynamic Programming (MTG-DP), and the simple heuristic as Many-to-one Traffic Grooming using Simple Approach (MTG-SA).

We generated a traffic matrix randomly with the following parameters, and used it for all the experiments:

- The number of sessions destined to each node is generated uniformly between 0 and 3.
- 50% of the sessions carry many-to-one traffic, while the remaining sessions carry unicast traffic.
- For each many-to-one session, the source set size is uniformly distributed between 2 and 8.
- For each many-to-one session terminating at a destination, the sources are chosen randomly among all the nodes, excluding the destination, for both unicast and many-to-one sessions.
- The generated traffic, for both unicast and many-to-one sessions, is an integer multiple of OC-1, and is uniformly chosen from the set \{1,3,9,12,18,24,36,48\}. These values represent the recommended rates for OC-1 streams.

Since the type and amount of aggregation can be application dependent, we use three different ways to compute the aggregation ratios, as follows.

- Aggregation Type 1: All aggregation ratios are unity, i.e., \( r_{C_j}^a = 1, \forall f \). Hence when two or more streams are aggregated, no capacity reduction will take place.

- Aggregation Type 2: Aggregation ratios decrease linearly with a step \( \sigma \), i.e., \( r_{C_j}^{ca} = r_{C_j}^{ca-1} - \sigma, 2 \leq f \leq |S_{ca}| \). We assume that \( |S_{ca}| < 10 \), in this case we have chosen \( \sigma \) to be 0.1.

- Aggregation Type 3: Aggregation ratios are computed using the formula, \( r_{C_j}^{p} = 1 - \frac{\ln(f)}{\rho} \), which provides logarithmically decreasing values. We have chosen \( \rho \) to be 5. Hence, quantitatively, when the number of aggregated streams increases, the difference
in the reduction of the capacity of each stream decreases. In other words, the benefit we
achieve, in terms of reduction in capacity, by aggregating two streams over one stream,
is more than that of aggregating three streams over two aggregated streams.

The three aggregation types are graphically shown in Figure 5.7. Note that aggregation type
3 provides the maximum reduction in per stream traffic if the number of the aggregated streams
are 2 or 3, but if the number of the aggregated streams are more than 3 then aggregation type
2 provides more reduction.

![Aggregation Types](image)

Figure 5.7 Different aggregation types used for collecting experimental results.

We have conducted experiments using four scenarios, where for each scenario, we assumed
that all many-to-one connections have the same aggregation type and ratios. Now we will
discuss experiments in each scenario.

The experiments in scenario 1 are conducted to study the effect of the aggregation factor
on the network cost. We conducted three experiments using the three aggregation types.
The results are shown in Table 5.3. MTG-DP outperformed the MTG-SA in all cases. The
comparison between both heuristics shows the benefit of extra intelligence and computation
we are carrying out in MTG-DP. For aggregation types 1, the MTG-DP resulted in 17 fewer LTs than the MTG-SA. Given that the price of a single LT is on the range of tens of thousands of dollars, this saving translates into hundreds of thousands of dollars. For aggregation type 1, 2 and 3, the saving in the network in terms of the number of LTs is almost 16%, 21%, and 8%, respectively. Although, in most experiments we observed that aggregation type 3 performs better than aggregation type 2, notice that in this scenario aggregation type 2 outperformed aggregation type 3. However, this is achieved at the expense of an additional wavelength with aggregation type 2. Notice also that in this scenario the number of the LTs required using aggregation type 1 and 3 are the same. However, aggregation type 3 required fewer wavelengths.

The experiments in scenario 2 are conducted to study the effect of traffic granularity on the cost of the network. We modified the original traffic matrix by revisiting each of the sources and replacing its current traffic amount with the next higher traffic amount in the vector \{1,3,9,12,18,24,36,48\}, unless the traffic was already 48 units, in which case it was not changed. For example, if a source was previously sending out 18 units of traffic, it will now be sending out 24 units of traffic. We used this modified traffic matrix to run three different experiments with aggregation types 1, 2 and 3, respectively. The results are shown in Table 5.4. Again, the proposed heuristic outperformed the simple approach. Also we noticed that more aggressive aggregations, i.e., smaller aggregation ratios result in offsetting the increase in the traffic granularity.

The experiments in scenario 3 are conducted to study the effect of the grooming factor on the cost of the network. The results are shown in Table 5.5. We used the traffic matrix
Table 5.4 Results of experiments in scenario 2; grooming factor is 48 and traffic granularity is increased.

<table>
<thead>
<tr>
<th></th>
<th>Agg. type 1</th>
<th>Agg. type 2</th>
<th>Agg. type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTG-DP</td>
<td>91</td>
<td>88</td>
<td>84</td>
</tr>
<tr>
<td>wavelengths</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>MTG-SA</td>
<td>99</td>
<td>102</td>
<td>106</td>
</tr>
<tr>
<td>wavelengths</td>
<td>19</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 5.5 Results of experiments in scenario 3; grooming factor is 96.

<table>
<thead>
<tr>
<th></th>
<th>Agg. type 1</th>
<th>Agg. type 2</th>
<th>Agg. type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTG-DP</td>
<td>72</td>
<td>64</td>
<td>63</td>
</tr>
<tr>
<td>wavelengths</td>
<td>9</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>MTG-SA</td>
<td>92</td>
<td>87</td>
<td>90</td>
</tr>
<tr>
<td>wavelengths</td>
<td>9</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

with the original load, but this time we set the grooming factor to 96. Hence the capacity of each participating wavelength is now doubled. As a result, we see a substantial decrease in the number of LTs, though not exactly halved. The number of the wavelengths, however, is almost halved. Moreover, the results show that using non-unity aggregation ratios (aggregation types 2 and 3) result in a significant cost saving in terms of the number of LTs over using aggregation type 1 (for aggregation types 2 and 3, the savings are almost 14% and 15%, respectively, over that of aggregation type 1). One of the possible explanation of the results could be that as we are maintaining a non-bifurcated solution, earlier in scenario 1 and 2 not so many traffic streams from different sources of a many-to-one connection were having a chance to share the ride on the same wavelength and hence the benefit of aggregation was not very profound. In scenario 3, however, by increasing the wavelengths' bandwidth, but maintaining the same traffic demands as that of scenario 1, more streams can be accommodated especially after stream aggregation.

The experiments in scenario 4 are conducted to study the effect of varying the size of the source set on the network cost. We modified the original traffic matrix by adding 3 additional
Table 5.6 Results of experiments in scenario 4; 3 additional sources per connection are added to the original traffic matrix.

<table>
<thead>
<tr>
<th></th>
<th>Agg. type 1</th>
<th>Agg. type 2</th>
<th>Agg. type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTG-DP</td>
<td>LTs</td>
<td>138</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>wavelengths</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>MTG-SA</td>
<td>LTs</td>
<td>183</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>wavelengths</td>
<td>23</td>
<td>22</td>
</tr>
</tbody>
</table>

Sources per session. We employ aggregation types 1, 2 and 3 for three different experiments. The results are shown in Table 5.6. Similar to previous scenarios, the proposed heuristic MTG-DP is offering substantial savings over the simple heuristic MTG-SA in terms of the number of LTs. We also noticed that the performance with aggregation type 3 is better than that with aggregation type 2. This could be due to the reason that in most of the cases streams from only 2 to 3 sources got a chance to be aggregated on a wavelength before the capacity of the wavelength is completely consumed.

5.5 Chapter Summary

In this chapter we addressed the design of optical network which support many-to-one traffic grooming, using optimal as well as heuristic approaches. We developed a mixed integer linear program of an otherwise non-linear problem, by exploiting the specifics of routing and aggregation sub-problems. The model used is quite generic and supports arbitrary many-to-one connections. Moreover, at merging points in many-to-one trees, a fraction of the traffic from each source can be selected such that the fraction itself depends on the number of the streams being aggregated. Furthermore, we ensured the non-bifurcation of the traffic. We also designed a Dynamic Programming style heuristic that developed the solution through a number of stages. At each stage the heuristic selected a number of best partial solutions from a large number of candidate partial solutions. Experimental results show that the MILP can be used for small to medium sized examples, while the heuristic approach can be used for large sized networks.
CHAPTER 6 SUMMARY AND FUTURE WORK

6.1 Summary

In this dissertation we proposed a number of optimal and heuristic solutions for the design and provisioning of WDM networks with traffic grooming. Our objective was to reduce the cost of the network by reducing the number of higher layer electronic components.

In Chapter 1 we provided some background on the optical networks, and also briefly covered some challenges posed by these networks. We then provided the motivation for the design and provisioning of WDM networks for traffic grooming, and discussed the challenges posed by this problem. We finally discussed the contribution of our work provided in this dissertation.

In Chapter 2 we provided an extensive review of the traffic grooming literature. We divided the work into the following categories: (1) static traffic grooming on ring topologies (both uniform and non-uniform traffic), (2) static traffic grooming on mesh topologies, (3) dynamic traffic grooming, (4) survivable traffic grooming, and (5) Multipoint traffic grooming. For each category we reviewed the work available in the literature. We summarized the contributions of each work and discussed the implications of some of the studies.

The thesis contribution to the area of traffic grooming in WDM networks is in the following three chapters. Chapter 3, we addressed the grooming of the non-uniform traffic on unidirectional and bidirectional rings. Our approach to grooming arbitrary traffic, while minimizing the number of LTs and wavelengths, was different than the previous approaches in that we first mapped the unidirectional and bidirectional rings onto a linear topology, and then developed a two-step approach to solve the grooming problem on the mapped topology. For the first step, traffic demands were compacted in the horizontal dimension by employing the proposed heuristic, MIN-STRINGS. For the second step, the traffic demands were compacted in the
vertical dimension by employing another proposed heuristic, GROUPING. The net effect of both steps was the compaction of the non-uniform traffic streams in a way that reduces the required number of LTs and wavelengths. Moreover, with the help of an example, we showed that for bidirectional rings using shortest path would not always lead to the minimum number of LTs. We then proposed few routing approaches which can reduce a considerable number of the required wavelengths and the LTs. We also conducted a large number of experiments to assess the performance of our approaches.

In this thesis, the area of multipoint traffic grooming was also pioneered. In Chapter 4, we proposed a unified framework for the optimal network dimensioning and channel provisioning for multicast traffic grooming. First, with the help of an example we showed that using a Steiner Minimum Tree, to route multicast requests, will not necessarily lead to the minimum number of LTs. We then considered the generic multicast problem in which a source node transmits to a set of destinations. All destinations need to be reached, and with the same amount of traffic. Moreover, we explored two variants of the multicast traffic grooming problem, namely, partial destination reachability and traffic thinning. We formulated optimal solution techniques in terms of a Mixed Integer Linear Programs for the generic multicast traffic grooming problem and both of its variants. Beside the optimal approach we also provided a heuristic approach. The main idea was to construct an initial solution and then improve it iteratively while exploring different routes. We conducted many experiments to demonstrate the efficacy of our techniques. The results confirmed the intuition that both variants of the generic multicast traffic grooming problem, by either providing service to only a select group of destinations in each destination set, or providing service to all the destinations but with different traffic rates, incur lower network cost. We specifically concluded that multicast traffic grooming model is more economical, in terms of the number of LTs and wavelengths, than using multiple unicast connections. We also concluded that when operating under a budget constraint, it is helpful to divide the members of each destination set into critical and non-critical nodes, and then serve non-critical members only if doing this does not increase the network cost.

In Chapter 5, we addressed the many-to-one traffic grooming problem. We mentioned the
difference between the multicast and many-to-one delivery models, and argued that a different solution approach is required for the many-to-one service model. We then provided optimal and heuristic solutions for the many-to-one traffic grooming problem. One of the unique features of the model was that at merging points in a many-to-one tree, a fraction of the traffic from each source was selected such that the fraction itself depended on the number of streams being aggregated. This inherently made the optimal formulation non-linear. However, we exploited the routing and aggregation sub-problems in such a way that allowed us to successfully develop a Mixed Integer Linear formulation. For the heuristic approach, we used a Dynamic Programming style approach. The final solution was achieved after going through a number of stages. At each stage the heuristic selected a number of best partial solutions from a large number of candidate partial solutions. Each candidate partial solution itself was determined after exploring many wavelength assignments and selected one that would cost the least in terms of the number of LTs and wavelengths. Finally, we provided many scenarios by conducting a number of experiments. The experimental results showed that instead of treating a many-to-one connection as multiple unicast connections, the network cost can be substantially reduced by exploiting the many-to-one service model. Also, we concluded that higher aggregation ratios help offset, in terms of the network cost, the effects of increased traffic load.

6.2 Future Directions

Keeping in mind the current trends in the field of the traffic grooming in the WDM networks, we can make the following observations. A significant amount of work on traffic grooming on ring topologies using uniform traffic already exists. However, as the real world is operating with non-uniform traffic, hence one needs to focus more on arbitrary non-uniform traffic. The ring topology is still a popular choice among researchers, and many real networks in fact do operate over such topologies. However, with the deployment of WDM networks on mesh networks, arbitrary topologies need to be under more consideration. Many researchers too have picked up this point and hence we are seeing more and more work in traffic grooming.
on mesh topologies. Moreover, besides static traffic grooming, dynamic traffic grooming has a significant practical appeal. Hence, in the near future we can expect a significant amount of research efforts directed to in the field of dynamic traffic grooming. Finally, although survivable traffic grooming is still in its infancy, given the importance of survivability in optical networks we can foresee a significant amount of progress in this area too in the near future.

Multipoint traffic grooming is a new area. With the work presented in this dissertation we just touched the tip of the iceberg, while many potential research problems are still hidden under water. There are many future directions that can be taken to extend this work. We here only provide a brief list:

- Our multicast traffic grooming model assumed that if a lightpath at a node needs to split, then it has to be terminated at that node, thus employing splitters at the electronic layer. Exploring architectures where optical splitting too is allowed may reduce the total number of required LTs. However, a detailed study is required to determine the overall cost of the network for such an alternative. We do, however, expect that this could lead to an interesting study involving the trade-offs between optical splitting and electronic splitting in the realm of traffic grooming.

- Multicast traffic grooming switch architectures needs to be explored. We assumed that replication of traffic is implemented at the electronic level. The implementation of the data replication need to be investigated. Also the effect of the cost of such a replication on the total network cost needs to be studied.

- In case of many-to-one traffic grooming, in order to ensure non-bifurcation, our model, for optimal solutions, assumed that the total amount of traffic from all sources of a many-to-one connection is less than the bandwidth of a single wavelength. Although, this might be a reasonable assumption for many practical scenarios, when the size of the source set increases such an assumption could be a limitation. Hence, removing this restriction should also be explored. However, to come up with a linear formulation without this restriction could be a challenging task.
• Although, optimal approaches provide a baseline to evaluate the performance of heuristics, they take an enormous amount of computation time even for medium sized networks. Many steps can be taken to solve this problem. For example, tighter bounds need to be developed for mesh topologies with non-uniform traffic. However, the bounds reported in the network even for unicast connections, and on simple ring topologies for non-uniform traffic are very loose. The ILPs and MILPs are solved using branch and bound techniques. Another possible direction can be to explore the application of the parallel branch and bound techniques for the parallel implementations of the proposed MILPs. In branch and bound techniques many computation nodes can work independent of each other, thus providing a lot of opportunities to exploit the inherent parallelism. The field of parallel branch and bound techniques is quite mature and a lot of literature exists on such techniques. For a taxonomy, please refer to reference [91]. Currently parallel machines with hundreds of physical nodes exist. We believe that our success in formulating the problems in a linear fashion has far reaching effects. These formulations now can be used to solve the multipoint traffic grooming problem for even medium and large sized networks in reasonable time using parallel branch and bound techniques. The details of which, however, need to be investigated.

• A significant body of work on performance evaluation of ATM or optical networks that employ full-wavelengths already exists in the literature. However, there is a dearth of performance evaluation studies in traffic grooming, and is open to further exploration.

• A natural extension of the work presented in this dissertation is to consider dynamic multicast traffic grooming. We are aware of no such work in the literature.

• The main thrust of the work in this dissertation was to groom traffic in a way that reduces the network cost by reducing the number of required components. However, for some application other factors such as delay may be of prime significance. An optimized network in terms of the number of the components may indeed violate such constraints. Hence, for such applications, an integrated approach that incorporates all constraints
need to be developed.

- The models developed in this dissertation may prove helpful in traffic grooming in MPLS/GMPLS networks. However, many additional constraints, such as QoS, need to be considered.
APPENDIX A  Optimality of MIN-STRINGS Algorithm for a Linear Topology

Lemma 1: \(|\psi_j| \leq \theta\) for \(1 \leq j \leq |C|\)

Proof: \(|\psi_j| > \theta\) implies that at some link, say \(l\), where \(l = X_{\text{min}}(a_j)\), the number of segments traversing the link is greater than \(\theta\), which contradicts the definition of \(\theta\). Hence \(|\psi_j| \leq \theta\).

Corollary: \(|\pi_i| \leq \theta\) for \(1 \leq i \leq N - 1\)

Proof: By definition, \(\pi_i = \psi_m\) for \(1 \leq m \leq |C|\), or \(\pi_i = \emptyset\). However, from Lemma 1 we have \(|\psi_m| \leq \theta\). Hence, \(|\pi_i| \leq \theta\).

Lemma 2: In each iteration, \(k\), of the MIN-STRINGS algorithm (lines 3-18, Figure 3.2), exactly one member of each nonempty \(\pi_i, 1 \leq i \leq N - 1\), will be selected for inclusion in string \(k\).

Proof: As MIN-STRINGS first sorts all the segments in ascending order with respect to their \(X_{\text{min}}\) coordinates, in each iteration \(k\) (lines 3-18, Figure 3.2), the sequence \(\Pi\) will be inspected in order, and if the element, \(\pi_i\), is not empty then:

(a) Either the segment that was chosen in the previous nonempty element, \(\pi_j\), is not a member of \(\pi_i\), and therefore does not overlap with the members in \(\pi_i\). In this case, a new segment in \(\pi_i\) can be chosen for inclusion in the string, and will be removed from set \(\pi_i\). Or,

(b) the segment, \(a_k\), that was last chosen from the previous non-empty member, \(\pi_j\), is also a member of \(\pi_i\), and will also be removed from \(\pi_i\).

In both cases, the size of all the nonempty sets, \(\pi_i\), which are members of the sequence \(\Pi\), will be reduced by 1.

Theorem 1: MIN-STRINGS algorithm is optimal in the number of strings.

Proof: As \(\theta\) is the lower bound on the number of strings, we will prove that the number
of strings obtained by MIN-STRINGS algorithm is equal to \( \theta \), i.e., \(|R| = \theta\), and is therefore optimal. Since \(|\pi_i| \leq \theta, 1 \leq i \leq (N - 1)\), then by lemma 2, in \( \theta \) or less iterations we will be able to select all the segments \( a_k \in \pi_i \), for the strings. Let \( k^* \) be the link with density \( \theta \). Then \(|\pi_{k^*}| = \theta\). Hence the total number of iterations required to select all the segments is \( \theta \). Since each iteration corresponds to a string, then \(|R| = \theta\). ■
APPENDIX B  Traffic Splitting May Reduce the Number of Wavelengths and LTs

In this appendix with the help of an example, we will demonstrate that performing traffic splitting during traffic grooming may reduce the number of required wavelengths and LTs.

An example is shown in Figure B.1.

![Figure B.1 Traffic grooming with and without traffic splitting.](image)

Two units of traffic streams exist between each of the following pair of nodes, (1,2), (1,3), (1,4), (2,4) and (3,4). Let the grooming factor \( g \) be 3. When splitting was not allowed (Figure B.1(a)), the number of wavelengths required to accommodate all the traffic was 3 and the number of LTs were 8. However, when splitting was allowed (Figure B.1(b)), the number of wavelengths required is reduced to 2 and the number of LTs to 6. Thus splitting may help to reduce both the number of wavelengths and the number of LTs. Many applications, especially
real-time applications, require that their traffic would better be kept intact, i.e., without de-
multiplexing at the source, independent switching at intermediate nodes, and multiplexing at
the destination. Such operations increase the cost of handling the traffic, and also introduce
delay jitter.
APPENDIX C  Earlier Non-Bifurcation Formulation in the Literature Leads to Bifurcation

Reference [65] has presented an ILP formulation for traffic grooming in a mesh network with an arbitrary topology. The problem was formulated in order to constrain a traffic demand to be carried in its entirety on a single lightpath, i.e., without bifurcation. However, the actual constraints of the problem do not guarantee such a condition. In this appendix, with the help of a counter example, we show how the condition of non-bifurcation is violated.

First, we present a few variables to facilitate the discussion.

- $s$: A variable that indicates the source node.
- $d$: A variable that indicates the destination node.
- $C$: Capacity of each wavelength channel. In the examples in reference [65], $C$ was taken as 48.
- $\Lambda$: Traffic matrix set. $\Lambda = \{\lambda_y\}$, where $y$ can be any allowed low-speed stream. In reference [65], $y$ was restricted to the values in the set $\{1, 3, 12, 48\}$. The parameter $\lambda_{y,sd}$ is the number of $y$ connection requests between the node pair $(s, d)$.
- $V_{ij}$: Number of lightpaths from node $i$ to node $j$ in the virtual topology. This variable is an integer, which is greater than or equal to 0. (Note that multiple lightpaths from $i$ to $j$ may exist, and even using the same wavelength, but must therefore use different physical routes.)
- $\lambda_{ij,y}^{sd,t}$. The $t$th $y$ low-speed traffic request from node $s$ to node $d$ employing lightpath $(i, j)$ as an intermediate virtual link.
\( S_{sd}^{y,t} \): A binary variable that is equal to 1 if and only if the \( t \)th \( y \) low-speed connection request from node \( s \) to node \( d \) has been successfully routed.

Consider the network shown in Figure C.1, which consists of two nodes, S and D, where there is only a single fiber from S to D. There are two wavelength channels on the fiber, and each channel has a capacity, \( C \), of three units. Let the traffic demands from S to D be in the form of three streams, where each stream requires two units.

Therefore, two lightpaths from S to D (shown by the dashed lines in the Figure) will be created on the fiber between S and D. These lightpaths will accommodate all three traffic demands such that one instance of the traffic demand will be on the first wavelength, the second instance will be on the second wavelength, while the third one will be split between the two lightpaths, such that each wavelength channel carries a one unit stream. The variables will therefore assume the following values:

- \( \Lambda_{2,SD} = 3. \)
- \( S_{SD}^{2,1} = S_{SD}^{2,2} = S_{SD}^{2,3} = 1. \)
- \( \lambda_{SD,1}^{SD,1} = \lambda_{SD,2}^{SD,2} = \lambda_{SD,2}^{SD,3} = 1. \)
- \( V_{SD} = 2. \)
which satisfy all the constraints provided in [65], while

\[
\sum_{2,t=1,2,3} 2 \times \lambda_{SD,2}^{SD,t} \leq V_{SD} \times C = 6.
\]

Note that enforcing non-bifurcation of traffic must result in \( S_{SD}^{2,3} = \lambda_{SD,2}^{SD,3} = 0 \), and the carried traffic will be only 4 units instead of 6 units. Note also that this problem was not exhibited in the examples in [65] since the lightpath capacity, 48, was an integer multiple of the individual traffic demands, which were in the set \( \{1, 3, 12, 48\} \).
# APPENDIX D  Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADM</td>
<td>Add Drop Multiplexer.</td>
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<tr>
<td>APS</td>
<td>Automatic Protection Switching.</td>
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<tr>
<td>BLSR</td>
<td>Bidirectional Line-Switched Ring.</td>
</tr>
<tr>
<td>DCS</td>
<td>Digital Cross Connect.</td>
</tr>
<tr>
<td>DP</td>
<td>Dynamic Programming.</td>
</tr>
<tr>
<td>GMPLS</td>
<td>Generalized Multi-Protocol Label Switching.</td>
</tr>
<tr>
<td>ILP</td>
<td>Integer Linear Program.</td>
</tr>
<tr>
<td>IP</td>
<td>Internet Protocol.</td>
</tr>
<tr>
<td>LP</td>
<td>Light Path.</td>
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<tr>
<td>LT</td>
<td>Light Terminal.</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed Integer Linear Program.</td>
</tr>
<tr>
<td>MPEG</td>
<td>Moving Picture Experts Group.</td>
</tr>
<tr>
<td>MPLS</td>
<td>Multi-Protocol Label Switching.</td>
</tr>
<tr>
<td>NSF</td>
<td>National Science Foundation.</td>
</tr>
<tr>
<td>OC</td>
<td>Optical Channel.</td>
</tr>
<tr>
<td>OEO</td>
<td>Optical-Electrical-Optical.</td>
</tr>
<tr>
<td>OXC</td>
<td>Optical Cross Connect.</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service.</td>
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<tr>
<td>RWA</td>
<td>Routing and Wavelength Assignment.</td>
</tr>
<tr>
<td>SDH</td>
<td>Synchronous Digital Hierarchy.</td>
</tr>
<tr>
<td>SONET</td>
<td>Synchronous Optical Network.</td>
</tr>
<tr>
<td>UPSR</td>
<td>Unidirectional Path-Switched Ring.</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength Division Multiplexing.</td>
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