Separation of crack growth signals is of fundamental importance for detecting, locating, and determining the significance of an internal flaw. The difficulty associated with modeling acoustic emission is not only in providing an accurate representation of the source mechanism, but also in determining the effect of the specimen geometry and the sensor on the acoustic emission signal.

An influence function is used to develop an integral equation to model the near tip dynamic stress due to a prescribed crack growth event. The propagation of the crack greatly influences the stress field in the vicinity of the crack tip, causing stress waves to radiate into the body and on the crack surface; it is the displacement caused by these stress waves that is being modeled. Acoustic emission testing detects stress waves at the body's surface and relates these back to crack propagation events. An advantage of the analysis presented is that the source for the acoustic emission signature is an actual crack propagation event and not a simple point source model.

The velocity of the moving crack tip and the time dependent displacement due to the crack growth event are measured using a crack propagation gage and an interferometric displacement/velocity sensor respectively. The displacements being measured are acoustic emissions from the dynamic crack growth. These displacements are the benchmark comparison to the analytical model. The velocity measurements are input parameters for the analytical model.

In the next section, the analytical method is developed and discussed. Then the experimental procedure is explained, and these results are compared with the analytical model in the last section.
ANALYTICAL METHOD

First, the dynamic Mode I stress caused by a semi-infinite crack propagating with a prescribed velocity is determined, and then the displacements at any point are calculated. The method for determination of the stresses, summarized in [1], uses an influence function (or Green's function) method to formulate an integral equation in two variables, a spatial coordinate \( x \) and time \( t \). The influence function \( U_{yy}(x-x', t-t') \), obtained in closed form using the Cagniard-de Hoop method, is the vertical displacement of an elastic half-space subjected to a unit concentrated impulse acting at a point normal to its edge.

Assume that a crack exists at time \( t=0 \) with its tip located at \( x=a(0) \) and \( y=0 \). For time \( t>0 \), the crack tip moves from \( x=a(0) \) to \( x=a(t) \). The two relevant boundary conditions are that the newly formed crack faces are stress free and that the vertical displacement in front of the moving crack tip is zero. These boundary conditions are satisfied by:

(a) Removing the existing known static stress, \( \sigma_{yy} = P(x) \), and assuring that a new unknown time-dependent stress, \( \sigma_{yy} = F(x,t) \), develops.

(b) Requiring that this new stress distribution be developed such that there is vertical displacement continuity in front of the moving crack tip.

The continuity boundary condition can now be expressed in terms of the influence function, \( U_{yy}(x-x', t-t') \) as:

\[
\int_{-\infty}^{\infty} P(x') U_{yy}(x-x', t-t') \, dx' \, dt' + \int_{-\infty}^{\infty} F(x', t') U_{yy}(x-x', t-t') \, dx' \, dt' = 0
\]

The above expression is a Volterra integral equation of the first kind in the variables \( x \) and \( t \). To provide a simple solution of this integral equation, assume some spatial form of the unknown stress distribution, \( F(x', t') \), that contains a square root singularity at its tip location, \( a(t') \). This unknown stress in front of the moving crack tip is assumed to have the spatial form of a static crack with its tip located at \( a(t') \), multiplied by some unknown time function, \( K(t') \):

\[
F(x', t') = K(t') / \sqrt{2\pi} \sqrt{x'-a(t')}
\]

It should be noted that the assumed spatial stress distribution exists instantaneously for all values of \( x'>a(t') \) for any time, \( t' \).

For the steady state case of a crack propagating with a constant velocity, the calculated value of \( K(t') \) is a constant that is only a function of the crack tip velocity. As the crack tip speed increases, the corresponding constant value of \( K(t') \) decreases. The results for the case of a crack that suddenly stops after propagating is that the calculated value of \( K(t') \) discontinuously jumps to the value of the corresponding static stress; there is no transition zone and the stress never increases above the value of an equivalent static crack. The dynamic stress results computed here compare well with those calculated using the Wiener-Hopf Technique (Freund [2] and Rose [3]).

The displacement at any point in the infinite body is determined using the above dynamic stress, \( F(x', t') \), and the influence functions,
These influence functions \( U_{x,y}(x-x',y,t-t') \) and \( U_{x,y}(x-x',y,t-t') \) represent the displacement \( u_x \) and \( u_y \) at any point \((x,y)\) in an elastic half-space. The solution for these influence functions is again accomplished using integral transform techniques. Using the previously calculated dynamic stress distribution, convolution integrals can be written to calculate the vertical and horizontal displacement as a function of time for any point \((x,y)\) within the body. For numerical simplicity, the displacement is determined at a point, \( x \), in the plane of the crack \((y=0)\). At any point on this plane, the displacements are given by:

\[
\begin{align*}
u_y(x,t) &= 0 \text{ (because of symmetry)} \quad (3) \\
u_x(x,t) &= -\int_0^t \int_{-\infty}^{\infty} P(x') U_{xy}(x-x',y=0,t-t') \, dx' \, dt' \\
&\quad + \int_0^t \int_{-\infty}^{\infty} F(x',t') U_{xy}(x-x',y=0,t-t') \, dx' \, dt' \quad (4)
\end{align*}
\]

The proposed solution procedure is illustrated for the case of a crack tip that instantaneously reaches a constant velocity, \( c_a' \), propagates for a length of time, \( t_c' \), and instantaneously stops. Figure 1 illustrates the time dependent displacement at a point, \( x \), caused by a crack that propagates at a velocity of approximately 20% of the Rayleigh wave velocity \( (c_p) \) for a short time duration. The effect of the arrival of the longitudinal wave fronts from the starting and stopping phases of the crack propagation \((t_L(\text{start})\) and \( t_L(\text{stop}) \)) is clearly seen. After the arrival of the transverse wave from the stopping event \((t_T(\text{stop}))\), the displacement reaches a quasi-static value larger than the original static value. The small non-zero displacement prior to the arrival time of the fastest wave from the initial crack tip \((t_L(\text{start}))\) is due to the assumed spatial stress distribution of Eqn. (2). This assumption implies the physically unrealistic case that an instantaneous stress exists in front of the moving crack tip. The stress distribution assumed in Eqn. (2) contains a square root singularity at the moving crack tip which dominates the dynamic stress field while the stress values away from the moving crack tip are negligible in comparison. Except for this discrepancy in arrival times, the assumed stress distribution models the actual dynamic stresses. Figure 2 shows the change in displacement due to variations in crack velocity while Figure 3 illustrates the effect of the duration of crack propagation. The results indicate that the maximum displacement increases for increasing values of constant velocity or duration of propagation and that the displacement gradients increase for increasing values of constant velocity.

**EXPERIMENTAL PROCEDURE**

The experimental procedure examined the incremental propagation of an existing crack in a modified compact tension specimen. Polymethyl methacrylate (PMMA) material was used to fabricate one-half inch thick specimens that have varying length and width dimensions. The material’s low fracture toughness and nearly brittle failure mode allowed cracks to be easily propagated. Optical transparency permitted the size, geometry and location of cracks to be readily determined. The crack propagation gages were located at the initial crack tip to measure crack tip velocities. A high sensitivity heterodyne interferometer was used to detect acoustic events resulting from crack growth in the specimen. A schematic of the instrumentation is shown in Figure 4. The device used in these studies permits high fidelity localized measurement of displacements resulting from acoustic emission events arriving at various
Figure 1 Displacement in Solid Due to an Abrupt Crack Extension.

Figure 2 Variation of Displacement with Crack Velocity ($c_A$).
Figure 3 Variation of Displacement with Duration of Crack Propagation ($t_c$).

Figure 4 Interferometer Schematic.
locations on the sample surface. Since this type of measurement does not acoustically load the sample, the event being observed is undisturbed by the measurement process.

The polished face of the specimen, opposite the crack, serves as one mirror of the interferometer. The beam striking this face is approximately 1.5 mm in diameter and samples the average displacement over this region, which is much smaller than the wavelength of the acoustic events being observed. The sample is designed so that initial acoustic emission events leaving the crack tip will arrive at this face prior to reflection from other faces of the specimen.

It should be noted that the analytical procedure being developed describes events prior to the arrival of stress waves reflected from the test specimen's boundary and is invalid for the time period after the fastest reflected waves interfere with the direct signal from the crack propagation event.

The operation of the heterodyne interferometer is similar to that described elsewhere [4]. Briefly, single frequency laser light is split into two components using an acousto-optic modulator. These two components, which are separated in frequency by 40 MHz, are sent along two arms of an interferometer one of which contains the sample to be monitored. The beams are recombined on the surface of a photodetector producing a beat frequency of 40 MHz. Phase shifts in the light reflected from the sample surface result in proportional phase shifts in the beat signal. As a result the 40MHz signal acts as a carrier that can be demodulated to determine the time dependent displacement occurring at the sample surface. The signal from the photodetector was demodulated in real time using an FM discriminator with a bandwidth of 10MHz. The demodulated output signal is proportional to the normal surface velocity of the specimen and can be integrated to determine its time dependent displacement.

The initial crack velocity is determined by measuring the change in resistance of the crack gage as a function of time. Both the acoustic emission waveform and the crack velocity profile are acquired on a pretriggered dual channel, digital oscilloscope. The time difference between the start of the crack growth event and the arrival of its signal at the observation point is easily determined with the pretriggering feature of the scope.

DISCUSSION AND CONCLUSIONS

A characteristic crack emission is shown in Figure 5. Here, a delay of 11.4μsec exists between the start of the gage and acoustic emission signal, which compares well with a calculated time lag of 11.2μsec. The first reflected wave from the specimen boundary is calculated to arrive 10.4μsec after the arrival of the initial signal. By using an estimated velocity profile for the final stage of growth, the last transverse wave should arrive approximately 70μsec after the arrival of the starting event. The velocity profile, curve fitted from the crack propagation gage data, is shown in Figure 6, while the integrated acoustic emission signal is shown in Figure 7. This integrated signal represents the measured displacement at the observation point, x, due to the actual crack propagation event. This displacement curve is needed to verify the analytical results being developed. In the early time period, Figures 1 and 7 show fairly good qualitative agreement between the analytically predicted and experimentally measured displacements. The two curves begin to deviate from each other at later times as the finite specimen geometry dominates the experimentally obtained signal.
Figure 5 Characteristic Surface Velocity from a Crack Emission.

Figure 6 Crack Tip Velocity Through Crack Gage (Initial time, $t=0$, occurs when the first gage wire is broken).
Figure 7 Displacement at Surface Perpendicular to Crack Path.

Anomalies in the fracture behavior of the specimen included out of plane growth and crack tunneling. Tunneling was caused by the stress gradient through the specimen thickness. In some cases crack growth was initiated on the surface opposite the propagation gage and there was a reduced time lag between the start of propagation and the arrival of its signal. More experimental development is necessary to reduce the influence of the specimen geometry on the measured acoustic emission waveforms. The analytical model must also be refined to diminish the effect of the assumed spatial stress distribution.

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REFERENCES


