ANISOTROPIC GAUSS-HERMITE BEAM MODEL APPLIED TO THROUGH-TRANSMISSION INSPECTIONS OF DELAMINATIONS IN COMPOSITE PLATES

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INTRODUCTION

Manufactured parts containing composite materials can present challenging ultrasonic inspection problems. The inherent anisotropy of such materials acts to distort propagating ultrasonic beams, leading in turn to an associated distortion of defect images. Such distortions complicate the task of estimating the physical dimensions of a defect from its ultrasonic image. In the present work we demonstrate how these difficulties can be overcome by appropriately modelling the ultrasonic inspection process, and using the model to analyze defect images. To illustrate the approach, we consider a normal incidence through-transmission inspection of a flat uniaxial composite plate with an internal delamination. We begin by reviewing our model of the inspection process which incorporates the Gauss-Hermite model for beam propagation in anisotropic materials. The inspection model requires as inputs certain parameters which characterize the transducers, and others which characterize the composite material. We demonstrate how these parameters can be obtained from simple beam-mapping experiments. We then present experimental C-scan images of a seeded circular delamination in a composite plate, and compare these to images predicted by the model. Finally, we demonstrate how the model can be used to accurately size a delamination from its ultrasonic image.

MODEL OF THE INSPECTION PROCESS

Consider the through-transmission inspection geometry depicted in Figure 1a. A plate of uniaxial graphite/epoxy composite (AS-4/3502) is immersed in water and positioned between transmitting (T) and receiving (R) transducers. The transducers are aligned such that their beams are coaxial and normal to the plate. The two transducers are then locked together and scanned in tandem, each transducer moving parallel to the upper surface of the plate. We employ a coordinate system in which the x-axis is along the central ray emanating from the transmitter, and the z-axis is parallel to the graphite fibers in the plate. If a delamination is present within the plate, the through-transmitted signal will be modified. Let \( \Gamma_{rr} (f) \) denote the spectral component at frequency \( f \) of the outgoing electrical signal in the coaxial cable of the receiver when no flaw is present in the plate. \( \Gamma_{flaw} (f) \) similarly denotes the spectral component when a delamination is present. By beginning with Auld's reciprocity relationship and applying a Kirchhoff scattering approximation, one can relate the ratio \( \Gamma_{flaw}/\Gamma_{rr} \) to ultrasonic displacement fields incident upon the flaw plane. Details of the derivation are contained in Ref. [1]. One finds:
Here \( U^i(x,y,z)e^{i\omega t} \) is the \( z \)-component of the ultrasonic displacement field incident upon point \( (x,y,z) \) in the flaw plane when a time-harmonic electrical signal of angular frequency \( \omega = 2\pi f \) and power \( P \) is input to transducer \( T \). Similarly, \( U^r(x,y,z)e^{i\omega t} \) is the incident field component which appears when the same electrical signal is input to transducer \( R \) (i.e., when \( R \) acts as the transmitter and \( T \) is switched off). The integral in the numerator of Eq. (1) is over the upper surface of the delamination, and the denominator integral is over the entire \( xy \) plane in which the flaw resides. Factors involving transducer efficiencies, fluid and solid attenuations, and interface transmission coefficients cancel in the ratio and need not be computed.

To evaluate the displacement fields appearing in Eq. (1), we use the anisotropic Gauss-Hermite beam model of Newberry and Thompson [2]. Each transducer is modelled as an ideal circular piston source, and the displacement field at the transducer is expanded in a truncated set of Gauss-Hermite (GH) basis functions. These expansion functions are propagated through space using the anisotropic wave equation and a Fresnel approximation; the functions are propagated through interfaces using paraxial rules derived from Snell's Law. In each case, a basis function maintains its functional form as it propagates, but certain parameters which describe the amplitude, phase, width, and radius of phase curvature change during propagation. To evaluate the displacement field at a point in space, we sum the propagated basis functions appropriately weighted by their expansion coefficients. The Fresnel approximation employed in the beam model uses a paraboloid to represent the forward portion of the slowness surface within a given material layer. For propagation through an anisotropic layer along a principal symmetry direction, as is the case here, the exact slowness surface is replaced by

\[
S_{\pm} = S_0 + C_x S_S^2 + C_y S_S^2
\]

(2)

Here \( S_0 \) is the inverse phase speed for propagation along the central ray direction (\( z \)-axis), and \( S_i = k_i/\omega \) is the \( i \)-th component of the slowness vector for a plane wave component propagating in a nearby direction \( \vec{k} \). The paraboloid curvature parameters \( C_x \) and \( C_y \), or their dimensionless counterparts

\[
\Delta_x = -2C_x S_0; \quad \Delta_y = -2C_y S_0
\]

(3)

effectively determine the rates of beam divergence in the \( xz \) and \( yz \) planes, respectively. If the elastic constants of the material are known,
the Christoffel equation can be used to generate the exact slowness surface of the appropriate wave type. $\Delta_x$ and $\Delta_y$ can then be determined by fitting a paraboloid to the forward portion of this surface. In our case, the composite layer possesses hexagonal symmetry about the x-axis, and the propagating wave within the layer is predominantly quasi-longitudinal (QL). If the exact QL slowness surface is expanded in a Taylor's series about the point \((S_x, S_y, S_z) = (0, 0, S_z)\) and only quadratic terms are retained, one finds:

$$\Delta_x = \frac{C_{33}}{C_{33}} + \frac{(C_{33} + C_{53})^2}{C_{33}(C_{33} - C_{53})}; \quad \Delta_y = 1$$

(4)

The corresponding analysis for an isotropic layer leads to $\Delta_x = \Delta_y = 1$. The curvature parameters given by Eq. (4) may not be optimal in a given application of the GH model to beam propagation an anisotropic layer. When the beam contains a wide angular spectrum of plane wave components, $\Delta_x$ deduced from a more global fitting of the paraboloid to the exact slowness surface may be more appropriate [3].

In order to evaluate the right hand side of Eq. (1) using the Gauss-Hermite beam model, a number of model inputs must be specified. These include the effective piston radii of the two transducers, the thicknesses of the water and composite layers, the size of the delamination and its location relative to the transducers, and pertinent material parameters for each layer (namely, the phase propagation speed in the forward direction $V_p = 1/S_z$, and the curvature parameters $\Delta_x$ and $\Delta_y$). In addition one must specify the number of basis functions to be retained in the expansion. In this work we have used $V_p = 0.1485 \text{cm/\mu s}$ and $\Delta_x = \Delta_y = 1$ for each water layer, and the measured speed $V_p = 0.309 \text{cm/\mu s}$ for QL waves in the composite. The effective piston radii of the transducers ($a_T$ and $a_R$, respectively) and the effective slowness surface curvature parameters of the composite were deduced from beam-mapping experiments to be described shortly. In the computation of displacement fields we retained all (even) basis functions through 24-th order in the Hermite polynomials, which was sufficient to guarantee three digits of converged precision. The surface integrals in Eq. (1) were computed numerically.

**TRANSDUCER CHARACTERIZATION**

In our flaw imaging experiments, we used commercial broadband planar transducers for the transmitter and receiver. Each transducer had a nominal diameter of $2a = 0.25$ inches, and a center frequency of 5 MHz. The two transducers were of the same design, and bore nearly sequential serial numbers. The measurement geometry shown in Fig. 1c was used to determine the effective piston radius of each transducer. The transducer was fixed in water, excited by an applied voltage spike, and the resulting ultrasonic field was then scanned laterally using a "point" receiver. The point transducer was supplied by Battelle NW, and had an effective diameter of approximately 0.05 cm. The lateral scans were performed in an xy plane 7.0 cm from the face of the transducer. This waterpath was representative of the effective waterpaths used in the imaging experiments described in this work, and in additional experiments using curved composite sections which will be presented elsewhere. (At $z = 7.0 \text{ cm}$, the dimensionless farfield distance parameter $zV_p/\alpha^2$ is approximately 2 at the center frequency of 5 MHz.) At each position of the point transducer, the received broadband time-domain signal was digitized (at a sampling rate of 100 MHz), its Fourier spectrum was computed, and selected frequency components were stored. This procedure allowed us to display single-frequency transverse profiles of the emitted ultrasonic beam. One such measured profile, for a scan through the center of the beam along the line \((x, y, z) = (x, 0, 7 \text{ cm})\), is shown in Fig. 2a. The main lobe and first side lobes can be seen. The solid curve in Fig. 2a is the
prediction of the GH beam model at the same frequency (2.93 MHz) assuming a piston radius of $a = 0.3316$ cm for the transmitter. For this choice of piston radius, the measured and predicted profiles have the same full-width at 1/e of the peak amplitude (FWE), and are generally in good agreement elsewhere. To characterize each planar transducer, we scanned the emitted field along the lines $(x, y, z) = (0, y, 7$ cm) and $(x, y, z) = (x, 0, 7$ cm), in each case determining the FWE of the main lobe at several frequencies. At each frequency, the GH beam model was used to locate the piston radius $(a)$ whose predicted field had the same FWE at $z = 7.0$ cm. The resulting piston radii are displayed in Fig. 2b. In our frequency range of interest ($1.5$ MHz $\leq f \leq 5$ MHz) we found that the two transducers behaved similarly, with each exhibiting a weak dependence of radius upon frequency. As may be seen in Fig. 2b, the effective piston radius of each transducer was found to be approximately

$$a_r = a_x = 0.301 + 0.091/f \quad (1.5 \text{MHz} \leq f \leq 5 \text{MHz})$$

where $a$ is in cm and $f$ is in MHz. Eq. (5) was used to determine input transducer radii for all ensuing model calculations.

**MATERIAL CHARACTERIZATION**

There are two approaches for determining the slowness curvature parameters, $\Delta_1$ and $\Delta_2$, which are required for model calculations of displacement fields within the composite layer. One can measure the independent elastic constants of the composite, determine the exact slowness surface and fit a paraboloid to it. Alternatively, one can infer the effective values of $\Delta_1$ and $\Delta_2$ from measurements which are sensitive to the rates of beam diffraction in the xz and yz planes. In this work, we compare the two approaches. Measurements of longitudinal and shear wave speeds for propagation along principal symmetry directions were made in coupons cut from a sample of the composite material. This allowed us to determine four of the five independent elastic constants of the composite. The fifth constant was then determined from an off-diagonal Poisson's ratio measured by the manufacturer. We found $C_{11}$, $C_{33}$, $C_{44}$, $C_{55}$, $C_{12}$ and $C_{23} = C_{32} - 2C_{44}$ to be 128.2, 14.95, 3.81, 6.73, 6.90 and 7.33 GPa, respectively. Using Eq. (4) we then obtained $\Delta_1 = 1.96$ and $\Delta_2 = 1$. The effective values of $\Delta_1$ and $\Delta_2$, as functions of frequency were also
deduced from beam-mapping experiments similar to those used to determine the piston radii. The measurement geometry is displayed in Fig. 1d. The beam from one of the characterized transducers was transmitted through a 2.56-cm thick plate of the uniaxial composite material at normal incidence. With the transmitter and plate fixed in position, the emerging beam was then scanned using the point receiver. The thicknesses of the water layers were 5.0 cm above the plate and 0.06 cm below it. Again the received broadband signals were processed to determine the FWE of the emerging beam components in the xz and yz planes. The experiment was repeated at each of four regions in the plate; the presence of local inhomogeneities caused the measured FWE at a given frequency to vary slightly along the plate. The measured FWE is displayed as a function of frequency in Fig. 3a. As expected, the emerging beam was considerably broader along the fiber direction [i.e., along the scan line (x, y, z) = (x, 0, 7.62 cm)], than it was in the transverse direction [along (x, y, z) = (0, y, 7.62 cm)]. To determine the effective values of the slowness curvature parameters, within the context of the GH beam model, we carried out a series of model calculations. At each frequency, the beam model was applied to the three-layer problem (water-composite-water) of Fig. 1d to predict the beam profile in the plane of the receiver. The model inputs \( \Delta_x \) and \( \Delta_y \) were varied until the predicted profile had the same pair of FWE values as the measured profile. The results of the fitting are shown in Fig. 3b together with the values of \( \Delta_x \) and \( \Delta_y \) obtained from Eq. (4). The scatter of the measured FWE values about their mean led to a similar scatter in the deduced curvature parameters. The curvature parameters deduced from the beam-mapping experiment are seen to be approximately independent of frequency and in good agreement with Eq. (4). The mean values of the deduced curvature parameters, averaged over frequency, were used in all ensuing model calculations (\( \Delta_x = 1.96; \Delta_y = 1.00 \)).

MEASURED AND PREDICTED C-SCAN IMAGES

A 0.75 cm thick uniaxial composite plate containing a simulated delamination in its midplane was supplied to us by LTV Aerospace, Dallas, Texas. The delamination was circular with a nominal diameter of 0.25". A through-transmission C-scan of the flawed plate was carried out using the two characterized transducers in the geometry of Fig. 1a ("thin plate geometry"). The waterpath on each side of the plate was 5.0 cm. Let (x', y') denote the lateral position of the beam (i.e., the lateral position of the ray joining the centers of the transducer faces) relative to the center of the flaw. At each transducer position (x', y'), the

Figure 3 Material characterization. (a): measured 1/e full widths of a beam emerging from a 1"-thick plate of uniaxial composite; (b): deduced values of the slowness surface curvature parameters \( \Delta_x \) and \( \Delta_y \). Values obtained from Eq. (4) appear as horizontal lines.

1543
Fourier spectrum $\Gamma_{\text{flow}}(f)$ of the received broadband signal was computed and selected frequency components were stored. In addition, a reference signal $\Gamma_{\text{ref}}(f)$ was acquired with the transducers positioned far from the flaw. Single-frequency C-scan images were generated by displaying the measured $|\Gamma_{\text{flow}}(f)/\Gamma_{\text{ref}}(f)|$ as a function of $(x', y')$. Predicted C-scan images were generated using Eq. (1) assuming a circular delamination of diameter 0.25" located in the midplane of the plate. Measured and predicted images are compared in Fig. 4a-b for the 1.95-MHz spectral component. Notice that the darkest portion of the experimental image (i.e., minimum transmitted spectral component) does not occur when the beam is centered on the flaw. Rather, there is an ultrasonic bright spot in the center of the image resulting from constructive interference at the receiver of waves diffracting around the delamination. This is analogous to Poisson's bright spot seen in the optical shadow cast by a circular disk. Measured and predicted values of $|\Gamma_{\text{flow}}/\Gamma_{\text{ref}}|$ for the thin plate geometry are further compared in Fig. 5, and are seen to be in excellent agreement. The curves in Fig. 5 depict slices through the centers of the single-frequency C-scan images parallel to the fiber direction. The central bright spot is clearly evident at the lower two frequencies. As the frequency increases, the beam diameter decreases and the delamination is able to block a greater fraction of the incident beam; this results in a smaller value of $|\Gamma_{\text{flow}}/\Gamma_{\text{ref}}|$ when the beam is centered on the flaw.

In the thin-plate geometry, the effects of material anisotropy are small, as evidenced by near-circular C-scan images. To simulate the inspection of a much thicker plate containing a similar defect, we used the "sandwich geometry" of Fig. 1b. The 0.75-cm thick flawed plate was

![Figure 4](image-url) Measured (a,c,f) and predicted (b,d,f) C-scan images of a 0.25" diameter circular delamination. $A = |\Gamma_{\text{flow}}/\Gamma_{\text{ref}}|$ is displayed as a function of transducer coordinates $x'$ (horizontal, along fibers) and $y'$ (vertical). The region scanned in each case measured 1.52 cm x 1.52 cm. The gray scale ranges from black ($A \leq 0.4$) to white ($A \geq 0.9$). The minimum value of $A$ is 0.45, 0.46, 0.59, 0.60, 0.40, 0.42 in images a, b, c, d, e, and f, respectively.
Figure 5  Measured (a) and predicted (b) values of $A = |I_{\text{ref}}/I_{\text{ref}}|$ at six frequencies for a 0.25" circular delamination in the thin plate geometry. $x'$ denotes the lateral separation (along the fiber direction) between the centers of the beam and the flaw.

Figure 6  Flaw sizing method. (---): predicted values of $A = |I_{\text{ref}}/I_{\text{ref}}|$ at 2.93 MHz for six circular delaminations in the thin plate geometry having diameters of 1/16", 2/16", ..., 6/16", respectively. (△): measured value of $A$ at 2.93 MHz.

placed between two 2.41-cm thick unflawed plates of the same composite material. The fiber directions (x-axes) of the three plates were aligned. Exterior waterpaths (between each transducer and the nearest thick plate) were 3.0 cm on each side, and interior waterpaths were 0.63 cm. Measured and predicted images of the delamination in the sandwich geometry are compared at two frequencies in Fig. 4. Again the predictions of the model agree well with experiment. The images are clearly elongated along the fiber direction, due to the greater beam diffraction rate in that direction. The central bright spot is evident at both frequencies, and the darkest portion of each image now is found on either side of the bright spot along the fiber direction.

FLAW SIZING

Since the inspection model can accurately predict through-transmitted signal components, the model can be used to size defects from measured
Table 1

Delamination diameter (cm) deduced by fitting the model to selected through-transmission data. The nominal diameter of the seeded circular delamination was 0.635 cm.

<table>
<thead>
<tr>
<th>THIN-PLATE GEOMETRY</th>
<th>SANDWICH GEOMETRY</th>
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<tbody>
<tr>
<td>( f ) (MHz)</td>
<td>1.37 1.95 2.93 3.91 5.08</td>
</tr>
<tr>
<td>Scan along fibers</td>
<td>0.626 0.628 0.642 0.636 0.648</td>
</tr>
<tr>
<td>Scan perp. to fibers</td>
<td>0.622 0.642 0.630 0.648 0.662</td>
</tr>
</tbody>
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spectral values. One sizing algorithm is demonstrated in Fig. 6. For the thin-plate geometry, the model has been used to predict the ratio \( \frac{|I_{\text{low}}|}{I_{\text{ref}}} \) at 2.93 MHz as a function of beam position \((x')\) for a \((y' = 0)\) scan through the center of the flaw along the fiber direction. This was done for six circular delaminations having different assumed diameters \((D)\), resulting in six predicted curves. The measured values of \( |I_{\text{low}}|/I_{\text{ref}} | \) at 2.93 MHz have been plotted on the same figure, and a good match is observed with the \( D = 4/16'' \) theory curve. Thus the flaw diameter is estimated to be near 1/4", as expected. This comparison process can be made more quantitative by using a fitting program which adjusts the assumed flaw diameter in the model to minimize the mean-squared difference between the measured and predicted \( |I_{\text{low}}|/I_{\text{ref}} | \) curves. Estimated flaw diameters obtained in this manner from model fits to selected data are listed in Table 1. For example, an estimated flaw diameter of 0.628 cm was obtained using data acquired at 2.93 MHz in the sandwich geometry when the beam was scanned through the center of the flaw along the fiber direction. Using data at other frequencies or other scan directions leads to slightly different estimated diameters, but all are within a few percent of the nominal diameter of \( D = 0.25'' = 0.635 \) cm.

SUMMARY

The Gauss-Hermite model for beam propagation in anisotropic media has been used to simulate normal-incidence through-transmission inspections of uniaxial composite plates. Predicted C-scan images of a circular delamination were in good agreement with experiment. Central bright spots in the defect images, which arise from wave interference, and image elongation in the fiber direction were correctly reproduced by the model.

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REFERENCES