INTRODUCTION

Current research efforts have concentrated on developing an instrumentation approach to test green-state powder metal (P/M) compacts by exploiting the conductivity or resistivity variations in the samples under test [1]. A novel multi-pin sensor is used to inject current in various directions through an array of outer contacts, and a matrix of 7 by 7 pointed needles record the voltage response of the P/M sample. Initial tests with production samples as well as a few controlled samples have shown good sensitivity and the ability to detect flaws of various sizes [6].

The multi-pin sensor developed permits semi-quantitative measurements on the size and orientation of surface-breaking and subsurface flaws. A more quantitative investigation with a range of flaw configurations turned out to be excessively time-consuming to fabricate. For this reason, numerical methods centered around the Finite Element Method (FEM) were employed to investigate the current flow-flaw interaction. Owing to the very small defect size of 200 micron surface openings and less, and the need to investigate the current flow pattern in three dimensions, finite elements proved to be computationally expensive in terms of creating suitable meshes and solving the resulting very large matrix systems. Since P/M samples can be modeled as a homogeneous body except for the flaw location, an integral equation approach similar to the boundary element approach is a viable approach and was
chosen for this study. The numerical model is custom-tailored such that it solves a surface-breaking defect of a given depth, width, and length. In addition, arbitrary current injection points can be chosen in an effort to provide sufficient flexibility to investigate a wide range of source flaw configurations. The exceptional quality of the numerical model was established by comparing measurements from controlled samples versus theoretical predictions.

THEORY

Starting point of the analysis is the electrostatic integral equation for the potential [2,3]

\[
V(x) = -2 \int_S V(y) \frac{\partial}{\partial n_y} g(x, y) \, ds_y + 2 \int_S g(x, y) \frac{\partial}{\partial n_y} V(y) \, ds_y + \frac{4\pi}{\sigma} \int_W g(x, x') q^0(x') \, dx' \tag{1}
\]

with \( S \) denoting surface and \( W \) volume integration. Here the free-space Green's function is \( g(x, y) = \frac{1}{4\pi |x - y|} \) and \( x \) and \( y \) are observer and source locations, respectively. The term \( q^0 \) indicates a current source. Equation (1) follows from Poisson equation for the interior

\[
\nabla^2 V(x) = -\frac{2\pi}{\sigma} q^0(x) \tag{2}
\]

after applying Green's theorem. The factor of \( 2\pi \) is used instead of \( 4\pi \) since our focus is on the surface potential. Once (1) is solved, i.e. the surface potential is found, the potential in the interior is obtained by

\[
V(x) = -2 \int_S V(y) \frac{\partial}{\partial n_y} g(x, y) \, ds_y + \frac{4\pi}{\sigma} \int_W g(x, x') q^0(x') \, dx' \tag{3}
\]

where the flux over the surface is set to zero (Neumann problem). To avoid solving a mixed Neumann-Dirichlet boundary problem, we position the current injection points not exactly on the body surface, but at some interior points \( x_0, x_1 \in W \setminus S \) situated slightly beneath the surface. Given the physical configuration, such an assumption does not appreciably affect the accuracy of our calculation. The external source function \( q^0 \) is represented as follows

\[
q^0(x) = I_0 \delta(x - x_0) - I_1 \delta(x - x_1), \quad x_0, x_1 \in W \setminus S \tag{4}
\]

where \( I_0, I_1 \) are two arbitrary constant amplitude factors \( (I_0 = I_1) \) and \( \delta(x - x_i), i = 0,1 \) denotes shifted delta functions. Substitution of (4) into (3) finally yields

\[
V(x) = -2 \int_S V(y) \frac{\partial}{\partial n_y} g(x, y) \, ds_y + \frac{1}{\sigma} \left[ I_0 \frac{1}{|x - x_0|} - I_1 \frac{1}{|x - x_1|} \right] \tag{5}
\]
On the surface (5) can be rewritten as

\[ LV = g(x) \]  

(6)

where \( L \) is a linear integral operator in the form

\[ L = 1 + 2 \int_{\Sigma} \frac{\partial}{\partial n_y} g(x, y) \, ds_y \]  

(7)

and the current sources are given by

\[ g(x) = \frac{1}{\sigma} \left[ I_0 \frac{1}{|x - x_0|} - I_1 \frac{1}{|x - x_1|} \right] \]  

(8)

From (6) we obtain the expression

\[ V = (V - \alpha LV) + \alpha g \]  

(9)

which after discretization can be cast in a linear system of equations in the form

\[ \tilde{V} = \tilde{A} \cdot \tilde{V} + \tilde{b} \]  

(10)

with the matrix \( \tilde{A}(\alpha) \) and column vector \( \tilde{b} \). The factor \( \alpha \) denotes a relaxation parameter which has to be chosen in an appropriate manner [4, 5]. The starting potential \( V^0 \) (zeroth iteration and simultaneously the surface potential in the absence of the flaw) follows from

\[ V^0(x) = g(x) = \frac{1}{\sigma} \left[ I_0 \frac{1}{|x - x_0|} - I_1 \frac{1}{|x - x_1|} \right] \]  

(11)

The relative error at the \( N^{th} \) iteration is quantified to be

\[ E_N = \frac{\|LV^N - g\|}{\|LV^0 - g\|} \]  

(12)

with the Euclidean norm

\[ \|V\| = \sqrt{\int_{S} V^2(y) ds_y} \]  

(13)

which is calculated at every iteration step.
SIMULATIONS

In this section simulations are conducted to quantify the electrostatic behavior in the vicinity of a 3D surface-breaking defect. A practically useful display results when the change in voltage $V^S$ is considered which is the total voltage distribution of the flawed sample $V^F$ minus the total voltage distribution of the unflawed sample $V^0$

$$V^S = V^F - V^0$$  \hspace{1cm} (14)

For the simulations, the following modeling data are selected: the surface is a square 2x2 surface area in arbitrary units (a.u.); $x_0, x_1$ are the locations (-0.4, 0, 0) and (0.4, 0, 0) in (a.u.), $I_0, I_1$ are set to $-1/(2\pi)$ (Fig. 1), with the conductivity normalized to unity, $\sigma = 1$. As seen in Figure 1, one can observe the current injection points followed by the voltage on the right for a flawed surface.

Figure 2 shows the change in voltage $V^S$ as an infinite length flaw of width $2w = 0.04$ and depth of 0.12 is introduced. The iterative algorithm requires three iterations to converge to a relative error of less than 5% based on the error criterion (12).

In Figure 3 a flaw of finite extent in y-direction is simulated. The flaw shape in Cartesian coordinates $X, Y, Z$ (cf. Fig. 1) is controlled by the function

$$Z = -h \frac{\exp(-b(Y - c)^2)}{1 + \left(\frac{|X|}{w}\right)^n}$$  \hspace{1cm} (17)

where we have used the following notations: $h$ is the flaw depth, $w$ is the flaw half-width, and $n$ is the flaw "power" affecting the sharpness transition in X-direction. The parameters $b$ and $c$ characterize variations along the flaw axis in Y-direction.

EXPERIMENTAL CONFIRMATION

The accuracy of the simulations can be tested by carrying out a direct comparison with experimental measurements on controlled green-state P/M compacts. Such a comparison was conducted for rectangular P/M blocks with a surface area of 3 by 4 inches and 2 inches in depth with the following P/M material specifications: atomized stainless steel powder, density of 6.7 g/cm$^3$, and containing 0.75% lithium stearate lubricant. The test involved a novel multiple probe sensor with an eight by eight planar array of point probes [1]. A current of 0.5 A is injected through the outer pins over a distance of 27.4 mm, whereas the inner set of probes records the differential signal between the voltage probes. In Figure 4, the measured absolute voltages are directly compared with the theoretical predictions for an unflawed specimen. The model discretizes a 2 by 2 a.u. surface domain. As seen, the agreement is excellent except for the locations close to the current injection points at +/- 0.8 a.u. (which corresponds to a distance of 27.4 mm) where the theoretical model assumes delta function distributions.
Figure 1. Generic problem geometry of a surface breaking defect.

Figure 2. Voltage distribution $V_S$ (right) over the surface of a sample (left) with flaw width $2w=0.04$, depth $d=0.12$ and infinite extent along the $y$-axis.
Figure 3. Finite size flaw with parameters $2w = 0.04$, $h = 0.2$, and $b = 8$ in equation (17).
Finally, Figure 5 shows for measured differential voltage responses and the numerical predictions for a 200 micron surface-breaking defect. The defect consists of a hard plastic material which was inserted into the P/M sample prior to compaction. The flaw is situated between voltage recording points 3 and 4 with a 2.5 mm depth and approximately 2.0 cm length.
CONCLUSIONS

The aim of this theoretical investigation is the development of a theoretical testbed capable of simulating realistic voltage distributions over the surface of planar green-state P/M compacts containing surface-breaking defects. Since modeling configurations are inherently three-dimensional, an integral formulation is proposed resulting in a linear matrix equation which can be solved iteratively. Measurements conducted with controlled samples achieve remarkable agreement. This theoretical model will ultimately serve as an indispensable tool in designing an appropriate instrument sensor to address such important issues as number, spacings, and current strengths of the multi-pin system.

Future developments will focus on modeling subsurface defects as well as extensions to handle more complex surface geometries simulating complex P/M compacts used industrially.

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