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A new trilevel optimization algorithm for the two-stage robust unit commitment problem

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A new trilevel optimization algorithm for the two-stage robust unit commitment problem

by

Bokan Chen

A thesis submitted to the graduate faculty in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Industrial Engineering

Program of Study Committee:
Lizhi Wang, Major Professor
Sigurdur Olafsson
Zhijun Wu

Iowa State University
Ames, Iowa
2013

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DEDICATION

I would like to dedicate this thesis to my parents Qibing Chen and Fangwei Shi without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance during the writing of this work.
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ABSTRACT

We present a new trilevel optimization algorithm to solve the robust two-stage unit commitment problem. In a robust unit commitment problem, first stage commitment decisions are made to anticipate the worst case realization of demand uncertainty and minimize operation cost under such scenarios. In our algorithm, we decomposed the trilevel problem into a master problem and a sub-problem. The master problem can be solved as a mixed-integer program and the sub-problem is solved as a linear program with complementary constraints with the big-M method. We then designed numerical experiments to test the performance of our algorithm against that of the Benders decomposition algorithm. The experiments show that our algorithm performs consistently better than the Benders approach.
CHAPTER 1. OVERVIEW

1.1 Introduction to Unit Commitment

In recent years, the power generation industry has seen considerable growth. Due to the increase of economy and productivity, the usage of electricity is rising. According to the estimation of U.S. Energy Information Administration (EIA) published in [2], the total electricity generation in the year 2011 was 4,105.7 billion KWh. Comparing to the 296.1 billion KWh in the year 1949, we can see a thirteen-time increase. With the increasing importance of the role the power sector plays in the modern society, a lot of effort has been put into developing a secure, reliable and economic power supply. Unit commitment is crucial in realizing this goal.

The power generation industry utilizes unit commitment (UC) and economic dispatch to help make generation scheduling decisions. In a unit commitment problem, decisions about which unit to interconnect are made for the day-ahead market. Independent system operators (ISO) are responsible for coordinating, controlling and monitoring the operation of power systems. Most ISOs today run the UC problem 24 hours before the real time market. The objective of a running a UC problem is to identify a schedule of committing units to minimize the joint cost of unit commitment and economic dispatch, while at the same time meet the forecasted demand.

Because of uncertainties associated with load forecast error, unexpected generator and transmission line failure and outages, during real-time operations, operators have to deviate from decisions prescribed by UC programs and take expensive corrective actions including committing flexible fast generators, load shedding or forced outages to maintain system security. In face of global warming and environmental issues caused by fossil fuels, the industrial is shifting their focus to renewable energy resources. The proliferation of renewable energy resources that
are intermittent in nature such as solar energy and wind power introduces more uncertainty into the system and poses new challenges to grid management and generation scheduling. Under such circumstances, there is a tremendous need for a unit commitment process that can handle more uncertainty in the system.

The current practice the power generation industry employs to cope with uncertainty is imposing reserve constraints to the UC problem [31]. Researches have been focusing on analyzing the effect of different levels of reserves [27, 25]. This approach has a few disadvantages. The requirements for reserves are usually very conservative, making it economically inefficient. In addition, the standards of determine reserve levels are based on the expected demand, which means the solution yielded by UC program with reserve constraints might still not be able to meet the real-time demand due to significant deviation from forecasted data.

Another very popular method to cope with uncertainty in the power system is stochastic programming. In [32, 39], the authors explored the benefits of combining the stochastic programming framework with reserve constraints. Stochastic programming has been proved to be able to improve the solution compared to solution with expected value [14], however, a few disadvantages exist with this framework. In a typical stochastic programming application, huge emphasis is placed on scenario generation where multiple scenarios are generated to approximate the actual probability distribution of uncertainty realizations. In addition, to improve the reliability of the stochastic solution, a large number of scenarios should be generated, making it computationally challenging to solve the large scale unit commitment problem. Although scenario reduction procedures can be employed to reduce the size of the problem, the performance of the resulted solution cannot be guaranteed.

As an alternative modeling approach in dealing with uncertain parameters in optimization, robust optimization is gaining more popularity over the years. It provides a framework that uses parametric sets to describe uncertainty [11]. Such uncertainty sets can be constructed simply by using information such as the mean and variance of a random variable, which are easier to obtain than the probability distribution. By changing the uncertainty set, robust optimization can provide a tradeoff between the conservativeness of the solution and its robustness. Another very desirable feature of robust optimization is that it can produce a set of solutions that is robust
against all uncertainty realizations, which means it can anticipate the worst case scenarios and take preventive measures against such scenarios. Such a feature is very important in unit commitment problems because any infeasible solution that might lead to a blackout is unacceptable in our current society.

Research in Robust Unit Commitment is very limited. In [22], the authors proposed a two stage robust unit commitment model and provided a separation algorithm under Benders decomposition framework to solve the problem. In [9], the researchers decomposed the problem into an outer level problem and an inner level problem, solved by Benders decomposition and outer approximation respectively. In the two researches, the solution schemes can not guarantee a global optimal solution. In addition, Benders decomposition algorithms can take a long time to converge.

1.2 Introduction to Robust Optimization

In traditional mathematical programming problems, the input data of a developed model is usually assumed to be deterministic. However, as is illustrated by the case studies in [7], it is possible that some uncertainty exists in data, rendering the optimal solution derived by the mathematical program sub-optimal or even infeasible. Thus the need for an optimization tool that can immunize the solution against the uncertainty in data arises in real world applications.

In [34], a linear optimization as follows is proposed:

$$\begin{align*}
\max & \quad c^\top x \\
\text{s.t.} & \quad \sum_{j=1}^{n} A_j x_j \leq b, \forall A_j \in K_j, j = 1, \leq n \\
& \quad x \geq 0
\end{align*}$$

where the uncertainty sets $K_j$ are convex. One issue of this formulation is that it is too conservative, which means much of the optimality is sacrificed to ensure robustness [7].

To address the problem of over conservatism, in [7, 19, 18], the researchers proposed a linear programming model with ellipsoidal uncertainties. The formulation proposed in [7] is as
follows:

\[
\begin{align*}
\text{max} & \quad c^T x & (1.4) \\
\text{s.t.} & \quad \sum_j a_{ij} x_j + \sum_{j \in J_i} a_{ij}^2 y_{ij} + \Omega_i \sqrt{\sum_{j \in J_i} a_{ij}^2 y_{ij}^2} \leq b_i \quad \forall i \\
& \quad -y_{ij} \leq x_j - z_{ij} \leq y_{ij} \quad \forall i, j \in J_i & (1.5) \\
& \quad l_j \leq x_j \leq u_j \quad \forall i \in J_i & (1.6) \\
& \quad y_{ij} \geq 0 \quad \forall i, j \in J_i & (1.7)
\end{align*}
\]

This approach is less conservative and with carefully chosen ellipsoids, more complex uncertainty sets can be approximated. The authors also derive the upper bound of the probability the \( i \)th constraint is violated. With this formulation, however, although the problem remains convex, it is no longer linear, making it less tractable in terms of computation.

In this thesis, we use the robust optimization framework proposed in [11], where the uncertainty in data is characterized by polyhedral uncertainty sets. This approach has the advantage of keeping the linear structure of the problem, making it more tractable computationally. Another advantage of this approach is that the degree of conservatism can be controlled by adjusting parameters of the uncertainty set.

### 1.3 Research Objective

Many studies [27, 39, 25, 32, 22] have analyzed the methodologies to handle uncertainty in power systems. Most of the studies focus on the effect of setting different levels of spinning and non-spinning reserves. In this research, we are focusing on robust optimization as a framework to deal with uncertainty.

The objective is to develop an effective algorithm to solve the robust unit commitment problem. To facilitate algorithm development, we reformulated the problem into a trilevel optimization problem. In [26], the difficulty of solving a bilevel linear optimization program is discussed and the researchers presented several heuristics to solve simpler problems. In [43], a trilevel optimization model is proposed for the power network defense problem and a heuristic algorithm is presented. General trilevel optimization programs are extremely difficult to solve.
However, the robust unit commitment problem has a special structure that can be exploited to simplify the solving procedure. We try to propose a two level iterative algorithm for the robust unit commitment problem. In addition, in order to demonstrate the performance of our algorithm, we try to conduct numerical experiments where we compare our algorithm with the one presented in [9].

1.4 Thesis Organization

This thesis is organized as follows: Chapter 2 reviews relevant literatures about unit commitment, robust optimization and multilevel optimization. In Chapter 3, we present the robust unit commitment formulation and the solution methods. We presented the data and results of our case study in Chapter 4. Finally, we conclude this thesis with Chapter 5.
CHAPTER 2. REVIEW OF LITERATURE

In this chapter, we review some literatures on unit commitment problems and robust optimization.

2.1 Unit Commitment

A lot of emphasis has been placed on the unit commitment problem because of its high impact in the power industry. The objective function of the UC problem is in fact nonlinear and is approximated by piecewise linear functions. In [42], a rigorous segment partitions method is proposed to derive a tighter approximation. In [15], the authors formulated the thermal constraints and inter temporal constraints in a more computationally efficient manner. A lot of researches have been done in improving the efficiency of solution methods of unit commitment problems [23, 20, 28]. In recent years, researchers are paying more attention to uncertainty in power systems. The effect of pre-specified reserve requirements imposed on generation scheduling is studied in [30, 33]. In [3, 41, 40], power balance under both normal and contingency states is explicitly imposed. The effects of different levels of reserve is studied in [25, 27]. Imposing reserves can lead to sub optimal solutions or even infeasible solutions. To mitigate this problem, stochastic programming is applied to unit commitment problems [37, 38, 29]. In [32, 39], the researches explored the benefit of combining stochastic programming with reserve requirements. Robust optimization is a popular alternative of stochastic programming in dealing with uncertainty. However, relatively less research has been done in robust unit commitment. In [35], a bilevel optimization model is proposed to include generator availability as the source of uncertainty in constraints. A two stage robust unit commitment model is proposed in [22]. In [9], a Benders decomposition algorithm is proposed to solve the
two-stage robust unit commitment problem.

## 2.2 Robust Optimization

Research in robust optimization is gaining momentum over the years. The first robust optimization model is proposed in [34]. To address the problem of over conservatism, ellipsoid uncertainty set is proposed in [7, 6, 19, 18]. In [11], the researchers proposed a robust optimization framework where a bilevel optimization structure is assumed by the robust optimization model in which the uncertainty set is polyhedral. With this approach, the conservatism can be adjusted by changing a parameter in the uncertainty set. In [8, 16], risk measurements are utilized to construct uncertainty sets. In [10, 12], the researchers analyzed the properties of the solutions and the tractability of different robust optimization models. They also extend robustness to more general conic problems. In [36], the authors formulate the robust optimization problem as a stochastic program with controlled second stage variability and solve it with a variant of the L-shape method. Robust optimization require controlled performance for data realizations in the uncertainty set, while in [4], the authors proposed an approach that also control the performance deterioration for data outside the uncertainty set. In [17], the authors compared the performance of stochastic programming and robust optimization. In addition, they propose an approach to construct uncertainty sets using deviation measures for random variables. A lot of researchers have been doing research applying robust optimization in many areas. In [13], the authors propose a numerically tractable robust optimization methodology to solve the optimal control problem of a supply chain subject to stochastic demand. In [5], the authors use the affinely adjustable robust counterpart methodology to solve the min-max retailer-supplier flexible commitment problem with uncertain demand. The authors of [24] propose a robust optimization model for the multi-site production planning problem with uncertain data. Research in robust unit commitment problems are limited, [22, 9] among the first to research this topic.
CHAPTER 3. METHODS AND PROCEDURES

In this section, we present the robust unit commitment model, which consists of three levels of optimization. At the first level, a set of binary commitment decisions are made to decide which generators should be operating during the next 24 hours so as to minimize the total operating cost under the worst case scenario. At the second level, the objective is to identify the worst case uncertainty realization. Then at the third level, given the first level commitment decision and the second level uncertainty realization, economic dispatch decisions are made to minimize the production cost. The notations used in this chapter are summarized in Section 3.1 and the robust optimization model is presented in 3.2.

3.1 Notations

Sets

- $\mathcal{D}$: The polyhedron uncertainty set of demand.
- $\mathcal{L}_m$: The set of load at bus $m$.
- $\mathcal{M}$: Set of buses, $\mathcal{M} = \{1, 2, \ldots, M\}$. $M$ is the number of buses.
- $\mathcal{N}_m$: Set of generating units at bus $m$.
- $\mathcal{Q}$: Set of transmission lines, $\mathcal{Q} = \{1, 2, \ldots, Q\}$. $Q$ is the number of transmission lines.
- $\mathcal{T}$: Set of time periods, $\mathcal{T} = \{1, 2, \ldots, T\}$.

Parameters

- $\lambda_i^m$: Startup offer cost for unit $i$ at bus $m$, in $\$$.
- $C_{i,m}$: The coefficients of the linear fuel cost function of unit $i$ at bus $m$, in $\$/MWh.
• $F^q$: The line flow limit of transmission line $q$, in MWh.

• $G_{i,m}^m$: The number of hours unit $i$ at bus $m$ has to be on line due to its initial state, no unit.

• $L_{i,m}^m$: The number of hours unit $i$ at bus $m$ has to be off line due to its initial state, no unit.

• $p_{i,0}^m$: The initial power output of unit $i$ at bus $m$, in MW.

• $PL_{i,m}^m$: Minimum power output of unit $i$ at bus $m$ if it is on, in MW.

• $PU_{i,m}^m$: Maximum power output of unit $i$ at bus $m$ if it is on, in MW.

• $RU_{i,m}^m$: Ramp-up limit of unit $i$ at bus $m$, in MW.

• $RD_{i,m}^m$: Ramp-down limit of unit $i$ at bus $m$, in MW.

• $f_{m,q}$: Shift factors.

• $TD_{i,m}^m$: Minimum down time of unit $i$ at bus $m$, in hour.

• $TD_{i,0}^m$: The time unit $i$ at bus $m$ has been off line at the beginning of the first time period, in hour.

• $TU_{i,0}^m$: The time unit $i$ at bus $m$ has been on line at the beginning of the first period, no unit, in hour.

• $TU_{i,m}^m$: Minimum up time of unit $i$ at bus $m$, in hour.

• $u_{i,0}^m$: Binary parameter indicating the initial on and off status of unit $i$ at bus $m$. $u_{i,0}^m = 1$ means the unit is on initially.

**Decision Variables**

• $d_{l,m,t}^m$: The demand of load $l$ at bus $m$ at period $t$, in MW.

• $p_{n,m,t}^m$: Power output of unit $n$ at bus $m$ at period $t$, in MW.

• $pn_{m,t}$: Net power injection at bus $m$ at period $t$, in MW.
• $s_{i,t}^m$: Binary variable indicating the startup status of unit $i$ at bus $m$ at time $t$. $s_{i,t}^m = 1$ means the unit is started at this hour.

• $u_{i,t}^m$: Binary variable representing the on and off status of unit $i$ at bus $m$ at period $t$. $u_{i,t}^m = 1$ means the unit is on at this hour.
### 3.2 Robust Unit Commitment Model Formulation

\[
\begin{align*}
\min_{s,u,p} & \quad \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{i\in \mathcal{N}_m} \lambda_i^m s_{i,t}^m + \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{i\in \mathcal{N}_m} C_{i,m} p_{n,t}^m \\
\text{s.t.} & \quad s_{i,t}^m \geq u_{i,t}^m - u_{i,t-1}^m, \forall m,i,t \\
& \quad \sum_{t=1}^{T} (1 - u_{i,t}^m) = 0, \forall i \\
& \quad \sum_{t=k}^{k+TU_i^m-1} u_{i,t}^m \geq TU_i^m (u_{i,k}^m - u_{i,k-1}^m), \forall m,i,k \in [2, \ldots, T] \\
& \quad \sum_{t=k}^{T} [u_{i,t}^m - (u_{i,k}^m - u_{i,k-1}^m)] \geq 0, \forall m,i,k \in [T - T U_i^m + 2, \ldots, T] \\
& \quad \sum_{t=k}^{T} [1 - u_{i,t}^m - (u_{i,k}^m - u_{i,k-1}^m)] \geq 0, \forall m,i,k \in [T - T D_i + 2, \ldots, T] \\
& \quad u_{i,t}^m \text{ binary} \\
\max_{d} & \quad \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{i\in \mathcal{N}_m} C_{i,m} p_{n,t}^m \\
\text{s.t.} & \quad D_{l,m} \leq d_{l,t}^m \leq U_{l,m}, \forall m,t \\
& \quad \sum_{m=1}^{M} \sum_{l\in \mathcal{L}_m} \tau_{l,t} p_{n,t}^m \leq \pi, \forall t \\
\min_{p} & \quad \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{i\in \mathcal{N}_m} C_{i,m} p_{n,t}^m \\
\text{s.t.} & \quad P L_i^m u_{i,t}^m \leq p_{i,t}^m \leq P U_i^m u_{i,t}^m, \forall m,i,t \\
& \quad p_{i,t}^m - p_{i,t-1}^m \leq (2 - u_{i,t-1}^m - u_{i,t}^m) P L_i^m + (1 + u_{i,t-1}^m - u_{i,t}^m) R U_i^m, \\
& \quad \forall m,i,t
\end{align*}
\]

\[(3.1)\]  
\[(3.2)\]  
\[(3.3)\]  
\[(3.4)\]  
\[(3.5)\]  
\[(3.6)\]  
\[(3.7)\]  
\[(3.8)\]  
\[(3.9)\]  
\[(3.10)\]  
\[(3.11)\]  
\[(3.12)\]  
\[(3.13)\]  
\[(3.14)\]  
\[(3.15)\]
\[ p_{i,t}^m - p_{i,t-1}^m \leq (2 - u_{i,t-1}^m - u_{i,t}^m)PL_i^m + (1 - u_{i,t-1}^m + u_{i,t}^m)RD_i^m, \forall m, i, t(3.16) \]

\[ -F^q \leq \sum_{m=1}^{M} f_{m,q} P_{m,t} \leq F^q, \forall q, t \quad (3.17) \]

\[ p_{m,t} = \sum_{n \in N_m} p_{n,t}^m - \sum_{l \in L_m} d_{l,t}^m, \forall m, t \quad (3.18) \]

\[ \sum_{m=1}^{M} \sum_{n \in N_m} p_{n,t}^m = \sum_{m=1}^{M} \sum_{l \in L_m} d_{l,t}^m, \forall t. \quad (3.19) \]

The upper level problem is defined by Equations (3.1)-(3.19), the middle level problem is defined by Equations (3.10)-(3.19) and the bottom level problem is defined by Equations (3.13)-(3.19). The objective function of the upper level problem is Equation (3.1), which includes the startup cost and the economic dispatch cost. The objective function of the middle level and the bottom level problems are Equations (3.10) and (3.13) respectively and they both mean the dispatch cost.

Constraint (3.2) specifies the startup status of a unit based on its on and off status at two consecutive periods. Constraints (3.3)-(3.5) specify the minimum up time of all the units. The parameter \( G_i^m \) can be calculated by \( G_i = \min \{ T, (TU_i - TU_i^0)u_i, 0 \} \). Constraints (3.6)-(3.8) specify the minimum down time of all the units. The parameter \( L_i^m \) can be calculated by \( L_i = \min \{ T, (TD_i - TD_i^0)(1 - u_{i,0}) \} \). Constraint (3.14) specifies the output limit of units. Constraints (3.15)-(3.16) specify the ramp up and ramp down limits of units respectively. Constraint (3.17) is the transmission limits constraint. Equation (3.18) calculates the net injection at nodes with respect to positive power injection and negative power injection. Equation (3.19) is the supply and demand balance constraint.

### 3.3 Introduction to The Benders Decomposition Approach

In this section, we briefly introduces the Benders decomposition approach presented in [9]. Suppose the robust unit commitment model is formulated as follows:

\[
\begin{align*}
\min_x & \quad c^T x + \max_{d \in \mathcal{D}} \min_{y \in \Omega(x,d)} b^T y \\
\text{s.t.} & \quad Fx \leq f \\
& \quad x \text{ binary}
\end{align*}
\]  

\[ (3.20) \]

\[ (3.21) \]

\[ (3.22) \]
where
\[ \Omega(x, d) = \{ y : \begin{array}{l} H y \leq h \\ Ax + By \leq g \\
J y = d \end{array} \} \] (3.23)
and
\[ D = \{ d : D d \leq k \}. \] (3.26)

The middle and bottom level of the problem is equivalent with a biliner optimization problem if we write out the dual of the bottom level problem:

\[ I(x) = \max_x -h^\top \alpha + (Ax - g)^\top \beta + d^\top \lambda \] (3.27)

s.t. \[ -H^\top \alpha - B^\top \beta + J^\top \lambda = b \] (3.28)
\[ Dd \leq q \] (3.29)
\[ \alpha \geq 0, \beta \geq 0. \] (3.30)

The algorithm decomposes the whole problem into an outer problem, which is solved by Benders decomposition and an inner level problem, which is solve by outer approximation.

### 3.3.1 The Outer Problem: Benders Decomposition

\[ (x^*, d^*, y^*) = \text{Alg}^{\text{BD}}(c, b, F, f, H, h, A, B, g, J, D, k) \]

**Step 0:** Initialization. Denote the lower bound as \( LB = -\infty \) and the upper bound as \( UB = \infty \). Set the iteration count \( C = 0 \). Choose the termination tolerance \( \epsilon \). Go to Step 1.

**Step 1:** Solve the following master problem:

\[ \max_{x, \zeta} \quad c^\top x + \zeta \] (3.31)

s.t. \[ Fx \leq f \] (3.32)
\[ \zeta \geq -h^\top \alpha_l + (Ax - g)^\top \beta_l + d_l^\top \lambda_l, \forall l \leq C \] (3.33)
\[ x \text{ binary.} \] (3.34)
Let \((x_C, \zeta_C)\) be the optimal solution. Update the lower bound \(LB = c^\top x_C + \zeta_C\). Increase the iteration count \(C\) by 1. Go to Step 2.

**Step 2:** Solve the inner problem \(I(x_C)\). Denote the optimal solution as \((d_C, \alpha_C, \beta_C, \lambda_C)\). Update the upper bound \(UB = \min(UB, c^\top x_C + I(x_C))\).

if \(UB - LB > \epsilon\) then
    Go to Step 1.
else
    Calculate the dispatch variable \(y_C\) given \(x_C\) and \(d_C\). Return. Output the optimal solution \((x_C, d_C, y_C)\).
end

3.3.2 The Inner Problem: Outer approximation

\[(d^*, \alpha^*, \beta^*, \lambda^*) = \text{AlgOA}(c, b, F, f, H, h, A, B, g, J, D, k, x_C)\]

**Step 0:** Initialization. Pass the current commitment decision from the outer problem \(x_C\) to the inner problem. Find an initial uncertainty realization \(d_1 \in \mathbb{D}\). Set the lower bound as \(LOA = -\infty\) and the upper bound as \(UOA = \infty\). Set the iteration count \(j = 1\) the termination tolerance \(\theta\). Go to Step 1.

**Step 1:** Solve the following sub-problem:

\[
S(x_C, d_j) = \max \quad -h^\top \alpha + (Ax_C - g)^\top \beta + d_j^\top \lambda \quad (3.35)
\]

s.t. \[-H^\top \alpha - B^\top \beta + J^\top \lambda = b \quad (3.36)\]

\[
\alpha \geq 0, \beta \geq 0. \quad (3.37)
\]

Denote the optimal solution as \((\alpha_j, \beta_j, \lambda_j)\). Define the linearization of the bilinear term \(d_j^\top \lambda_j\) as \(L_j(d_j, \lambda_j) = d_j^\top \lambda_j + (d - d_j)^\top \lambda_j + (\lambda - \lambda_j)^\top d_j\). Update the lower bound as \(LOA = \max(S(x_C, d_j), LOA)\). Go to Step 2.
Step 2: Solve the following master problem:

\[
U(d_j, \lambda_j) = \max \ -h^\top \alpha + (Ax_C - g)^\top \beta + \zeta \\
\text{s.t.} \quad \zeta \leq L_i(d_i, \lambda_i), \forall i \leq j \\
\quad -H^\top \alpha - B^\top \beta + J^\top \lambda = b \\
\quad Dd \leq k \\
\quad \alpha \geq 0, \beta \geq 0.
\] (3.38)

Increase the iteration count \(j\) by 1. Denote the optimal solution as \((d_j, \alpha_j, \beta_j, \lambda_j, \zeta)\).

Update the upper bound as \(UOA = \min(UOA, U(d_j, \lambda_j))\).

\[
\begin{align*}
\text{if } UB - LB &> \theta \text{ then} \\
& \quad \text{Go to Step 1.} \\
\text{else} & \\
& \quad \text{Return. Output the optimal solution } (d_j, \alpha_j, \beta_j, \lambda_j, \zeta).
\end{align*}
\]

3.3.3 Limitations of The Benders Approach

The Benders decomposition approach described above has several limitations. Firstly, the master problem depends on the dual variables of the sub-problem, which means the sub-problem cannot have integer variables. In addition, the outer approximation algorithm cannot guarantee a global optimal solution. It means the true worst case scenario might not be identified. Finally, the Benders decomposition approach converges very slowly, especially for large scale problems. Given all those limitations, we propose a new trilevel optimization algorithm.

3.4 Trilevel Optimization Algorithm

To facilitate the comparison of our algorithm with the Benders approach, we use the same abstract model as the one introduced in [9]. In order to describe our algorithm more concisely,
we reformulate the model as a trilevel optimization problem as follows.

\[
\begin{align*}
\min_x & \quad c^\top x + b^\top y \\
\text{s.t.} & \quad Fx \leq f \\
& \quad x \text{ binary} \\
\max_d & \quad b^\top y \\
\text{s.t.} & \quad Dd \leq k \\
\min_y & \quad b^\top y \\
\text{s.t.} & \quad Ax + By \leq g \\
& \quad Hy \leq h \\
& \quad Jy = d.
\end{align*}
\]

Equation (3.44) represents the constraints on first stage commitment variables. Equation (3.49) represents the constraints that couples the commitment and dispatch variables. Equation (3.50) is the constraints on dispatch variables. Equation (3.51) represents the constraints that couples the dispatch variables with demand variables. Equation (3.47) describes the uncertainty set for demands.

The above trilevel model can be equivalently reformulated as a single level model as follows:

\[
\begin{align*}
\min_{x,\phi} & \quad c^\top x + \phi \\
\text{s.t.} & \quad Fx \leq f \\
& \quad x \text{ binary} \\
& \quad \phi \geq b^\top y, \forall y \in \mathcal{Y}_D
\end{align*}
\]

where \(\mathcal{Y}_D = \{y|Ax + By \leq g, Hy \leq h\} \cap \{y|Jy = d, d \in \mathcal{D}\}\).

If we have \(\Omega \subset \mathcal{D}\) and \(\mathcal{Y}_\Omega = \{y|Ax + By \leq g, Hy \leq h\} \cap \{y|Jy = d, d \in \Omega\}\), then we will have \(\mathcal{Y}_\Omega \subset \mathcal{Y}_D\). With this, we can conclude that the following program is a relaxation of
problem (3.52)-(3.55):
\[
\begin{align*}
\min_{x, \phi} & \quad c^\top x + \phi \\
\text{s.t.} & \quad Fx \leq f \\
& \quad x \text{ binary} \\
& \quad \phi \geq b^\top y, \forall y \in \mathcal{Y}_\Omega.
\end{align*}
\] (3.56)

With that in mind, we decompose the trilevel problem into a master problem and a sub-problem. The master problem $M$ is a relaxation of the trilevel problem as follows:
\[
\begin{align*}
\min_{x, y, \zeta} & \quad c^\top x + \phi \\
\text{s.t.} & \quad Fx \leq f \\
& \quad x \text{ binary} \\
& \quad \phi \geq b^\top y^i, \forall i = 1, \ldots, |\Omega| \\
& \quad H y^i \leq h, \forall i = 1, \ldots, |\Omega| \\
& \quad A x + B y^i \leq g, \forall i = 1, \ldots, |\Omega| \\
& \quad J y^i = d^i, \forall i = 1, \ldots, |\Omega|
\end{align*}
\] (3.60)

where we iteratively add elements into the set $\Omega$. Solving the master yields a lower bound of the actual optima.

Then we can find the dispatch cost under the worst-case scenario by solving the following bilevel sub-problem $S(x)$
\[
\begin{align*}
\max_d & \quad b^\top y \\
\text{s.t.} & \quad D d \leq k \\
\min_y & \quad b^\top y \\
\text{s.t.} & \quad H y \leq h \\
& \quad B y \leq g - A x \\
& \quad J y = d.
\end{align*}
\] (3.67)

Solving the sub-problem yields an upper bound of the actual optima. Global optimality is obtained when the lower bound and upper bound coincide.
The complete steps for the algorithm is developed in the following sections and a flow chart of the algorithm can be seen from Figure 3.1.

3.4.1 An Iterative Algorithm For The Trilevel Optimization Problem

\[(x^*,d^*,y^*,\zeta^*) = \text{Alg}^{\text{TMILP}}(c,b,F,f,H,h,A,B,g,J,D,k)\]

**Step 0:** Initialization. Denote the lower bound as \(LB = -\infty\) and the upper bound as \(UB = \infty\). Create an empty set \(\Omega\). Go to Step 1.

**Step 1:** Solve the master problem \(\text{M}\). The solution of the problem is \((x^M,\zeta^M,y^M)\). Then update the lower bound of the algorithm \(LB = \max(LB,c^T x^M + \zeta^M)\). Go to Step 2.

**Step 2:** Solve the sub problem \(\text{S}(x^M)\). The solution to the problem is \((y^S,d^S)\). Update the upper bound of the objective \(UB = \min(UB,c^T x^M + b^T y^S)\) and the set \(\Omega = d^S \cap \Omega\).

\[\text{if } UB - LB > 0 \text{ then}\]
\[| \quad \text{2(a): Go to Step 1.}\]
\[\text{else}\]
\[| \quad \text{2(b): Find } i = \arg \max_i b^T y^i. \text{ Calculate the total cost } \zeta = c^T x^M + b^T y^i. \text{ Return.}\]
\[\text{Output optimal solution as } (x^M,y^i,\zeta).\]

![Figure 3.1 Diagram of Alg^{\text{TMILP}}](image)

3.4.2 Convergence Proof

**Theorem** In two iterations \(k\) and \(l\) where \(k < l\), suppose the optimal solutions to the master problem are \((x^k,\zeta^k,y^{1,k}, y^{2,k}, \ldots, y^{k,k})\) and \((x^l,\zeta^l,y^{1,l}, y^{2,l}, \ldots, y^{l,l})\). If \(x^k = x^l\), then the algorithm terminates.
Proof: If \( x^k = x^l \), then we will know by solving the sub-problem, the solution \( (d^l, y^{l,s}) \) of iteration \( l \) is the same with the optimal sub-problem solution \( (d^k, y^{k,s}) \) of iteration \( k \).

We have the upper bound of the optimal solution \( UB = c^\top x^l + b^\top y^{l,s} \). We will also have \( \zeta^l \geq b^\top y^{k,l} \). Because \( x^l = x^k, d^l = d^k \), we can get \( y^{l,s} = y^{k,s} = y^{k,l} \). Then we have \( UB = c^\top x^l + b^\top y^{l,s} \leq LB = c^\top x^l + \zeta^l \), which satisfies the termination criteria of the algorithm.

The theorem proves that when the same first-stage solution occurs, the algorithm terminates. We could also see the first-stage variable is in a set with finite number of elements so the master problem will not generate different solutions infinitely. Thus, the algorithm terminates finitely.

In this model, we added slack variables to make it a relatively complete recourse problem, which means the second-stage problem is always feasible given a first-stage solution regardless of the uncertainty realization. In addition, all the variables and constraints in the UC problem is bounded. So when the algorithm terminates and the lower bound and upper bound meet, we can get the optimal solution.

3.4.3 Solving the Master Problem

As can be seen from the formulation of the Master Problem \( M \), the size of the Mixed-Integer program increases fast with the increase of the number of iterations. We have two solutions to this problem.

Firstly, it is easy to see the similarities between the Master problem and traditional stochastic programming problem in terms of problem structure. This means varies stochastic programming solution techniques including the L-shape method and other heuristics can be readily applied to solving the Master problem.

In addition, we can see not every element in the set \( \Omega \) can affect the solution of the Master Problem, which means intelligent methods can be used to reduce the size of the problem.
3.4.4 Solving the Sub-Problem

Currently, the bilevel sub-problem is solved by the big-M method for Linear Program with Complimentary Constraints (LPCC) in our algorithm. Firstly, by using the KKT condition, we can change the form of the sub-problem into

\[
\begin{align*}
\max & \quad \mathbf{b}^\top \mathbf{y} \\
\text{s. t.} & \quad D\mathbf{d} \leq k \\
& \quad J\mathbf{y} = d \\
& \quad -H^\top \mathbf{\alpha} - B^\top \mathbf{\beta} + J^\top \mathbf{\gamma} = \mathbf{b} \\
& \quad 0 \leq h - H\mathbf{y} \perp \mathbf{\alpha} \geq 0 \\
& \quad 0 \leq g - A\mathbf{x} - B\mathbf{y} \perp \mathbf{\beta} \geq 0.
\end{align*}
\]

By introducing auxiliary binary variables, we can reformulate the above problem into

\[
\begin{align*}
\max & \quad \mathbf{b}^\top \mathbf{y} \\
\text{s. t.} & \quad D\mathbf{d} \leq k \\
& \quad -H^\top \mathbf{\alpha} - B^\top \mathbf{\beta} + J^\top \mathbf{\gamma} = \mathbf{b} \\
& \quad H\mathbf{y} \leq h \\
& \quad A\mathbf{x} + B\mathbf{y} \leq g \\
& \quad H\mathbf{y} + M\mathbf{z} \geq h \\
& \quad A\mathbf{x} + B\mathbf{y} + M\mathbf{w} \geq g \\
& \quad \mathbf{\alpha} + M\mathbf{z} \leq M \\
& \quad \mathbf{\beta} + M\mathbf{w} \leq M \\
& \quad \mathbf{\alpha} \geq 0, \mathbf{\beta} \geq 0; \ z, w \text{ binary.}
\end{align*}
\]

Then, the above problem can be solved as a mixed-integer linear optimization problem.

In the ideal situation, \( M \) should be infinity. However, this is not possible in computer programs. Here, \( M \) is a large constant and should be chosen with caution. If \( M \) is too small, then potential feasible regions might be cut off and the solution obtained might not be
optimal. If $M$ is too large, then computational errors might occur. One approach of deriving a valid $M$ is to solve the relaxed LP of the original problem and find bounds on the variables and constraints[21]. In the robust unit commitment problem, dispatch variables are usually bounded, making it easy to obtain a valid $M$ that does not cut off the optimal solution.
CHAPTER 4. NUMERICAL RESULTS

In this case study, we tested our algorithm against the Benders Decomposition approach with outer Approximation proposed in[21]. Both algorithms are implemented in MATLAB using TOMLAB/CPLEX as the MILP solver. Computational experiments are executed on a desktop with Intel Core(TM)2 Quad 2.40 GHz CPU and 4 GB RAM.

We tested the algorithms on an IEEE 30-bus system with six generators and thirty-one transmission lines. The system topology can be seen from Figure 4.1. We tested the two algorithms with six instances. The comparison of computational time of the two algorithms for the six instances is summarized in Table 4.4. In the first three instances, we omit the power flow constraints and use the system data summarized in Table 4.1. The three instances are generated by adjusting the upper bound and lower bound of uncertain demand and the upper bound of the sum of the 24-hour demands. For the next three instances, we include the power flow constraints and use the data summarized in Table 4.2 and Table 4.3. The three instances are generated in the same way as the first three instances. The convergence process of the two algorithms for the six instances are demonstrated in Figure 4.2-4.13.

From the computational results, we can see that our iterative trilevel optimization algorithm consistently terminates with less iterations than the Benders decomposition approach. In the

<table>
<thead>
<tr>
<th>Unit i</th>
<th>Bus m</th>
<th>$\lambda_i^m$</th>
<th>$C_{i,m}$</th>
<th>$PU_i^m$</th>
<th>$PL_i^m$</th>
<th>$TU_i^m$</th>
<th>$TD_i^m$</th>
<th>$RU_i^m$</th>
<th>$RD_i^m$</th>
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</thead>
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<td>30</td>
<td>2</td>
<td>2</td>
<td>50</td>
<td>50</td>
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<td>18.1</td>
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<td>2</td>
<td>50</td>
<td>50</td>
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<td>30</td>
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<td>4</td>
<td>4</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>G5</td>
<td>23</td>
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<td>17.8</td>
<td>60</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>G6</td>
<td>27</td>
<td>15</td>
<td>16.0</td>
<td>70</td>
<td>20</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>
Table 4.2 Generator Data 2 of The IEEE 30-bus System

<table>
<thead>
<tr>
<th>Unit</th>
<th>Bus</th>
<th>$m$</th>
<th>$C_{i,m}$</th>
<th>$PU_{i}^m$</th>
<th>$PL_{i}^m$</th>
<th>$TU_{i}^m$</th>
<th>$TD_{i}^m$</th>
<th>$RU_{i}^m$</th>
<th>$RD_{i}^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
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<td>13</td>
<td>19</td>
<td>2,400</td>
<td>600</td>
<td>3</td>
<td>2</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>G2</td>
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<td>19</td>
<td>18</td>
<td>2,000</td>
<td>500</td>
<td>2</td>
<td>3</td>
<td>500</td>
<td>500</td>
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<td>3</td>
<td>3</td>
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<td>300</td>
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<td>9</td>
<td>22</td>
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<td>4</td>
<td>4</td>
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<td>500</td>
<td>3</td>
<td>3</td>
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</tr>
</tbody>
</table>

The trilevel optimization algorithm, the lower bound and upper bound are updated with each iteration, closing the gap between them much faster. While in the Benders cutting plane framework, multiple cuts must be generated to cut off infeasible solutions and the lower and upper bound remain unchanged in many consecutive iterations. In addition to outperforming the Benders approach in terms of convergence time for most of the instances, the trilevel optimization algorithm can identify the real worst case cost. We also find the range of the uncertainty set has significant influence on the rate of termination on both algorithms.
### Table 4.3  Network Data of The IEEE 30-bus System

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<th>Line $q$</th>
<th>From</th>
<th>To</th>
<th>$X$ (p.u.)</th>
<th>$F^q$</th>
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<tr>
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</tr>
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<td>41</td>
<td>6</td>
<td>28</td>
<td>0.06</td>
<td>640</td>
</tr>
</tbody>
</table>
Table 4.4 Comparison of Computational Speed

<table>
<thead>
<tr>
<th>Instance</th>
<th>Convergence Time</th>
<th>Number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trilevel</td>
<td>Benders</td>
</tr>
<tr>
<td>1</td>
<td>9,200s</td>
<td>&gt;30,000s</td>
</tr>
<tr>
<td>2</td>
<td>1s</td>
<td>27,600s</td>
</tr>
<tr>
<td>3</td>
<td>1s</td>
<td>354s</td>
</tr>
<tr>
<td>4</td>
<td>12s</td>
<td>35s</td>
</tr>
<tr>
<td>5</td>
<td>20s</td>
<td>14s</td>
</tr>
<tr>
<td>6</td>
<td>22s</td>
<td>24s</td>
</tr>
</tbody>
</table>

Figure 4.1 IEEE 30-bus test system [1]
Figure 4.2 Convergence of Alg^{TMILP} for Instance 1
Figure 4.3 Convergence of the Benders approach for Instance 1
Figure 4.4 Convergence of Alg\textsuperscript{TMILP} for Instance 2
Figure 4.5  Convergence of the Benders approach for Instance 2
Figure 4.6 Convergence of Alg$^{\text{TMILP}}$ for Instance 3
Figure 4.7  Convergence of the Benders approach for Instance 3
Figure 4.8 Convergence of $\text{Alg}^{\text{TMILP}}$ for Instance 4
Figure 4.9 Convergence of the Benders approach for Instance 4
Figure 4.10 Convergence of Alg$^{\text{TMLP}}$ for Instance 5
Figure 4.11  Convergence of the Benders approach for Instance 5
Figure 4.12 Convergence of $\text{Alg}_{}^\text{TMLP}$ for Instance 6
Figure 4.13  Convergence of the Benders approach for Instance 6
CHAPTER 5. CONCLUSION AND DISCUSSION

5.1 Conclusion

In this paper, we reformulate the two-stage robust unit commitment problem as a trilevel optimization problem and present a novel iterative algorithm. In our algorithm, we decompose the trilevel problem into a master problem and sub problem, where the master problem identifies a commitment decision and the sub problem identifies the worst case cost under the current commitment decision. The information about the worst case uncertainty realization is then passed to the master problem and the commitment decision is modified accordingly. The master problem is solved as a mixed-integer programming problem and the sub-problem is solved as a linear program with complimentary constraints with the big-M method. In our computational experiments, we compared the performance of our algorithm with that of the Benders decomposition framework nested with outer approximation. In comparison, our algorithm has the ability to identify an exact optimal solution and can terminate after relatively small number of iterations.

In our numerical experiments, we only considered the uncertainty caused by load forecasts. However, uncertainty from other sources such as wind power can be easily incorporated into the model without affecting the algorithm. In addition, because the master problem does not depend on the dual variables of the sub-problem, integer decision variables such as the commitment of fast-starting units can be introduced to the sub-problem. To make this possible, a new bilevel optimization algorithm that does not rely on complementarity constraints should be developed.
5.2 Future Work

In our future work, we will focus on several directions. Firstly, we have not done much research on how to construct effective uncertainty sets to capture the probabilistic characteristics of uncertainty realizations. In our future study, we will spend more effort on the construction of uncertainty sets and their effects on the performance of the robust unit commitment model. In addition, other sources of uncertainty including renewable energy could be incorporated into the model.

Another very important direction for future work is improving the algorithm. The bilevel sub-problem needs a more efficient algorithm that does not depend on information of the dual variable so it can handle the mixed-integer case. In addition, heuristics can be developed to increase the speed of the algorithm.

Finally, we would like to obtain more data so we could conduct a more extensive case study. In our future work, we would like to compare the performance of the robust unit commitment model with that of the deterministic model currently employed by the industry via simulation. In that way, we can understand the impact of the robust unit commitment model more comprehensively.


