ABSTRACT

The scattering and reflection of ultrasonic pulses is the basis for ultrasonic non-destructive testing techniques. However problems are encountered in the interpretation of experimental results in many systems especially those which have target dimensions of the order of a wavelength, where there are no analytical solutions. It is shown that the lack of an adequate analytical theory can be overcome by the use of numerical models based on finite difference approximations of the basic elastic equations of motion. Finite difference methods, which have previously had a successful history in seismology, are introduced to study non-destructive testing problems and provide a complete description of the interactions of elastic waves, including mode-conversion as an intrinsic part of the formulations. A review is given of the types of problem, that are of interest to non-destructive testers and which finite difference methods are best suited to solve. Specific examples of the technique are shown applied to pulsed Rayleigh, compressional and shear wave scattering by such features as open slots. The descriptions given by the numerical models are confirmed by laboratory experiments on both aluminium and steel blocks. The prediction of a new family of mode-conversion techniques for crack detection and sizing, suggested by the numerical models, is confirmed by experimental results.

INTRODUCTION

Ultrasonic methods of non-destructive testing have been developed considerably in recent years, both in terms of the range of systems that are examined and the importance placed on the results that are obtained. The importance of pre-service and in-service inspection is going to increase in the future as the consequences of the failure of a system, such as a nuclear plant or an aircraft, increase in magnitude. However although considerable progress has been made in improving ultrasonic testing techniques, many problems remain, especially in the interpretation of experimental results. These problems are in part due to the lack of adequate theories for the elastic wave propagation and scattering processes involved.

If ultrasonic non-destructive testing methods are to become increasingly reliable and quantitative it is necessary to develop a well-founded theory for such problems as the interaction of pulsed waves with small targets, particularly those with dimensions of the order of a wavelength.\cite{1} The lack of adequate analytical theories for such problems has caused the attention of some workers to turn to consider numerical methods. It is found that numerical methods, which give full-wave descriptions of the interaction of elastic waves with structures that are of relevance to engineers, can be found in the seismological literature.\cite{2}

The activities within the research group at The City University fall into three related areas. Firstly there is the development of practical ultrasonic non-destructive testing techniques, in particular using short pulses and spectroscopy. Secondly, there is the development of instrumentation required for broad-band and spectroscopic studies, together with a transducer development programme which includes using visualisation techniques. Thirdly, there are the studies of wave propagation and scattering using numerical models supported by experimental measurements.

This paper is based on the work in the third area of study and considers finite difference methods and the application of such techniques to problems which form the basis for ultrasonic non-destructive testing.

BASIC EQUATIONS AND BOUNDARY CONDITIONS

The basic elastic equations for wave propagation and scattering have been presented by many authors, including Graff \cite{3}, so only a brief summary is included in this paper.

For a general heterogeneous, linear, isotropic perfectly elastic medium, an elastic wave can be described by the equation\cite{4}

\[
p\frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + \mu)\nabla (\nabla \cdot \mathbf{u}) + \mu \nabla \mathbf{u} + \nabla \times (\nabla \times \mathbf{u}) + 2(\nabla \cdot \nabla \mathbf{u})
\]

(1)

where $\mathbf{u}$ is the displacement, $\rho$ is the density, $t$ is the time and $\lambda$ and $\mu$ are the Lamé parameters of the medium.
With a restriction to two spatial dimensions and homogeneity, the basic equations which describe the displacements in the system are

\[
\begin{align*}
\frac{\partial^2 u_1}{\partial t^2} &= V_c^2 \frac{\partial^2 u_1}{\partial x_1^2} + V_s^2 \frac{\partial^2 u_1}{\partial x_2^2} + (V_c^2 - V_s^2) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \\
\frac{\partial^2 u_2}{\partial t^2} &= V_s^2 \frac{\partial^2 u_2}{\partial x_3^2} + V_c^2 \frac{\partial^2 u_2}{\partial x_2^2} + (V_c^2 - V_s^2) \frac{\partial^2 u_1}{\partial x_1 \partial x_3}
\end{align*}
\]  
(2)

where \(u_1\) and \(u_2\) are displacements parallel to the \(x_1\) and \(x_3\) axes respectively. \(V_c\) is the compressional wave velocity, \(V_s\) is the shear wave velocity.

The velocities are related to the Lamé parameters of a medium by the relations

\[
V_c = \sqrt{\frac{\lambda + 2\mu}{\rho}}; \quad V_s = \sqrt{\frac{\mu}{\rho}}
\]  
(3)

The boundary conditions for free surfaces and interfaces can be defined in terms of components of Cartesian stress tensor \(\tau\).

\[
\tau = \begin{bmatrix}
T_{11} \\
T_{13} \\
T_{33}
\end{bmatrix}
\]  
(4)

and

\[
\begin{align*}
T_{11} &= \frac{V_c^2}{c} \frac{\partial u_3}{\partial x_1} + \frac{(V_c^2 - 2V_s^2)}{s} \frac{\partial u_1}{\partial x_3} \\
T_{13} &= T_{31} = \frac{V_s^2}{s} \left( \frac{\partial u_3}{\partial x_3} + \frac{\partial u_1}{\partial x_1} \right) \\
T_{33} &= \frac{V_c^2}{c} \frac{\partial u_1}{\partial x_3} + \frac{(V_c^2 - 2V_s^2)}{s} \frac{\partial u_3}{\partial x_1}
\end{align*}
\]  
(5)

where for the cases of: (in two spatial dimensions) no bodily rotation, \(T_{13} = T_{31}\), stress free horizontal surface, \(T_{31} = T_{33} = 0\) and at the horizontal interface, \(u_1, u_2, T_{31}\) and \(T_{33}\) are all continuous.

**FINITE DIFFERENCE METHODS**

Finite difference methods replace the basic elastic equations which describe the system under study with an incremental approximation. Although the detailed difference forms used can vary, they give both the time development of the system and the full wave solution, in the spatial dimensions considered, as intrinsic parts of the formulation. They allow free surface and interface boundary conditions to be handled. They can also handle transients and pulses and from the view of a potential user, who is not a mathematician, they have a good history of successful applications to a range of hyperbolic problems. All these advantages has been demonstrated, in the late 1960's, by the work of the mathematical seismology group of the late Professor Alterman [5].

Two different approaches have been used for constructing finite-difference representations for the elastic equations. The most common approach is to treat the elastic medium as a collection of homogeneous regions, each characterized by constant values of density and elastic parameters. Motion in each region is described by a finite difference approximation to the elastic wave equation for a homogeneous region (equation 2) and the boundary conditions across all interfaces are satisfied explicitly [2].

An alternative approach, based on finite difference representations for the more general elastic equation 1, incorporates boundary conditions implicitly. This idea has been applied to a scalar wave propagation problem by Boore[6], by the association of different values of density and the elastic parameters with every grid point. Such a formulation permits the treatment of complex subsurface geometries. For this reason it has been used for a number of seismological problems [7]. If the density is assumed constant, equation 1 in rectangular coordinates has the same form as for the homogeneous case given as equation 2.

In the work, reported in this paper, by the author and his coworkers it is the former technique which has been used.[8,9]

A model using finite difference methods consists of four parts, which are the finite difference nodal formulations, the initial impulse, the artificial internal boundaries and the material data. Each of these parts of the problem is now considered in turn.

**FINITE DIFFERENCE FORMULATIONS**

The incremental forms of the basic elastic equations consist of combinations of displacements at two time levels and these are used to give the new displacements at a future time. The detailed mathematical technique used in this study has been described in detail elsewhere.[2,10] Here only the main features of the techniques are indicated.

Body nodes: The basic body node formulation, for two spatial dimensions, is obtained by the application of centred-difference forms to equation 2. The basic coordinate scheme used is shown in fig. 1.
The resulting nodal formulation obtained is:

\[ u(i,j,k + 1) = 2u(i,j,k) - (u(i,j,k - 1) + S^2(F(u))) \]  

where \( F(u) \) is an explicit function of displacements and velocities. However this formulation applies only to the body of the medium, so that at boundaries and interfaces special forms are required.

**Boundary nodes:** The boundary nodal forms can be either of first or second order type: The first order schemes use the concept of pseudo-nodes, a line of imaginary or fictitious nodes introduced outside the boundary of the materials as shown in fig.2.

The fictitious nodes allow the boundary conditions to be satisfied and the equations used for the horizontal free surface are obtained from the stress tensor components, equation 4. When centered-difference forms are used the resulting equation is

\[ u(i,j,1) + 1, k) = 2u(i,j + 1, k) + C(u(i + 1, j, k) - u(i - 1, j, k)) \]

where \( C \) is a constant, which for the vertical component of displacement has the form:

\[ \frac{(V^2_z - 2V^2_x)}{V^2_z} \]

The use of equation 7 to give the displacements at the pseudo-nodes enables the application of the body-node formulation, equation 6, to the node \( \text{A}(i,j,k) \), to give the time development of the node.

The second-order schemes have the time development as an intrinsic part of the formulation. These are obtained by direct solution of the basic equations subject to the relevant boundary conditions and use only nodes within the body of the medium or on the surface. The equation for a horizontal free surface has the form:

\[ u(i,j,k + 1) = 2A(i,j + 1,k) - u(i,j,k - 1) + S^2[B(u)] \]  

where \( A \) is a constant and \( B(u) \) is a function of displacements at time level \( k \) and velocities. \( B \) can take various forms, the original derivation of which was performed by Ilan, Ungar and Alterman[12].

**ACCURACY AND STABILITY OF FINITE DIFFERENCE SCHEMES**

The limits of accuracy and a stability for a finite difference scheme are of considerable importance, and they are found not only to be affected by the nodal formulation but also by the form of input pulse used and the material data.

For the body node formulation, equation 6, the limit to the size of the time increments is set by the von Neumann criterion[11].

\[ S \leq \frac{h}{\sqrt{V^2_z + V^2_x}} \]

where \( h \) is the spatial increment for a uniform grid.

The limit set by equation 9 is found for many schemes to be insufficient. The limits of stability for four sets of boundary node formulations have been investigated for the half and quarter spaces respectively by Ilan and Loewenthal [13] and Ilan [14]. The schemes have limits in the Poisson's Ratio value up to which they are stable, and for most schemes this is about \( \alpha = 0.4 \).

One parameter which is found to be of considerable importance is the number of nodes per wavelength. This is especially so when impulses or short pulses are used. Various workers [6,15] have shown that there is a lower limit of 7 nodes per wavelength at the shortest wavelength present and it has been found that a compressional wave impulse must be spread over 18 grid units if aliasing errors and numerical dispersion are not to affect the results.

The problems of scheme stability and accuracy have now been considered by a number of workers, and, although further work is required, working guidelines have been established. [10,15,16]
INPUT PULSES

The initial conditions for a model require the specification of the displacements at all grid points, for two time levels, separated by the time increment, as set by stability considerations.

Two techniques have been used to obtain pulses. The first, which was used for a pulse of Rayleigh waves is after Boore[6] and Munasinghe[16], and the sets of displacements are obtained by applying a fast Fourier transform to an analytically obtained wave number spectrum for each time level[8,10]. The basic type of pulse obtained is shown in fig.3.

![Fig. 3. Short pulse of Rayleigh waves.](image)

The second technique, which has been used for body wave impulses is obtained by a procedure adapted from Boore[6], and this computes a set of displacements from a Dirac delta function which is integrated five times (G_1 to G_5) and then five consecutive central finite differences of G_5 are taken over an arbitrary interval \( \Delta \). The function is then normalized to unity at \( \xi = 0 \). This gives a smoothed \( \delta \)-function.

\[
\langle \delta(\xi) \rangle = \begin{cases} 
G_5(\xi + 5\Delta) - 5G_5(\xi + 3\Delta) + 10G_5(\xi + \Delta) & \\
- 10G_5(\xi - \Delta) + 5G_5(\xi - 3\Delta) & \\
G_5(\xi - 5\Delta)/(230\Delta^4) & 
\end{cases}
\]

(10)

which has the shape shown in fig.4.[9].

The basic impulse, shown in fig. 4 has been used to give line compressional[9] and shear impulses, impulses of limited extent and as a cylindrical source.

![Fig.4. Shape of basic impulse.](image)

ARTIFICIAL GRID BOUNDARIES

In almost all models, irrespective of the size of computer available, as it is not possible to model a semi-infinite medium, artificial internal boundaries must be set at some distance from the region of special interest in the calculations. These boundaries can be considered in one of several ways. These include producing an absorbing nodal formulation, as is done in the finite element model by Lysmer and Drake[17]; keeping a larger iteration space[2]; by specifying that the internal boundaries have zero displacement; or by using symmetry considerations and/or an analytical solution to follow the development of the input pulse at the boundaries as is done by Ilan, Bond and Spivack[9].

There is no universally applicable procedure for the artificial boundary nodes. In the studies performed at The City University, all the procedures, except absorbing nodes, have been applied in various studies. In all studies it is necessary to be sure that the grid used is large enough to provide results that are not contaminated by artificial reflections, up to the times of interest.

For example when an impulsive line source is used on a semi-infinite half-space containing a slot[9] the artificial boundary nodal displacements were calculated using equations for the analytical solution on a half-space[18]. This solution is accurate until the first scattered pulse reaches a boundary node.

MATERIAL DATA

The requirements for the basic material data are that enough data should be given to enable the calculation of a consistent set of parameters, such as elastic moduli. In the studies at The City University the material data which are used are the shear wave velocity, the compressional wave velocity and the density. All other necessary parameters are calculated using relationships between elastic moduli and velocities[10].

![Material Data](image)
TYPES OF WAVE PROBLEM SUITABLE FOR SOLUTION USING FINITE DIFFERENCE METHODS.

Finite difference methods have proved themselves as a technique for solving specific types of wave-propagation problems in seismology and geophysics. It is important to understand what these are when applying them to problems of ultrasonic wave propagation and scattering for non-destructive testing. Problems which can be solved are restricted to the following types:

(a) Problems that can be reduced to two spatial dimensions and time.

(b) Pulsed-wave and transient-impulse problems in both solids and fluids for which basic equations of motion can be formulated.

(c) Problems for which a wave source can be completely specified at two times.

(d) Problems that can fit into grids, the size limits for which are set by the computer available, using about 10 nodes per wavelength, (practically, about 300 x 100 nodes for the system at The City University).

(e) Problems for which a basic data set of compressional wave velocity ($V_p$), shear wave velocity ($V_s$) and density ($\rho$) or equivalent elastic constants can be specified for all media involved.

(f) Problems for which the geometry can be reduced to smooth curves, or straight lines and their intersections, for the free surfaces and the material interfaces.

Rayleigh wave pulses on:-

- down step
- open slot

Compressional impulses on:-

- half-space
- open slot

Shear impulse on:-

- half-space
- open slot

Cylindrical line source on:-

- half-space
- plate

Fig.5. Examples of basic models considered at The City University.

The six constraints above may appear to be very restrictive: They are so in some senses, but the method can be applied to many problems of importance.

Constraint (a) provides for the reduction to systems with two spatial dimensions. This reduction results in two families of problems, one including the horizontal shear (SH) wave and the other the vertical shear (SV) wave. To solve systems requiring both SH and SV waves or mode conversion between the two families requires the full 3-D system which is only possible if a very large computing system is available. The studies at The City University have only considered the two dimensional problems involving shear (vertical), compressional and Rayleigh waves, and some examples of the basic configurations considered are shown in fig.5.

Other material properties such as internal friction[19] and piezoelectricity[20] have been added by some workers and some models have also been constructed using cylindrical and spherical coordinate systems, with restriction to reduce problems to two dimensions[2].

MODEL STUDIES

The basic finite difference model formulations are combined, subject to the basic model-initial conditions, to give a computer program for a basic configuration, with a particular type of input wave pulse.

Each computer program is written in Fortran and run on the University of London Computer Centre's CDC 7600 computer. The models require in excess of 32K of core and have run times between 20 and about 200 seconds. The exact requirements vary considerably, depending mainly on the size of the basic grids of nodes, the velocities of the main wave or waves of interest, the graphics output produced and the peripheral calculations performed. Models in which either the shear or the Rayleigh wave is under investigation require computer program run times about twice those required for the same configuration and grid size when studying compressional waves. The limits on the size of system which can be modelled are set only by the size of the computing facilities available.

Two types of system have been studied; these are firstly the near field systems and secondly the far field systems. Examples of the models which have been produced at The City University are now presented.

RAYLEIGH WAVE MODELS

The original finite difference models developed at The City University were used to study pulsed Rayleigh wave propagation and scattering. This work provided an opportunity for the collection and development of sets of boundary node formulations and an investigation of basic model accuracy and stability[10].

The problem of scattering by open slots was approached by first considering individual corners and their combinations into steps with depths of the order of a wavelength. This work is illustrated by a pulse at a down step shown in fig.6.
Rayleigh pulse at a half wavelength deep down step, using aluminium data and 35 nodes per wavelength. System after, (a) 20, and (b) 200 interations.

(c) Main pulse identification:
C Compressional wave, S shear wave
R_r Reflected and R_t Transmitted Rayleigh waves.

Fig. 6.

The energy in the various pulses generated is found to vary considerably as the depth of the step changes. The reflection coefficients were measured and this was compared with experimental results and is shown in Fig. 13.

LINE BODY WAVE SOURCES

The analytical solutions for the problems of line impulses of compressional and shear (SV) waves on semi-infinite half spaces are well known. These solutions provide a means of testing the accuracy of the finite difference solutions to the same problems. It is found that finite difference models can be produced which give solutions that are indistinguishable from the analytical cases.

The example of a line compressional impulse on a half space of material with Poisson's ratio of 0.25 is shown in Fig. 7. The angle of reflection for the mode converted shear wave given by the theory is 65° 54' and that given by the finite difference model is 66°, and the energies are also in good agreement.

The scattering of the impulse shown in fig. 4, in both compressional and shear form, by open slots was studied for slots with a range of widths and depths and with a range of angles of incidence (Fig. 8). The case of a compressional impulse at 45° angle of incidence is shown in Fig. 9. The amplitude and direction of individual modal displacements is clearly shown by the vector-visualisation which is after Harumi.[2].

Fig. 7. Line Compressional wave impulse incident on a half-space incident angle 45°.

Fig. 8. Basic model for body wave scattering by open slots.
Fig. 9. A line compressional impulse incident on an open slot, for the case of 45° angle of incidence.

For compressional wave scattering it is found that complex patterns of scattered and mode-converted pulses are generated, the energy is each mode being dependent on slot dimensions and material properties. For all angles of incidence between 0° and 90°, strong pulses of Rayleigh waves are seen to be generated, the wavelength of which appears to be related to the slot dimensions. [9]

The interaction of a line shear impulse with 80° angle of incidence and an open slot is shown in fig. 10. As for a compressional pulse it is found that a complex pattern of scattered and mode-converted pulses is generated.

In addition to the presentation of results in various forms of visualisation, synthetic time domain signals are generated. These provide results which can be directly compared to the time domain signals from transducers. Examples of this type of results for surface points is the model shown as fig. 10 and are shown as fig. 11.

A cylindrical line source of compressional waves based on the smoothed σ-function shown in fig. 4 is shown on a half-space by both numerical and vector visualisations in fig. 12. Both the vectors and grid show only every fourth node in the scheme.
COMPARISON BETWEEN NUMERICAL MODEL AND EXPERIMENT

In parallel with the production of the computer models a series of experimental measurements have been made to test the model results on simple configurations.

The experimental equipment used is the Central Ultrasonics Test Equipment (CUTE) of The City University Ultrasonics Group which includes a spectrum analyser[22], and is on line to a PDP 11/34 computer. In outline, the experiments consist of using broadband pulses (0.5 - 6 MHz) generated by piezoelectric transducers on both aluminium and steel blocks and looking at the reflected, transmitted and mode converted pulses resulting from interactions with corners, steps and slots.

The reflection coefficients from both models and experiments for a set of measurements made at down step are shown in fig.13, together with experimental results obtained by Frost et al [23]. The curve plotted for the model using data for media with Poisson's ratio = 0.24 is identical with that given by Munasinghe[16], who used a finite difference model to obtain his results.

![Reflection coefficients](image)

**Fig. 13.** Transmission and reflection coefficients at down steps, with depth (d) measured in wavelengths (λ).

Numerical and experimental results for the case of compressional impulses normally incident on an open slot[9], are shown in fig. 14.
Fig. 14 Compressional impulse normal to a slot. Vertical displacements below a slot given by: (a) the numerical model (b) experiments.

All the experimental measurements performed have provided results which are in good agreement with those given by the numerical models, subject only to limitations set by transducer coupling, surface roughness and pulse width (normal to direction of propagation).

NEW MODE-CONVERSION TECHNIQUES FOR NDT

The numerical model predicts that, when compressional waves are incident on a surface-breaking defect, they cause it to oscillate and generate Rayleigh waves. This is clearly visible in Figs. 9 and 15 and is strikingly shown in the computer-generated 16 mm film[25]. This type of mode conversion has been previously used for bulk-surface wave mode conversion in Acoustic Surface Wave devices[24], the fundamentals of which are shown in Fig. 15, together with comparisons between numerical-model and experimental results. The energy found in the mode-converted pulses measured by this technique is large, typically about 20 dB above noise levels when using normal commercial transducers on undamped blocks, this being increased to between 30 and 40 dB above noise levels when damping material is added around the test block and the maximum pulse used to excite the transducer.
The potential of the new technique was investigated by a set of time domain and spectral measurements performed on pulses scattered by slots 0.3 (±.02) mm wide and with a range of depths. An example of a time domain signal and its spectrum are shown in fig. 16a. For each slot studied the peak frequency in the mode converted Rayleigh wave was measured and it is shown plotted against slot depth as in fig. 16b.

$$\text{Slot Depth } \alpha \frac{1}{f^2} \quad (11)$$

where $f$ is the peak frequency in the spectrum of the slot generated Rayleigh wave. It would therefore appear to offer a new method for crack sizing, in addition to just detection. [26].

In general the observation of mode-converted pulses generated at defects would appear to form the basis for a new range of non-destructive testing techniques. The use of the crack-tip-generated shear wave has previously been investigated by Silk[27] and there are indications from the present studies that several of the different types of mode-converted pulses can be used for crack detection. It also appears that for the mode-converted pulses both pulse spectral content and their relative energy can be related to crack size.

**CONCLUSIONS**

The finite difference method provides a means of studying systems which have no analytical solutions. When used to follow ultrasonic wave propagation and scattering, it has given models which include mode conversions as an intrinsic part of the formulation, which have specific elastic properties, and which are free of coupling and other practical problems of laboratory models. These numerical models give both numerical and visual results which provide material for direct comparison with laboratory results.

In addition to providing insight into many scattering problems, a new family of crack detection methods have been discovered and it is shown that using compressional/Rayleigh mode conversion, slots can be sized by taking the spectrum of the slot-generated Rayleigh wave pulse generated by the slot.

**ACKNOWLEDGEMENTS**

This work forms part of a programme of research which seeks to provide an adequate theory for the understanding of ultrasonic non-destructive testing problems. It has been carried out with the support of the Procurement Executive, Ministry of Defence (U.K.), the Natural Environment Research Council, The Science Research Council, and the British Gas Corporation.

Thanks are due to all the members of the group who have been involved in this work, especially Dr. Aimoga Ilan (formerly of Tel-Aviv University), for her contribution as a colleague during her 15 month stay at The City University and also by correspondence.
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SUMMARY DISCUSSION
(Leonard Bond)

James Krumhansl (Session Chairman--Cornell University): I would like to start off the questioning. The frequency $F$ in your Quad $S$ versus one over $F$ squared, is that the center?

Leonard Bond: That's the center frequency.

Wolfgang Sachse (Cornell University): Have you made a comparison between this model or calculations to the exact solution, say the lambda problems or the epicenter knock-off problem?

Leonard Bond: The cylindrical epicenter on half space, yes, we have been comparing that. And where we have got a lambda solution, we do compare them. The results are in good agreement. So, the case of line sources on a half space, which is in fact how we solve the artificial boundaries for all the slots once they're in excellent agreement, the angles are the same and amplitudes are very close.

Gordon Kino (Stanford University): Do you have any feel from these solutions as to what the sharpness of the corners are? We, for instance, in experiments look at a corner and we look at Cornell's theory, which is an early forerunner of this, I think, and the results, therefore, are agreed, sometimes they do, sometimes they don't with the kinds of theories that people have already done, and we have always (inaudible) to that corner. Do you have any feel for this?

Leonard Bond: With the Rayleigh wave work, yes, we did start looking at the corners because we found on the 90-degree corner, if the radius of the corner was greater than twice the wavelength, the wave just didn't see it. And as you came in, there was a critical point where it became much sharper, and suddenly got to a deflection coefficient. For the 270 corner we had one set of steps that were cut, and we were getting very funny results. The pulse was just going around them. And so, yes, the corner sharpness does have a considerable effect. And that does come into the model because if you change the modes for wavelength and take it down too far, you start getting (inaudible).

Gordon Kino: Can the model really reproduce the corner well? Because it's a numerical procedure, this finite differences. What do you do about these sharp edges?

Leonard Bond: If you keep the nodes for the wavelength above 16 nodes per wavelength, it's okay because we have compared it with experiments, and we use both. We use mechanical modeling and lab experiments, the two working together.

James Krumhansl: May I supplement that question? Is there a technique in which you use a different grid, different mesh size in the vicinity of some artifact which you know to be critical, so you could use a much finer mesh size and map it onto the--I'm asking a question about--

Leonard Bond: You can change your grid size. It increases the complexity of the programming, and there's a group that has been using different grid sizes. We tended to stick with a very fast algorithm for uniform grid, the problem is every time you change grid size, unless you're very careful, you can introduce an artificial impulse at this artificial grid change.

J. D. Achenbach (Northwestern University): I'm sure every time you give these talks somebody brings up the finite element matter, and it would seem that in this connection the finite element might have some advantage in the sense you could construct a special element that would take into account singularities that appear at corners and at crack tips. Is that true, or?

Leonard Bond: Basically, you can do static problems either with finite elements and finite differences and your grid special elements are very useful. When it comes to this type of pulse problem, hyperbolic problem, some people have had some success with finite elements. As yet they really have got to prove themselves for general hyperbolic problems. You end up with a problem where you have to invert, albeit band spars matrices, and you got size limitations on the grid. In principle, you can probably get it out eventually using finite elements, but the algorithms are much more complicated.

(continued)
Leonard Bond (discussion continued)

George Herrmann (Stanford University): Have you been interested in problems where you have a physical boundary relatively far away from your (inaudible) and you're interested in the reflected wave? There is a problem there, obviously, that you cannot model the whole body because the computer capability is insufficient, and you have to try to match analytically a solution in the interested region, and it turns out it's rather difficult to do that because of artificial wave reflections at the interfaces.

Leonard Bond: It is difficult indeed to match the two together.

George Herrmann: Do you have any experience in that?

Leonard Bond: We tended to do it on (inaudible) where we have been comparing experiments, and we mainly use the finite difference for looking at local interactions. If you want to follow propagations in the body of the material, we follow the array theory besides the seismological ones.

# #