Fabrication and angle compensation analysis of skew rolling mill

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Fabrication and angle compensation analysis of skew rolling mill

by

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A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Mechanical Engineering

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Iowa State University
Ames, Iowa
2014

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NOMENCLATURE

$h$, thickness

$h_{max}$, maximum thickness

$h_0$, specimen initial thickness

$h_f$, specimen final thickness

$w$, specimen width

$R$, roller radius

$x$, unknown parameter in the equation

$S_p$, deformed area under parallel rolling

$S_s$, deformed area under skew rolling

$\mu$, coefficient of friction

$\theta$, crossed angle between the upper roller and bottom roller

$P$, pressure on the specimen

$R_a$, area ratio

$F_{xp}$, calculated force in $X$ axis direction in the center under parallel rolling

$F_{yp}$, calculated force in $Y$ axis direction in the center under parallel rolling

$F_{zp}$, calculated force in $Z$ axis direction in the center under parallel rolling

$F_{zd}$, recorded force in $Z$ axis direction at one end of the bottom roller by dynamometer

$F_{yd}$, recorded force in $Y$ axis direction at one end of the bottom roller by dynamometer

$F_{zd}$, recorded force in $Z$ axis direction at one end of the bottom roller by dynamometer

$F_{xs}$, calculated force in $X$ axis direction in the center under skew rolling

$F_{ys}$, calculated force in $Y$ axis direction in the center under skew rolling

$F_{zs}$, calculated force in $Z$ axis direction in the center under skew rolling
ACKNOWLEDGEMENTS

First of all, I would like to thank Dr. Gap-Yong Kim and Dr. Abhijit Chandra, my two advisors, for their guidance, patience, and especially, for offering me the opportunity to work on these projects. Second, I would like to thank my committee member, Dr. Kaitlin Bratlie, for her efforts and contributions to this work. I would also like to thank Wyman Martinek, Craig Severson, and Jim Dautremont in the department who provided significant help for this work. I want to offer my appreciation to my colleagues and my friends as well, especially Mina Bastwros. The work by Jack B. Lorch, Letian Yang, and Jianan Wu is greatly acknowledged.
The rolling process is an excellent method for processing new high performance polymers. Skew rolling, as a highly accurate and highly efficient rolling technology, has been commonly used to enhance mechanical properties and productivity. However, the complexity of the mechanics in the skew rolling process of polymers is a significant challenge for manufacturers and researchers. In this work, models for calculating the deformed area under parallel rolling and skew rolling have been developed. Based on the models, interactive force models in the rolling process have also been developed. Finite element analysis was used to simulate the rolling process. Experimental results were compared with the force model and simulation results. A study on angle compensation based on the influences of angle and friction conditions on force ratio $F_{xd}/F_{zd}$ in the skew rolling process was carried out. The results showed the accuracy of models and equations, and the work offered the possibility of angle compensation for the two polymers employed in this study.
CHAPTER 1. INTRODUCTION

1.1. Motivation

New rolling mills, such as the continuous variable crown (CVC) and pair cross (PC) [1-3], as well as work rolls crossing and shifting (RCS) [4] mills have been introduced and developed to improve the strip shape and profile. The PC mill was first used commercially at Nippon Steel’s Kimitsu Works in 1991 [1], and it has been commonly used in hot rolling mills for plating and stripping as a finishing mill. In the development of highly accurate and highly efficient rolling technology, the PC mill significantly improved the capability of the shape and crowns control, reduced the edge drop, and also enhanced mechanical properties and productivity [1]. This system has been applied not only to hot strip mills [5, 6] but also to plate mills and cold strip mills and the areas of application are still increasing. Research has focused on analyzing the rolling load or rolling torque generated by pair cross mills [7, 8], and on the strip profile control capability of the roll crossing mill[9], and so forth.

Polymers have increasingly been used to replace metallic parts in a wide range of applications over the past few decades. The rolling process is one of the best methods for processing new high performance polymers. The process allows for fabrication of more highly valued products than products produced by simple injection [10] and it can produce components more accurately and effectively [11]. The rolling process has been used for many other materials such as polypropylene (PP), polyethylene and polyoxymethylene due to the possibility of achieving high molecular orientation [12].

However, only a small amount of research literature was found on the skew rolling process of polymers. The complexity of the mechanics of deformation is a significant
challenge to the skew rolling process of polymers. The motivation of this research is to study the influence of crossed angle and friction conditions on the mechanics of polymers in the skew rolling process. In this work, a geometric model of deformed specimens will be built and forces will be analyzed. Methods to control the forces in the skew rolling process will be proposed based on the investigation of forces.

1.2. Research framework and objectives

The objective of this research is to study the influence of crossed angle and friction conditions on the force ratio $F_{xd}/F_{zd}$ in order to offer methods to control axial force. A simplified force model in parallel rolling was applied in order to compare the measured forces via dynamometer in order to determine the friction conditions for the two different materials. A force model in skew rolling was developed based on the friction conditions in the parallel rolling. Meanwhile, measured data were utilized to validate the models in skew rolling. With validated models, the influence of crossed angles and friction conditions on the force ratio $F_{xd}/F_{zd}$ was analyzed.

1.2.1. General design of a skew rolling mill

This section focuses on the design and construction of the skew rolling mill to produce a skew rolling system and to measure forces in three axes via a dynamometer, as well as to include the basic functions of traditional rolling mills. This section also describes the calibration process for the dynamometer in detail.
1.2.2. Experimental validation of force analysis

Forces in parallel rolling and skew rolling are fundamental characteristics in the rolling process. Forces characteristics in the rolling process affect the rolling stability, potentially causing concern for practical applications. This section simplifies and analyzes force models of parallel rolling and skew rolling. Forces were measured and experimental outcomes were compared to those models to provide validation.

1.2.3. Study of the angle compensation

Based on the validated models in skew rolling, the influence of a) rolling process parameters, b) the crossed angle, and c) friction conditions, on the force ratio $F_{xd}/F_{zd}$ were analyzed. This section captured the angles and friction conditions on the force ratio in order to demonstrate the feasibility of compensating angles in order to minimize the axial forces on bearings.

1.3. Dissertation organization

The remainder of this dissertation is divided into five chapters. In Chapter 2, a literature survey with regard to rolls, skew rolling, and friction in the rolling process is conducted. In Chapter 3, details of the skew rolling mill are illustrated. Design concepts to realize required functions are described in detail. In Chapter 4, force models of parallel rolling and skew rolling were built and compared with experimental behaviors. In Chapter 5, the influence of angles and friction conditions on the force ratio is studied and the angle
compensation is analyzed based on the results. A summary and conclusion are presented in Chapter 6.
CHAPTER 2. LITERATURE SURVEY

2.1. Rolling

Rolling is a process of reducing the thickness or changing the cross section of long workpieces through compressive forces applied by a set of rolls; thus the process is similar to rolling dough with a rolling pin to reduce the dough’s thickness. Rolling is widely used due to its high rate of production and the accurate control of final product [13]. Rolling is used to produce slabs, sheets, strips, and foils with a dense attractive surface finish and strengthened mechanical properties. Rolling processes can be divided into hot rolling process and cold rolling process, based on the rolling temperature.

Hot rolling is a metalworking process that occurs above the recrystallization temperature of the material. After the grains deform during the rolling process, they recrystallize and prevent the metal from hardening while being worked. Hot rolled metals have little directionality in general in terms of their mechanical properties; their residual stresses are induced by deformation. But in certain instances non-metallic inclusions would impart a certain amount of directionality, and workpieces less than 20 mm thick often have some directional properties. Also, a lot of residual stresses to those shapes that have a non-uniform cross-section are induced by non-uniform cooling. While the finished product is of good quality, the surface is covered in mill scale, which is the anoxide that forms at high temperatures. It is usually removed through pickling or by a smooth clean surface process, which reveals a smooth surface [14]. Dimensional tolerances are usually two to five percent of the overall dimension [14].
Cold rolling is a metalworking process that occurs when the metal’s temperature is below its recrystallization temperature (usually at room temperature); this process increases the strength by strain hardening. This process also improves the surface’s finish. Products commonly produced by cold rolling include sheets, bars, strips, and rods; these products are usually smaller than products that are produced by hot rolling. Due to the smaller size of the workpieces and their greater strength compared to hot rolled stock, four-high or cluster mills are used [14]. It is worth noting that in a single pass, cold rolling cannot reduce the thickness of a workpiece as much as hot rolling can.

Other shapes can be cold-rolled if the cross-section is relatively uniform and the transverse dimension is relatively small. Cold rolling shapes requires a series of shaping operations, usually along the lines of sizing, breakdown, roughing, semi-roughing, semi-finishing, and finishing.

Although about 90% of all metals produced by the metalworking processes are produced through a rolling process [15], the rolling process has also been used for many other materials including polypropylene (PP), polyethylene and polyoxymethylene due to the possibility of achieving a high level of molecular orientation [12]. Rolling is an excellent method for processing new high performance polymers. It can be used to manufacture products that are more highly valued than products produced by simple injection [10]. The rotary forming process has attracted significant interest in recent years because this process can produce components more accurately and effectively [11]. Polymers have increasingly been used to replace metallic parts in a wide range of applications over the past few decades. Polymers have been used in rolling processes to produce materials with various performances characteristics [16].
Ring rolling is a special forging process that has demonstrated the feasibility of cold processing of certain polymers [11]. Research on polymers subjected to cold rolling has been done extensively: plastic deformation and morphology change of crystalline polymer materials by rolling processes were investigated by J. Qiu et al. [10, 12]. Amorphous transitions in glassy polymers subjected to cold rolling was studied by D. Cangialosi et al. [17]. Elastic-recovery properties and dimensional changes followed by the cold rolling of rods of thermoplastics of various kinds were studied [18]. Fatigue crack growth in polycarbonate (PC) and polyvinylchloride (PVC) have been studied using anisotropic sheets oriented by cold rolling [19]. Deformation of semicrystalline polymericics subjected to cold rolling were studied by T. Asano and Y. Fujiwara [20]. The tensile behavior of high density polyethylene subjected to cold rolling was studied by R. M. Caddell et al. [21]. The effects of cold rolling on crazing of polycarbonate was studied by G. O. Shonaike and P. E. Reed [22].

2.2. Friction in rolling

Friction is due to the interactions between the opposite asperities of the two sliding surfaces. From the interactions, friction is divided into two categories: adhesion and deformation [23]. Adhesion is the characteristic that some asperities of the contact surfaces will adhere when there is load in the two objects. There would be a junction between the two objects that will impede the motion of the two objects. Friction is generated as a result of the relative motion of one object over another object. The second kind is deformation in the macroscopic interaction. The deformation will cause the harder surface to plough grooves in the softer surface.
K. L. Johnson studied the friction in rolling contact in detail [24]. In Johnson’s opinion, all the motions between two contact bodies can be divided into sliding, rolling, and spinning. The resultant forces can always be resolved into a normal force, which acts in the direction of the normal on the initial contact point of the two contact bodies, and a tangential force, which acts on the tangential plane. If the normal force is zero, the motion is known as free rolling, while the other is called tractive rolling. In free rolling, the contact stress and deformation can be calculated using Hertz theory [25].

F. W. Carter [26], H. Poritsky [27], K. L. Johnson [24] and R. D. Arnell [28] have studied the tractive rolling of elastic cylinders in detail. In the contact zone of tractive rolling, there is a central “stick” area and two outer “slip” areas. This is possible because of the deformable nature of materials. Thus, if the normal force is constant while the tangential force is increased from zero, microslip occurs at the edges of the contact zone and spreads inward as the tangential force increases; when the tangential force reaches the limit value, the two zones of slip meet in the center and gross sliding begins.

Rolling resistance also comes from the adhesive and deformation losses even though the magnitude is smaller comparatively. Deformation loss (as opposed to adhesive loss) is the major loss in the resistance of free rolling. In tractive rolling, deformation loss is also the dominant factor of resistance.

2.3. Skew rolling

The skew rolling process, evolved from the transverse rolling process, is a unique rolling process that produces near-net-shape cylindrical or annular metal components and the
process has the ability to contour the outside diameter. The name “skew rolling” reveals one of the main characteristics of this process: that the axes of the rolls are arranged obliquely to each other with a small angle [29]. Fig. 2.1 shows an image of the skew rolling with grooved rollers. Worth mentioning here is that in our setup, the cylindrical rollers are plain surfaces without grooves.

![Fig. 2.1 Skew rolling mill with groves](image)

Skew rolling is a new technology for producing various shafts. Compared to traditional casting, forging, and machining, this process has many remarkable advantages in productivity, material consumption, overall mechanical properties and service life [30]. Various kinds of products ranging from balls, rollers, stepped shafts, to anchor rods with threads are being manufactured by skew rolling [30].

However, the complexity of the forming process is a notable challenge for all manufacturers when developing skew rolling processes. Theories that explaining the mechanics and tool workpiece interactions are mainly based on simplified assumptions and empirical results. Numerous variables in the process are another challenge the manufacturers face.
To tackle those challenges, the most powerful and most widely employed tool is the finite element method. The finite element method is widely used to characterize the workpiece material stress, strain, and deformation behavior [31]. Another method is to analyze the skew rolling method through mathematical models derived and simulated based on envelope theories and assumptions [32]. To design the parameters in the skew rolling process, mathematical algorithms are also widely employed to optimize the design process [33].
CHAPTER 3. GENERAL DESIGN OF A SKEW ROLLING MILL

3.1. General design

According to the aim of the experiments for cold skew rolling, the rolling mill should have the following abilities:

1. The ability to roll the specimens at an adjustable speed;
2. The ability to change the gap between two rollers in the range of 1mm-5mm;
3. The ability to skew the rollers along a range of angles from $-15^\circ$-$15^\circ$;
4. The ability to measure the forces along three axes.

3.2. Design of the power source with speed adjustment

To provide power for the rolling mill, a motor (Marathon electric) and gearbox (Baldor Electric Co.) are applied and connected by a coupler (Lovejoy) to the roller shaft as seen in Fig. 3.1. Power is transmitted through the coupler to the bottom roller which makes the bottom roller the drive roller. When specimens are inserted between the two rollers, the top roller turns due to the friction between the specimens and the top roller; this makes the top roller the driven roller.
To adjust the rotation speed, a variable frequency drive (VFD) (Toshiba, VF-S15) is employed. By connecting this VFD to the motor, the rotation speed of the rollers can be controlled. Rotational speed adjustment is the only function that is required for the current research, although this VFD offers a variety of functions which can be explored. Details of the VFD can be seen in Fig. 3.2.
3.3. **Design to change reduction ratios**

The gap between the two rollers can be adjusted by various methods. Since skewness is one of the main functions of this setup, common designs to adjust the gap cannot be applied here. In our setup, slotted shims with various thicknesses were inserted underneath the screws of the top roller. Various gaps of the two rollers can be achieved by alternating the number of shims along with various thicknesses of shims, thus raising the height of top roller.
By changing the gap, the reduction ratios can be while the specimens’ thicknesses are held constant. Fig. 3.3 shows the shape of shims used in the setup.

![Shims used in the setup to adjust gap between two rollers](image)

**Fig. 3.3** Shims used in the setup to adjust gap between two rollers

### 3.4. Design to realize the skewness function of the setup

The most common rolling mills are parallel rolling mills, but in this study the top roller can have an angle positioned up to ±15° oblique to the bottom roller. The power supply for the top roller could be difficult given this fact. To achieve the goal of the skew roller, the upper plates that support the top rollers at both ends are made with arc slots that create the angles that can be seen in Fig. 3.4. The angle is adjusted by loosening the screw; moving the pillow blocks along the arc slots to a desired angle; and retightening the screw.
Fig. 3.4 Solidworks drawing for the upper plate to achieve required angle

3.5. Design to acquire forces

3.5.1. Dynamometer

To acquire forces in the rolling process, a piezo-dynamometer (Kistler 9257B) has been installed underneath the pillow block at one end, close to the motor of the bottom roller, as can be seen from Fig. 3.1. The dynamometer is designed to have compactness, great rigidity, and high natural frequency. The dynamometer setup is corrosion-resistant and is protected against of spray water and cutting fluids. Forces of three axes can be measured by this dynamometer with the specifications in Table 3.1.
Table 3.1 Specifications of the dynamometer employed in the setup

<table>
<thead>
<tr>
<th></th>
<th>X axis</th>
<th>Y axis</th>
<th>Z axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Range (kN)</td>
<td>-5-5</td>
<td>-5-5</td>
<td>-5-10</td>
</tr>
<tr>
<td>Sensitivity (pC/N)</td>
<td>7.94</td>
<td>7.93</td>
<td>3.71</td>
</tr>
</tbody>
</table>

Sensitivities of the three axes are listed, and these will be applied to calibrate the dynamometer. Ranges of the three axes were seen to be wide in the later experiments.

3.5.2. Data acquisition interface

An amplifier was employed to amplify the dynamometer’s signal. Physical data were collected through the data acquisition card inserted in the computer. A LabVIEW program was created to record data and for the researchers to operate. Fig. 3.5 shows the force measurement system in this setup.

Fig. 3.5 Force measurement setup flowchart in the rolling mill

Fig. 3.6 and Fig. 3.7 present the LabVIEW interfaces that are designed to record forces.
Fig. 3.6 LabVIEW interface front panel. The white, red, greed curves stand for X, Y, and Z forces.

Fig. 3.7 Block diagram behind front panel of the LabVIEW program.
The white, red, and green curves in the three windows from top to bottom in Fig. 3.6 are the real-time forces of the X, Y, and Z axes. The curves can be exported to other files for further investigation. Fig. 3.7 shows the block diagram behind the front panel to portray how the physical signals are converted to forces. Signals from the data acquisition card are read by this program and are converted to forces based on the settings of the amplifier. Three channels of force data are illustrated by waveform charts.

3.5.3. Dynamometer calibration

To obtain accurate force data, the dynamometer needs to be calibrated. The common method for calibration is to use standard weights in three axes. The standard weights are compared with the readings from the dynamometer to find the adjustment factor. Due to space limitation, two axes in this rolling setup could not be accomplished. The amplifier in this system, however, offers a second way to calibrate.

Fig. 3.8 Dynamometer shows the sensitivities and amplification factors of three axes
As shown in Fig. 3.5, the dynamometer is connected to an amplifier that is employed to enhance the ability to acquire data. To calibrate the dynamometer, the sensitivities of three axes from Table 3.1 are applied in the first row of amplifier. To meet various ranges of the three axes, amplification factors in three axes are adjusted both in the amplifier and in the block diagram. Values of amplification factors in the amplifier should remain consistent with values in the equations in order to calculate forces in three axes in the block diagram. Details can be seen in Fig. 3.8. As the figure shows, the three values of the first row are the sensitivities of the X, Y, and Z axes. The second row of the amplifier is the amplification factors of three axes. Values are consistent from Table 3.1, Fig. 3.7, and Fig. 3.8. After the calibration, standard weights were placed on the dynamometer to validate the whole procedure and the results showed that the calibration was effective and successful.

CHAPTER 4. FORCE ANALYSIS AND GEOMETRY MODELS IN THE ROLLING PROCESS

4.1. Force analysis and geometry model of parallel rolling

Rolling is the process of reducing the initial thickness by compressive forces that are applied by rolls. Parallel rolling is a process in which the axes of two rollers are parallel to each other. Fig. 4.1 shows the schematic deformation of the specimens in between the two rollers. $h_0$ is the thickness of the specimen. $h_f$ stands for the final thickness, which is equal to the gap between two rollers. $w$ is the width of the specimen. $R$ is the radius of rollers. The two rollers in our setup have the same radius.
Fig. 4.1 Deformation scheme in parallel rolling process

The reduction is relatively small compared to the radius of the roller; the deformed area $S_p$ in XY plane seen in Fig. 4.2 can be calculated as:

$$S_p = w \sqrt{R(h_0 - h_f)}$$  \hspace{1cm} (1)
The force in Z axis $F_{zp}$ that is due to the reduction can be easily calculated as the product of the pressure $P$ in the Z axis and the deformed area in the XY plane.

$$F_{zp} = P \times S_p$$  \hfill (2)

The force in the Y axis $F_{yp}$ can be represented as the product of the coefficient of friction $\mu$ and the force in the Z axis.

$$F_{yp} = \mu \times F_{zp}$$  \hfill (3)

In the parallel rolling process, there is no component force in the X axis.

Fig. 4.3 shows the force analysis for the bottom roller. For simplicity, the pressure distribution is assumed to be uniform along the width of the roller. The dynamometer is placed underneath one of the two ends of the bottom roller as seen from Fig. 3.1.

---

Fig. 4.3 Forces analysis on bottom roller in parallel rolling process
$F_{zd}$ and $F_{yd}$ are the forces measured by the dynamometer, and they are shown in the LabVIEW program. From the forces scheme on the bottom roller shown in Fig. 4.3, it would be straightforward to find the equation (4) (5) (6) (7) due to the symmetry:

\[
F_{zd} = \frac{1}{2} F_{zp} \quad (4)
\]
\[
F_{zd} = \frac{1}{2} PW \sqrt{R(h_0 - h_f)} \quad (5)
\]
\[
F_{yd} = \frac{1}{2} F_{yp} \quad (6)
\]
\[
F_{yd} = \frac{1}{2} \mu PW \sqrt{R(h_0 - h_f)} \quad (7)
\]

In a later study, the forces shown on the LabVIEW program are compared with the forces calculated following these relations above.

### 4.2. Force analysis and geometry model of skew rolling

The top roller can be rotated to have an angle $\theta$ ranging from $-15^\circ$ to $15^\circ$ obliquely to the bottom roller through the angle guidance. A specimen is positioned and rolled as shown in Fig. 4.4. $x$ is the contact width of which the value should be no greater than the specimen width $w$. 
Fig. 4.4 Specimen in between the two rollers, the red rectangle is the maximum area it can deform, the arc shaped area is an approximate area assumed.

From the Fig. 4.4, it is simple to determine the maximum deformed area, which is shown as a rectangle $S_x$:

$$S_x = \tan\left(\frac{\theta}{2}\right) x^2$$ \hspace{1cm} (8)

The edges of the red rectangles would not be in contact with either roller in the actual experiments. The actual contact area would be more accurately represented as an arc shaped area. Assume the curves are parts of circles; and then the arc shaped area would be:

$$S_a = \tan\left(\frac{\theta}{2}\right) x^2 - \frac{x^2 \theta}{4 \sin^2\left(\frac{\theta}{2}\right)} + \frac{x^2}{2 \tan\left(\frac{\theta}{2}\right)} - \frac{\tan^2\left(\frac{\theta}{2}\right) x^2 (\pi - \theta)}{4 \cos^2\left(\frac{\theta}{2}\right)} + \frac{x^2 \tan^3\left(\frac{\theta}{2}\right)}{2}$$ \hspace{1cm} (9)

The area ratio $R_a$ is defined as:

$$R_a = \frac{S_x}{S_a}$$ \hspace{1cm} (10)
Due to various materials properties, deformation would be different when different materials were used. Consequently, deformed areas would be different even with the same reduction ratio. An area factor would have to be introduced to calculate the real deformation area.

The contact curve from the center point to the edge of the specimen between the bottom roller and the lower surface of the specimen can be simplified as seen in Fig. 4.5:

![Diagram](image.png)

Fig. 4.5 Specimen in contact with the bottom roller surface. Contact curve is represented as the ellipse dot line
Points on the ellipse curve should fall into the equation (11):

\[
\left( \frac{x}{2 \cos \left( \frac{\theta}{2} \right)} \right)^2 + \left( \frac{R - h + h_f}{R} \right)^2 = 1
\]  

(11)

\( h \) is the distance from any point on the curve to the bottom line of the top roller, as can be seen in Fig. 4.5.

To calculate the area \( S_x \), \( x \) needs to be determined. Firstly, let \( x \) equal \( w \), the specimen width, to obtain the maximum height \( h_{\text{max}} \). Compare the specimen thickness \( h_0 \) with \( h_{\text{max}} \). If \( h_0 \) is smaller than \( h_{\text{max}} \), it indicates that part of the specimen is not in contact with the bottom roller’s surface. The value \( x \) can be obtained by solving the equation (11), under \( h = h_0 \). If \( h_0 \) is larger than \( h_{\text{max}} \), it indicates that even the edge of the specimen is deformed. The contact width is \( w \) while the thickness of the specimen at the edge is not \( h_0 \), but \( h_{\text{max}} \).

Due to the skewness of the rollers, there exists a component force in the X axis in the bottom roller; and this is a distinct difference compared to parallel rolling.
As shown in Fig. 4.6, $F_{zs}$ is the pressure force on bottom roller that is due to the reduction. $F_{zs}$ can be simply represented by the product of the pressure $P$ in the $Z$ axis and the deformed area in XY plane.

$$F_{zs} = P \times S_a$$  \hspace{1cm} (12)$$

$F_{ys}$ is the component friction force caused by the pressure.

$$F_{ys} = \mu F_{zs} \cos \left( \frac{\theta}{2} \right)$$ \hspace{1cm} (13)$$

$F_{xs}$ is also the component friction force caused by the pressure.
\[ F_{zs} = \mu F_{zs} \sin\left(\frac{\theta}{2}\right) \]  

(14)

\( F_{zd} \) and \( F_{yd} \) are the forces measured by the dynamometer which are shown in the LabVIEW program. From the forces scheme on the bottom roller shown in Fig. 4.3, it would be straightforward to find the equation (15) (17) due to the symmetry:

\[ F_{zd} = \frac{1}{2} F_{zs} \]  

(15)

\[ F_{zd} = \frac{1}{2} \mu p \left[ \tan\left(\frac{\theta}{2}\right) x^2 - \frac{x^2 \tan^2 \left(\frac{\theta}{2}\right)}{2 \tan \left(\frac{\theta}{2}\right)} \right] \]  

(16)

\[ F_{yd} = \frac{1}{2} F_{ys} \]  

(17)

\[ F_{yd} = \frac{1}{2} \mu p \cos\left(\frac{\theta}{2}\right) \tan\left(\frac{\theta}{2}\right) x^2 - \frac{x^2 \tan^3 \left(\frac{\theta}{2}\right)}{2 \tan \left(\frac{\theta}{2}\right)} \]  

(18)

However, due to the over constraint of bottom roller in the X axis direction, the relationship between \( F_{zd} \) and \( F_{zs} \) is not determined.
4.3. Experimental validation of force analysis

4.3.1. Experimental data, calculations and simulations of parallel rolling

To validate the analyzed forces, some experiments were conducted. Two materials, PETG (polyethylene terephthalate-glycol) and HDPE (high-density polyethylene) were chosen to be rolled with different reductions. A finite element analysis (FEA) simulation was also performed to obtain the resultant forces. The drawing in Abaqus was based on the original Solidworks files for fabrication. The top roller and bottom roller were set as analytic rigid parts in order to save simulation time. For later experiments, only plastics were inserted in between the two rollers. Because of this, neglecting the deformation of the rollers would not introduce significant error.

![Fig. 4.7 Rolling model created in Abaqus](image-url)
The position of rollers was locked. All displacements and rotations were set to zero except the rotation about the X axis in Fig. 4.7. The specimen was initially positioned in between the two rollers and an angular velocity of the bottom roller was introduced when the specimen started being rolled. Properties of the specimens followed the two curves in Fig. 4.10 and Fig. 4.11. Resultant forces were obtained after the rolling.

<table>
<thead>
<tr>
<th></th>
<th>Final thickness, $h_f$ (mm)</th>
<th>Thickness, $h_0$ (mm)</th>
<th>Reduction, $h_0-h_f$ (mm)</th>
<th>Width, w (mm)</th>
<th>Average pressure, $P$ (MPa)</th>
<th>Roller radius, R (mm)</th>
<th>Coefficient of friction, $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE-1</td>
<td>1.0922</td>
<td>1.5241</td>
<td>0.4319</td>
<td>50.8</td>
<td>23.4</td>
<td>69.85</td>
<td>0.1</td>
</tr>
<tr>
<td>HDPE-2</td>
<td>2.286</td>
<td>3.048</td>
<td>0.762</td>
<td>50.8</td>
<td>23.4</td>
<td>69.85</td>
<td>0.1</td>
</tr>
<tr>
<td>PETG-1</td>
<td>1.27</td>
<td>1.397</td>
<td>0.127</td>
<td>50.8</td>
<td>52</td>
<td>69.85</td>
<td>0.05</td>
</tr>
<tr>
<td>PETG-2</td>
<td>2.54</td>
<td>2.794</td>
<td>0.254</td>
<td>50.8</td>
<td>52</td>
<td>69.85</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Fig. 4.8 and Fig. 4.9 show the forces measured by the dynamometer. Due to the unbalanced weight of the roller itself, there are some fluctuations when there were no specimens in between the rollers. Compared to the measured values, the error caused by the imbalance can either be deducted from the total values or it can be neglected due to its small magnitude.

The curves in Fig. 4.8 and Fig. 4.9 are not flat when the specimen is being rolled and measured. One reason might be small vibrations since the rollers are not perfectly rigid; the second reason might be the specimens didn’t move straight which made the deformed area
varying in the rolling process. The average values of the curves when the specimens were being rolled were recorded on the table.

Fig. 4.8 Measured Y axis force under parallel rolling with different reductions

Fig. 4.9 Measured Z axis force under parallel rolling with different reductions
Forces of HDPE-2 and PETG-2 were much larger than forces of HDPE-1 and PETG-1 both in the Z axis and the Y axis directions. From equation (3), it can be seen that the Y axis force is proportional to the Z axis forces. The thickness of HDPE-2 and PETG-2 was reduced more, leading to more deformation and more force. Fig. 4.8 and Fig. 4.9 show the forces in the Z axis have increased approximately by 2000 N for both materials. Due to different frictional conditions for two materials, the forces increase in the Y axis direction by different amounts.

To calculate the force by applying equations (5) and (7), two more parameters need to be fully understood: The average pressure and the coefficient of friction. The coefficient of friction varies greatly with different conditions. It would not be accurate enough to find the values from literature since it would be difficult to find exactly the same conditions in other operations. Due to this limitation, it’s reasonable to assign values and validate the assigned values using other conditions. In our setup, the coefficients of friction for PETG and HDPE in contact with the low carbon steel roller are assumed to be 0.05 and 0.1 respectively.

The rolling is a process under plane stress. Compressive stress strain curves were found in the literature to offer the deformation information for the current experiments. The following two figures Fig. 4.10 and Fig. 4.11 from Moura et al. [34], as well as Dupaix and Boyce [35] are applied to show the compressive stress strain relation of HDPE and PETG.
Fig. 4.10 Compressive stress strain curve for HDPE, green curve in the graph is applied.

Fig. 4.11 Compressive stress strain curve for PETG, 0.5/s plane in the graph is applied.

The average pressure is obtained by calculating the strains of the deformed specimen and the corresponding stress from those curves. The average pressure is the average Y axis value in Fig. 4.10 and Fig. 4.11. All of the pressures are listed on the table. After the forces equations in the parallel rolling are determined, $F_{yp}$ and $F_{zp}$ can be calculated.
Table 4.2 Comparison between the measured forces and the calculated forces in parallel rolling

<table>
<thead>
<tr>
<th></th>
<th>Experiment data</th>
<th>Calculation data</th>
<th>Simulation data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_{yd}$ (N)</td>
<td>Error (N)</td>
<td>$F_{yd}$ (N)</td>
</tr>
<tr>
<td>HDPE-1</td>
<td>320</td>
<td>29.7</td>
<td>3815</td>
</tr>
<tr>
<td>HDPE-2</td>
<td>500</td>
<td>71.2</td>
<td>5600</td>
</tr>
<tr>
<td>PETG-1</td>
<td>175</td>
<td>29.2</td>
<td>3900</td>
</tr>
<tr>
<td>PETG-2</td>
<td>281</td>
<td>42.5</td>
<td>6238</td>
</tr>
</tbody>
</table>

The calculated values are plotted in Fig. 4.12 and Fig. 4.13. As can be seen, the calculated forces are close to the measured values either in the Y axis or the Z axis. Experimental and simulation data are close to each other in the Z axis forces in Fig. 4.13. However, calculation data are generally smaller. The calculation assumes the average pressure in the deformed area, which may result in smaller values. For the comparison in Fig. 4.12, the simulation is smaller than that of the experiment data of HDPE-2 while the calculation falls into the range of the experimental data. The simulation of HDPE-2 is smaller than the experiment data and this can be explained by the fact that the theoretical friction condition in the simulation may not accurately represent the real experiment. The friction in actual situation for HDPE might be higher than an assumed single value in the simulation. For the PETG in Fig. 4.12, the experimental data and calculation data are close, while the simulation data is larger. Deflection is the main reason behind this phenomenon. Since the rollers are considered to be rigid in the simulation, simulation data are usually larger than experimental data. Since the PETG is harder than HDPE, the deflection would be more evident for PETG rather than for HDPE. As a result, the friction assumed in the simulation
factor dominated the difference rather than the rigidity of HDPE. Consequently, the difference between the simulation and experiment of PETG would be more distinct than the difference between the simulation and experiment for HDPE. In the figure, it can be seen the Z axis force of PETG-2 is higher than the experimental force, which leads to a higher force in the Y axis.

Overall, the results shown in the figures can lead to the conclusion that the assumed coefficient of friction can correctly represent the actual friction conditions in the rolling setup, since different reductions were carried out for two materials in which experimental data, calculated data, and simulated data precisely met each other.

The validation of the coefficients of friction for two different materials in turn proved the correctness of the equations and the assumptions behind those equations.

Fig. 4.12 Comparison among the measured forces, the calculated forces and simulated forces in Y axis in parallel rolling
4.3.2. Experimental data, calculations and simulations of skew rolling process

After the validation of the parallel rolling, the coefficients of friction of both materials are demonstrated to be accurate. In the skew rolling, the coefficients of friction are assumed to be the same and known.

In the skew rolling, experiments were conducted under three different angles between two rollers: 15°, 10°, and 6°. Other parameter settings can be seen from Table 4.3.
Table 4.3 Experiment setting of skew rolling process with PETG and HDPE

<table>
<thead>
<tr>
<th>Trials</th>
<th>PETG-15°</th>
<th>PETG-10°</th>
<th>PETG-6°</th>
<th>HDPE-15°</th>
<th>HDPE-10°</th>
<th>HDPE-6°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final thickness, $h_f$ (mm)</td>
<td>1.143</td>
<td>1.143</td>
<td>1.143</td>
<td>1.143</td>
<td>1.143</td>
<td>1.143</td>
</tr>
<tr>
<td>Thickness at the edge, $h$ (mm)</td>
<td>1.397</td>
<td>1.2827</td>
<td>1.1938</td>
<td>1.4968</td>
<td>1.2827</td>
<td>1.1938</td>
</tr>
<tr>
<td>Contact width, $x$ (mm)</td>
<td>42.672</td>
<td>50.8</td>
<td>50.8</td>
<td>50.8</td>
<td>50.8</td>
<td>50.8</td>
</tr>
<tr>
<td>Average pressure, $P$ (MPa)</td>
<td>58</td>
<td>66</td>
<td>62</td>
<td>22</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Roller radius, $R$ (mm)</td>
<td>69.85</td>
<td>69.85</td>
<td>69.85</td>
<td>69.85</td>
<td>69.85</td>
<td>69.85</td>
</tr>
<tr>
<td>Coefficient of friction, $\mu$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum deformed area, $S_x$ (mm$^2$)</td>
<td>239.73</td>
<td>225.78</td>
<td>135.25</td>
<td>339.75</td>
<td>225.78</td>
<td>135.25</td>
</tr>
<tr>
<td>Arc shaped area, $S_a$ (mm$^2$)</td>
<td>139.08</td>
<td>136.73</td>
<td>84.98</td>
<td>197.07</td>
<td>136.73</td>
<td>84.98</td>
</tr>
<tr>
<td>Ratio, $R_a$</td>
<td>1.72</td>
<td>1.65</td>
<td>1.59</td>
<td>1.72</td>
<td>1.65</td>
<td>1.59</td>
</tr>
<tr>
<td>Area factor</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Fig. 4.14, Fig. 4.15, Fig. 4.16 and Fig. 4.17 show the measured results of the dynamometer in the Y axis and the Z axis.

![Fig. 4.14 Measured force in Y axis of PETG under different angles](image-url)
Fig. 4.15 Measured force in Y axis of HDPE under different angles

Fig. 4.16 Measured force in Z axis of PETG under different angles
Forces in skew rolling are different when compared with the forces in parallel rolling, because there would have a component force which would cause a slight slip in the X axis. As a result, the deformation area would change with the motion. This is the main reason why the curves in the figures are not flat when the specimens were being rolled.

The motion of the specimen under skew rolling contributes to the inclination of the curves in the Z axis. The specimen would move away from the center and, as a result, the deformation area would be smaller. The reduction of the area would result in a smaller force, as can be seen from the equation: The force is the product of the pressure and the deformed area.

Table 4.4 shows the calculated forces in comparison to the forces measured.
Table 4.4 Comparison between the measured forces and the calculated Y axis and Z axis forces for PETG and HDPE at three different angles

<table>
<thead>
<tr>
<th></th>
<th>Experiment data</th>
<th>Calculation data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F_yd (N)</td>
<td>Error (N)</td>
</tr>
<tr>
<td>PETG-15°</td>
<td>320</td>
<td>55.6</td>
</tr>
<tr>
<td>PETG-10°</td>
<td>300</td>
<td>26.0</td>
</tr>
<tr>
<td>PETG-6°</td>
<td>190</td>
<td>28.3</td>
</tr>
<tr>
<td>HDPE-15°</td>
<td>380</td>
<td>64.4</td>
</tr>
<tr>
<td>HDPE-10°</td>
<td>330</td>
<td>39.7</td>
</tr>
<tr>
<td>HDPE-6°</td>
<td>270</td>
<td>38.9</td>
</tr>
</tbody>
</table>

Table 4.4 is plotted based in Fig. 4.18 and Fig. 4.19.

Fig. 4.18 Comparison between the measured forces and the calculated forces in Y axis in skew rolling for PETG and HDPE at three different angles
As can be seen in Fig. 4.18 and Fig. 4.19, the calculated Z axis forces of HDPE were smaller than the experimental forces. Because HDPE is more easily deformable than PETG, under the skew rolling, the contact area would be larger than the contact area of PETG. The difference between the experimental observations with the calculation of HDPE could be the area consideration. The contact area in the actual situation could be larger than assumed. Since the Y axis force is proportional to the Z axis force, the Y axis force of the HDPE is also smaller than the experimental force.

For the comparison of PETG, the contact area under a skew angle of 10° would be larger than the contact area under a skew angle of 15° based on the geometry of the model. From the point of the geometry model, part of the specimen was not rolled under a skew angle of 15°. From the equations used to calculate forces, forces under a skew angle of 10° would be larger than forces under a skew angle of 15°. Evidently, the experimental
observations did not follow the trend. However, the experimental data under skew angles of 10° and 15° are close. One explanation could be that the slip in the X axis caused a change in the contact area and this variance was not considered in the equations. The experimental Z axis forces are smaller than those calculated. This could be due to the fact that the PETG is harder than HDPE. With the same reduction, PETG would be harder to deform. The deformed area was smaller than expected. Also in the skew rolling process, the specimens were supposed to be fed through in the centerline between two rollers. As the specimen might not be fed in accurately along the centerline of two rollers, the specimen might have a greater component force in the Y axis than expected. That contributed to larger experimental data than calculations.

Overall, Fig. 4.18 and Fig. 4.19 successfully demonstrate the force models under skew rolling. Due to the setup’s rigidity, there would be deflections in the rolling process that could contribute to the errors in the attempts to predict the exact values. The forces would drive the specimen away from the center, but the area calculation in the model is based on the contact center of the two rollers. This difference will also cause some errors compared to the actual data.
CHAPTER 5. EXPERIMENTAL STUDY ON THE ANGLE COMPENSATION

In the skew rolling process, the component force in the axial direction would be generated due to the crossed angle. This component force is the force that pushes the bearings in the rolling setup. The influence of this force on the bearings needs to be understood. The influence of rolling process parameter, the crossed angle, and the friction conditions, on the force ratio $F_{xd}/F_{zd}$ was analyzed.

5.1. Experimental procedure and results

Two specimens, PETG and HDPE, were rolled under eight different angles. Forces in the X axis $F_{xd}$ and the Z axis $F_{zd}$, were measured. Simulation has been performed under the same conditions. Measured data were compared to simulated data and discussed in the following context.

Table 5.1 is the experimental setting for the two specimens.

<table>
<thead>
<tr>
<th></th>
<th>Final thickness, $h_f$ (mm)</th>
<th>Thickness, $h_0$ (mm)</th>
<th>Width, $w$ (mm)</th>
<th>Roller radius, $R$ (mm)</th>
<th>Coefficient of friction, $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE</td>
<td>1.016</td>
<td>1.4732</td>
<td>50.8</td>
<td>69.85</td>
<td>0.1</td>
</tr>
<tr>
<td>PETG</td>
<td>1.016</td>
<td>1.397</td>
<td>50.8</td>
<td>69.85</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Fig. 5.1 to Fig. 5.4 are some examples of forces at various angles. Results of other angles are listed in the table.

Fig. 5.1 X axis force of HDPE at an angle of 6°

Fig. 5.2 Z axis force of HDPE at an angle of 3°
Fig. 5.3 X axis force of PETG at an angle of 3°

Fig. 5.4 Z axis force of PETG at an angle of 8°
Forces in the Z axis were generally flatter at different angles. One reason was that the error of the unbalanced weight of the rollers was relatively small compared with the rolling forces. Another reason might be that forces in the XY plane could cause the vibration, with the vibration contributing to the difficulty of recording forces in the X or the Y axis.

Table 5.2 Experiment X and Z axes forces of HDPE and PETG at eight different angles

<table>
<thead>
<tr>
<th></th>
<th>HDPE</th>
<th></th>
<th>PETG</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F_{zd} (N)</td>
<td>Error (N)</td>
<td>F_{zd} (N)</td>
<td>Error (N)</td>
</tr>
<tr>
<td>1°</td>
<td>139</td>
<td>24</td>
<td>3450</td>
<td>350</td>
</tr>
<tr>
<td>2°</td>
<td>210</td>
<td>38</td>
<td>3561</td>
<td>389</td>
</tr>
<tr>
<td>3°</td>
<td>202</td>
<td>26</td>
<td>3522</td>
<td>298</td>
</tr>
<tr>
<td>4°</td>
<td>234</td>
<td>16</td>
<td>3705</td>
<td>460</td>
</tr>
<tr>
<td>5°</td>
<td>241</td>
<td>19</td>
<td>3653</td>
<td>197</td>
</tr>
<tr>
<td>6°</td>
<td>220</td>
<td>25</td>
<td>3700</td>
<td>200</td>
</tr>
<tr>
<td>8°</td>
<td>229</td>
<td>33</td>
<td>3772</td>
<td>194</td>
</tr>
<tr>
<td>10°</td>
<td>197</td>
<td>22</td>
<td>3811</td>
<td>214</td>
</tr>
</tbody>
</table>

Fig. 5.5, Fig. 5.6, Fig. 5.7 and Fig. 5.8 are the plotted based on Table 5.2.
Fig. 5.5 X axis forces of HDPE at different angles

Fig. 5.6 Z axis forces of HDPE at different angles
Although the forces did not change too much, it could still be found out that rolling forces increase slowly as the angle increases. From equation (9), an increasing angle would lead to a larger contact area. With the force as a product of area and stress, rolling forces
were expected to increase. Forces in the axial direction were not only affected by the rolling forces, but were also affected by the crossed angle. Component force $F_{xd}$ in the axial direction would decrease with the increasing angle, but the increased angle would have a larger rolling force that would increase the component force. From Fig. 5.5 and Fig. 5.7, forces in the axial direction increased from the beginning but decreased at larger angles.

![Graph comparing simulated and experimental forces](image)

**Fig. 5.9** Comparison between simulation and experiment data of $F_{xd}/F_{zd}$ of HDPE
5.2. Discussion

From the results in section 5.1, the force ratio increased with the angle significantly from zero to around 0.06 for HDPE and 0.05 for PETG. It can be seen in Fig. 5.9 and Fig. 5.10 that the blue curve and the green curve, which represented the simulation and experimental data respectively, plateaued after 2° for HDPE and 6° for PETG. In Fig. 5.9, both the blue curve and green curve showed that an angle of 2° was the threshold for HDPE specimen.

This indicates the force ratio in the rolling process of HDPE was sensitive in the range 0-2° and that it would increase significantly with a slight angle change. Beyond the threshold angle of 2°, the force ratio would be less sensitive to the angle which was reflected from the figure as an almost flat curve with increasing angles. In Fig. 5.10, both simulation and experimental data showed a similar trend for the PETG specimen. Although the
Experimental curve was off by the simulation curve 0.02, they shared the same trend, and it could be seen from the curve that the ratio increased significantly from 0° to 2°, and then slightly increased from 2° to 6°. After 6°, the curve became flat. The experimental curve was less sensitive as the angle changed from 2° to 6° compared to the simulation curve. The deflection of the rollers in experiment processing should be taken into consideration to explain the slight variations between the simulation and the experiments. Other factors include the position in which specimens were inserted, the variation of the thickness of specimens, the gap change due to deflection, and so forth.

Simulation data with different frictional conditions were also plotted as the red curves in both specimens. As can be seen from the Fig. 5.9 and Fig. 5.10, if the coefficient of friction doubled, the force ratio would be 100% larger, which was expected since the axial force would increase due to the coefficient of friction. The red curves didn’t show the threshold angles, but did show a continuous increase.

The axial force in the rolling system is the force used to push the bearings, it would be meaningful to understand and control the variations. The study carried out in this chapter showed that for a lower coefficient of friction, the force ratio varied little after 2° for HDPE and 8° for PETG. The force ratio can be minimized by compensating for this threshold angle instead of for a larger one because they have the same force ratio. As shown in Fig. 5.9, the force ratio under an angle of 2° had the same value as with an angle of 10°. To minimize the axial force under 10°, only the 2° needed to be compensated. In Fig. 5.10, the threshold angle would be 6°. This trend which could be used to the experimenter’s advantage disappeared when there were larger coefficients of friction. That means that the exact angle needs to be compensated for, in order to minimize the axial forces. Trend difference between
larger coefficients of friction and lower coefficients of friction indicated that lubricants would benefit the compensation process by minimizing the force ratios.
CHAPTER 6. CONCLUSIONS

1. Hardware design and fabrication of the rolling mill provided the functions needed to make the gap as small as 0.25 mm; the roller can be rotated at speeds ranging from 0 to 30 RPM; a dynamometer was successfully calibrated and could accurately measure the forces in the ranges of three axes; angles could be adjusted as accurately as 1°, and the range could be up to 15°.

2. Deformation area models for parallel rolling and skew rolling have been built. Experiment data were measured and compared to the calculated values. Coefficients of friction for PETG and HDPE materials in this rolling setup were determined to be 0.05 and 0.1 respectively. Calculations and experimental data showed that the rolling force can be calculated as a product of stress and the deformed area. The thrust force can be calculated as a product of rolling force and the coefficient of friction. Based on the friction conditions determined in the former section, equations in skew rolling were shown to be validated. Calculations and experimental data showed that the rolling force can be calculated as a product of stress and the deformed area. The deformed area followed the geometry built in the former chapter. Forces in other two axes can be calculated as two component forces of a product of rolling force and coefficient of friction.

3. Throughout this study, the influence of rolling process parameter, the crossed angle, and the friction conditions on the force ratio $F_{xd}/F_{zd}$ was analyzed. The study carried out showed that for a lower coefficient of friction, the force ratio varied little after 2° for high density polyethelene (HDPE) and 6° for polyethylene terephthalate glycol-modified (PETG). The force ratio can be minimized by compensating for these threshold angles instead of
actual larger angles because they have the same force ratio. Force ratios with larger coefficients of friction losing this similar trend suggest that attempts to lower the coefficient of friction were necessary. As the axial force is the force that pushes the bearings in the setup, minimizing this force is necessary. The trend in Fig. 5.9 and Fig. 5.10 can be obtained with a lower coefficient of friction, and angle compensation can be carried out based on this trend. Analysis of the force ratio provided methods to minimize the forces, which is through lubricants and angle compensation. This study can also be expanded to other materials under different friction conditions to obtain threshold angles for control purposes.
CHAPTER 7. BIBLIOGRAPHY


