Thermal, radiation and mechanical analysis of cylindrical oxide fuel elements of fast reactor in unsteady state

Chi-kang Cheng
Iowa State University

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THERMAL, RADIATION AND MECHANICAL ANALYSIS OF CYLINDRICAL OXIDE FUEL ELEMENTS OF FAST REACTOR IN UNSTEADY STATE

by

Chi-kang Cheng

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major Subject: Nuclear Engineering

Approved:

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In Charge of Major Work

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Dean of Graduate College

Iowa State University
Ames, Iowa

1970
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOMENCLATURE</td>
<td>vii</td>
</tr>
</tbody>
</table>

<p>| I. INTRODUCTION                     | 1    |
| II. THERMAL ANALYSIS                | 8    |
| A. Temperature Distribution in Cladding | 8    |
| B. Temperatures Through the Bonding Gap | 9    |
| C. Temperature Distribution in Fuel | 11   |
| III. RADIATION ANALYSIS             | 23   |
| A. Introduction                     | 23   |
| B. General Descriptions for the Model | 25   |
| C. Fission-product Swelling         | 29   |
| D. Gas Release                      | 42   |
| IV. THERMAL, RADIATION AND STRESS ANALYSIS | 46   |
| A. Introduction                     | 46   |
| B. Basic Assumptions                | 47   |
| C. Equations and Solutions of Stress Analysis | 48   |
| D. The Method of Successive Approximation | 68   |
| V. COMPUTER PROGRAM AND NUMERICAL EXAMPLE | 70   |
| A. Introduction to Computer Program | 70   |
| B. Results                          | 71   |
| VI. DISCUSSION                      | 94   |
| VII. CONCLUSIONS                    | 98   |
| VIII. LITERATURE CITED              | 101  |
| IX. ACKNOWLEDGMENTS                 | 106  |</p>
<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>X.</td>
<td>APPENDIX A: COMPUTER PROGRAM ISUNE-1</td>
<td>107</td>
</tr>
<tr>
<td>XI.</td>
<td>APPENDIX B: TABLES OF RESULTS</td>
<td>143</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Input data</td>
<td>74</td>
</tr>
<tr>
<td>10.1</td>
<td>Important notations in computer program other than those for input listed in Table 5.1</td>
<td>136</td>
</tr>
<tr>
<td>11.1</td>
<td>Time variations of temperatures and radii at region surfaces, fuel-clad gap, and pressures in central void and gap</td>
<td>144</td>
</tr>
<tr>
<td>11.2</td>
<td>Swelling strain and plastic strains in fuel at time t=1 hour</td>
<td>145</td>
</tr>
<tr>
<td>11.3</td>
<td>Swelling strain and plastic strains in fuel at time t=125 hours</td>
<td>146</td>
</tr>
<tr>
<td>11.4</td>
<td>Swelling strain and plastic strains in fuel at time t=729 hours</td>
<td>147</td>
</tr>
<tr>
<td>11.5</td>
<td>Swelling strain and plastic strains in fuel at time t=1000 hours</td>
<td>148</td>
</tr>
<tr>
<td>11.6</td>
<td>Temperature distribution, temperature gradients and thermal strains in fuel at time t=1 hour</td>
<td>149</td>
</tr>
<tr>
<td>11.7</td>
<td>Temperature distribution, temperature gradients and thermal strains in fuel at time t=125 hours</td>
<td>150</td>
</tr>
<tr>
<td>11.8</td>
<td>Temperature distribution, temperature gradients and thermal strains in fuel at time t=729 hours</td>
<td>151</td>
</tr>
<tr>
<td>11.9</td>
<td>Temperature distribution, temperature gradients and thermal strains in fuel at time t=1000 hours</td>
<td>152</td>
</tr>
<tr>
<td>11.10</td>
<td>Dimensionless strains and stresses in fuel at time t=1 hour</td>
<td>153</td>
</tr>
<tr>
<td>11.11</td>
<td>Dimensionless strains and stresses in fuel at time t=125 hours</td>
<td>154</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Table 11.12.</td>
<td>Dimensionless strains and stresses in fuel at time t=729 hours</td>
<td>155</td>
</tr>
<tr>
<td>Table 11.13.</td>
<td>Dimensionless strains and stresses in fuel at time t=1000 hours</td>
<td>156</td>
</tr>
<tr>
<td>Table 11.14.</td>
<td>Temperature distribution, thermal strain and plastic strains in cladding at time t=1 hour</td>
<td>157</td>
</tr>
<tr>
<td>Table 11.15.</td>
<td>Temperature distribution, thermal strain and plastic strains in cladding at time t=125 hours</td>
<td>158</td>
</tr>
<tr>
<td>Table 11.16.</td>
<td>Temperature distribution, thermal strain and plastic strains in cladding at time t=729 hours</td>
<td>159</td>
</tr>
<tr>
<td>Table 11.17.</td>
<td>Temperature distribution, thermal strain and plastic strains in cladding at time t=1000 hours</td>
<td>160</td>
</tr>
<tr>
<td>Table 11.18.</td>
<td>Dimensionless strains and stresses in cladding at time t=1 hour</td>
<td>161</td>
</tr>
<tr>
<td>Table 11.19.</td>
<td>Dimensionless strains and stresses in cladding at time t=125 hours</td>
<td>162</td>
</tr>
<tr>
<td>Table 11.20.</td>
<td>Dimensionless strains and stresses in cladding at time t=729 hours</td>
<td>163</td>
</tr>
<tr>
<td>Table 11.21.</td>
<td>Dimensionless strains and stresses in cladding at time t=1000 hours</td>
<td>164</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>2.1</td>
<td>Typical cross section of an irradiated, low density, oxide fuel rod</td>
<td>17</td>
</tr>
<tr>
<td>5.1</td>
<td>General flowchart for the code</td>
<td>72</td>
</tr>
<tr>
<td>5.2</td>
<td>Flowchart for computing stresses and strains by the method of successive approximations</td>
<td>73</td>
</tr>
<tr>
<td>5.3</td>
<td>Temperature varies with time at surfaces of columnar and equiaxed grain regions</td>
<td>80</td>
</tr>
<tr>
<td>5.4</td>
<td>Time variations of radii of columnar and equiaxed grain regions</td>
<td>81</td>
</tr>
<tr>
<td>5.5</td>
<td>Temperature distribution in the fuel element</td>
<td>83</td>
</tr>
<tr>
<td>5.6</td>
<td>Time variations of fuel-clad gap and radius of central void</td>
<td>84</td>
</tr>
<tr>
<td>5.7</td>
<td>Time variations of pressure in central void and fuel-clad gap</td>
<td>86</td>
</tr>
<tr>
<td>5.8</td>
<td>Radial, tangential and axial stress distributions in clad at irradiation times of 1 and 1000 hours</td>
<td>87</td>
</tr>
<tr>
<td>5.9</td>
<td>Radial and tangential strain distributions in clad at irradiation times of 1 and 1000 hours</td>
<td>88</td>
</tr>
<tr>
<td>5.10</td>
<td>Thermal and plastic strain distributions in clad at irradiation time of 1000 hours</td>
<td>89</td>
</tr>
<tr>
<td>5.11</td>
<td>Radial, tangential and axial stress distributions in fuel zone vary with time</td>
<td>90</td>
</tr>
<tr>
<td>5.12</td>
<td>Radial and tangential strain distributions in fuel zone vary with time</td>
<td>91</td>
</tr>
<tr>
<td>5.13</td>
<td>Thermal, radiation and plastic strain distributions in fuel zone at radiation time of 729 hours</td>
<td>93</td>
</tr>
</tbody>
</table>
NOMENCLATURE

**Primary English Letter Symbol**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Van der Waals correction term</td>
</tr>
<tr>
<td>c</td>
<td>coolant specific heat</td>
</tr>
<tr>
<td>$C_0$</td>
<td>average volumetric contribution of solid fission products per fission per unit volume</td>
</tr>
<tr>
<td>d</td>
<td>diameter of gas bubble</td>
</tr>
<tr>
<td>$d_{cg}$</td>
<td>critical diameter of bubble for movement in grain</td>
</tr>
<tr>
<td>D</td>
<td>grain size after an annealing time $t$</td>
</tr>
<tr>
<td>$D_e$</td>
<td>equivalent diameter of coolant channel</td>
</tr>
<tr>
<td>$D_0$</td>
<td>initial grain size; coefficient</td>
</tr>
<tr>
<td>$D_s$</td>
<td>surface diffusion coefficient</td>
</tr>
<tr>
<td>e</td>
<td>mean strain</td>
</tr>
<tr>
<td>$e_c$, $e_f$</td>
<td>emissivities of clad and fuel</td>
</tr>
<tr>
<td>$e_r$, $e_\theta$, $e_z$</td>
<td>radial, tangential and axial elastic strain deviators</td>
</tr>
<tr>
<td>$e^r$, $e^\theta$, $e^z$</td>
<td>radial, tangential and axial total strain deviators</td>
</tr>
<tr>
<td>$e^{tr}$, $e^{t\theta}$, $e^{tz}$</td>
<td>radial, tangential and axial modified total strain deviators</td>
</tr>
<tr>
<td>E</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>$E_c$, $E_f$</td>
<td>modulus of elasticity for clad and fuel</td>
</tr>
<tr>
<td>$\dot{F}$</td>
<td>fissions per unit volume per unit time</td>
</tr>
<tr>
<td>$F_{gb}$</td>
<td>maximum retarding force of grain boundary</td>
</tr>
<tr>
<td>$F_s$</td>
<td>driving force on bubbles due to thermal gradient</td>
</tr>
</tbody>
</table>
defined variable \( g = \frac{1}{3}(\varepsilon_r - \varepsilon_\theta - 3 \varepsilon_R) \)

heat transfer coefficient between coolant and clad

Boltzmann constant

constant

coolant thermal conductivity

clad thermal conductivity

gap thermal conductivity

bonding gas thermal conductivity

fuel thermal conductivity in region I, II, and III

average length across a grain

thickness of fuel-clad gap

number of fission-gas atoms in a bubble

molecular weights of fuel and primary vapor constituent in pore

number of time intervals for bubble migrating in grain

Avogadro's number

number of bubbles per unit area on grain boundary having been collided \( i \) times and having passed \( n \) time intervals

number of bubbles per unit volume

total number of bubbles per unit volume moving away from grain boundaries

Nusselt number

gas pressure inside a bubble

total pressure in fabricated pore

Peclet number
\( P_g \) pressure in fuel-clad gap

\( P_0 \) coolant pressure

\( P_v \) pressure in central void

\( q \) linear power

\( Q \) activation energy for equiaxed-grain growth

\( Q^* \) activation energy for surface diffusion

\( r_1 \) radius at outer surface of columnar-grain region

\( r_2 \) radius at outer surface of equiaxed-grain region

\( r_i, r_o \) radii of clad at inner and outer surfaces

\( r_s \) radius at fuel rod surface

\( r_v \) radius of central void

\( R \) universal gas constant

\( s \) average normal stress or hydrostatic stress

\( s_r, s_\theta, s_z \) radial, tangential and axial stress deviators

\( S_b \) Stefan-Boltzmann constant

\( t \) irradiation time

\( t_{cb} \) time when bubble attains its critical size for moving away from grain boundary

\( t_{cg} \) time when bubble attains its critical size for movement in grain

\( T_a \) absolute temperature

\( T_c \) bulk coolant temperature

\( T_g \) average temperature in fuel-clad gap

\( T_i, T_o \) temperature of clad at inner and outer surfaces
\( T_s \) temperature at fuel rod surface
\( u \) radial displacement
\( v \) coolant average velocity
\( V_b \) velocity of bubble leaving grain boundary
\( V_{cb} \) critical velocity for bubble leaving grain boundary
\( V_{cg} \) critical velocity for bubble moving in grain
\( V_g \) velocity of bubble moving in grain

**Primary Greek Letter Symbol**

\( \alpha \) linear coefficient of thermal expansion
\( \gamma \) surface tension of fuel material
\( \gamma_{gb} \) grain boundary surface tension
\( \varepsilon_I \) irradiation dilatation strain
\( \varepsilon_{pe} \) equivalent plastic strain
\( \varepsilon_{pr}, \varepsilon_{p\theta}, \varepsilon_{pz} \) radial, tangential and axial plastic strains
\( \varepsilon_r, \varepsilon_\theta, \varepsilon_z \) radial, tangential and axial total strains
\( \varepsilon_{te} \) equivalent total strain
\( \varepsilon_R \) thermal-irradiation dilatation strain
\( \varepsilon_i \) tangential strain of clad at \( r_i \)
\( \varepsilon_s \) tangential strain of fuel at \( r_s \)
\( \varepsilon_v \) tangential strain of fuel at \( r_v \)
\( \Delta H_v \) molar heat of vaporization
\( \left( \frac{\Delta V}{V} \right)_{gas} \) total swelling due to bubbles
\( \left( \frac{\Delta V}{V} \right)_{sgb} \) swelling due to bubbles on grain boundaries
\( \left( \frac{\Delta V}{V} \right)_{sgg} \) swelling due to anchored bubbles in grains
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\frac{\Delta V}{V})_{sgg}'$</td>
<td>swelling due to new bubbles anchored on original bubble sites in grains</td>
</tr>
<tr>
<td>$(\frac{\Delta V}{V})_{sgm}$</td>
<td>swelling due to bubbles moving in grains before reaching a grain boundary</td>
</tr>
<tr>
<td>$(\frac{\Delta V}{V})_{sld}$</td>
<td>swelling due to solid fission products</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time interval for bubble movement in grains</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>non-negative constant</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>$v$</td>
<td>number of diffusion molecules per unit surface area</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>cross-sectional radius for collision between matrix and vapor molecules</td>
</tr>
<tr>
<td>$\sigma'$</td>
<td>intrinsic stress of fuel</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>equivalent stress</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>hydrostatic stress</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>proportional limit</td>
</tr>
<tr>
<td>$\sigma_r, \sigma_\theta, \sigma_z$</td>
<td>radial, tangential and axial stresses</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>yield strength of 0.2% offset</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>coolant density</td>
</tr>
<tr>
<td>$\rho_I, \rho_{II}, \rho_{III}$</td>
<td>fraction of theoretical density in region I, II, and III</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>molecular volume of fuel</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

There is a widespread interest in the oxides of plutonium, uranium, and thorium, especially, in the system of \((U_{0.8}Pu_{0.2})\) as nuclear fuel materials in fast reactors. Notable advantages of their applications are high melting points, freedom from harmful phase transformation, high neutron utilization, stability in intensive radiation and high-temperature coolant media, and high burn-up potentiality. A breeder fuel in either the system \(\text{ThO}_2-\text{UO}_2\), \(\text{UO}_2-\text{PuO}_2\) or \(\text{ThO}_2-\text{UO}_2-\text{PuO}_2\) would be feasible because these oxides are isomorphic in structure, and difference in lattice constants of the three oxides is such that the formation of solid solutions would be indicated (1). Many of the physical properties of these oxides have been evaluated, and currently, these oxides are being investigated as candidate fuels for fast reactors.

Solid cylindrical fuel elements used in fast reactors have the advantages of low fabrication cost, compact core, and high capacity of fission-gas retention.

Recently, the radiation and creep solutions for solid and tubular cylindrical fuel elements at steady state (2-4), the radiation growth and swelling analysis for solid cylindrical fuel elements at unsteady state (5,6), the computer program CYGRO-2 for stress analysis of the growth of concentric cylinders with gas bubbles (7), the computer program KER-4 for
thermal analysis of cylindrical fuel rods (8), the computer program NUKER for thermal and mechanical analysis of fuel rods (9), and the computer program CRASH for the evaluation of the creep and plastic behavior of fuel pin sheaths (10) have been given.

The objective of this work is to analyze and calculate stresses and strains due to thermal, radiation and mechanical effects for cylindrical oxide fuel elements of fast reactors in unsteady state during a constant linear power operation. The variables and parameters dealt with in this work will have significant and important effects on the fuel element performance, reactor safety, reliability, and economic operation of oxide fast breeders.

In order to achieve a long operating life of the fuel element, low density oxide fuels are normally used in a fast reactor. The configuration of the low-density cylindrical oxide fuel element with high temperatures and large temperature gradients is usually changed to form a central void in the axial direction in the early operating life (11-13). This phenomenon is generally believed to be the results of the migration of fabricated pores up the temperature gradients by a vapor transport mechanism to the centerline of the fuel element, and the sintering of equiaxed-grain growth. Thus, from inside to outside, except the central void, the cross-sectional area of the fuel exhibits three regions (or zones):
the columnar grain region, the equiaxed grain region and the unaffected grain region. The densities for the regions are different and so are the heat power generations and the thermal conductivities. The formation of the three regions is time-dependent. In this unsteady condition the temperature distribution, a primary factor to produce stresses in fuel elements, has not been formulated in existing literatures. This temperature distribution will be obtained in Chapter II.

As a result of neutron bombardments of fissionable materials, fission products are produced in a nuclear fuel element. Some of the products are solid atoms and others are gas atoms (most are inert gases). Both can cause swelling in the fuel. This is another factor to produce stresses in the fuel elements. The swelling of fission solids is due to the increase of total atomic volumes. The swelling of fission gases is due to the easy nucleation and formation of gas bubbles and due to the collisions of bubbles with one another to form new bubbles which result in larger volumes. The behavior of fission-gas bubbles in oxide fuel is very complicated and no simple model is available to give the resulted swelling on a quantitative base. Barnes and Nelson have discussed the basic ideas of the nucleation, trapping, and migration of the bubbles (14). They have also given models on a qualitative base (15). Nichols has analyzed
the problem with a great effort and has formulated a semi-quantitative model (16, 17). A model to analyze and calculate the swelling and the gas release will be given in Chapter III of this work. This model will satisfy most of their analytical results and is suitable for the computations by a digital computer.

When the thermal and radiation analyses are obtained, the analysis and calculation of stresses and strains can be carried out.

The method to be used for the calculation of stresses and strains is an iteration method of successive approximations. The steps in the procedure are:

(1) assuming three plastic strains in the three principal directions,

(2) calculating stresses and strains from equations of equilibrium and compatibility with the known values of thermal and swelling strains and the assumed plastic strains to change the nonlinear problem to a linear solution,

(3) finding the equivalent stress or strain with the calculated stresses and strains and a suitable yielding criterion,

(4) fitting the obtained equivalent stress or strain in the experimental stress-strain curves to obtain three new plastic strains which can be used in the
next iteration, and

(5) iterating successively to obtain the converged values of stresses and strains desired.

This method has been used a great deal by Mendelson (18) and by Manson (19). In its application here, the principles are the same as theirs but different techniques will be developed as can be seen in Chapter IV.

The whole problem is a compound and complex iteration. The stresses and strains in the clad and in the fuel are, of course, iterated values; the temperature of the fuel rod surface and the gap thickness between the fuel and clad are also iterated results. The pressures in the gap and in the central void are treated corresponding to the stress-strain iterations. The sizes of gas bubbles in fuel, which can be influenced by the hydrostatic stress surrounding them due to external load or thermal stresses, is also iterated in the stress-strain calculations of the fuel (20). The whole system may be seen in the general flow charts for the computer program as shown in Figures 5.1 and 5.2.

The convergence of the method of successive approximation has been tested and discussed in details by Mendelson (18) and by Manson (19). A rigorous proof of the convergence problem of the complicated situation is beyond the scope of this thesis. Experience and observations are therefore the guide.
The computer code is called ISUNE 1 (Iowa State University Nuclear Engineering) and the program in FORTRAN IV language is given in Appendix A.

The general physical conditions may be described as follows:

The fuel element is a long thin cylinder with a metal cladding and a low density oxide fuel. The thermal bond material is an inert gas like helium which is preferred to sodium (21). The coolant material is a liquid metal like sodium or NaK.

The fuel element has been operated in a fast reactor for a short, early period of time passing through a preliminary state of cracking.

The gap between the clad and the fuel will not be closed during the analysis since this is a normal provision to avoid distortion of the cladding in fast reactors.

There is little neutron flux depression in the cross section of the fuel element and the volumetric heat rate is proportional to the mass and enrichment of the fuel present.

A plane strain state and an axial symmetry are assumed so that all variables vary only with radius and time.

The coolant temperature, coolant pressure and power level are input values. At a given time, the temperature distribution, irradiation swelling, gas release, fuel-clad gap thickness, gap pressure, radius of the central void, central void
pressure, strains and stresses are calculated or iterated for the analysis of the fuel element.
II. THERMAL ANALYSIS

The purpose of the thermal analysis is to find the temperature distribution in the cross section of the fuel element during a constant linear power operation. The analysis is started from the surface of the clad through the cladding material, the gap and the fuel to the central void.

A. Temperature Distribution in Cladding

For a fast reactor cooled with liquid metal sodium or NaK, the heat transfer coefficient between the metallic coolant and the cladding material can be found with the following experimental relation for turbulent flow in a long tube (22):

\[ \text{Nu} = 7 + 0.025 \text{Pe}^{0.8}, \]  

(2-1)

where

\[ \text{Nu} = \frac{hD_e}{K}, \text{ the Nusselt number} \]

\[ \text{Pe} = \frac{D_e \rho_c c}{K}, \text{ the Peclet number} \]

\[ D_e = \text{equivalent diameter of the coolant channel} \]

\[ v = \text{coolant average velocity} \]

\[ \rho_c = \text{coolant density} \]

\[ K = \text{coolant thermal conductivity} \]

\[ c = \text{coolant specific heat} \]

\[ h = \text{heat transfer coefficient between coolant and clad.} \]

Then

\[ h = \frac{K}{D_e} \left( 7 + 0.025 \text{Pe}^{0.8} \right). \]  

(2-2)
Mean values of $K$, $\rho_C$, and $c$ in the temperature range may be used for the calculation.

For a given bulk coolant temperature $T_C$ the surface temperature $T_o$, the temperature distribution $T(r)$ and the inner surface temperature $T_i$ of the cladding can be calculated from the basic heat transfer equations:

\[ T_o = T_C + \frac{q}{2\pi r_o h} \]
\[ T(r) = T_o + \frac{q}{2\pi K_C} \ln\left(\frac{r_o}{r}\right) \]
\[ T_i = T_o + \frac{q}{2\pi K_C} \ln\left(\frac{r_o}{r_i}\right) \]

where $q$ is the linear power, $K_C$ is the thermal conductivity of the cladding material, and $r_o$ and $r_i$ are the outer and inner surface radii of the cladding, respectively.

B. Temperatures Through the Bonding Gap

The inner surface temperature of the clad is given by Equation (2-5). Similarly, the temperature at the surface of the fuel pin, $T_s$, is given by

\[ T_s = T_i + \frac{q}{2\pi K_{\text{gap}}} \ln\left(\frac{r_i (1+\epsilon_{\theta_i})}{r_s (1+\epsilon_{\theta_s})}\right) \]

where $r_s$ is the surface radius of the fuel rod, $K_{\text{gap}}$ the thermal conductivity of the gap, $\epsilon_{\theta_i}$ the tangential strain of clad at $r_i$ and $\epsilon_{\theta_s}$ the tangential strain of fuel at $r_s$. 
The average gap temperature $T_g$ may be written

$$T_g = \frac{T_s + T_i}{2}. \quad (2-7)$$

$K_{\text{gap}}$ in Equation (2-6) may be written in the form below (23):

$$K_{\text{gap}} = K_{\text{gas}} + \frac{4L_gS_b}{\left(\frac{1}{e_c} + \frac{1}{e_f} - 1\right)}(T_g)^3 a. \quad (2-8)$$

where $K_{\text{gas}}$ is the thermal conductivity of the gaseous medium separating the fuel and the clad, $e_c$ and $e_f$ are the emissivities of the clad and fuel respectively and are functions of temperature, $L_g$ is the gap thickness, $S_b$ is the Stefan-Boltzmann constant, and $(T_g)^a$ is the absolute temperature of $T_g$.

Additional equations for the calculation of $K_{\text{gap}}$ are given by

$$e_f = C_{g1} + C_{g2} T_s \quad (2-8a)$$
$$e_c = C_{g3} + C_{g4} T_i \quad (2-8b)$$
$$K_{\text{gas}} = C_{g5} T_g \quad (2-8c)$$
$$L_g = r_i(1 + \epsilon_{gi}) - r_s(1 + \epsilon_{gs}). \quad (2-8d)$$

where $C_g$'s are constants.

$T_s$ and $K_{\text{gap}}$ are obtained by the combination of equations (2-6) and (2-8) through iteration process since tangential strains are involved.
C. Temperature Distribution in Fuel

In an oxide fuel the temperature is so high that two phenomena, the equiaxed grain growth and the columnar grain growth, (both are temperature and time dependent), should be considered. These structural changes make the problem quite complicated. A detailed stress analysis including time dependent effects has not been reported or published.

The grain growths sinter the fuel and a central void is formed in the axial direction. In an irradiated oxide fuel of low density, a typical cross section starting from the central void to the surface of the fuel exhibits three regions: The first, innermost region is the columnar grain annulus, the second region is the equiaxed-grain annulus, and the last, outermost region is the unaffected grain annulus. Since the densities are different from one another in these regions as are the volumetric heat sources and the thermal conductivities, special techniques should be applied to obtain the temperature distribution. Sayles has formulated the three-region temperature distribution (24) based on Robertson's method (13), but has not considered the time-dependent property. The mechanisms of these phenomena will be examined in order to formulate the temperature distribution.

The mechanism for columnar grain formation in oxide
fuel is not entirely clear. There is evidence, however, that the growth is connected with the presence of porosity-fabricated pores. These pores appear to be moving up the temperature gradients and this migration results in the formation of columnar grains. The migration is usually treated by an evaporation-condensation mechanism that is a mechanism of the evaporation on the hot side followed by the condensation on the cold side of the pore (12,13). Nichols has presented a quantitative analysis for the migration of fabricated pores (12). He derived an equation for the velocity $V$ of the pore with the assumption that the velocity of migration of a non-spherical pore will not differ greatly from that of a spherical pore. The equation is given by

$$
V = \frac{9 \ C_0 \ \Omega \ \Delta H_v \ [N(M_1+M_2)]^{1/2} \ exp \left[-\frac{\Delta H_v}{(RT_a)}\right]}{16 \ \pi \ \sigma^2 \ T_a^{3/2} \ (2\pi k M_1 M_2)} \ \frac{dT}{dr}, \quad (2-9)
$$

where

$C_0$ = constant

$\Omega$ = molecular volume of fuel

$\Delta H_v$ = molar heat of vaporization

$N$ = Avogadro's number

$M_1, M_2$ = molecular weights of fuel and primary vapor constituent, respectively

$R$ = universal gas constant
P = total pressure in pore
\( \sigma = \) cross-sectional radius for collisions between matrix and vapor molecules
k = Boltzmann constant
\( T_a = \) absolute temperature
\( \frac{dT}{dr} = \) radial temperature gradient.

It is seen that the velocity is independent of pore size.

With further assumptions that the temperature distribution in fuel is parabolic and the average temperature gradient is that at 1/3 of the fuel rod radius, from Equation (2-9) the centerline temperature required for "significant" motion (defined as 1/3 of fuel rod radius) in the allotted time is calculated. Thus he obtained the "threshold" temperature for columnar grain formation as well as concomitant central void formation as a function of time. The expression can be written as

\[
- \frac{r_s}{3t} = \frac{C_1 \exp(-C_2/T_a)}{T_a^{3/2}} \left( \frac{dT}{dr} \right)_{\text{ave}}, \tag{2-10}
\]

where \( C_1 \) and \( C_2 \) are constants evaluated from the average or constant values of the parameters in Equation (2-9), \( \left( \frac{dT}{dr} \right)_{\text{ave}} \) is the temperature gradient obtained at the position of 1/3 of \( r_s \), the radius at the fuel rod surface, and \( t \) is the time. Thus the threshold centerline temperature in a specific time can be calculated from Equation (2-10). If the centerline
temperature is above the threshold value, the radius of the columnar grain, the temperature at this radius, and radius of the concomitant central void at the specified time can be calculated from the combination of Equation (2-10), the equation of parabolic temperature distribution and the given value of porosity. By this method he obtained results in good agreement with both in-pile and out-of-pile measurements on UO₂ (12).

In the present calculation, Equation (2-10) is coupled with equations for finding the temperature distribution (which is not necessarily parabolic) and the radii of the three regions and the central void as shown later.

Since the porosity in the columnar region has almost moved completely to the central void, a density of about 99% of the theoretical value of fuel may be assumed in this region.

Now the mechanism for the equiaxed grain growth is to be examined.

Burke assumed that the average grain diameter is related to the average radius of curvature of the grain boundaries and that the drag force for the boundary migration during grain growth is the surface tension of the boundary. He deduced the following expression for grain growth (25):

\[ D^2 - D_0^2 = k_0 t^n \exp\left(-\frac{Q}{RT_a}\right) \]  

(2-11)
where

\[ D_0 = \text{initial grain size} \]
\[ D = \text{grain size after an annealing of time } t \]
\[ k_0 = \text{a constant} \]
\[ Q = \text{activation energy for the process} \]
\[ n = \text{time exponent} = 1, \text{theoretically} \]

Grain growth does not normally proceed in exact accordance with Equation (2-11) (11,26,27,28). For oxide fuel UO₂, Lyons et al. analyzed the grain growth data and obtained a cubic equation for the best representation (28) as

\[ D^3 - D_0^3 = k_0 t \cdot \exp\left(-\frac{Q}{RT_a}\right). \quad (2-12) \]

There is a correlation between the porosity and grain size. MacEwan reported that pronounced grain growth does not occur in sintered UO₂ until the porosity decreases to about 3% of the volume of the specimen and then grain growth occurs with no further decrease in porosity (11). From the experimental data a minimum grain size of 15 µm at the outer radius of the equiaxed grain region and a density of about 97% of the theoretical value may be specified to this region for UO₂ (11).

With the specified value of minimum grain size for the extent of equiaxed grain in the fuel and the given value of time and initial grain size, one can calculate the temperature at the outer radius of the equiaxed grain region from Equation
This relation will also be coupled with the temperature distribution equations.

Now comes the time to formulate the temperature distribution in the fuel.

The three regions in the cross section of the fuel rod after an irradiation time \( t \) at constant linear power \( q \) of the reactor operation are designated by I, II, and III as shown in Figure 2.1. The radius of the central void \( r_v \), the outer radius of columnar-grain region \( r_1 \), the outer radius of equiaxed-grain region \( r_2 \), and the radius at the fuel rod surface \( r_s \) are also indicated. The corresponding temperature at these surfaces are designated by \( T_v \), \( T_1 \), \( T_2 \), and \( T_s \), respectively.

Since fuel pins in fast reactors are very thin, the neutron-flux depression in the pins may be neglected. Therefore, the heat generation is proportional to the density of the fuel when the fuel enrichment is given. For the given linear power \( q \) the volumetric heat rates in the three regions are

\[
Q_{\text{III}} = \frac{q}{\pi r_s^2} \quad (2-13)
\]

\[
Q_{\text{II}} = Q_{\text{III}} \frac{\rho_{\text{II}}}{\rho_{\text{III}}} \quad (2-14)
\]

\[
Q_{\text{I}} = Q_{\text{III}} \frac{\rho_{\text{I}}}{\rho_{\text{III}}} \quad (2-15)
\]
Figure 2.1. Typical cross section of an irradiated, low density, oxide fuel rod.

I = columnar grain region
II = equiaxed grain region
III = unaffected grain region

Fuel-clad gap
where \( \rho_I, \rho_{II}, \) and \( \rho_{III} \) are known values of fraction of theoretical density in the three regions, respectively.

To consider the thermal conductivity as a function of temperature and porosity, Robertson employed the concept of \( \int K(T) dT \) in heat transfer equations for the fuel, where \( K(T) \) is the thermal conductivity at temperature \( T \) and the porosity is considered constant in the temperature range of the integration (13). In a recent experimental work, Asamoto et al. give the following expression for the thermal conductivity of \( UO_2 \) (29):

\[
K_f = 0.0130 + \{T(0.4848 - 0.4465 \rho)\}^{-1} \frac{w}{cm^0C} \quad (2-16)
\]

where \( T \) is temperature in centigrade and \( \rho \) is the fraction of theoretical density. A corresponding expression for the general case may be written as

\[
K_f(T,\rho) = C_3 + \{T(C_4 - C_5\rho)\}^{-1} \quad (2-17)
\]

For the long cylindrical fuel rod, the equation of heat conduction is

\[
\int Q(r) 2\pi r \, dr = -K_f(T,\rho) 2\pi r \frac{dT}{dr} \quad (2-18)
\]

If \( Q(r) \), the volumetric heat source, and \( \rho \) are constant in the integration range, the above equation becomes

\[
Q \frac{K}{r} = -K(T) \frac{dT}{dr} \quad (2-19)
\]

Therefore, in region III it is
\[ Q_{III} \frac{r}{2} = -k_{III}(T) \frac{dT}{dr} \] (2-20)

Integration gives
\[
\int_{T_s}^{T} k_{III}(T) dT = \frac{Q_{III} r_s^2}{4} [1-(\frac{r}{r_s})^2] 
\] (2-21)

or
\[ C_3(T-T_s) + (C_4-C_5 \rho_{III})^{-1} \ln \frac{T}{T_s} = \frac{Q_{III} r_s^2}{4} [1-(\frac{r}{r_s})^2] \] (2-22)

When \( T = T_2 \), it gives
\[ C_3(T_2-T_s) + (C_4-C_5 \rho_{III})^{-1} \ln \frac{T_2}{T_s} = \frac{Q_{III} r_s^2}{4} [1-(\frac{r}{r_s})^2] \] (2-23)

Similarly, in region II the heat conduction equation is
\[
\frac{(Q_{III}-Q_{II}) r_2^2}{2r} + \frac{Q_{II} r}{2} = -k_{II}(T) \frac{dT}{dr} 
\] (2-24)

Integration yields
\[
\int_{T_2}^{T} k_{II}(T) dT = \frac{Q_{III} r_2^2}{4} [1-(\frac{r}{r_2})^2 + (1-\frac{Q_{III}}{Q_{II}}) \ln(\frac{r}{r_2})^2] 
\] (2-25)

or
\[ C_3(T-T_2) + (C_4-C_5 \rho_{II})^{-1} \ln \frac{T}{T_2} 
\]
\[ = \frac{Q_{II} r_2^2}{4} [1-(\frac{r}{r_2})^2 + (1-\frac{Q_{III}}{Q_{II}}) \ln(\frac{r}{r_2})^2] \] (2-26)

When \( T=T_1 \), the substitution gives
\[ C_3(T_1 - T_2) + (C_4 - C_5 \rho_{II})^{-1} \ln \frac{T_1}{T_2} \]

\[ = \frac{Q_{II}r_2^2}{4} \left[ 1 - \left( \frac{r_1}{r_2} \right)^2 + \left( 1 - \frac{Q_{III}}{Q_{II}} \right) \ln \left( \frac{r_1}{r_2} \right) \right] \quad (2-27) \]

In region I these equations are

\[ \frac{Q_I}{2} \frac{r^2 - r_v^2}{r} = -K_I(T) \frac{dT}{dr} \quad (2-28) \]

and

\[ \int_{T_1}^{T} K_I(T) dT = \frac{Q_{II}r_1^2}{4} \left[ 1 - \left( \frac{r_1}{r_v} \right)^2 + \left( \frac{r_v}{r_1} \right)^2 \ln \left( \frac{r_1}{r_v} \right) \right] \quad (2-29) \]

or

\[ C_3(T - T_1) + (C_4 - C_5 \rho_{II})^{-1} \ln \frac{T}{T_1} \]

\[ = \frac{Q_Ir_1^2}{4} \left[ 1 - \left( \frac{r_1}{r_v} \right)^2 + \left( \frac{r_v}{r_1} \right)^2 \ln \left( \frac{r_1}{r_v} \right) \right] \quad (2-30) \]

Substitution of \( T_v \) for \( T \) results

\[ C_3(T_v - T_1) + (C_4 - C_5 \rho_{II}) \ln \frac{T_v}{T_1} \]

\[ = \frac{Q_Ir_1^2}{4} \left[ 1 - \left( \frac{r_1}{r_v} \right)^2 \left[ 1 - \ln \left( \frac{r_v}{r_1} \right)^2 \right] \right] \quad (2-31) \]

From the equality of power output before and after the formation of regions, an equation can be written as

\[ r_v^2 = \frac{r_2^2(Q_{II} - Q_{III})}{Q_I} + \frac{r_1^2(Q_I - Q_{II})}{Q_I} \quad (2-32) \]

In the beginning of irradiation the whole fuel cross section is in region III. Equation (2-20) can be used to find
the average temperature gradient, i.e., the temperature gradient at 1/3 of the fuel rod radius. Upon substituting this into Equation (2-10) an expression for solving $T_1$ at time $t$ can be obtained

$$K_{III}(T_1) = C_1 t \exp\left(-C_2/(T_1)^{a_1}\right)/\left[2(T_1)^{3/2}Q_{III}\right] \quad (2-33)$$

Also, if there were no central void, a centerline temperature $T_{CL}$ could be found from Equation (2-22). In this case Equation (2-22) becomes

$$C_3(T_{CL} - T_S) + (C_4 - C_5 \rho_{III})^{-1} \ln \left(\frac{T_{CL}}{T_S}\right) = \frac{Q_{III}r_s^2}{4} \quad (2-34)$$

At a certain time $t$, $T_1$ and $T_2$ are found from Equations (2-33) and (2-12), respectively. Then $r_2$ can be calculated from Equation (2-23) and then $r_1$ from Equation (2-27). Now, $r_v$ and $T_v$ can be found from Equations (2-32) and (2-31).

The temperature distribution can then be obtained from Equations (2-22), (2-26), and (2-30).

If the temperature $T_1$ calculated from Equation (2-33) is greater than the temperature $T_{CL}$ calculated from Equation (2-34), $T_1$ is above the 'threshold' value as mentioned before and region I is not formed. $T_2$ may also be greater than $T_{CL}$ and region II will not occur. Then the problem becomes a two-region or one-region problem. It is easy to write equations for these cases and they will be considered in the
computer program.

The temperature gradients are calculated by Equations (2-20), (2-24), and (2-28).

It should be noted that in solving the nonlinear equations, use has been made of the Muller's iteration method in the computer program (30,31).
III. RADIATION ANALYSIS

A. Introduction

Fission products are new atoms formed during fissioning process in reactor fuels. Some of them are solids which can cause swelling due to the difference in total atomic volumes before and after fissioning. The probable average swelling of fission solids in oxide fuel $\text{UO}_2$ or $(\text{U}_{0.80}\text{Pu}_{0.20})\text{O}_2$ was calculated to be $\Delta V/V = 0.35\%$ per $10^{20}$ fissions/cm$^3$ of fuel (32). The other fission products, about 10% of the total, are inert gases xenon and krypton.

The behavior of fission gases in oxide fuels during irradiation is very complex and no simple model is available for describing the complex behavior on a quantitative base. In the early time a simple diffusion model assumed that the fission-gas diffused slowly in the grains of the fuel until it emerged from the surface of the grain and then escaped rapidly through the interconnected channels leading to the specimen surface. This model has been proved incorrect for oxide fuel as discussed by Carroll (33). A widely used model for swelling analysis presented by Greenwood and Speight (34) can not be applied to oxide fuels as discussed by Nichols (16). Still many others (35, 36, 37) are limited by conditions and do not appear to be applicable for the purposes.
Recent results of work (14,38,39) have led to the conclusion that a trapping process during the diffusion of bubbles of fission gases controls the swelling and release of fission gases.

Barnes and Nelson have proposed the basic ideas of fission-gas swelling in oxide fuels and have given a qualitative model for it (14,15). Along the proposed general lines Nichols has also given a semi-quantitative model for the swelling (16).

In the present analysis these models will be referred to from time to time. Hence, the outlines of these models are given as follows:

1. The Barnes and Nelson model

   (1) If the solubility of the inert gases during irradiation is small, then bubbles will be formed and will migrate under the influence of the temperature gradient.

   (2) The dislocation lines and the grain boundaries will trap the bubbles, and only when their radii are greater than 650 Å and 5500 Å on the respective positions will the temperature gradients drag them off.

   (3) Precipitates can also anchor bubbles smaller than about 5500 Å.

   (4) The gas in the outer lower temperature region of the fuel rod is not likely to be released until very high
contents are attained. The gas in the inner region, where bubbles can migrate up the temperature gradient, however, will escape from the grains to the grain boundaries or rod center.

2. The Nichols model

(1) Bubbles are trapped by dislocation lines during short exposures and the fission-gas swelling is due to the increase of bubble size at their fixed positions.

(2) As the bubbles grow, it is assumed that they reach a critical size at which the thermal gradient drags them off the dislocations and they migrate with relatively few collisions until they reach a grain boundary. Here the swelling is felt to be primarily due to the capture of incoming bubbles by those already on the grain boundaries.

(3) When the bubbles reach a second critical size, they are dragged from the grain boundaries and migrate up the thermal gradient to the central void and release their gas there.

B. General Descriptions for the Model

General descriptions for the proposed model suitable for the calculation of fission-product swelling and fission gas releasing in a digital computer are given below:

(1) The solubility of inert gases during irradiation
is very small and easily precipitate to form gas bubbles on suitable nuclei - dislocations \((14,15,16)\).

(2) Bubbles are ready to form and to reach their equilibrium pressure as a result of vacancy diffusion since in the fissioning process there are always enough vacancies introduced in accompanying with fission-gas atoms \((15,16)\).

(3) Bubbles are trapped on dislocations and grain boundaries and retarded from motion by the retarding forces of dislocations and grain boundaries \((15,16)\).

(4) When grain growth occurs, all fission gases trapped are assumed to be released to the central void for low density oxide fuels. This assumption is based on the phenomenon that fission gas will escape during any process involving gross reordering of the microstructure \((33,39)\), and the bubble concentration for \(\text{UO}_2\) extrapolated from the experimental result of Cornell et al. \((40)\) approaches to zero at about \(1600^\circ\text{C}\), above which the grain growth is usually considered to occur.

(5) As the bubble size increases with time at dislocations due to the increase of fission-gas atoms, there will be a critical size at which the bubble can be dragged off from the dislocation by the driving force due to thermal gradients, which dominate bubble mobility in an oxide fuel \((15,16)\).

(6) The migration of bubbles up the thermal gradients
is treated by surface diffusion mechanism. Usually, there are three mechanisms for calculation of the migration velocity, i.e., surface diffusion, volume diffusion, and vapor transport mechanisms. Barnes and Nelson showed that the volume diffusion mechanism was not important for oxide fuels (14). Vapor transport is important only at very high temperature at which, however, grain growth occurs in a very short time. Therefore, only the surface diffusion mechanism is considered in this analysis.

(7) The shape of bubbles is assumed to remain spherical during migration. This is normally assumed for swelling analysis.

(8) If the velocity of a bubble calculated from surface diffusion does not enable the bubble to travel one hundredth of the grain diameter in the period after it began to move to the time of analysis, the bubble is assumed still anchored on the dislocation (15).

(9) After bubbles leave the anchored sites, new bubbles will nucleate and grow again on these sites as time proceeds. The moving bubbles will migrate with very few collisions and eventually arrive at the grain boundaries since they leave the sites at approximately the same size and so the same velocity (16). It is assumed that all newly generated gas atoms are absorbed by the new bubbles. In other words, the moving bubbles and the bubbles arriving
at the grain boundaries will not absorb new gas atoms. Although there is some swelling loss for the moving bubbles and those bubbles at the grain boundaries, yet there is some swelling gain for the new bubbles at the original sites.

(10) The swelling at the grain boundaries is primarily due to the capture of incoming bubbles by those already on the grain boundaries (16). There the bubble size increases until reaching another critical size at which the bubble is dragged off the grain boundary by the thermal gradient force and migrates all the way to the central void. This is because the velocity varies exponentially with temperature. But, if the velocity does not enable the bubble to travel one grain diameter in the time under consideration, the bubble is assumed still at the grain boundary (15).

(11) Fission products escaped to the gap between the clad and the fuel is neglected since the trapping process is quite effective in the outer low temperature region (33).

(12) The total fission-product swelling is the sum of the swellings due to solid fission products, gas bubbles anchored in the grains, gas bubbles moving in the grains, and gas bubbles anchored at the grain boundaries. The solid-product swelling is proportional to the fission rate and time as mentioned early in the introduction.

(13) The total gas atoms released to the central void
is the sum of the gas atoms from fabricated pores and gas bubbles as a result of the grain growths, and the gas atoms in bubbles leaving grain boundaries from the outer region.

C. Fission-product Swelling

Fission-product swelling consists of fission-solid swelling and fission-gas swelling. The appropriate formulas will be given in this section.

1. Fission-solid swelling

With a fission rate \( \dot{F} \), fissions per unit volume per unit time, in an irradiation time \( t \), the total number of fissions per unit volume is \( \dot{F}t \). If the average volumetric contribution of the solid fission products is known, the solid volume swelling is given by

\[
(\Delta V/V)_{sld} = C_6 \dot{F}t
\]

where

\[
(\Delta V/V)_{sld} = \text{fission-solid swelling}
\]

\[
C_6 = \text{the average volumetric contribution per fission per unit volume.}
\]

The fission rate can be related to the volumetric heat rate by

\[
\dot{F} = Q/C'_1
\]

where \( C'_1 \) is a conversion constant.
2. Anchored gas-bubble swelling in grain

Fission-gas atoms of xenon and krypton are known to be extremely insoluble and easily precipitate to form gas bubbles on suitable nuclei like dislocations as assumed. The bubbles will be anchored there by the retarding force exerted by the tension of the dislocation lines. In calculating the swelling due to these anchored bubbles, one should know the size and number of the bubbles. If there are $N_0$ bubbles with diameter $d$ in a unit volume, then the swelling is given by

$$ (AV/V)_{sgg} = \frac{1}{6} \pi d^3 N_0 $$

(3-2)

where $(AV/V)_{sgg} =$ swelling of anchored gas bubbles in grain.

In this equation $N_0$ can be obtained from experimental data and then the problem is to find the diameter of the bubble. There are two relations involving another unknown, the pressure inside the bubble, for evaluating the diameter. One is the pressure equilibrium condition and the other is the state equation of gas.

The pressure equilibrium condition can be written as

$$ p + \sigma_H + \sigma' = 4\gamma/d $$

(3-3)

where

- $p =$ gas pressure inside the bubble
- $\gamma =$ surface tension of fuel material
\( \sigma_H = \) hydrostatic stress in the material around the bubble due to externally applied forces or thermal and radiation effects (positive value for tension)

\( \sigma' = \) intrinsic stress of the fuel.

The hydrostatic stress \( \sigma_H \) is the average value of the three principal stresses in the fuel. It should be noted that the swelling of gas bubbles contributes the stress and the stress, in turn, affects bubble size. Therefore, both \( \sigma_H \) and \( d \) are values obtained through iteration process (20).

The intrinsic stress \( \sigma' \) can generally be neglected for oxide fuel due to high temperature (14). Then Equation (3-3) becomes

\[
p + \sigma_H = 4\gamma/d \quad (3-4)
\]

The state equation of an ideal gas is

\[
p\left(\frac{1}{6}\pi d^3\right) = mkT_a \quad (3-5)
\]

where

- \( m \) = number of the fission-gas atoms in the bubble
- \( k \) = Boltzmann constant.

The total number of fission-produced atoms per unit volume is about \( 2\hat{F}t \) of which about 10% are the inert gases, xenon and krypton. Therefore, in Equation (3-5) the number of fission-gas atoms in one bubble is

\[
m = 0.2 \hat{F}t/N_o \quad (3-6)
\]
Eliminating \( p \) from Equations (3-4) and (3-5), it gives

\[
d^3 - \frac{4\gamma}{\sigma_H} d^2 - \frac{6mk}{\pi \sigma_H} T_a = 0 \tag{3-7}
\]

For bubbles with diameters less than about \( 10^{-5} \) cm, the inert gases may not behave ideally and Van der Waals correction should be applied (35). Then the gas equation of state becomes

\[
p \left( \frac{1}{6} \pi d^3 - mb \right) = mk T_a \tag{3-8}
\]

where \( b \) is the Van der Waals correction term (a value of \( 8.3 \times 10^{-23} \) cm\(^3\) may be used in the computations (20,41)).

Again, eliminating \( p \) results

\[
d^4 - \frac{4\gamma}{\sigma_H} d^3 + \frac{6m}{\pi \sigma_H} (k T_a - \sigma_H b)d + \frac{24 mb}{\pi \sigma_H} = 0 \tag{3-9}
\]

This equation is more general than Equation (3-7) and is to be used to compute the diameter of a bubble for the swelling of bubbles at fixed positions. If the value of \( \sigma_H \) in equation is negative, i.e., compressive stress, the solution of the diameter required is the unique positive root. If \( \sigma_H \) is positive, namely tensile stress, there are two positive real roots. It is clear that the smaller one is the desired solution.

If \( \sigma_H \) is very small in comparison with \( 4\gamma / d \), it may be neglected and Equation (3-9) reduces to
\[ d^3 - \frac{3mkT_a}{2\pi\gamma} d - \frac{6mb}{\pi} = 0 \]  

(3-10)

Here the only positive real root is the solution.

3. Fission-gas swelling after movement of bubbles

The driving force due to thermal gradient has been obtained from thermal diffusion theory (42, 43). For surface diffusion mechanism this force can be written as

\[ F_s = \frac{\pi Q^*_S d^3 \text{d}T}{6\Omega T_a \text{d}r} \]  

(3-11)

where \( Q^*_S \) is the activation energy for surface diffusion, \( \Omega \) is the molecular volume of the fuel, and \( F_s \) is the driving force.

The maximum retarding force exerted by a dislocation in an oxide fuel is approximately \( 10^{-4} \) dyne (14). When the driving force is equal to this value, a critical diameter of bubble for movement can be written as

\[ d_{CG} = \left( \frac{6\Omega T_a 10^{-4}}{\pi Q^*_S \text{d}T/\text{dr}} \right)^{\frac{1}{3}} \]  

(3-12)

When the bubble begins to move, its velocity derived from surface diffusion mechanism is given by (43)

\[ V_g = \frac{6D_s v \Omega Q^*_S}{kT_a^2 d} \frac{\text{d}T}{\text{d}r} \]  

(3-13)

where

\[ D_s = \text{the surface diffusion coefficient} \]

\[ v = \text{number of diffusion molecules per unit surface area}. \]
The surface diffusion coefficient may be written as

\[ D_s = D_o \exp\left(-\frac{Q_S}{kT_a}\right) \quad (3-13a) \]

where

- \( D_o \) = coefficient
- \( Q_S \) = activation energy.

Also, there is a critical velocity as mentioned in the general description. It can be expressed as

\[ V_{cg} = \frac{D}{100(t-t_{cg})} \quad (3-14) \]

where \( D \) is the diameter of grain and \( t_{cg} \) is the time when the bubble attains its critical size. This critical time can be evaluated approximately by the combination of Equations (3-4), (3-5), and (3-6).

The result is

\[ t_{cg} = \pi N_o \left(\frac{4\gamma}{d_{cg}} - \sigma_H\right) \frac{d_{cg}^3}{(1.2 \hat{P} k T_a)} \quad (3-15) \]

The value of the hydrostatic stress, \( \sigma_H \) in the above equation, is obtained by iteration without considering the movement of bubbles. This is why Equation (3-15) is approximate. Since \( \sigma_H \) varies little with time during the constant linear power operation as can be seen from the calculations, thus the approximation is reasonable.
When the velocity is smaller than the critical velocity, it is considered that the bubble will still remain in the grain; otherwise, without this consideration, there will be 1\% of the bubbles arriving at the grain boundaries, and the total swelling will not be much different for these two cases.

When the velocity is greater than the critical velocity, bubbles are now moving up the thermal gradient to reach the grain boundaries. The swelling at the grain boundaries is primarily due to the collisions of the incoming bubbles with those already on the grain boundaries. This kind of swelling is the result of volume increase in the formation of a new bubble due to collision and coalescence of two bubbles. The diameter of the new bubble can be obtained from the condition of pressure equilibrium and the conservation of gas atoms.

If one bubble with diameter $d_o$ and number of gas atoms $m_o$ and another with $d_i$ and $m_i$ collide with each other and form the new bubble with diameter $d$, the equation of state for the new bubble is

$$p \left(\frac{1}{6} \pi d^3\right) = (m_o + m_i)k T_a \quad (3-16)$$

or

$$Pd^3 = P_o d_o^3 + p_i d_i^3. \quad (3-17)$$

By substitution of $p$'s from Equation (3-4), the equation
for solving $d$ is found to be

$$d^3 - \frac{4\gamma}{\sigma_H} d^2 - \left( d_o^3 + d_i^3 \right) + \frac{4\gamma}{\sigma_H} \left( d_o^2 + d_i^2 \right) = 0. \quad (3-18)$$

If $\sigma_H$ is negative, there is always a solution that is the only positive root. If $\sigma_H$ is positive, the solution is then the smaller one of the two positive roots and $\sigma_H$ must be

$$\sigma_H \leq \frac{8\gamma}{3d}. \quad (3-19)$$

Otherwise, there will be no equilibrium value of $d$, i.e., Equation (3-18) gives no solution. Actually, this is not likely to happen in the analysis since the hydrostatic tension stress is produced only in the outer, lower temperature region of the fuel rod where the bubble diameters are small in the period of interest. This may also occur to Equation (3-9) and the discussion has been omitted for the same reason.

If $\sigma_H$ is small, the diameter is then

$$d = \left( d_o^2 + d_i^2 \right)^{1/2} \quad (3-20)$$

Now for the swelling of bubbles at grain boundaries, consider a small parallelepiped across the grain in the direction of the temperature gradient with cross-section area $\Delta A$ and an average length $L$ given by

$$L = \frac{\pi}{4} D \quad (3-21)$$

where $D$ is the diameter of the grain.
Since the bubbles are evenly distributed with \( N_0 \) bubbles per unit volume as assumed, the number of bubbles per unit length is then \( N_0^{1/3} \). The time required for a bubble to travel across the parallelepiped is

\[
  t_g = \frac{L}{V_g} \tag{3-22}
\]

where \( V_g \) is given by Equation (3-13).

If \( t_g \) is divided into \( n_g \) intervals where \( n_g \) is the number of bubbles on the length \( L \), the time interval \( \Delta t \) can be written as

\[
  \Delta t = \frac{t_g}{n_g} \tag{3-23}
\]

where \( n_g = \frac{L}{V_g N_0^{1/3}} \). \tag{3-24}

Then,

\[
  \Delta t = \frac{1}{(V_g N_0^{1/3})} \tag{3-25}
\]

It is seen that the time interval is just the time for a bubble to travel one bubble spacing. Thus an approximation may be made that the moving bubbles within each time interval will not collide with one another as they arrive at the grain boundary.

The critical time \( t_{cg} \) for the movement of bubbles from dislocations was given by Equation (3-15). At any time \( t \) before the last bubble arriving the grain boundary, the time-interval number is

\[
  n = \frac{t - t_{cg}}{\Delta t}, \quad t_{cg} < t < (t_{cg} + t_g) \tag{3-26}
\]
When \( n=1 \), the number of bubbles per unit area at the small boundary area \( \Delta A \) is

\[
N_{0,1} = N_0 \nu_g \Delta t \Delta A / \Delta A = N_0^{2/3} \tag{3-27}
\]

The first subscript of \( N_{0,1} \) represents the number of collisions for the bubbles at the grain boundary having been collided with incoming bubbles and the second subscript is the number of time intervals passing by after the critical time \( t_{cg} \).

It is clear that in each time interval there will be \( N_{0,1} \) bubbles per unit area arriving at \( \Delta A \).

The probability for each of the bubbles with diameter \( d_i \) on the grain boundary to be collided is \( \pi (d_o + d_i)^2 N_{0,1} \) where the subscript of the diameter is the number of collisions having been collided and \( d_o \) or \( d_{cg} \) is the critical diameter given by Equation (3-12). The diameter of a bubble after collision can be calculated from Equation (3-18) or (3-20).

When \( n=2 \), the number of bubbles per unit area at \( \Delta A \) having been collided once is

\[
N_{1,2} = \frac{\pi (d_o + d'_o)^2}{4} N_{0,1} N_{0,1} \tag{3-28}
\]

and the number of uncollided bubbles per unit area at \( \Delta A \) is

\[
N_{0,2} = 2N_{0,1} - 2N_{1,2} \tag{3-28a}
\]
When \( n=3 \)

\[
N_{2,3} = \frac{\pi}{4} (d_o + d_1)^2 N_{0,1} N_{1,2} \\
N_{1,3} = \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,2} + N_{1,2} - N_{2,3} \\
N_{0,3} = N_{0,1} + N_{0,2} - N_{2,3} - 2 \left( \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,2} \right)
\]

(3-29)

When \( n=4 \)

\[
N_{3,4} = \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{2,3} \\
N_{2,4} = \frac{\pi}{4} (d_o + d_1)^2 N_{0,1} N_{1,3} + N_{2,3} - N_{3,4} \\
N_{1,4} = \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,3} + N_{1,3} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{1,3} \\
N_{0,4} = N_{0,1} + N_{0,3} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{2,3} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{1,3} \\
- 2 \left( \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,3} \right)
\]

When \( n=5 \)

\[
N_{4,5} = \frac{\pi}{4} (d_o + d_3)^2 N_{0,1} N_{3,4} \\
N_{3,5} = \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{2,4} + N_{3,4} - N_{4,5} \\
N_{2,5} = \frac{\pi}{4} (d_o + d_1)^2 N_{0,1} N_{1,4} + N_{2,4} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{2,4} \\
N_{1,5} = \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,4} + N_{1,4} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{1,4} \\
N_{0,5} = N_{0,1} + N_{0,4} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{3,4} - \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{2,4} \\
- \frac{\pi}{4} (d_o + d_3)^2 N_{0,1} N_{1,4} - 2 \left( \frac{\pi}{4} (d_o + d_2)^2 N_{0,1} N_{0,4} \right)
\]

(3-31)
With definitions:

\[ C_{i,n} = \frac{\pi}{4} (d_0 + d_i)^2 N_{0,1} N_{i,n} \quad \text{when } -1 < i \leq n \]

\[ C_{i,n} = N_{i,n} = 0 \quad \text{when } i = n \quad (3-32) \]

\[ C_{i,n} = N_0,1 - \sum_{k=0}^{n} C_{k,n} \quad \text{when } i = -1, \]

a general recurrence relation can be expressed as

\[ N_{i,n} = C_{i-1,n-1} + N_{i,n-1} - C_{i,n-1} \quad (3-33) \]

where \( i \) is the number of collisions for a bubble on the grain boundary and \( n \) is the number of time intervals having passed. Then, the swelling due to bubbles on grain boundaries is

\[ \left( \frac{\Delta V}{V} \right)_{\text{sgb}} = \frac{1}{L} \sum_{i=0}^{n-1} \left\{ N_{i,n} \left( \frac{1}{6} \pi d_i^3 \right) \right\} \quad (3-34) \]

The swelling due to new bubbles nucleated and anchored on the bubble sites is

\[ \left( \frac{\Delta V}{V} \right)_{\text{sgg}} = \frac{1}{6} \pi d^3 N_0 \quad (3-35) \]

where \( d' \) is the diameter of the new bubbles obtained from Equation (3-9) or (3-10) in which \( m \) is given by

\[ m = 0.2 \frac{F(t-t_{cg})}{N_0} \quad (3-36) \]

The swelling due to bubbles still moving in the grains is
The total gas-bubble swelling after the movement of bubbles in the grains is given by the sum of the swellings due to new bubbles nucleated on the bubble sites, bubble collisions at grain boundaries, and bubbles still moving in the grains. That is

\[
\left( \frac{\Delta V}{V} \right)_{\text{sgm}} = \left( \frac{\Delta V}{V} \right)_{\text{gb}} + \left( \frac{\Delta V}{V} \right)_{\text{gb}'} + \left( \frac{\Delta V}{V} \right)_{\text{sgm}}
\]

(3-38)

where

\[
\left( \frac{\Delta V}{V} \right)_{\text{gas}} = \text{the total gas-bubble swelling}
\]

\[
\left( \frac{\Delta V}{V} \right)_{\text{gb}} = \text{the swelling due to bubbles on grain boundaries}
\]

\[
\left( \frac{\Delta V}{V} \right)_{\text{gb}'} = \text{the swelling due to new bubbles anchored on the bubble sites}
\]

\[
\left( \frac{\Delta V}{V} \right)_{\text{sgm}} = \text{the swelling due to bubbles still moving in the grains}.
\]

It should be noted that all hydrostatic stresses involved in the calculations of bubble diameters after the movement of bubbles in grains are obtained by iteration processes without considering the movement of bubbles.

If bubbles at grain boundaries attain the critical conditions for migrating away from the grain boundaries at xth collision, then all bubbles after the xth collision will migrate away. The swelling due to bubbles on the grain boundaries given in Equation (3-34) becomes
If \( n > n_g \) or \( t > (t_{cg} + t_g) \), the swelling due to moving bubbles in Equation (3-37) is then equal to zero.

The calculations of bubble number and diameters are given in the computer program.

The operating time \( t \) in the analysis may not exceed \( 2t_{cg} \), a period of time more than 8 months estimated from the calculations of a numerical example in this work.

D. Gas Release

The fission-gas release consists of two parts: (1) due to grain growth and (2) due to bubbles moving away from grain boundaries. These can be formulated in the following:

1. The region of grain growth in the fuel rod is the circular area with radius \( r_2 \) indicated in Figure 2.1. At a time \( t \), the total number of fission-gas atoms released to the central void due to this part can be written as

\[
m_{gg} = 0.2 \dot{F}t(\pi r_2^2) \tag{3-40}
\]

2. Bubbles on grain boundaries will migrate away under two conditions as assumed in the model:

(a) They must attain a critical size at which the force due to temperature gradients equals the retarding
force exerted by grain boundaries. The latter is given by (16)

\[ F_{gb} = \frac{\pi}{2} \gamma_{gb} d \]  

(3-41)

where \( F_{gb} \) = the maximum retarding force of grain boundary
\( \gamma_{gb} \) = grain boundary surface tension.

The former is given by Equation (3-11). From the equality of these two forces, a critical diameter for bubbles at grain boundaries being able to move away is given by

\[ d_{cb} = \frac{3\Omega \gamma_{gb} T_a}{\sqrt{g_s (dT/dr)}} \]  

(3-42)

where

\( d_{cb} \) = the critical diameter for bubbles at grain boundaries being able to move away.

If the bubbles start movement at xth collision at which the diameter of a collided bubble obtained from Equation (3-18) equals or exceeds \( d_{cb} \) for the first time, then the corresponding critical time is

\[ t_{cb} = t_{cg} + x \cdot \Delta t \]  

(3-43)

where \( t_{cg} \) and \( \Delta t \) are obtained from Equations (3-15) and (3-25), respectively.

(b) The moving bubbles must move quickly enough so that the higher and higher temperatures can accelerate them all the way to the central void. This critical velocity
based on the concept of Barnes and Nelson (15) can be written as

\[ V_{cb} = D/(t-t_{cb}) \]  \hspace{1cm} (3-44)

The velocity of the moving bubbles is given by Equation (3-13) for the surface diffusion mechanism, i.e.,

\[ V_b = \frac{6 D_s \cdot \Omega \cdot Q_s}{k T_a} \frac{dT}{dr} \]  \hspace{1cm} (3-45)

where \( V_b \) is the velocity of the moving bubbles leaving grain boundaries.

If \( V_b < V_{cb} \), the bubbles are assumed to be still anchored on the grain boundaries.

If \( V_b > V_{cb} \), the total number of moving bubbles per unit volume migrating away is

\[ N_t = \frac{1}{L} \sum_{k=x+1}^{n} N_{x,k} \]  \hspace{1cm} (3-46)

where \( N_{x,k} \) is the number of bubbles per unit area moving away from the grain boundary after \( k \) time-intervals and having been collided \( x \) times with incoming bubbles. The combination of Equations (3-4) and (3-5) gives the total number of gas atoms released to the central void from bubbles at station \( j \) of the fuel rod, i.e.,

\[ m_b = N_t \left( \frac{4 \gamma}{d_{cb}} - \sigma_H \right) \left( \frac{\pi d_{cb}^3}{6 k T_a} \right) \pi (r_j^2 - r_{j-1}^2) \]  \hspace{1cm} (3-47)
The calculations for gas release under the conditions are shown in the computer program.
IV. THERMAL, RADIATION AND STRESS ANALYSIS

A. Introduction

After the temperature distribution, the thermal expansion and the irradiation swelling are determined, the next task is to formulate the problem of stress analysis.

Because of the high thermal stress, the yield points of both clad and fuel of an oxide fuel element in a fast reactor may be exceeded. The equations of equilibrium and compatibility for stress analysis must be combined with nonlinear stress-strain relations instead of the linear Hooke’s law. To solve the nonlinear equations a method of successive approximations is used in this analysis. This method is an extension of Picard’s method (44) of successive approximations to nonlinear equations and was first used by Ilyushin (45) in his treatment of a thin shell.

In the earlier days this method may be used only for relatively simple problems because of the cumbersome calculations. Recently, the high-speed computing machinery came into existence, and it becomes a powerful method in the stress calculations involving the strain-hardening exhibited in experimental stress-strain curves and has been used in various problems by Manson (19) and Mendelson (18). Usually, problems of stress analysis solved in the elastoplastic range are simplified by assumptions such as the elastic strains to be neglected and the material
to be perfectly plastic in the plastic range. The method of successive approximations makes these assumptions unnecessary.

In the stress analysis of nuclear reactor fuel elements the special phenomenon of radiation effects and the variable thermal conductivity in the fuel make the analysis different from ordinary problems.

B. Basic Assumptions

In this analysis the following basic assumptions are made:

1. Both fuel and cladding materials under irradiation are approximately isotropic.
2. The fuel element is in a plane strain state.
3. It is also in an axial symmetry.
4. The Prandtl-Reuss equations are applicable (46,47).
5. The von Mises yield criterion is to improve the assumptions (48).
6. The Bauschinger effect is ignored.

The first assumption implies that the material properties are functions of radius but not of direction during the irradiation.

Since the cylindrical fuel element in a fast reactor is long and thin and the axial displacement could be ignored, the basic assumption 2 is sound.
The third basic assumption implies that all variables vary only with radius and time. The Prandtl-Reuss relation is a more general case for elastoplastic stress-strain expressions, and has been confirmed experimentally by Lode (49).

Two criteria for the yielding, the von Mises and the Tresca yield conditions, have received appreciable acceptance in engineering applications. The Mises yield criterion, however, has been indicated the more general validity and it becomes the choice in this analysis.

The last basic assumption means that both the fuel and cladding materials have the strength equally in tension and in compression. This is a most frequently used assumption in stress analysis.

C. Equations and Solutions of Stress Analysis

Since the cylindrical-type fuel element is in axial symmetry, the three principal axes are coincided with the cylindrical coordinates. The stresses are designated by $\sigma_r$, $\sigma_\theta$ and $\sigma_z$ and the corresponding strains by $\varepsilon_r$, $\varepsilon_\theta$ and $\varepsilon_z$ in the cylindrical coordinates. The body force is neglected and the plane strain as assumed. At a given time the equilibrium equation is then

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (4-1)$$
The strain-displacement relations are

\[ \varepsilon_r = \frac{du}{dr} \]  
\[ \varepsilon_\theta = \frac{u}{r} \]  

where \( u \) is the radial displacement at \( r \), and the corresponding compatibility equation is

\[ \frac{d\varepsilon_\theta}{dr} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 . \]  

The strain consists of four parts: the elastic strain, the thermal expansion strain, the irradiation dilatation strain and the plastic strain. The equations are

\[ \varepsilon_r = \frac{1}{E}[\sigma_r - \mu(\sigma_\theta + \sigma_z)] + \alpha T + \varepsilon_I + \varepsilon_{\text{pr}} \]  
\[ \varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \mu(\sigma_r + \sigma_z)] + \alpha T + \varepsilon_I + \varepsilon_{\text{p}\theta} \]  
\[ \varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_r + \sigma_\theta)] + \alpha T + \varepsilon_I + \varepsilon_{\text{pz}} \]  

where \( \alpha \) is the linear coefficient of thermal expansion, \( T \) is the temperature rise with respect to a reference temperature, \( E \) is the modulus of elasticity, \( \mu \) is Poisson's ratio, \( \varepsilon_I \) is the irradiation dilatation strain and \( \varepsilon_{\text{pr}}, \varepsilon_{\text{p}\theta}, \varepsilon_{\text{pz}} \) are the plastic strains.

For plane strain state the axial total strain \( \varepsilon_z \) may be neglected

\[ \varepsilon_z = 0 \]  

(4-6)
50

The thermal and irradiation dilatation strains may be combined in a single term, \( \varepsilon_R \) (2). Equations (4-5) become

\[
\varepsilon_r = \frac{1}{E} [\sigma_r - \mu (\sigma_\theta + \sigma_z)] + \varepsilon_R + \varepsilon_{pr}
\]

\[
\varepsilon_\theta = \frac{1}{E} [\sigma_\theta - \mu (\sigma_z + \sigma_r)] + \varepsilon_R + \varepsilon_{p\theta}
\]

\[
\varepsilon_z = \frac{1}{E} [\sigma_z - \mu (\sigma_r + \sigma_\theta)] + \varepsilon_R = \varepsilon_{pz}
\]

where

\[
\varepsilon_R = \alpha T + \varepsilon_r. \tag{4-7a}
\]

The basic assumption 4 implies the material to be incompressible in the plastic range, i.e.

\[
\varepsilon_{pr} + \varepsilon_{p\theta} + \varepsilon_{pz} = 0 \tag{4-8}
\]

Then, the mean strain \( \varepsilon \) is

\[
\varepsilon = \frac{1}{3} (\varepsilon_r + \varepsilon_\theta - 3\varepsilon_R). \tag{4-9}
\]

The average normal stress is

\[
s = \frac{1}{3} (\sigma_r + \sigma_\theta + \sigma_z). \tag{4-10}
\]

The combination of Equations (4-7), (4-8), (4-9) and (4-10) results

\[
s = \frac{E}{1 - 2\mu} \varepsilon \tag{4-11}
\]

The elastic strain deviators are
\[ e_r = (\varepsilon_r - \varepsilon_R - \varepsilon_{pr}) - e \]
\[ e_\theta = (\varepsilon_\theta - \varepsilon_R - \varepsilon_{p\theta}) - e \]
\[ e_z = (-\varepsilon_R - \varepsilon_{pz}) - e \]  

and the stress deviators are

\[ s_r = \sigma_r - s = \frac{E}{1+\mu} e_r \]
\[ s_\theta = \sigma_\theta - s = \frac{E}{1+\mu} e_\theta \]  
\[ s_z = \sigma_z - s = \frac{E}{1+\mu} e_z . \]  

If another function \( g \) is defined as

\[ g = \frac{1}{3}(\varepsilon_r - \varepsilon_\theta - 3\varepsilon_R), \]  

and if \( \varepsilon_R \) and plastic strains are known, all other variables of interest may be expressed in terms of \( e \) and \( g \) as follows:

Combination of Equations (4-2), (4-3), (4-9) and (4-14) gives

\[ u = \frac{3}{2}(e-g)r \]  

and then

\[ \varepsilon_r = \frac{du}{dr} = \frac{3}{2}(e+g+2 \varepsilon_R) \]
\[ \varepsilon_\theta = \frac{u}{r} = \frac{3}{2}(e-g) \]  

The elastic strain deviators become
\[
e_r = \frac{1}{2}(e+3g+4 \varepsilon_R^{-2} \varepsilon_{pr})
\]
\[
e_\theta = \frac{1}{2}(e-3g-2 \varepsilon_R^{-2} \varepsilon_{p\theta}) \tag{4-18}
\]
\[
e_z = -\varepsilon_R^{-\varepsilon_{pz}} - e
\]

The average normal stress has already been in the right form given by Equation (4-11). The stress deviators in Equation (4-13) are then
\[
s_r = \frac{E}{2(1+\mu)} (e+3g+4 \varepsilon_R^{-2} \varepsilon_{pr})
\]
\[
s_\theta = \frac{E}{2(1+\mu)} (e-3g-2 \varepsilon_R^{-2} \varepsilon_{p\theta}) \tag{4-19}
\]
\[
s_z = \frac{E}{1+\mu} (-\varepsilon_R^{-\varepsilon_{pz}} - e).
\]

The three components of stress can now be written as
\[
\sigma_r = \frac{3E}{2(1+\mu)(1-2\mu)} e + \frac{3E}{2(1+\mu)} g + \frac{E}{1+\mu} (2 \varepsilon_R^{-\varepsilon_{pr}}) \tag{4-20}
\]
\[
\sigma_\theta = \frac{3E}{2(1+\mu)(1-2\mu)} e - \frac{3E}{2(1+\mu)} g - \frac{E}{1+\mu} (\varepsilon_R^{+\varepsilon_{p\theta}}) \tag{4-20a}
\]
\[
\sigma_z = \frac{3\mu E}{(1+\mu)(1-2\mu)} e - \frac{E}{1+\mu} (\varepsilon_R^{+\varepsilon_{pz}}). \tag{4-20b}
\]

By substitution of the stresses and strains from Equations (4-16), (4-17), (4-20) and (4-20a) into the equilibrium Equation (4-1) and the compatibility Equation (4-4), new equilibrium and compatibility equations with the defined variables e and g as dependent variables can be obtained. With suitable boundary conditions, e and g can be solved easily from these equations.
If the modulus of elasticity and Poisson's ratio vary moderately along the radius, their average values in the temperature range may be used. Then the coefficients of the equations are constants and a closed-form solution can be obtained. This fact will be applied to the thin cladding material in the analysis. As for the oxide fuel region, the temperature gradient and the density changes due to irradiation are so large that the coefficients cannot be assumed constant any more. In this case a method of finite difference will be used to solve the differential equations.

1. **Closed-form solution for clad**

Consider the clad a hollow cylinder with inner radius \(r_i\) and outer radius \(r_o\), subjected to an internal pressure \(p_g\) which is the pressure in the gap between the fuel and the clad, an external pressure \(P_\theta\) due to coolant, and a radial temperature distribution \(T(r)\). All the stresses and strains are functions of radius \(r\) at a given time. The modulus of elasticity \(E\) and the Poisson ratio \(\nu\) are assumed to be constant.

Upon substitution of \(\sigma_r\), \(\sigma_\theta\), \(\varepsilon_r\) and \(\varepsilon_\theta\) from Equations (4-20), (4-20a), (4-16) and (4-17) into (4-1) and (4-4), the equilibrium Equation (4-1) becomes

\[
\frac{d}{dr} \left[ \frac{3E}{2(1+\mu)(1-2\mu)} \varepsilon_r + \frac{3E}{2(1+\mu)} \varepsilon_\theta + \frac{E}{1+\mu} (2 \varepsilon_R - \varepsilon_{pr}) \right] \\
+ \frac{3E}{1+\mu} \varepsilon_\theta + \frac{E}{(1+\mu)r} (3\varepsilon_R - \varepsilon_{pr} + \varepsilon_{p\theta}) = 0
\]  

(4-21)
or

\[
\frac{1}{1-2\mu} \frac{de}{dr} + \frac{dg}{dr} = - \frac{2(g+\varepsilon_R)}{r} - \frac{4}{3} \frac{d\varepsilon_R}{dr} + \frac{2}{3} \frac{d\varepsilon_{pr}}{dr} + \frac{2(\varepsilon_{pr}-\varepsilon_{p\theta})}{3r}
\]

(4-21a)

and the compatibility Equation (4-4) changes to

\[
\frac{de}{dr} - \frac{dg}{dr} = \frac{2g+\varepsilon_R}{r}
\]

(4-22)

Solving for \(e\) and \(g\) from the combination of Equations (4-21a) and (4-22), one obtains

\[
e = \frac{1-2\mu}{3(1-\mu)} [\varepsilon_{pr} - 2 \varepsilon_R + \int_{r_i}^r \frac{\varepsilon_{pr} - \varepsilon_{p\theta}}{r} dr] + C_1
\]

(4-23)

\[
g = -\frac{2(1+\mu)}{3(1-\mu)} \frac{1}{r^2} \int_{r_i}^r r \varepsilon_R dr + \frac{1-2\mu}{3(1-\mu)} \varepsilon_{pr} - 2 \varepsilon_R - \frac{1}{r^2} \int_{r_i}^r r (\varepsilon_{pr} + \varepsilon_{p\theta}) dr + \frac{C_2}{r^2}
\]

(4-24)

where \(C_1\) and \(C_2\) are integral constants to be determined.

The boundary conditions for the clad are

\[
\begin{align*}
    r &= r_i, & \sigma_r &= -P_g \\
    r &= r_o, & \sigma_r &= -P_o
\end{align*}
\]

(4-25)

where the coolant pressure \(P_o\) is a given value and the gap pressure \(P_g\) will be formulated in the next section.

With these, \(C_1\) and \(C_2\) are determined

\[
C_1 = \frac{2(1+\mu)(1-2\mu)}{3E(r_o^2-r_i^2)} [r_i^2 P_g - r_o^2 (P_o+G)]
\]

(4-26)
\[ C_2 = \frac{2r_i^2 r_o^2 (1+\mu)}{3E(r_o^2 - r_i^2)} [-p_g + p_o + G] \quad (4-27) \]

where

\[
G = \frac{E}{1-\mu} \left[ \frac{1}{2(1+\mu)} \int_{r_i}^{r_o} \frac{\varepsilon p_r - \varepsilon p_\theta}{r} \, dr - \frac{1}{r_o^2} \int_{r_i}^{r_o} \varepsilon_R \, dr \right] - \frac{1-2\mu}{2(1+\mu)r_o^2} \int_{r_i}^{r_o} r(\varepsilon p_r + \varepsilon p_\theta) \, dr . \quad (4-28)
\]

The integrals can be evaluated by numerical methods if the values of the integrands at each station are known.

2. Finite-difference method for fuel zone

The fuel has an outer surface radius \( r_s \) and may have an inner void along the axial axis with a radius \( r_v \) as a result of irradiation. There are also internal and external pressures \( p_v \) and \( p_g \). The fuel is divided into \( N \) intervals (not necessarily equal) along the radius and so there are \( N+1 \) stations. The first station is at the center for a solid fuel rod, or at the inner radius \( r_v \) for the fuel with a central void. In an application of central difference between station \( i \) and \( i-1 \) \((50)\), the compatibility Equation \((4-22)\) can be written as

\[
\frac{e_i - e_{i-1}}{r_i - r_{i-1}} - \frac{g_i - g_{i-1}}{r_i - r_{i-1}} = \frac{1}{2} \left[ \frac{2(g_i + \varepsilon_R i)}{r_i} + \frac{2(g_{i-1} + \varepsilon R_{i-1})}{r_{i-1}} \right] \quad (4-29)
\]

or

\[
e_i - A_i g_i = e_{i-1} - B_i g_i + C_i \quad (4-29a)\]
where

\[ A_i = 1 + a_i \]
\[ B_i = 1 - b_i \]
\[ C_i = a_i \epsilon_{R,i} + b_i \epsilon_{R,i-1} \quad (4-30) \]
\[ a_i = \frac{r_i - r_i-1}{r_i} \]
\[ b_i = \frac{r_i - r_i-1}{r_i-1} \]

In the same manner the equilibrium Equation (4-21) can be written as

\[ D_i e_i + F_i g_i = U_i e_i - 1 + V_i g_i - 1 - W_i \quad (4-31) \]

where

\[ D_i = \frac{c_i}{1 - 2 \mu_i} \]
\[ F_i = c_i (1 + a_i) \]
\[ U_i = D_i - 1 \]
\[ V_i = c_i - 1 (1 - b_i) \quad (4-32) \]
\[ W_i = \frac{1}{3} (f_i - f_i - 1 + a_i h_i + b_i h_i - 1) \]
\[ c_i = \frac{3E_i}{2 (1 + \mu_i)} \]
\[ f_i = 2c_i (2 \epsilon_{R,i} - \epsilon_{pr,i}) \]
\[ h_i = c_i (-\epsilon_{pr,i} + \epsilon_{pr,i} + 3 \epsilon_{R,i}) \]
Combination of Equation (4-29a) and (4-31) gives \( e \) and \( g \) at ith station in terms of those at the (i-1)st station, i.e.,

\[
e_i = K_i e_{i-1} + L_i g_{i-1} + M_i \quad (4-33)
\]

\[
g_i = K'_i e_{i-1} + L'_i g_{i-1} + M'_i \quad (4-34)
\]

where

\[
K_i = \frac{F_i + A_i U_i}{F_i + A_i D_i} \quad K'_i = \frac{U_i - D_i}{F_i + A_i D_i}
\]

\[
L_i = \frac{A_i V_i - F_i B_i}{F_i + A_i D_i} \quad L'_i = \frac{V_i + D_i B_i}{F_i + A_i D_i} \quad (4-35)
\]

\[
M_i = \frac{F_i C_i - A_i W_i}{F_i + A_i D_i} \quad M'_i = \frac{-W_i - C_i D_i}{F_i + A_i D_i}
\]

By successive application of Equations (4-33) and (4-34), the values of \( e \) and \( g \) at the ith station can be correlated to the values of \( e \) and \( g \) at the first station. Let this linear relation be written as

\[
e_i = \alpha_i e_1 + \beta_i g_1 + \gamma_i \quad (4-36)
\]

\[
g_i = \alpha'_i e_1 + \beta'_i g_1 + \gamma'_i \quad (4-37)
\]

Also

\[
e_{i-1} = \alpha_{i-1} e_1 + \beta_{i-1} g_1 + \gamma_{i-1} \quad (4-38)
\]

\[
g_{i-1} = \alpha'_{i-1} e_1 + \beta'_{i-1} g_1 + \gamma'_{i-1} \quad (4-39)
\]
where the coefficients $\alpha_i$, $\beta_i$, $\gamma_i$, ... are to be determined.

Substituting Equations (4-36), (4-37), (4-38) and (4-39) into Equations (4-33) and (4-34), and rearranging, one obtains

\[
(\alpha_i - K_i \alpha_{i-1} - L_i \alpha_{i-1}) e_i + (\beta_i - K_i \beta_{i-1} - L_i \beta_{i-1}) g_i
+ (\gamma_i - K_i \gamma_{i-1} - L_i \gamma_{i-1} - M_i) = 0 \quad (4-40)
\]

\[
(\alpha_i' - K_i' \alpha_{i-1}' - L_i' \alpha_{i-1}') e_i + (\beta_i' - K_i' \beta_{i-1}' - L_i' \beta_{i-1}') g_i
+ (\gamma_i' - K_i' \gamma_{i-1}' - L_i' \gamma_{i-1}' - M_i') = 0 \quad (4-41)
\]

Because $e_i$ and $g_i$ depend on boundary conditions and can be completely arbitrary; Equations (4-40) and (4-41) are therefore valid for all values of $e_i$ and $g_i$. It follows that the coefficients in these equations must be zero, or

\[
\alpha_i = K_i \alpha_{i-1} + L_i \alpha_{i-1}
\]

\[
\beta_i = K_i \beta_{i-1} + L_i \beta_{i-1}
\]

\[
\gamma_i = K_i \gamma_{i-1} + L_i \gamma_{i-1} + M_i \quad (4-42)
\]

\[
\alpha_i' = K_i' \alpha_{i-1}' + L_i' \alpha_{i-1}'
\]

\[
\beta_i' = K_i' \beta_{i-1}' + L_i' \beta_{i-1}'
\]

\[
\gamma_i' = K_i' \gamma_{i-1}' + L_i' \gamma_{i-1}' + M_i' .
\]
If the kinds of Equations (4-33), (4-34), (4-36) and (4-37) are written for the second station, one gets

\[ e_2 = K_2 e_1 + L_2 g_1 + M_2 \]
\[ g_2 = K'_2 e_1 + L'_2 g_1 + M'_2 \]  \hspace{1cm} (4-43)

and

\[ e_2 = \alpha_2 e_1 + \beta_2 g_1 + \gamma_2 \]
\[ g_2 = \alpha'_2 e_1 + \beta'_2 g_1 + \gamma'_2 \]  \hspace{1cm} (4-44)

Comparison of Equations (4-43) and (4-44) gives

\[ \alpha_2 = K_2 \quad \beta_2 = L_2 \quad \gamma_2 = M_2 \]  \hspace{1cm} (4-45)
\[ \alpha'_2 = K'_2 \quad \beta'_2 = L'_2 \quad \gamma'_2 = M'_2 \]

Subsequently all the other \( \alpha' \)'s, \( \beta' \)'s,... can be computed successively by the recurrence Equations (4-42). If \( e_1 \) and \( g_1 \) are obtained, all the other values of \( e \) and \( g \) can then be found by Equations (4-36) and (4-37). To evaluate \( e_1 \) and \( g_1 \), use is made of the boundary conditions. At the \( (N+1) \)st station the radial stress is equal to the gap pressure, i.e.

\[ \sigma_{r,N+1} = -P_g = \frac{-P_o (r_i^2 - r_s^2) (T_g') a}{[r_i^2 (1 + \varepsilon_i')^2 - r_s^2 (1 + \varepsilon_s')^2] (T_o') a} \]  \hspace{1cm} (4-46)

where

\[ P_o = \text{initial pressure} \]
\[ (T_o') a = \text{initial absolute temperature} \]
\[ (T_g') a = \text{absolute temperature of gap} \]
\(\varepsilon_{6i} = \text{tangential strain of clad at } r_i\)

\(\varepsilon_{8s} = \text{tangential strain of fuel at } r_s\)

and from Equation (4-20)

\[
\sigma_{r,N+1} = \frac{3E_{N+1}}{2(1+\mu_{N+1})(1-2\mu_{N+1})} \varepsilon_{N+1} + \frac{3E_{N+1}}{2(1+\mu_{N+1})} \eta_{N+1}
\]

\[+ \frac{E_{N+1}}{1+\mu_{N+1}} (2 \varepsilon_{R,N+1} - \varepsilon_{pr,N+1}) \quad (4-47)\]

or

\[-P_g = D_{N+1} \varepsilon_{N+1} + C_{N+1} \eta_{N+1} + \frac{2}{3} C_{N+1} (2 \varepsilon_{R,N+1} - \varepsilon_{pr,N+1}) \quad (4-48)\]

Equations (4-36) and (4-37) can be written for the last station as

\[
\varepsilon_{N+1} = \alpha_{N+1} \varepsilon_1 + \beta_{N+1} \eta_1 + \gamma_{N+1}
\]

\[\eta_{N+1} = \alpha'_{N+1} \varepsilon_1 + \beta'_{N+1} \eta_1 + \gamma'_{N+1} \quad (4-49)\]

Substituting in Equation (4-47) and solving for \(\varepsilon_1\) result

\[
\varepsilon_1 = P_{N+1} \eta_1 + Q_{N+1} \quad (4-50)\]

where

\[
P_{N+1} = \frac{-(D_{N+1} \beta_{N+1} + C_{N+1} \beta'_{N+1})}{D_{N+1} \alpha_{N+1} + C_{N+1} \alpha'_{N+1}}
\]

\[
Q_{N+1} = \frac{-(P_g + D_{N+1} \gamma_{N+1} + C_{N+1} \gamma'_{N+1} + \frac{1}{3} f_{N+1})}{D_{N+1} \alpha_{N+1} + C_{N+1} \alpha'_{N+1}} \quad (4-51)\]
For a solid fuel rod the boundary condition at the first station is

$$\sigma_{r,1} = \sigma_{\theta,1}. \quad (4-52)$$

$\sigma_{r,1}$ and $\sigma_{\theta,1}$ can be obtained from Equations (4-20) and (4-20a), i.e.,

$$\sigma_{r,1} = D_1 e_1 + C_1 g_1 + \frac{2}{3} C_1 (2 \varepsilon_{R,1} - \varepsilon_{pr,1}) \quad (4-53)$$

$$\sigma_{\theta,1} = D_1 e_1 - C_1 g_1 - \frac{2}{3} C_1 (\varepsilon_{R,1} + \varepsilon_{p\theta,1}) \quad (4-54)$$

Thus the value of $g_1$ for the boundary condition is obtained:

$$g_1 = \frac{1}{3} (\varepsilon_{pr,1} - \varepsilon_{p\theta,1} - 3 \varepsilon_{R,1}) \quad (4-55)$$

For a fuel rod with a central void, the boundary condition at the first station is

$$\sigma_{r,1} = -P_v = -\left[ \frac{P_0 (T_v)_a}{(T_0)_a (1+\varepsilon_{\delta v})^2} + \frac{(0.2 \pi r_2^2 \dot{T}_t + M_b) k(T_v)_a}{\pi r_v^2 (1+\varepsilon_{\delta v})^2} \right] \quad (4-56)$$

where

$$(T_v)_a = \text{absolute temperature in central void at time } t.$$  

$$\varepsilon_{\delta v} = \text{tangential strain at } r_v$$

The value of $g_1$ can then be found from the combination of Equations (4-50), (4-53) and (4-56)

$$g_1 = \frac{-P_v - D_1 Q_{N+1} - \frac{1}{3} \varepsilon_1}{C_1 + D_1 P_{N+1}} \quad (4-57)$$

After $g_1$ is determined by Equation (4-55) or (4-57) for
the different boundary conditions, \( e_1 \) can then be computed from Equation (4-50).

For the case of solid fuel rod, the value of \( r \) at the first station is zero and some of the coefficients become infinite. Therefore, a relatively small value may be assigned to \( r_1 \) in order to avoid the situation in the computation for this case.

As soon as \( e \) and \( g \) are solved from the equilibrium and the compatibility equations in either the closed form solution or the finite-difference method described above, all stresses and strains can then be determined.

In solving \( e \) and \( g \), plastic strains are assumed to be known. In the method of successive approximations, values of plastic strains at stations will be assumed at first and then solutions of stresses and strains are obtained. Hereafter the problem is to find new plastic strains to continue the successive calculations till convergence is satisfied.

When plastic strains occur, a yield criterion and a flow rule will bring the triaxial problem to be associated with uniaxial problem. The equivalent quantities will be compared to actual experimental results thus to obtain new plastic strains. These will be developed as follows:

The flow rule of Prandtl-Reuss relations describes that the plastic strain increment at any instant of loading is proportional to the instantaneous stress deviator. That is
\[
\frac{d \varepsilon_{pr}}{s_r} = \frac{d \varepsilon_{p\theta}}{s_{\theta}} = \frac{d \varepsilon_{pz}}{s_z} = d\lambda \quad (4-58)
\]
or
\[
\frac{\varepsilon_{pr}}{s_r} = \frac{\varepsilon_{p\theta}}{s_{\theta}} = \frac{\varepsilon_{pz}}{s_z} = \lambda \quad (4-59)
\]

where \(\lambda\) is a non-negative constant.

According to Mises' yield criterion, the distortion energy stored in a unit volume of the material at yielding is a constant quantity. The result may be expressed as

\[
\sqrt{s_r^2 + s_{\theta}^2 + s_z^2} = K \quad (4-60)
\]

where \(K\) is the constant. For the case of simple tension,

\[
K = \frac{2}{3} \sigma_e \quad (4-61)
\]

where \(\sigma_e\), the equivalent stress, is the axial stress in the simple tension case. Then the triaxial stresses can be related to the equivalent stress by

\[
\sigma_e = \frac{3}{2} \sqrt{s_r^2 + s_{\theta}^2 + s_z^2} \quad (4-62)
\]

The equivalent stress deviator corresponding to the equivalent stress is

\[
s_e = \frac{2}{3} \sigma_e \quad (4-63)
\]

Hence the equivalent plastic strain is
\[ \varepsilon_{pe} = \lambda \varepsilon_e = \frac{2}{3} \lambda \varepsilon_e \quad (4-64) \]

The combination of Equations (4-59), (4-62) and (4-64) gives

\[ \varepsilon_{pe} = \sqrt{\frac{2}{3}(\varepsilon_{pr}^2 + \varepsilon_{p\theta}^2 + \varepsilon_{pz}^2)} \quad (4-65) \]

The total strain deviators are

\[ e_{tr} = \varepsilon_r - \varepsilon = e_r + \varepsilon_R + \varepsilon_{pr} \]
\[ e_{t\theta} = \varepsilon_\theta - \varepsilon = e_\theta + \varepsilon_R + \varepsilon_{p\theta} \]
\[ e_{tz} = \varepsilon_z - \varepsilon = e_z + \varepsilon_R + \varepsilon_{pz} \quad (4-66) \]

If three modified total strain deviators are defined as

\[ e'_{tr} = e_{tr} - \varepsilon_R = e_r + \varepsilon_{pr} \]
\[ e'_{t\theta} = e_{t\theta} - \varepsilon_R = e_\theta + \varepsilon_{p\theta} \quad (4-67) \]
\[ e'_{tz} = e_{tz} - \varepsilon_R = e_z + \varepsilon_{pz} \]

or

\[ e'_{tr} = \frac{1}{2}(e + 3g + 4 \varepsilon_R) \]
\[ e'_{t\theta} = \frac{1}{2}(e - 3g - 2 \varepsilon_R) \quad (4-67a) \]
\[ e'_{tz} = -e - \varepsilon_R \]

the following expressions can be obtained by substituting \( e_r \), \( e_\theta \) and \( e_z \) from Equation (4-13) into (4-67) and using the rela-
tions from Equation (4-59):

\[ e'_{tr} = \left( \frac{1+\mu}{E\lambda} + 1 \right) \varepsilon_{pr} \]
\[ e'_{t\theta} = \left( \frac{1+\mu}{E\lambda} + 1 \right) \varepsilon_{p\theta} \]  \hspace{1cm} (4-68)
\[ e'_{tz} = \left( \frac{1+\mu}{E\lambda} + 1 \right) \varepsilon_{pz} \]

Squaring both sides of the above three equations, adding together, multiplying with a value of 2/3 on either side and then taking square root, one obtains

\[ e'_{te} = \left( \frac{1+\mu}{E\lambda} + 1 \right) \varepsilon_{pe} \]  \hspace{1cm} (4-69)

where

\[ e'_{te} = \sqrt{\frac{2}{3} (e'_{tr}^2 + e'_{t\theta}^2 + e'_{tz}^2)} \]  \hspace{1cm} (4-70)
\[ \varepsilon_{pe} = \sqrt{\frac{2}{3} (\varepsilon_{pr}^2 + \varepsilon_{p\theta}^2 + \varepsilon_{pz}^2)} \]  \hspace{1cm} (4-65)

Here \( e'_{te} \) is the corresponding modified total equivalent strain defined.

Combination of Equations (4-68) and (4-69) results

\[ \varepsilon_{pr} = \frac{\varepsilon_{pe}}{e'_{te}} e'_{tr} \]
\[ \varepsilon_{p\theta} = \frac{\varepsilon_{pe}}{e'_{te}} e'_{t\theta} \]  \hspace{1cm} (4-71)
\[ \varepsilon_{pz} = \frac{\varepsilon_{pe}}{e'_{te}} e'_{tz} \]

If \( \lambda \) obtained from Equation (4-64) is substituted into (4-69), the result is
\[
e_{te} = e_{pe} + \frac{2(1+\mu)\sigma_e}{3E} ~ (4-72)
\]

Here, \(\sigma_e\) and \(e_{pe}\) can be related to the actual stress-strain curves obtained in simple tension or compression test. The equation representing these curves may be written in a general form as

\[
e_{te} = e_{pe} + \frac{\sigma_e}{E} = f(\sigma_e, T) ~ (4-73)
\]

where \(e_{te}\) is the total equivalent strain which can be expressed as a function of equivalent stress and temperature.

To represent the stress-strain curve for the cladding material, an empirical equation due to Ramberg and Osgood (51) may be used, i.e.,

\[
e_{te} = \frac{\sigma_e}{E_c} + K_c \left( \frac{\sigma_e}{E_c} \right)^h \text{ or } e_{pe} = K_c \left( \frac{\sigma_e}{E_c} \right)^h ~ (4-74)
\]

where \(h\), a function of temperature, may be written as

\[
h = c_{c1} + c_{c2} T_c + c_{c3} ~ (4-76)
\]

where \(c_{c1}, c_{c2}, \text{ and } c_{c3}\) are constant, \(T_c\) is the temperature in clad, \(E_c\) is the modulus of elasticity of clad, and \(K_c\) can be written as (52)

\[
K_c = \frac{3}{7} \left( \frac{E_c}{\gamma} \right)^{m-1} ~ (4-77)
\]
where $\sigma_y$ is the assumed yield strength of 0.2% offset and is a function of temperature, i.e.,

$$\sigma_y = C_{c4} - C_{c5} T_c$$  \hspace{1cm} (4-78)

in which $C_{c4}$ and $C_{c5}$ are constant.

For the oxide fuel material the stress-strain curve beyond the proportional limits may be represented by

$$\varepsilon_{te} = K_f \left( \frac{\sigma_p}{\sigma_p} \right)^{n_f}$$  \hspace{1cm} (4-79)

where

$$K_f = \frac{\sigma_p}{E_f}$$  \hspace{1cm} (4-80)

$$E_f = (C_{14} - C_{15} T_f) (1 - C_{16} P - C_{17} P^2)$$  \hspace{1cm} (4-81)

$$\sigma_p = C_{18} / (T_f - C_{19})$$  \hspace{1cm} (4-82)

$$n_f = C_{20} (T_f - C_{21})^2 + C_{22}$$  \hspace{1cm} (4-83)

the $C$'s are constant, $\sigma_p$ is the proportional limit, $E_f$ is the modulus of elasticity and $T_f$ is the temperature in the fuel, and $P$ is the porosity given for each zone in the fuel.

It should be noted that the oxide fuel material at high temperature in a fast reactor during normal operation is assumed to be plastic. The ductile-brittle transition occurs in the temperature range of 600°C to 800°C for UO$_2$ (54).

From the combination of Equations (4-72) and (4-73), an equivalent plastic strain associated with experimental stress-
strain curves can be obtained. Inserting this value of $\varepsilon_{pe}$ into Equations (4-71), one can obtain the new plastic strains required for the iteration process.

D. The Method of Successive Approximation

The principle of the successive-approximation method is to solve the elastoplastic problem through iterations in which each time involves essentially the solution of a linear problem, when the plastic strains are presumed.

The steps in the procedure to carry out the successive approximations are:

(a) Assuming three plastic strains $\varepsilon_{pr}$, $\varepsilon_{p\theta}$, and $\varepsilon_{pz}$ for each station along the radius.

(b) With the known values of plastic strains, thermal expansion strains and irradiation dilatation strains, solving the variables $e$ and $g$ from the combination of the equilibrium Equation (4-21) and the compatibility Equation (4-22) in closed-form solution for clad or in finite-difference solution for fuel.

(c) Calculating the modified total strain deviators $e'_{tr}$, $e'_{t\theta}$ and $e'_{tz}$ from Equation (4-67a)

(d) Calculating the corresponding modified total equivalent strain $e'_{te}$ from Equation (4-70) (a result of the application of Prandtl-Reuss relations and the von Mises yield criterion).
(e) Solving equivalent plastic strain $\varepsilon_{pe}$ from the combination of Equations (4-72) and (4-73) (stress-strain curves being associated).

(f) Calculating new plastic strains from Equation (4-71) with obtained values in steps (c), (d) and (e). These new plastic strains are to be used in the next iteration.

(g) Repeat steps (b), (c), (d), (e) and (f). The iteration proceeds till desired convergence is obtained.

Experience shows that the iterated results are converged rapidly.
V. COMPUTER PROGRAM AND NUMERICAL EXAMPLE

A. Introduction to Computer Program

A computer code, ISUNE 1, written in Fortran IV language and adapted for the IBM 360 computer at the Iowa State University Computer Center was developed to perform the thermal, radiation and stress analysis for oxide cylindrical fuel pins or fuel elements of fast reactors in unsteady state.

The coolant is a liquid metal. The clad is a metal tube. The bonding material is an inert gas. For the fuel, pure UO$_2$ or a mixture of PuO$_2$·UO$_2$ (or PuO$_2$·ThO$_2$·UO$_2$) is applicable.

The function of the Code is to analyze and calculate several important variables as the functions of radial space and irradiation time of the fuel elements:

1. The irradiation swelling, fission-gas release and fuel burnup,
2. The temperature distributions and thermal gradients in the cladding and in the fuel zone,
3. The changes of fission-gas pressure in the fuel elements,
4. The fuel-clad gap thickness and the radii of the central void, the columnar-grain region and equi-axed-grain region,
5. The stresses and strains produced in the fuel elements.

The complete computer program is presented in Appendix A.
Schematic flowcharts of the code are shown in Figures 5.1 and 5.2. Important notations in the program are given in Tables 5.1 and 10.1.

The input data which must be given to the computer including geometric parameters, thermal parameters, radiation parameters, elastic and plastic parameters and all constants involved. These parameters, constants and their values for the numerical example are tabulated with names arranged in alphabetical order in Table 5.1.

B. Results

In the numerical example, the fuel rod is UO$_2$ of 90% smeared density with an outer diameter of 0.215 inch, the clad tube is Type 316 stainless steel with an outer diameter of 0.250 inch and an inner diameter of 0.220 inch, and the fuel element is cooled with sodium, bonded with helium and irradiated at 15 kw/ft of constant linear power. All other values of parameters and constants for performing the calculation are listed in Table 5.1.

Some important results of the numerical example are presented in Figures 5.3 through 5.13, and also tabulated in Tables 11.1 through 11.21, Appendix B.

Figures 5.3 and 5.4 show the time variation with the temperature and radii at the surfaces of the columnar-grain and equiaxed-grain regions in the fuel zone respectively. The
Start

Read input

Calculate volumetric heat rate, porosity, and fission rate

Calculate temperature distribution in clad

Set MEMO=13

Calculate fission-gas swelling and release after the movement of bubbles

Are bubbles moving?

Yes

No

Is MEMO=13?

Yes

No

Set time

Initialize number of iteration, NITRL=0, and fuel surface temperature, \( T_{sl} \)

Iterate stresses and strains in clad (see Figure 5.2)

More time to be set?

Yes

No

Calculate fuel surface temperature \( T_s \) and gap

Calculate and print all the information interested

Yes

More time to be set?

No

Is NITRL=0?

Yes

No

\[ T_{sl} = \frac{T_s + T_{sl}}{2} \]

Compare \( T_s \) with \( T_{sl} \), converged?

Yes

More time to be set?

No

NITRL=NITRL+1

Determine three region radii and calculate temperature distribution, temperature gradient, and other functions of temperature in fuel

Stop

Figure 5.1. General flowchart for the code
Assume plastic strains $\varepsilon_{pr}$, $\varepsilon_{p\theta}$ and $\varepsilon_{pz}$

Calculate thermal and swelling strains

Calculate variables $e$ and $g$ from equations of equilibrium and compatibility

Calculate modified total strain deviators $e_{tr}'$, $e_{t\theta}'$ and $e_{tz}'$ from Equations (4-67a)

Calculate modified equivalent total strain $e_{te}'$ from Equation (4-70)

Solve equivalent plastic strain $\varepsilon_{pe}$ from Equations (4-72) and (4-73)

$\langle$ Converged? $\rangle$ Yes

No

Calculate new plastic strains $\varepsilon_{pr}'$, $\varepsilon_{p\theta}'$ and $\varepsilon_{pz}$ from Equations (4-71)

Figure 5.2. Flowchart for computing stresses and strains by the method of successive approximations
Table 5.1. Input data

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Data</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABST</td>
<td>Absolute temperature conversion constant</td>
<td>273.0</td>
<td>°C</td>
</tr>
<tr>
<td>ACTEGD</td>
<td>Activation energy for grain growth(^a)</td>
<td>8.38x10(^{-12})</td>
<td>erg/mol.</td>
</tr>
<tr>
<td>AMIG</td>
<td>Molecular volume of the matrix(^b)</td>
<td>4.09x10(^{-23})</td>
<td>cm(^3)</td>
</tr>
<tr>
<td>AVCV</td>
<td>Coolant average velocity(^c)</td>
<td>793.0</td>
<td>cm/sec</td>
</tr>
<tr>
<td>AVGDN</td>
<td>Avogadro number</td>
<td>6.025x10(^{23})</td>
<td>mol./mole</td>
</tr>
<tr>
<td>BOLZ</td>
<td>Boltzmann constant</td>
<td>1.380x10(^{-16})</td>
<td>erg/°K/mol.</td>
</tr>
<tr>
<td>CDEN</td>
<td>Coolant density(^c)</td>
<td>0.867</td>
<td>gm/cm(^3)</td>
</tr>
<tr>
<td>CELSMD</td>
<td>Clad elastic modulus</td>
<td>24.10(^{13})</td>
<td>1000psi</td>
</tr>
<tr>
<td>CEXCL</td>
<td>Thermal linear expansion coefficient of clad</td>
<td>19.26x10(^{-6})</td>
<td>cm/cm-°C</td>
</tr>
<tr>
<td>CLDK</td>
<td>Clad thermal conductivity</td>
<td>0.16748</td>
<td>w/cm-°C</td>
</tr>
<tr>
<td>CNAVST</td>
<td>Minimum significant hydrostatis stress</td>
<td>1000.0</td>
<td>dynes/cm(^2)</td>
</tr>
<tr>
<td>CNER</td>
<td>Control error</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td>CNT1</td>
<td>Conversion constant for fission rate</td>
<td>3.206x10(^{-11})</td>
<td>w-sec</td>
</tr>
<tr>
<td>CNT2</td>
<td>Constant for calculation of thermal conductivity of fuel(^d)</td>
<td>0.0130</td>
<td>-</td>
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\(^a\)Calculated from experimental data (27).
\(^b\)Data given by Nichols (12).
\(^c\)Data provided by Ma (5).
\(^d\)Empirical constants (29).
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<thead>
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<th>Data</th>
<th>Units</th>
</tr>
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<tr>
<td>CNT3</td>
<td>Constant for calculation of thermal conductivity of fuel&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.4848</td>
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</tr>
<tr>
<td>CNT4</td>
<td>&quot;</td>
<td>0.4465</td>
<td>-</td>
</tr>
<tr>
<td>CNT5</td>
<td>Constant for calculation of grain size&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.2306</td>
<td>-</td>
</tr>
<tr>
<td>CNT6</td>
<td>Constant for calculation of solid fission-product swelling&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0.35x10^-22</td>
<td>-</td>
</tr>
<tr>
<td>CNT7</td>
<td>Constant for calculation of linear expansion coefficient of fuel&lt;sup&gt;f&lt;/sup&gt;</td>
<td>6.0x10^-6</td>
<td>-</td>
</tr>
<tr>
<td>CNT8</td>
<td>&quot;</td>
<td>2.0x10^-9</td>
<td>-</td>
</tr>
<tr>
<td>CNT9</td>
<td>&quot;</td>
<td>1.7x10^-12</td>
<td>-</td>
</tr>
<tr>
<td>CNT10</td>
<td>Fraction of fission gas atoms</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>CNT11</td>
<td>Constant for determining bubble number per unit volume&lt;sup&gt;g&lt;/sup&gt;</td>
<td>3.83x10^17</td>
<td>-</td>
</tr>
<tr>
<td>CNT12</td>
<td>&quot;</td>
<td>0.66x10^14</td>
<td>-</td>
</tr>
<tr>
<td>CNT14</td>
<td>Constant for calculation of elastic modulus of fuel&lt;sup&gt;h&lt;/sup&gt;</td>
<td>26.</td>
<td>-</td>
</tr>
<tr>
<td>CNT15</td>
<td>&quot;</td>
<td>0.0025</td>
<td>-</td>
</tr>
<tr>
<td>CNT16</td>
<td>&quot;</td>
<td>1.9</td>
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<sup>a</sup>Experimental result (32).
<sup>b</sup>Empirical constants (23).
<sup>c</sup>Obtained from experimental data (40).
<sup>d</sup>Constants in the empirical equation in reference (53).
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<th>Units</th>
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<tr>
<td>CNT17</td>
<td>Constant for calculation of elastic modulus of fuel(^h)</td>
<td>0.9</td>
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<tr>
<td>CNT18</td>
<td>Constant for fitting stress-strain curves of fuel(^i)</td>
<td>(10^4)</td>
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<td>CNT19</td>
<td>&quot;</td>
<td>375.</td>
<td>-</td>
</tr>
<tr>
<td>CNT20</td>
<td>&quot;</td>
<td>(6.17\times10^{-6})</td>
<td>-</td>
</tr>
<tr>
<td>CNT21</td>
<td>&quot;</td>
<td>400.</td>
<td>-</td>
</tr>
<tr>
<td>CNT22</td>
<td>&quot;</td>
<td>4.</td>
<td>-</td>
</tr>
<tr>
<td>CNTC1</td>
<td>Constant for fitting stress-strain curves of clad(^j)</td>
<td>5.5</td>
<td>-</td>
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<tr>
<td>CNTC2</td>
<td>&quot;</td>
<td>(7.11\times10^{-20})</td>
<td>-</td>
</tr>
<tr>
<td>CNTC3</td>
<td>&quot;</td>
<td>7.03</td>
<td>-</td>
</tr>
<tr>
<td>CNTC4</td>
<td>&quot;</td>
<td>40.60</td>
<td>-</td>
</tr>
<tr>
<td>CNTC5</td>
<td>&quot;</td>
<td>0.01785</td>
<td>-</td>
</tr>
<tr>
<td>CNTG1</td>
<td>Constant for calculating fuel emissivity(^f)</td>
<td>0.986</td>
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<tr>
<td>CNTG2</td>
<td>&quot;</td>
<td>(1.625\times10^{-4})</td>
<td>-</td>
</tr>
<tr>
<td>CNTG3</td>
<td>Constant for calculating clad emissivity(^k)</td>
<td>0.544</td>
<td>-</td>
</tr>
<tr>
<td>CNTG4</td>
<td>&quot;</td>
<td>(3.540\times10^{-4})</td>
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\(^i\) From experimental results (54).

\(^j\) From experimental results (55,56).

\(^k\) From reference (57).
Table 5.1 (Continued)

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<th>Name</th>
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<tr>
<td>CNTG5</td>
<td>Constant for calculating thermal conductivity of gas in fuel-clad gap&lt;sup&gt;f&lt;/sup&gt;</td>
<td>6.875x10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td></td>
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<tr>
<td>CNTG6</td>
<td>&quot;</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>CPR</td>
<td>Total vapor pressure in fabricated pores&lt;sup&gt;b&lt;/sup&gt;</td>
<td>10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>dynes/cm&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>CPSNRT</td>
<td>Poisson's ratio of clad</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>CSRH</td>
<td>Cooland specific heat&lt;sup&gt;c&lt;/sup&gt;</td>
<td>1.377</td>
<td>w-sec/gm-°C</td>
</tr>
<tr>
<td>CSRC</td>
<td>Cross-sectional radius for collisions between matrix and vapor molecule&lt;sup&gt;b&lt;/sup&gt;</td>
<td>3x10&lt;sup&gt;-8&lt;/sup&gt;</td>
<td>cm</td>
</tr>
<tr>
<td>CTHK</td>
<td>Thermal conductivity of coolant&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.817</td>
<td>w/cm-°C</td>
</tr>
<tr>
<td>DENFR&lt;sup&gt;(1)&lt;/sup&gt;</td>
<td>Fraction of fuel theoretical density in columnar grain region</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>DENFR&lt;sup&gt;(2)&lt;/sup&gt;</td>
<td>Fraction of fuel theoretical density in equiaxed grain region</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>DENFR&lt;sup&gt;(3)&lt;/sup&gt;</td>
<td>Fraction of fuel theoretical density in unaffected grain region</td>
<td>0.90</td>
<td></td>
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<tr>
<td>DGG2</td>
<td>Diameter of grain at equiaxed-grain region surface</td>
<td>15x10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>cm</td>
</tr>
<tr>
<td>DGGI</td>
<td>Initial diameter of grain</td>
<td>5x10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>cm</td>
</tr>
<tr>
<td>DSLF</td>
<td>Dislocation retarding force</td>
<td>10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>dynes</td>
</tr>
<tr>
<td>EQDIA</td>
<td>Equivalent diameter of coolant channel&lt;sup&gt;c&lt;/sup&gt;</td>
<td>3.20</td>
<td>cm</td>
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Table 5.1 (Continued)

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<tr>
<td>GBTF</td>
<td>Grain boundary retarding force</td>
<td>300.</td>
<td>dyne/cm</td>
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<tr>
<td>HTVP</td>
<td>Heat of vaporization</td>
<td>9.91x10^{-12}</td>
<td>erg/mol.</td>
</tr>
<tr>
<td>MMAX</td>
<td>A maximum number for setting time</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>MNITR</td>
<td>Maximum number of iteration</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>NINTC</td>
<td>Number of interval in clad</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>NINTT</td>
<td>Number of interval in fuel</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>PRESF</td>
<td>Coolant pressure</td>
<td>0.0147</td>
<td>1000psi</td>
</tr>
<tr>
<td>PSNRT</td>
<td>Poisson's ratio of fuel</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>PWRL</td>
<td>Linear heat power</td>
<td>492.</td>
<td>w/cm</td>
</tr>
<tr>
<td>RMT</td>
<td>Room temperature</td>
<td>25.</td>
<td>°C</td>
</tr>
<tr>
<td>RRIN</td>
<td>Internal surface radius of clad tube</td>
<td>0.2790</td>
<td>cm</td>
</tr>
<tr>
<td>RROU</td>
<td>Outer surface radius of clad tube</td>
<td>0.3175</td>
<td>cm</td>
</tr>
<tr>
<td>RRS</td>
<td>Surface radius of fuel</td>
<td>0.2730</td>
<td>cm</td>
</tr>
<tr>
<td>SACTE</td>
<td>Surface diffusion activation energy¹</td>
<td>6.606x10^{-12}</td>
<td>erg/mol.</td>
</tr>
<tr>
<td>SCACTE</td>
<td>Activation energy for surface diffusion coefficient¹</td>
<td>6.606x10^{-12}</td>
<td>erg/mol.</td>
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</table>

¹From reference (16).
<table>
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<th>Description</th>
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<tr>
<td>SPR</td>
<td>Constant</td>
<td>$1.64 \times 10^{14}$</td>
<td>dynes/cm$^2$</td>
</tr>
<tr>
<td>SRDFCO</td>
<td>Coefficient of surface diffusion coefficient</td>
<td>$10^4$</td>
<td>cm$^2$/sec</td>
</tr>
<tr>
<td>STBO</td>
<td>Stefan-Boltzmann constant</td>
<td>$5.71 \times 10^{-12}$</td>
<td>W/cm$^2$°K</td>
</tr>
<tr>
<td>SURT</td>
<td>Surface tension of fuel matrix</td>
<td>$10^3$</td>
<td>dyne/cm</td>
</tr>
<tr>
<td>TCLMAX</td>
<td>Maximum allowable centerline temperature</td>
<td>2800</td>
<td>°C</td>
</tr>
<tr>
<td>TFLU</td>
<td>Coolant temperature</td>
<td>550</td>
<td>°C</td>
</tr>
<tr>
<td>UNIVGC</td>
<td>Universal gas constant</td>
<td>$1.3804 \times 10^{-16}$</td>
<td>erg/°K/mol.</td>
</tr>
<tr>
<td>VNDWC</td>
<td>Van der Waals correction term</td>
<td>$8.3 \times 10^{-23}$</td>
<td>cm$^3$</td>
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<tr>
<td>WTML1</td>
<td>Molecular weight of fuel matrix</td>
<td>270</td>
<td>gm/mole</td>
</tr>
<tr>
<td>WTML2</td>
<td>Molecular weight of vapor constituent</td>
<td>4</td>
<td>gm/mole</td>
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</table>
Figure 5.3. Temperature varies with time at surfaces of columnar and equiaxed grain regions.
Figure 5.4. Time variations of radii of columnar and equiaxed grain regions

$r_1$ = columnar region radius
$r_2$ = equiaxed region radius
$r_s$ = fuel surface radius

$= 0.2730$ cm
variations are rapid during the first hundred hours, and reach almost constant values after about 500 hours of the normal operation at the given linear power.

Figure 5.5 shows the temperature distributions in the cross section of the fuel element at different times. The clad outer surface temperature is 588.6°C and its inner surface temperature is 649.1°C. The temperature drop of 60.5°C across the clad corresponds to the linear power of 15 kw/ft. Temperatures in the outer region (about 1/3) of the fuel rod change very little with time. Those in the inner region decrease significantly in the early stage of the operation due primarily to the grain growths in the fuel. Thus, the effect of change of fuel rod surface temperature upon the time-dependent property of the temperature distribution in fuel may be neglected in the analysis.

In Figure 5.6, the fuel-clad gap increases with time during the first hundred hours and decreases hereafter. The increase in the early stage is due to the temperature decrease in the fuel as a result of grain growths, while the decrease is due to the effects of the fission-product swelling and the increase of pressure in the central void as soon as the grain growths are not significant. Also, Figure 5.6 shows the time variation of the radius of central void, increasing continuously and approaching almost constant after about 500 hours of the operation.
Figure 5.5. Temperature distribution in the fuel element
Figure 5.6. Time variations of fuel-clad gap and radius of central void
Figure 5.7 shows the time variation of pressures in the fuel-clad gap and in the central void. The pressure in the central void increases with time due to the increase of fission gas atoms escaped to it. The pressure in the fuel-clad gap decreases during the first hundred hours and then increases with time because of the variation on the gap thickness.

Figures 5.8 and 5.9 show the stress and strain distributions in the clad. The distributions remain little changed after 1000 hours of the operation. The radial stresses are so small that they may be neglected in the analysis. The tangential stresses vary from compression at the inner surface to tension at the outer surface.

Figure 5.10 gives the plastic strain and thermal strain distributions in the clad at 1000 hours of the operation.

The stress and strain distributions in the fuel zone at different times are shown in Figures 5.11 and 5.12. During the operation the changes in the distributions are relatively insensitive. The tangential stresses change from compression to tension with zero stress at about 0.7 of the fuel rod radius. The radial stresses at the outer and inner surfaces are very small. This implies that the pressures in the central void and in the fuel-clad gap may be neglected in comparison with the thermal stresses in the analysis. The radial strains decrease with radius for a given time and increase
Figure 5.7. Time variations of pressure in central void and fuel-clad gap
Radial, tangential and axial stress distributions in clad at irradiation times of 1 and 1000 hrs

- Radial stress $\sigma_r$
- Tangential stress $\sigma_\theta$
- Axial stress $\sigma_z$

$r_s = 0.273$ cm

Figure 5.8. Radial, tangential and axial stress distributions in clad at irradiation times of 1 and 1000 hrs
Figure 5.9. Radial and tangential strain distributions in clad at irradiation times of 1 and 1000 hrs.
Figure 5.10. Thermal and plastic strain distributions in clad at irradiation time of 1000 hrs
Figure 5.11. Radial, tangential and axial stress distributions in fuel zone vary with time
Figure 5.12. Radial and tangential strain distributions in fuel zone vary with time

Radial strain $\varepsilon_r$
Tangential strain $\varepsilon_\theta$

$r_s = 0.273$ cm

Radial and tangential strains, $\varepsilon_r$ and $\varepsilon_\theta$ (10^{-2})

Fuel radius $r/r_s$
with time in the outer region. The tangential strains increase to a maximum and then decrease with radius as shown in Figure 5.12.

Figure 5.13 shows the swelling, thermal and plastic strain distributions in the fuel rod at 729 hours of the operation. The plastic strains are comparative to the thermal strains. The swelling strains are small at this time in comparison with the plastic and thermal strains. The gas-bubble swelling in the unaffected region and the change of density passing through the surface of the equiaxed-grain region have marked effect on the swelling and plastic strains as shown in Figure 5.13.
Figure 5.13. Thermal, radiation and plastic strain distributions in fuel zone at radiation time of 729 hours
Equation (2-9) gives the migration rate of fabricated pores. Nichols used this equation to find the extent of the columnar grains (12). The trouble for him was the unknown temperature profile in the oxide fuel. Hence he had to assume a parabolic temperature distribution and found the results are not very sensitive to the choice of fuel surface temperature. Thus he set this temperature at 800°K and calculated the extent of the columnar grains as a function of time. In this analysis Equation (2-9) has also been used to couple with the heat transfer equations for oxide fuels. Here the fuel surface temperature is an iterated value. The temperature distribution is not necessary to be parabolic. The computed radius of the columnar-grain region is in good agreement with that obtained from Nichols. For example, at 1000 hours of irradiation the radius of columnar-grain region is 0.5719 of the fuel surface radius in this analysis, while in Nichols' analysis, the fictitious centerline temperature (a centerline temperature without considering grain growths) is 2829°K and so the radius is about 0.58 of the fuel surface radius. The agreement confirms that variation of the temperature distribution with irradiation time is primarily due to the grain growths. Nichols reported recently that the observations of cross section of a ThO₂·UO₂ fuel rod after irradiation (supplied by J. E. McCauly) were in good agreement
with his theoretical predictions (17). In the prediction of the radius of the central void, he did not consider the sintering of equiaxed grain growth, yet his assumption of a 100% of theoretical density in the columnar-grain region, which may not be so high, made a compensation. In the present analysis the sintering of equiaxed grain growth has been considered and the fraction of theoretical density in the columnar-grain zone has been assumed in the general accepted range, 98% to 99%. This analysis is shown to be applicable.

In the radiation analysis, fission gases in the regions where grain growth took place were assumed releasing to the central void. This assumption was made neither in the Barnes and Nelson model nor in the Nichols model. However, it has a sound basis due to the recent experimental work by Carroll et al. (39). Their result is quoted as below:

"Equiaxed grain growth in UO₂ occurs at 1650°C and fission gas, normally trapped in a grain boundary, can then migrate along the mobile grain boundary. The fission-gas release rate is thus greatly increased during the time of grain growth, but the increase in grain size has little influence on the subsequent gas release at lower temperatures." Because of this assumption, the analysis of fission-gas swelling after movement of bubbles from their anchored sites, dislocations, can seldom be applied except at very high
burnups since anchored bubbles can exist only in the unaffected zone or the outerperiphery of the fuel rod in which the movement of bubbles is very unlikely as shown by Barnes and Nelson (15) and in the illustrative calculations of this analysis.

Oxide cracking may take place in an early period of irradiation time when the reactor was brought up to the normal operation, and not occur again until high burnups (58). Numerous experiments have shown that at temperatures where oxide material shows plastic or viscous flow, thermal stress failure occurred in fuel elements is negligible. In this work, it is assumed that the fuel element has been operated in a fast reactor for a short period of early time passing through a preliminary state of cracking and the fuel has been in a ductile state. Therefore, the effect of oxide cracking has not been taken into account in this analysis.

In the method of successive approximations for the stress analysis, there are several ways to relate the experimental stress-strain curves with plastic strains. These ways have thoroughly been discussed from the views of convergence by Mendelson (18). The way given in this analysis has avoided the situations of slow convergence or divergence. In applying the method of successive approximations, the major unusual techniques used are (a) The equilibrium and the compatibility
equations are solved in terms of the defined variables e and g. (b) In the fuel, the bubble size, which can be influenced by the hydrostatic stress (or average stress), is iterated in connection with the stress iterations. The number of iterations for convergence accurate to $10^{-3}$ is no more than 5 for clad and 6 for fuel in the illustrative calculations. These are very good results.

In iterating the fuel surface temperature, the average value of the preceding and the present calculated surface temperature is used for the next iteration. This simple device accelerated the convergence a great deal. By doing this, an accuracy of $10^{-2}$ can be obtained in two iterations for most the cases; otherwise, it may need more than eight iterations.

In addition to the present analysis the computer program is versatile for the design of oxide fuel elements. One can vary the fuel rod radius, the fuel-clad gap, the clad thickness, the fuel porosity, the power level and the coolant temperature and pressure to investigate problems concerning safety, economics and optimum parameters. The computing time is sufficiently short for carrying out the studies.

Discussions about results in this analysis have been given in Chapter V.
VII. CONCLUSIONS

The thermal, radiation and mechanical analysis for oxide cylindrical fuel pins or fuel elements of fast reactors in unsteady state has been developed. The computer program ISUNE 1 designed for performing the analysis has been proved useful to analyze and calculate several important variables as the functions of radial space and irradiation time of the fuel elements:

1. The irradiation swelling, fission-gas release and fuel burnup,
2. The temperature distributions and thermal gradients in the cladding and in the fuel zone,
3. The changes of fission-gas pressure in the fuel elements,
4. The fuel-clad gap thickness and the radii of the central void, the columnar-grain region and equiaxed-grain region,
5. The stresses and strains produced in the fuel elements.

These variables are significantly related to the fuel element performance as well as the reactor safety and operation economics.

The calculation as a whole is a compound and complex iteration. Experience shows that all the iteration results are converged rapidly.
Based on the results calculated from the numerical example using the computer code, the following main conclusions are drawn:

1. The radii of the central void, columnar-grain region and equiaxed-grain region near the mid-plane of the fuel element increase very rapidly during the first hundred hours of irradiation time; then gradually slow down, and reach almost constant values of radii after about 500 hours of normal operation at the given linear power.

2. The change of temperature distribution in the fuel depends greatly on the grain growths and not much on the surface temperature of the fuel rod during an early period of the constant linear power operation.

3. The thickness of fuel-clad gap increases with time during the first hundred hours and decreases hereafter.

4. The pressure in the central void increases with time, while the pressure in the fuel-clad gap decreases during the first hundred hours and then increases with time.

5. The plastic strains are comparable to the thermal strains in the fuel element.

6. The gas-bubble swelling in the unaffected region and the change of density passing through the surface of
the equiaxed-grain region have marked effects on the swelling and plastic strains.

7. After a sufficiently long period of constant linear power operation (about 1000 hours) the changes in stress and strain distributions in the fuel element are relatively insensitive.
VIII. LITERATURE CITED


IX. ACKNOWLEDGMENTS

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X. APPENDIX A: COMPUTER PROGRAM ISUNE-1
ISUNE-1 is a FORTRAN IV computer program for mechanical, radiation and thermal analysis of cylindrical oxide fuel elements of fast reactor in unsteady state at constant linear power.

EXTERNAL FCT1, FCT2, FCT3, FCT4, FCT5, FCT6, FCT7, FCTD
EXTERNAL FCTG1
EXTERNAL FCTC4
DIMENSION PORST(3), RDR(3), T(21), FSNRT(3), EQSTS3(21)
DIMENSION CPLSTR(11), CPLSTT(11), CPLSTA(11), CTHSTN(11), FCTC1(11),
1 FCTC2(11), FCTC3(11), ZINVL(11), ZINV2(11), ZINV3(11), SEC(11),
2 SGC(11), CTTNDR(11), CTTNDT(11), CTTNDA(11), CAVSTS(11), CESTS1(11),
3 CESTS2(11), CEPLS1(11), CEPLS2(11), CETSTN(11), CRAT1(11), CRAT2(11),
4 CERR1(11), CERR2(11), TC(11), RC(11), CSTSR(11), CSTST(11), CSTSA(11),
5 CSTNR(11), CSTNT(11)
DIMENSION STSR(21), STST(21), STSA(21), STNR(21)
DIMENSION PLSTR(21), PLSTT(21), PLSTA(21), ELSMD(21), ALPH(21),
1 BETA(21), GAMMA(21), ALPHP(21), BETAP(21), GAMAP(21), SG(21), SE(21),
2 TSTNDR(21), TSTNDT(21), TSTNDA(21), EQTSTN(21), EQPLS1(21), EQPLS2(21),
3 RATIO2(21), ERR2(21), STN(21), VELG(21), VELCG(21), TIMEG(21),
4 SWESTN(21), PPILMT(21), FEXPN(21), FCOEFK(21), DTEMP(21), EQSTS1(21),
5 STSDVR(21), STSDUT(21), STSDVA(21), EQSTS2(21)
COMMON /FCTG4X/ CCOEFK, CEXPN, CPSNRT, ALSE, CELSMD
COMMON /FCTG1X/R, TC, CSTNR, CSTNT, CSTSR, CSTST, CSTNT, CSTSA,
1 CSTG3, CSTG4, CSTG5, CSTG6
COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)/FCT123/I, R(21)
1/FCT15/R1, R2/FC/Tempi/FCT24/Temp2, RR2/FCT34/RSS/FCT3X/TEPS
COMMON /FCT4X/ EQTSTN, PPILMT, ELSMD, FCOEFK, FEXPN, PSNRT, J
COMMON /FCT6X/TIME, CNTC, HP, ABST
COMMON /FCTDX/ C0EF(5), MNTR
COMMON /HAPPY1/ NUTIME, GASN(21), DTIME(21), TIMECG(21), DG(21),
1 DBCG(21), CNTF, TGDN(21), SCAGTE
2/HAPPY2/AVSTS(21), ATEMP(21), BNU(21), SURT, CNAVST, SGB(21)
3/HAPPY3/THSWST(21), GATMN(21), BATMN(21), THSTN(21), NSTNT, NINT(3),
\[
\begin{align*}
4SGG(21), SGGM(21), VNDWC, SWSLD(21), DB(21)/& HAPPY4/BOLZ \\
FPRESA(\text{AT}, \text{STNT1}, \text{RR2}, \text{FSNRT3}, \text{TIME}, \text{GATMN}, \text{RRV}) &= \left(\frac{\text{CPR} \times \text{AT}}{(\text{RMT} + \text{ABST}) \times (1. + \text{STNT1})^{2}}\right) + \left(\frac{\text{RR2} \times \text{FSNRT3} \times \text{TIME} \times \text{CNT10} \times \text{PI} + \text{GATMN}}{(\text{PI} \times \text{RRV} \times (1. + 2 \times \text{STNT1})^{2}) \times \text{BOLZ} \times \text{AT}}\right) \\
\text{PRESG}(\text{AT}, \text{STNC}, \text{STNF}) &= \frac{\text{CPR} \times (\text{RRIN}^{2} - \text{RRS}^{2})}{(\text{RMT} + \text{ABST}) \times \text{AT} / (\text{RRIN} \times (1. + \text{STNC})^{2} - \text{RRS} \times (1. + \text{STNF})^{2})} \\
\text{FK}(\text{TEM}, \text{DENF}) &= \frac{\text{CNT4} \times \text{ACTED} \times \text{ABST} \times \text{WRL} \times \text{TCLMAX}}{\text{2CNT1} \times \text{UNIVG} \times \text{DGG2} \times \text{DGG1}, \text{MMAX}, \text{NIINTT}, \text{MNITR}} \\
\text{READ (1, 1011)} &= \text{EQDIA, AVCV, CDEN, CSPH, CTHK, TFLU, RROU, Rrin,} \\
\text{1CLDK, CPSNR}, \text{PRESF, CNTC1, CNTC3, CNTC5, CNER, NIINTC,} \\
\text{2CEXCL, CELSMOD, CNTC2, CNTC4} \\
\text{READ (1, 1012)} &= \text{STBO, CNTG1, CNTG2, CNTG3, CNTG4, CNTG5, CNTG6} \\
\text{READ (1, 1001)} &= \text{RMT, DEFR(1), DEFR(2), DEFR(3), RRS, CNT2, CNT3,} \\
\text{1CNT4, CNT5, ACTED, ABST, WRL, TCLMAX,} \\
\text{2CNT1, UNIVG}, \text{DGG2}, \text{DGG1}, \text{MMAX, NIINTT, MNITR} \\
\text{READ (1, 1002)} &= \text{SPR, AMIG, HTVP, AVGDN, CPR, CSRC} \\
\text{READ (1, 1003)} &= \text{BOLZ, WML1, WML2} \\
\text{READ (1, 1004)} &= \text{CNT7, CNT6, CNT8, CNT9, CNT10, CNT11, CNT12} \\
\text{READ (1, 1005)} &= \text{CNT13, CNT14, CNT15, CNT16, CNT17, CNT18, CNT19} \\
\text{READ (1, 1006)} &= \text{CNT20, CNT21, CNT22} \\
\text{READ (1, 1008)} &= \text{SURT, VNDWC, SRDFCO, DFHLN, SCACTE, DSLF,} \\
\text{1SACTE, GBTF, PSNR}, \text{CNAVST} \\
\text{1011 FORMAT (8F10.4/7F10.5, I5/4E12.5)} \\
\text{1012 FORMAT (E12.5, F10.4, E12.5, F10.4, 2E12.5, F10.4)} \\
\text{1001 FORMAT (7F10.5/2F10.5, 4F10.2/4E12.5, 3I5)} \\
\text{1002 FORMAT (6E12.4)} \\
\text{1003 FORMAT (E12.5, 2F10.3)} \\
\text{1004 FORMAT (E12.5, E10.3, 2E12.5, F10.4, 2E12.5, E12.4)} \\
\text{1005 FORMAT (5F10.4, E12.4, F10.4)} \\
\text{1006 FORMAT (E12.4, 2F10.4)} \\
\text{1008 FORMAT (6E12.5/E12.5, 3F10.4)} \\
\text{PI} &= 3.14159 \\
\text{CNTCA} &= \text{PWRL / (2. * PI * CCLK)} \\
\text{CNTCB} &= \frac{(1. - 2. \times \text{CPSNRT})}{3. \times (1. - \text{CPSNRT})} \\
\text{NNTCC} &= \frac{2. \times (1. - \text{CPSNRT})}{3. \times (1. - \text{CPSNRT})} \\
\text{CNETC} &= \text{CELSMD / (1. - 2. \times \text{CPSNRT})} \\
\text{CNETCH} &= \text{CELSMD / 2. / (1. + \text{CPSNRT})} \\
\text{CNETC} &= \frac{1. / 2. \times 9. \times \text{SPR} \times \text{AMIG} \times \text{HTVP} \times \text{SQR} \text{(AVGDN} \times \text{(WML1+WML2))}}{(16. \times \text{CPR} * \text{WML1})^2.}}
\]
1CSRC**2*SQRT(2.*PI*BOLZ*WTML1*WTML2))

C CALCULATION OF VOLUMETRIC HEAT SOURCE
C
PWRV(3)=PWRL/PI/RRS**2
PWRV(2)=PWRV(3)*DENFR(2)/DENFR(3)
PWRV(1)=PWRV(3)*DENFR(1)/DENFR(3)

C CALCULATION OF FISSION RATE AND POROSITY
C
DO 2 J=1,3
  PORST(J)=1.-DENFR(J)
  FSNRT(J)=PWRV(J)/CNT1

C CALCULATION OF HEAT TRANSFER COEFFICIENT
C
PECLET=EQDIA*AVCV*CDEN*CSPH/CTHK
HC0EF=CTHK/EQDIA*(7.0+0.025*PECLET**0.8)**0.8
TOU=TFLU+PWRL/(2.*PI*RROU*HC0EF)
TIN=TOU+ALOG(RROU/RRIN)/CLDK*PWRL/(2.*PI)

C SET TIME
C
M=2
1 TIME=3600.*FLOAT(M)**3
MT=M**3

C INITIALIZE NUMBER OF ITERATION OF FUEL SURFACE TEMPERATURE
C
NITR1=0

C CALCULATION OF TEMPERATURE DISTRIBUTION IN CLAD
C CALCULATION OF THERMAL STRAINS IN CLAD
C
NSTNC=NINTC+1
RDC=(RROU-RRIN)/FLOAT(NINTC)
RC(1)=RRIN
DO 10 I=2,NSTNC
RC(I)=RC(I-1)+RDC
10 CONTINUE
TC(NSTNC)=TOU
TC(I)=TIN
DO 15 I=1,NSTNC
TC(I)=TOU+ALOG(RROU/RC(I))*CNTCA
CTHSTN(I)=CSEXCL*(TC(I)-RMT)
15 CONTINUE

C MECHANICAL ANALYSIS FOR CLAD
C
CSTNT(1)=0.
NSTNT=NINTT+1
STNT(1)=0.
STNT(NSTNT)=0.
ATGAP=(TIN+700.)/2.+AUST
PRESGP=PRESG(ATGAP,CSTNT(1),STNT(NSTNT))

C INITIALIZE NUMBER OF ITERATION FOR CLAD STRESS AND STRAINS
C
21 NITR3=0
DO 20 J=1,NSTNC
CPLSTR(J)=0.
CPLSTT(J)=0.
CPLSTA(J)=0.
CEPLS1(J)=0.
CEESTS1(J)=0.
20 CONTINUE

22 DO 30 I=1,NSTNC
FCTC1(I)=(CPLSTR(I)-CPLSTT(I))/RC(I)
FCTC2(I)=RC(I)*CTHSTN(I)
FCTC3(I)=RC(I)*(CPLSTR(I)+CPLSTT(I))
CALL QTFE (RDC,FCTC1,ZINVL1,I)
CALL QTFE (RDC,FCTC2,ZINVL2,I)
CALL QTFE (RDC,FCTC3,ZINVL3,I)
30 CONTINUE
CALCULATION OF INTEGRATION CONSTANTS

BNCG = CELSMO / (1 + CPSNRT) * (ZINVL1(NSTNC) / 2 + (1 + CPSNRT) - 
1ZINVL2(NSTNC) / RC(NSTNC) ** 2 - (1 - 2 * CPSNRT) * ZINVL3(NSTNC) / (2 * (1 + 
2 * CPSNRT) * RC(NSTNC) ** 2))

BNCC1 = 2 * (1 + CPSNRT) / (3 + CELSMO * (RC(NSTNC) ** 2 - RC(1) ** 2) + 
(1 + CPSNRT) * PRESGP - RC(NSTNC) ** 2 * (PRESF + BNCG))

BNCC2 = 2 * RC(1) ** 2 * RC(NSTNC) ** 2 / (3 + CELSMO * (RC(NSTNC) ** 2 - 
RC(1) ** 2) - PRESGP + PRESF + BNCG)

CER = 0.

DO 40 I = 1, NSTNC
SEC(I) = CNTCB * (CPLSTR(I) - 2 * CTHSTN(I) + ZINVL1(I)) + BNCC1
SGC(I) = CNTCC * (ZINVL2(I) / RC(I) ** 2 + CNTCB * (CPLSTR(I) - 2 * CTHSTN(I) - 
1ZINVL3(I) / RC(I) ** 2)) + BNCC2 / RC(I) ** 2

CALCULATION OF MODIFIED EQUIVALENT TOTAL STRAIN

CTTNUR(I) = 1 / 2 * (SEC(I) + 3 * SGC(I) + 4 * CTHSTN(I))
CTTNDT(I) = 1 / 2 * (SEC(I) - 3 * SGC(I) - 2 * CTHSTN(I))
CTTNDA(I) = -SEC(I) - CTHSTN(I)
CETSTN(I) = SQRT(2 / 3 * (CTTNDR(I) ** 2 + CTTNDT(I) ** 2 + CTTNDA(I) ** 2))
CAVSTS(I) = CNTCE * SEC(I)
CEXP = CNTC1 + CNTC2 * TC(I) ** CNTC3
CGOEFK = 3 * 7 / (CELSMO / (CNTC4 - CNTC5 * TC(I))) ** (CEXP - 1.)
ALSE = CETSTN(I)

CALCULATION OF EQUIVALENT STRESS AND STRAIN

CALL RTMI(CESTS, FCTC4V, FCTC4, 0, 100, 1.E-07, 50, IERC4)
CESTS2(I) = CESTS
CEPLS2(I) = CESTS(I) - 2 * (1 + CPSNRT) * CESTS2(I) / 3 / CELSMO

CALCULATION OF RATIOS AND ERRORS FOR CONVERGENCE

CRAT(I) = CESTS1(I) / CESTS2(I)
CERR1(I) = ABS(CRAT(I) - 1.)
CER = AMAX1(CER, CERR2(I))
IF (CPLS2(I) LT 0.002) GO TO 35
CRAT2(I) = CEPLS1(I) / CEPLS2(I)
CERR2(I) = ABS(CRAT2(I) - 1.)
CER = AMAX1(CER, CERR2(I))
35 CESTS1(I) = CESTS2(I)
CEPLS1(I) = CEPLS2(I)
40 CONTINUE
NITR3 = NITR3 + 1
IF (NITR3 EQ MNI) GO TO 1000
IF (GER LT CNER) GO TO 60

C CALCULATION OF PLASTIC STRAINS
C
DO 50 J = 1, NSTNC
CPLSTR(J) = CEPLS2(J) / CETSTN(J) * CTTNDR(J)
CPLSTT(J) = CEPLS2(J) / CETSTN(J) * CTTNDT(J)
CPLSTA(J) = -CPLSTR(J) - CPLSTT(J)
50 CONTINUE
GO TO 22

C CALCULATION OF FUEL SURFACE TEMPERATURE
C
60 R(INSTNT) = RRS
IF (NITR1 GT 0) GO TO 61
TEMPS = 700.
GO TO 199
61 CALL RTMI(TEMPS, FCTG1V, FCTG1, TIN, 2600., 0.0001, MNI, IERG1)

C TEST CONVERGENCE OF FUEL SURFACE TEMPERATURE
C
GRATIO = TEMPS1 / TEMPS
GER = ABS(GRATIO - 1.)
IF (GER LE 0.01) GO TO 990
TEMPS = (TEMPS1 + TEMPS) / 2.
199 TEMPS1 = TEMPS
C CALCULATION OF GAP THICKNESS

GAP = RC(1)*(1. + CSTNT(1)) - R(NSTNT)*(1. + CSTNT(NSTNT))
ATGAP = (TIN + TEMPS)/2 + ABST
NITR1 = NITR1 + 1
WRITE (3,1073) TEMPS, ATGAP, GAP, NITR1, NITR3
1073 FORMAT (' TEMPS=', F8.3, ' ATGAP=', F8.3, ' GAP=', F9.6, ' NITR1',
1=", I2, ' NITR3=', I2)
IF (GAP .LE. 0.) GO TO 1000
IF (NITR1 .EQ. 10) GO TO 1000

C CALCULATION OF T2, T1, R1, R2, RV, AND TV

I = 1
CALL RTMI(TEMPC, FCT3V, FCT3, TEMPS, 5000., 0.0001, MNITR, IERC)
WRITE (3,1050) TEMPC, IERC
1050 FORMAT (' TEMPC=', F8.2, ' IERC=', I2)
IF (TEMPC .GT. TCLMAX) GO TO 1000
IF (IERC-1) 200, 1000, 1000
200 TEMP2 = AACTEGD/UNIVGC/ALOG(CNT5*TIME/(DG2**3-DG1**3))-ABST
RR2 = RRS*SQRT(1. - 4./(PWRV(3)*RRS**2)*(CNT2*(TEMP2-TEMPS)+ALOG(TEMP2/
1/TEMPS)/(CNT3-CNT4*DENFR(3))))
K0 = 1
CALL RTMI(TEMPC, FCT6V, FCT6, TEMPC, 0.0001, MNITR, IER6)
RRX = RR2*SQRT(1. - PWRV(3)/PWRV(2))
CALL RTMI(RR1, FCT7V, FCT7, RRX, RR2, 0.0001, MNITR, IER7)
IF((IER7 .EQ. 2).AND. (TEMP2 .GE. TEMPO) GO TO 208
IF (IER7 .EQ. 2) GO TO 207
IF (TEMP2 .GE. TEMPO GO TO 205
206 RRV = SQRT(RR2**2*PWRV(2)-PWRV(3))/PWRV(1)+RR1**2*(PWRV(1)-PWRV(2))
1/PWRV(1))
CALL RTMI(TEMPV, FCT5V, FCT5, TEMPI, 2600., 0.0001, MNITR, IER5)
WRITE (3,1051) TEMPV, TEMPI, TEMP2, RRV, RR1, RR2, IER5, IER6, IER7
1051 FORMAT (' THREE REGIONS', ' TEMPV=', F8.2, ' TEMP1=', F8.2, ' TEMP2=', F8.2,
1 ' T1=', F8.2, ' T3=', F8.2, ' R1=', F8.6, ' R2=', F8.6, ' RRV=', F8.6, ' RR1=',
1 ' RR2=', F8.6, ' IER5 (FOR TEMPV)=', I1, ' IER6 (FOR TEMPI) =', I1, ' IER7')
3 'IER7(FOR RR1)=,I1)
   IF (IER5-1) 210,1000,1000
   C CONSIDERATION OF TWO REGIONS
   205 RR2=RR1
       TEMP2=TEMP1
       DENFR(2)=DENFR(1)
       PWRV(2)=PWRV(1)
       FSNRT(2)=FSNRT(1)
       P0RST(2)=P0RST(1)
   207 K0=2
       RRV=SQR((RR2**2*(PWRV(2)-PWRV(3)))/PWRV(2))
       I=1
       R(I)=RRV
       CALL RTMI(TEMPV,FCT2V,FCT2,TEMP2,TEMPC,0.0001,MNITR, IER2)
       RR1=RRV
       TEMP1=TEMPV
       WRITE (3,1052) TEMPV,TEMP2,RRV,RR2,IER7
1052 FORMAT ('OFOR TWO REGIONS',4X,'TEMPV=',F8.2,3X,'TEMP2=',F8.2,3X,
1*RRV=',F8.6,3X,'RR2-',F8.6/ ' ',15X,'IER7=',I1,3X,'IF IER7=2, NO CO
2LUNNAR GRAIN ZONE')
   GO TO 210
208 K0=3
   RRV=0.
   RR1=0.000001
   RR2=0.000001
   TEMPV=TEMPC
   TEMP1=TEMPC
   WRITE (3,1052) TEMPV,TEMP2,RRV,RR2,IER7
1052 FORMAT ('OFOR TWO REGIONS',4X,'TEMPV=',F8.2,3X,'TEMP2=',F8.2,3X,
1*RRV=',F8.6,3X,'RR2-',F8.6/ ',15X,'IER7=',I1,3X,'IF IER7=2, NO CO
2LUNNAR GRAIN ZONE')
   GO TO 210
210 RD=(RRS-RRV)/FLOAT(NINTT)
   NINT(1)=IFIX((RR1-RRV)/RD)+1
   NINT(2)=IFIX((RR2-RR1)/RD)
   RDR(1)=(RR1-RRV)/FLOAT(NINT(1))
   RDR(2)=(RR2-RR1)/FLOAT(NINT(2))
220 IF (K0.NE.2) GO TO 225
   C IN CASE OF TWO REGION, NINT(1)=0.
   NINT(1)=0
   GO TO 230
225 IF(K0.NE.3) GO TO 230
NINT(1)=0
NINT(2)=0
230 NINT(3)=NINTT-NINT(1)-NINT(2)
   RDR(3)=(RRS-RR2)/FLOAT(NINT(3))
   NSTN1=NINT(1)+1
   NSTN2=NSTN1+NINT(2)
   NSTNT=NSTN2+NINT(3)
   WRITE (3,1053) (NINT(I),I=1,3),NSTN1,NSTN2,NSTNT,{RDR(I),I=1,3}
1053 FORMAT ('NUMBER OF INTERVALS IN REGION 1 IS ',I2,3X,'IN REGION 2 IS ',I2,3X,'IN REGION 3 IS ',I2,3X,'STATION NUMBER AT R1 IS ',I2,3X,'AT R2 IS ',I2,3X,'AT RS IS ',I2,3X,'RADIAL DIFFERENCE IN THE THREE REGIONS ARE (1) ',F8.6,2X,(2) ',F8.6,2X,(3) ',F8.6)

C CALCULATION OF TEMPERATURE DISTRIBUTION IN FUEL
C
NSTNA=2
NSTNB=1
R(1)=RRV
T(1)=TEMPV
ROI=R(1)
IF (R(1).GT.0.) GO TO 300
R(1)=0.000001
RRV=0.000001
300 DO 390 NR=1,3
   NSTNB=NSTNB+NINT(NR)
   DO 340 I=NSTNA,NSTNB
   R(I)=R(I-1)+RDR(NR)
   GO TO (310,320,330),NR
310 CALL RTMI(T(I),FCT1V,FCT1,TEMP1,2600.,0.0001,MNITR,IER1)
   GO TO 340
320 CALL RTMI(T(I),FCT2V,FCT2,TEMP2,TEMP1,C.0001,MNITR,IER2)
   GO TO 340
330 CALL RTMI(T(I),FCT3V,FCT3,TEMP3,TEMP2,0.0001,MNITR,IER3)
   CONTINUE
NSTNA=NSTNB+1
390 CONTINUE
C TEMPERATURE GRADIENTS AND FUNCTIONS OF TEMPERATURE
C
GATMM=0.
NSTNA=1
NSTNB=1
400 DO 490 NR=1,3
NSTNB=NSTNB+NINT(NR)
DO 480 I=NSTNA,NSTNB
SGB(I)=0.
SGG(I)=0.
GATNM(I)=CNT10*FSNRT(NR)*TIME
GO TO (410,420,430),NR
410 TGDN(I)=PWRV(1)*(R(I)**2-RRV**2)/(2.*FK(T(I),DENFR(1))*R(I))
   BNU(I)=0.
   SGG(I)=0.
   GO TO 440
420 TGDN(I)=((PWRV(3)-PWRV(2))*RR2**2+PWRV(2)*R(I)**2)/(2.*R(I)*
   1FK(T(I),DENFR(2)))
   SGG(I)=0.
   BNU(I)=0.
   GO TO 440
430 TGDN(I)=PWRV(3)*R(I)/2.*FK(T(I),DENFR(3))
   GASNM(I)=0.
440 ATMP(I)=T(I)+ABST
   BNU(I)=CNT11-CNT12*T(I)
   DTMPI(I)=T(I)-RMT
   ShSDL(I)=CNT6*FSNRT(NR)*TIME
   THSTN(I)=(CNT7+CNT8+T(I))+CNT9*T(I)**2)*T(I)
   DG(I)=(DG(I)**3+CNT5*TIME*EXP(-ACTEGD/(UNIVGC*ATMP(I))))**(1./3.)
   ELSDM(I)=(CNT14-CNT15*T(I))*(1.-CNT16*PORST(NR)-CNT17*PORST(NR)**2
   1)*1.0E+03
   PPTLMT(I)=CNT18/(T(I)-CNT19)
   FEXPN(I)=CNT20*(T(I)-CNT21)**2+CNT22
   FCoeff(I)=PPTLMT(I)/ELSDM(I)
460 CONTINUE
NSTNA=NSTNB+1

CONTINUE

MECHANICAL ANALYSIS IN FUEL

DO 500 J=1,NSTNT
PLSTR(J)=0.
PLSTT(J)=0.
PLSTA(J)=0.
EQSTS3(J)=0.
AVSTS(J)=0.
CONTINUE

500 CONTINUE

NITR2=0
MEMO=0

CALCULATION OF PressURES IN GAP AND IN CENTRAL VOID

PRESA=FPRESA(ATEMP(1),STNT(1),RR2,FSNRT(3),TIME,GATMNM,RRV)
PRESGP=PRESG(ATGAP,CSTNT(1),STNT(NSTNT))

CALL THSW

NSTNA=2
NSTNB=1
SC1=3./2.*ELSMD(1)/(1.+PSNRT)
SF1=2.*SC1*(2.*THSWS(1)-PLSTR(1))
SH1=SC1*(-PLSTR1)+PLSTT(1)+3.*THSWS(1))
CD1=SC1/(1.-2.*PSNRT)
SC11=SC1
SF11=SF1
SH11=SH1
CD11=CD1
DO 590 NR=1,3
NSTNB=NSTNB+NINT(NR)
DO 580 I=NSTNA,NSTNB
SA=RDR(NR)/R(I)
SB=RDR(NR)/R(I-1)
SC2=3.*ELSMD(I)/(2.*(1.+PSNRT))
SF2=2.*SC2*(2.*THSWS(I)-PLSTR(I))
\begin{align*}
SH2 &= SC2 \cdot (-PLSTR(I) + PLSTT(I) + 3 \cdot THSWST(I)) \\
CA &= 1 \cdot SA \\
CB &= 1 \cdot SB \\
CC &= SA \cdot THSWST(I) + SB \cdot THSWST(I-1) \\
CD2 &= SC2/(1 - 2 \cdot PSNRT) \\
CF &= SC2 \cdot CA \\
CU &= CD1 \\
CV &= SC1 \cdot CB \\
CW &= 1 \cdot 3 \cdot (SF2 - SF1 + SA \cdot SH2 + SB \cdot SH1) \\
\text{DENOM} &= CF + CA \cdot CD2 \\
CK &= (CF + CA \cdot CU) / \text{DENOM} \\
CL &= (CA \cdot CV - CF \cdot CB) / \text{DENOM} \\
CM &= (CF \cdot CC - CA \cdot CW) / \text{DENOM} \\
CKP &= (CU - CD2) / \text{DENOM} \\
CLP &= (CV + CD2 \cdot CB) / \text{DENOM} \\
CMP &= (-CW - CC \cdot CD2) / \text{DENOM} \\
\text{IF} \ (I > 2) \ \text{GO TO 560} \\
\text{ALPH}(I) &= CK \\
\text{BETA}(I) &= CL \\
\text{GAMA}(I) &= CM \\
\text{ALPH}(I) &= CKP \\
\text{BETA}(I) &= CLP \\
\text{GAMA}(I) &= CMP \\
\text{GO TO 570} \\
560 \quad \text{ALPH}(I) &= CK \cdot \text{ALPH}(I-1) + CL \cdot \text{ALPH}(I-1) \\
\text{BETA}(I) &= CK \cdot \text{BETA}(I-1) + CL \cdot \text{BETA}(I-1) \\
\text{GAMA}(I) &= CK \cdot \text{GAMA}(I-1) + CL \cdot \text{GAMA}(I-1) + CM \\
\text{ALPH}(I) &= CKP \cdot \text{ALPH}(I-1) + CLP \cdot \text{ALPH}(I-1) \\
\text{BETA}(I) &= CKP \cdot \text{BETA}(I-1) + CLP \cdot \text{BETA}(I-1) \\
\text{GAMA}(I) &= CKP \cdot \text{GAMA}(I-1) + CLP \cdot \text{GAMA}(I-1) + CMP \\
570 \quad SC1 &= SC2 \\
SF1 &= SF2 \\
SH1 &= SH2 \\
CD1 &= CD2 \\
580 \quad \text{CONTINUE} \\
\text{NSTNA} &= \text{NSTNB} + 1 \\
590 \quad \text{CONTINUE}
\end{align*}
\begin{verbatim}
QNPL1 = (-PRESG - CD2 * GAMA(NSTNT) - SC2 * GAMAP(NSTNT) - 1./3. * SF2) /(CD2 * ALPH(NSTNT) + SC2 * ALPH(NSTNT))
PNPL1 = (-CD2 * BETA(NSTNT) + SC2 * BETAP(NSTNT)) /(CD2 * ALPH(NSTNT) + SC2 * ALPH(NSTNT))

IF (RO1.GT.0.) GO TO 650
SG(1) = 1./3.* (PLSTR(1) - PLSTT(1) - 3.* THSWST(1))
GO TO 660

650 SG(1) = (-PRESA - CD11 * QNPL1 - 1./3. * SF11) /(SC11 + CD11 * PNPL1)
660 SE(1) = PNPL1 * SG(1) + QNPL1
DO 670 I = 1, NSTNT
SE(I) = ALPH(I) * SE(1) + BETA(I) * SG(1) + GAMAP(I)
SG(I) = ALPH(I) * SE(1) + BETAP(I) * SG(1) + GAMAP(I)
670 CONTINUE
ER = 0.
DO 680 J = 1, NSTNT
AVSTS(J) = ESLMD(J)/(1. - 2.* PSNRT) * SE(J) *(6.91E+07)
STSDVR(J) = ESLMD(J)/(2.* (1. + PSNRT)) *(SE(J) + 3.* SG(J) + 4.* THSWST(J) - 2.* PLSTR(J))
STSDVT(J) = ESLMD(J)/(2.* (1. + PSNRT)) *(SE(J) - 3.* SG(J) - 2.* THSWST(J) - 2.* PLSTT(J))
STSDVA(J) = ESLMD(J)/(1. + PSNRT) * (-SE(J) - THSWST(J) + PLSTR(J) + PLSTT(J))
EQSTS1(J) = SQRT(1./5.*(STSDVR(J)**2 + STSDVT(J)**2 + STSDVA(J)**2))
IF (EQSTS1(J) .LT. PPTLMT(J)) GO TO 680

C C C
C CALCULATION OF MODIFIED EQUIVALENT TOTAL STRAIN
C
TSTNDR(J) = 1./2.* (SE(J) + 3.* SG(J) + 4.* THSWST(J))
TSTNRT(J) = 1./2.* (SE(J) - 3.* SG(J) - 2.* THSWST(J))
TSTNDA(J) = -SE(J) - THSWST(J)
EQTSTN(J) = SQRT(2./3.* (TSTNDR(J)**2 + TSTNRT(J)**2 + TSTNDA(J)**2))

C C C
C CALCULATION OF EQUIVALENT STRESS AND STRAIN
C
676 PLT1 = PPTLMT(J)/1.5
PLT2 = PPTLMT(J)*8.
CALL RTMI(SIG, FCT4V, FCT4, PLT1, PLT2, 1.E-07, MNITR, IER4)
EQSTS2(J) = SIG
\end{verbatim}
EQPLS2(J) = EQTSTN(J) - 2/3*(1 + PSNRT)*EQSTS2(J)/ELSMD(J)

TEST CONVERGENCE

RATIO2(J) = EQSTS3(J)/EQSTS2(J)
ERR2(J) = ABS(RATIO2(J) - 1.0)
ER = AMAX1(ER, ERR2(J))
EQSTS3(J) = EQSTS2(J)

CONTINUE
NITR2 = NITR2 + 1
IF (ER .LT. CNER) GO TO 695
IF (NITR2 .EQ. 20) GO TO 1000

CALCULATION OF PLASTIC STRAINS

DO 690 J = 1, NSTNT
PLSTR(J) = EQPLS2(J)/EQTSTN(J)*TSTNDR(J)
PLSTT(J) = EQPLS2(J)/EQTSTN(J)*TSTNDR(J)
PLSTA(J) = -PLSTR(J) - PLSTT(J)
690 CONTINUE
GO TO 505

IF (MEMO.EQ.13) GO TO 796

DETERMINE WHETHER BUBBLES ARE IN MOTION

NSTNA = NINT(1) + NINT(2) + 1
NSTNB = NSTNT
N = NSTNA - 1
CNTD = (6.*AMIG*DSLF/(PI*SACTE))**(1./3.)
CNTE = 6.*SRDFCO*DFMNLN*AMIG*SACTE/B0LZ
CNTF = SQRT(3.*AMIG*GTTF/SACTE)
DO 890 I = NSTNA, NSTNB
DBCG(I) = CNTD*(ATEMPC(I)/TGDN(I))**(1./3.)
IF (DBCG(I) .GT. DB(I)) GO TO 880
VELG(I) = CNTE*TGDN(I)/(ATEM(I)**2*DBCG(I))*EXP(-SACTE/(BOLZ*1ATEMP(I)))
TIMECG(I) = PI*BNUI(I)*(4.*SURT/DBCG(I) - AVSTS(I))*DBCG(I)**3/6.
890 CONTINUE
1*CNT10*FSNRT(3)*DOLZ*ATCMP(I))
VELCG(I)=DG(I)/100./(TIME-TIMLCG(I))
IF (VELCG(I).GT.VELG(I)) GO TO 880
DTIME(I)=1./(VELG(I)*BNU(I)**(1./3.))
TIMEG(I)=PI*DG(I)/(4.*VELG(I))
IF (TIME-TIMECG(I)-TIMEG(I)) .GT. 820,830,830
820 NUTIME=(TIME-TIMECG(I))./DTIME(I)
SGGM(I)=(TIME-TIMECG(I))./TIMEG(I)*PI/6.*DBCG(I)**3*BNU(I)
GO TO 840
830 NUTIME=(TIMEG(I))./DTIME(I)
SGGM(I)=0.
840 NUT=NUTIME*2
WRITE (3,1058)I,DBG(I),VELG(I),TIMECG(I),VELCG(I),NUTIME
1058 FORMAT (///STATION*,5X, 'DBCG',12X, 'VELG',10X, 'TIMECG',10X,
'VELCG',10X, 'NUTIME'///',3X,12,2X,4E15.5, 110)
CALL SGASB(DBG,NUT,SGBC,SGBN,I,TIME)
GATMN=GATMN+GASNM(I)*4./PI/DG(I)*(PI*(R(I)**2-R(I-1)**2))
GATMN(I)=GATMN(I)*(TIME-TIMECG(I))/TIME
880 N=N+1
890 CONTINUE
IF (N.EQ.NSTNB) GO TO 795
MEMO=13
GO TO 505
795 WRITE (3,1059)NT
1059 FORMAT (/// TILL THE TIME*,16 , ' HOURS, THERE IS NO MOTION
1 OF BUBBLES')
796 WRITE (3,1060) MEMO
1060 FORMAT (/// MEMO='*,I2)
CSTNT(1)=3./2.*SEC(1)-SGC(1))
STNT(1)=3./2.*SE(1)-SG(1))
STNT(NSTNT)=3./2.*(SE(NSTNT)-SG(NSTNT))
RRRV=RRV*(1.+STNT(I))
WRITE (3,1056) NITR2,PRESA,PRESGP,RRRV
1056 FORMAT (/// NITR2='*,I2,5X,'PRESSURE IN THE VOID='*,F10.5,5X,'PRESS
SURE IN THE GAP='*,F10.5,5X,'RADIUS OF VOID='*,F10.6)
IF (NITR1.EQ.1) GO TO 61
GO TO 21
990 FFSNTT=FSNRT(3)*TIME/(1.E+20)
    BURNUP=PWRV(3)*TIME/(8.64+E+05)
    WRITE (3,993) FFSNTT,BURNUP,TIME
993 FORMAT ('// TOTAL FISSIONS = ',E12.5,2X,'FISSIONS/CM**3/10**20'//
      1' BURNUP = ',E12.5,2X,'MW-DAY/TUNNE',5X,'TIME = ',E12.2,' SECONDS')
DO 2000 I=1,NSTNT
2000 SWESTN(I)=1.3*(SWSLD(I)+SGG(I)+SGB(I)+SGGM(I))
    WRITE (3,1072) (I,RC(I),TC(I),CTHSTN(I),CPLSTR(I),CPLSTT(I),
    ICPLSTA(I),I=1,NSTNT)
1072 FORMAT ('// TABLE'/I,23X,' TEMPERATURE DISTRIBUTION, THERMAL'
               1STRAIN AND PLASTIC STRAINS IN CLADDING'//
               2RADIUS TEMPERATURE THERMAL AXIAL',
               3/, 72X,'PLASTIC PLASTIC PLASTIC'/' ',39X,'(CM)', 7X,
               4'(C)' ,8X,' STRAIN STRAIN STRAIN'//
               54X,'(CM)',5X,'(C)',9X,'(C/CM)'//
               1054 FORMAT ('// TABLE'/I,48X,TABLE//
               1 ',23X,' TEMPERATURE DISTRIBUTION, TEMPERATURE GRADIENTS'
               ' AND THERMAL STRAINS IN FUEL'//
               2STATION RADIAL TEMPERATURE THERMAL'//
               358X,'GRADIENT STRAIN'/',35X,'(CM)',10X,'(C)',9X,'(C/CM)'//
               1103 FORMAT ('// TABLE'/I,48X,TABLE//
               1 ',30X,'SWELLING STRAIN AND PLASTIC STRAINS IN FUEL'
               ' STATION SWELLING RADIAL TANGENTIAL'
               2AXIAL ',33X,' STRAIN PLASTIC PLASTIC'/'
               3,47X,' STRAIN STRAIN STRAIN'//
               J=NSTNT
    STSR(J)=ELSMD(J)/2./(1.+PSNRT)*(SE(J)+3.*SG(J)+4.*THSWST(J)-2.*
    PLSTR(J)+AVSTS(J)/(6.91E+07)
    STNR(J)=1.5*(SE(J)+SG(J)+2.*THSWST(J))
    STNT(J)=1.5*(SE(J)-SG(J))
    ERF=STNR(J)
    ETF=STNT(J)
    SRF=STSR(J)
CSTSR(I) = (SEC(I) + 3. * SGC(I) + 4. * CTHSTN(I) - 2. * CPLSTR(I)) * CNTCH + ICAVSTS(I)
SRO = CSTSR(I)
I = NSTNC
CSTNR(I) = 1.5 * (SEC(I) + SGC(I) + 2 * CTHSTN(I))
CSTNT(I) = 1.5 * (SEC(I) - SGC(I))
ETO = CSTNT(I)
ERO = CSTNR(I)
WRITE (3, 995)

995 FORMAT ('!*', 30X, 'STRAINS AND STRESSES IN CLADDING MATERIAL', 1
            '210X,' 'CSTSR', '10X,' 'CSTST', '10X,' 'CSTSA' //</
DO 991 I = 1, NSTNC
CSTNR(I) = 1.5 * (SEC(I) + SGC(I) + 2 * CTHSTN(I))
CSTNT(I) = 1.5 * (SEC(I) - SGC(I))
CSTSR(I) = (SEC(I) + 3. * SGC(I) + 4. * CTHSTN(I) - 2. * CPLSTR(I)) * CNTCH + ICAVSTS(I)
CSTST(I) = (SEC(I) - 3. * SGC(I) - 2 * CTHSTN(I) - 2. * CPLSTT(I)) * CNTCH + ICAVSTS(I)
CSTSA(I) = 2. * CNTCH * (-SEC(I) - CTHSTN(I) - CPLSTA(I)) * CAVSTS(I)
WRITE (3, 996) I, RC(I), CSTNR(I), CSTNT(I), CSTSR(I), CSTST(I), CSTSA(I)

996 FORMAT ('!*', 15X, '!', 4X, 6E15, 5)
RC(I) = RC(I) / RRS
CSTST(I) = CSTST(I) / (ABS(SRO))
CSTSA(I) = CSTSA(I) / (ABS(SRO))
CSTSR(I) = CSTSR(I) / (ABS(SRO))
CSTNR(I) = CSTNR(I) / (CSTNR(NSTNC))
CSTNT(I) = CSTNT(I) / (CSTNT(NSTNC))

CONTINUE

994 FORMAT ('!', 30X, 'STRAINS AND STRESSES IN FUEL MATERIAL', 1
            '211X,' 'CSTSA' //</
DO 992 J = 1, NSTNT
STNR(J) = 1.5 * (SE(J) + SG(J) + 2 * THSWST(J))
STNT(J) = 1.5 * (SE(J) - SG(J))
STSR(J) = ELSMO(J) / 2 * (1 + PSNRT) * (SE(J) + 3 * SG(J) + 4 * THSWST(J) - 2 *
1PLSTR(J) = AVSTS(J) / (6.91E+07)
STST(J) = ELSDM(J) / 2. / (1. + PSNR) * (SE(J) - 3.*SG(J) - 2.*THSWST(J)) - 2.*
1PLSST(J) = AVSTS(J) / (6.91E+07)
STSA(J) = ELSDM(J) / 2. / (1. + PSNR) * (-SE(J) - THSWST(J) - PLSTA(J)) + AVSTS(J) / 
1(6.91E+07)
WRITE (3,998) J,R(J),STNRIJ),STNT(J),STSR(J),STST(J),STSACJ)
998 FORMAT (• ',15X,I4,6E15.5)
R(J) = R(J)/RRS
STST(J) = STST(J) / ABS(STSR(INSTNT)))
STSA(J) = STSA(J) / ABS(STSR(INSTNT)))
STSR(J) = STSR(J) / ABS(STSR(INSTNT)))
STNR(J) = STNR(J) / ABS(STSR(INSTNT)))
STNT(J) = STNT(J) / ABS(STSR(INSTNT)))
992 CONTINUE
WRITE (3,997) (I »RC(I),CSTNR(I),CSTNT(I),CSTSR(I),CSTST(1),
CSTSA(I),I=1,NSTNC)
997 FORMAT ('1'/////// ',54X,'TABLE'/// ',
1 40X,'DIMENSIONLESS STRAINS AND STRESSES IN
1CLADDING'/////// ',24X,'STATION',3X,'
1R/RO ER/ERO ET
2/EZO SR/|SRC| ST/|SRO| SA/|SRO| '//'(',24X,14,6X,
34F12.4,2F12.3)
WRITE (3,2001) RRS, ERO, ETO, SRO
2001 FORMAT (/// ',23X,'NOTE: R=RADIUS ',15X,'RO=RADIUS AT THE FUEL S
1URFACE=',F6.4,' CM',15X,'ERO=RADIAL STRAIN ',15X,'ERO=RADIAL STRAIN AT CLAD OUTER SURFACE=',F10.7,' ',31X,'ET=TANGENTIAL STRAIN
3ETO=TANGENTIAL STRAIN AT CLAD SURFACE=',F10.7,' ',31X,'SR=RADIAL STRESS
4LSTRESS ST=TANGENTIAL STRESS AT CLAD INNER SURFACE, SRO= '
5|SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT CLAD INNER SURFACE, SRO= '
6,F8.5,'X1000 PSI')
WRITE (3,999) (I,R(U,STNR(I),STNT(I),STSR(I),STST(I),STSA(I),1=1,
INSTNI)
999 FORMAT ('I'/////// ',54X,'TABLE'/// ',
1 40X,'DIMENSIONLESS STRAINS AND STRESSES IN
1FUEL'/////// ',24X,'STATION',3X,'
1R/RO ER/ERO ET/ETO
2 SR/|SRC| ST/|SRO| SA/|SRO| '//'(',24X,14,6X,
32F12.3)
WRITE (3,2002) RRS, ERF, ETF, SRF
2002 FORMAT
11// '*23, 'NOTE: R=RADIUS, RO=RADIUS AT THE FUEL SURFACE='
1,F6.4,' CM/', 31X,'ER=RADIAL STRAIN ER0=RADIAL STRAIN AT FUEL SURFACE=
3,F6.4,' CM/', 31X,'ET=TANGENTIAL STRAIN ET0=TANGENTIAL STRAIN AT FUEL SURFACE=',
31X,'ST=TANGENTIAL STRESS ST0=TANGENTIAL STRESS AT FUEL OUTER SURFACE=
6|SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT FUEL OUTER SURFACE, SRO='7,F8.5,'X1000 PSI'

IF (M.GE.MMAX) GO TO 1000
M=M+1
GO TO 1
1000 STOP
END
FUNCTION FCTG4(ES)

TO OBTAIN EQUIVALENT STRESS IN CLAD

COMMON /FCTC4X/ CCOEFK,CEXPN,CPSNRT, ALSE,CELSMD
FCTC4=(ALSE-2.*(1.+CPSNRT)*ES/3./CELSMD-CC0EFK*(ES/CELSMD)**CEXPN)/1*100.
RETURN
END
FUNCTION FCTG1(TX)

TO CALCULATE FUEL SURFACE TEMPERATURE

DIMENSION RC(11),TC(11),CSTNT(11),STNT(21)
COMMON /FCTG1X/RC,TC,CSTNT,STNT,
PWRL,STBO,CNTG1,CNTG2,
1CNTG3,CNTG4,CNTG5,CNTG6/FCT123/1Q,R(21)/HAPPY3/DUMMY2(84),NSTNT,
2NINT(3)/FCT6X/DIM(3),ABST
EMIS1=CNTG1+CNTG2*TX
EMIS2=CNTG3+CNTG4*TC(1)
GASK=CNTG5*((TC(1)+TX)/2)***CNTG6
GAPK=GASK+4.*(WC(1)*(1.+CSTNT(1))-R(NSTNT)*(1.+STNT(NSTNT)))*1ST60/(1./EMIS1+1./EMIS2-1.)*(TC(1)+TX+2.*ABST)/2.)*3
FCTG1=TX-TC(1)-ALOG((RC(1)*1.+CSTNT(1))/R(NSTNT)*1.+STNT(NSTNT))
11)/GAPK*PWRL/(2.*3.14159)
FUNCTION FCT1(X)

TO CALCULATE FUEL TEMPERATURES IN REGION 1

COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)/FCT123/I, R(21)
1/FCT15/RRV, RR1, TEMP1
FIK=CNT2*(X-TEMP1)+AL0G(X/TEMP1)/(CNT3-CNT4*DENFR(1))
FCT1=FIK-PWRV(1)*RR1**2/4.*(1.-(R(I)/RR1)**2+(RRV/RR1)**2*
1AL0G((R(I)/RR1)**2))
RETURN
END
FUNCTION FCT2(X)

TO CALCULATE FUEL TEMPERATURES IN REGION 2

COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)/FCT123/I, R(21)
1/FCT24/TEMP2, RR2
FIK=CNT2*(X-TEMP2)+AL0G(X/TEMP2)/(CNT3-CNT4*DENFR(2))
FCT2=FIK-PWRV(2)*RR2**2/4.*(1.-(R(I)/RR2)**2+(1.-PWRV(3)/PWRV(2))**2*
1AL0G((R(I)/RR2)**2))
RETURN
END
FUNCTION FCT3(X)

TO CALCULATE FUEL TEMPERATURES IN REGION 3

COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)/FCT123/I, R(21)
1/FCT3X/TEMP3, FCT34/RRS
FIK=CNT2*(X-TEMP3)+AL0G(X/TEMP3)/(CNT3-CNT4*DENFR(3))
FCT3=FIK-PWRV(3)*RRS**2/4.*(1.-(R(I)/RRS)**2)
RETURN
END
FUNCTION FCT4(SIG)

TO OBTAIN EQUIVALENT STRESS IN FUEL
DIMENSION PPTLMT(21), FEXPN(21), FCOEFSK(21), EQTSTN(21), ELSMD(21)
COMMON /FCT4X/ EQTSTN, PPTLMT, ELSMD, FCOEFSK, FEXPN, PSNRT, J
GOD = (SIG / PPTLMT(J)) ** (FEXPN(J))
FCT4 = 100. * (EQTSTN(J) - FCOEFSK(J)) * GOD - (2. * PSNRT - 1.) * SIG / 3. / ELSMD(J)
RETURN
END
FUNCTION FCT5(X)
FOR CALCULATION OF TEMPERATURE IN CENTRAL VOID
COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)
FCT5 = CNT2 * (X - TEMP1) + ALOG(X / TEMP1) / (CNT3 - CNT4 * DENFR(1))
RETURN
END
FUNCTION FCT6(X)
FOR CALCULATION OF T1
COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)
FCT6 = FK - TIME * CNTC * EXP(-HTVP / BOLZ / (X + ABST)) / SQRT(X + ABST) ** 3 * PWRV(3)
RETURN
END
FUNCTION FCT7(FRR1)
FOR CALCULATION OF R1
COMMON CNT2, CNT3, CNT4, DENFR(3), PWRV(3)
FCT7 = FK - PWRV(1) * RR1 ** 2 / 4. * (1. - (RRV / RR1) ** 2) ** 2 * (1. - ALOG((RRV / RR1) ** 2))
RETURN
END
RETURN
END

FUNCTION FCTD(D)

C TO CALCULATE BUBBLE DIAMETERS

COMMON /FCTDX/COEF(5)
FCTD=(COEF(5)*D**4+COEF(4)*D**3+COEF(3)*D**2+COEF(2)*D+COEF(1))/
1(COEF(1)/10.)
RETURN
END

SUBROUTINE QTFE(H,Y,Z,NDIM)

C TO EVALUATE INTEGRATIONS

DIMENSION Y(l),Z(l)
SUM2=0.
IF (NDIM-1)4,3,1
1 HH=5*H
DO 2 I=2,NDIM
SUM1=SUM2
SUM2=SUM2+HH*(Y(I)+Y(I-1))
2 SUM1=SUM1
3 Z(NDIM)=SUM2
4 RETURN
END

SUBROUTINE THSW

C TO CALCULATE TOTAL THERMAL AND SWELLING STRAINS

EXTERNAL FCTD
COMMON /FCTDX/COEF(5),MNITR
2/HAPPY2/AVSTS(21),ATEMP(21),BNU(21),SURT,CNAVST,SGB(21)
3/HAPPY3/THSWST(21),GAMTN(21),BATM(21),THSTN(21),NSTNT,NINT(3),
4/SGG(21),SGGM(21),VNDWC,WSWL(21),DB(21)/HAPPY4/BOLZ
PI=3.14159
IA=NINT(1)+NINT(2)+1
IB=NSTNT
DO 770 I=1A,IB
BATMN(I)=GATMN(I)/BNU(I)
IF (ABS(AVSTS(I)) GT CNAVST) GO TO 710
COEF(1)=-6.*BATMN(I)*VNDWC/PI*(1.0E+24)
COEF(2)=-3.*BATMN(I)*BOLZ*ATEMP(I)/(2.*SURT*PI)*(1.0E+16)
COEF(3)=0.
COEF(4)=1.
COEF(5)=0.
DY=SQRT(-COEF(2)/3.)
DX=(-COEF(1)-COEF(2)*1000.)**(1./3.)
CALL RTMI(DIAM,FCTDV,FCTD,DY,DX,0.01,MNTR,IERD1)
GO TO 720
710 COEF(1)=24.*SURT*BATMN(I)*VNDWC/PI/AVSTS(I)*(1.0E+32)
COEF(2)=6.*BATMN(I)*(BOLZ*ATEMP(I)-AVSTS(I)*VNDWC)/PI/AVSTS(I)
1*(1.0E+24)
COEF(3)=0.
COEF(4)=-4.*SURT/AVSTS(I)*(1.0E+08)
COEF(5)=1.
IF (AVSTS(I).GT.0.) GO TO 715
DY=SQRT(COEF(2)/COEF(4)/(-3.))
DX=100.
CALL RTMI(DIAM,FCTDV,FCTD,DY,DX,0.01,MNTR,IERD2)
GO TO 720
715 DX=-3./4.*COEF(4)
CALL RTMI(DIAM,FCTDV,FCTD,0.,DX,0.01,MNTR,IERD3)
720 DB(I)=DIAM*(1.0E-08)
SGG(I)=1./6.*PI*DB(I)**3*BNU(I)
770 CONTINUE
DO 780 I=1,NSTNT
THSWST(I)=1./3.*(SWSLD(I)+SGG(I)+SGB(I)+SGGM(I))+THSTN(I)
780 CONTINUE
RETURN
END

SUBROUTINE SGASB(DBGB,KMAX,SGBC,SGBN,I,TIME)
C TO CALCULATE SWELLING AT GRAIN BOUNDARIES
EXTERNAL FCTD  
DIMENSION VELB(21), TIMECB(21), VELCB(21), SGBC(KMAX), 
LSGBN(KMAX), DBGB(KMAX), 
COMMON /HAPPY1/ NUTIME, GASNM(21), DTIME(21), TIMECG(21), DG(21), 
1 DBGC(21), CNTF, TGDN(21), SCACTE/HAPPY4/BQLZ 
2/HAPPY2/AVSTS(21), ATEMP(21), BNU(21), SURT, CNAVST, SGB(21) 
COMMON /FCTDX/COEF(5), MNITR 
840 DBGB(1)=DBCG(1) 
DO 860 K=2,NUTIME 
IF (AVSTS(I).GT.CNAVST ) GO TO 850 
DBGB(K)=SQRT(DBGB(1)**2+DBGB(K-1)**2) 
GO TO 855 
850 COEF(1)=4.*SURT/AVSTS(I)**2+DBGB(K-1)**2)*(1.0E+24) 
1(DBGB(1)**3+DBGB(K-1)**3)*(1.0E+24) 
COEF(2)=0. 
COEF(3)=-4.*SURT/AVSTS(I)**2*(1.0E+08) 
COEF(4)=1. 
COEF(5)=0. 
IF (AVSTS(I).GT.0.) GO TO 853 
DX=6000.  
CALL RTMI(DIAM,FCTDV,FCTD,0.,DX, 0.01, MNITR,IERD4) 
WRITE (3,3004)I,IERD4,K 
3004 FORMAT (' STATION ',I2,5X,'IERD4='I1,5X,'K='I4) 
GO TO 854 
853 DX=-2./3.*COEF(3) 
CALL RTMI(DIAM,FCTDV,FCTD,0.,DX, 0.01, MNITR,IERD5) 
WRITE (3,3005)I,IERD5,K 
3005 FORMAT (' STATION ',I2,5X,'IERD5='I1,5X,'K='I4) 
854 DBGB(K)=DIAM*(1.0E-08) 
855 DBCB(I)=CNTF*SQR(ATEMP(I)/TGDN(I)) 
WRITE (3,3007) DBGB(K) 
3007 FORMAT (' DIAMETER OF BUBBLE AT GRAIN BOUNDARY='G10.5,'CM') 
IF (DBCB(I).GT.DBGB(K)) GO TO 860 
VELB(I)=CNTF*TGDN(I)/(ATEMP(I)**2*DBCB(I))*EXP(-SCACTE/BQLZ/ 
1ATEMP(I)) 
TIMECB(I)=DTIME(I)*K+TIMECG(I)
VELCB(I) = DG(I)/(TIME-TIMECB(I))
IF (VELCB(I).GT.VELB) GO TO 860
KTIMEC = K
GO TO 870
860 CONTINUE
KTIMEC = NUTIME+1
870 KMAX = NUTIME+2
PI = 3.14159
SG8(I) = 0.
KC = KTIMEC+2
GASNM(I) = 0.
BNUS = BNU(I)**2/3.
BNUSPI = BNUS*PI/4.
SGBC(1) = BNUS
SGBC(2) = 0.
SGBN(2) = 0.
900 DO 940 K = 3, KMAX
IF (K.LE.KC) GO TO 910
M = KC-1
GASNM(I) = GASNM(I) + SGBN(KC-1)*(4.*SURT/DBGB(KC-2)-AVSTS(I))*1./6.*PI*(DBGB(KC-2)**3)/(BOLZ*ATEMP(I))
GO TO 920
910 M = K-1
920 SGBN(M) = SGBC(M-1) + SGBN(M) - SGBC(M)
SGBC(M) = BNUSPI*(DBGB(M)+DBGB(M-1))**2*SGBN(M)
M = M-1
IF (M.GT.1) GO TO 920
SGBC(K) = 0.
SGBN(K) = 0.
SGB(1) = BNUS
DO 930 J = 3, K
SGB(C) = SGB(C)-SGBC(J-1)
930 CONTINUE
940 CONTINUE
NMAX = KC-2
DO 950 N = 2, NMAX
SGI = SGB(I) + SGBN(N)**1./6.*PI*DBGB(N-1)**3
950 CONTINUE
SUBROUTINE RTM1 (X, P, FCT, XL, XR, EPS, IEND, IER)

C TO SOLVE NONLINEAR EQUATIONS

IER=0
XL=XL
XR=XR
X=XL
TOL=X
F=FCT(TOL)
IF(F)1,16,1
1 FL=F
X=XR
TOL=X
F=FCT(TOL)
IF(F)2,16,2
2 FR=F
IF(SIGN(1.,FL)+SIGN(1.,FR))25,3,25
3 I=0
TOLF=100.*EPS
4 I=I+1
DO 13 K=1, IEND
X=.5*(XL+XR)
TOL=X
F=FCT(TOL)
IF(F)5,16,5
5 IF(SIGN(1.,F)+SIGN(1.,FR))7,6,7
6 TOL=XL
XL=XR
XR=TOL
TOL=FL
FL=FR
FR=TOL
7 TOL=F-FL
A=F*TOL
A=A+A
IF(A-FR*(FR-FL))8,9,9
8 IF(I=IEND)17,17,9
9 XR=X
FR=F
TOL=EPS
A=ABS(XR)
IF(A=1.)11,11,10
10 TOL=TOL*A
11 IF(ABS(XR-XL)-TOL)12,12,13
12 IF(ABS(FR-FL)-TOL)14,14,13
13 CONTINUE
IER=1
14 IF(ABS(FR)-ABS(FL))16,16,15
15 X=XL
F=FL
16 RETURN
17 A=FR-F
DX=(X-XL)*FL*(1.+F*(A-TOL)/(A*(FR-FL)))/TOL
XM=X
FM=F
X=XL-DX
TOL=X
F=FCT(TOL)
IF(F)18,16,18
18 TOL=EPS
A=ABS(X)
IF(A=1.)20,20,19
19 TOL=TOL*A
20 IF(ABS(DX)-TOL)21,21,22
21 IF(ABS(F)-TOL)16,16,22
22 IF(SIGN(1.,F)+SIGN(1.,FL))24,23,24
23 XR=X
FR=F
GO TO 4
24 XL=X
FL=F
XR=XM
FR=FM
GO TO 4
25 IER=2
RETURN
END
Table 10.1. Important notations in computer program other than those for input listed in Table 5.1

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPH(I)</td>
<td>$\alpha$, used in stress analysis for fuel</td>
</tr>
<tr>
<td>ALPHP(I)</td>
<td>$\alpha'$,</td>
</tr>
<tr>
<td>ATEMP(I)</td>
<td>Absolute temperature</td>
</tr>
<tr>
<td>ATGAP</td>
<td>Absolute temperature in fuel-clad gap</td>
</tr>
<tr>
<td>AVSTS(I)</td>
<td>Average stress in fuel</td>
</tr>
<tr>
<td>BATMN(I)</td>
<td>Number of gas atoms in one bubble</td>
</tr>
<tr>
<td>BETA(I)</td>
<td>$\beta$, used in stress analysis for fuel</td>
</tr>
<tr>
<td>BETAP(I)</td>
<td>$\beta'$,</td>
</tr>
<tr>
<td>BNCC1</td>
<td>$C_1$, integral constant for clad stress analysis</td>
</tr>
<tr>
<td>BNCC2</td>
<td>$C_2'$,</td>
</tr>
<tr>
<td>BNCG</td>
<td>Calculated constant $G$ in clad stress analysis</td>
</tr>
<tr>
<td>BNU(I)</td>
<td>Number of gas bubble</td>
</tr>
<tr>
<td>CAVSTS(I)</td>
<td>Average stress in clad</td>
</tr>
<tr>
<td>CEPLS1(I)</td>
<td>Equivalent plastic strain in clad</td>
</tr>
<tr>
<td>CESTS1(I)</td>
<td>Equivalent stress in clad</td>
</tr>
<tr>
<td>CETSTN(I)</td>
<td>Modified equivalent total strain in clad</td>
</tr>
<tr>
<td>CEXPN</td>
<td>Exponential constant $n_c$ in the expression for clad stress-strain curves</td>
</tr>
<tr>
<td>CPLSTA(I)</td>
<td>Axial plastic strain in clad</td>
</tr>
<tr>
<td>CPLSTR(I)</td>
<td>Radial plastic strain in clad</td>
</tr>
<tr>
<td>CPLSTT(I)</td>
<td>Tangential plastic strain in clad</td>
</tr>
<tr>
<td>CSTNR(I)</td>
<td>Radial strain in clad</td>
</tr>
<tr>
<td>CSTNT(I)</td>
<td>Tangential strain in clad</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------------------------------------------</td>
</tr>
<tr>
<td>CSTSA(I)</td>
<td>Axial stress in clad</td>
</tr>
<tr>
<td>CSTSR(I)</td>
<td>Radial stress in clad</td>
</tr>
<tr>
<td>CSTST(I)</td>
<td>Tangential stress in clad</td>
</tr>
<tr>
<td>CTHSTN(I)</td>
<td>Thermal strain in clad</td>
</tr>
<tr>
<td>CTTNDA(I)</td>
<td>Modified axial total strain deviator in clad</td>
</tr>
<tr>
<td>CTTNDR(I)</td>
<td>Modified radial total strain deviator in clad</td>
</tr>
<tr>
<td>CTTNDT(I)</td>
<td>Modified tangential total strain deviator in clad</td>
</tr>
<tr>
<td>DB(I)</td>
<td>Diameter of bubble in grain</td>
</tr>
<tr>
<td>DBCB(I)</td>
<td>Critical diameter of bubble for movement at grain boundary</td>
</tr>
<tr>
<td>DBCG(I)</td>
<td>Critical diameter of bubble for movement in grain</td>
</tr>
<tr>
<td>DBGB(K)</td>
<td>Diameter of bubble at grain boundary after Kth collision</td>
</tr>
<tr>
<td>DG(I)</td>
<td>Diameter of grain</td>
</tr>
<tr>
<td>DTEMP(I)</td>
<td>Temperature rise with respect to a reference temperature</td>
</tr>
<tr>
<td>DTIME(I)</td>
<td>Time interval for swelling calculation after movement of bubbles</td>
</tr>
<tr>
<td>ELSMD(I)</td>
<td>Elastic modulus in fuel</td>
</tr>
<tr>
<td>EQSTSL(I)</td>
<td>Equivalent stress in fuel</td>
</tr>
<tr>
<td>EQTSTN(I)</td>
<td>Modified equivalent total strain in fuel</td>
</tr>
<tr>
<td>FCT1</td>
<td>Function subprogram for calculating temperature in region I</td>
</tr>
<tr>
<td>FCT2</td>
<td>Function subprogram for calculating temperature in region II</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>FCT3</td>
<td>Function subprogram for calculating temperature in region III</td>
</tr>
<tr>
<td>FCT4</td>
<td>Function subprogram for calculating equivalent stress in fuel</td>
</tr>
<tr>
<td>FCT5</td>
<td>Function subprogram for calculating central void temperature of fuel</td>
</tr>
<tr>
<td>FCT6</td>
<td>Function subprogram for calculating temperature at the outer surface of columnar grain region</td>
</tr>
<tr>
<td>FCT7</td>
<td>Function subprogram for calculating temperature at the outer surface of equiaxed grain region</td>
</tr>
<tr>
<td>FCTC4</td>
<td>Function subprogram for calculating equivalent stress in clad</td>
</tr>
<tr>
<td>FCTD</td>
<td>Function subprogram for calculating diameter of bubble</td>
</tr>
<tr>
<td>FCTGl</td>
<td>Function subprogram for calculating temperature at fuel surface</td>
</tr>
<tr>
<td>FEXPN</td>
<td>Exponential constant $n_f$ in the expression for fuel stress-strain curves</td>
</tr>
<tr>
<td>FSNRT(I)</td>
<td>Fission rate per unit volume</td>
</tr>
<tr>
<td>GAMA(I)</td>
<td>$\gamma$, used in stress analysis for fuel</td>
</tr>
<tr>
<td>GAMAP(I)</td>
<td>$\gamma'$, used in stress analysis for fuel</td>
</tr>
<tr>
<td>GAP</td>
<td>Thickness of fuel-clad gap</td>
</tr>
<tr>
<td>GASNM(I)</td>
<td>Number of gas atoms in bubbles moving away from grain boundaries at Ith station</td>
</tr>
<tr>
<td>GATMN(I)</td>
<td>Number of gas atoms per unit volume</td>
</tr>
<tr>
<td>GATMNM</td>
<td>Total number of gas atoms in bubbles moving away from grain boundaries in region III</td>
</tr>
<tr>
<td>NINT(I)</td>
<td>Number of intervals in region I</td>
</tr>
</tbody>
</table>
Table 10.1 (Continued)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>NITR1</td>
<td>Number of iteration for fuel surface temperature</td>
</tr>
<tr>
<td>NITR2</td>
<td>Number of iteration for fuel stress analysis</td>
</tr>
<tr>
<td>NITR3</td>
<td>Number of iteration for clad stress analysis</td>
</tr>
<tr>
<td>NSTN1</td>
<td>Station number at the outer surface of columnar grain region</td>
</tr>
<tr>
<td>NSTN2</td>
<td>Station number at the outer surface of equiaxed grain region</td>
</tr>
<tr>
<td>NSTN3</td>
<td>Station number at the fuel surface</td>
</tr>
<tr>
<td>NSTNC</td>
<td>Total number of stations in clad</td>
</tr>
<tr>
<td>NSTNT</td>
<td>Total number of stations in fuel</td>
</tr>
<tr>
<td>NUTIME</td>
<td>Number of time intervals for bubble movement</td>
</tr>
<tr>
<td>PECLET</td>
<td>Peclet number</td>
</tr>
<tr>
<td>PI</td>
<td>3.14159</td>
</tr>
<tr>
<td>PLSTA(I)</td>
<td>Axial plastic strain in fuel</td>
</tr>
<tr>
<td>PLSTR(I)</td>
<td>Radial plastic strain in fuel</td>
</tr>
<tr>
<td>PLSTT(I)</td>
<td>Tangential plastic strain in fuel</td>
</tr>
<tr>
<td>PNPL(I)</td>
<td>$P_{N+1}$, used in stress analysis for fuel</td>
</tr>
<tr>
<td>PORST(I)</td>
<td>Porosity</td>
</tr>
<tr>
<td>PPTLMT(I)</td>
<td>Proportional limit</td>
</tr>
<tr>
<td>PRESA</td>
<td>Pressure in central void</td>
</tr>
<tr>
<td>PRESG</td>
<td>Pressure in fuel-clad gap</td>
</tr>
<tr>
<td>PWRV(I)</td>
<td>Volumetric heat power</td>
</tr>
<tr>
<td>QNPL(I)</td>
<td>$Q_{N+1}$, used in stress analysis for fuel</td>
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</table>
Table 10.1 (Continued)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>QTEF</td>
<td>Subroutine subprogram to evaluate integrals by trapezoid rule</td>
</tr>
<tr>
<td>R(I)</td>
<td>Radius in fuel</td>
</tr>
<tr>
<td>RC(I)</td>
<td>Radius in clad</td>
</tr>
<tr>
<td>RDC</td>
<td>Radial difference in clad</td>
</tr>
<tr>
<td>RDR(J)</td>
<td>Radial difference in region J of fuel</td>
</tr>
<tr>
<td>RR1</td>
<td>Radius at outer surface of columnar grain region</td>
</tr>
<tr>
<td>RR2</td>
<td>Radius at outer surface of equiaxed grain region</td>
</tr>
<tr>
<td>RRS</td>
<td>Radius at fuel surface</td>
</tr>
<tr>
<td>RRV</td>
<td>Radius of central void</td>
</tr>
<tr>
<td>RTMI</td>
<td>Subroutine subprogram to solve general nonlinear equations of the form ( f(x) = 0 ) by means of Mueller-S iteration method</td>
</tr>
<tr>
<td>SE(I)</td>
<td>The variable ( e ) in fuel</td>
</tr>
<tr>
<td>SEC(I)</td>
<td>The variable ( e ) in clad</td>
</tr>
<tr>
<td>SG(I)</td>
<td>The variable ( g ) in fuel</td>
</tr>
<tr>
<td>SGASB</td>
<td>Subroutine subprogram to calculate fission-gas swelling at grain boundaries</td>
</tr>
<tr>
<td>SGB(I)</td>
<td>Fission-gas swelling at grain boundaries</td>
</tr>
<tr>
<td>SGBC(M)</td>
<td>The defined variable ( C_{i,R} ) in the calculation of fission-gas swelling at grain boundaries</td>
</tr>
<tr>
<td>SGBN(M)</td>
<td>The defined variable ( N_{i,R} ) in the calculation of fission-gas swelling at grain boundaries</td>
</tr>
<tr>
<td>SGC</td>
<td>The variable ( g ) in clad</td>
</tr>
<tr>
<td>SGGM(I)</td>
<td>Fission-gas swelling due to moving bubbles in grains before reaching a grain boundary</td>
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</table>
Table 10.1 (Continued)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
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<td>STNR(I)</td>
<td>Radial strain in fuel</td>
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<tr>
<td>STNT(I)</td>
<td>Tangential strain in fuel</td>
</tr>
<tr>
<td>STSA(I)</td>
<td>Axial stress in fuel</td>
</tr>
<tr>
<td>STSR(I)</td>
<td>Radial stress in fuel</td>
</tr>
<tr>
<td>STST(I)</td>
<td>Tangential stress in fuel</td>
</tr>
<tr>
<td>STSDVA(I)</td>
<td>Axial stress deviator in fuel</td>
</tr>
<tr>
<td>STSDVR(I)</td>
<td>Radial stress deviator in fuel</td>
</tr>
<tr>
<td>STSDVT(I)</td>
<td>Tangential stress deviator in fuel</td>
</tr>
<tr>
<td>SWSLD(I)</td>
<td>Solid fission-product swelling</td>
</tr>
<tr>
<td>T(I)</td>
<td>Temperature in fuel</td>
</tr>
<tr>
<td>TC(I)</td>
<td>Temperature in clad</td>
</tr>
<tr>
<td>TEMP1</td>
<td>Temperature at outer surface of columnar grain region</td>
</tr>
<tr>
<td>TEMP2</td>
<td>Temperature at outer surface of equiaxed grain region</td>
</tr>
<tr>
<td>TEMPC</td>
<td>Fictitious centerline temperature</td>
</tr>
<tr>
<td>TEMPS</td>
<td>Temperature at fuel surface</td>
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<tr>
<td>TEMPV</td>
<td>Temperature in central void</td>
</tr>
<tr>
<td>TGDN(I)</td>
<td>Temperature gradient</td>
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<tr>
<td>THSTN(I)</td>
<td>Thermal strain in fuel</td>
</tr>
<tr>
<td>THSW</td>
<td>Subroutine subprogram to calculate thermal and irradiation strains</td>
</tr>
<tr>
<td>THSWST(I)</td>
<td>Thermal and irradiation dilatation strain</td>
</tr>
<tr>
<td>TIME</td>
<td>Irradiation time</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>TIMECB(I)</td>
<td>Critical time for movement of bubbles trapped on grain boundaries</td>
</tr>
<tr>
<td>TIMECG(I)</td>
<td>Critical time for movement of bubbles anchored on dislocations in grains</td>
</tr>
<tr>
<td>TIMEG(I)</td>
<td>Time required for a bubble to travel across a grain</td>
</tr>
<tr>
<td>TIN</td>
<td>Temperature at clad inner surface</td>
</tr>
<tr>
<td>TOU</td>
<td>Temperature at clad outer surface</td>
</tr>
<tr>
<td>TSTNDA(I)</td>
<td>Modified axial total strain deviator in fuel</td>
</tr>
<tr>
<td>TSTNDR(I)</td>
<td>Modified radial total strain deviator in fuel</td>
</tr>
<tr>
<td>TSTNDT(I)</td>
<td>Modified tangential total strain deviator in fuel</td>
</tr>
<tr>
<td>VELB(I)</td>
<td>Velocity of bubble at grain boundary</td>
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<tr>
<td>VELCB(I)</td>
<td>Critical velocity for bubbles moving away from grain boundaries</td>
</tr>
<tr>
<td>VELCG(I)</td>
<td>Critical velocity for bubbles moving away from anchored sites in grains</td>
</tr>
<tr>
<td>VELG(I)</td>
<td>Velocity of bubble in grain</td>
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</tbody>
</table>
XI. APPENDIX B: TABLES OF RESULTS
Table 11.1 Time variations of temperatures and radii at region surfaces, fuel-clad gap, and pressures in central void and gap

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>Temperatures (°C)</th>
<th>radii</th>
<th>fuel-clad gap (mm)</th>
<th>pressures (1000 psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(T_v)</td>
<td>(T_1)</td>
<td>(T_2)</td>
<td>(T_s)</td>
</tr>
<tr>
<td>1</td>
<td>2398.</td>
<td>2340.</td>
<td>2013.</td>
<td>744.9</td>
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<tr>
<td>8</td>
<td>2331.</td>
<td>2149.</td>
<td>1846.</td>
<td>754.8</td>
</tr>
<tr>
<td>27</td>
<td>2295.</td>
<td>2050.</td>
<td>1759.</td>
<td>759.8</td>
</tr>
<tr>
<td>64</td>
<td>2269.</td>
<td>1985.</td>
<td>1701.</td>
<td>762.0</td>
</tr>
<tr>
<td>125</td>
<td>2247.</td>
<td>1936.</td>
<td>1658.</td>
<td>761.7</td>
</tr>
<tr>
<td>216</td>
<td>2228.</td>
<td>1898.</td>
<td>1624.</td>
<td>759.0</td>
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<td>343</td>
<td>2209.</td>
<td>1867.</td>
<td>1597.</td>
<td>753.7</td>
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<td>512</td>
<td>2190.</td>
<td>1841.</td>
<td>1574.</td>
<td>745.4</td>
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<td>729</td>
<td>2169.</td>
<td>1819.</td>
<td>1555.</td>
<td>733.8</td>
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<td>1000</td>
<td>2145.</td>
<td>1799.</td>
<td>1537.</td>
<td>718.5</td>
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<td>STATION</td>
<td>SWELLING STRAIN</td>
<td>RADIAL PLASTIC STRAIN</td>
<td>TANGENTIAL PLASTIC STRAIN</td>
<td>AXIAL PLASTIC STRAIN</td>
</tr>
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Table 11.4. Swelling strain and plastic strains in fuel at time $t=729$ hours

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Table 11.6. Temperature distribution, temperature gradients and thermal strains in fuel at time $t=1$ hour

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Table 11.7. Temperature distribution, temperature gradients and thermal strains in fuel at time t=125 hours

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Table 11.8. Temperature distribution, temperature gradients and thermal strains in fuel at time \( t = 729 \) hours

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Table 11.9. Temperature distribution, temperature gradients and thermal strains in fuel at time $t=1000$ hours

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Table 11.10. Dimensionless strains and stresses in fuel at time t=1 hour

| STATION | R/RO  | ER/ERO | ET/ETO  | SR/|SRO|   | ST/|SRO|   | SA/|SRO|   |
|---------|-------|--------|---------|-----|-----|---|-----|---|-----|---|
| 1       | 0.1447| -7.7686| 0.9766  | -0.7517 | -172.172 | -129.791 |
| 2       | 0.1791| -6.7554| 1.3797  | -31.9040| -144.179 | -135.770 |
| 3       | 0.2135| -5.1339| 1.5927  | -46.9873| -127.217 | -138.034 |
| 4       | 0.2479| -5.6769| 1.7137  | -59.1911| -115.508 | -138.345 |
| 5       | 0.2823| -5.2097| 1.7831  | -65.5852| -106.477 | -137.504 |
| 6       | 0.3304| -4.7760| 1.8292  | -70.9153| -96.258  | -135.634 |
| 7       | 0.3785| -4.2816| 1.8363  | -73.5470| -86.062  | -131.736 |
| 8       | 0.4266| -3.7886| 1.8184  | -74.4229| -75.701  | -126.685 |
| 9       | 0.4748| -3.2931| 1.7833  | -74.0095| -64.579  | -120.436 |
| 10      | 0.5229| -2.7984| 1.7356  | -72.6054| -52.336  | -112.950 |
| 11      | 0.5663| -2.2494| 1.6830  | -70.7483| -44.296  | -108.952 |
| 12      | 0.6096| -1.7419| 1.6222  | -68.3144| -28.402  | -98.661 |
| 13      | 0.6530| -1.2664| 1.5560  | -65.0984| -10.808  | -86.911 |
| 14      | 0.6964| -0.8270| 1.4861  | -61.1143|  8.920   | -73.660 |
| 15      | 0.7397| -0.4273| 1.4143  | -56.3548| 31.544   | -58.771 |
| 16      | 0.7831|  0.0708| 1.3416  | -50.7554| 58.320  | -41.888 |
| 17      | 0.8265|  0.2601| 1.2694  | -44.1843|  91.232 | -22.318 |
| 18      | 0.8699|  0.5036| 1.1984  | -36.3994| 133.335 | -1.176 |
| 19      | 0.9132|  0.7185| 1.1294  | -27.0314| 189.201 |  30.544 |
| 20      | 0.9566|  0.8843| 1.0631  | -15.5153| 265.746 |  68.774 |
| 21      | 1.0000|  1.0000| 1.0000  | -1.0000| 374.479 | 120.540 |

**NOTE:**
- **R** = RADIUS
- **RO** = RADIUS AT THE FUEL SURFACE = 0.2730 CM
- **ER** = RADIAL STRAIN
- **ERO** = RADIAL STRAIN AT FUEL SURFACE = -0.0145211
- **ET** = TANGENTIAL STRAIN
- **ETO** = TANGENTIAL STRAIN AT FUEL SURFACE = 0.0350265
- **SR** = RADIAL STRESS
- **ST** = TANGENTIAL STRESS
- **SA** = AXIAL STRESS
- **|SRO|** = ABSOLUTE VALUE OF RADIAL STRESS AT FUEL OUTER SURFACE; SR0 = -0.16109

1000 PSI
| STATION | R/RO  | ER/ERO | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|-------|--------|--------|----------------|----------------|----------------|----------------|----------------|
| 1       | 0.1966 | -8.4190| 0.9624 | -1.1417        | -188.035       | -150.203       |
| 2       | 0.2347 | -7.5139| 1.2487 | -30.0147       | -162.353       | -156.560       |
| 3       | 0.2728 | -6.8672| 1.4158 | -47.5550       | -144.744       | -159.322       |
| 4       | 0.3109 | -6.3358| 1.5167 | -58.7861       | -131.277       | -159.885       |
| 5       | 0.3490 | -5.8558| 1.5764 | -66.1601       | -119.913       | -158.940       |
| 6       | 0.3872 | -5.3972| 1.6085 | -70.9897       | -109.498       | -156.842       |
| 7       | 0.4253 | -4.9458| 1.6210 | -74.0221       | -99.295        | -153.745       |
| 8       | 0.4634 | -4.4957| 1.6188 | -75.6944       | -88.830        | -149.690       |
| 9       | 0.5015 | -4.0457| 1.6054 | -76.2891       | -77.762        | -144.669       |
| 10      | 0.5397 | -3.5970| 1.5831 | -75.9866       | -65.831        | -138.647       |
| 11      | 0.5866 | -2.9909| 1.5455 | -74.5427       | -49.538        | -130.085       |
| 12      | 0.6336 | -2.4115| 1.4986 | -71.9758       | +29.788        | -118.507       |
| 13      | 0.6806 | -1.9105| 1.4455 | -68.3627       | -9.046         | -106.429       |
| 14      | 0.7205 | -1.3324| 1.3958 | -64.5423       | 10.780         | -94.629        |
| 15      | 0.7605 | -0.8299| 1.3417 | -59.8534       | 39.304         | -77.022        |
| 16      | 0.8004 | -0.3829| 1.2850 | -54.0991       | 72.402         | -57.144        |
| 17      | 0.8403 | -0.0075| 1.2269 | -47.1547       | 112.183        | -34.383        |
| 18      | 0.8802 | 0.3409 | 1.1686 | -38.8181       | 161.840        | -7.603         |
| 19      | 0.9201 | 0.6174 | 1.1109 | -28.7443       | 225.998        | 25.045         |
| 20      | 0.9601 | 0.8372 | 1.0545 | -16.403        | 311.443        | 66.315         |
| 21      | 1.0000 | 1.0000 | 1.0000 | -1.0000        | 428.942        | 120.327        |

NOTE: R=RADIUS
RO=RADIUS AT THE FUEL SURFACE=0.2730 CM
ER=RADIAL STRAIN
ERO RADIAL STRAIN AT FUEL SURFACE=-0.0114324
ET=TANGENTIAL STRAIN
ETO TANGENTIAL STRAIN AT FUEL SURFACE= 0.0339097
SR=RADIAL STRESS
ST=TANGENTIAL STRESS
SA=AXIAL STRESS
|SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT FUEL OUTER SURFACE; SR=0=-0.12993
1000PSI
Table 11.12. Dimensionless strains and stresses in fuel at time t=729 hours

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**NOTE:**

- R=RADIUS
- RO=RADIUS AT THE FUEL SURFACE=0.2730 CM
- ER=RADIAL STRAIN
- ERO=RADIAL STRAIN AT FUEL SURFACE=-0.0061277
- ET=TANGENTIAL STRAIN
- ETO=TANGENTIAL STRAIN AT FUEL SURFACE= 0.0352096
- SR=RADIAL STRESS
- ST=TANGENTIAL STRESS
- SA=AXIAL STRESS
- |SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT FUEL OUTER SURFACE, SRO=-0.19090 1000PSI
### Table 11.13. Dimensionless strains and stresses in fuel at time $t=1000$ hours

| STATION | R/RO  | ER/ERO | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|-------|--------|--------|-----|-----| |-----|-----| |-----|-----| |
| 1       | 0.2039| -22.7198| 0.9617 | -1.4228 | -93.121 | -76.721 |
| 2       | 0.2407| -20.5658| 1.1842 | -14.7692 | -81.081 | -79.576 |
| 3       | 0.2775| -18.9748| 1.3186 | -23.1247 | -72.650 | -80.877 |
| 4       | 0.3143| -17.6432| 1.4020 | -28.6062 | -66.136 | -81.221 |
| 5       | 0.3511| -16.4298| 1.4528 | -32.2877 | -60.628 | -80.920 |
| 6       | 0.3879| -15.2661| 1.4813 | -34.7597 | -55.595 | -80.112 |
| 7       | 0.4247| -14.1189| 1.4938 | -36.3655 | -50.701 | -78.873 |
| 8       | 0.4615| -12.9745| 1.4941 | -37.3239 | -45.720 | -77.234 |
| 9       | 0.4983| -11.8295| 1.4850 | -37.7606 | -40.490 | -75.207 |
| 10      | 0.5351| -10.6868| 1.4684 | -37.7612 | -34.872 | -72.761 |
| 12      | 0.6157| -8.0352 | 1.4122 | -36.4854 | -20.346 | -65.789 |
| 13      | 0.6594| -6.6353 | 1.3723 | -35.0912 | -10.462 | -60.629 |
| 14      | 0.7032| -5.6562 | 1.3322 | -33.4441 | -7.436  | -62.946 |
| 15      | 0.7456| -4.6842 | 1.2919 | -31.5695 | 7.499   |    54.980 |
| 16      | 0.7880| -3.2796 | 1.2460 | -28.9881 | 25.599  |    46.106 |
| 17      | 0.8304| -2.0592 | 1.1975 | -25.6528 | 47.406  |    35.985 |
| 18      | 0.8728| -1.0224 | 1.1476 | -21.4502 | 74.736  |    24.246 |
| 19      | 0.9152| -0.1678 | 1.0974 | -16.1833 | 110.334 |    10.218 |
| 20      | 0.9576| 0.5064  | 1.0480 | -9.5342  | 158.410 |       7.171 |
| 21      | 1.0000| 1.0000  | 1.0000 | -1.0000  | 226.144 |       29.448 |

**NOTE:**
- $R =$ RADIUS
- $R_0 =$ RADIUS AT THE FUEL SURFACE = 0.2730 CM
- $ER =$ RADIAL STRAIN
- $ERO =$ RADIAL STRAIN AT FUEL SURFACE = -0.0039700
- $ET =$ TANGENTIAL STRAIN
- $ETO =$ TANGENTIAL STRAIN AT FUEL SURFACE = 0.0359726
- $SR =$ RADIAL STRESS
- $ST =$ TANGENTIAL STRESS
- $SA =$ AXIAL STRESS
- $|SRO| =$ ABSOLUTE VALUE OF RADIAL STRESS AT FUEL OUTER SURFACE, $SRO = -0.25706$
Table 11.14. Temperature distribution, thermal strain and plastic strains in cladding at time t=1 hour

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<th>RADIUS (CM)</th>
<th>TEMPERATURE (C)</th>
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<th>RADIAL PLASTIC STRAIN</th>
<th>TANGENTIAL PLASTIC STRAIN</th>
<th>AXIAL PLASTIC STRAIN</th>
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Table 11.15. Temperature distribution, thermal strain and plastic strains in cladding at time \( t = 125 \) hours

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Temperature distribution, thermal strain and plastic strains in cladding at time \( t = 729 \) hours

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Table 11.17. Temperature distribution, thermal strain and plastic strains in cladding at time $t=1000$ hours

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Table 11.18. Dimensionless strains and stresses in cladding at time t=1 hour

| STATION | R/RO  | ER/ERO | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|-------|--------|--------|-----------------|-----------------|-----------------|-----------------|
| 1       | 1.0220| 1.2191 | 1.0016 | -1.0000         | -20.417         | -256.787        |
| 2       | 1.0361| 1.1942 | 1.0027 | -1.2261         | -15.202         | -257.148        |
| 3       | 1.0502| 1.1701 | 1.0034 | -1.3760         | -9.930          | -257.333        |
| 4       | 1.0643| 1.1466 | 1.0038 | -1.4515         | -4.601          | -257.339        |
| 5       | 1.0784| 1.1239 | 1.0040 | -1.4554         |  0.776          | -257.172        |
| 6       | 1.0925| 1.1018 | 1.0039 | -1.3897         |  6.194          | -256.833        |
| 7       | 1.1066| 1.0803 | 1.0036 | -1.2555         | 11.647          | -256.325        |
| 8       | 1.1207| 1.0594 | 1.0030 | -1.0577         | 17.130          | -255.650        |
| 9       | 1.1348| 1.0390 | 1.0022 | -0.7954         | 22.637          | -254.813        |
| 10      | 1.1489| 1.0193 | 1.0012 | -0.4721         | 28.162          | -253.818        |
| 11      | 1.1630| 1.0000 | 1.0000 | -0.0894         | 33.702          | -252.668        |

**Note:**
- **R=RADIUS**
- **RO=RADIUS AT THE FUEL SURFACE=0.2730 CM**
- **ER=RADIAL STRAIN**
- **ERO=RADIAL STRAIN AT CLAD OUTER SURFACE= 0.0151023**
- **ET=TANGENTIAL STRAIN**
- **ETO=TANGENTIAL STRAIN AT CLAD SURFACE= 0.0168744**
- **SR=RADIAL STRESS**
- **ST=TANGENTIAL STRESS**
- **SA=AXIAL STRESS**
- |SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT CLAD INNER SURFACE, SRO=-0.16111 1000PSI
Table 11.19. Dimensionless strains and stresses in cladding at time t=125 hours

| STATION | R/RO  | ER/ERO  | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|-------|---------|--------|-----|------| |------|------| |------|------| |
| 1       | 1.0220 | 1.2193  | 1.0012 | -1.0000 | -27.020 | -318.240 |
| 2       | 1.0361 | 1.1944  | 1.0023 | -1.3074 | -20.559 | -318.705 |
| 3       | 1.0502 | 1.1702  | 1.0031 | -1.5193 | -14.026 | -318.949 |
| 4       | 1.0643 | 1.1467  | 1.0036 | -1.6371 | -7.426  | -318.974 |
| 5       | 1.0784 | 1.1240  | 1.0038 | -1.6671 | -0.767  | -318.782 |
| 6       | 1.0925 | 1.1018  | 1.0037 | -1.6102 | 5.945   | -318.377 |
| 7       | 1.1066 | 1.0803  | 1.0034 | -1.4684 | 12.70  | -317.763 |
| 8       | 1.1207 | 1.0594  | 1.0029 | -1.2446 | 19.491  | -316.942 |
| 9       | 1.1348 | 1.0391  | 1.0021 | -0.9418 | 26.314  | -315.919 |
| 10      | 1.1489 | 1.0193  | 1.0012 | -0.5627 | 33.158  | -314.700 |
| 11      | 1.1630 | 1.0000  | 1.0000 | -0.1110 | 40.020  | -313.289 |

NOTE:  
R=RADIUS  
R0=RADIUS AT THE FUEL SURFACE=0.2730 CM  
ER=RADIAL STRAIN  
ERO=RADIAL STRAIN AT CLAD OUTER SURFACE= 0.0151211  
ET=TANGENTIAL STRAIN  
ETO=TANGENTIAL STRAIN AT CLAD SURFACE= 0.0168502  
SR=RADIAL STRESS  
ST=TANGENTIAL STRESS  
SA=AXIAL STRESS  
|SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT CLAD INNER SURFACE, SRO=0.13029  
1000PSI
| STATION | R/RO | ER/ERO | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|------|--------|--------|-----------------|------------------|------------------|------------------|------------------|
| 1       | 1.0220 | 1.2189 | 1.0020 | -1.0000 | -16.025 | -215.872 |
| 2       | 1.0361 | 1.1941 | 1.0030 | -1.1724 | -11.639 | -216.165 |
| 3       | 1.0502 | 1.1699 | 1.0037 | -1.2809 | -7.203 | -216.309 |
| 4       | 1.0643 | 1.1465 | 1.0041 | -1.3278 | -2.721 | -216.303 |
| 5       | 1.0784 | 1.1238 | 1.0042 | -1.3139 | 1.800 | -216.152 |
| 6       | 1.0925 | 1.1017 | 1.0041 | -1.7433 | 6.357 | -215.857 |
| 7       | 1.1066 | 1.0802 | 1.0037 | -1.1157 | 10.944 | -215.419 |
| 8       | 1.1207 | 1.0593 | 1.0031 | -0.9332 | 15.555 | -214.842 |
| 9       | 1.1348 | 1.0390 | 1.0022 | -0.6978 | 20.188 | -214.129 |
| 10      | 1.1489 | 1.0192 | 1.0012 | -0.4114 | 24.835 | -213.282 |
| 11      | 1.1630 | 1.0000 | 1.0000 | -0.0757 | 29.494 | -212.306 |

**NOTE:**
- **R** = RADIUS
- **RO** = RADIUS AT THE FUEL SURFACE = 0.2730 CM
- **ER** = RADIAL STRAIN
- **ERO** = RADIAL STRAIN AT CLAD OUTER SURFACE = 0.0150841
- **ET** = TANGENTIAL STRAIN
- **ETO** = TANGENTIAL STRAIN AT CLAD SURFACE = 0.0168980
- **SR** = RADIAL STRESS
- **ST** = TANGENTIAL STRESS
- **SA** = AXIAL STRESS
- |SRO| = ABSOLUTE VALUE OF RADIAL STRESS AT CLAD INNER SURFACE, SRO = -0.19122 1000PSI
Table 11.21. Dimensionless strains and stresses in cladding at time t=1000 hours

| STATION | R/RO | ER/ERO | ET/ETO | SR/|SRO| | ST/|SRO| | SA/|SRO| |
|---------|------|--------|--------|-----|-----|-----|-----|-----|-----|
| 1       | 1.0220 | 1.2185 | 1.0028 | -1.0000 | -9.971 | -159.549 |
| 2       | 1.0361 | 1.1937 | 1.0037 | -1.0984 | -6.726 | -159.747 |
| 3       | 1.0502 | 1.1696 | 1.0043 | -1.1504 | -3.444 | -159.834 |
| 4       | 1.0643 | 1.1463 | 1.0046 | -1.1568 | -0.127 | -159.813 |
| 5       | 1.0784 | 1.1236 | 1.0046 | -1.1206 | 3.218 | -159.683 |
| 6       | 1.0925 | 1.1015 | 1.0044 | -1.0418 | 6.589 | -159.448 |
| 7       | 1.1066 | 1.0801 | 1.0040 | -0.9216 | 9.983 | -159.108 |
| 8       | 1.1207 | 1.0592 | 1.0033 | -0.7619 | 13.395 | -158.665 |
| 9       | 1.1348 | 1.0390 | 1.0024 | -0.5635 | 16.822 | -158.123 |
| 10      | 1.1489 | 1.0192 | 1.0013 | -0.3276 | 20.260 | -157.482 |
| 11      | 1.1630 | 1.0000 | 1.0000 | -0.0563 | 23.706 | -156.745 |

NOTE:  
R=RADIUS  
RO=RADIUS AT THE FUEL SURFACE=0.2730 CM  
ER=RADIAL STRAIN  
ERO=RADIAL STRAIN AT CLAD OUTER SURFACE= 0.0150437  
ET=TANGENTIAL STRAIN  
ETO=TANGENTIAL STRAIN AT CLAD SURFACE= 0.0169502  
SR=RADIAL STRESS  
ST=TANGENTIAL STRESS  
SA=AXIAL STRESS  
|SRO|=ABSOLUTE VALUE OF RADIAL STRESS AT CLAD INNER SURFACE, SRO= -0.25745 1000PSI