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The valuation of performance of educational systems: prescriptive models

Yildirim Omurtag

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The valuation of performance of educational systems: Prescriptive models

by

Yildirim Omurtag

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Engineering Valuation

Approved:

Signature was redacted for privacy.

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I. INTRODUCTION

Everyone associated with state universities in Iowa is aware of the successful attempts by state legislators to reduce the annual increase of funds allocated to these institutions (1) (2). The problems faced by educational institutions are in many respects similar to allocation problems faced by industrial organizations. In both cases decisions have to be made in allocating limited resources among alternative and often competing ends or uses.

Unlike a firm, which is said to be price guided, since its inputs and outputs are paid for in market prices and whose objective of long term profit maximization is well recognized, an educational system is quite complicated in terms of quantitative measures of its inputs and outputs. Its objectives are general and its processes are at best traditional or experimental.

But this does not mean that educational systems should not be subjected to similar analyses and examinations that industries are. In fact, educational systems, because they serve and concern such a large portion of our society, should be closely evaluated in terms of utilization of resources allocated to them.
This concept has met with widespread and renewed interest and acceptance in recent years. In educational terms we now talk about "accountability" (3).

A. Accountability in Education

The idea of accountability is not a new one. The philosopher teachers of ancient Greece and the professors of European universities in the Middle Ages derived much of their support from their students. In the early history of American education the 1817 Georgia law applying to poor schools for low income families stated that (4):

"The commissioners are forbidden to pay a teacher any salary if an examination shows that his students have not made good progress in that quarter."

A recent sign of the growing importance of accountability is found in an article on Yale University (5):

"At the urging of Kingman Brewster, Jr., a committee of trustees has been selected to review his tenure as president of Yale University ... Last fall Mr. Brewster recommended that Yale adopt a new policy of accountability to guard against incompetent and unresponsive administration."

The concept of accountability was an important item in President Nixon's special message to Congress March 3, 1970, on educational reform, when he called upon school systems to "begin the responsible, open measurement of how well the educational process is working." He further stated that (6):
"School administrators and teachers alike are responsible for their performance, and it is in their interest as well as in the interests of their pupils that they be held accountable..."

Later on he pointed out that:

"In opposing some mythical threat what we have too often been doing is avoiding accountability for our own local performances. We have as a nation, too long avoided thinking of the productivity of schools. This is a mistake because it undermines the principle of local control of education. Ironic though it is, the avoidance of accountability is the single most serious threat to a continued and even more pluralistic educational system."

It seems no longer possible to answer demands for accountability by denying economic rationality to such a serious matter as education even if "...higher education is the world of humane learning, scholarly inquiry and freedom of the spirit" (7).

A recent official affirmation along these lines is found in the following passage taken from the Report on Higher Education March, 1971 (8, p. 86):

"It is apparent that with multimillion-dollar budgets and a growing questioning by the public, higher education can no longer afford the luxury of avoiding consideration of how effectively it uses its resources. How can skill in resource utilization become a factor in the system of academic rewards? The challenging intellectual task of finding more effective learning patterns by better utilization of resources must become a legitimate campus concern."

This concern is a natural outcome of the fact that our resources are limited. When we examine the economic situation the picture becomes even more clear (9). Faculty
salaries for full professors in 1958 averaged just over 8,000 dollars. In 1968 the American Association of University Professors reported comparable salaries to be over 16,000 dollars. Educational allocations have been rising by about 10.6% a year for the last decade, but the estimated improvement produced by these outlays has been reported to be around 8% for public schools during the interval 1955-65.

This is not a very favorable figure when compared to other types of organizations. For example, in American industry labor costs have more than doubled in the last two decades, but so has productivity. Communication costs have gone down due to the advancement in technology. American farmers can now produce at a much higher rate than just a decade ago. In the total private economy, productivity as measured in production per man hour has almost doubled since 1947.

Accountability now seems to be not only desirable, but also mandatory for any public service system. However, quantification of outputs in educational systems has proven to be very difficult due to the complexity of the processes involved. We can measure the actual performance of a mechanical system and compare it with the intended performance criteria. If it is not within the limits, then we find a way to get it working properly. How do we subject an educational system to such scrutiny? Is it not true that an ed-
ucational system is also a creation of human intellect and culture as much as any other complex organizational system? It is supposed to serve the people by fulfilling a function, meeting some objectives set forth by people. Thus, it should be possible to find a way to measure how well this system is meeting the performance criteria.

Without measurement of the performance of a system, one cannot achieve accountability. Accountability rests on the principle stated by Kelvin that "Whatever exists must exist in some quantity and therefore can be measured" (10, p.14). In order to achieve accountability we must be able to measure the quantity called "education" in a reliable and relevant fashion. Only when we are able to measure this quantity can the prediction that, "the 1970's will become the age of accountability in American education", come true (11).

In measurement one starts with a subjective evaluation of the thing to be measured. The next step is to develop objective indices which correlate with these subjective evaluations. This leads into the development of objective measures of the actual quantity. These measures are first evaluated subjectively until they become validated. In time, however, confidence is placed on objective measurements completely disregarding subjective evaluations. This is the ultimate level in science of measurement. However, in many cases for reasons of economy one may develop objective indices to use
in place of actual measurements. This process of developing measures when applied to educational systems shows the necessity of answering the question of what education is in operational terms. To identify such a concept one must go to its observable effects. Thus, one can understand what education is by measuring its effects. But what are the effects of education? This is where philosophical considerations must be dealt with to find a viable answer. This will be taken up in the next section.

In summarizing this section one may repeat the premise that our resources are limited no matter how inexhaustible they may seem. Limited resources require wise allocation among competing ends. When it is the people who carry the ultimate burden of producing resources, they have the right to demand accountability and the educational systems cannot escape this responsibility. However, accountability rests on the principle of reliable measurement even when this is not easy. But, measurement becomes feasible when we identify that which we wish to measure, thus leading to the core of the problem at hand.

B. Philosophical Considerations

Interest in education as an important factor in economic and social development has increased all over the world in
the past few years. The emphasis placed on economic development by the so-called developing nations and political and social pressures everywhere have stimulated interest in this subject. Also, trends in looking at educated human resources as capital to be used for the well-being of a nation have gained importance in recent years.

In addition, the effects of the critics of the contemporary society who have included educational institutions among their targets must be mentioned.

Marcuse states "Utopian possibilities are inherent in the technical and technological forces of advanced capitalism and socialism, the rational utilization of these forces on a global scale would terminate poverty and scarcity within a very foreseeable future." (12, p.4). And that these utopian possibilities can be accomplished "...as a result of enlightenment, education, political practices in a sense indeed as a result of organization" (12, p.89).

Illich, another critic goes as far as to make the following prediction (13, p.128):

"I expect that by the end of this century, what we now call school will be a historical relic, developed in the time of the railroad and private automobile and discarded along with them. I feel sure that it will soon be evident that the school is as marginal to education as the witch doctor is to public health."

He also demands that "...the absurd discrimination in favor of the person (one who has the degree) who learns a given
skill with the largest expenditure of public funds..." be eliminated. These criticisms cannot go unanswered. A reassessment of the current ideas in education is required.

One has to start with a clear understanding of that which he tries to change or measure. Therefore the question of definition of education must be raised first.

Herman Horne defines education as "...the external process of superior adjustment of the physically and mentally developed, free, conscious, human being to God" (14, p. 285).

John Dewey states "education may be defined as a continuous reconstruction of experience with the purpose of widening and deepening its social content" (15).

According to Pope Pius XI, "education consists essentially in preparing man for what he must be and for what he must do here below in order to attain the sublime end for which he was created...The subject of education is man whole and entire, soul united to body in unity of nature, with all his faculties natural and supernatural, such as right reason and revelation show him to be" (16).

Another definition is "...the translation of knowledge into reality, of humanistic values into humane conditions of existence." The objective of a university then becomes "...to provide the student with the conceptual instrument for a solid and thorough critique of the material and intellectual culture...the development of a true
consciousness" (12, p.56).

These definitions are not suitable for a quantitative analysis of education. This is why one who attempts to work on educational systems must have his own philosophy well established before hand. His philosophy must clarify his basic beliefs, to analyze and organize premises upon which he conducts his research. He "...cannot criticize existing educational ideals and policies or suggest new ones without considering ... the nature of the good life, to which education should lead, the nature of man, because it is man who is educated, the nature of society, because education is a social process..." unless he has a philosophical framework of his own to serve as a springboard (17, p.22).

Thus it is up to the philosophy to:

1. Provide speculative theories about the nature of man and society,
2. Test the logic and consistency of these theories,
3. Prescribe the ends that education should follow.

It is especially the last item which is relevant to the quest for a valuation function for education. That is, the formulation of quantitative goals which will withstand the abuse and test of time, by being consistent with the nature of man and society.

General philosophy provides three basic outlooks; namely, Idealism, Realism, and Pragmatism. In the Idealistic
view, man is a spiritual being with free will and an ultimate destiny or "sublime end" to fulfill. He cannot be studied as an object since he is spiritual. Reality is an illusion, things don't really exist except in ideas from which they spring.

Realism has its basis on the reality of matter. Belief that things exist in reality by themselves and are not mere illusions based on certain ideas in the mind, constitutes the basic principle of realism.

Pragmatism is related to the empiricist tradition of the realist philosophy and emphasizes the sense experience in the act of knowing. The principles of pragmatism include the following:

1. Change is real; nothing is permanent.
2. Values are relative.
3. Man has a social as well as a biological nature.

According to the pragmatist view, the material world which exists independent of the mind is not permanent or independent of man. Reality is an interaction between man and his environment, thus making both man and the environment responsible in the interpretation of it. Therefore, the world has meaning only to the extent that man can interpret it. There is no ultimate purpose in the universe; even if there were, as long as man cannot understand it or experience it by his senses, it cannot be interpreted and therefore be a part of
his philosophy.

Peirce and Dewey, leading Pragmatist philosophers, emphasized that the facts of reality are established primarily by the scientist. Thus the truth is obtained after one has carefully observed and investigated the evidence. Since truth depends on the data and the observer as well as the method used, it is always subject to revision and expansion (18).

The history of philosophy shows that it has a continuing interest in the interpretation of the totality of life. This is often known in contemporary terms as the systems approach, which is an effort to bring all aspects of human existence into some kind of meaningful relationship to achieve "Weltanschauung", a unified view of the world (19).

Brameld gives an extensive discussion of various philosophical choices available to people when they are concerned about incompatible beliefs they hold and try to remove this incompatibility by critically analyzing their thinking to establish this unified view. As a result of such analysis several choices may be made by an individual involved.

The most primitive level of choice is to ignore the existence of disharmony in our beliefs after the first moment of realization. We resent the disturbance caused in our belief pattern and develop a habit of automatically ignoring recurrence of such disturbance. Many people make this choice
which is identified as the complacent choice.

The next alternative is to analyze our beliefs upon realization of the inconsistency in them such that we may keep examining them without any revision or conclusion. This negative choice easily could lead to the skeptical choice where we are sure of our disbeliefs; or the agnostic choice where we, on examination of various sets of beliefs, just can't make up our minds as to which one to prefer, thus remaining as neutral as possible.

The next step would be to agree with bits and pieces of various sets of beliefs without making a harmonious set of new beliefs from these and hold one or the other according to the need. This choice would be called the eclectic.

Another possibility is that upon careful analysis we may become more convinced that on the whole our patterns of belief are sound and that we should protect and preserve them intact in spite of the incompatibilities. This decision would lead to the transmissive choice or the essentialist philosophy in education.

We may decide to gradually change our thought patterns by modification and improvement if we have made the moderative choice which is called the progressive philosophy in education. Or, we may conclude that the disequilibrium is caused by the modern approaches, thus returning to the widely followed patterns of the past because it seems to us more
permanently satisfactory. This choice is called the restorative choice and exhibits itself as the perennialist philosophy of education.

Finally we may in fact synthesize a new pattern which was not tried before. Not an improved version but an entirely new solution synthesis based on the totality of human experience. The educational philosophy based on this approach is known as the reconstructionist philosophy.

It can easily be shown that people are more often drawn to settle for the complacent level and less often upon the transformative.

Educational philosophy formalized under various choice conditions explained above, provides the following specific alternatives to give meaning to one's interpretation of education. Essentialism holds that schools must be based on essentials, that is upon the tried and tested heritage of skills, facts, laws of knowledge which have been inherited by society, to be transmitted from generation to generation (20).

Perennialism emphasizes the absolute nature of truth, thus concluding that the basic principles of education must be changeless. It calls for a return to absolute and universal principles and to the restoration of the spirit which governed education during the Middle Ages (21).
Progressivism is based on pragmatist philosophy and maintains that the main purpose of education is to stimulate people to think effectively (22).

Reconstructionism is perhaps not as well established as the first three. This term as a philosophy of education holds that formal or informal experiences of learning and teaching are inclusive processes which both transmit and innovate culture (23). It may be regarded as an extension of pragmatic philosophy in that it regards experience as the key to reality, but interprets mind as the functional capacity by which man is able to solve problems. Since man is capable of using his mind in this way, he can then look into the future and prevent some of the problems before they are created. This is the relevant aspect of reconstructionism today. It proposes that the schools must become the tools to reconstruct society. Man has the ability to use his mind and design his own future, so why not let education serve as the means to bring these changes about?

The essentialist philosophy would create unquestioned acceptance of the inherited culture. Problems can only be suppressed by this method. If educational systems are to be studied from the essentialist point of view, only the symptoms of real problems would be cured without substantial alterations in the existing patterns. An individual who has this philosophy would not question the established patterns
of educational systems such as teaching methods, lectures, courses, curriculum, etc. He would simply try to design systems so that more of the same could be accomplished faster and cheaper. No design modifications would be proposed in the essential features of schools.

The perennialist philosophy would be unacceptable for analysis of education simply because "self evidence of truth" and the "perennial nature of standards" just would not work in a changing world.

The progressive view would work to a degree because it has the experimental spirit of open-minded, tolerant consideration of all sides of questions, but does not help in setting up requirements for the direction of progress. Trial and error methods of evolutionary processes are acceptable when there is a lot of time, and no better method is available. But science and engineering have provided us with sophisticated tools and most importantly, we don't have much time. Trial and error methods must be abandoned in favor of set goals and clear cut processes. This is the most important principle of scientific management. A philosophy which commits itself to this urgency is the only one which can be used.

Brameld states this urgency as follows (23, p.23):

"Education during these waning decades of the twentieth century is confronted by one imperative before all others. This is to engage in a radical
Thus the future-centered approach and the interpretation of the mind as a functional capacity to solve problems, provide two principles upon which an operational definition of education can be formulated leading to quantification and measurement of system performance.
II. LITERATURE REVIEW

In this chapter some of the most relevant studies in the field of quantitative analysis of educational systems particularly related to performance evaluation will be reviewed.

In general, the studies in the subject may be divided into two basic categories each reflecting a certain approach to the problem.

1. The quantitative approach taken by the economists and system scientists using recent techniques and methods of operations research and systems analysis.

2. Methods of measurement and evaluation used and proposed by the educators.

The first category may further be divided into several sections with a large degree of overlap. Since an economist uses the techniques of OR and an OR scientist uses the principles of economics in his work, it is not necessary to make a clear distinction between these subgroups. However for familiarity with the subject matter, the existence of econometric models at micro and macro levels, flow models sometimes referred to as the consistency models, cost benefit or cost effectiveness models, planning models, activity analysis models and simulation models should be mentioned as some of the most common approaches found in this category.

In econometric models the process of education is gener-
ally linked to the economic development of a nation and the educational system is treated as a part of a more inclusive economic system of the country.

Flow models deal with the passage of students from grade to grade or from one stage to another in terms of numbers and types. These are used in forecasting future needs in terms of staff and facilities for planning purposes.

Benefit/cost models are used to evaluate the educational programs in terms of the economic objectives of the society.

A. Overview

Classical economists provide a comprehensive treatment of the subject of education and economics. From their writings one may obtain the following conclusions (24):

1. Education is a capital good far superior to the physical capital.

2. Education is a profitable investment return of which being manifested in terms of higher earnings and productivity.

3. Education is a consumer durable, an end product in that it contributes to the greater satisfaction of the individual, his family and the nation.

4. Education carries many externalities, an economic term which means indirect advantages or disadvan-
tages to the society. In this sense it is a social product.

5. Education is a "peculiar commodity in a peculiar market" since the separation of the owner of education and his commodity is impossible.

6. Education is a factor enhancing social mobility and income distribution.

7. Education is a political good closely tied with democracy and government.

8. Education is an area calling for government intervention for all of the reasons given above.

The last item in the list may be best explained by the following quotation (25):

"Laissez-faire has no place in this field largely because of external economies, imperfect competition and absence of market value or valuation, the invisible hand would undoubtedly fail to bring about the right types and amounts of schooling (education)."

The excerpts from President Nixon's message given in the previous chapter and the various federal and state interventions in the affairs of schools support the conclusions of the classical economists.

In this connection Bowles predicted that by 1990 education in the U.S. will have lost its importance as a major local political issue, but will have become a major government activity measured in terms of its share of the national budget (26).
Modern day economists too have put a lot of time and effort into the analysis of education. Prior to 1960 there were very few quantitative models of educational systems. The proliferation of the literature in this field since the early sixties is indicated by the large amount of reference material collected in the following three sources.

Hufner lists 1333 recent sources in an article on the economics of higher education published in 1969 (27). Among these thirty deal with the university as an enterprise, over sixty deal with cost analysis of higher education where the systems of higher education are treated as productive units with certain inputs and outputs. Concepts of efficiency and productivity occupy an important portion of the study of educational systems in these writings. The definitions and quantitative identification of inputs and outputs are based on economical interpretations and are controversial. Stoikov (28) and Intriligator and Smith (29) formulate a model for the optimal allocation of scientists between teaching and research in an attempt to cope with the complex nature of the outputs. Blaug and Woodhall attempt empirical measurements of productivity in British Universities through the years 1938-62 (30).

Benveniste's work (31) is another example of a study on the "efficient resource utilization". Schultz (32) provides estimates of the "opportunity cost" involved in education.
which was first discussed by Walsh (33). A criticism of the concept of opportunity cost is found in Vaizey (34).

Organization, financing both public and private, socio-economic determinants and consequences in education, human capital concept and educational planning are among other topics covered by Hufner.

Blaug provides 1358 sources on the economics of education bringing the subject coverage up to 1969. This annotated bibliography contains selected articles and views the subject under two main headings, a) developed countries and, b) developing countries. It then divides each group into several sections and covers the general survey of the subject along with details of the economic aspects, educational planning, and socio-political aspects of education (35).

Blaug discusses the profitability of educational investments as compared to other available alternatives. To answer such a question he suggests we compare the costs of education with the expected future earnings due to the education gained. The calculation of an internal rate of return on the basis of the present worth of the prospective earnings is found unsatisfactory since it treats the purchase of education the same as the purchase of any capital asset. This situation is not quite true because education is partly purchased for consumption and the future earnings do not
solely depend on the education one gains (35, p.19).

Along these lines Denison has suggested that only three fifths of the observed income differentials among males of the same age can be attributed to their formal education (36).

This was later confirmed by several workers in the field using multiple regression techniques (37)(38).

Another comprehensive source is the Systems Analysis for Educational Planning published by the Organisation for Economic Co-operation and Development. This annotated bibliography brings the systems analysis aspects of educational planning up to 1969 by including 306 annotated sources plus many non-abstracted references. Partly written in French it classifies the subject matter under eight categories, and gives a detailed coverage of each (39).

Another noteworthy review of the subject of economics of higher education is by Witmer (40). A relatively concise but comprehensive coverage is provided with evaluation of various approaches found in the literature.

Professor Tinbergen's work in constructing econometric models of a national economy as early as 1939 must be cited for the important contributions to the field. In 1962 Tinbergen and Correa published their article on the quantitative application of education to accelerated economic growth (41). This basic model was later improved and tested
by applying to six countries under the Organisation for Eco-
nomic Co-operation and Development Mediterranean Regional
Project (42).

The Tinbergen model is a planning model representing the
link between economic development and the educational system
of a nation. It takes the following basic premises into ac-
count:

1. Economic life requires a stock of educated manpower.
2. Education consists of a series of successive steps.
3. Part of the stock of qualified manpower must be used
   in the process of education as seed is used in
   agriculture.
4. Qualified manpower may be imported for rapid devel-
opment.

On the basis of these facts, the model distinguishes
secondary and higher education stages as two consecutive
stages to be considered and assumes the primary schooling to
be of no problem in supplying graduates to the secondary
level.

Six equations were expressed to relate the various pa-
rameters of the relationship between education and economic
growth. These equations represent the following assumptions.

1. The labor force with a secondary education only is
   used for production and must increase with the
   volume of national production.
2. The number of newcomers to the labor force with a secondary education is equal to the number of students one time unit earlier minus the number of students now in level three.

3. The number of newcomers to the labor force with a third level education is equal to the number of third level students one time unit earlier.

4. The labor force consists of those already in it one time unit earlier and those who joined it during the last time unit. Also a certain proportion leaves the labor force due to retirement and death.

5. The labor force with a third level education consists of those in production and those teaching at both levels of education. Also the volume of production and the number of students are proportional to those numbers.

This model with several degrees of refinement has been applied to Spain, Turkey and Greece with very useful planning implications even though there are many areas requiring more study before the model can become a realistic planning tool (43). The lack of data or the difficulties of obtaining reliable data to determine the model parameters constitute the greatest obstacle to the effective utilization of this model. However it has served as an educational tool to stimulate work in this area.
A benefit/cost model to evaluate an educational system is given by Spiegelman, where a mathematical model is skillfully applied to the evaluation of a Title I Program of the Elementary and Secondary Education Act of 1965 (44).

This benefit/cost model identifies three distinct classes of benefits as:

1. Increases in lifetime earnings due to attainment of higher levels of education,
2. Intergeneration benefits accruing to the next generation,
3. Reduction of juvenile delinquency and related losses to society.

Thirteen separate equations are used to estimate these benefits. The basic nature of the model can be expressed by the following equation:

\[ B = \sum_{0}^{n} V_i(E) \cdot P_i(E) - \sum_{0}^{n} V_i(E) \cdot P_i(E) \]

which simply states the expected net gain expressed as benefits to be equal to the difference between the expected value of various levels of educational attainment with and without the individual having been through the Title I program.

This model has been applied to a Title I ESEA program in San Francisco during the 1966-67 school year to measure the performance of the system. The model parameters such as the
probabilities of various levels of educational attainment, reduction in juvenile crime, and prospective earnings have been based on census data for that area. Markov chain methods are used in estimating the eventual educational level attained by the students who participated in the program when they were in grade school.

Findings of this study indicate that the total benefits of Title I program to a male black student is $1580 and to a non-black the same figure is $3610. The benefit/cost ratios were found to be 2 to 1 for male blacks and 6.8 to 1 for non-blacks for personal gains, and 5.2 to 1 for blacks and 9.6 to 1 for non-blacks for social gains. Similar figures were obtained for females also. The discrepancy between the figures for black and non-blacks is attributed to racial discrimination.

Social benefits include all the intergeneration benefits, that is the earnings and increased educational attainability of the next generation plus the reduced costs of juvenile delinquency and crime. Private benefits are those which accrue to the individual directly, mostly as increased lifetime earnings.

Such a quantitative assessment of an educational process system has obvious desirability. If we could indeed measure reliably the benefits of a program and determine benefit/cost ratios as was done in this study, we would be in a very good
position to make policy decisions in education. However, to attribute the reduction in the numbers of hubcaps stolen by a ghetto youth to the Title I program, he participated in while he was in grade school seems to be too restrictive if not over optimistic. One could see the effects of many undetected social and political forces in play in changing the behavior of such an individual.

In spite of its acute shortcomings, this study must be carefully analyzed for the many keen insights it brings to the problem of performance measurement in educational systems.

A typical systems analysis model is a flow model constructed by Blot and expanded by Durstine (45) (46). These models generally deal with the passage of students from one level to another in a sequential arrangement of educational levels, thereby allowing great flexibility in system boundary definition and providing all levels of aggregation from a single student to all the students in a national school system.

A basic equation of a flow model may be given as follows:

\[ E_{in} = r_{i(n-1)} E_{i(n-1)} + P_{(i-1)(n-1)} E_{(i-1)(n-1)} \]

where

- \(E_{in}\) = enrollment in stage \(i\) during period \(n\)
- \(r_{in}\) = fraction of \(E_{in}\) repeating \(ith\) stage during period
n+1 called the repetition coefficient

\[ P_{in} = \text{fraction of } E_{in} \text{ in stage } i+1 \text{ in period } n+1 \text{ called} \]

the passage coefficient

Graduates and the dropouts are given by the following equations:

\[ P_{in} = P_{in} E_{in} \]
\[ F_{in} = (1 - P_{in} - R_{in}) E_{in} \]

where

\[ P_{in} = \text{graduates from stage } i \text{ during period } n \]
\[ F_{in} = \text{dropouts from stage } i \text{ during period } n \]

This set of equations give a descriptive flow model of a successive educational system provided the following conditions are met:

1. There are no dropouts between stages
2. \( r \) and \( p \) are independent of the enrollment composition.
3. Flow is strictly serial in nature.
4. There is no net migration in or out.
5. \( r \) and \( p \) are constant from period to period.

On the basis of these simplifying assumptions and the equations stated above, a general analytical model in considerable detail considering only a single stage is developed. From such a unit it is then suggested that we can build
modular blocks to represent highly complex systems.

The enrollment function is determined for the single stage and investigated under the effect of varying forcing functions and the transient and steady state curves are plotted against time.

After the single stage is examined in detail a modular block diagram is constructed representing a more complex system. This system is treated mathematically with proposals to obtain approximations of the input and output functions and the determination of upper and lower limits.

Later on some possible uses are suggested and parameter values are given from data on South American countries.

Finally, on the basis of the mathematical possibilities provided by the model three different performance measures are suggested.

1. Fraction of all entrants (or of all those eligible to enter) who eventually graduate.
2. Total number of periods spent in the system by all entrants whether eventually graduated or not.
3. Mean number of periods spent in the system per graduate.

These measures as suggested by Blot take the following forms when applied to a single stage, constant coefficient system:

1. Fraction who graduate
\[
W = \frac{p}{1 - r} \quad \text{(measures production)}
\]

2. Total periods of instruction used

\[
Y = \frac{1}{1 - r} \quad \text{(measures cost or effort)}
\]

3. Periods of instruction per graduate

\[
Z = \frac{1}{p} \quad \text{(measures efficiency)}
\]

The attractiveness of these measures is obvious in view of the fact that they are quantitative extrapolations of performance of the system into the future as well as the present, provided the \( p \) and \( r \) values are reliable, meaningful and constant wherein lies the problem.

Durstine points to the shortcomings by stating that (46, p.434):

"They serve as interim measures and as a beginning. They must eventually be developed to take into account quality of product, variations in the unit costs with quality, enrollment, and time and efficiency in terms of realistic constraints, not as a simple ratio. These needs do not nullify the above measures but indicate a need to go beyond them."

Alper (47) provides an interesting analysis of flow models referring to them as consistency models from the point of view of the insight they give to the planner. According
to Alper the consistency models discussed in various sources are basically the same in spite of their apparent diversity in the subjects they discuss. He criticizes the use of Markov chain input-output model because it does not indicate to the planner what to do to control the course of events before they actually happen. A planner using this model will have nothing to do "other than idly observe future events over which he has no control."

Also, systems with capacity limitations do not lend themselves to this treatment due to the assumptions of the model given earlier, and yet we would be most interested in these systems for utilization and performance evaluation, simply because these are the systems which will normally have problems in need of solutions.

With basic criticisms of this kind directed against Markov chain type descriptive models, Alper sees little use for another similar model which expresses in 352 equations the same stage to stage flow as determined by passage or repetition constants not only between stages but also, educational qualification coefficients, manpower coefficients and other similar coefficients. These models consequently become "bookkeeping" models based on "tautological arithmetic".

Finally it is stated that education still needs, desperately, models which explain the "physics" of the educational system. Logically consistent models do not necessari-
ly fill this need. Excessive elaboration and mathematical disaggregation and symbolism may act only to conceal rather than reveal what is actually happening.

In reviewing the literature in educational systems at first it may not seem relevant to include quantitative treatments encountered in communication theory. However, the following sources will show the usefulness of such an action in arriving at a basis for theory development.

The publication of Shannon's Mathematical Theory of Communication brought instant light to the quantification of information and its repercussions in communication (48). Mathematical significance of Shannon's definition of information as a related quantity to entropy will be used in the next chapters. In this section only a non-mathematical review of the significance and interpretations of it will be given.

Weaver in the same book defines communication as all the procedures by which one mind may affect another (48, p.95). Since education may be summarized as a sequence of communications "to alter the pupils' behavior in specific desirable directions" (49, p.19), one may conclude that communication or education affects one's mind which is ascertained by the observed behavior of the recipient of the communication (or education).

Weaver's definition of communication carries the following problems along with it:
1. The technical problem, which is the problem of transmission from one mind to the other those symbols which will change the latter's overt behavior.

2. The semantic problem, which deals with the precision in the meaning of the symbols transmitted.

3. The effectiveness problem, that is how effective the communication is in affecting behavior.

The Mathematical Theory of Communication only deals with the physical transmitting of signals. The semantics problem and especially the effectiveness problem need to be worked on for educational system evaluation.

Ackoff, on the basis of information and decision theory principles derived quantitative measures of effectiveness in the process of communication. Information is conceptually defined to be related to the decision problems of the recipient. Behavior determining elements in an individual's purposeful state are identified in the form of a decision matrix with objectives, values and courses of action available to the individual. The probability matrix thus obtained constitutes the description of the purposeful state of an individual (50).

Assuming that communication is concerned with the acts of humans which affect the decisions of other humans, the amount of information, instruction and motivation in a
purposeful state can be explicitly defined in terms of the decision matrices of individuals involved. Thus the amount of information is defined in terms of probabilities of choice of courses of action, the amount of instruction is defined in terms of the efficiencies of the course of action available, and the amount of motivation is defined in terms of the values of the objectives.

On the basis of this analysis comparison between the two purposeful states before and after a communication gives the amounts of information, motivation and instruction carried by the message.

The mathematical development of the theory was put to test by Martin and Ackoff in trying to develop a measure of the value of scientific information (51).

In this experiment the amount of instruction in hubits and the amount of information in inbits were measured, but no value calculations or performance evaluation of the communication system was undertaken.

In a more recent paper, Ackoff brings in this interpretation of education in relation to strategic planning of education and gives an operational definition of education as (52):

"any communication or demonstration process which increases the recipient's expected utility in one or more problem situations."
Professor Ackoff further observes that, this approach can be used to measure education but that it would be "too costly and time consuming" at the present, to evaluate educational processes this way. He also points out that the quantitative measures cited "...make it possible to determine the extent to which much-used scores on achievement tests correlate with the measure of education.... Such a determination would give achievement-test scores something to be an index of."

This definition and the original explanation of the decision theory model has not been used elsewhere in evaluating system performance so far as ascertainable by this writer. In a recent proposal Professor Ackoff has suggested subjective weighing by judges to evaluate the quality of the output of a college (53).

Educators have made many attempts to deal effectively with the problem of evaluation and assessment of school systems. The 1940 edition of the Evaluative Criteria published by the General Committee in charge of the cooperative study of secondary school standards is an example of a method very extensively laid out for the sole purpose of evaluating a given secondary school system (54).

The approach followed in this and more recent editions of the Evaluative Criteria is basically one of preparing extensive check lists and subjectively evaluating the system on
the basis of these check lists. The weighted scores of one through five for each item are then summed and compared with a standard to obtain a measure of the investigated school's performance (55) (56).

According to the cooperative study group, the experience with these methods and procedures are used for three distinct purposes, namely (57):

1. For accreditation - as a satisfactory method for forming a valid judgement of the quality of a school.

2. For stimulation - as a means of making a detailed and helpful diagnosis of a school's standing in a large number of significant features, thus furnishing an incentive to continuous growth and improvement.

3. For professional development of the participating school administrators.

The results of the extensive subjective evaluation are checked against three different scales represented graphically as "educational temperatures" in one of the publications of the cooperative study group (57).

Alpha Scale is to be used for standard evaluation processes. Beta and Gamma scales are reserved for a quick indication of performance of a system without a detailed study.
Evaluations are to be made on the basis of personal observation and judgement of an inspection team, in the light of the checklists, and of all the other available evidence using the five-point rating scale where 1 and 5 denote very inferior and very superior respectively.

The 4th edition of the Evaluative Criteria has been revised and considerably improved by the expansion of the method of self evaluation for schools. This extensive evaluative source includes 10 sections and 18 subsections under the Curriculum title with several hundred items to be rated before a diagnosis can be made as to the relative rank of a school compared to the national standards (58).

An earlier study by Mort and Cornell is also a checklist method using subjective evaluations requiring yes or no answers only (59). These answers are then transferred to a score sheet where adjustments are made on the basis of observations and a score is obtained for each item. The sum of the scores is compared with the maximum possible score of 1000 to determine the relative rating of a school.

As a result of their studies, authors Mort and Cornell have estimated quantitative performance standards for schools on the basis of their scores. For example, school systems spending $1850 in current expense per classroom unit scored 300 on the average (out of 1000), whereas schools spending around $3000 for the same purpose scored 580 on the average.
In spite of the subjective nature of the method used, the establishment of quantitative criteria is noteworthy.

Bloom in a paper discusses the importance of testing in relation to measurement and evaluation of education. This importance is expressed by stating that; "to control the matriculation examinations in a country is to control its educational system; to develop tests that are widely used for selection and prediction purposes is to determine which human qualities are prized and which are neglected" (60).

Furthermore he states that "it is no great exaggeration to compare the power of testing, on human affairs, with the power of atomic energy. Both are capable of great positive benefit to all mankind and both contain equally great potential for destroying mankind."

Having cautioned the reader about the importance of testing in general, Bloom identifies three basic approaches to the problem of quantifying educational system performance:

1. Measurement, which he defines as the determination of the responses of an individual to standard stimuli, tasks and questions in terms of speed and accuracy.

2. Evaluation, meaning the appraisal of the changes in students' behavior.

3. Assessment, which refers to the assessment of the
characteristics of individuals in relation to a particular environment.

Each of these approaches are considered to be a partial view of the whole man, his environment and the nature of the observed evidence. Consequently it is proposed that a synthesis of all three is needed for a sound theory of educational measurement. An attempt is made for such a synthesis by laying the ground rules and basic principles. The synthetic theory suggested is discussed without showing any experimental evidence as to its applicability to real problems in educational systems.

Brown, an economist as well as an educator, emphasizes the timeliness, for educators, of defining objectives precisely, developing reliable measures of performance and constructing a quantitative model for judging the success or failure of a given system in attaining desired goals (61).

After perhaps facetiously stating that it would be very easy if we could measure the output by the number of blue-eyed graduates, number of words published on pink paper or the number of times Playboy magazine was checked out of the library, he re-emphasizes the importance of defining objectives for meaningful measurement.

Brown identifies five growth objectives in relation to higher education:

1. Whole Man Growth
2. Specialized Man Growth
3. Growth in the Pool of Knowledge
4. Growth in society at large
5. The joy of growing and of being in an educational environment.

On the basis of these objectives the growth measurements are established as a result of assigned weights to forty items included in a detailed table. In this table there are twenty specific goals of higher education and forty distinct items listed as the relevant measures for these goals. The nature of evaluation is akin to the process of index making in economics and many of the measures are based on the testimony of students, staff and similar subjective evaluations.

Indexes thus obtained are to be used in evaluating the whole educational system similar to a benefit/cost type approach commonly used by economists as reviewed earlier.

Eiss discusses the subject of instructional systems evaluation comprehensively by including the behavioral objectives and the philosophical aspects of educational systems (62). He also points out the importance of feedback to provide continuous control and improvement in the system without delay. The book includes evaluative methods prepared in form of checklists where certain statements have to be agreed or disagreed with in evaluating affective outcomes on a five-
point scale for quantification.

Another checklist allows frequency counts on observed behavior where an undesirable behavior may be indicated by a minus sign.

There is also a 53 question checklist for evaluating an instructional system on the basis of its philosophy, goals and objectives, the system content, process and facilities. Each question is to be subjectively evaluated and assigned a value from zero to five. The total points scored would indicate where the evaluated system falls in the given ranking scale.

Finally we must say a few words about the work of interaction analysts. It seems reasonable to think that by observing teacher-pupil interaction behavior in a systematic fashion one can develop an insight to the nature of processes involved in education. Flanders, the forerunner of the interaction analysis field, has developed a systematic frequency categorization of behavior patterns in classrooms (63). Flanders' Interaction Analysis Categories (FIAC) developed between 1955 and 1960 divides teacher-pupil interaction into three basic categories, namely:

1. Teacher talk
2. Pupil talk
3. Silence or confusion
Each category is subdivided further to provide a total of ten interaction categories. A trained observer sits in a class and takes a reading every 3-5 seconds as to which category behavior is taking place. These readings are then tabulated in a sequence matrix and by statistical techniques evaluated in terms of common behavior patterns.

From the interpretation of the interaction matrix it is possible to compute certain behavior indices and then compare them with long time averages established for a given course.

Table 1. Interaction analysis indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Norm, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRR</td>
<td>Teacher response ratio</td>
<td>42</td>
</tr>
<tr>
<td>TQR</td>
<td>Teacher question ratio</td>
<td>26</td>
</tr>
<tr>
<td>PIR</td>
<td>Pupil initiation ratio</td>
<td>34</td>
</tr>
<tr>
<td>CCP</td>
<td>Cross content ratio</td>
<td>55</td>
</tr>
<tr>
<td>SSR</td>
<td>Steady state ratio</td>
<td>50</td>
</tr>
<tr>
<td>PSSR</td>
<td>Pupil steady state ratio</td>
<td>35-40</td>
</tr>
</tbody>
</table>

For example, teacher response ratio (TRR) indicates the teacher's tendency to react to the ideas and feelings of the pupils. The teacher question ratio (TQR) is a measure of the tendency of a teacher to use questions in guiding the stu-
dents through a class discussion. A pupil initiation ratio (PIR) is proposed as an index of pupils' tendency to initiate talk in class. These and other indexes are given in Table 1, with the average values of each as determined by Flanders.

Since the technique of interaction analysis is based mostly on verbal interactions happening in class, it may seem to have limited potential in the analysis of education. However by increasing the complexity of categories, as has already been done by Flanders and others in the field, one can obtain as much precision as desired (64) (65) (66). Especially by the incorporation of computer capabilities in reducing the data into meaningful index numbers one can come up with very useful insight to the processes of education thus providing prescriptive standards of behavior. In engineering courses this technique has not been used as of this date. However, in the light of its rapid development, this writer is convinced that interaction analysis is a valuable method which could be used by anyone interested in improving educational system performance. As a powerful tool for the analysis of complex social interaction systems, it can be used for industrial applications as well as for schools (67) (68).

At last but not least mention should be made of a little book by Stephens on the process of schooling (69). This book is addressed to the question of determining the basic forces in education and to support the theory subscribed, which may
be summarized as the spontaneity of educational processes, the author compares a large number of conflicting research findings in such diversified areas as attendance, programmed instruction, independent study, correspondence courses, size of class, counseling, individualized instruction, increasing the student's involvement, amount of time spent in study, size of school, distraction, teacher selection and training, easing the teacher's load, ability grouping, differences in philosophy of education, discussion vs. lecture, group-centered vs. teacher-centered, use of quizzes, etc.

In the last chapter, the author suggests several possible tests to conduct in order to isolate the nature of education and test the hypotheses of spontaneity in educational processes. This is a good source for anyone interested in working on educational systems since it gives a comprehensive picture of the past in a comparative fashion allowing the reader to note dead-end streets, blind alleys, and the productive areas of research in education.

B. Evaluation

The review, so far presented, has hardly made a dent in the vast amount of literature found in the subject of educational systems evaluation. The various bibliographical references suggested above and the several books and articles
specifically emphasized constitute a good start for one who is interested in research in educational systems.

This limited review has helped the writer form the following opinions about various approaches to the subject matter:

1. It may be concluded that, the economists' approach to education from a benefit/cost study point of view has run into difficulties since the measure of the value of the output has not gone beyond controversial stage. Costs in general have been easier to determine than the benefits, but the general tendency to measure educational benefits in terms of an adjusted life time earnings has not been completely satisfactory though no doubt extremely useful in many applications.

2. In general, economists have taken an external observer's point of view in measuring or evaluating the benefits of education by interpreting it in terms of what it is worth to society or some other external reference. In that their analysis of an educational system is very similar to an industrial process where the "raw material" comes in, and the "finished product" goes out, "value" being added in the process.

This similarity may be said to exist to an
extent but it must not be carried too far, for the simple reason that in the case of industrial processes the materials have no "say" about what is done to them other than their physical requirements.

With education it is different in that the individuals involved can be negative, indifferent, enthusiastic, etc., about what is being done to them. Students have been known to take over the operation of schools in protest of some aspects of education, but no one has heard of rolls of sheet steel demonstrating against a "mean press" wanting to form them into a decorative non-bumper on a contemporary automobile.

This criticism does neither mean that we advocate a "hands off" policy in the analysis of educational systems, nor do we agree with the critics of the "cult of efficiency" in American education (70).

Subscribing to the principle that everything worth doing is worth doing well (or in an optimum way), our main concern is with an accurate identification of that which we are interested in measuring, so that we know how well it is done.

3. On the basis of above discussion it seems that a personal view of the value of education must be the
basis of a model constructed for the purpose of gaining insight to the meaning and measurement of education. The model must be able to account for the differences in the values attached to the apparently similar educational outcomes by various individuals.

A personal viewpoint would provide benefits to be calculated separately for each individual from his point of view, then summed to obtain the total gain from a given educational process. Thus, the idea of using gross social product (GSP) in place of GNP as proposed and elaborated by Professor Fox is an important step (71).

4. The evaluation methods proposed by educators have remained mostly on subjective levels even though techniques have been developed to a high degree of sophistication. Among them the method of interaction analysis seems to have the greatest potential in obtaining descriptive models of educational processes and eventually isolating the basic elements of education to help construct prescriptive models. This technique also lends itself to operational description of education, thereby allowing the utilization of computer technology in analyzing enormous amounts of data which must be handled.
5. Another point which was completely neglected by the literature reviewed is the idea of determining a measure of efficiency in a thermodynamical sense. This approach is based on the comparison of the performance of a given system under ideal vs. real conditions.

In thermodynamics, the Carnot Cycle performance is used as an ultimate to be accomplished for a real engine working under similar conditions. Similarly the Air Standard Analysis of Otto and Diesel cycles are useful in interpreting the performance of real internal combustion engines designed and built to operate following these cycles.

This approach would require the derivation of a Theoretical Maximum obtainable from an educational system under given conditions to be used as a prescriptive standard for comparing with the actual results.

6. Finally it must be pointed out that there is a need for some non-monetary measure of the value attached to education.

The decision theory principles and the concept of entropy as used in information theory and statistical thermodynamics seem to have some applicability in the valuation of education.
These points will be pursued in more detail in the following chapter.
III. MODEL DEVELOPMENT

Two basic equations can be written on the basis of what may be called a "thermodynamic model" of an educational system as suggested by Sisson (72). The first which might be called the General Equation of Education Balance is as follows:

\[ S_a = \sum_i^G E_i + \sum_d^D E_j - \sum_k^N E_k + \Delta E_o \]

Where:
- \( S_a \) = education imparted to the pupils
- \( E_i \) = education content of the \( i \)th successful leaver
- \( E_j \) = education content of the \( j \)th unsuccessful leaver
- \( E_k \) = education content of the \( k \)th entering student
- \( G \) = number of graduates
- \( D \) = number of dropouts
- \( N \) = number of entrants
- \( \Delta E_o \) = increase in education content of the system.

A more general and simplified form of this equation may be given as,

\[ S_a = \sum_{out} e_{out} - \sum_{in} e_{in} + \Delta E_o \]
For stable conditions corresponding to steady flow in a similar thermodynamic system we obtain:

\[ S_a = E_{out} - E_{in} \]

which is the basic equation of energy balance under stable conditions that is assuming no change in the system over the period of observation.

The second equation may be called the student flow equation which assumes that all entering students must exit eventually. Thus the following equation for a steady flow system is stated:

\[ \sum n_{in} = \sum n_{out} \]

The flow models reviewed in the literature are based on the above equations which are similar to the energy and mass balance equations of thermodynamics (73) (74).

A. Educational Attainment Method

For a meaningful measurement of a system performance a ratio between desired output and required input, both expressed in the same units is needed. The following may be
listed as some of the desired outputs of an educational system:

1. Increased ability of the products to solve the problems of the society in which they live.
2. Increased economic contribution of the products to the society. (Or reduced economic burden of the products to the society if the reduction of crime and poverty is an outcome).
3. Increased total knowledge of the society as a result of the education products receive.
4. Increased educational level of the society (presumably indicates a more advanced culture whatever its worth).
5. Number of degree holders (related to all of the above).
6. Number of credits completed (related to all of the above).
7. Number of terms of schooling completed, or number of hours of exposure to a subject, etc., (related to all of the above).
8. Number of quality points completed as a measure of 1, 2, 3, and 4 above.
9. Increased ability of the teachers, and the research and extension benefits.
Judy gives the following list of the outputs for a University (75):

a. Graduates from the various programs.
b. Failures and dropouts of such programs.
c. Research done and papers published in various fields.
d. Professional development of the staff.
e. Information stored on paper and other media.
f. Public and miscellaneous services.

This list is in agreement with the one developed above with the difference in the emphasis placed on problem solving capability of the products.

If one considers only the teaching-learning aspects of an educational system for simplifying the analysis, it may then be concluded that the increased ability of the graduates to solve societies' problems and the economic contribution, seem to be the most distinct desired results. In other words, presumably schools "give education" to students so that they are better able to solve problems of the society and they contribute more to the social welfare. But how do you measure one's increased ability to solve problems?

In evaluating economic benefits present worth of life time earnings of the products has been used extensively with the following tacit assumptions:

1. That the economic worth of a person is equal to his
2. Earning power is a result of the schooling. The validity of these assumptions has been questioned by many workers in this field as not being completely satisfactory. The third and fourth desirable outputs of an educational system, namely the total knowledge and the increased educational level are equivalent in that they imply it to be desirable if society knows more about "anything" with or without inherent value to society. When the nature of schooling and the reasons and ways of their evolution in the society are examined, it can be seen that "level of knowledge" or the "educational level" of the society is and has been indeed an important concern of educational systems throughout the history of mankind (69, p.30).

Then the question becomes, how does one measure the "educational level" of the product of such systems? The answer to this question may be given by items 5-8 above. However, in the opinion of this writer, Total quality points (item 8) is the most suitable measure of educational level, if one must use this sort of a quantification of benefits.

Total quality points can be further improved by incorporating some correction factors for the type of course and school or department etc., to compensate for the differences in subjective evaluations of these factors.
Therefore, it is proposed that the desired output of an educational system for the society (in the limited sense as specified above; that is, not including research and extension work) is the increased educational attainment of the leavers of the system, measured in quality points. Other measures listed in items 5-8 above could be used, depending on the desired emphasis of the analysis.

An examination of the required inputs may yield the following list:

1. Cost in dollars to run the system.
2. Number of teachers required.
3. Physical plant required.
4. Intellectual effort required, etc.
5. A composite of all of the factors listed above.

Judy provides a similar list for the inputs of a University including the following (75):

a. Students enrolled.
b. Teaching load of the staff.
c. Time spent on administration.
d. Time spent on student counseling.
e. Time spent on library services.
f. Physical plant, including computer facilities.
g. Equipment, maintenance and supplies.
h. Travel and other effort.
Input requirements are frequently measured in dollar terms. Any other system parameter listed above could be used in the ratio, output/input, depending on the purposes of the analysis. However, to obtain an index of productivity akin to an efficiency in thermodynamics, one must express input requirements in the same units as the outputs. It is proposed that educational attainment per dollar, per staff or per square foot of classroom are not the only ways of measuring the inherent efficiency of an educational system even though they are extremely valuable.

The idea of determining system efficiency by comparing output with ideal conditions simplifies the problem of quantification and the need to express both in the same units.

The following simplified outline is presented to show how efficiency of an educational system can be measured.

Define:

1. System: any educational process unit
2. Output: change in any desirable characteristic of the products.

Then

$$\text{The efficiency of a system} = \frac{\text{Actual desired output}}{\text{Max. theoretical output}}$$
or

\[ \text{Efficiency} = \frac{\text{Actual schooling produced per term}}{\text{Max. theoretical producible per term}} \]

\[ E = \frac{S_a}{S_t} \]

Where:

\[ E = \text{educational system efficiency} \]

\[ S_a = \text{actual schooling produced by the system} \]

\[ S_t = \text{theoretical max. schooling producible by the system} \]

Now the problem is how to determine the quantity \( S_t \), the theoretical maximum producible by a system. Two different approaches, both based on the principles of engineering valuation and requiring determination of some optimum quantities jointly by the teachers of various subjects, school administrators and students are proposed in an attempt to answer this question.

Starting with one hundred students at time zero with zero educational level and following their progress through the system while plotting the number in the system versus the Total Quality Points for a given term, one may expect a survival curve or educational attainment curve similar to
survivor curves found in actuarial work. This approach implies that if one starts with \( N \) students and they all get A's this would give the maximum theoretical curve.

For a detailed theoretical explanation of the survivor curve development see reference (76). For one detailed application of this approach to the study of student performance see reference (77).

The limitation in this approach is that its maximum is determined by the number of students and it does not include the effect of class or staff size. Thus the mathematical approach which follows may be preferred:

We define:

\[
E = \frac{S_a}{S_t}
\]

and,

\[
S_a = \sum_{i}^{n} C_i Y_i G_i
\]

where:

\( n \) = number of classes (sections of courses)

\( C_i \) = credit value of \( i \)th class

\( Y_i \) = number of students in \( i \)th class

\[
G_i = \frac{\sum_{j}^{Y_i} G_{ij}}{Y_i} = \text{Average grade of the } i\text{th class}
\]
\[ S_t = \lambda \sum_{i=1}^{n} C_i X_i \]

where:

- \( X_i \) = optimum number of students to have in each class so as to be able to teach them the subject matter such that all of them could get A grades
- \( \lambda \) = equivalent numeric value of highest grade such as \( \lambda = 4 \).

This assumes that in a course, tests are of equal difficulty term after term and that, \( X_i \) number of average students spends the proper amount of time and could score in the 90's in all exams and quizzes and earn A grades. (If the assumption that all must get \( \lambda \) is too rigid, then an optimum distribution of grades may be sought for this purpose).

Thus from the equations given above for the quantities \((S_a)\) and \((S_t)\) one can calculate the system efficiency as follows:

\[ E = \frac{\sum_{i=1}^{n} C_i Y_i G_i}{\lambda \sum_{i=1}^{n} C_i X_i} \]
This equation provides a means of evaluation of educational system efficiency without monetary terms.

B. Analytical Method

The question of why we act the way we do has puzzled philosophers and scientists all through history. In recent years a common approach has been to make a list of various drives which apparently motivate men. In general, most of these attempts were not found to be satisfactory because they were too long and arbitrary, did not consider all of the non-physiological factors and because the drives were often contradictory.

The theory of economic man; Maslow's theory of the hierarchy of needs; Herzberg's motivation-hygiene theory; McGregor's Theory X and Theory Y to explain certain aspects of human nature; Skinner's explanations of human behavior; the field theory as proposed and expanded by Lewin, Freudian and Pavlovian approaches are all among the numerous explanations for the question presented above (78) (79) (80) (81).

In spite of various differences there are some basic similarities between many of the above explanations. In fact, one gets the impression that the different explanations were the product of the varied backgrounds of the proponents rather than being inherent in the nature of their explana-
tions. What are some of the characteristics common to all of these theories? It seems that one such commonality is the existence of at least three components in any human activity. These components are:

1. The individual
2. The environment
3. The connection between the individual and the environment

Since education appears to be aimed at influencing one's behavior in a given environment, the nature of the interrelationships between these factors should be examined carefully to understand the process of education.

1. State of awareness

When an individual has an interaction with his environment how does a connection form? From physiology we know that the senses provide these connections. If the physical environment of an individual is not sensed by one of the senses, there is no way a connection can be formed. This connection by sense perceptions is required, but it may not be sufficient to alter our actions. The reason for this may be that our sense channels are open to such signals, but our "mind" is busy with something else. Or that we have "turned our minds off" to those signals. This leads to the conclusion that the important behavioral component to explain human
actions, that is the means by which a connection takes place between the individual and his environment, rests on what may be called a state of awareness. When an individual is not aware of the environment, there can be no interdependent connection between the individual and his environment.

This connection between the individual and his environment is better stated as the process of communication.

Webster's dictionary defines communication as, "a transmitting, a giving or giving and receiving of information." Communication is a social activity and means a process of "sharing" of elements of behavior or modes of life by the existence of a set of rules. A more specific definition given by a leading psychologist says, "communication is the discriminatory response of an organism to a stimulus" (82). This statement needs clarification, however, because it is the relationship between the receipt of stimulus and the derivation of response which constitutes the communication and not the response itself. Every communication adds to a person's accumulation of experiences; thus as a result, he is continually becoming a different person.

Many philosophers and scientists emphasize the goal seeking or purposeful nature of the behavior of man; such an assumption consequently implies the availability of choice among several alternatives of actions for fulfilling his goals. That is why the goal-seeking behavior of servo-
mechanisms make them appear functionally similar to living organisms (83) (84).

When we communicate, what we transmit is the representations of our thoughts and ideas using the language we know, and not the ideas and thoughts themselves. Statistical laws govern the structure of a language, thus communication is enhanced by virtue of the individual departing from predicted messages. This is another way of saying that you cannot communicate by nagging because they already know or can predict with almost complete certainty what you will say. Information can be received only when there is an uncertainty about the outcome of a situation. This implies the existence of choice among alternatives. We are continuously making choices, mostly sub-consciously, as a result of a basic animal characteristic. This is the fact that the act of discrimination is the simplest performable operation for organisms (85).

Communication is established by means of signs which one transmits to affect the behavior of another; but a more general statement is to say that this affects the state of awareness of another. Certainly it is possible to obtain the same physical state for an individual by sending a message or physically forcing him into it. But the state of awareness of the individual will be different in the two cases as ascertained from the difference of his subsequent behavior.
Cherry uses "Subjective probabilities", "degrees of belief", "state of mind", "state of preparedness", "mental state", and "psychological set" basically to say the same thing as we have called the state of awareness in explaining the change in one's behavior as a result of communication (86). Similarly, Ammerman has used the term "image state" to denote all of the accumulated and organized knowledge that a person has about himself and his environment (87).

It would be difficult to dispute the fact that the "state" of an individual must depend upon inborn and environmental factors. Thus education changes the state of awareness of an individual, also causes certain changes in his overt behavior and adds to his store of life experiences which cause him to become a different person.

In Kurt Lewin's field theory basically the same conclusions are reached as to the importance of the state of awareness as determining the future actions of an individual (88). According to the field theory all behavior is conceived to be the result of the field of an individual at a given time. In psychology this field is the "life space" of each individual. This life space consists of the individual and his psychological environment as he perceives it to exist at a given time.

Psychologically, Lewin describes the whole situation by using the Stimulus-Response connection. Thus behavior is a
function of a stimulus, but since all events are dependent on the state of the person and his environment, we revise the functional relationship from

\[ B = f(s) \]

to

\[ B = F(P,E) \]

Which says that behavior is a function of the individual person and his environment. The person and his environment are expressed as the totality of facts which determine the behavior at a given moment for an individual. This is another definition for the "life space". Thus Lewin conceives life space as the "whole situation" expressed in "possibilities" (89).

The expression applies to "emotional outbreaks" as well as "purposive activities" since in the formula \( P \) and \( E \) are implied to be dependent on each other. Thus:

\[ B = f(Lsp) \]

or behavior is a function of life space. Lewin then identifies the problems as:

1. Finding a scientific representation of the life-space (Lsp)
2. Determining a function which links the behavior to life space

If found, this function would be the law bringing predictability to the question "Why do we act the way we do?" submitted at the beginning of this section.

Ackoff, has defined a purposeful state (s) using decision theory principles. He starts with the premise that only "purposeful entities" can engage in communication, then defines "a purposeful state" by referring to a purposeful entity (I) in an environment (N), such that it may be summarized by saying that if an individual has objectives he wishes to reach and commands alternative ways with varying degree of competence to get to them, then he is in a purposeful state.

When a person is in a state of awareness the totality of his future has been identified by a set of probabilities in the face of a problem situation. Every choice process is a problem situation and the outcomes open to him are either desirable, undesirable or indifferent. This assumption although not required, will be found useful in valuation of the state of awareness. Thus an individual in a field, behaves according to his field parameters (set variables) in such a way as to first decide on a response; second, work according to his competence, and finally arrive at a consequence.
Response — Competence — Consequence — Response

form the chain of transitions for an individual along a time axis. Before pursuing this line of thought further let us pause and introduce another important aspect of the educational process.

It has been said that (62, p.3):

"...primary purpose of an educational system is to carry out the educational process. (and that) Education consists of helping the individual learn to use his brain effectively. Signals from outside go to the brain and in return the brain sends signals to other parts of the body that result in action."

If this is so, then study of the action is the only way we have of knowing whether or not the signals caused any change in the desired direction. Education is a sum of many communications aimed at behavior modification. In order to describe what should be accomplished as a result of education, it is necessary to set goals in general, and specify objectives. These are the statements in behavioral terms of the desired ends of education, better known as the "behavioral objectives" in education (90).

In Taxonomy of Educational Objectives, Bloom and Krathwohl classified educational goals into three areas—cognitive, affective and psychomotor domains, each divided into several levels (Appendix E.).

The cognitive domain pertains to knowledge of facts, un-
derstanding of ideas and relationships, synthesis and evaluation of ideas (91). The affective domain pertains to attitudes, appreciation, interest and values whereas the psychomotor domain is related to the muscular control of body and other skills (92).

The affective domain acts as a filter to the conscious. Many sensory stimuli never reach the cognitive stage because we are not aware of them. An individual is conscious of a stimulus when it penetrates the affective level of awareness. If the individual has no interest in the stimulus or is indifferent towards its outcomes, he does not respond to it. This is why it is very important to work on the affective domain in education. That is also why it is important to define a state of awareness as the state which needs to be examined, described, and measured to obtain a valuation of educational processes.

The following tentative definition for the state of awareness can now be made:

An individual is said to be in a state of awareness if a stimulus from the environment constituting his physical or mental world has penetrated his awareness level of the affective domain and he finds himself compelled toward some objectives valued by him in response to the stimulus.

The variables defining this state of awareness on the basis of decision theory principles are:
\( v_j \) = relative values of the set of consequences

\( p_i \) = probability of the \( i \)th response

\( C_{ij} \) = competence in \( i \)th response for \( j \)th consequence

The following table describes the properties of a state of awareness of an individual in the face of a given stimulus.

**Table 2. Description of the state of awareness**

<table>
<thead>
<tr>
<th>State</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
<th>( p_4 )</th>
<th>( \ldots )</th>
<th>( p_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( v_j )</td>
<td>( v_1 )</td>
<td>( v_2 )</td>
<td>( v_3 )</td>
<td>( v_4 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( R_i )</td>
<td>( 0_j )</td>
<td>( 0_1 )</td>
<td>( 0_2 )</td>
<td>( 0_3 )</td>
<td>( 0_4 )</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>

The three parameters mentioned above need to be discussed further. Let us begin with the values attached to each objective, \((v_j)\). These need not be absolute values of objectives since this would be impossible to determine.
Also the objectives and their values must be determined from a specific point of view. The same state may have different values for an individual and his family or the society in which he lives. This is a natural result of the process of valuation. The value of something has meaning only if the concerned parties are known.

Probabilities (Pi) relate to one's knowledge of facts of subjective evaluation of the outcomes of his responses. While driving a car, a misinformed person may take the wrong turn with great confidence, thus causing himself delay and trouble. Nevertheless (Pi) values are based on how one evaluates his chances of getting to the desired objective or avoiding the undesirable consequences when he makes a response. This is related to the cognitive domain in educational objectives.

Cij are the probabilities of reaching given objectives when a certain action is taken. Thus they represent a measure of the skill or competence one has in proceeding along a given action which is presumably taken because the individual believes that he can get to the desired objective taking that action. These relate to the psychomotor domain. Thus, having described the state of awareness in quantitative terms, it is now possible to examine a property called entropy of such a state in quest of a measure of effectiveness of educational processes.
2. **Entropy of a state**

"In the huge manufactory of natural processes, the principle of entropy occupies the position of manager for it dictates the manner and method of the whole business, whilst the principle of energy merely does the bookkeeping, balancing credits and debits (93)."

The term entropy has long been known to be essentially statistical in nature but attached to it there has always been a sense of mystery and a sort of mathematical fiction that was somehow unreal or very hard to grasp. And yet the change of entropy constitutes one of the most important characteristics of natural processes. The fact that entropy of the universe is always increasing must have philosophical implications, but few philosophers have addressed themselves to such an elusive subject (94)(95).

Clausius coined the term entropy from a Greek word meaning transformation (96). He defined it as the algebraic sum of the transformations necessary to bring a body into its existing state. It was early recognized that increase in entropy meant degradation of energy and the statement of the second law of thermodynamics in the form of:

\[ TdS = dQ \]

led to the identification of entropy as a factor of energy long before its statistical meaning was understood (97).

Maxwell's demon experiment in which an imaginary being
could violate the second law by separating "hot" and "cold" particles in a gas contained within the walls of a single container stimulated a lot of thought on the statistical nature of entropy (98) (99). But it was Boltzmann and later Planck who clearly demonstrated the statistical meaning of entropy (100).

Before going into the details let us begin by discussing the meaning of the "state" of a system. Planck defines the "state" of a physical system as "the conception as a whole of all those mutually independent magnitudes which determine the sequence of events occurring in the system so far as they are accessible to measurement; the knowledge of state is therefore equivalent to a knowledge of the initial conditions" (101, p.54).

For a system composed of many micro-state components, such as a volume of gas at a given temperature, there will be a large number of "complexions" which will produce the same macro-state, namely the same temperature and pressure under constant volume. Another illustration of this situation may be given as follows. Suppose we are throwing two ordinary, cubical dice. The "state of sum of four" can be obtained in any one of the following combinations or in Planck's terms "complexions";

First cube 1, the second cube 3
First cube 2, the second cube 2
First cube 3, the second cube 1

If we want to get the "state" where the sum is three, then we have the following "complexions":

First cube 1, second cube 2
First cube 2, second cube 1

Boltzmann assumes all complexions to be equally probable as in the illustration above and therefore the number of complexions in a state determine the probability of that state such that the probability of throwing a sum of four is 1 1/2 times greater than of throwing a sum of three.

Thus the connection between entropy and the probability of states was made by Boltzmann in his "H-theorem" first, but the statement which has been adopted belongs to Planck who gave the formula:

\[ S = k \log W + K \]

where

\[ k = \text{dimensional constant} \]
\[ W = \text{number of complexions in a state} \]
\[ K = \text{an arbitrary constant which may be omitted}. \]

In the case of two dice illustration above, the entropy of the two states may be calculated as follows:

\[ S(4) = k \ln 3 \]
For a state which has three complexions and that each occurrence has a probability of $1/3$, the entropy of that state can be found by the following equation:

$$S = -k \sum P_j \log P_j$$

$$= -k \sum \frac{1}{3} \log \frac{1}{3}$$

$$= -3k \left( \frac{1}{3} \right) (-1.1)$$

$$= (1.1) k$$

As can be seen from this example the ($-$) sign in front of the expression provides the same value of $S$ as was obtained with the number of complexions. This is similar to the definition of information given by Shannon discussed in the review of literature.

When the complexion occurrences are not equal as in the outcomes expected in a state of awareness, the use of the second entropy equation will be made. The logarithmic nature of the expression can be explained in the following way.

Let us first realize that entropy is an extensive property, that is, its value for a system equals the sum of its values for the parts of the system. Therefore; the entropy of system $A$ composed of subsystems $B$ and $C$ is given by:
\[ S(A) = S(B) + S(C) \]

At the same time the probability of State A is given by the number of "complexions" of states B and C as follows:

\[ W_a = W_b \cdot W_c \]

This can only be possible if

\[ S = k \log W \]

because

\[ S(A) = S(B) + S(C) \]
\[ = k (\log W_b + \log W_c) \]
\[ = k \log (W_b \cdot W_c) \]
\[ = k \log W_a \]

The following statements are given to interpret the meaning of the entropy of a state:

1. Entropy is a universal measure of the disorder in the configuration of a system.
2. Growth of entropy is from less probable to more probable states.
3. Growth of entropy is a passage from a regulated or ordered to a less regulated or ordered state.
4. Growth of entropy implies an increase in the disorder of a system.
5. Any mathematical function whose time variation always has the same sign until a certain state is reached and is then zero may be called an entropy function.

This last statement is another way of saying that the entropy of the universe always increases.

"The increase in entropy, then, simply means the passage from a more easily distinguishable state to a less easily distinguishable state, or, in terms of the generalized space, from a less probable to a more probable configuration" (102, p.182).

3. Valuation of a state

Having established the ground work for the idea of the "state of awareness" for an individual and the theory behind the important concept of entropy, one can now make the connection between the two in terms of educational processes.

As discussed above, entropy in a statistical sense is a measure of the randomness of the arrangement of the particles comprising a gas and as such it is a function of the probabilities of the formation of various configurations (complexions) of a state.

Similarly, the state of awareness as described by the probability matrix shown in Table 3 can be said to be composed of several configurations each with different probability of occurrence as represented by a consequence set relative to the given situation.
The consequence set is a set of outcomes resulting from the responses of an individual providing a complete description of his probable future states. It has different probabilities attached to each consequence (Oj) of a response (Ri) selected by the individual who has a competence factor of (Cij) in achieving (Oj).

Based on the (pi) and (Cij) values we can calculate the probability (Pj) of occurrence of each configuration describing a given state of awareness. Then using the equations shown above we can calculate the entropy of a state of awareness.

The change in the entropy of a state of awareness as a result of an educational experience, should provide a convenient measure of the "education" to that individual.

The theory would predict that as a result of education an individual would reduce the entropy of his state of awareness since he would be equipped with better cognitive, affective and psychomotor qualities assuming his consequence set remained the same during a given educational experience.

If only the affective domain is influenced during an educational experience, the change in (vj) values may cause the consequence set to vary. This variation affects the individual's motivation towards his cognitive and psychomotor interests, thus influencing the (pi) and (cij) values which result in a different set of probabilities (Pj) and different
entropy.

In general it may be said that education reduces the entropy of the state of awareness of an individual at time \( t \) compared to a previous state by altering \((p_i)\) and \((C_{ij})\), and as a result \((P_j)\) probabilities. An educated person has a high degree of sophistication in his values, shown by his consistency in preferring ethical outcomes, has developed self confidence which is reflected in his probability \((p_i)\) values for alternative responses and that he is competent in his work. Such an individual will have a low entropy when he is in a state of awareness in a given environment.

The quantitative nature of entropy of a state of awareness can be used in various ways to obtain a measure of the educational system performance. The next section will deal with the utilization of this concept in valuation of a state of awareness.

The inherent value of a state of awareness for an individual is related to the values he attaches to each of the consequences in the consequence set.

If we assume a symmetrical value function, then we can postulate that all of the consequences in a set can be categorized into three mutually exclusive and exhaustive areas namely:

1. Desirable, valued at 1.0;
2. Undesirable, valued at -1.0;
3. Indifferent, valued at zero.

If we can specify all of the consequences and rank them according to the importance to the valuator, we could then assign fractional values to each and still obtain a workable measure.

The value of a state of awareness therefore is the expected value of an individual's ability to get to all of the outcomes (Oj) by all of the responses (Ri) available to him at a point in time.

Thus:

\[ V = \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{ij} C_{ij} V_{j} \]

gives a Value of a State of Awareness to that individual.

Let us define education tentatively as:

"any interaction process of an individual with his physical and mental environment which affects his state of awareness."

From this definition of the meaning of education and the state of awareness, a "thermodynamic" model of education and its valuation can be obtained as shown in Fig. 1.

This allows us to establish a valuation function for education in operational terms:

\[ V(ED)_{1-2} = V_{2} - V_{1} \]
This analysis implies that a state of awareness has no inherent absolute value which is independent of the future. Rather, the value of a state of awareness to an individual is determined by his ability to respond to his environment and arrive at consequences including his objectives, which have positive value to the individual.

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} (p_i c_{ij} v_j) - \sum_{i=1}^{m} \sum_{j=1}^{n} (p_j c_{ij} v_j)
\]

Fig. 1. A valuation model of the process of education.

Two measures related to educational processes are now proposed:

1) The change in the entropy of the state of awareness.
2) The change in the value of the state of awareness.
These can be used to obtain indices of performance for an educational system in the following way:

\[
I_v = \frac{V_2 - V_1}{V_1} \quad \text{Value index}
\]

\[
I_s = \frac{S_1 - S_2}{S_1} \quad \text{Entropy index}
\]

The following examples have been prepared to show applicability of the theory.

Let us assume that an individual (I), in an environment (E), has the following state of awareness parameters as shown in Fig. 2. For seven different values of the variables involved, the calculation of the state values (V) and the state entropies (S) are given in Table 3.
Fig. 2. Description of a state of awareness with two alternative responses and three mutually exclusive outcomes.
Table 3. Values and Entropies of a state of awareness as shown in Fig. 2.

<table>
<thead>
<tr>
<th>State</th>
<th>( p_i )</th>
<th>( c_{ij} )</th>
<th>( p_j )</th>
<th>( S )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>0.30</td>
<td>0.50</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>5</td>
<td>0.70</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>0.90</td>
<td>0.10</td>
<td>0.90</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
C. Procedural Summary

Based on the analytical method developed, three approaches were made to its application to the valuation of performance of an educational system.

The first approach involved a class where the experimenter was teaching mathematics and a review of the basic slide rule manipulations were required for the efficient conduct of the class.

The educational system was defined as the students in the class and the instructor during the summer of 1971 at Iowa State University. The specific educational experience to be measured was the slide rule instruction given to the students. The following steps outline the nature of the experiment in behavioral terms:

1. Present students with a stimulus by giving them problems consisting of multiplication and division of numbers, thus getting them in a state of awareness.
2. Motivate them properly towards the consequence set of each problem.
3. Allow two responses open, the use of a slide rule or long hand to perform given operations.
4. From the performance results calculate state value and entropy for the class.
5. Expose them to slide rule instruction.

6. Construct another state of awareness matrix under the same conditions as existed previously.

7. Calculate value and the entropy of the state after this "education".

8. Compare with State 1 values and calculate Entropy and Value indexes.

The second approach involved the use of final class lists in a selected course in engineering. The responses were defined as the major department of the student and the consequence set consisted of the ultimate grades at the end of the quarter. In this case, the values of the consequences were ranked on the basis of letter grades. As a result D, C, B, A grades were the ranked subsets of the desirable, X grade indicated the neutral and F grades corresponded to the undesirable consequence. Thus the values of the consequence set were as follows:

<table>
<thead>
<tr>
<th>Desirable consequences</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D grade</td>
<td>0.1</td>
</tr>
<tr>
<td>C &quot;</td>
<td>0.2</td>
</tr>
<tr>
<td>B &quot;</td>
<td>0.3</td>
</tr>
<tr>
<td>A &quot;</td>
<td>0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Neutral consequence</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X grade</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Undesirable consequence</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>F grade</td>
<td>-1.0</td>
</tr>
</tbody>
</table>
Two sequence courses taught at Iowa State University were used to obtain data for the state of awareness of the first and second states to evaluate the performance of the educational system defined by the second course in the sequence. From these again the entropy and relative value were calculated for the class as a whole for both states, then the value index and the entropy index were determined.

The third experiment involved hypothetical data using the final grades students received in a three course sequence all in the same subject. The system evaluated was the second course in the sequence. The intention here was to obtain more meaningful values for the probabilities in the state of awareness matrix. The grades at the end of the first course were used to determine the pi values at state 1. Similarly the grades at the end of the second course were used to identify the pi values for state 2. The competence factors Cij were determined on the basis of the grades the students obtain at the end of the second and third courses for states 1 and 2 respectively.
## IV. RESULTS

Three main experiments were conducted in an attempt to apply the analytical model developed in the last chapter to evaluate the performance of an educational system. All the data and calculations underlying the results shown in this chapter are included in the Appendix.

The first experiment, called the slide rule teaching experiment gave the following results:

<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
<th>Entropy</th>
<th>Iv</th>
<th>Is</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.221</td>
<td>(1.268)k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.479</td>
<td>(1.214)k</td>
<td>1.17</td>
<td>0.043</td>
</tr>
</tbody>
</table>

This experiment was evaluated a second time using the analytical model. In this interpretation, the consequence set was determined by the number of correct answers given by each student divided into 8 groups of three answer increments. Each group was valued on the basis of its ranking in a scale of 0-1.0, with the following results:

<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
<th>Entropy</th>
<th>Iv</th>
<th>Is</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.247</td>
<td>(0.246)k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.461</td>
<td>(0.246)k</td>
<td>0.84</td>
<td>0</td>
</tr>
</tbody>
</table>
The second experiment was based on the actual scores of two mathematics courses taught at Iowa State University during the winter and spring quarters in 1971. The first course was a college algebra course leading into the calculus course. The grades of 41 students were analyzed with the following results:

<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
<th>Entropy</th>
<th>Iv</th>
<th>Is</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.266</td>
<td>(1.264)k</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.215</td>
<td>(1.458)k</td>
<td>-0.190</td>
<td>-0.130</td>
</tr>
</tbody>
</table>

The third experiment was intended to show a refinement in the second experiment by the use of data pertaining to a three course sequence. In this experiment the educational system evaluated was the second course of the sequence and the following results were obtained:

<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
<th>Entropy</th>
<th>Iv</th>
<th>Is</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.173</td>
<td>(1.433)k</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.189</td>
<td>(1.376)k</td>
<td>0.091</td>
<td>0.021</td>
</tr>
</tbody>
</table>
V. SUMMARY AND CONCLUSIONS

In this study, it has been shown that a quantitative and non-monetary valuation of the performance of educational systems is possible and feasible. The introduction of the problem from a resource allocation point of view was expanded to show the need for a definition of the quantity called "education" in operational terms so as to allow identification and measurement.

Review of the literature, although limited in view of the enormous amount of published material in the field of education, showed that definition of the outputs of an educational system still remains to be resolved.

Economists' approach to performance evaluation from a benefit/cost ratio point of view requires the valuation of benefits in monetary terms. The "present worth of life-time earnings" as the value of an "education" has not been completely satisfactory even though it has been very useful. It leaves three important determinants, namely; the income pattern, the applicable discount rate and the number of years involved, to arbitrary decision. Also the relationship between earnings and education has not been substantiated in many cases.
Two distinct methods for the valuation of the performance of an educational system have been developed.

The first, called the educational attainment method developed a measure of the efficiency of an educational system similar to a thermodynamical interpretation of the concept. Thus a measure of performance is derived in the form of a ratio of the actual performance divided by the theoretical maximum obtainable.

The analytical method developed as the second model for the valuation of an educational system effectiveness was also aimed at a non-monetary measure.

The nature of education is first defined in behavioral terms and interpreted as the effects of a set of communications with the intention of behavior modification in the recipient. Thus the measure of education is found in the change of the overt behavior of the recipient.

From this interpretation, and an analysis of the various domains of educational objectives, a definition of a State of Awareness was made for an individual in a given environment. This term was coined in preference to others because, it takes the first level of affective domain in education that is, awareness, as the requirement for determination of a consequence set in terms of decision and value theory principles.
Such a state of awareness, once operationally defined in terms of available responses, their probabilities of being made, the consequence set with their relative values and the competence factors of the individual related to each response and the consequence set, allows the determination of two valuation parameters. One such parameter is the Value of a state and the other is the Entropy of a state.

Using these two parameters and treating the educational process as an input-output system, two performance indices are developed.

The Value Index shows the percent change in the value of a state of awareness, and the Entropy Index shows the change in the entropy of a state. If education is to be successful it must increase the value and decrease the entropy.

The application of the analytical method to four different evaluations has shown that such measures are quite flexible in applicability and feasible in a variety of valuation problems in education. Since value calculations are based on subjective weighing of the consequence set, some variation is to be expected. Also it is reasonable to expect the weights to change over time. In fact a totally different set of values given to the same consequence set can produce drastically different results.

However, the entropy measure does not depend on any subjective weights. It is an inherent measure of the order-
disorder in the state of the system, depending primarily on the probability values of the consequence set which are determined by the cognitive and competence factors of the individual.

This fact shows that a non-monetary measure of performance in terms of the change in the entropy of an individual's state of awareness in a given environment is indeed feasible and should be preferred to other measures based on subjective weighing.

The meaning of the change in entropy in a state of awareness can be interpreted as being desirable if it is decreasing thus indicating that the individual is less confused about his actions, more capable of accomplishing one of the consequences in the set and that he is more confident and competent in accomplishing it.

On the basis of this explanation it may be said that an educated person is one who has the lowest total entropy when confronted by a given number of environmental stimuli in a given period of time. An expert in a given subject might have a very low entropy in that area; but the same person may be confused in another area thus exhibiting a higher overall entropy.

In conclusion, the following specific points for further research are proposed:

1. More sophisticated experiments are needed to apply
the models developed. The educational attainment model can be used if a large number of student records are available in computerized form. Many schools already have this facility and many others will also have it in the near future. Due to the nature of the method a large number of students need to be treated for reliable measures.

2. More research is needed in the area of determining a theoretical maximum obtainable from a given educational system to be used as a prescriptive standard for comparison of actual system performance.

3. Interaction analysis methods can be used to derive such standards of performance in a variety of educational experiences at its present stage of development. However, it must be modified and extended to become more productive in the case of engineering and design oriented teaching where additional categories of behavior are needed for adequate analyses.

4. A personal view of valuation of education needs to be taken such as suggested by Professor Fox in terms of happiness function of individuals. This provides a new orientation in the measurement of benefits of education.

5. Concepts of State of Awareness and the Entropy of such a state should be developed further for appli-
cation to complex educational systems.

6. Prescriptive standards for Value index and Entropy index should be established for given educational systems. This requires determination of a set of stimuli for the creation of a state of awareness in the individuals for the purpose of valuation before and after an educational experience. This set may be construed to be some kind of a test but the evaluation of the test is to be made on the basis of entropy and value change rather than the well known concept of pre-test and post-test score comparisons.

In closing, it must be said that great changes are needed in the world if mankind is to survive on this spaceship called the earth. It is in the hands of the world leaders and the educators, if civilization is to survive and flourish or to explode and perish. This world predicament needs transformative actions without delay, especially in the field of education. This fact was emphasized nearly 40 years ago, by Atatürk, one of the leaders of all time when he said (103):

"If we want a continuing peace we must make international efforts to improve the conditions of the masses. The prosperity of all mankind must replace hunger and oppression. World citizens must be educated away from jealousy, greed and hatred."
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VII. ACKNOWLEDGMENTS

I wish to thank everyone who helped with the efficient completion of this work. In particular, I would like to express my gratitude to Professor A. C. Kleinschmidt and all the other members of my committee who so generously gave their time and energy.

Special thanks are due to Professor W. L. Larsen for his interest in the subject and constructive criticisms, Professor P. W. Barcus and Professor K. L. McRoberts for their illuminating comments, Professors J. K. Walkup and H. M. Black for the many useful suggestions and evaluations during the whole program of study.

I wish to thank Janet who had to do the more difficult part of this joint project by being both a mother and a father to Tijen and Meral during the time it took to complete this work.

Finally, I would like to extend my sincere appreciations to Pat Swan for helping with the computerization of this thesis.
VIII. APPENDIX A:
SLIDE RULE TEACHING EXPERIMENT
CALCULATIONS AND DATA FOR THE FIRST EVALUATION

**Entropy Index**

\[
\frac{S_1 - S_2}{S_1} = \frac{1.268 - 1.214}{1.268} = \frac{0.054}{1.268} = 0.0426
\]

**Value Index**

\[
\frac{V_2 - V_1}{V_1} = \frac{0.479 - 0.221}{0.221} = \frac{0.258}{0.221} = 1.17
\]
Table 4. State of awareness matrix for state one.

<table>
<thead>
<tr>
<th>State</th>
<th>$P_j$</th>
<th>0.249</th>
<th>0.182</th>
<th>0.450</th>
<th>0.119</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$v_j$</td>
<td>1.0</td>
<td>0.5</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>$P_i$</th>
<th>Correct Answer</th>
<th>Cor. Digits Answer</th>
<th>No Answer</th>
<th>Incorrect Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long hand</td>
<td>0.194</td>
<td>0.411</td>
<td>0.041</td>
<td>0.512</td>
<td>0.036</td>
</tr>
<tr>
<td>Slide rule</td>
<td>0.806</td>
<td>0.210</td>
<td>0.216</td>
<td>0.435</td>
<td>0.139</td>
</tr>
</tbody>
</table>

$P_1 = (0.194)(0.411) + (0.806)(0.210) = 0.249$
$P_2 = (0.194)(0.041) + (0.806)(0.216) = 0.182$
$P_3 = (0.194)(0.512) + (0.806)(0.435) = 0.450$
$P_4 = (0.194)(0.036) + (0.806)(0.139) = 0.119$

**Entropy of State 1:**

$S = -k(\ln 0.249 + \ln 0.182 + \ln 0.450 + \ln 0.119)$
$S = -k(-0.346 - 0.310 - 0.359 - 0.253)$
$S = (1.268)k$

**Value of State 1:**

$V = 0.194 [0.411 + (0.041)(0.5) + 0.036 (-1)]$
$+ 0.806 [0.21 + (0.095)(0.216) + (0.139)(-1)]$
$V = 0.194 (0.411 + 0.021 - 0.036)$
$+ 0.806 (0.21 + 0.108 - 0.139)$
$V = 0.194 (0.396) + 0.806 (0.179)$
$V = 0.077 + 0.144$
$V = 0.221$
Table 5. State of awareness matrix for state two.

<table>
<thead>
<tr>
<th>State</th>
<th>Pj</th>
<th>0.465</th>
<th>0.178</th>
<th>0.282</th>
<th>0.075</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>vj</td>
<td>1.0</td>
<td>0.5</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td>Oj</td>
<td>Correct</td>
<td>Cor.dgts.</td>
<td>No</td>
<td>Incorrect</td>
</tr>
<tr>
<td>Ri</td>
<td>Pi</td>
<td>answer</td>
<td>only</td>
<td>answer</td>
<td>answer</td>
</tr>
<tr>
<td>Long hand</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Slide rule 1.0</td>
<td>0.465</td>
<td>0.178</td>
<td>0.282</td>
<td>0.075</td>
<td></td>
</tr>
</tbody>
</table>

**Entropy of State 2:**

\[ S = -k \left( 0.465 \ln 0.465 + 0.178 \ln 0.178 + 0.282 \ln 0.282 + 0.075 \ln 0.075 \right) \]
\[ = -k \left( -0.356 - 0.307 - 0.357 - 0.194 \right) \]
\[ = (1.214) \ k \]

**Value of State 2:**

\[ V = 0.465 + (0.5) (0.178) - 0.075 \]
\[ = 0.465 + 0.089 - 0.075 \]
\[ = 0.479 \]
IX. APPENDIX B:
SLIDE RULE TEACHING EXPERIMENT
CALCULATIONS AND DATA FOR THE SECOND EVALUATION

\[ \text{Entropy Index:} \]
\[ \frac{S_1 - S_2}{S_1} \]
\[ \frac{0.246 - 0.246}{0.246} \]
\[ 0 \]

\[ \text{Value Index:} \]
\[ \frac{V_2 - V_1}{V_1} \]
\[ \frac{0.461 - 0.25}{0.25} \]
\[ 0.21 \]
\[ \frac{0.21}{0.25} \]
\[ 0.84 \]
Table 6. State of awareness matrix for state one.

<table>
<thead>
<tr>
<th>State</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
<th>P_5</th>
<th>P_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>vi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.21</td>
<td>0.33</td>
<td>0.46</td>
<td>0.58</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome Classification</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.H.</td>
<td>0.20</td>
<td>0.0</td>
<td>0.14</td>
<td>0.29</td>
<td>0.43</td>
<td>0.0</td>
</tr>
<tr>
<td>S.R.</td>
<td>0.80</td>
<td>0.45</td>
<td>0.21</td>
<td>0.28</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Entropy of State 1:

\[ S = -k \left( 0.360 \ln 0.360 + 0.196 \ln 0.196 + 0.282 \ln 0.282 \\
+ 0.110 \ln 0.110 + 0.024 \ln 0.024 + 0.028 \ln 0.028 \right) \]

\[ = \left( \frac{1.477}{6} \right) k \]

\[ = 0.246 k \]

Value of State 1:

\[ V = 0.2 \left[ 0.14(0.21) + 0.29(0.33) + 0.43(0.46) + 0.14(0.71) \right] \\
+ 0.8 \left[ 0.45(0.08) + 0.21(0.21) + 0.28(0.33) \\
+ 0.03(0.46) + 0.03(0.58) \right] \]

\[ = 0.2 \left( 0.422 \right) + 0.8 \left( 0.204 \right) \]

\[ = 0.084 + 0.163 \]

\[ = 0.247 \]
Table 7. State of awareness matrix for state two.

<table>
<thead>
<tr>
<th>State</th>
<th>Pj</th>
<th>0.03</th>
<th>0.14</th>
<th>0.17</th>
<th>0.28</th>
<th>0.22</th>
<th>0.14</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>v_2</td>
<td>0.08</td>
<td>0.21</td>
<td>0.33</td>
<td>0.46</td>
<td>0.58</td>
<td>0.71</td>
<td>0.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>O_j</th>
<th>Outcome</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_i</td>
<td>P_i</td>
<td>1</td>
</tr>
<tr>
<td>L.H</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>S.R.</td>
<td>1.0</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Entropy of State 2:

\[ S = -k \left( 0.03 \ln 0.03 + 0.14 \ln 0.14 + 0.17 \ln 0.17 + 0.22 \ln 0.22 + 0.02 \ln 0.02 \right) \]

\[ = \left( \frac{1.723}{7} \right) k \]

\[ = 0.246 \text{ k} \]

Value of State 2:

\[ V = 0.03(0.08) + 0.14(0.21) + 0.17(0.33) + 0.28(0.46) + 0.22(0.58) + (0.14)(0.71) + (0.02)(0.83) \]

\[ = 0.461 \]
Data was the same as the first evaluation in previous pages, but the consequence set was defined, as follows:

<table>
<thead>
<tr>
<th>Consequence, Oj</th>
<th>No. correct</th>
<th>Value, Vj</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 3</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>4 - 6</td>
<td>0.21</td>
</tr>
<tr>
<td>3</td>
<td>7 - 9</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>10-12</td>
<td>0.46</td>
</tr>
<tr>
<td>5</td>
<td>13-15</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>16-18</td>
<td>0.71</td>
</tr>
<tr>
<td>7</td>
<td>19-21</td>
<td>0.83</td>
</tr>
<tr>
<td>8</td>
<td>22-24</td>
<td>1.00</td>
</tr>
</tbody>
</table>

R₁ = Long Hand method
R₂ = Slide Rule method

**State 1:** Before teaching slide rule, \( p_1 = 0.2, p_2 = 0.8 \)
**State 2:** After teaching slide rule, \( p_1 = 0, p_2 = 1.0 \)

Length of instruction — 1 hour lecture, 1 hour practice.

Average score of class:

**State 1:** 5.98, Consequence \( O_2 \)
**State 2:** 11.50, Consequence \( O_4 \)

Gain in Score = 0.46 - 0.21 Based on consequence
= 0.25 score rankings.

<table>
<thead>
<tr>
<th>No.</th>
<th>No. correct</th>
<th>No. correct dgts only</th>
<th>No. incorrect</th>
<th>No. blank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>10</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>13</td>
</tr>
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<td>6</td>
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<td>0</td>
<td>8</td>
<td>14</td>
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<td>1</td>
<td>2</td>
<td>18</td>
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<td>8</td>
<td>1</td>
<td>10</td>
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<td>13</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>10</td>
</tr>
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<td>12</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
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<td>14</td>
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<td>15</td>
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<td>6</td>
</tr>
<tr>
<td>17</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>13</td>
<td>0</td>
<td>8</td>
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<td>8</td>
</tr>
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<td>1</td>
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<td>13</td>
</tr>
<tr>
<td>27</td>
<td>13</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>28</td>
<td>9</td>
<td>1</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>29</td>
<td>8</td>
<td>6</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Totals 146  150  97  303
% 0.210  0.216  0.139  0.435

| No. | No. correct | No. correct
gts only | No. incorrect | No. blank |
<table>
<thead>
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<th></th>
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</tr>
<tr>
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<td>1</td>
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<td>3</td>
<td>10</td>
<td>0</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
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Totals 69

% 0.411  0.041  0.036  0.512
Data 3. All students using Slide Rule, State 2.

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Totals 402   154   65   243
% 0.465  0.178  0.075  0.282
X. APPENDIX C:
CALCULATIONS AND DATA
FOR THE EVALUATION OF A COURSE IN MATHEMATICS

Entropy Index

\[
\frac{S_1 - S_2}{S_1} = \frac{1.264 - 1.458}{1.264} = -0.194 \\
\frac{1.264}{1.264} = 1 \\
-0.133
\]

Value Index

\[
\frac{V_2 - V_1}{V_2} = \frac{0.215 - 0.266}{0.266} = -0.190
\]
Table 8. State of awareness matrix for state one.

<table>
<thead>
<tr>
<th>State</th>
<th>Pj</th>
<th>0.244</th>
<th>0.244</th>
<th>0.439</th>
<th>0.073</th>
<th>0.00</th>
<th>0.00</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>v1</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Oj</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>X</th>
<th>F</th>
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<td>Ri</td>
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<td>grade</td>
<td>grade</td>
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<td>grade</td>
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<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>0.268</td>
<td>0.364</td>
<td>0.182</td>
<td>0.272</td>
<td>0.182</td>
<td>0.0</td>
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<tr>
<td></td>
<td>P2</td>
<td>0.512</td>
<td>0.143</td>
<td>0.285</td>
<td>0.524</td>
<td>0.048</td>
<td>0.0</td>
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<tr>
<td></td>
<td>P3</td>
<td>0.220</td>
<td>0.333</td>
<td>0.222</td>
<td>0.445</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\[ P_4 = 0.268(0.364) + 0.512(0.143) + 0.220(0.333) = 0.244 \]
\[ P_2 = 0.268(0.182) + 0.512(0.285) + 0.220(0.222) = 0.244 \]
\[ P_3 = 0.268(0.272) + 0.512(0.524) + 0.220(0.445) = 0.439 \]
\[ P_4 = 0.268(0.182) + 0.512(0.048) = 0.073 \]

**Value of State 1:**

\[ V = (0.268) [(0.4)(0.364) + (0.3)(0.182) + 0.2)(0.272) \]
\[ + (0.1)(0.182)] + (0.512) [(0.4)(0.143) \]
\[ + (0.3)(0.285)(0.2)(0.524) + (0.1)(0.048)] \]
\[ + (0.220)(0.4)(0.333) + (0.3)(0.222) \]
\[ + (0.2)(0.445) ] \]
\[ = (0.268)(0.1456 + 0.055 + 0.054 + 0.018) \]
\[ + (0.512)(0.057 + 0.086 + 0.1048 + 0.0048) \]
\[ + (0.222)(0.1332 + 0.0666 + 0.0890) \]
\[ = 0.073 + 0.129 + 0.063 \]
\[ = 0.266 \]

**Entropy of State 1:**

\[ S = -k[(0.244 \ln 0.244) + 2 + 0.439 \ln (0.439 + 0.073 \ln 0.073)] \]
\[ = (1.264)k \]
Table 9. State of awareness matrix for state two.

<table>
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<th>State</th>
<th>( P_j )</th>
<th>0.243</th>
<th>0.200</th>
<th>0.342</th>
<th>0.142</th>
<th>0.049</th>
<th>0.024</th>
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<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
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<tr>
<td></td>
<td>( O_j )</td>
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<td>C</td>
<td>D</td>
<td>X</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>( P_i )</td>
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<td>grade</td>
<td>grade</td>
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<td>0.268</td>
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<td>0.091</td>
<td>0.455</td>
<td>0.082</td>
<td>0.0</td>
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<td>E.T.</td>
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<td>0.512</td>
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<td>0.334</td>
<td>0.238</td>
<td>0.095</td>
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<td>C.T.</td>
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<td>0.220</td>
<td>0.222</td>
<td>0.445</td>
<td>0.222</td>
<td>0.111</td>
<td>0.0</td>
</tr>
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</table>

\[ P_1 = 0.268(0.364) + 0.512(0.190) + 0.22(0.222) = 0.243 \]
\[ P_2 = 0.268(0.091) + 0.512(0.143) + 0.22(0.445) = 0.200 \]
\[ P_3 = 0.268(0.455) + 0.512(0.334) + 0.22(0.222) = 0.342 \]
\[ P_4 = 0.268(0.00) + 0.512(0.238) + 0.22(0.111) = 0.142 \]

**Value of State 2:**

\[ V = 0.268[0.4(0.364) + 0.3(0.091) + 0.2(0.455) - 0.09] \]
\[ + 0.512[0.4(0.190) + 0.3(0.143) \]
\[ + 0.2(0.334) + 0.1(0.238)] + 0.22[0.4(0.222) \]
\[ + 0.3(0.445) + 0.2(0.222) + 0.1(0.111)] \]
\[ = 0.268(0.2639) + 0.512(0.2095) + 0.22(0.2778) \]
\[ = 0.268(0.2639) + 0.512(0.2095) + 0.22(0.2778) \]
\[ = 0.2150 \]

**Entropy of State 2:**

\[ S = -k[0.243 \ln 0.243 + 0.2 \ln 0.2 + 0.342 \ln 0.342 \]
\[ + 0.142 \ln 0.142 + 0.049 \ln 0.049 \]
\[ + 0.024 \ln 0.024] \]
\[ = 1.4576 \text{ k} \]
Data 4. Math class grades.

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<td>A</td>
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<td>B</td>
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</table>
XI. APPENDIX D:
CALCULATIONS AND DATA FOR THE EVALUATION OF
THE SECOND COURSE IN A THREE COURSE SEQUENCE

**Entropy Index**

\[ \frac{S_4 - S_2}{S_1} \]

\[ \frac{1.402 - 1.376}{1.376} \]

0.0214

**Value Index**

\[ \frac{V_2 - V_1}{V_1} \]

\[ \frac{0.1891 - 0.1734}{0.1734} \]

0.0905
Table 10. State of awareness matrix for state one.

<table>
<thead>
<tr>
<th>State</th>
<th>Pj</th>
<th>0.200</th>
<th>0.333</th>
<th>0.267</th>
<th>0.067</th>
<th>0.067</th>
</tr>
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<tbody>
<tr>
<td>vj</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>-1.0</td>
<td></td>
</tr>
<tr>
<td>Oj</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.134</td>
<td>0.75</td>
<td>0.25</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>B</td>
<td>0.266</td>
<td>0.25</td>
<td>0.625</td>
<td>0.125</td>
<td>0.0</td>
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<tr>
<td>C</td>
<td>0.400</td>
<td>0.083</td>
<td>0.333</td>
<td>0.417</td>
<td>0.084</td>
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<td>D</td>
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<td>0.0</td>
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<td>F</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\[
P_1 = 0.134(0.75) + 0.266(0.25) + 0.4(0.083) = 0.200 \\
P_2 = 0.134(0.25) + 0.266(0.625) + 0.4(0.333) = 0.333 \\
P_3 = 0.266(0.125) + 0.4(0.417) + 0.5(0.134) = 0.267 \\
P_4 = 0.40(0.083) + 0.134(0.25) = 0.067 \\
P_5 = 0.40(0.083) + 0.134(0.25) = 0.067
\]

**Entropy of State 1:**

\[
S = -k[0.200 \ln 0.200 + 0.333 \ln 0.333 + 0.267 \ln 0.267 + 2(0.067 \ln 0.067)] \\
= 1.402 \text{ k}
\]

**Value of State 1:**

\[
V = 0.134 [(0.75)(0.4) + 0.25(0.3)] + 0.266 [0.25(0.4) + 0.625(0.3) + 0.125(0.2)] + 0.4[0.083(0.4) + 0.333(0.3) + 0.417(0.2) + 0.084(0.1)] - 0.083 + 0.134[0.5(0.2) + 0.25(0.1) - 0.25] \\
= 0.0502 + 0.0832 + 0.0568 - 0.0168 \\
= 0.1734
\]
Table 11. State of awareness matrix for state two.

<table>
<thead>
<tr>
<th>State</th>
<th>Pj</th>
<th>0.250</th>
<th>0.388</th>
<th>0.250</th>
<th>0.072</th>
<th>0.036</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>vj</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rj</th>
<th>Pj</th>
<th>grade</th>
<th>grade</th>
<th>grade</th>
<th>grade</th>
<th>grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.214</td>
<td>0.67</td>
<td>0.33</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>B</td>
<td>0.357</td>
<td>0.30</td>
<td>0.5</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>0.286</td>
<td>0.0</td>
<td>0.25</td>
<td>0.375</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>D</td>
<td>0.071</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>F</td>
<td>0.072</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\[ P_1 = 0.214(0.67) + 0.357(0.3) = 0.250 \]
\[ P_2 = 0.214(0.33) + 0.0357(0.5) + 0.25(0.286) = 0.388 \]
\[ P_3 = 0.071 + 0.286(0.375) + 0.2(0.375) = 0.250 \]
\[ P_4 = 0.286(0.25) = 0.072 \]
\[ P_5 = 0.286(0.125) = 0.036 \]

**Entropy of State 2:**

\[ S = -k \left[ (0.25 \ln 0.25) + 0.388 \ln 0.388 + 0.076 \ln 0.076 \right] 
+ 0.036 \ln 0.036 \]
\[ = 1.376 \, k \]

**Value of State 2:**

\[ V = 0.214 \left[ 0.67(0.4) + 0.33(0.3) \right] 
+ 0.357 \left[ 0.3(0.4) + 0.5(0.3) + 0.2(0.2) \right] 
+ 0.286 \left[ 0.25(0.3) + 0.357(0.2) + 0.25(0.1) - 0.125 \right] 
+ 0.071(0.2) \]
\[ = 0.0785 + 0.0821 + 0.0143 + 0.0142 \]
\[ = 0.1891 \]
## Data 5. Grades in a three course sequence

<table>
<thead>
<tr>
<th>Student No.</th>
<th>Course 1</th>
<th>Course 2</th>
<th>Course 3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>A</td>
<td>A</td>
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<tr>
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<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
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<td>C</td>
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<td>B</td>
<td>A</td>
</tr>
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<td>A</td>
<td>A</td>
</tr>
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<td>B</td>
<td>B</td>
</tr>
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<td>C</td>
<td>B</td>
</tr>
<tr>
<td>14</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
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<td>C</td>
<td>C</td>
</tr>
<tr>
<td>16</td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
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<td>C</td>
<td>B</td>
<td>B</td>
</tr>
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<td>18</td>
<td>C</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>19</td>
<td>C</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>20</td>
<td>C</td>
<td>F</td>
<td>-</td>
</tr>
<tr>
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<td>C</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>22</td>
<td>C</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>23</td>
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<td>C</td>
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<td>F</td>
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<td>D</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>29</td>
<td>F</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>F</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
XII. APPENDIX E:
THE TAXONOMY OF EDUCATIONAL OBJECTIVES

COGNITIVE DOMAIN

1.00 Knowledge

1.10 Knowledge of Specifics
   1.11 Knowledge of Terminology
   1.12 Knowledge of Specific Facts

1.20 Knowledge of Ways and Means of Dealing with Specifics
   Organization and classification of information
   1.21 Knowledge of Conventions
   1.22 Knowledge of Trends and Sequences
   1.23 Knowledge of Classification and Categories
   1.24 Knowledge of Criteria
   1.25 Knowledge of Methodology

1.30 Knowledge of the Universals and Abstractions in a Field
   1.31 Knowledge of Principles and Generalizations
   1.32 Knowledge of Theories and Structures

2.00 Comprehension
2.10 Translation

2.20 Interpretation

2.30 Extrapolation

3.00 Application

The application of a theory or a principle to a specific situation

4.00 Analysis

The breakdown of an idea into its constituent parts

4.10 Analysis of Elements

4.20 Analysis of Relationships

4.30 Analysis of Organizational Principles

5.00 Synthesis

Putting together parts of elements
5.10 Production of a Unique Communication
5.20 Production of a Plan or Proposed Set of Operations
5.30 Derivation of a Set of Abstract Relations

6.00 Evaluation

Judgments about the value of materials and/or methods

6.10 Judgments in Terms of Internal Evidence
6.20 Judgments in Terms of External Criteria

AFFECTIVE DOMAIN

1.0 Receiving

This category is closely allied with the category of knowledge in the cognitive domain, and implies only awareness or tolerance of the stimulus produced by the situation that causes the response.

1.1 Awareness

This sub-category almost parallels knowledge in the
cognitive domain, but emphasis is on consciousness of a condition, rather than on pure recall.

1.2 Willingness to Receive

1.3 Controlled Attention

2.0 Responding

In addition to awareness, described in 1.0, responding involves a more active treatment of the stimulus.

2.1 Acquiescence in Responding

2.2 Willingness to Respond

2.3 Satisfaction in Response

3.0 Values

Here, we reach the realm of attitudes and values. The individual is now motivated from his sense of worth of the stimulus, rather than from obedience or from a sense of duty.

3.1 Acceptance of a Value
3.2 Preference for a Value

3.3 Commitment

4.0 Organization

This category deals with the individual's attempts to group related values.

4.1 Conceptualization of a Value

4.2 Organization of a Value System

5.0 Characterization by a Value or a Value Complex

The individual has developed an internally consistent system of values that he uses as the basis for action decisions. Evidence of this category may be found in the individual's philosophy and the consistency of his value system.

5.1 Generalized Set

5.2 Characterization
PSYCHOMOTOR DOMAIN

1.0 "Learned" motor activities, habits

2.0 Skills

3.0 Conscious motor activities, "learning" to do something