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Fatigue effects in the wear of polymers

Vinod Kumar Jain

Iowa State University

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Fatigue effects in the wear of polymers

by

Vinod Kumar Jain

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1980
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Scanning electron microscopy revealed that the worn surfaces of poly(methyl methacrylate) and high density polyethylene pins were covered with arced ripples stretching across the transverse direction. In the case of poly(vinyl chloride) there were signs of considerable plastic deformation on the worn surface. The fracture surfaces of notched fatigue samples of poly(methyl methacrylate) exhibited well-defined striation markings which were obscure, ill-defined and discontinuous in the case of high density polyethylene and poly(vinyl chloride). The final stage of fatigue fracture in the latter two materials was accompanied with plastic deformation. Topographical analysis of the sliding surfaces was performed using a data acquisition system. The arithmetic average and root-mean-square surface roughness, slope and radius of curvature of asperities, standard deviation and distribution
of profile ordinates and slopes, radii of curvatures and heights of asperities were computed using a FORTRAN IV program. It was found that the asperity heights and ordinate heights followed a Gaussian distribution whereas the asperity slopes remained unchanged during the steady state wear as was also the case with the average interface temperature.
NOMENCLATURE

\( A_0 \) = nominal area of contact, mm\(^2\)
\( A_1 \) = area of a discrete contact zone, mm\(^2\)
\( A_r \) = real area of contact, mm\(^2\)
\( a_l \) = radius of a discrete contact zone, mm
\( b \) = Weibull shape parameter; slope of regression line
\( C \) = wear particle thickness, mm
\( d \) = distance between the reference planes of contacting surfaces, mm
\( E \) = Young's modulus of elasticity, N/mm\(^2\)
\( F_n (h) \) = dimensionless integral
\( F(x) \) = cumulative density function
\( f \) = coefficient of sliding friction
\( f(x) \) = probability density function
\( H \) = Brinell hardness, N/mm\(^2\)
\( h \) = normalized distance between the reference planes of contacting surfaces
\( K \) = wear factor
\( k \) = proportionality constant
\( L \) = sliding distance, m
\( l_p \) = asperity pitch, mm
\( m \) = asperity slope
\( N \) = number of cycles-to-failure
\( N_R \) = asperity encounter rate
\( N_w \) = number of wear particles
\( n \) = number of ordinates
\( n_0 \) = number of discrete contact zones

\( P \) = normal load between the sliding surfaces, N

\( P_1 \) = normal load supported by a discrete contact zone, N

\( p \) = contact pressure, N/mm\(^2\)

\( p_0 \) = maximum pressure at the center of a discrete contact zone, N/mm\(^2\)

\( R \) = universal gas constant

\( R_a \) = arithmetic average of surface roughness, \( \mu m \)

\( R(\ell) \) = auto-correlation function, \( \mu m^2 \)

\( R_q \) = root-mean-square (r.m.s.) surface roughness, \( \mu m \)

\( S \) = engineering normal stress, N/mm\(^2\)

\( S_u \) = engineering ultimate strength, N/mm\(^2\)

\( S_o \) = failure stress for a single stress cycle, N/mm\(^2\)

\( s \) = normalized asperity height from the reference plane

\( t \) = power exponent of fatigue curve

\( U \) = activation energy of bond failure, N.m

\( V \) = wear volume, mm\(^3\)

\( V_R \) = wear rate, mm\(^3\)/s

\( v \) = sliding speed, m/s

\( v_P \) = average volume of a wear particle, mm\(^3\)

\( x \) = random variable

\( x_o \) = expected minimum value of the variable \( x \)

\( z \) = asperity height

\( \beta \) = radius of curvature of asperities, \( \mu m \)

\( \gamma \) = surface energy, N.m/m\(^2\)

\( \epsilon \) = elongation-to-break, mm
\( \lambda \) = sampling interval, \( \mu m \)

\( \eta \) = surface density of asperities

\( \eta_L \) = line density of asperities

\( \nu \) = Poisson's ratio

\( \theta \) = characteristic value of the random variable, \( x \)

\( \theta_1 \) = temperature, \( ^{\circ}C \)

\( \theta_2 \) = asperity slope, rad

\( \omega \) = compliance, mm

\( \sigma \) = standard deviation of asperity heights, \( \mu m \)

\( S_Y \) = engineering yield strength, \( N/mm^2 \)

\( \phi(s) \) = standardized probability density function of asperity heights

\( \phi(z) \) = probability density function of asperity heights
1. INTRODUCTION

1.1. Literature Review

1.1.1. Wear of polymers

The use of polymers in a wide variety of tribological applications, viz., seals, brakes, prosthetic joints, gears, tires, dry bearings, etc., has been continuously increasing during the past decade. The polymer wear studies are therefore important from both the scientific and technological viewpoints. Wear is a complex phenomenon, resulting from a combination of physical and physico-chemical processes that take place in the sliding surfaces and boundary layers.

There are four basic mechanisms commonly used to explain the wear of materials. These are the adhesion, abrasion, corrosion and surface fatigue. Of these the adhesive and abrasive mechanisms have most often been used to explain the wear of polymers. In an adhesive wear situation, adhesive bonding at asperity contact locations occurs between the two surfaces in sliding contact and the fragments from the weaker of the two materials are generally removed. The abrasive wear is produced as a result of the penetration and ploughing of the softer polymer by asperities on the harder counterface. The corrosive wear arises due to the thermal or thermo-oxidative degradation of polymers resulting in the formation of highly-reactive low-molecular weight compounds (1). The fatigue wear is usually associated with rolling, but the localized fatigue on an asperity scale is being increasingly recognized now as an important factor in sliding (2). The separation of adhesive and fatigue processes is almost impossible; there are
grounds to believe that the same wear processes previously categorized as adhesive may involve a large contribution from fatigue (2,3).

In addition to the above wear mechanisms, Suh (4) proposed the delamination theory for the wear of metals. It postulates that the wear is caused by sub-surface deformation followed by the nucleation and propagation of a crack. The theory appears to be a precursor to the fatigue theory of wear.

In real sliding situations, a considerable amount of interplay exists between the various processes which account for fracture in the substrate leading to the formation of wear debris. For example, adhesion influences the stress distribution around a localized contact contributing to fatigue. Similarly, thermal or thermo-oxidative degradation may change the mechanical properties of surface layers thereby affecting the process of wear by abrasion or fatigue. The lack of understanding of these basic mechanisms, along with their interactions in real sliding situations, have presumably been responsible for the diverse correlations of wear with different material properties and sliding parameters.

1.1.1.1. Adhesive wear Here the formation of wear particles occurs due to adhesive bonding and subsequent rupture of these bonds. Due to adhesion the transfer of material may also occur from one surface to the other. The theories of wear based on adhesion predict a direct proportionality between wear rate, load and sliding distance.

The most widely quoted adhesive wear model is that due to Archard (5). It considers that the real area of contact between sliding members is determined by the flow pressure or hardness of the softer material and that the wear particles of hemispherical shape are removed
as a consequence of the localized adhesion. On the basis of these considerations, Archard derived the following equation

\[ V = \frac{KPL}{H} \]  

(1)

where \( V \) is the wear volume, \( P \) the normal load, \( L \) the sliding distance, \( H \) the Brinell hardness of the softer material, and \( K \) a proportionality constant. The latter, in some obscure way, implies the probability of producing a wear particle per asperity encounter. The experimental values of \( K \) fall in the range of \( 10^{-3} \) to \( 10^{-8} \). Archard's equation, though originally proposed for metal pairs, has also been sparingly applied to polymer-metal pairs. The disadvantages of his equation are that the assumptions made in deriving it have no experimental basis and then there is no way to estimate the factor \( K \) except through a wear experiment. Furthermore, the equation involves "Brinell hardness" as a material property which is meaningless due to creep occurring in thermoplastic materials under ambient conditions.

Kar and Bahadur (6) developed the following equation for adhesive wear of polymers correlating the experimental wear data obtained from unfilled and PTFE-filled polyoxymethylene pins sliding against a steel disc using dimensionless parameters (π-terms)

\[ V = 1.5 \frac{k_1 \gamma^{1.775} p^{1.47} L^{1.25}}{E^{3.225}} \]  

(2)

where \( V \), \( \gamma \) and \( E \) are the wear volume, surface energy and elastic modulus of the polymeric material, respectively, \( p \) the nominal contact pressure, \( L \) the sliding distance, and \( k_1 \) the proportionality constant. The elastic
modulus and surface energy terms in the wear equation emphasize the dominating effect of adhesion in the wear process.

1.1.1.2. Abrasive wear The essential requirement for this type of wear is that the deformation around asperity contacts is of a plastic nature. The abrasive wear of polymers has been extensively studied by a number of workers (7-12). Ratner et al. (8) have shown that the abrasive wear rate is inversely proportional to the product of the nominal tensile fracture stress and the elongation-to-break. For thermoplastic polymers Giltrow (13) found the wear rates to be inversely proportional to the square root of their cohesive energy. Lontz and Kumnick (14) reported the wear rate of polytetrafluoroethylene being directly proportional to its flexure modulus and inversely proportional to its yield strain. Warren and Eiss (15) have shown that the wear rates of poly(vinyl chloride) and polychlorotrifluoroethylene are inversely proportional to their energy-to-rupture.

Lancaster (12) observed that the abrasive wear rate of an amorphous polymer such as poly(methyl methacrylate) was a minimum near glass transition temperature. For the crystalline polymers, viz., polyamides and polytetrafluoroethylene, he found that the change in wear rate with temperature was less marked than for the amorphous polymers until near the crystalline melting point. At such a high temperature, the mobility of polymer molecules increases so that the material softens, the strength decreases and the abrasive wear rate increases.

A number of expressions have been proposed for the abrasive wear of polymers. For example, Ratner et al. (8) suggested the following equation
where \( V \) is the wear volume, \( L \) the sliding distance, \( P \) the normal load, \( f \) the coefficient of sliding friction, \( H \) the indentation hardness, \( S_u \) the breaking strength, \( \varepsilon \) the elongation-to-break and \( k^2 \) is a proportionality constant.

Assuming the metal surface to be covered with cone shaped asperities, Rabinowicz (16) derived the following equation for the abrasive wear volume

\[
V = \frac{P \tan\theta_2}{H^2}
\]

(4)

where \( \theta_2 \) is the asperity slope and \( \bar{\tan\theta_2} \) the weighted average of the \( \tan\theta_2 \) values of all individual cones.

1.1.1.3. Fatigue wear The evidence of fatigue as a possible mechanism for the wear of polymers has been accumulating steadily in recent years, particularly from the Russian work (7,9,10,17-20). Kraghelsky and Nepomnyashchi (19) proposed five possible types of friction bonds in a contact situation and pointed out that the moderate wear which occurs in service is mostly caused by fatigue. The fact that extensive chemical (oxidation) degradation, mechano-chemical and other changes that are observed in the layers of sliding surfaces also occur in the dynamic fatigue of polymers (21) supports the fatigue nature of the wear mechanism.

Based on the concept of fatigue, Kraghelsky and Nepomnyashchi (18) derived a wear equation for rubbers. They considered one of the contacting surfaces to be smooth and another rough, and characterized
the latter by its bearing area curve. On the basis of fatigue and rolling experiments, they estimated that the tensile principal stress responsible for fatigue failure in the contact zone was 7-8 times the specific rolling friction force (real shear stress). Instead of considering the deformation and failure due to fatigue at individual asperity contacts, they assumed that the total volume of rubber deformed in a contact situation was removed from the surface after a certain number of cycles which depended upon the tensile stress, as estimated above, and the fatigue properties of the material. This is a gross simplification of the actual loading and unloading process of asperities in sliding. Furthermore, it inhibits the use of fracture mechanics approach to estimate the number of cycles needed for fracture of an individual asperity. Their equation is originally in terms of the real pressure which is changed to normal load assuming an ideal plastic flow in the contact zone. This is contrary to the initial assumption of elastic deformation as applicable to rubbers.

Ratner and Lur'e (9) have applied the concept of activation energy to the wear process and have suggested a relationship of the form

\[ \text{Wear rate} \propto e^{-\frac{(U-kP)}{R\theta_1}} \]

where \( U \) is the activation energy of the bond failure, \( P \) the load, \( R \) the universal gas constant, \( \theta_1 \) the temperature and \( k \) is a constant that depends upon the material properties. The consideration of fatigue as a thermally activated process leads to the idea that anything that prevents degradation under the influence of light, heat or oxidation should improve the fatigue life. In view of the above, the wear
resistance should also be increased. The study related to the effect of stabilizers on wear reinforces the above concept (10).

Halling (22) has also proposed a model for the wear of metals utilizing the concept of fatigue and simple plastic deformation failure (which is considered as fatigue failure in one loading cycle). His analysis assumes one of the surfaces to be smooth and another rough, ignores the interfacial shear stress, and considers the strain produced in an asperity being responsible for fatigue failure irrespective of the nature of strain. This is again an oversimplified view of the sliding contact situation and may be more realistically applicable to a rolling situation where the coefficient of friction is very small. The equation provides wear being proportional to sliding distance and load and inversely proportional to hardness. This result is analogous to that given by Archard's equation and has, therefore, the same limitations as mentioned earlier.

1.1.1.4. Other mechanisms of wear A number of hybrid models for the wear of metals have recently been proposed. For example, Suh and coworkers (4,23) derived two equations based on their delamination theory. One of these assumes that a strong junction is formed at a fraction of the asperity contacts and sliding causes the junction to be sheared, thereby producing a wear sheet. The latter is created solely as a result of the interaction of one set of asperities. The second equation considers the creation of a wear sheet as a cumulative process which results when the metal is sheared a small amount by each passing asperity. The creation of a wear sheet will occur, however, only after a
large number of asperities have passed over each point on the surface. The reduced form of these equations is similar to that of the Archard's equation.

Hornbogen (24) has proposed a model for the wear of metals on the basis of fracture toughness. According to this, if the strain induced in an asperity encounter is smaller than the critical strain for crack growth, the wear rate is independent of the toughness and Archard's equation is followed. If, on the other hand, the applied strain is larger than the critical strain, the probability of crack growth is increased and the wear rate is therefore higher. This implies that in the former case a subcritical crack growth can be expected. This type of crack growth is typical of fatigue, thermal fatigue, stress corrosion cracking and corrosion fatigue. A quantitative correlation based on this model remains to be done.

1.1.2. Surface topography and wear

It is now universally recognized that real surfaces are rough and are comprised of an aggregation of micro- and macro-asperities. The area of real contact between the surfaces in sliding contact is thus the summation of the areas of discrete contact. The contact locations as well as the areas at these contact spots are governed by the shape, size and distribution of asperities, elastic moduli of contacting materials and normal load under which the contact occurs. The wear rate would be influenced by the modifications in the surface topography of the counterface which may be produced by corrosion from the surrounding environment, polishing (or abrasion) by hard particles incorporated in the polymer or generated during the wear process, and by the development
of a film of transferred material (25). The changes occurring in surface
topography due to this last aspect have not hitherto been studied quan­
titatively and so deserve special consideration because of the universal
nature of the phenomenon.

1.1.2.1. Topographical analysis Several methods are available
for estimating the micro- and macro-features of surface geometry. These
include the optical methods which use electron, interference or reflec­
tion microscopy, and the mechanical methods such as oblique sectioning
and profilometry. The optical methods have the advantage of providing
a three-dimensional image of the surface, but the quantitative analysis
by these methods is very tedious. This difficulty is overcome by the
use of profilometers which provide a high resolution of the surface
irregularities in a plane normal to the surface over a representative
length. Since its introduction by Abbott and Firestone (26) in 1933,
the profilometry has developed into one of the most powerful tools in
surface analysis work.

The most commonly used profilometry method consists of obtaining
surface profiles by a stylus instrument. Here a pointed diamond stylus
is traversed over the surface and the resulting vertical movement of the
stylus over an appropriate datum on amplification provides a measure of
the roughness of the surface in terms of the voltage output. The sur­
face analysis from the profilometer record, unaided by the computer is
very time-consuming. This difficulty has been overcome, in practice, by
feeding the output of the stylus instrument through an analog-to-digital
converter into a sampling unit and then into a digital computer (27-29).
Thus, the surface is represented here in terms of a series of
regularly-spaced ordinate readings along the profile of the surface. This approach to surface analysis is relatively new and is considered superior to the other methods because of its quantitative character and the ease of handling.

1.1.2.2. **Surface parameters** The surface parameters are measured relative to a mean line which is defined by the American Standards Association (30) as a line parallel to the general trend of the profile such that the area of the profile above it is equal to the area below it. In order to quantify the surface profile by a single number, the arithmetic average of profile ordinates, \( R_a \), is obtained. It is referred in the British Standard 1134 (31) as the center-line-average (c.l.a.) and in the American Standard B46.1 (30) as the arithmetic average (A.A.). Many other measures of the profile are also recommended and used. For example, I.S.O. (32) recommends the ten point height as a useful measure. In Germany the commonly used parameter is the maximum peak-to-valley height within the sampling length. An alternative parameter is the root-mean-square (r.m.s.) of the surface roughness which is almost out of the practical usage but is considerably used in the theoretical work (28). Abbott and Firestone (26) suggested the use of the bearing length or bearing area curve, which represents the percentage of contact lying above a certain height from the base line. In addition to the above, there are in use the hybrid parameters, such as the average slope of asperities, the radius of curvature of peaks and the valleys, etc. It is, therefore, important to note that the choice of parameters for specifying the surface texture is controlled by the functional requirements of the surface, that is, by measurement of those features of the surface
which are significant in a practical situation.

With the advent of digital techniques in recent years, the deterministic descriptions of surface profiles have largely been replaced by the statistical descriptions where the surface profile is considered as a stationary random process. Here the r.m.s. surface roughness is the standard deviation of the ordinate height distribution and the bearing area curve is regarded as a plot of the cumulative height distribution. In their analysis of the surfaces of bead-blasted aluminum and mild steel rubbed against a copper flat in oleic acid, Greenwood and Williamson (33) found that the ordinate heights and the peak heights of these surfaces followed a Gaussian distribution. In the case of a ground stainless steel surface, Thomas and Probert (34) found that the ordinates, peaks and valleys of the profile had a Gaussian distribution but the radii of curvature and slopes of asperity peaks followed an exponential distribution. It has been shown through mathematical derivation by Peklenik (35) that the profile slope distribution is identical to the ordinate height distribution.

There is yet another approach used to specify a surface profile which considers surface roughness as a random process. Here the entire behavior is represented by two parameters, viz., the r.m.s. roughness and an auto-correlation function (35-37). The latter is defined as

$$R(\lambda) = \lim_{L \to \infty} \frac{1}{L} \int_0^L z(x) z(x+\lambda) \, dx$$

where $L$ is the length of the profile, $z(x)$ the height at a given coordinate $x$ along the mean line and $z(x+\lambda)$ the height an interval $\lambda$ apart from the previous point. The auto-correlation function thus provides a measure
of the dependence of a part of the profile on the other part. The general decay of the function indicates the decrease in correlation among the profile ordinates. Peklenik (35) has analyzed the surfaces produced by a number of machining processes and found that surfaces can be described by five different shapes of the auto-correlation function curve.

1.1.2.3. **Effect of surface topography on wear** Lancaster and coworkers (11,25) found that polymer wear rate decreases with increasing average radius of curvature of asperities and decreasing c.l.a. roughness of the metallic counterface. Rabinowicz (16) showed a direct proportionality between the wear rate and the average asperity slope.

Several workers have also studied the effect of running-in under lubricated conditions on the surface topography of metallic components. For example, working with grease-lubricated plain bearings, Rowe et al. (38) found that roughness of the softer metal surface decreases through a process of wear and plastic deformation. Thus, the initial surface finish of the bearing member is not as important as that of the hard journal because the latter plays a dominating role in the running-in process. Masouros et al. (39) showed that a high initial bearing surface roughness resulted in a larger initial wear of its surface and, finally, steady state wear law was established.

Endo and Kotani (40) studied the topographical changes occurring in steel surfaces sliding under lubricated conditions. They concluded that the surfaces were initially roughened due to adhesion but then attained a stable condition after sliding had continued for some time. The roughness pitch (which is the reciprocal of asperity density) and the asperity radius of curvature changed with varying load conditions.
Stout et al. (41) recently investigated the topographical changes occurring at the surface of a phosphor bronze pin rubbing against a case hardened steel disc under lubricated conditions. They found that the distribution of surface profile ordinate heights changed drastically during the running-in process. For the case of rubbing between a tool steel pin and a medium carbon steel cylinder under lubricated conditions, Ostvik and Christensen (42) found that during the running-in process the profile ordinate distribution of the cylinder surface remained unchanged but the gradient of the distribution curve changed.

The studies on surface topography changes occurring during sliding related to polymer-metal systems are comparatively scarce. In most of the cases, the transfer of a thin film of the polymer on the metallic surface is observed (43-48). Hollander and Lancaster (25) noted that the transfer of brittle polymers is in the form of irregular lumps, so that the arithmetic average roughness and the average radius of curvature of asperities on the metal surface are increased. In the case of ductile polymers, smoothening of the counterface occurs due to the transfer of thin polymer films.

The phenomenon of film transfer during sliding of polymers against metal and glass surfaces has been investigated by a number of workers. For example, a massive transfer of polytetrafluoroethylene on clean glass surfaces was reported by Makinson and Tabor (44). Pooley and Tabor (45) observed the lumps of polytetrafluoroethylene and high density polyethylene transferred to the glass and polished metal surfaces. Bowers et al. (46) noted the transfer of a thin film of polytetrafluoroethylene on steel surfaces and showed by electron diffraction that the
film was oriented in the direction of sliding. Tanaka and Uchiyama (47) inferred from electron diffraction the transfer of a thin film of low density polyethylene on a steel disc. Tanaka and Miyata (48) observed thin films of polytetrafluoroethylene and a few other crystalline polymers transferred on a clean glass plate during sliding.

1.1.3. Fatigue of polymers and its effect on wear

The mechanism of fatigue in polymers is very complex because polymers are highly sensitive to temperature and strain rate and suffer from hysteresis effects. In contrast to metals, polymers have a low thermal conductivity and high damping, both of which lead to a significant temperature rise in a polymeric specimen during cyclic loading. A continuous increase in temperature results in failure of the polymer fatigue sample due to thermal softening. However, under low frequency and small stress amplitude conditions, the temperature rises initially but later attains a steady value. In this case, a fatigue crack initiates and propagates in steps resulting finally in a fatigue fracture (49,50). Thus, the deformation or damage which a polymeric material suffers under cyclic loading is of two types (51). One of them includes continuing deformation resulting from imperfect recovery in successive cycles, creep and stress relaxation effects and rise in temperature owing to the dissipation of energy in cyclic deformation. Another type includes conventional fatigue due to the growth of a crack initiated in a region of high localized deformation and general weakening of the material due to structural damage and/or chemical degradation.

The fatigue behavior of polymers is further complicated by the evolution of heat in crack propagation due to crack tip straining (52).
The rate of heat generation depends on the rate of straining. The change in temperature affects the mechanical response of the material locally, which in turn influences the rate of heat generation. The resulting temperature rise is also governed by the conduction of heat away from the crack tip. The generation of heat in the crack tip region of glassy polymers under cyclic loading seems to play an important role (50,53,54). There has been an extensive softening observed in these polymers to the extent that failure occurs by excessive deformation. Thus, crack propagation becomes virtually impossible because the material flows rather than fractures.

The fatigue behavior of polymers is influenced by a number of factors which are the cyclic frequency, loading, wave form, stress intensity factor, morphology and molecular weight of the polymer, etc. The loading frequency effect on polymer fatigue (50,55) arises from two sources: (a) environmental effect of temperature rise and (b) strain rate effect (56). The heat generated which depends on the loading frequency and the stress amplitude decreases the fatigue resistance whereas the strain rate tends to increase it. Thus there are two counteracting effects where the deleterious effect of temperature rise predominates.

With the square wave form loading, the fatigue strength is lower than with the sinusoidal wave form (57). The adverse effect in the case of square wave form is due to the greater energy dissipation in each loading cycle that causes higher temperature rise in the polymer specimen.
The increase in crystallinity results in an improvement of the fatigue strength, as observed by Riddell et al. (55) for polytetrafluoroethylene and Bucknall et al. (57) for nylon 66. It is so because with increased crystallinity the damping is reduced (thereby causing smaller temperature rise) and the modulus is increased slightly.

An increase in molecular weight of the polymeric material may mean the reduction in chain ends which serve as defects in the crystalline order and may also act as sources of microcracks. The increase in intermolecular bonding due to increased molecular weight renders a more uniform distribution of stress in the material. A higher molecular weight allows for a greater degree of chain orientation in plastically deformed regions around the tips of microcracks. The molecular orientation would result in local strengthening of the material, thus facilitating the transmission of load despite the presence of a crack and would mitigate the stress concentration effect caused by the crack. All of these factors contribute to the improvement in fatigue strength with increased molecular weight, as observed by Foden et al. (58) and Sauer et al. (59) for polystyrene and polyethylene, and by Kim et al. (60) for poly(methyl methacrylate).

The effect of stress raisers and notches on the fatigue of polymers has been investigated by several workers (61-63). Crawford and Benham (61) investigated the effect of a sharp notch on the fatigue behavior of poly(methyl methacrylate), polycarbonate, polypropylene, etc., under uniaxial loading and at several frequencies. They found that the notch decreased the fatigue strength of these polymers. At the same time a reduction in the heat generated was also observed. Thus, in the presence
of a notch, a conventional fatigue failure was observed in polypropylene under the stresses which would have otherwise produced thermal softening. Furthermore, the fatigue behavior of notched samples was found to be unaffected by the variations in test frequency. Hutchinson and Benham (63), who tested poly(vinyl chloride) sheets with holes under constant strain fatigue loading, also found a reduction in fatigue strength due to the stress concentration from holes. The decrease in strength was more severe at lower stress values.

It may thus be noted that a bimodal failure mechanism exists for thermoplastic polymers under cyclic loading. Cracks propagate incrementally from existing defects and stress raisers leading to conventional fatigue failure, provided this is not preceded by thermal softening at some section. The thermal softening has been shown to be related to damping, frequency, crystallinity, and amplitude of cyclic stress.
2. EXPERIMENTAL PROCEDURE

2.1. Test Equipment

2.1.1. Wear test set-up

The sliding experiments were performed in a pin-on-disk type of wear machine, as shown in Fig. 2.1. The machine was capable of providing wear data, average interface temperature and friction force corresponding to different sliding speeds, loads and times. Here the disk was made of AISI 4340 steel, hardened (55 Rc) and ground, and a cylindrical polymer pin was secured to a vertical arm. The disk was keyed to a motor shaft whose speed could be varied continuously from 50 to 2500 rpm. A magnetic pick-up and a digital display device were used for monitoring the revolutions of the disk. The load on the polymer specimen was applied by dead weights supported at the end of a hinged horizontal beam. The cylindrical vertical arm, carrying the polymer specimen, was secured rigidly to the beam and had the strain gages mounted on it to provide a measure of the friction force between the rotating disk and the stationary polymer pin. The dead weight of the horizontal beam assembly was counterbalanced by a balancing weight whose position could be adjusted on the rear end of the beam. In order to measure the temperature rise at the sliding interface, two iron-constantan thermocouple junctions, made from 0.075 mm diameter wires, were embedded at two diametrically opposite locations in the steel disk at a depth of about 0.5 mm below the rubbing surface. The thermocouple leads were connected to a Leeds and Northrup temperature recorder through a slip ring assembly. A linear differential transformer (LVDT) was used
Fig. 2.1. Schematic diagram of a pin-on-disk wear machine.
to monitor continuously the reduction in length of the polymer pin, and the latter was related to the wear volume through a calibration process. The outputs from the LVDT and the strain gages were fed into a Fishers OmniscrIBE Recorder to obtain a measure of the wear volume and the friction force.

2.1.2. System for surface analysis

A data acquisition system for surface analysis work was developed. It consists of a microcomputer (KIM-1), two random access memory (RAM) boards of 4K (KIM-2) and 8K (KIM-3) bytes each, a mother board (KIM-4), and an analog-to-digital convertor, as shown in Fig. 2.2. The system is connected to a teletype for recording the data on a tape. The analog signal obtained from the profilometer, as the diamond stylus traverses over the specimen surface, can be discretized at any time interval in the 80 to 266 μsec range, converted to the digital signal, and stored in the system memory. It can store up to a maximum of 12,000 data points. The latter are printed and punched on a paper tape by the use of a teletype. An assembly language program which is included in Appendix A was written to program the microprocessor. The paper tape is processed on an ITEL AS/6 computer so as to punch the data on cards which are later used in the computation of various surface parameters.

2.2. Material Selection

Three polymeric materials, viz., high density polyethylene, poly-vinyl chloride) and poly(methyl methacrylate), were selected for the present work. The selection was based on the diversity in structural, mechanical, and wear characteristics of the materials so that the
Fig. 2.2. Line diagram of data acquisition system for surface analysis.
correspondence between fatigue and wear could be investigated in a
general way. Of the three polymers selected, high density polyethylene
is highly crystalline (% crystallinity ~98) and is in the rubbery state
under ambient conditions (glass transition temperature -85°C). The other
two polymers are amorphous and glassy (glass transition temperature 100°C
for PMMA and 70°C for PVC). As for the fatigue properties, poly(methyl
methacrylate) is highly susceptible to thermal softening in comparison
to the other two polymers as reported by Constable et al. (50). The
wear rate of high density polyethylene is about two orders of magnitude
smaller than that of the other two polymers and so is one of the most
commonly used materials in tribological applications (2,3).

2.3. Test Procedures

2.3.1. Specimen preparation for sliding experiments

The polymer pins, 6.3 mm diameter 25.0 mm long, were machined from
9-12 mm thick sheets that were bought from the Cadillac Plastics and
Chemical Co. The end of the cylindrical pin was finished by polishing
with 600 grade emery paper under running water. The specimen was washed
in distilled water and methanol, dried and stored overnight in a desic­
cator before testing. The disk was machined from a 12 cm diameter rod
of AISI 4340 HR steel and was then austenitized and oil quenched to
55 Rc. It was ground while mounted on the motor shaft so as to make
sure that the disk was concentric with the shaft. It helped to reduce
vibrations of the horizontal beam in the wear test set-up and obtain
reliable wear data using the linear differential transformer.
2.3.2. Wear tests

The wear tests for poly(methyl methacrylate) and poly(vinyl chloride) were performed at the sliding speeds of 0.5, 1, 2.0, 2.5, 3, and 3.5 m/s and at a normal load of 8.83 N. Sliding was performed for a duration of 10 hours and the wear volume, friction force and substrate temperature were recorded continuously. Each test was repeated at least twice to ensure reliability in the measured quantities.

In the case of high density polyethylene, the wear tests were performed at a speed of 1.75 m/s and a load of 47 N only. Whereas the wear data for poly(methyl methacrylate) and poly(vinyl chloride) were obtained using LVDT, the wear for high density polyethylene was determined by weighing polymer pins at the end of each test. The latter was found necessary because the wear of high density polyethylene was lower by about two orders of magnitude as compared to the other two materials. As such the decrease in polyethylene pin length was counteracted by the expansion of steel disk due to interface temperature rise, thus nullifying the LVDT core movement. After weighing each time for wear, a fresh polymer specimen was used duplicating the earlier part of sliding and continuing it for an extended time, because the interruption in sliding for taking weight changed thermal conditions at the rubbing surface. Since this made the test very time-consuming, the wear data on high density polyethylene with varying time were obtained for one sliding speed only.

2.3.3. Surface topography evaluation

The topographical changes produced in the metal disk and the polymer pin rubbing surfaces were investigated by evaluating the surface profiles at the end of sliding for 1, 2, 3, 4, 6, 8 and 10 hours. After each
interruption in sliding for surface analysis, a new polymer sample was used and the metal disk surface was thoroughly cleaned. It was considered necessary because the change in temperature due to stoppage of the test could provide a false picture of the surface topography. In every wear test, the final lay of the abraded pin surface was kept perpendicular to the sliding direction. The profiles of the polymer pin and the metal disk sliding surfaces were analyzed, using a Bendix profilometer and a diamond stylus with a tip radius of 2.5 μm, both along and perpendicular to the direction of sliding. The surface waviness was filtered using a 0.75 mm cut-off filter.

The profile ordinate data for the case of poly(methyl methacrylate) sliding against the metal disk were extracted manually from the profile records. Here the surface profiles were recorded using a Brush Mark 220 recorder. The chart speed was set at 125 mm/s while the pilotor was traversing the surface at a speed of 7.62 mm/s. The profile records for the case of metal disk were obtained from 6.3 mm and 4.75 mm lengths along and perpendicular to the sliding direction, respectively, and for the polymer pin from 3.15 mm length corresponding to both the directions. The surface records were enlarged photographically by 3X and the profile ordinates were read at 0.5 mm intervals which corresponded to 10 μm spacing on the surface.

The surface analysis in the case of other two materials was made using the data acquisition system. Here, the data for both the disk and the pin were obtained at a much shorter interval of 2 μm from a length of 2 mm in each of the two directions.

2.3.4. Fatigue tests

The fatigue tests were performed using a Krouse rotating cantilever
beam fatigue testing machine. Due to the hysteresis loss in polymers and the resulting temperature rise, the fatigue strength of an unnotched polymer sample depends on the test frequency. On the other hand, as described earlier in Section 1.1.3., the fatigue strength of notched polymer specimens has been found to be independent of the frequency of loading (61,62). Thus, in order to obtain the conventional fatigue data, notched fatigue specimens (Fig. 2.3) were used. These were machined from 12.5 - 45.0 mm diameter extruded rods. All of the fatigue tests were performed at 1800 cycles per minute as per ANSI/ASTM D671-71 recommendations (64). Optical microscopy of fatigue fracture surfaces indicated no sign of melting in these tests.

2.4. Surface Examination

The unrubbed and rubbed metal disk surfaces were examined in a scanning electron microscope to facilitate the explanation of the changes occurring with sliding in the disk surface topography. It was done only for the case of poly(methyl methacrylate) sliding against the metal disk. Since the regular metal disk could not be admitted in a scanning electron microscope chamber due to its large size, sliding experiments were run for 1, 2 and 8 hours on identically machined and ground metal disks. Small portions were then cut from the rubbed disks for microscopic examination.

The worn surfaces of polymer pins, as well as the fractured surfaces of fatigue specimens, were examined by optical and scanning electron microscopy to study the resemblance in fracture features, if any. Transmission electron microscopy of the worn surfaces of polymer pins
ALL DIMENSIONS IN mm.

Fig. 2.3. Geometry of a notched fatigue specimen.
was also carried out for better insight.

The metal disk and polymer specimen surfaces were prepared for scanning electron microscopic examination by sputtering with Au under vacuum to reduce charging. An accelerating voltage of 5 to 10 KV was used to minimize the radiation damage. Stereophoto-micrographs were obtained in some cases to provide a three dimensional image of the surfaces. A conventional single-stage replication technique using polyacrylic acid was employed for transmission electron microscopy work.
3. ANALYTICAL METHODS

3.1. Development of the Fatigue Wear Equation

In a sliding process, the contact occurs between the asperities on the mating surfaces, as shown in Fig. 3.1. Consider, at some instant, the asperities 2, 3 and 5 on the upper surface loaded against the asperities 7, 8 and 10 on the lower surface, respectively. With sliding, the contact positions will change, causing some of these asperities to interact with other asperities on the opposite surface. The asperities on the two surfaces are being continuously loaded and unloaded. The loading of an asperity in an encounter is of a complex nature and involves a normal load as well as a tangential force due to friction. The stresses in a sliding circular contact zone produced by the contact of a spherical indentor with a plane surface have been analyzed by Hamilton and Goodman (65). The analysis shows that two of the principal stresses induced in the contact zone are largely compressive in nature. As such, these would not be expected to contribute to the initiation of a fatigue crack. The third principal stress, in the direction of sliding, whose variation in the contact region is shown in Fig. 3.2, is tensile in nature ahead of the contact zone and compressive behind it. Therefore, the region around an asperity in a loading interaction will be subjected to tensile as well as compressive stresses. The stress reversal just described constitutes fatigue. It should be noted that the term fatigue is being used here to imply a mode of fracture where nonapparent damage is accumulated over a large number of loading and unloading cycles, resulting finally in the apparent damage, i.e., the
Fig. 3.1. Schematic representation of sliding contact between two rough surfaces.
Fig. 3.2. Contact between a moving hemispherical indentor and a plane surface:
(i) Schematic view of circular contact.
(ii) Variation of stress $S$ (in sliding direction) around the center of contact for varying values of coefficient of friction, $f$. Here $(3P_1/2a_1^2)$ is the maximum pressure at the center of contact under a static load $P_1$ (65).
formation and separation of a wear particle through the initiation and propagation of a fatigue crack. It is this third principal stress that will later be used for estimating the life of the contact zone under repetitive loading.

In order to derive a wear equation on the basis of the concept of repetitive loading of asperities, the following assumptions are made:

1. The asperity heights on both the surfaces in contact vary randomly.
2. The asperities have spherical tips.
3. The asperities on one surface are aligned with those on the other surface and have the same pitch in the sliding direction. This assumption is based on the experimental results of the present investigation.
4. The deformation in contact zones is of an elastic nature. This is in agreement with the experimental studies reported in the literature for polymer-metal contacts (66, 67).
5. The discrete contact zones are well-separated to act independently of each other.

3.2. Contact Parameters

In order to determine the real area of contact and the number of discrete contact zones, the analysis by Greenwood and Williamson (33) for a rough surface loaded against a smooth plane is used here. The expressions are later modified to account for roughness of both the surfaces.
For elastic deformation in the contact of a sphere with a smooth plane, the Hertzian equations (68) provide the following expressions for the contact radius $a_1$, area $A_1$, and load $P_1$

$$a_1 = \frac{1}{2^2} \frac{1}{\omega^2}$$  \hspace{1cm} (5)  

$$A_1 = \pi \beta \omega$$  \hspace{1cm} (6)  

$$P_1 = \frac{4}{3} \frac{1}{E'} \frac{1}{\beta^2} \frac{3}{\omega^2}$$  \hspace{1cm} (7)  

where $\frac{1}{E'} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$, $\nu$ is the Poisson's ratio, $\omega$ is the compliance (distance by which the points outside the contact zone move closer due to deformation), $\beta$ the radius of sphere (that is the tip of the spherical asperity) and $E$ the modulus of elasticity. The subscripts 1 and 2 refer to the two surfaces in contact.

Using the above relationships for a single contact and the assumptions stated above, Greenwood and Williamson (33) derived the following expressions for the number of discrete contact zones $n_o$, the real area of contact $A_r$, and the load $P$, for the case of contact between a rough and a smooth surface, as shown in Fig. 3.3.

$$n_o = \eta \int_0^\infty A_0 \phi(z) \, dz$$  \hspace{1cm} (8)  

$$A_r = \pi \eta A_0 \beta \int_0^\infty (z-d) \phi(z) \, dz$$  \hspace{1cm} (9)
Fig. 3.3. Contact between a smooth and a rough surface. The load is supported by the asperities (shaded) whose heights are greater than the separation between the reference planes (33).
\[ P = \frac{4}{3} \eta A_o E' \beta^2 \int_0^\infty (z-d)^2 \phi(z) dz \]  

where \( \eta \) is the surface density of asperities, \( A_o \) the nominal area of contact, \( z \) the asperity height, \( \phi(z) \) the distribution of asperity heights, and \( d \) the distance between reference planes of the surfaces in contact.

Introducing the normalized variables \( h \) and \( s \) where \( h = d/\sigma \) and \( s = z/\sigma \), \( \sigma \) being the standard deviation of asperity height distribution, the above equations are reduced to

\[ n_o = \eta A_o F_o(h) \]  

\[ A_r = \pi \eta A_o \beta \sigma F_1(h) \]  

\[ P = \frac{4}{3} \eta A_o E' \beta^2 \sigma^2 F_3(h) \]

where

\[ F_n(h) = \int_h^\infty (s-h)^n \phi(s) ds \]

and \( \phi(s) \) is the standardized height distribution.

Greenwood and Tripp (69) have shown that the two contacting rough surfaces can be replaced by a rough surface in contact with a smooth plane where the asperity radius of curvature \( \beta \), and standard deviation of asperity height distribution \( \sigma \), of the equivalent rough surface will be given by
\[ \beta = \frac{\beta_1 \beta_2}{\beta_1 + \beta_2} \] (14)

and

\[ \sigma = \frac{1}{(\sigma_1^2 + \sigma_2^2)^2} \] (15)

Here \( \beta_1 \) and \( \beta_2 \) are the average radii of curvatures of asperity tips on the two surfaces and \( \sigma_1 \) and \( \sigma_2 \) the standard deviations of asperity height distributions. Thus, equations (11) to (13) provide the contact parameters for two rough surfaces in contact when \( \beta \) and \( \sigma \) are calculated from equations (14) and (15).

3.3. Stresses in a Contact Zone

The expression, as derived by Hamilton and Goodman (65), for the principal stress responsible for fatigue failure is

\[ S = -\left( \frac{3P_r}{2\pi a_1} \right) \frac{x\pi (4+\nu)}{8} + \left( \frac{3P_r}{2\pi a_1} \right) \left[ -2\nu (a_1^2 - r^2) \frac{1}{2} \\
+ (1-2\nu) \left[ \frac{1}{3} (a_1^2 - r^2)^{\frac{3}{2}} r^{-2} - \frac{2}{3} x^2 r^{-4} (a_1^2 - r^2)^{\frac{3}{2}} \\
- x^2 r^{-2} (a_1^2 - r^2)^{\frac{1}{2}} - \frac{1}{3} a_1^3 r^{-2} + \frac{2}{3} x^2 r^{-4} a_1^3 \right] \right] \] (16)

where \( r = (x^2 + y^2)^{\frac{1}{2}} \), \( x \) and \( y \) being the distances from the center of contact in the contact plane along \( x \) and \( y \) directions, and \( P \) is the coefficient of friction.
As seen from Fig. 3.2, the tensile stress has a maximum value at $x = -a_1$ and $r = a_1$, and so its magnitude is given by

$$S = \left[ \frac{3P_1}{2\pi a_1^2} \right] \left[ \frac{f}{8} (4+\nu) \pi + \left( \frac{1-2\nu}{3} \right) \right]$$

$$= P_o k, \text{ say} \tag{17}$$

where

$$P_o = \frac{3P_1}{2\pi a_1^2} \tag{18}$$

$$k = \left[ \frac{f}{8} (4+\nu) \pi + \left( \frac{1-2\nu}{3} \right) \right] \tag{19}$$

It is seen from the above that the computation of the tensile stress $S$ requires the determination of the load $P_1$ and the radius of the contact zone $a_1$ for discrete contacts. These may be determined using the relationships for $n_o$ and $A_x$ as given by equations (11) and (12).

Greenwood (70) has shown that the average size of discrete microcontacts remains fairly constant for an elastic contact situation. Therefore, the load $P_1$ and the contact radius $a_1$ may be expressed as

$$P_1 = \frac{P}{n_o}$$

$$= \frac{P}{\eta A_o F_o (h)} \tag{20}$$

$$a_1 = \left[ \frac{A_x}{\pi n_o} \right]^{\frac{1}{2}}$$
Substituting for $P_1$ and $a_1$ from the above in equation (17), the maximum value of the tensile stress $S$ is

$$S = \frac{3kP}{2\pi \eta A_0 \beta F_1(h)}$$

where $k$ is given by equation (19).

3.4. Criterion for Asperity Fracture

In order to determine the number of stress reversals required to cause failure in an asperity, the fatigue failure criterion is used. According to this, the fracture of an asperity under loading in sliding contact occurs when it is subjected to the same number of cycles as would be required for fracture of a fatigue specimen of the material in reversed bending.

The fatigue properties of a polymeric material can be represented by Wöhler's curve (71), the equation for which is

$$N = \left[ \frac{S_o}{S} \right]^t$$

where $N$ is the number of cycles-to-failure, $S_o$ the failure stress corresponding to the application of a single stress cycle, $S$ the applied cyclic stress and $t$ a material constant. The parameters $S_o$ and $t$ for a material can be determined from a plot of its fatigue data.
Combining equations (22) and (23), the number of loading cycles \( N \) needed for fracture of an asperity so as to produce a wear particle is given by

\[
N = \left( \frac{2\pi \sigma_o \beta \delta F_1(h)}{3kP} \right)^t
\]

(24)

It should be noted that this equation is in terms of the sliding surface parameters, material fatigue parameters, normal load, coefficient of friction and Poisson's ratio.

3.5. Wear Equation

Consider sliding motion between two rough surfaces as shown in Fig. 3.1. At any instant, the number of discrete contact points \( n_o \) in a contact region of nominal area \( A_o \) is given by equation (11) to be equal to \( n_o A_o F_o(h) \).

As the sliding occurs, these contacts cease and the new ones occur, and this sequence of events continues. If \( \eta_L \) be the line density of the asperities on the moving surface, then \( n_o \) asperity interactions will occur in sliding over a distance of \( 1/\eta_L \). Thus, the number of asperity interactions in a sliding distance \( L \) will be equal to \( \eta_L n_o A_o F_o(h) \) and so the asperity encounter rate \( N_R \) for a sliding speed \( v \) will be given by

\[
N_R = \eta_L n_o A_o v F_o(h)
\]

(25)

According to the fatigue theory of wear, a wear particle is produced after \( N \) asperity interactions. Therefore, the number of
wear particles formed per unit time, \( N_w \), will be given from equations (24) and (25) as

\[
N_w = \frac{N^R}{N} = \frac{\eta_v F(h) (kp)^t (\eta A_o)^{1-t}}{\left( \frac{2\pi}{3} S_o \beta \sigma F_1(h) \right)^t} \quad (26)
\]

If \( v^P \) be the average volume of a wear particle, the volume wear rate \( V_R \) then becomes

\[
V_R = \frac{\eta_v L P F(h) v (kp)^t (A_o \eta)^{1-t}}{\left( \frac{2\pi}{3} S_o \beta \sigma F_1(h) \right)^t} \quad (27)
\]

and the wear volume \( V \) for a sliding distance \( L \) is

\[
V = \frac{\eta_v L P L F(h) (kp)^t (A_o \eta)^{1-t}}{\left( \frac{2\pi}{3} S_o \beta \sigma F_1(h) \right)^t} \quad (28)
\]

Substituting for \( P \) from equation (13), we get

\[
V = \frac{K_1 P L \eta_v L P}{2S_o} \left[ \frac{f}{8} (4+\nu) \pi + \left( \frac{1-2\nu}{3} \right) \right] \quad (29)
\]

where

\[
K_1 = \left( \frac{2kE'}{\pi S_o} \right)^{t-1} \left( \frac{1}{2} \right) \left( \frac{\sigma}{2} \right)^{t-3} \left[ \frac{F_3(h)}{F_1(h)} \right]^t \left[ \frac{F_o(h)}{F_1(h)} \right] \quad (30)
\]
The above equation indicates that the wear depends upon the fatigue property and modulus of elasticity of the polymeric material, topography of sliding surfaces, normal load, coefficient of friction, and sliding distance.

3.6. Computation of Surface Parameters

The prediction of wear from the equation derived above on the concept of fatigue involves the surface parameters, viz., the asperity density, the asperity radius of curvature and the distribution and the standard deviation of asperity heights. As such, for the prediction of wear from this equation as well as for developing an understanding of the surface topographical changes in sliding, a surface topography analysis was performed. Here surface profile data from two perpendicular directions were obtained where one of the directions coincided with the sliding direction. The data so obtained were assumed to be representative of the surface roughness in the respective directions. Of the many attributes of the surface geometry, namely, the roughness, waviness and the general errors of form, the roughness alone was analyzed. It provides an idea of the primary texture of the surface due to irregularities resulting from the inherent action of production processes, and is normally used for surface analysis. The waviness and the general errors of form, produced due to machine tool vibrations, were filtered out by using a low pass filter with a cutoff of 0.75 mm.

The quantitative analysis of a surface profile was performed computing a number of parameters pertinent to the wear process,
namely, the r.m.s. and c.l.a. roughness, asperity density, the mean, standard deviation, and distribution of asperity heights, asperity radii of curvatures, asperity slopes and ordinate heights. The profile ordinate heights were treated as positive or negative with respect to a datum which was generated by the vertical movement of the stylus relative to the skids sliding over the test surface. The data were later interpreted with respect to an arbitrary datum so that all the ordinates could be treated as positive numbers. A mean line was fitted to the data by the least squares method (Fig. 3.4) and the surface ordinates \( z'_i \) with respect to this line were obtained. The center-line-average \( R_a \) and the r.m.s. roughness \( R_q \) were calculated as follows

\[
R_a = \frac{1}{n} \sum_{i=1}^{n} |z'_i| \tag{31}
\]

\[
R_q = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (z'_i)^2} \tag{32}
\]

where \( n \) is the number of ordinates considered for profile analysis.

A three-point analysis was used to define a peak (28, 29, 36). According to this criterion, if the central ordinate of three contiguous ordinates is higher than the other two, then this constitutes a peak. Thus, a peak at the \( i^{th} \) ordinate exists if \( z_{i-1} < z_i > z_{i+1} \).

The absolute slope at a point, \( m_i \), was found from a three-point central-difference formula as below

\[
m_i = \left| \frac{z_{i+1} - z_{i-1}}{2\lambda} \right| \tag{33}
\]
Fig. 3.4. Two-dimensional representation of a surface profile.
where $\lambda$ is the sampling interval.

The radius of curvature, $\beta$, of a peak was computed from the formula

$$\beta = \left[ 1 + \left( \frac{dz}{dx} \right)^2 \right]^{3/2} \frac{dz}{dx^2}$$

(34)

where the second derivative $\frac{dz}{dx^2}$ is given by the central-difference formula as

$$\left( \frac{dz}{dx^2} \right)_i = \frac{2z_i - z_{i-1} - z_{i+1}}{\lambda^2}$$

(35)

In order to determine the distributions of ordinate heights, peak heights, asperity slopes and peak radii of curvature, a Weibull distribution was fitted. This distribution was originally proposed for the interpretation of fatigue data (72,73). Mischke (74) has used it in studying the other mechanical properties of metals. For a Weibull distribution, the probability density function is given by

$$f(x) = \frac{b}{\theta} \left( \frac{x-x_0}{\theta} \right)^{b-1} \exp \left[ - \left( \frac{x-x_0}{\theta} \right)^b \right]$$

(36)

and the cumulative density function by

$$F(x) = 1 - \exp \left[ - \left( \frac{x-x_0}{\theta} \right)^b \right]$$

(37)

where $x$ is the random variable, $f(x)$ the probability density function, $x_0$ the least possible value of $x$, $b$ the Weibull shape parameter, and $\theta$ the Weibull characteristic value of $x$. 
The shape parameter $b$ has a pronounced effect on the probability density, as shown in Figure 3.5. For $b = 1$, the Weibull distribution becomes an exponential distribution, and for $b = 3.3085$, it approximates the Gaussian distribution (75). The distribution curve is positively skewed for $b < 3.3085$ and negatively skewed for $b > 3.3085$. Thus the shape parameter $b$ gives sufficient information about the distribution of the variable $x$.

The parameter $b$ was determined from equation (37) which may be simplified as

$$\ln \ln \left[ \frac{1}{1-F(x_i)} \right] = b \ln (x_i - x_0) - b \ln \theta$$

where $x_i$ designates $x_1, x_2, x_3$, etc. when the values of $x$ are arranged in an ascending order. An unbiased estimator of $F(x)$ is given by the following equation (76)

$$F(x_i) = \frac{i - 0.3}{n + 0.4}$$

or

$$1 - F(x_i) = \frac{n - i + 0.7}{n + 0.4}$$

where $x_0$ is the minimum value of $x$ and has to be determined by trial and error. The latter involves assigning different values, lying between zero and the lowest value of $x_i$, to $x_0$. A straight line is then fitted through the data points corresponding to $\ln \ln \left[ \frac{1}{1-F(x_i)} \right]$ and $\ln (x_i - x_0)$ and the correlation coefficient determined. This is repeated for each value of $x_0$. The final value of $x_0$ selected is the one that gives the best correlation coefficient. The shape parameter $b$ of interest is the one corresponding to this value of $x_0$. 
Fig. 3.5. Effect of Weibull shape parameter $b$ on the variation of probability density function (75).
The distributions of ordinate-heights, asperity slopes, peak heights and radii of curvature of the asperities were determined by calculating the shape parameter for each case using the above procedure. A FORTRAN IV program was written for the purpose and is given in Appendix B.
4. RESULTS AND DISCUSSIONS

4.1. Mechanism of Fracture in Wear

4.1.1. Markings on fracture surfaces

The markings on a fracture surface are characteristic of the mechanism that operates in giving rise to a particular mode of fracture. These features depend upon the type of loading, shape and size of the component, deformation characteristics of the materials, etc. In complicated situations where the interplay of a number of factors is involved, it is only the fracture pattern that provides an indication of the predominant mechanism responsible for fracture. In practice, a number of typical markings, as described below, are observed on fracture surfaces.

The chevron or herringbone patterns (Fig. 4.1) are observed on rectangular specimens with large width-to-thickness ratio and subjected to static loads. These markings are associated with an unstable and a relatively rapid crack propagation. Their appearance is the result of the general direction of crack propagation and the inherent tendency of a propagating crack to take a direction that gives the shortest path to a free surface. Normally, the fracture starts inside and terminates at the surface (Fig. 4.1a). In some cases, the fractures initiate on the opposite free surfaces due to the presence of scratches on them and terminate in the middle. Here the markings are known as reversed chevron markings (77) (Fig. 4.1b).

The beach, clamshell, concoidal or arrest marks (Fig. 4.2) are normally produced in case of fractures occurring in service but are
Fig. 4.1. Chevron (also called "herringbone") markings on the fracture surface of a cellulose acetate plate: (a) regular (b) reversed. The crack propagated from left to right (77).
Fig. 4.2. Optical micrograph of the fracture surface of an AISI 1050 steel shaft showing beach and ratchet marks (78).

Fig. 4.3. Optical micrograph of 7076-T6 aluminum alloy showing fatigue striations (78).
scarce on laboratory test specimens due to short loading times and uniform loading conditions. These are associated with stable crack propagation. Here the individual marks represent the successive positions of the crack front arrest. Although these marks are usually associated with fatigue crack propagation, their absence does not necessarily mean that the fracture was not by fatigue (78).

Striations (Fig. 4.3) are produced by successive propagation of the fatigue crack and are found to exist within the fine structure of individual beach marks. Their presence is a definite evidence of fatigue-crack propagation but the reverse is not true. Since a striation is produced in every loading cycle, counting of striations has been used to estimate the crack propagation part of the fatigue life (78).

Ratchet marks (Fig. 4.2) are macroscopic features that are observed on fatigue fracture surfaces of shafts. These are the result of multiple fatigue crack origins, each producing a separate fatigue crack zone. The separately initiated cracks in a shaft are normally propagated on planes slightly inclined to the plane of shaft diameter. As two approaching cracks meet, a small step called the ratchet mark is formed. The occurrence of ratchet marks requires virtually simultaneous initiation of multiple fatigue cracks which is favored by high stress conditions.

A dimpled appearance (Fig. 4.4) arises from a non-repetitive loading producing fracture or in cases where the fracture is due to tearing. Here plastic flow initiates microvoids at inclusions or discontinuity sites. As the plastic strain increases, the existing microvoids grow, new ones are initiated and the enlarged microvoids grow
Fig. 4.4. Scanning electron micrograph showing dimples on the fracture surface of AISI 1020 steel failed in uniaxial tension (78).

Fig. 4.5. Parabolic markings on the fracture surface of an acrylic (Trade name "Lucite") plate. The fracture progressed from left to right (77).
into close enough proximity so that the thin ridges, or membranes separating them, rupture and fracture occurs. The resulting fracture surfaces have numerous cup-like depressions or "dimples."

Parabolic markings (Fig. 4.5) are caused by interaction between the primary and secondary fracture fronts where the planes of the two fronts are separated by a small distance. A discontinuity, which is normally the origin of the fracture, lies at the focus of the parabola and is generally microscopic in size. These markings are commonly observed on poly(methyl methacrylate) surfaces and their presence does not necessarily mean fatigue fracture.

4.1.2. Fracture markings on a plane surface loaded by a sliding hemispherical indentor

The sliding contact between a hemispherical indentor and a plane surface represents hypothetically the contact between an asperity with a round tip and a smooth surface. In real situations, the load between the sliding surfaces is supported by a large number of such contacts. The hypothetical case of a single contact is discussed here to evolve an understanding of the type of fracture markings that are likely to develop in sliding situations.

For a hemispherical indentor sliding on a plane surface, the stress ahead of the contact zone is compressive and is tensile behind it (65,79). The locus of the tensile stress is a part of the rear half of the contact circle and so the fracture is likely to result from arcuate cracks (Fig. 4.6). Since the axis of the maximum tensile stress below the plane surface is inclined to the horizontal axis, the form of subsurface cracking would be hyperbolic. The above hypothesis has been confirmed by Preston (80) who found a series of arcuate
Fig. 4.5. Development of surface cracks on a plane surface due to the sliding of a hemispherical indentor on it.
flaws transverse to the direction of sliding in his sliding experiment between a steel ball and a glass surface (Fig. 4.7). In spite of the initiation and propagation of flaws, no detachment of material from the glass surface was observed. The latter was probably for the reason of a single sliding traversal. In case of repeated sliding, the fracture followed by separation of wear particles from the glass surface would be expected. In real sliding contact situations, the markings on the wear surface would be similar though not identical to those described above, since these are produced by the action of multiple asperities.

4.1.3 Microscopic examination of fatigue fracture surfaces

The fracture surfaces of notched and unnotched round fatigue samples of poly(methyl methacrylate), high density polyethylene and poly(vinyl chloride) subjected to reversed bending stresses were examined. The surface features were observed to be similar for both the notched and unnotched samples of the same material but were different for different materials. The fatigue fracture details for an unnotched poly(methyl methacrylate) sample are shown in Fig. 4.8-4.12, arranged in the order of increasing crack growth. Fig. 4.8 shows the region of crack initiation which looks mirror-like smooth. The specimen surface ahead of this region was found at higher magnification to be covered with a series of fine striae of about 1 μm spacing (Fig. 4.9). The striation spacing increases in the direction of crack growth. Fig. 4.10 obtained from a region well ahead of it shows much larger striation spacing (15 μm approximately). Fig. 4.11 exhibits the ribs produced between successive crack arrest positions indicating that the
Fig. 4.7. Optical micrograph showing cracks on a glass surface due to the sliding of a hemispherical indentor on it. Sliding direction is shown by the arrow (80).
Fig. 4.8. Scanning electron micrograph showing the region of crack initiation and growth on a fatigue fracture surface of poly(methyl methacrylate). Crack propagation is from left to right. Tilt angle 30°.

Fig. 4.9. Scanning electron micrograph of the same surface as in Fig. 4.8, showing the fatigue striation. Tilt angle 30°.
Fig. 4.10. Scanning electron micrograph of the same surface as in Fig. 4.8, but from a different location well ahead in the direction of crack propagation. Tilt angle 30°.

Fig. 4.11. Scanning electron micrograph of the same surface as in Fig. 4.8, showing the ribs produced between two striae. Tilt angle 30°.
Fig. 4.12. Scanning electron micrograph of the same surface as in Fig. 4.8, showing parabolic markings. Tilt angle 30°.
crack does not propagate through the cross-section of the specimen in one cycle (81). Fig. 4.12 shows parabolic markings in the last stage of fatigue fracture implying that this part of the fracture was brought about by excessive static loading.

The features on a high density polyethylene fatigue fracture surface are shown in Fig. 4.13-4.15, again arranged in the order of increasing crack growth. These are not as clearly defined as were in the case of poly(methyl methacrylate), and are discontinuous in nature. The fracture surface gets rougher with increasing crack growth (Fig. 4.14). Here the surface has a layered structure with thin fibers being pulled out of it. The latter is an indication of extensive plastic deformation occurring in the fracture zone. The final stage of fracture is due to a sudden rupture from excessive static loading and the fracture surface has a very rough appearance (Fig. 4.15).

The changes in surface features for poly(vinyl chloride) fatigue sample in the crack growth direction are shown in Fig. 4.16-4.18. Fig. 4.16 shows markings in the region immediately after crack initiation. These are obscure, discontinuous and seem to follow no general pattern. In the direction of crack propagation, the fracture surface tends to get rougher and gradually develops a layered structure (Fig. 4.17) which is similar to the case of high density polyethylene. There is the evidence of considerable plastic deformation occurring at the fracture surface. The final part of the fracture exhibits dimples (Fig. 4.18) which indicate failure by the coalescence of micro-voids produced under a single application of load.
Fig. 4.13. Scanning electron micrograph of fatigue fracture surface of high density polyethylene showing the initial stage of crack growth. Crack propagation is at 45° to the horizontal. Tilt angle 30°.

Fig. 4.14. Scanning electron micrograph of the same surface as in Fig. 4.13, obtained from a different location in the direction of crack propagation, showing the layered structure and the occurrence of plastic deformation. Tilt angle 30°.
Fig. 4.15. Scanning electron micrograph of the same surface as in Fig. 4.13, showing the final stage of fracture. Tilt angle 30°.

Fig. 4.16. Scanning electron micrograph of fatigue fracture surface of poly(vinyl chloride). Showing the region immediately after crack initiation. Crack propagation is from left to right. Tilt angle 30°.
Fig. 4.17. Scanning electron micrograph of the same surface as in Fig. 4.16, obtained from a different location in the direction of crack propagation, showing plastic deformation at the surface. Tilt angle 30°.

Fig. 4.18. Scanning electron micrograph of the same surface as in Fig. 4.16, showing the development of dimples in the final fracture zone. Tilt angle 30°.
4.1.4. Microscopic examination of wear surfaces

The worn surfaces of polymer pins were examined with a view to studying the probable mode of fracture in sliding and investigating the degree of resemblance with fatigue fractures, if any. This type of investigation is hampered by the complication that the fracture features produced by the separation of wear particles are instantaneously marred by the following sliding action. The resolution of the fracture mechanism operating at the sliding interface, is, therefore, fraught with many uncertainties and complications.

The micrographs of the unrubbed surfaces of poly(methyl methacrylate) and high density polyethylene pins, finished with 600 grade emery paper under running water, are given in Fig. 4.19 and 4.20 respectively. These show grooves running in the horizontal direction that were produced due to the abrasion process. The finished surface of poly(vinyl chloride) was similar in appearance to that of high density polyethylene and so is not being shown here.

Fig. 4.21 is a scanning electron micrograph of poly(methyl methacrylate) pin surface rubbed against a lapped steel disk (0.025 μm AA surface roughness) for six hours (which corresponds to the steady state wear condition). It may be noted that the wear process changes the surface features drastically and there is a complete disappearance of the abrasion marks. The surface is now covered with bands of arced ripples which are stretched transverse to the sliding direction. The formation of these ripples can be explained on the basis of the surface model discussed earlier in Section 4.1.2. According to this, an asperity on the metal disk in passing over the polymer pin surface would
Fig. 4.19. Scanning electron micrograph of poly(methyl methacrylate) pin surface prepared by abrasion against 600 grade emery paper under running water.
Fig. 4.20. Scanning electron micrograph of high density polyethylene pin surface prepared by abrasion against 600 grade emery paper under running water.
Fig. 4.21. Scanning electron micrograph of poly(methyl methacrylate) pin surface rubbed against a lapped steel disk for 6 hours, showing the bands of arced ripples. Sliding conditions: speed 1 m/s; load 8.83 N.
produce a series of arcuate flaws, as shown in Fig. 4.6. The repeated interaction between the rubbing surfaces would result in cracking and separation along the hyperbolic flaw surfaces of wear particles. The upper portion of Fig. 4.21 shows the worn polymeric material packed on the pin surface and so the surface markings discussed above are being obscured except in the background. Fig. 4.22 shows a transmission electron micrograph from the packed region. Here the packing is almost continuous with discrete wear particles sitting side by side.

The features on the worn surface of high density polyethylene pin rubbed against the lapped steel disk periphery for 10 hours are shown in Fig. 4.23-4.25. Since the pin was mounted with abrasion marks from the finishing operation oriented perpendicular to the sliding direction, it is evident that sliding has completely changed the pattern of markings on the surface. As for the case of poly(methyl methacrylate) discussed earlier, the surface is again covered with bands of arcuate ripples which are elongated transverse to the direction of sliding (Fig. 4.23-4.24). The ripples are quite shallow and their spacing and size are variable probably due to the varying loading conditions at discrete contacts. In addition, the wear grooves oriented parallel to the sliding direction are observed. These have been produced due to the abrasion of polymeric material by asperities on the harder metallic surface. Fig. 4.25 shows a scanning electron micrograph (out of a stereo-micrograph pair) obtained from the wear surface of high density polyethylene. It exhibits a number of wear particles which appear to have been separated along the edges marked by arrows but are still attached to the surface on the farther side. A few
Fig. 4.22. Transmission electron micrograph of poly(methyl methacrylate) pin surface rubbed against a lapped steel disk. Sliding conditions: same as in Fig. 4.21; sliding direction 45° to the horizontal.
Fig. 4.23. Scanning electron micrograph of high density polyethylene pin surface rubbed against a lapped steel disk for 10 hours, showing the bands of arced ripples. Sliding conditions: speed 1.75 m/s; load 47 N. Tilt angle 30°.
Fig. 4.24. Scanning electron micrograph of the same surface as in Fig. 4.23, but from a different location. Tilt angle 30°.

Fig. 4.25. Scanning electron micrograph of the same surface as in Fig. 4.23. Showing the separation of wear particles occurring along the edges marked by arrows. Tilt angle 30°.
more loading encounters would cause complete separation between the wear particle and the substrate. Thus, it provides an additional evidence of the phenomenon of cumulative damage in wear.

Figure 4.26 is a micrograph reported by Atkinson et al. (3) from the wear surface of ultra-high molecular weight polyethylene pin rubbing at 0.25 m/s against a lapped metallic disk of comparable roughness. It shows the ductile areas comprised of pulls and smears. Hastings (82) has recently reported the wear surface micrograph (Fig. 4.27) of the same material from a knee prosthesis. It should be noted that both of these wear surface micrographs are very much similar to the fatigue fracture surface micrographs given earlier in Fig. 4.13 and 4.14.

The features on poly(vinyl chloride) worn surface are shown in Fig. 4.28-4.30. Contrary to the cases discussed above, the surface markings here are completely obscured. Fig. 4.28 shows a surface with a fibrous structure and a few elongated dimples indicating considerable plastic deformation on the wear surface. Such features were also observed on the fatigue fracture surfaces of this material (Fig. 4.17-4.18). Fig. 4.29 shows plastic deformation and Fig. 4.30 also shows a few shallow pits on the wear surface. The edges of these pits have thin curled films adhering to them. It appears that the material under repeated asperity encounters was removed from these pits in the form of thin films. The latter finally broke leaving small fragments curling back along the edges of pits.

A number of workers (83-85) have reported the surface features of worn metal surfaces believed to be undergoing fatigue wear. Those
Fig. 4.26. Scanning electron micrograph of ultra-high molecular weight polyethylene pin rubbed against a lapped stainless steel disk, showing ductile areas which constitute the pulls and smears. Sliding direction: horizontal (3).

Fig. 4.27. Scanning electron micrograph of a worn surface of ultra-high molecular weight polyethylene obtained from a knee prosthesis (82).
Fig. 4.28. Scanning electron micrograph of poly(vinyl chloride) pin surface rubbed against a lapped steel disk for 6 hours, showing fibrous structure and dimples. Sliding conditions: speed 1 m/s; load 8.83 N. Tilt angle 30°.
Fig. 4.29. Transmission electron micrograph of poly(vinyl chloride) pin surface rubbed against a lapped steel disk, showing plastic deformation at the sliding surface. Sliding conditions: same as in Fig. 4.28.

Fig. 4.30. Scanning electron micrograph of the same surface as in Fig. 4.28, showing shallow pits on the sliding surface. Tilt angle 30°.
micrographs provide a marked degree of resemblance with the surface features observed on worn polymer pin surfaces. Furthermore, the metallographic studies of rolling contact wear (86) show fatigue cracks transverse to the direction of sliding which would also provide markings similar to those observed.

From the above discussion, it is seen that the wear surfaces of polymers do not show conventional fatigue marks, viz., striation and beach marks. The absence of these marks does not mean the absence of fatigue because these do not appear necessarily on fatigue fracture surfaces of polymers either. The wear surfaces do not exhibit parabolic markings which would provide a definite evidence of rapid fracture (as involved in static loading). A considerable amount of plastic deformation is noticed on the wear surfaces of high density polyethylene and poly(vinyl chloride) similar to that observed on the fatigue surfaces of these materials. The wear surfaces mostly show ripple markings stretching transverse to the sliding direction. Their geometry is consistent with the stress distribution in contact zone between a moving hemispherical indentor and a plane surface. The experimental evidence suggests that the separation of a wear particle cannot occur in a single traverse, as seen in Fig. 4.25. Thus, the mechanism of formation of a wear particle involves the initiation of a crack behind an asperity contact due to excessive tensile principal stress and the progression of it by repetitive loading action. This phenomenon of cumulative damage has been termed in this work as fatigue.
4.2. Polymer Wear

The wear of poly(methyl methacrylate), high density polyethylene and poly(vinyl chloride) pins sliding against the metal disk was measured for 10 hours at different sliding speeds. The wear data were needed in order to verify the fatigue wear equation derived in Section 3.5 and to explore the probable correlation between the topographical parameters of the sliding surfaces and the steady and unsteady wear states. The variation of polymer wear volume with time for sliding between poly(methyl methacrylate), high density polyethylene and poly(vinyl chloride) pins against the metal disk is shown in Fig. 4.31-4.33. Here the sliding conditions were chosen so as to get the first two states of wear and avoid the catastrophic wear failure. Contrary to the other two polymers, the wear data for high density polyethylene are reported for one sliding speed only. It is so because the data for this material were obtained by weighing, each time starting the wear test from the beginning and so it was very time-consuming. For all the materials, corresponding to every sliding condition two data points were obtained. Whereas the curved portion representing the unsteady state of wear was drawn by the free-hand method, the latter portion was obtained by fitting a straight line through the data points by the method of least squares and represents the steady state wear. The correlation coefficients which have been marked on every regression line are close to unity in almost all the cases.

The wear rate for poly(methyl methacrylate) and high density polyethylene decreased continuously with time in the unsteady state condition but later assumed a steady constant value. Except in the case of
Fig. 4.31. Variation of wear volume with time for poly(methyl methacrylate) pin sliding against a steel disk at various speeds and at 8.83 N load.
Fig. 4.32. Variation of wear volume with time for high density polyethylene pin sliding against a steel disk.
Sliding conditions: speed 1.75 m/s; load 47 N.
Fig. 4.33. Variation of wear volume with time for poly(vinyl chloride) pin sliding against a steel disk at various speeds and at 8.83 N load.
sliding at 3 and 3.5 m/s for poly(methyl methacrylate), the steady state wear rates were always smaller than the unsteady state wear rates. The higher initial wear rate is caused by abrasion of the softer polymer pin from the asperities on the harder metallic disk. The decreasing wear rate in the unsteady state condition is attributed to a continuing modification of the counterface topography from the transfer of polymer to the metal disk periphery. The steady state wear rate is restored after a stable thickness of the polymer film builds up.

A reversed trend is noted for poly(vinyl chloride) in that the wear rates in the steady state condition are higher than those in the unsteady state condition (Fig. 4.33). It was initially suspected that this uncommon behavior was due to the error in wear measurement from the use of a linear differential transformer. The wear data for this polymer were therefore obtained by weighing of the polymer pin and a similar behavior was again observed (Fig. 4.34).

The investigation of the behavior described above was further pursued in terms of the counterface topography resulting from the transfer of polymer film. It was observed that the material transfer was in the form of thin films which were seen adhering to the disk. It resulted in increasing the counterface roughness which was verified by the topographical analysis of metal disk surface rubbed against poly-(vinyl chloride) pin. The increased surface roughness is believed to be responsible for the increased wear rate in the steady state condition.

The wear rates for both poly(methyl methacrylate) and poly(vinyl chloride) increased with sliding speed which agrees with the past experimental observations.
Fig. 4.34. Variation of wear volume (obtained by weighing of polymer pins) with time for poly(vinyl chloride) pin sliding against a steel disk. Sliding conditions: speed 1.5 m/s; load 8.83 N. (The t test indicates that the slopes of the lines A and B are different at the 5% significance level.)
The variation of average disc temperature rise with time for poly(methyl methacrylate), high density polyethylene and poly(vinyl chloride) pins sliding against a steel disk at different speeds is shown in Fig. 4.35-4.37, respectively. In all the cases, the temperature rises initially with increasing time and then after about two hours assumes a fairly constant value. The initial period of increasing temperature rise seems to be associated with the changing wear rate condition and the latter condition of constant temperature with the steady state wear. The temperature rise is always proportional to the sliding velocity, which is in agreement with the common observations. The maximum temperature observed during sliding, within the range of sliding parameters used in this investigation, was well below the melting points of the respective polymers, thus precluding any possibility of polymer melting in the wear tests.

The friction force between the polymer pins and the metal disk was measured using strain gages and the average value of the coefficient of friction in the steady state wear condition for each sliding speed is indicated within parentheses on the corresponding wear curve in Fig. 4.31-4.33. It is noted that the friction coefficient for any material remains almost unchanged in the range of sliding speeds used.

4.3. Microscopic Examination of Metal Disk Surfaces

In order to explain the variation occurring in surface topography during sliding, it was found necessary to obtain the three-dimensional image of the metal disk surface before and after sliding by stereo scanning electron microscopy. The metal disks in the unrubbed
Fig. 4.35. Variation of average disk surface temperature rise with time in sliding between poly(methyl methacrylate) pin and steel disk at various speeds. Sliding conditions: same as in Fig. 4.31.
Fig. 4.36. Variation of average disk surface temperature rise with time in sliding between high density polyethylene pin and steel disk. Sliding conditions: same as in Fig. 4.32.
Fig. 4.37. Variation of average disk surface temperature rise with time in sliding between poly(vinyl chloride) pin and steel disk at various speeds. Sliding conditions: same as in Fig. 4.33.
condition as well as rubbed against poly(methyl methacrylate) pins for 1, 2 and 8 hours were therefore examined in the microscope. Since the disks could not be admitted directly in the scanning electron microscope chamber due to their size, they were cut into smaller sections for surface examination. As this was a very time-consuming process, such microscopic investigation was limited to one polymer, viz., poly(methyl methacrylate) only.

An examination of the SEM micrographs of the unrubbed disk surface (Fig. 4.38) revealed that the disk surface was comprised of a number of parallel wedge-shaped asperities with their axes running along the periphery of the disk. The pitch of the asperities appeared to be quite uniform. The characteristic asperity shape described above was attributed to the grinding process used for finishing the disk.

The sliding between the metal disk and poly(methyl methacrylate) pin resulted in the transfer of polymer to the metal surface. The scanning electron micrograph of the disk surface rubbed for one hour (Fig. 4.39) showed that the transfer occurred in the form of a large number of fragmented and irregular polymer particles which filled the asperity crevices in a number of locations and were also scattered all over the surface. The latter could result in a slight increase in the roughness of the disk surface along the sliding direction.

In the case of the disk rubbed for two hours (Fig. 4.40), the deposition of polymer fragments was much heavier, filling the valleys and even causing accumulation over the surface in discrete locations. Here the particles appeared to be much larger in size which was
Fig. 4.38. Scanning electron micrograph of an un-rubbed (ground) metal disk showing wedge-shaped asperities.
Fig. 4.39. Scanning electron micrograph of metal disk rubbed against poly(methyl methacrylate) pin for 1 hour. Sliding conditions: speed 1 m/s; load 8.83 N.
Fig. 4.40. Scanning electron micrograph of metal disk rubbed against poly(methyl methacrylate) pin for 2 hours. Sliding conditions: same as in Fig. 4.39.
probably due to the agglomeration of the small fragmented particles. The surface roughness was expected to increase further in this case.

Fig. 4.41 shows the scanning electron micrograph of the disk surface that had been in rubbing contact for eight hours. The continuous sliding for an extended time had almost completely filled the valleys and deposited a uniform layer of polymer on top of it. Thus, sliding in such a condition was essentially between the deposited film of polymer and the polymer pin. The extent of deposition for sliding times of 2 and 4 hours was essentially the same and was probably responsible for the steady state wear.

4.4. Surface Topography

From the profile ordinate data obtained along and perpendicular to the sliding directions for both the unrubbed and rubbed metal disk and polymer pin surfaces, a number of surface topographical parameters pertinent to the wear process and their distributions were computed. The variation of these parameters with sliding time is discussed below with the help of the conceptual image of surface roughness provided by stereo scanning electron microscopy.

4.4.1. Arithmetic average of surface roughness, $R_a$

The wear of polymers depends upon the roughness of counterface (25). The studies relating to the variation of counterface roughness with sliding time are, therefore, important and relevant to the wear situation. The variation of arithmetic average roughness $R_a$ of the metal disk surface with sliding time, rubbing against poly(methyl methacrylate), poly(vinyl chloride) and high density polyethylene pins is shown in Figs. 4.42-4.44.
Fig. 4.41. Scanning electron micrograph of metal disk rubbed against poly(methyl methacrylate) pin for 8 hours. Sliding conditions: same as in Fig. 4.39.
Fig. 4.42. Variation of metal disk surface arithmetic average roughness with time for poly(methyl methacrylate) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: speed 1 m/s; load 8.83 N.
Fig. 4.43. Variation of metal disk surface arithmetic average roughness with time for poly(vinyl chloride) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: speed 1 m/s; load 8.83 N.
Fig. 4.44. Variation of metal disk surface arithmetic average roughness with time for high density polyethylene pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: speed 1.75 m/s; load 47 N.
respectively. Here the portions of the curves in broken lines represent
the unsteady state wear part whereas the continuous lines represent the
steady state wear. The latter were drawn using the method of least
squares. The t tests indicated that the slopes of the continuous lines
did not differ from zero at the 5 percent significance level.

It is noted from Fig. 4.42 that for poly(methyl methacrylate), $R_a$
in the direction of sliding increases initially but then decreases and
maintains a fairly constant value for sliding times of three hours and
more. The initial increase in the disk surface roughness is attributed
to the pile-up of polymeric material at discrete and isolated locations.
After the sliding has occurred for more than two hours, the fragments of
poly(methyl methacrylate) have filled up the crevices and result in a
uniform film of deposited material (Fig. 4.41). The latter decreases
the surface roughness to a minimum value achievable by the transfer of
a stable film of poly(methyl methacrylate). The sliding now occurs
between the deposited film of polymer and polymer pin, a situation that
corresponds to steady state wear.

In a direction perpendicular to sliding, the surface roughness
decreases initially somewhat with sliding time. This is due to the
deposition of fragmented polymer particles at the bottom of crevices
so that the effective asperity heights are reduced. For periods in
excess of two hours, the arithmetic average surface roughness remains
unaltered due to a uniform deposition of the polymer fragments all over
the surface.

In the case of poly(vinyl chloride), $R_a$ of the metal disk in both
the directions increases slightly in the beginning but attains a constant
value with sliding later on. The steady state value of the roughness $R_a$ is here more than the initial roughness of the metal disk, a fact which is probably responsible for the higher wear rate of poly(vinyl chloride) in the steady state than the wear rate in the unsteady state. The higher surface roughness in the steady state is attributed to the transfer of polymer which occurs in the form of thin films that were seen tenaciously adhering to the metal surface.

The variation of the disk surface roughness for sliding against high density polyethylene is shown in Fig. 4.44. The behavior is similar to that observed for rubbing against poly(vinyl chloride) and so the same explanation is offered.

The variation of the arithmetic average surface roughness $R_a$ for the sliding surfaces of polymer pins is shown in Figs. 4.45-4.47. In plotting this variation, the data for the initial two hours of sliding have not been included because the surface topography during this period is greatly affected by the abrasion marks left over from the finishing operation. The straight lines have been drawn by the method of least squares and the slope $b$ of each line is indicated on the respective curve. The $t$ tests indicated that the slopes of the lines do not differ from zero at the 5 percent level of significance. Thus, in the steady state condition, the surface roughness of the polymer remains fairly unchanged. The roughness of the polymer surface is always lower than that of the counterface which indicates some amount of abrasion occurring in the sliding process.

4.4.2. Radius of curvature of asperity tips, $\beta$

The investigation of the variation of radius of curvature of asperity
Fig. 4.45. Variation of poly(methyl methacrylate) pin surface arithmetic average roughness with time for the pin sliding against the metal dish (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.42.

Fig. 4.46. Variation of poly(vinyl chloride) pin surface arithmetic average roughness with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.43.
Fig. 4.47. Variation of high density polyethylene pin surface arithmetic average roughness with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.44.
Fig. 4.48. Variation of average radius of curvature of metal disk asperities with time for poly(methyl methacrylate) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.42. (The values of average asperity radius of curvature in this figure were obtained manually and are about 40 times the corresponding values obtained by the data acquisition system).
Fig. 4.49. Variation of average radius of curvature of metal disk asperities with time for poly(vinyl chloride) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.43.
Fig. 4.50. Variation of average radius of curvature of metal disk asperities with time for high density polyethylene pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.44.
tips in a sliding situation is important because the real area of contact and the wear rate depend upon the size and shape of asperities (25,33,66). Such a variation for the metal disk is shown in Figs. 4.48-4.50. The portions of these curves in continuous lines were drawn by the method of least squares and the slopes were found not to differ from zero at the 5% significance level. It may be seen that the asperity radius of curvature for the disk sliding against poly(methyl methacrylate) and high density polyethylene increases initially but then stabilizes to a more or less constant value. The increase in asperity radius implies flattening of the asperity tips which occurs due to the transfer and deposition of polymer on the counterface. It has already been reported that the arithmetic average of surface roughness and the wear rate for these two cases are smaller in the steady state than in the unsteady state. It agrees with the experimental results of Hollander and Lancaster (25) who found that the polymer wear rate was inversely proportional to the average radius of curvature of counterface asperities and directly proportional to the arithmetic average.

Contrary to the above, the radius of curvature of disk asperities sliding against poly(vinyl chloride) initially decreases slightly and then attains a steady value. This behavior is again consistent with the variation of the arithmetic average of surface roughness and wear rate.

The variation of the radius of curvature of asperity tips on the worn surfaces of poly(methyl methacrylate), poly(vinyl chloride) and high density polyethylene pins is shown in Fig. 4.51-4.53. The straight lines have been drawn using the method of least squares. It may be seen that the asperity radii in both the directions are approximately same
Fig. 4.51. Variation of average radius of curvature of poly(methyl methacrylate) pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.42.

Fig. 4.52. Variation of average radius of curvature of poly-vinyl chloride) pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.43.
Fig. 4.53. Variation of average radius of curvature of high density polyethylene pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.44.
except for high density polyethylene, where the radii in the direction of sliding are slightly larger than in the perpendicular direction. The t tests were performed which indicated that the slopes of the lines do not differ from zero at the 5 percent level of significance. Thus, the radius of curvature of pin asperities does not vary with sliding time in the steady wear state.

The observation of the asperity tip radii being equal in the orthogonal directions justifies the assumption of the asperity tips approximating a spherical shape in the derivation of the wear equation.

The average radii of curvature of the asperities on pin and disk surfaces for sliding against poly(methyl methacrylate) are larger by a factor of about 40 than for the other two materials. It is so because the profile ordinate data for poly(methyl methacrylate) were extracted manually from the profilometer records, whereas, in the cases of the other two polymers, they were obtained using the data acquisition system. Since the calculation of the radius of curvature involves the sum of the differences between three ordinates of comparable magnitude (Equation 35) which appears in the denominator of the radius of curvature equation, a slight error can make the curvature values off by a large factor. For this reason the profile ordinate data for poly(methyl methacrylate) sliding against the metal disk for four and ten hours were obtained using the data acquisition system and the surface parameters were computed as given in Table 4.1.
Fig. 4.54. Variation of average slope of metal disk asperities with time for poly(methyl methacrylate) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.42.
Fig. 4.55. Variation of average slope of metal disk asperities with time for poly(vinyl chloride) pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.43.
Fig. 4.56. Variation of average slope of metal disk asperities with time for high density polyethylene pin sliding against the disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.44.
Table 4.1. Surface parameters for poly(methyl methacrylate) sliding against the steel disk computed using the profile ordinate data obtained by the data acquisition system. Sliding conditions: same as in Fig. 4.42.

<table>
<thead>
<tr>
<th>Surface parameter</th>
<th>Sliding time, hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>$\beta_1$ (disk), $\mu$m</td>
<td>44.92</td>
</tr>
<tr>
<td>$\beta_2$ (pin), $\mu$m</td>
<td>45.31</td>
</tr>
<tr>
<td>$\sigma_1$ (disk), $\mu$m</td>
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</tr>
<tr>
<td>$\sigma_2$ (pin), $\mu$m</td>
<td>0.58</td>
</tr>
<tr>
<td>$\eta_{L1}$ (disk), no./mm</td>
<td>34</td>
</tr>
<tr>
<td>$\eta_{L2}$ (disk), no./mm</td>
<td>93</td>
</tr>
</tbody>
</table>
4.4.3. **Average slope of asperities**

The average slope of asperities (also known as the "profile slope") is yet another important surface parameter which affects the wear process (12,25). The variation of the disk profile slope with time for sliding against poly(methyl methacrylate), poly(vinyl chloride) and high density polyethylene pins is shown in Figs. 4.54-4.56, respectively. The variation is similar to that of the arithmetic average roughness and so the same explanation applies. The same is true for the asperity slopes of polymer pin surfaces (Figs. 4.57-4.59). The continuous lines in both the cases have been drawn using the method of least squares. As was the case with arithmetic average roughness, the slopes of the lines were also not found to be different from zero at the 5 percent significance level. It should be noted that the asperity slopes of pin surfaces are almost equal in the two directions.

The similar variation between the arithmetic average surface roughness and the profile slopes raised the possibility of a relationship between these two parameters. The plots in Figs. 4.60-4.62 showed that the average asperity slope is directly proportional to the r.m.s. roughness. The above variation agrees with the findings of Endo and Kotani (40) who reported a similar relationship between the two parameters. A straight line correlation could not be obtained between the arithmetic average of surface roughness and the average asperity slope.
Fig. 4.57. Variation of average slope of poly(methyl methacrylate) pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.42.

Fig. 4.58. Variation of average slope of poly(vinyl chloride) pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.43.
Fig. 4.59. Variation of average slope of high density polyethylene pin asperities with time for the pin sliding against the metal disk (a) along the sliding direction; (b) normal to the sliding direction. Sliding conditions: same as in Fig. 4.44.
Fig. 4.60. Variation of average asperity slope with r.m.s. roughness for poly(methyl methacrylate) sliding against the metal disk. Sliding conditions: same as in Fig. 4.42. Regression is of the form $y = bx$. 

\[ b = 0.0165 \]
\[ b_{\text{max}} = 0.0208 \]
\[ b_{\text{min}} = 0.0122 \]
\[ \text{C.C.} = 0.94 \]

5% SIGNIFICANCE LEVEL
\[ b = 0.034 \]
\[ b_{\text{max}} = 0.039 \] 5\% SIGNIFICANCE
\[ b_{\text{min}} = 0.029 \]
\[ \text{C.C.} = 0.96 \]

Fig. 4.61. Variation of average asperity slope with r.m.s. roughness for poly(vinyl chloride) sliding against the metal disk. Sliding conditions: same as in Fig. 4.43.

\[ b = 0.04 \]
\[ b_{\text{max}} = 0.0614 \] 5\% SIGNIFICANCE
\[ b_{\text{min}} = 0.0186 \]
\[ \text{C.C.} = 0.89 \]

Fig. 4.62. Variation of average asperity slope with r.m.s. roughness for high density polyethylene sliding against the metal disk. Sliding conditions: same as in Fig. 4.44.
A plot between the r.m.s. surface roughness and the standard deviation of asperity heights is shown in Fig. 4.63. The slope of the straight line drawn through the data points is 0.99 (significant at 0.1 percent level) which means that for a surface the r.m.s. roughness value is approximately equal to the standard deviation of asperity heights. It should be very helpful in practice because the r.m.s. roughness which can be measured directly by a profilometer can be used in place of the standard deviation of asperity heights in calculating the real area of contact and later the wear rate.

4.4.4. Distribution of surface parameters

The distributions of surface parameters are needed in the computation of the real area of contact and the wear rate. For example, the distribution of profile ordinates is required in the computation of the bearing-area curve and the distribution of peak heights in estimating the real area of contact and the number of discrete contact zones (33, 69, 70). The distributions of profile ordinates, peak heights, asperity slopes, and peak radii of curvatures have been determined in this work using a Weibull approach. In order to determine the shape parameter, a regression line was fitted to the data-sets, $\ln(x_i - x_o)$ and $\ln\ln[1/(1-F(x_i))]$, in Equation (38). The values of $x_o$ and $F(x)$ were determined as per procedure described in Section 3.6. The data sets for the determination of the peak height distributions of the unrubbed and rubbed disk surfaces are plotted in Figs. 4.64-4.67 so as to examine the variation of the data-sets and their relationship with the regression line.
Fig. 4.63. Variation of r.m.s. roughness with standard deviation of asperity heights.
Fig. 4.64. Plot of $\ln \left[ \ln \left[ \frac{1}{1-F(x)} \right] \right]$ vs $\ln(x-5)$ for asperity heights ($x$) on the unrubbed metal disk surface in the direction of sliding.
SHAPE PARAMETER $b=5.20$
CC. = 0.91

Fig. 4.65. Plot of $\ln \left( \ln \left( \frac{1}{1-F(x)} \right) \right)$ vs $\ln(x-8.25)$ for asperity heights $(x)$ on the metal disk surface (in the direction of sliding) rubbed against poly-(methyl methacrylate) for 4 hours. Sliding conditions: same as in Fig. 4.42.
SHAPE PARAMETER $b = 3.83$
C.C. = 0.95

Fig. 4.66. Plot of $\ln \left[ \frac{1}{1-F(x)} \right]$ vs $\ln(x-8.50)$ for asperity heights ($x$) on the metal disk surface (in the direction of sliding) rubbed against poly(vinyl chloride) for 8 hours. Sliding conditions: same as in Fig. 4.43.
SHAPE PARAMETER $b=4.81$

C.C. = 0.98

Fig. 4.67. Plot of $\ln[\ln(1/(1-F(x)))$ vs $\ln(x-8)$ for asperity heights ($x$) on the metal disk surface (in the direction of sliding) rubbed against high density polyethylene pin for 8 hours. Sliding conditions: same as in Figure 4.44.
The shape parameter values were calculated for both the pin and the disk surfaces and the average values corresponding to different sliding times are given in Tables 4.2-4.4. These distributions are mostly negatively skewed \((b > 3.30)\) where the skewness provides an idea of the symmetry of the curve about the mean line. For example, a peaky surface has a distribution curve with positive skewness whereas a negative skewness indicates a scratchy surface \((41)\). There is no general trend discernible in the variation of skewness for the profile ordinate and peak height distribution curves. The distribution of asperity slopes is approximately exponential and those of the radii of curvatures of asperity tips is either exponential or positively skewed.

4.4.5. Asperity density

The variation of asperity density (number of asperities per mm) with sliding time was also determined since it is used in the calculation of the real area of contact, the number of discrete contacts and the wear rate between sliding bodies. The asperity density values for the unrubbed and rubbed metal disk and polymer pin surfaces are given in Table 4.5. It may be seen that the variation in asperity density in any direction is small considering the large changes that are occurring at the sliding surfaces due to the material transfer and wear processes.

Topographical analysis for poly(vinyl chloride) sliding against the metal disk at 2 m/s was also made with the intent of computing the surface parameters for verifying the wear equation and the pertinent parameters are given in Table 4.6.
Table 4.2. Weibull shape parameter $b$ for metal disk sliding against poly(methyl methacrylate) pin for different sliding times.

<table>
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<th>Sliding time, hours</th>
<th>Surface Parameter</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
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<tr>
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<td>0.94</td>
<td>0.80</td>
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<tr>
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<td>4.13</td>
<td>1.62</td>
<td>2.94</td>
<td>2.54</td>
<td>3.10</td>
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<tr>
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<td>0.93</td>
<td>0.99</td>
<td>0.92</td>
<td>0.98</td>
<td>0.99</td>
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Table 4.3. Weibull shape parameter \( b \) for metal disk sliding against poly(vinyl chloride) pin for different sliding times.

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<th>Sliding time, hours</th>
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<th>6</th>
<th>8</th>
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<td></td>
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<td>14.03</td>
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<td>0.97</td>
<td>0.95</td>
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<td>0.94</td>
<td>0.96</td>
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<tr>
<td></td>
<td>Pin Surface (Along sliding)</td>
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</tr>
<tr>
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<td>1.60</td>
<td>1.16</td>
<td>1.42</td>
<td>1.48</td>
<td>1.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pin Surface (Normal to sliding)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Ordinate height</td>
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<td>3.53</td>
<td>8.50</td>
<td>4.90</td>
<td>4.15</td>
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<td>1.65</td>
<td>1.44</td>
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Table 4.4. Weibull shape parameter $b$ for metal disk sliding against high density polyethylene pin for different sliding times.

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<th>Sliding time, hours</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Disk Surface (Along sliding)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<tr>
<td><strong>Pin Surface (Normal to sliding)</strong></td>
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<td>4.64</td>
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<td>2.25</td>
<td>1.46</td>
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<td>Profile orientation relative to sliding direction</td>
<td>Surface item</td>
<td>Sliding time, hours</td>
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</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Disk</td>
<td>12.0</td>
<td>13.7</td>
<td>10.9</td>
<td>11.3</td>
<td>11.1</td>
<td>12.4</td>
<td>13</td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td>Pin</td>
<td>9.76</td>
<td>12.9</td>
<td>11.7</td>
<td>14.5</td>
<td>12.9</td>
<td>12.9</td>
<td></td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Pin</td>
<td>15.1</td>
<td>25.2</td>
<td>18.6</td>
<td>16.1</td>
<td>11.7</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td><strong>Pin material:</strong> Poly(methyl methacrylate)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td>Disk</td>
<td>32.5</td>
<td>46</td>
<td>36</td>
<td>40</td>
<td>42</td>
<td>41</td>
<td>55</td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Disk</td>
<td>96</td>
<td>90.50</td>
<td>95</td>
<td>89</td>
<td>91.5</td>
<td>91</td>
<td>93</td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td>Pin</td>
<td>57</td>
<td>55.5</td>
<td>42</td>
<td>50</td>
<td>49.5</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Pin</td>
<td>65</td>
<td>75</td>
<td>59</td>
<td>61</td>
<td>62</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td><strong>Pin material:</strong> Poly(vinyl chloride)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td>Disk</td>
<td>32.5</td>
<td>30</td>
<td>51.5</td>
<td>48</td>
<td>43.5</td>
<td>47.5</td>
<td>56.5</td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Disk</td>
<td>96</td>
<td>82</td>
<td>94</td>
<td>96</td>
<td>92.5</td>
<td>93</td>
<td>86</td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td>Pin</td>
<td>28.50</td>
<td>28</td>
<td>34.50</td>
<td>31</td>
<td>29</td>
<td>26.50</td>
<td></td>
</tr>
<tr>
<td><strong>Perpendicular</strong></td>
<td>Pin</td>
<td>31.50</td>
<td>32</td>
<td>38.5</td>
<td>45.5</td>
<td>31.5</td>
<td>35.50</td>
<td></td>
</tr>
<tr>
<td><strong>Pin material:</strong> High Density Polyethylene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.6. Surface parameters for poly(vinyl chloride) sliding against the steel disk at a speed of 2 m/s and at 8.83 N load.

<table>
<thead>
<tr>
<th>Surface parameter</th>
<th>Sliding time, hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>( \beta_1 ) (disk), ( \mu m )</td>
<td>50.86</td>
</tr>
<tr>
<td>( \beta_2 ) (pin), ( \mu m )</td>
<td>38.22</td>
</tr>
<tr>
<td>( \sigma_1 ) (disk), ( \mu m )</td>
<td>0.44</td>
</tr>
<tr>
<td>( \sigma_2 ) (pin), ( \mu m )</td>
<td>0.92</td>
</tr>
<tr>
<td>( \eta_{L1} ) (disk), no./mm</td>
<td>47</td>
</tr>
<tr>
<td>( \eta_{L2} ) (disk), no./mm</td>
<td>97</td>
</tr>
</tbody>
</table>

In Section 4.4.4, it was pointed out that the distribution of asperity heights is negatively skewed. Such a distribution implies that a large fraction of the asperities have peak heights lower than the arithmetic average roughness value and so cannot participate in the wear process. On the other hand, the wear equation derived in Section 3.5 assumes a Gaussian distribution. In order to obtain a Gaussian distribution of the asperity heights from the wear surface profiles, a FORTRAN IV computer program (given in Appendix C) was written. This program deletes the asperities below a certain peak height in steps, calculates the distribution of asperity heights on this basis, and continues doing it until a Gaussian distribution is obtained. The asperity density values that result in the Gaussian distribution are then used in the calculation of the wear rates by the wear equation derived.
4.5. Polymer Fatigue

The fatigue behavior of poly(methyl methacrylate), high density polyethylene and poly(vinyl chloride) was obtained by testing the cylindrical, notched specimens in reversed bending mode. The size and shape of the notch and the type of loading used in testing the specimens resulted in an elastic stress concentration factor of 3.2 (87). The choice for the notched specimens was made upon the consideration that a conventional fatigue failure independent of the loading frequency was to be obtained (61,62). Previous studies have shown that the fatigue behavior of an unnotched specimen depends upon the loading frequency due to temperature rise (50,56). It was further concluded through experiments that it was impossible to obtain the conventional fatigue behavior for high density polyethylene using unnotched specimens due to its low modulus of elasticity. As such the use of notched fatigue specimens was mandatory in this work.

In the fatigue tests, the crack always initiated at the notch and progressed normal to the direction of principal stress. There was no sign of thermal failure. In order to plot the fatigue curves, the stress values were obtained by multiplying the maximum nominal bending stress by the elastic stress concentration factor. The stress amplitude vs. the number of cycles-to-failure (S-N) curves are plotted on log-log scale as shown in Figs 4.68-4.70. The curves indicate that the fatigue behavior of each material could be represented by a straight line. The large values of correlation coefficients for the lines fitted to the data by the method of least squares indicate that the scatter in the data
Fig. 4.68. Reversed bending fatigue failure curve for notched specimens of poly(methyl methacrylate).

Fig. 4.69. Reversed bending fatigue failure curve for notched specimens of poly(vinyl chloride).
Fig. 4.70. Reversed bending fatigue failure curve for notched specimens of high density polyethylene.

\[ b = -0.1866 \]
\[ b_{\text{max}} = -0.2296 \]
\[ b_{\text{min}} = -0.1436 \]

5% Significance level

C.C. = -0.92
128

is small. These plots led to the following relationships for the materials mentioned

\[
\text{poly(methyl methacrylate): } N = \left( \frac{135.18}{S} \right)^{20.44}
\]

\[
\text{poly(vinyl chloride): } N = \left( \frac{1244.89}{S} \right)^{3.18}
\]

\[
\text{high density polyethylene: } N = \left( \frac{496.29}{S} \right)^{4.51}
\]

where \(N\) is the number of cycles-to-failure and \(S\) the stress amplitude in \(\text{N/mm}^2\).

4.6. Computation of Wear from Fatigue Wear Equation

4.6.1. Evaluation of integral \(F_n(h)\)

In solving the wear equation (29) derived in Section 3.5 from the concept of the repetitive loading of asperities and progressive fracture, the evaluation of the integral \(F_n(h)\) is required. Here, consistent with the conclusions of surface studies in this work, a Gaussian distribution for asperity heights is considered. Thus the integral \(F_n(h)\) may be expressed as

\[
F_n(h) = \frac{1}{\sqrt{2\pi}} \int_h^\infty (s-h)^n e^{-\frac{1}{2} s^2} ds
\]

Substituting for the change in variable, \(s-h = x\) so that \(ds = dx\), we get

\[
F_n(h) = \frac{1}{\sqrt{2\pi}} \int_0^\infty x^n e^{-\frac{1}{2} (x+h)^2} dx
\]

\[
= \frac{1}{\sqrt{2\pi}} e^{-\frac{h^2}{2}} \int_0^\infty x^n e^{-\frac{x^2}{2} - hx} dx
\]

(40)
For such a case, the integral 3.462.1 from the Tables of

Integrals (88) states

\[ \int_0^\infty x^{v-1} e^{-\beta x^2-\gamma x} \, dx = (2\beta)^{-\frac{v}{2}} \Gamma(v) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-v}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \] (41)

for \( \beta > 0, v > 0 \).

The comparison between integrals (40) and (41) yields

\[ v = n + 1 \]
\[ \beta = \frac{1}{2} \]
\[ \gamma = h \]

so that the integral \( F_n(h) \) is reduced to

\[ F_n(h) = \frac{1}{\sqrt{2\pi}} e^{-\frac{h^2}{4}} \Gamma(n+1) D_{-(n+1)}(h) \] (42)

where \( D_{-(n+1)}(h) \) is related to the parabolic cylindrical function by

\[ D_{-(n+1)}(h) = U\left(n + \frac{1}{2}, h\right) \]

The values of \( U\left(n + \frac{1}{2}, h\right) \) have been tabulated by Miller (89) for different \( n \) and \( h \) values.

The various forms of the integral \( F_n(h) \) involved in equation (29) correspond to the three values of \( n \), viz., \( \frac{3}{2} \), 1, and 0. The substitution of these values in equation (42) yields

\[ F_{\frac{3}{2}}(h) = 0.5303 e^{-\frac{h^2}{4}} U\left(2, h\right) \] (43)

\[ F_1(h) = 0.3989 e^{-\frac{h^2}{4}} U\left(1.5, h\right) \] (44)
\[ F_0(h) = 0.3989 e^{-\frac{h^2}{4U(0.5,h)}} \] (45)

The values of these three integrals including those of \( F_{3/2}(h)/F_{1}(h) \) and \( F_0(h)/F_{1}(h) \) have been calculated for \( h = 0 \) to 3, and are tabulated in Appendix D.

For any sliding situation, \( F_{3/2}(h) \) is calculated from equation (13) because all other parameters relevant to the contact situation are known. The value of \( h \) corresponding to the calculated value of \( F_{3/2}(h) \) is read from Appendix D. For this particular value of \( h \), the other integrals can then be determined.

4.6.2. Asperity density and standard deviation for Gaussian distribution of asperity heights

Using the FORTRAN IV program (Appendix C) to reduce the population of asperity heights to a Gaussian distribution, as described in Section 4.4.5., the asperity density and the standard deviation of asperity heights for the pin and disk surfaces were computed. The mean values of the asperity density and standard deviation are given in Table 4.7 to be used later in the calculation of the wear rate.

4.6.3. Volume of a wear particle

The wear particles were assumed to be having a shape similar to a flattened sphere (special case of an ellipsoid) where the radius of the spherical portion of the particles was taken equal to the radius of a discrete contact zone. Thus the volume of a wear particle is given by
Table 4.7. Mean values of asperity density and standard deviation for Gaussian distribution of asperity heights.

<table>
<thead>
<tr>
<th>Polymer material in the steel disk-polymer pin pair</th>
<th>Sliding Speed m/s</th>
<th>Steel disk</th>
<th>Polymer pin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Along sliding</td>
<td>Normal to sliding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \eta_{L1} )</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>Unrubbed metal disk</td>
<td>-</td>
<td>28</td>
<td>0.35</td>
</tr>
<tr>
<td>High density polyethylene</td>
<td>1.75</td>
<td>51</td>
<td>0.61</td>
</tr>
<tr>
<td>Poly(methyl methacrylate)</td>
<td>1.0</td>
<td>37</td>
<td>0.39</td>
</tr>
<tr>
<td>Poly(vinyl chloride)</td>
<td>1.0</td>
<td>44</td>
<td>0.62</td>
</tr>
</tbody>
</table>
\[ v_p = \frac{2}{3} \pi a_1^2 C \]  

(46)

where \( a_1 \) is the radius of a discrete contact zone and \( C \) the thickness of the wear particle. The latter was calculated from the following expression (90)

\[ C = \frac{6E\gamma (1 + 3f^2)}{Sy^2} \]  

(47)

where \( f \) is the coefficient of friction and \( E, \gamma \) and \( Sy \) the modulus of elasticity, surface energy, and yield strength of the particle material, respectively.

4.6.4. Computation of wear rates

The steady state wear rates were calculated from Equation (29) for different sliding combinations. Table 4.8 lists the properties of the polymeric materials and Table 4.9 gives the data reported earlier in different figures and tables that will be used in calculations. In addition, the fatigue parameters, \( S_0 \) and \( t \), for the different polymeric materials were taken from Figs. 4.68-4.70.

In calculating the wear rate, first the values of \( \beta \) and \( \sigma \) were calculated from Equations (14) and (15). The value of \( F_0(h) \) was then calculated from Equation (15). It led to the determination of the values of \( h \), \( F_0(h) \) and \( F_1(h) \) by the use of the table in Appendix D, as discussed in Section 4.6.1. Thus \( K_1 \) can be calculated where \( k \) is determined using Equation (19). The volume of a wear particle, \( v_p \), is determined from Equations (21), (46) and (47). The wear rate can now be estimated from Equation (29).
Table 4.8. Properties of polymeric materials.

<table>
<thead>
<tr>
<th>Property</th>
<th>Reference No.</th>
<th>High density polyethylene</th>
<th>Poly(methyl methacrylate)</th>
<th>Poly(vinyl chloride)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface energy, N.m/m²</td>
<td>91</td>
<td>33.45 x 10⁻³</td>
<td>34.43 x 10⁻³</td>
<td>34.00 x 10⁻³</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>91</td>
<td>0.47</td>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>Modulus of elasticity, N/mm²</td>
<td>92</td>
<td>412.00</td>
<td>1825.00</td>
<td>2413.00</td>
</tr>
<tr>
<td>Yield strength, N/mm²</td>
<td>92</td>
<td>22.00</td>
<td>57.50</td>
<td>41.00</td>
</tr>
</tbody>
</table>
Table 4.9. Data for calculating wear rates.

<table>
<thead>
<tr>
<th>Sliding conditions:</th>
<th>Polymeric material in steel disk-polymer pin combination</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High density polyethylene</td>
<td>Poly(methyl methacrylate)</td>
</tr>
<tr>
<td>v, m/s</td>
<td>1.75</td>
<td>1.0</td>
</tr>
<tr>
<td>P, N</td>
<td>47</td>
<td>8.83</td>
</tr>
<tr>
<td>f</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>( \beta_1 ) (disk), ( \mu m )</td>
<td>49</td>
<td>46.67</td>
</tr>
<tr>
<td>( \beta_2 ) (pin), ( \mu m )</td>
<td>40</td>
<td>48.08</td>
</tr>
<tr>
<td>( \sigma_1 ) (disk), ( \mu m )</td>
<td>0.61</td>
<td>0.39</td>
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<tr>
<td>( \sigma_2 ) (pin), ( \mu m )</td>
<td>0.86</td>
<td>0.55</td>
</tr>
<tr>
<td>( n_{L_1} ) (disk), no./mm</td>
<td>51</td>
<td>37</td>
</tr>
<tr>
<td>( n ) (disk), no./mm(^2)</td>
<td>4488</td>
<td>3441</td>
</tr>
</tbody>
</table>
The wear rates for different sliding combinations were calculated as per procedure described above and are given in Table 4.10. This table also lists the corresponding experimental values for the sake of comparison. It is noted that the estimated wear rates are off by a maximum of 30% from the experimental values. The difference between them is within permissible limits as shown by the error analysis (Appendix E) performed considering a variation of ±1% to ±7% in the measured quantities. Some of the other reasons for the discrepancy between the wear rates are as follows:

1. The wear rate is affected by so many variables that are hard to control as a result of which a scatter of about 25% in the experimental wear data is very common.

2. In the computation of wear rates, the properties of bulk polymers have been used. On the other hand, the wear process is controlled by thin layers of the transferred polymeric material. The properties of these thin layers are hitherto unknown.

<table>
<thead>
<tr>
<th>Polymeric material</th>
<th>Sliding speed, m/s</th>
<th>Load, N</th>
<th>Wear rate mm$^3$/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>High density polyethylene</td>
<td>1.75</td>
<td>47</td>
<td>0.62</td>
</tr>
<tr>
<td>Poly(methyl methacrylate)</td>
<td>1.0</td>
<td>8.83</td>
<td>2.75</td>
</tr>
<tr>
<td>Poly(vinyl chloride)</td>
<td>1.0</td>
<td>8.83</td>
<td>3.74</td>
</tr>
<tr>
<td>Poly(vinyl chloride)</td>
<td>2.0</td>
<td>8.83</td>
<td>9.25</td>
</tr>
</tbody>
</table>
(3) During the sliding process, a moderate increase in temperature occurred. This could have changed the adhesional characteristics at the interface.

(4) The mechanical properties used correspond to normal slow rates of testing whereas in the sliding process very high strain rates are involved. The sliding speed to strain rate dependency is not yet known.

(5) The characterization of the topography of a surface with transferred films of a polymeric material on it has a considerable potential for error because of the likelihood of deformation and penetration of the soft polymeric material by the profilometer stylus. Furthermore, if the transfer is in the form of discrete patches, short traversals of profilometer may not be representative of the surface as a whole.

(6) The fatigue properties of discrete contacts may be different from the fatigue properties of the bulk material.

(7) The modelling of the asperities under adhesive contacts resulting in progressive fracture is at best approximate.
5. CONCLUSIONS AND SUGGESTIONS

5.1. Conclusions

1. This work has demonstrated that the concept of repetitive loading of surface asperities in sliding, causing initiation and propagation of a fatigue crack and resulting finally in the separation of a wear particle, is technically sound. Utilizing this concept, a wear equation has been derived from theoretical considerations. It is in terms of the sliding parameters, viz., load and sliding velocity, fatigue parameters and modulus of elasticity of the material being worn, and surface roughness parameters of the sliding surfaces. The equation has been verified experimentally for the case of three polymers, viz., poly(methyl methacrylate), poly(vinyl chloride) and high density polyethylene sliding against the metal disk.

2. The wear experiments demonstrated that the transition from unsteady state to steady state wear takes place in about two hours for sliding between a metal surface and poly(methyl methacrylate) and poly-(vinyl chloride) at 1 m/s and under 8.83 N load and for high density polyethylene it occurs at 1.75 m/s and under 47 N load. The average temperature at the sliding interface remained constant during the steady state wear.

3. Scanning electron microscopy of the steel disk surface sliding against poly(methyl methacrylate) revealed that in the unsteady state the transfer of polymer to the disk occurred in the form of a number of small fragments which finally built up a stable layer of polymer uniformly covering the disk. It resulted in changing the wear
process to steady state and there was no significant change in this layer observed during this state.

4. The transfer of polymer to the disk surface resulted in the modification of its surface topography. The measurement of the arithmetic average roughness, radius of curvature and slope of asperities with sliding time showed that these parameters were continuously changing during the unsteady state wear and were fairly constant in the steady state wear condition.

5. The surface analysis of worn surfaces of metal disk and polymer pins revealed that the distribution of asperity slopes was exponential. The profile ordinates and asperity heights were found negatively skewed, whereas the radii of curvatures of asperities were positively skewed.

6. The r.m.s. surface roughness of both the metal disk and polymer pins is approximately equal to the standard deviation of asperity heights. It is also directly proportional to the average asperity slope.

7. The fatigue failure of a notched polymer sample was of a conventional nature in that there was no thermal softening observed.

8. Scanning electron microscopy revealed the fatigue-fractured surfaces of poly(methyl methacrylate) to be covered with a series of striations. The striation spacing was observed to increase in the direction of crack propagation. The last stage of fracture was due to the rapid crack propagation mechanism.

The striations on high density polyethylene and poly(vinyl chloride) surfaces were obscure, discontinuous and unclear and the final stage of fracture was accompanied by considerable plastic deformation.
9. Scanning electron microscopy of the worn surfaces of poly-
(methyl methacrylate) and high density polyethylene revealed that they
were covered with bands of arced ripples stretched across the transverse
direction. The worn surfaces of poly(vinyl chloride) suffered severe
plastic deformation during sliding as discerned by the dimples observed
on them.

5.2. Suggestions for Future Work

The following suggestions are offered for future research work:

1. To explore the applicability of the fatigue wear equation
derived in this work for polymer composites and other polymers.

2. To develop a fatigue wear equation for sliding between metallic
surfaces considering plastic deformation and work hardening in the con-
tact zone.

3. To investigate the variation of surface topographical parameters
for sliding between metallic pairs and their influence on the wear process.

4. To develop a relationship for specifying the sliding conditions
necessary to avoid catastrophic wear in polymers.

5. To study the relationship between wear rate and the shape
and size of wear particles considering both the nature of the wear
process and the surface characteristics of the sliding members.

6. To perform the topographical analysis of surfaces produced by
various commercial machining processes and document the surface rough-
ness data for the use of designers for estimating the wear volume from
the fatigue wear equation.
6. ACKNOWLEDGMENTS

The author wishes to express his sincere and profound gratitude to Professor Shyam Bahadur for his continued guidance, encouragement and counseling throughout the course of this work.

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Thanks are also due to the technicians, Mr. Hap Steed and Mr. Larry Couture, for their assistance in the experimental work.

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The author is thankful to his wife Usha and son Abhinav for their inspiration and admirable patience during the course of this study.
7. REFERENCES


8. APPENDIX A: ASSEMBLY LANGUAGE PROGRAM
FOR THE KIM MICROCOMPUTER

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0036 D400 78 INPUT B410
0037 D404 9A TXS
0038 D40D EB INX
0039 D40E 8E 00 DO STX DATA
0040 D411 8D 01 DO STA CRA
0041 D414 20 80 D5 JSR COPY
0042 D417 20 76 D5 JSR CRLF
LDY $0
LDX $40
STX SELECT
INP300 BIT SAD
INP300 BHI INP300
LDA $0
INP400 CLC
ADC TIME
INP300 BIT TIMEOUT
BPL INP500
STA (POINTR),Y
JSR NEXT
BEQ INF600
LDA TIMER
JMP INF400
LDA (POINTER)
JSR NEXT
LDA $10
ST A (POINTER)
JSR NEXT
JSR COPY
LDY $0
CLD
LDA $10
STA LINCNT
JSR CRLF
JSR DELAY
LDY $0
OUT050
LDA $10
STA VALUE
STA DIGIT
JSR DELAY
JSR DELAY
LDY $0
OUT050
LDA $10
STA VALUE
STA DIGIT
JSR DELAY
JSR DELAY
LDY $0
OUT050
151

0141 D594 C9 FF  CMP  #$FF
0142 D596 D0 02  BNE  DEC
0143 D598 C6 06  DEC  DECR1
0144 D59A A5 05  DECR1 LDA  COUNT
0145 D59C C5 06  ORA  COUNT+1
0146 D59E 60  RTS

0150 D59F  LEADER JSR  CRLF
0151 D600 20 76 D5  LEADER JSR  DELAY
0152 D603 20 1B D6  JSR  DELAY
0153 D606 A2 E4  LDX  #$80
0154 D608 A9 00  LEAD1 LDA  $0
0155 D60A 20 A0 1E  JSR  OUTCH
0156 D60D A9 00  LDA  $0
0157 D60F 20 A0 1E  JSR  OUTCH
0158 D612 CA  DEX
0159 D613 D0 F3  BNE  LEAD1
0160 D615 20 5A 1E  JSR  GETCH
0161 D618 4C 4F 1C  JMP  MONITR

0163 D61B 20 1E D6  DELAY JSR  #$3
0164 D61E 20 21 D6  JSR  #$3
0165 D621 20 24 D6  JSR  #$3
0166 D624 A2 00  LDX  $0
0167 D626 A0 00  DELAY1 LDY  $0
0168 D628 88  DELAY2 DEY
0169 D629 D0 FD  BNE  DELAY2
0170 D62B CA  DEX
0171 D62C D0 F8  BNE  DELAY1
0172 D62E 60  RTS
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END OF ASSEMBLY(V5D)
9. APPENDIX B: COMPUTER PROGRAM FOR THE COMPUTATION OF SURFACE TOPOGRAPHICAL PARAMETERS

C MAIN PROGRAM SURFACE
C N=NO. OF DATA POINTS
C C=INITIAL VALUE OF X
C AINT=INTERVAL BETWEEN TWO DATA POINTS
C Y=HEIGHT OF A POINT ON THE PROFILE FROM A REFERENCE LINE (ORDINATE)
C Y0=MINIMUM EXPECTED VALUE OF THE ORDI NATE,Y
C M1=NUMBER OF Y0 VALUES
C YDOT=ASPERITY SLOPE AT A POINT
C YD0T=MINIMUM EXPECTED VALUE OF THE SLOPE,YDOT
C M2=NUMBER OF YDOT VALUES
C YP=PEAK Height
C YK=MINIMUM EXPECTED VALUE OF THE PEAK HEIGHT, API
C R=ARBITRARY NUMBER OF YK VALUES
C RAO=RA DIUS OF CURVATURE OF AN ASPERITY
C RAO=MINIMUM EXPECTED VALUE OF THE RADIUS OF CURVATURE OF AN ASPERITY,RA
C M4=NUMBER OF RA VALUES
C YPRIM=HEIGHT OF A POINT ON AN ASPERITY FROM THE MAIN LINE
C YPRIM=SQUARE OF YPRIM
C 8SL=SL=THE REGRESSION LINE
C AC=INTERCEPT OF REGRESSION LINE WITH Y-AXIS
C YPSUM=SUM OF YPRIM VALUES
C YPSUM=SQUARE OF YPRIM SQUARED
C YPSUM=SQUARE OF YPRIM SQUARED
C YPSUM=SQUARE OF YPRIM SQUARED
C YPSUM=SQUARE OF YPRIM SQUARED
C YPSUM=SQUARE OF YPRIM SQUARED
C L=NUMBER OF ASPERITIES
C K=NO. OF DATA POINT WHERE PEAK HAS OCCURRED
C YA=SAME AS Y YUT IN ASCENDING ORDER
DUE TO SUM X(I),Y(I),YPRIM(I),YDOT(I),YD0T(I),YK(I)
(1,2),Y(1,2),X(1,2),Y(1,2),Y(1,2),YPRIM(I),YDOT(I),YD0T(I),YD0T(1,2),YD0T(2,50)
2,YA(2,50)
READ (5) N,M1,M2,M3,M4,AINT
6 FORMAT(S11,2F14.4)
CRT1=1.0E-03
CRT2=5.0E-03
X(I)=C
WM(N)=N
DC 2 I=1,N
2 X(1+1)=X(1)+AINT
READ (5) YA(I),Y(I),M1
8 FORMAT(10F6.0)
WRITE (6,1800)
1800 FORMAT(*,**SURFACE DATA IN 41CRU M.****)
DU 11 10=1,N
Y(I)=1.0+0.0355*Y(I)-0.3549*10.00
11 CONTINUE
WRITE (6,90) Y(I),1=1,N
30 FORMAT(*,10G13.5)
DU 150 I=1,N
150 YA(I)=Y(I)
CALL ABSRT(N,YA)
READ(5,7) Y(I),1=1,N
8 FORMAT(10F6.0)
WRITE (6,1400)
1400 FORMAT(*,**DATA TO ESTIMATE THE DISTRIBUTION OF SURFACE**)
CALL JAIN(YA,Y,THETA,M1,N)
CALL VINUD(X,Y,ACEP,4SL,CORR,N)
R=3SL
A=ACEP
DO 15 I=1,N
YPRIM(I)=ABS(Y(I)-3*X(I)-A)
YP SQ(I)=YPRIM(I)**2
15 CONTINUE
YP SUM = YPRIM(1)
YP SQ = YPSQ(I)
DO 16 I=2,N
YP SUM = YPSUM+YPRIM(I)
YP SQ = YPSQ+YPSQ(I)
16 CONTINUE
YP SUM = YPSUM/N
YP SQ = YPSQ/N
RMSQ = SQRT(YPSQ)
WRITE(6,300)YP SUM, YPSQ, RMSQ
300 FORMAT(•O) •••• DATA TO ESTIMATE DISTRIBUTION OF ASPERITY SLOPE ••••
1* ARITHMETIC AVERAGE OF SURFACE ROUGHNESS IN MIC. ••••••••••••SI+$7/
2* RMS OF SURFACE ROUGHNESS IN MIC. ••• •••••••••• SI+$7)
NP1 = N-1
DO 18 K=2,NP1
K=K-1
YDOT(KI) = ABS((Y(KI+1)-Y(KI-1))/(2*AIN1))
IF(YDOT(K1)+4*CAT1)YDOT(KI)=2*C=04
YDABS(J) = YDOT(K1)
YDOT(KI) = ABS((Y(KI+1)-2*Y(KI)+Y(KI-1))/(9*AIN*2))
IF(YDOT(K1)+4*CAT2)YDOT(KI)=5*J=03
18 CONTINUE
WRITE(6,1500)
1500 FORMAT('O') •••• DATA TO ESTIMATE DISTRIBUTION OF ASPERITY SLOPE ••••
CALL ABSRT(N-2,YDABS)
CALL JAIN(YDABS,YDQ,YSUM,Y2,N-2)
L=O
WRITE(6,70)
70 FORMAT('O') •••• DATA TO ESTIMATE DISTRIBUTION OF ASPERITY SLOPE ••••
DO 17 K=2,NP1
IF(Y(KI)-Y(K))17,17,4
17 CONTINUE
17 FORMAT(2I15,2F15.5)
17 CONTINUE
WRITE(6,1700)
1700 FORMAT('O') •••• DATA TO ESTIMATE DISTRIBUTION OF ASPERITY SLOPE ••••
CALL ABSMT(L,AIP)
CALL JAIN(AIP,YK,AIPM,M5,L)
WRITE(6,1600)
1600 FORMAT('O') •••• DATA TO ESTIMATE DISTRIBUTION OF HAD. OF CURV. ••••
CALL ABSMT(L,RA)
CALL JAIN(RA,RAQ,RAQ,M4,L)
STOP
END
SUBROUTINE JAIN(Y,Y,THETA,M,N)
DIMENSION R(1250),RA(1250),H(1250),Y(1250),Y0(20)
SUM=O.
DO 3 I=1,N
5 SUM=Y(I)+SUM
THETA=SUM/N
SUM=0.
DO 74 I=1,N
SUM=SUM+((THETA-Y(I))*(THETA-Y(I)))
74 CONTINUE
STDY=SQR(SUM/N)
WRITE(0,78)THETA,STDY
78 FORMAT(0,1* THETA(ARITHMETIC AVERAGE) **, SL + 7, 
2* STANDARD DEVIATION **, SL + 7)
DO 100 I=1,N
100 R(I)=(N-FLOAT(I)+0.4)/(N+0.4)
DO 20 J=1,N
DO 200 I=1,N
RR(I)=ALGG(ALG((1.0/R(I))))
MM(I)=ALGG(Y(I)-YO(J))
200 CONTINUE
WRITE(6,1030)
1030 FORMAT(0,1* R(I) = ALGG(ALG((1.0/R(I)))) **)
WRITE(6,1100)(RR(I),I=1,N)
1100 FORMAT(0,1* RR(I) **)
WRITE(6,1100)(MM(I),I=1,N)
WRITE(6,1100)(Yo(J),J=1,N)
1000 FORMAT(0,1* R(I), J=1,N, 10X, Y(J) = ** F20.8)
CALL VINUD (MM, RR, AC1P, BS, CURR, N)
20 CONTINUE
RETURN
END
SUBROUTINE VINUD(X,Y,AC1P,BSL,CURR,N)
DIMENSION X(1250),Y(1250)
SUMX=0.
SUMY=0.
SUMXX=0.
SUMXY=0.
SUM1=0.
SUM2=0.
SUM3=0.
DO 2 I=1,N
SUMX=SUMX+X(I)
SUMY=SUMY+Y(I)
SUMXX=SUMXX+X(I)*X(I)
SUMXY=SUMXY+X(I)*Y(I)
2 CONTINUE
XBAR=SUMX/FLOAT(N)
YBAR=SUMY/FLOAT(N)
ANUM=SUMXX-SUMX*SUMY
BNUM=FLOAT(N)*SUMX-SUMX*SUMY
DENOM=FLOAT(N)*SUMXX-SUMX*SUMY
IF(DENOM.NE.0.) GO TO 1070
WRITE(6,106)
106 FORMAT(0,1* ERROR MESSAGE SUBROUTINE VINUD **)
1* INTERCEPT AND SLOPE INDETERMINATE, CHECK DATA*)
RETURN
1070 AC1P=ANUM/DENOM
BSL=BNUM/DENOM
DO 3 I=1,N
SUM1 = SUM1 + (X(i) - XBAR) * (X(i) - XBAR)
SUM2 = SUM2 + (Y(i) - YBAR) * (Y(i) - YBAR)
SUM3 = SUM3 + (X(i) - XBAR) * (Y(i) - YBAR)
3 CONTINUE
   IF(SUM1.NE.0.) GO TO 108
   WRITE(6,107)
   107 FORMAT(* ****ERROR MESSAGE SUBROUTINE VIND*****,/,
                1* ALL X-OBSERVATIONS SAME*,G15.7*, CHECK DATA*)
   GO TO 50
108 IF(SUM2.NE.0.) GO TO 110
   WRITE(6,109)
   109 FORMAT(* ****ERROR MESSAGE SUBROUTINE VIND*****,/,
                1* ALL Y-OBSERVATIONS SAME*,G15.7*, CHECK DATA*)
   110 CURRE=SUM3/SQRT(SUM1*SUM2)
   WRITE(6,8) ACER, BSL, CORK, SUMX, SUMY, SUMXX, SUMXY, SUM1, SUM2, SUM3
   8 FORMAT(*,
                1* ORDINATE INTERCEPT A OF REGRESSION LINE ...................*G14.7*/,
                2* SLOPE B OF REGRESSION LINE ........................................*G14.7*/,
                3* CORRELATION COEFFICIENT K ......................................*G14.7*/,
                4* SUM OF X ............................................................*G14.7*/,
                5* SUM OF Y ............................................................*G14.7*/,
                6* SUM OF X**2 ...........................................................*G14.7*/,
                7* SUM OF XY ............................................................*G14.7*/,
                8* SUM OF (X-XBAR)**2 ...................................................*G14.7*/,
                9* SUM OF (Y-YBAR)**2 ...................................................*G14.7*/,
                1* SUM OF (X-XBAR)*(Y-YBAR) ......................................*G14.7*/
50 RETURN
END
10. APPENDIX C: COMPUTER PROGRAM FOR THE REDUCTION OF A POPULATION OF RANDOM VARIABLES TO A GAUSSIAN DISTRIBUTION

C PROGRAM TO CALCULATE DISTRIBUTION OF A PARAMETER
C N=NO. OF DATA POINTS
C Y=PEAK HEIGHTS
C YO=MINIMUM EXPECTED VALUE OF THE PEAK HEIGHTS
C M=NUMBER OF Y VALUES
C BSL=SLOPE OF THE REGRESSION LINE
C ACO=INTERCEPT OF REGRESSION LINE WITH Y-AXIS
C YASAM=AS Y OUT IN ASCENDING ORDER
C DIMENSION X(500), Y(500), YO(20), YA(500)
C READ (5,6,N,M,BSL,YA)
C READ (5,7) (Y(I),I=1,N)
C FORMAT(5F20.8)
C FORMAT(8F10.3)
C DO 150 I=1,N
C YA(I)=Y(I)
C CALL AHSR1(N,YA)
C READ(5,7) (Y(I),I=1,N)
C FORMAT(5F20.8)
C WRITE(6,14J)
C FORMAT(*0*,,**DATA TU ASSIMATE THE DISTRIBUTION OF PARAMETER***)
C CALL JAIN(YA,YO,THETA,M,N,N1,BSL,TSTAT)
C *(BSL-3.35)+2.95 Y+X95
C N1=N+4
C GO TO 35
C 91 CALL JAIN(YA,YO,THETA,M,N,N1,BSL,TSTAT)
C *(BSL-3.35)+2.95 Y+X95
C 92 FORMAT(*X*,,**DATA T ASSIMATE THE DISTRIBUTION OF PARAMETER***)
C SUM=Y(I)+SUM
C SUM6=0.
C DO 74 I=N1,N
C SUM6=SUM6*(THETA-Y(I))*THETA-Y(I))
C CONTINUE
C STDV=SQRT(SUM6/(N-N1+1))
C WRITE(6,78)THETA,STDV
C FORMAT(*78)
C 78 FORMAT(*X*,,**THETAR=ARITHMETIC AVERAGE***)
C 78 FORMAT(*X*,,**STANDARD DEVIATION***)
C DO 100 I=N1,N
C 100 FORMAT(*10*,X+0.0)
IF(COFiM = DUMMY > 300.500% < 0)
20 DJMMy = COFR CONTINUE
300 I1 = J - 1 WRITE(6, 1010) I1, Y0(I1), DUMMY
1010 FORMAT(* J = *, I10, I10, * Y0(I) = *, F20.8, 10X, * CORR = *, F10.7)
RETURN
END

SUBROUTINE VINO( X, Y, BSL, CORR, N, NI, DUMMY, TSTAT)
D I M E N S I O N X(SOO), Y(SOO)
SUMX = 0. SUMY = 0.
SUMXX = 0.
SUMXY = 0.
SUM1 = 0.
SUM2 = 0.
SUM3 = 0.
SUM4 = 0.
DO 2 I = N1, N
SUMX = SUMX + X(I)
SUMY = SUMY + Y(I)
SUMXX = SUMXX + X(I)*X(I)
SUMXY = SUMXY + X(I)*Y(I)
2 CONTINUE
XBAR = SUMX/FLOAT(N- N1+ 1)
YBAR = SUMY/FLOAT(N- N1+ 1)
AN = SUMX*SUMY - SUMXX*SUMXY
DENOM = FLOAT(N- N1+ 1)*SUMXY - SUMXX*SUMYY
DENUM = FLOAT(N- N1+ 1)*SUMXY - SUMXX*SUMYY
RETURN
106 FORMAT(* Error message SUBROUTINE VINO**/,
  1' All X-OBSERVATIONS SAME, G15.7, ! Check DATA")
RETURN
1070 ACEP = ANU/M/DENUM
BSL = BNMJ/M/DENUM
DUMMY2 = BSL
DO 3 I = N1, N
SUM1 = SUM1 + (X(I) - XBAR)*(X(I) - XBAR)
SUM2 = SUM2 + (Y(I) - YBAR)*(Y(I) - YBAR)
SUM3 = SUM3 + (X(I) - XBAR)*(Y(I) - YBAR)
SUM4 = SUM4 + (Y(I) - YBAR)**2
3 CONTINUE
IF(SUM1. NZO.) GO TO 108
WRITE(6, 109)
108 IF(SUM2. NE. 0.) GO TO 110
RETURN
109 FORMAT(* Error message SUBROUTINE VINO**/,
  1' All Y-OBSERVATIONS SAME, G15.7, ! Check DATA")
RETURN
110 CORR = SUM3/SORT(SUM1*SUM2)
SYX = SORT(SUN1/FLOAT(N- N1- 1))
SB = SORT(SYX*SIX/SUM1)
BUP = BSL*TSTAT*SB
BLW = BSL*TSTAT*SB
WRITE(6, 8) ACEP, BSL, CORR, SUMX, SUMY, SUMXX, SUMXY, SUM1, SUM2, SUM3, SUP
1. BLOW
8 FORMAT(/.
1. ORDINATE INTERCEPT A OF REGRESSION LINE
2. SLOPE B OF REGRESSION LINE
3. CORRELATION COEFFICIENT R
4. SUM OF X
5. SUM OF Y
6. SUM OF X**2
7. SUM OF XY
8. SUM OF (X-XBAR)**2
9. SUM OF (Y-YBAR)**2
1. SUM OF (X-XBAR)*(Y-YBAR)
2. UPPER CONFIDENCE LIMIT ON SLOPE B
3. LOWER CONFIDENCE LIMIT ON SLOPE B
50 RETURN
END
11. APPENDIX D: TABLE OF INTEGRAL

\[ F_n(h) = \frac{1}{\sqrt{2\pi}} \int_{h}^{\infty} (s-h)^n e^{-\frac{1}{2}s^2} ds \]

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12. APPENDIX E: ERROR ANALYSIS

The calculation of wear volume from the fatigue wear equation involves several measured quantities, namely, sliding distance \( L \), coefficient of friction \( f \), standard deviation of asperity heights \( \sigma \), average radius of curvature of asperity tips \( \beta \), linear asperity density \( n^* \), and the fatigue parameters \( S_o \) and \( t \). Any error in the measurement of these will contribute to an error in the computed wear volume. Since \( f, \beta, \sigma, \) and \( \eta_L \) depend upon the sliding distance \( L \) (or sliding velocity) due to transfer of polymer to the counterface, the errors in these quantities are nonindependent but completely unrelated to \( S_o \) and \( t \). The parameters \( S_o \) and \( t \) are dependent on each other, hence the errors in this group will again be nonindependent but unrelated to the previous group. Thus the computation of wear volume is a case of mixed errors involving both the independent and nonindependent quantities.

The error in the calculated wear volume \( V \) may be expressed as (93)

\[
dV = \sqrt{(dV_1)^2 + (dV_2)^2}
\]

(E 1)

where

\[
dV_1 = \frac{3V}{3\eta_L} d\eta_L + \frac{3V}{3L} dL + \frac{3V}{3f} df + \frac{3V}{3\beta} d\beta + \frac{3V}{3\sigma} d\sigma
\]

(E 2)

and

\[
dV_2 = \frac{3V}{3S_o} dS_o + \frac{3V}{3t} dt
\]

(E 3)

The wear volume \( V \) for a sliding distance \( L \) is given by equation (29)
\[ V = \frac{\text{PL}_L \cdot \nu}{2} \left[ \left( \frac{f}{8} \cdot (4 + \nu) \pi + \frac{1 - 2\nu}{3} \right) \cdot t \left( \frac{1}{S_o} \right) \cdot t \left[ \frac{2E'}{\pi} \right] \cdot t^{-1} \left[ \frac{1}{\beta} \right] \cdot t^2 \left( \sigma \right) \cdot t^3 \left( \sigma \right) \right] \times \]

\[ \left[ \frac{F_3(h)}{F_1(h)} \right]^{t-1} \left[ \frac{F_0(h)}{F_1(h)} \right] \]

The differentiation of the above yields the following equations

\[ \frac{\partial V}{\partial n_L} = \frac{\text{PL}_L \cdot \nu}{2} \left[ \left( \frac{f}{8} \cdot (4 + \nu) \pi + \frac{1 - 2\nu}{3} \right) \cdot t \left( \frac{1}{S_o} \right) \cdot t \left[ \frac{2E'}{\pi} \right] \cdot t^{-1} \left[ \frac{1}{\beta} \right] \right] \cdot t^3 \left( \sigma \right) \times \]

\[ \left[ \frac{F_3(h)}{F_1(h)} \right]^{t-1} \left[ \frac{F_0(h)}{F_1(h)} \right] \]

(E 4)

\[ \frac{\partial V}{\partial L} = \frac{\text{PL}_L \cdot \nu}{2} \left[ \left( \frac{f}{8} \cdot (4 + \nu) \pi + \frac{1 - 2\nu}{3} \right) \cdot t \left( \frac{1}{S_o} \right) \cdot t \left[ \frac{2E'}{\pi} \right] \cdot t^{-1} \left[ \frac{1}{\beta} \right] \right] \cdot t^3 \left( \sigma \right) \times \]

\[ \left[ \frac{F_3(h)}{F_1(h)} \right]^{t-1} \left[ \frac{F_0(h)}{F_1(h)} \right] \]

(E 5)

\[ \frac{\partial V}{\partial f} = \frac{\text{PL}_L \cdot \nu}{2} \left[ \left( \frac{f}{8} \cdot (4 + \nu) \pi + \frac{1 - 2\nu}{3} \right) \cdot t \left[ \frac{4 + \nu}{8} \pi \left( \frac{1}{S_o} \right) \right] \cdot t \right] \times \]

\[ \left[ \frac{2E'}{\pi} \right]^{t-1} \left[ \frac{1}{\beta} \right] \cdot t^2 \left( \sigma \right) \cdot t^3 \left( \sigma \right) \times \]

\[ \left[ \frac{F_3(h)}{F_1(h)} \right]^{t-1} \left[ \frac{F_0(h)}{F_1(h)} \right] \]

(E 6)
\[
\begin{aligned}
(10.3) & \quad \left\{ \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] u_x + \frac{\partial u_y}{\partial z} \right\} \frac{z}{z^2-1} + g u_y \frac{z}{z^2-1} \\
& \quad - \left[ \frac{\mu}{\partial^2} \right] u_x + \frac{\partial u_y}{\partial x} - \left[ \frac{\varepsilon}{z^2-1} + \mu (\lambda + 4) \frac{\theta}{\varepsilon} \right] u_y \right\} \Lambda = \frac{3 \varepsilon}{\lambda \varepsilon} \\
(6.3) & \quad \left\{ \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \\
& \quad - \frac{\partial}{\partial z} \left[ \frac{\mu}{\partial^2} \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \frac{z}{z^2-1} = \frac{\partial}{\partial z} \frac{z}{z^2-1} \\
(8.3) & \quad \left\{ \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \frac{z}{z^2-1} \\
& \quad - \frac{\partial}{\partial z} \left[ \frac{\mu}{\partial^2} \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \frac{z}{z^2-1} = \frac{\partial}{\partial z} \frac{z}{z^2-1} \\
(7.3) & \quad \left\{ \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \frac{z}{z^2-1} \\
& \quad - \frac{\partial}{\partial z} \left[ \frac{\mu}{\partial^2} \right] \frac{z}{z^2-1} \left[ \frac{\partial}{\partial z} \left( \frac{z}{z^2-1} \right) \right] \frac{z}{z^2-1} \right\} \left\{ \left( \frac{\rho}{\varepsilon} \right) \frac{z}{z^2-1} \right\} \frac{z}{z^2-1} = \frac{\partial}{\partial z} \frac{z}{z^2-1} \\
\end{aligned}
\]
Considering the following variation in the measured quantities,

\( L = \pm 1\% \); \( S_o = \pm 1\% \); \( t = \pm 1\% \); \( \beta = \pm 5\% \); \( \eta = \pm 5\% \); \( \sigma = \pm 5\% \); \( f = 7\% \); we get

for the case of high density polyethylene sliding against the metal disk

\[ d\eta = \pm 2.55 \]

\[ dL = \pm 0.64 \times 10^5 \text{ mm} \quad \text{(for one hour sliding)} \]

\[ df = \pm 0.0266 \]

\[ d\beta = \pm 1.07 \times 10^{-3} \text{ mm} \]

\[ d\sigma = \pm 0.05 \times 10^{-3} \text{ mm} \]

\[ dS_o = \pm 4.944 \text{ N/mm}^2 \]

\[ dt = \pm 0.044 \]

Substituting the values from Tables 4.8 and 4.9 in equations (E 4) - (E 10) for high density polyethylene, we get

\[ \frac{\delta V}{\delta \eta} = 0.0180 \quad \quad \quad \quad \frac{\delta V}{\delta \sigma} = 657.70 \]

\[ \frac{\delta V}{\delta L} = 1.45 \times 10^{-7} \quad \quad \quad \quad \frac{\delta V}{\delta S_o} = -0.0083 \]

\[ \frac{\delta V}{\delta f} = 10.50 \quad \quad \quad \quad \frac{\delta V}{\delta t} = -2.19 \]

\[ \frac{\delta V}{\delta \beta} = -117.80 \]

Substitution of the above values and those from Equation (E 11) in Equations (E 2), (E 3), and (E 1) gives \( dV = 0.278 \text{ mm}^3/\text{hr} \). Similar calculations provided the error in wear rates as \( 1.20 \text{ mm}^3/\text{hr} \) at 1 m/s sliding speed for poly(methyl methacrylate) and \( 1.07 \text{ mm}^3/\text{hr} \) at 1 m/s and \( 2.67 \text{ mm}^3/\text{hr} \) at 2 m/s sliding speeds for poly(vinyl chloride).