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Heat transfer characteristics of simulated microelectronic chips under normal and enhanced conditions

Kyung-Am Park
Iowa State University

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HEAT TRANSFER CHARACTERISTICS OF SIMULATED MICROELECTRONIC CHIPS UNDER NORMAL AND ENHANCED CONDITIONS

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Heat transfer characteristics of simulated microelectronic chips under normal and enhanced conditions

by

Kyung-Am Park

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY
Major: Mechanical Engineering

Approved:

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For the Graduate College

Iowa State University
Ames, Iowa

1985
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LIST OF SYMBOLS

a  coefficient of boiling curve
A  cross section area
A_{\text{total}}  total surface area of the heat sink
A_{\text{base}}  base area of the heat sink
b  coefficient of boiling curve
C_1, C_2, C_3, C_4  coefficients of solution of differential equation
C_p  specific heat at constant pressure
d  diameter of microhole
D  diameter of cylinder
e  fin gap
G  vertical pitch between the heaters
g  gravitational acceleration
Gr^*  modified Grashof number \((g\beta q^4/\nu^2)\)
H  height of the test section
H'  non-dimensional height of the heater \((H[g(p_{\text{f}}-p_{\text{v}})/\sigma]^{0.5})\)
h  heat transfer coefficient
h_{\text{fg}}  latent heat
I  current
I  induced-convection parameter \((p_{\text{f}}\sigma/\mu^2)\)
I_0  modified Bessel function
k  thermal conductivity
k_p  thermal conductivity of the back side insulation
k_c  thermal conductivity of the cylinder
k_{\text{cu}}  thermal conductivity of copper
\( k_{cn} \) thermal conductivity of constantan
\( K_h \) thermal conductivity of heater
\( k_s \) thermal conductivity of the heat sink
\( k_t \) thermal conductivity of teflon
\( K_0 \) modified Bessel function
\( K_1 \) modified Bessel function
\( \varepsilon \) exponent of boiling curve
\( L \) length of the heat sink
\( L \) thickness of plexiglas insulation
\( L_1 \) characteristic length of thermocouple insulator
\( L_2 \) characteristic length of thermocouple insulator
\( LF \) heat loss factor (heat loss/heat generation)
\( n \) exponent of boiling curve
\( Nu_x \) Nusselt number at the center of heater (\( h_x/k \))
\( P \) horizontal pitch between the heaters
\( P \) perimeter
\( Pr \) Prandtl number (\( C_p\mu/k \))
\( q \) heat transfer rate
\( q'' \) heat flux
\( q''_{chf} \) critical heat flux
\( q''_{fc} \) heat loss through the foil
\( q'' \) heat generation rate per unit volume
\( r \) radius
\( R \) radius of cylinder
\( R' \) non-dimensional radius of cylinder (\( R\left[\frac{g(\rho_\ell-\rho_v)}{g}\right]^{0.5} \))
$r_w$  radius of single thermocouple wire without insulation

$r_1$  equivalent radius of thermocouple wires

$r_2$  equivalent radius of teflon insulation

$Ra_x^*$  local Rayleigh number ($Gr_x^* Pr$)

$s$  fin pitch

$t$  thickness of foil

$T$  absolute temperature

$T_b$  bulk temperature

$T_s$  saturation temperature

$T_w$  wall temperature

$T_w$  ambient temperature

$\Delta T$  temperature difference between wall and bulk fluid

$\Delta T_s$  superheat, $T_w - T_s$

$U$  overall heat transfer coefficient

$U_1$  overall heat transfer coefficient

$U_2$  overall heat transfer coefficient

$V$  voltage drop

$W$  width of the test section

$x$  space coordinate

$y$  space coordinate

$\alpha$  coefficient of $Ra_x$

$\beta$  coefficient of expansion

$\delta$  exponent of $Ra_x$

$\lambda$  wave length of ultrasonics

$\rho$  electric resistivity
\( \rho_f \)  

density of liquid

\( \rho_v \)  

density of vapor

\( \sigma \)  

surface tension

\( \mu \)  

dynamic viscosity of liquid

\( \nu \)  

kinematic viscosity of liquid

\( \xi \)  

wall temperature without thermocouple effect

**subscripts**

\( b \)  

bottom test section

\( bc \)  

heat loss to the back side (substrate)

\( i \)  

inside of the cylinder

\( m \)  

middle test section

\( o \)  

outside of the cylinder

\( \text{sub} \)  

subcooled

\( t \)  

top test section

\( tc \)  

heat loss to thermocouple wires

All properties are evaluated at the film temperature, \( (T_w + T_b)/2 \), unless otherwise noted.
I. INTRODUCTION

The development of microelectronic technology has required the resolution of complex thermal problems at both the component level and system level [1]. Overheating is a serious problem with microelectronic equipment. High operating temperatures decrease the lifetime of the components. Excessive temperature can also change the operating characteristics of the components in computers [2]. Current high packaging density designs maintain chip junction temperatures at acceptable levels by minimization of "internal" thermal resistance (conduction from junction to device surface), reduction in the "external" thermal resistance (convection from the device surface to larger, convectively cooled surfaces or by direct liquid or gas cooling of the device), and by maintaining a moderate "system-level" thermal resistance (temperature rise of the coolant) [3].

Although early programs at IBM [4, 5] and other companies [6] anticipated a need for direct liquid cooling (with phase change) of devices, it has not yet been necessary to use this technique in computers as the conduction cooling technology has evolved to accommodate high chip powers. An urgent problem has arisen, however, related to the cooling of devices during testing. In very general terms, the problem can be stated as the need to test individual chips and modules in a liquid bath at powers up to 10 W/chip (~ 5 mm x 5 mm) while maintaining junction temperatures below about 85 °C. This is compared to normal nucleate boiling limits of about 4 W/chip and 85 °C.
junction temperature. Testing takes places with the module (typically 100 chips) immersed in a bath of dielectric fluid. The present research has a number of objectives that will support the development of cooling technology for testing of microelectronic chips.

The first objective is to determine experimentally the heat transfer characteristics of plain chips (or simulated chips) under free convection or pool boiling conditions, individually or in arrays. Another objective is to study the enhancement of cooling during testing of chips and modules for supporting the development of systems which will reach the desired test power/junction temperature objectives within the specified constraints.

Fundamental studies of the effects of geometry on natural convection and boiling heat transfer are described in Chapter II. The important issue of critical heat flux is treated in Chapter III. Heat transfer from arrays of simulated microelectronic chips is discussed in Chapter IV. As enhancement techniques, ultrasonics and heat sinks were explored for microelectronic chip cooling. Ultrasonic agitation of the pool is reported in Chapter V. The heat sink work, involving microholes and microfins, is discussed in Chapters VI and VII. Heat sinks fabricated from commercial structured porous surfaces are considered in Chapter VIII. Finally, the entire study is summarized in Chapter IX.

The various chapters in this dissertation are basically self-contained. The background, literature, experimental apparatus, etc. are presented in each chapter.
II. EFFECT OF HEATER SIZE ON NATURAL CONVECTION AND BOILING

A. Introduction

Extreme miniaturization of electronic devices has reintroduced the thermal problem. The failure of microelectronic chips increases exponentially as the junction temperature increases. It is important in the packaging of these microelectronic chips to predict the heat transfer characteristics for all anticipated modes of heat transfer. Natural convection is an important mode of heat transfer for both air and liquid cooling.

Microelectronic chips should be modelled as uniform heat flux devices; due to their small size, the flow is laminar. The existing natural convection correlations, experimental and analytical, for uniform heat flux from vertical plates were obtained from heating surfaces that were large in at least one dimension (the width) [7, 8, 9]. The heat dissipation area of typical microchips is very small in two dimensions (about 5 mm x 5 mm). Also, a limited amount of data exists for such small distributed heat sources.

Baker [10, 11] got midpoint heat transfer coefficients for resistor chips, flush-mounted on a substrate, which were twice as wide as they were high: 4.6 mm x 2.3 mm, 9.8 mm x 4.9 mm, and 20 mm x 10 mm. The surfaces were placed vertically in air and liquid Freon-113 (R-113). His boiling curves with R-113, shown in Figure 1, illustrate the observed large increases in heat transfer coefficients above those
RESISTOR CHIPS
R-113
$T_s = 47.6 \, ^\circ C$

Figure 1. Baker's boiling curve [10]
expected for natural convection and boiling. Baker suggested qualitatively that the increases were due to leading edge and side flow effects. He also showed that conduction to the substrate was relatively unimportant.

Carey and Mollendorf [12] mentioned the increased complexity of the three dimensional flows associated with small heating elements. They investigated the temperature fields in an adiabatic wall above flush mounted circular and square heaters in water. Sufficient data were presented for one small square heater (4.7 mm x 4.7 mm) so that a single value of the heat transfer coefficient could be calculated. This coefficient was 110% higher than the predicted value.

The purpose of the present study was to clarify the dependence of heat transfer performance on heater size through a systematic experimental investigation in both natural convection and boiling.

B. Experimental Apparatus and Procedure

1. Experimental apparatus

Two working fluids, Freon-TF (R-113) and water, were used to determine the influence of test section size on natural convection heat transfer. The R-113 tests were extended to saturated boiling. R-113 was selected as its properties are similar to those of dielectric liquids commonly used for direct immersion cooling of microelectronic components. The R-113 experiments were done in an insulated tank 273 mm x 127 mm of 152 mm height with a plexiglas cover as shown in Figure 2.
Figure 2. Experimental apparatus
A pyrex beaker of 4 liter capacity (16 cm diameter) was used for experiments with distilled water. The distilled water was deaerated by preboiling. Room temperature conditions were reestablished throughout the pool before the natural convection experiments. The pool temperature was maintained close to room temperature (25 ~ 30 °C) during the experiments.

A circuit board (glass epoxy G-10) clad with copper on one side was the basis of the test section, as shown in Figure 3. The copper cladding in the center was removed by a razor blade to make room for the foil heater.

Three kinds of foil were used in this experiment: 25.4 μm thick steel, for R-113, and 10 μm thick constantan and 12.7 μm thick nichrome for both water and R-113. M610 epoxy resin (Measurements Group, Inc.) was used to bond the foil to the circuit board. The foil and copper cladding were then very carefully soldered to the 25.4 μm brass foil for the electrical connections. The resulting small protrusion of the power connections should not have affected the heat transfer. Direct-current power to the test section was supplied by a 50 ampere power supply. The final assembly simulated a chip flush with the substrate. The area of exposed surface was measured by an optical microscope. The surface was left in the original polished condition and cleaned with Freon TF degreaser. Plexiglas was added for insulation and to guide the thermocouple wires to the back side of the circuit board.
Figure 3. Test section details
A single thermocouple was placed at the back center of each foil: the junction was electrically insulated from the heater. The midpoint of the heaters was chosen as being representative of the thermal performance of a chip. Two types of thermocouples (iron-constantan 40 gauge and copper-constantan 36 gauge) were used for this experiment. The copper-constantan thermocouples were used in all but five test sections.

The main variable in this experiment was the width of the heated surface. Two different heights (5 mm and 10 mm) were employed with variations in width from about 2 mm to 70 mm. In all cases, a smooth leading edge was provided by a 50 mm section of the circuit board. A similar strip was at the trailing edge. All test sections were vertically oriented about 60 mm below the free surface of the pool.

A heater built into the side of the tank was used to keep the pool saturated during boiling experiments. The heater was controlled with a 0 ~ 140 V variac. The circulation of the working fluid induced by side heating could affect data for natural convection and even boiling at low heat flux. Accordingly, the side heater was off 30~40 seconds to reduce the circulation before the data were recorded. Since the circulation could not be entirely eliminated without waiting so long that the pool cooled down, the low heat flux, natural convection portions of the boiling curves were not included with the natural convection results.
2. **Data reduction**

The nominal heat flux, calculated from the voltage drop across the heater and current across a calibrated shunt, was corrected for the back side heat loss and heat loss to the electrical connections (Appendix). These heat loss corrections, evaluated at the midpoint of the test section, were about 10 ~ 15\% for natural convection (5 mm x 5 mm surface) and negligible for boiling. The main correction was heat loss to the back side. The very small temperature drop between the thermocouple junction and the outer foil surface was neglected (Appendix). It was confirmed by calculation and experiment for both types of thermocouples that the conduction loss through the leads was negligible (Appendix).

Heat flux profiles were not uniform because of the heat loss to electrical connections at the edges of the heater. With this type of heating, it is not possible to get uniform heat flux conditions for small heat sources. This is discussed in the Appendix. The Nusselt number was thus evaluated at the midpoint of the test section. In accordance with this local assessment of heat transfer performance, the modified Grashof number was also evaluated at the midpoint.

Properties of working fluids were evaluated at the film temperature, the average of the local surface and pool temperatures. The properties as functions of temperature were obtained from Jensen [13] for R-113 and Morcos and Bergles [14] for distilled water.
C. Experimental Results and Discussion

1. Natural convection data

The experimental data for the 5 mm and 10 mm high heaters in R-113 are shown in Figure 4. The base line in this figure is the Fujii and Fujii correlation of laminar boundary layer solutions for a vertical surface with constant heat flux [9]:

\[
\frac{\text{Nu}_x}{x} = \left(\frac{\text{Pr}}{4 + 9\sqrt{\text{Pr} + 10\text{Pr}}}\right)^{1/5}(\text{Gr}_x \times \text{Pr})^{1/5}
\]

For \( \text{Pr} = 7 \), the \( \text{Pr} \)-function is 0.59, essentially the same as the constant value of 0.6 found empirically by Vliet [8].

The data of Ma and Bergles [15] for a 5 mm x 5 mm test section, using a similar type of construction, are in very good agreement with the present data. The heat transfer coefficient clearly increases as the width is reduced. The chip size (5 mm x 5 mm) test section has a heat transfer coefficient 80 to 100% greater than that for the widest test section. A similar width effect was obtained for the 9.86 mm high test section. This suggests that the height effect is taken care of by the usual \( \text{Nu}_x \times \text{Ra}_x \) presentation. It is noted, however, that the heat transfer coefficients for the widest test section are 20% higher than predicted by Equation (1).

With water, the experimental results for varying width with two different heights are shown in Figure 5. The single data point of Carey and Mollendorf (4.7 mm x 4.7 mm heater) is somewhat higher than interpolated and extrapolated data of the present study. The width
FOIL HEATERS
R-113
$T_b \approx 27^\circ C$
Pr = 6.9 - 8.3

Figure 4. Natural convection data for test sections of various width (R-113)
Figure 5. Natural convection data for test sections of various width (water)
effect with water is much less pronounced than that for R-113. There was no effect of heating surface material on either set of natural convection data, as expected.

2. Qualitative explanation for wide plate deviation from the classical solution

It is apparent from Figures 4 and 5 that the present data are not in good agreement with the "textbook" boundary layer similarity solution. Consider first the 20% deviation of the widest test section. While not really addressed in the very limited experimental literature for uniformly heated test sections, the extensive literature for uniform temperature heaters does consider this deviation in great detail.

Baker [10, 11] assumed a uniform temperature chip and suggested that the leading edge effect is similar to that occurring in forced convection. This explanation does not seem to be correct.

Scherberg [16] has called attention to the inadequacy of the boundary layer similarity solution in the vicinity of the leading edge of real heaters. According to this simplified theory, the isotherms converge at the leading edge and an infinite heat transfer coefficient is predicted at this point. In reality, there is upstream conduction to nose pieces or, in the present case, the substrate. Furthermore, in this region of very low velocity, upstream conduction in the fluid is significant. The main effect is starting of the free convection flow below the heater; the velocities at the heater are higher, but the effect decreases with increasing distance from the leading edge.
Scherberg [16, 17, 18] obtained integral solutions that incorporated these effects; however, the emphasis was on the formidable mathematical details rather than predicting heat transfer coefficients.

Most investigators have considered only the fluid flow and not the conjugate problem involving the heater. Mathematically, the boundary layer solutions represent asymptotic solutions to governing equations that are valid at very large Grashof number. Accordingly, numerous perturbation solutions that are valid at small and moderate Grashof number have been attempted, e.g., Suriano et al. [19]. Increased heat transfer coefficients are predicted. Complications due to the discontinuous boundary condition at the plate edge are also recognized, e.g., Messiter and Linan [20].

The most recent summary of the analytical situation is presented by Martynenko et al. [21]. As shown in Figure 5, the average predicted coefficients (perturbation solution) for a uniform wall temperature plate increase when the interaction with the external flow is taken into account and increase further when the upstream induced flow, due to conduction in the fluid, is taken into account. Trailing edge and wake effects reduce the heat transfer coefficient, an effect that is opposite to that predicted by Messiter and Linan [20]. In any event, there is ample theoretical evidence for increased heat transfer coefficients and, as shown Figure 6, the data seem to support predictions. The origin of these data is not noted by Ede [22]; however, the other experimental evidence is considerable.
Figure 6. Comparison between calculation and experiments on the basis of the mean heat transfer coefficient: (1) boundary-layer theory, (2) with external flow taken into account, (3) with the leading edge effect taken into account, (4) with trailing edge and wake effects taken into account, (5) experiment [22] (from Martynenko et al. [21].)
Goldstein and Eckert [7] observed that the temperature of a uniformly heated plate, as obtained by an interferometer, did not agree with boundary layer predictions near the leading edge. This was attributed to conduction to the electrical connections, and the leading edge data were ignored when the comparison with boundary layer theory was made. Eichhorn [23] found that free convection boundary layer originated below a uniform temperature plate, due to heating of the upstream insulating strip. Brodowicz [24] studied uniform temperature plates with various leading edge geometries. He confirmed establishment of free convection upstream of the normal heated surface, but did not report heat transfer coefficients. Gryzgoridis [25] obtained local heat transfer coefficients interferometrically that conclusively demonstrated the increase in heat transfer coefficient at low Grashof number.

Qualitative experiments were conducted as a part of the present program. Dye was injected into the water with near-zero velocity at the nose of the heater. Dye streamers confirmed that there was an upstream velocity at the leading edge due to conduction to the substrate and fluid.

3. Correlation of the width effect

Although three-dimensional effects have been observed with large plates, no analyses or correlations have been proposed. In their tests of large uniformly heated, vertical plates, Fujii and Imura [26] observed that wall temperatures at the center were higher than toward
the sides. Wall temperatures at sides were measured at 15 mm (100 mm x 50 mm high) and 20 mm (150 mm x 300 mm high) from the side edges. They speculated that this was due to transverse conduction in the fluid.

Oosthuizen [27] observed that the average heat transfer coefficients for a vertical, isothermal heater (203 mm x 813 mm high) in air were higher than the predicted values. This was attributed to width effects even though the heater was quite wide. The width enhancement of the heat transfer coefficients was smaller as Ra increased. He correlated the width effects with nondimensional Ra based on the width. He further confirmed these effects with two isothermal heaters (254 mm high, 203 mm and 762 mm wide) [28]. Data for the 203 mm wide heater were higher than those for the 762 mm wide heater. Recently, Oosthuizen and Paul [29] numerically solved the boundary layer equations with additional terms for the transverse direction in the momentum and energy equations. In general, the width effects observed by Oosthuizen are qualitatively similar to the results of the present study. However, it is difficult to make use of his results due to the very large wide heaters used as compared with the present heaters.

Induced flow at the sides of the present small heaters was also observed with side injection of dye. A Bernoulli effect due to lowering of pressure in the plume induces a flow from the sides of the heater.

The width effect depends on the relation of the transverse conduction heat transfer to the energy convected in the vertical direction. The trends shown in Figures 5 and 6 are expected; that is,
the Nusselt number increases as the width decreases, but the percentage increase is less as the Rayleigh number increases. The experimental results show that the width effect is negligible at about 70 mm. The width was nondimensionalized by dividing it by this asymptotic width. This nondimensionalized width is adopted as the parameter characterizing the conduction effect. The general form of the correlation is chosen as

$$\text{Nu}_x = \alpha \text{Ra}_x^\delta$$  \hspace{1cm} (2)

The values of \(\alpha\) and \(\delta\) were evaluated by the Churchill-Usagi [30] technique so that the Nusselt number approaches the limiting value for \(W = 70\) mm as the width is increased. Marquart nonlinear regression (Statistical Analysis System package) was used to obtain optimized constants and exponents. For R-113

$$\alpha = 0.906[1 + \frac{0.00111}{(W/W_\infty)^{3.965}}]^{0.2745}$$  \hspace{1cm} (3)

$$\delta = 0.184[1 + \frac{2.64 \times 10^{-5}}{(W/W_\infty)^{9.268}}]^{-0.0362}$$  \hspace{1cm} (4)

where

\(W_\infty = 70\) mm

and for water

$$\alpha = 0.906[1 + \frac{0.09886}{(W/W_\infty)^{4.08}}]^{0.04654}$$  \hspace{1cm} (5)

$$\delta = 0.184[1 + \frac{2.219 \times 10^{-9}}{(W/W_\infty)^{9.834}}]^{-0.003966}$$  \hspace{1cm} (6)

The comparison between predicted and experimental \(\text{Nu}_x\) is shown in Figure 7. More than 95% of the data are within ± 10% of the correlation.
Figure 7. Comparison between predicted and experimental Nusselt number
It was not possible to obtain a single correlation for both R-113 and water. The reasons for this are not clear at the present time. More work with additional fluids is required as it would be desirable to have general correlation. In any event, the R-113 correlation should be valid for the dielectric fluids of interest in immersion cooling of microelectronic components.

4. Boiling

The boiling curve shown in Figure 8 for a 5.1 mm x 4.6 mm high surface was obtained under degassed saturated conditions in R-113 at 1 atm. As the heat flux was increased the first time, there was about 9 K overshoot. This overshoot is a characteristic of R-113, which tends to deactivate nucleation sites due to a near-zero contact angle; however, Baker did not report any overshoot, as shown in Figure 1. The fully developed boiling curve is in good agreement with other data of the present research program, considering the differences in surfaces (nichrome foil versus constantan foil) [15].

Boiling was observed first at the top of the heater, perhaps due to nucleation sites at the edge and the thick thermal boundary layer. Since the thermocouple was still in the nonboiling region, the measured superheats remained close to the natural convection values. Nucleation slowly propagated from top to bottom as the heat flux was gradually increased. Then, a relatively small overshoot (9 K) took place when boiling propagated to the location of thermocouple junction. Sometimes the superheat was increased without the nucleation at the top; then, the
FOIL HEATER
H = 4.6 mm, W = 5.1 mm
R-113
Tₜ = 46.4 °C
INCREASING POWER

BOILING STARTS AT TOP
PREDICTED NATURAL CONVECTION, EQ. (1)

Figure 8. Comparison of boiling curves for foil heaters
boiling took place suddenly over the entire heating surface and gave about 19 K overshoot, as shown in Figure 9.

On other occasions, a double overshoot was observed, as shown in Figure 10. This two-step overshoot could be due to lack of active boiling sites near the location of thermocouple junction at low heat flux. It is speculated that the first overshoot took place when there were a few boiling sites below the location of thermocouple. The second overshoot occurred when the boiling sites near the location of thermocouple finally became active. While decreasing power, sometimes the temperature increased in a "reverse overshoot" (Figure 10). This reverse overshoot was observed only a few times and was apparently due to deactivation of some nucleation sites.

The boiling curves with wider test sections (60 mm wide x 5.6 mm high and 40 mm wide x 5.6 mm high) are presented in Figure 9. There is negligible difference in the developed boiling curves for these two test sections, and the data are excellent agreement with the data for the 5.1 mm wide test section (Figures 8 and 9). This insensitivity to width is contrary to the result of Baker shown in Figure 1. Also, the boiling curves of the present studies are in good agreement with data for fluorocarbon liquids and surface-independent correlations derived from these data [31, 32]. Baker's superheat is uncommonly high for the largest heater, suggesting experimental errors.
Figure 9. Boiling data displaying large temperature overshoot
Figure 10. Comparison of boiling curves for different width heaters.
D. Conclusions

The effect of test section width in natural convection has been documented with small heaters in R-113 and water. The heat transfer coefficient increases with decreasing width, with the effect greater in R-113 than in water. The data for the widest test section are about 20% above the classical prediction. This is attributed to leading edge effects which have been documented analytically and experimentally. The present correlations of the data appear to be the first that have been attempted.

While increasing power during boiling tests, the temperature overshoot depends strongly on the locations of the boiling sites on the heating surface. Three types of overshoot were observed. Reverse overshoot was observed with decreasing power at low heat flux as nucleation sites were deactivated. The boiling curves for different widths in R-113 agreed very well; the width effect reported by Baker [10] is incorrect.
III. EFFECTS OF HEATER SIZE ON CRITICAL HEAT FLUX

A. Introduction

The prediction of the critical heat flux (CHF) is of considerable importance to estimate the maximum power of electronic components. CHF in pool boiling has been studied extensively with experimental and analytical methods as surveyed by Efimov [33] and Bergles [34].

The work done on CHF is divided into hydrodynamic and nonhydrodynamic aspects. The hydrodynamic model originated by Kutateladze [35] was analytically clarified with instability theory by Zuber [36] and Zuber et al. [37]. The hydrodynamic theory provides a reasonable prediction of critical heat fluxes for many situations of practical interest. Sun and Lienhard [38] and Lienhard and Dhir [39] proposed modifications to the theory to explain the significant increases in CHF as heater size is decreased. Of particular interest to this study, relatively few CHF points for the vertical heaters with one side insulated have been reported [39, 40, 41]. A semi-analytical correlation for CHF was deduced from the model for the cylinder of Sun and Lienhard [38] by Lienhard and Dhir [39]:

\[
\frac{q''_{chf}}{q''_{chf,z}} = \frac{1.4}{(H')^{1/4}} \quad \text{for } H' < 6 \tag{7}
\]

and

\[
\frac{q''_{chf}}{q''_{chf,z}} = 0.9 \quad \text{for } H' > 6 \tag{8}
\]

where

\[
H' = H\left[\frac{g(\rho_f - \rho_v)}{\sigma}\right]^{1/2}
\]

and
\[ q''_{\text{chf},z} = 0.131 \, h_{fg} \rho_v \left[ \sigma (\rho_f - \rho_v) g / \rho_v \right]^{1/4} \]

which is the CHF prediction of Zuber [36]. The data set of Adams [40] for wide heaters was used to justify the asymptotic value of the correlation for large \( H' \).

Nonhydrodynamic aspects have been discussed by many investigators [41, 42, 43]. It was reported that CHF depends on the properties, thickness, and surface condition of heaters as well as the type of power [34]. Carne and Charlesworth [41] reported CHF data for n-propanol at atmospheric pressure on four kinds of metal strips. CHF increased by 100% as much as the thermal conductivity and thickness of the strips increased. According to the data of Carne and Charlesworth, the CHF data of Lienhard and Dhir [39] should be affected since their heater was only 0.009 in. (0.23 mm) thick nichrome foil. But Lienhard and Dhir stated that they expected no premature burnouts due to low-thermal capacity effects.

Of particular interest to this study is the effect of width on CHF. Both the width and the height of small vertical heaters with one side insulated were varied systematically. Statistical correlations for height and width effects are suggested for these heaters.

B. Experimental Apparatus and Procedure

The main variables in this experiment were the width and height of the heaters as shown in Figure 11. A variation of the height of the
Figure 11. Test section for variations of (a) the height, (b) the width with fixed height, and (c) the height with fixed width.
heaters from 1 mm to 5 mm was tested for wide test sections. The width varied from 2.5 mm to 70 mm for a fixed height of about 5 mm. The tall heater with a fixed width of about 5 mm was tested with a variation of the height from 5 mm to 80 mm.

Visual observations were emphasized to observe the vapor formation near CHF. The test section was inserted in a plexiglas tank, 14 cm x 24 cm x 21 cm high as shown in Figure 2. The R-113 in the tank was kept at saturated conditions with a heating coil which circulated hot water from a constant temperature water bath. The front and one side of the tank were used to observe the boiling and to take pictures. The other sides were insulated with fiberglass.

Still photographs were obtained with a Canon AE-1 camera to which was attached a zoom lens 85 mm ~ 205 mm or 135 mm telephoto lens with close-up rings (Nos. 1, 2, 4). The shutter speed was 1/1000 second and the aperture was about f/4. Two 100 W frosted lights from the top and two 200 W flood lights from the bottom were used. With this arrangement, good pictures were obtained with Ektachrome 160 slide film.

The procedure of conducting experiments was similar to that described in Chapter II. Corrections to the nominal electrical determination of heat flux were negligible because of the high heat transfer coefficient at critical heat flux.

Each test section was used only one time to measure CHF. The power was shut off as soon as possible to prevent R-113 decomposition at high temperature after CHF data were obtained. After about four CHF tests
were conducted with approximately one-half gallon of R-113, new R-113 was used. The experimental results for CHF were repeatable.

C. Experimental Results and Discussion

One vapor column was observed until the width was reduced to less than 10 mm. There were two vapor columns in the 20 mm width, and four vapor columns in the 40 mm width test section. The number of columns agreed with the hydrodynamic model of Lienhard and Dhir except that columns were discontinuous and sometimes it was difficult to say how many columns there were. The shapes of the vapor masses differed according to the geometry. Big vapor masses covered first the top surface and then propagated toward bottom. Nucleate boiling took place at the bottom even after the CHF condition had occurred at the top.

There were black spots above the center on the nichrome or constantan foil test sections after the CHF was exceeded. These black spots resulted from carbon deposits on the heaters due to high temperature decomposition of R-113. The test sections were not physically destroyed except for the 1 mm and 2 mm high test sections. With these heaters, the wall temperature increased suddenly over about 100 K and the foil was separated from the substrate due to deterioration of the epoxy bond. All stainless steel heaters were physically destroyed.

Experimental data are tabulated in the Appendix. The experimental results for the short and wide strip test sections (W > 20 mm) generally
agreed with the data of Lienhard and Dhir [39], Adams [40], and Carne and Charlesworth [41]. Lienhard and Dhir did not compare the data set of Carne and Charlesworth with their correlation. All available experimental data are collected in Figure 12. A statistical correlation (Churchill-Ugasi type [30]) was obtained using Statistical Analysis System (SAS) package. The Marquart nonlinear regression method was used with $10^{-4}$ convergence criteria. The correlation of all the available data shown in Figure 12 is

$$\frac{Q^{n}_{chf}}{Q^{n}_{chf, z}} = 0.8604 \left[1 + \frac{152.2}{H'}^{3.291}\right] 0.1410$$

The asymptotic value of Lienhard and Dhir was 0.9, but the statistically accurate asymptotic value is 0.86. Also, Lienhard and Dhir compared their correlation with only a few experimental data with small dimensionless height ($H' < 6$). It is apparent that the differences between experimental data for R-113 and the prediction of Lienhard and Dhir become significant as the dimensionless height is reduced, as shown in Figure 12. It seems that there are no effects of thermal conductivity and thickness of the heaters in the present data sets because the data agree generally with the reported data [39, 40].

CHF data for the 4.5 mm ~ 5.1 mm high heaters were collected to determine the effects of variation of the width. The induced-convection parameter [35]

$$I = (p_f^{\nu_\sigma}/\mu^2)^{1/2}$$

(10)
Figure 12. Comparison between all available CHF data and correlation for the wide heaters.
was chosen to nondimensionalize the width. CHF increased significantly as the width of the test section decreased, as shown in Figure 13. Induced flow from sides due to the Bernoulli effect is a possible reason for increasing CHF for one vapor jet. Similar statistical methods were used to obtain the correlation for the width effect of the heater with a given asymptotic value (0.9320) calculated with equation (9) for the 5 mm height.

\[
\frac{q''_{chf}}{q''_{chf,z}} = 0.9320 \left[ 1 + \frac{52.22}{1.0177} \right]^{3.723}
\tag{11}
\]

The burnout location for the 5 x 10 mm high test section was at the top, but the burnout locations for the 20 mm, 40 mm, 60 mm, and 80 mm high test sections (about 5 mm width) were not the top, as depicted in Figure 14. The burnout locations were similar for 60 and 80 mm high heaters. As discrete bubbles moved up from the bottom of the heater, they coalesced on the heater. As this unstable vapor mass (one vapor jet) moved up, it prevented liquid from feeding the heater. Thus, burnout did not take place only at the top of the tall heater. Critical heat flux decreased as the height (5 mm width) increased, as shown in Figure 15. CHF approached 0.7 \( q''_{chf,z} \) as an asymptotic value. CHF data for the tall heater are about 30% lower than the hydrodynamically predicted value and about 20% lower than that for the wide heater. The statistical correlation for the height effect (about 5 mm width) is

\[
\frac{q''_{chf}}{q''_{chf,z}} = 0.6989 \left[ 1 + \frac{241300}{H^{4.030}} \right]^{0.08717}
\tag{12}
\]
Figure 13. Width effect for CHF (about 5 mm height)
Figure 14. Relative burnout position (a) 5 mm x 5 mm high,
5 mm x 10 mm high, (c) 5 mm x 20 mm high, (d) 5 mm
x 40 mm high, and (e) 5 mm x 80 mm high
Figure 15. Height effect for CHF for about 5 mm width
It has been reported that CHF for the vertical cylinder is about 20 ~ 30% lower than that for horizontal cylinders [42]. It is speculated that the mechanisms of burnout for the tall heater and vertical cylinder are similar. The CHF behavior for neither case can be explained by the hydrodynamic model.

One vapor mass is sufficient to interrupt liquid feed and cause CHF for the tall and narrow heater, but for the tall and wide heater there is adequate area for the electric current to flow even if one hot spot (high electric resistance) occurs. This is a possible reason for the difference of asymptotic values of the wide heater (0.86) and tall heater (0.7). Of course, this effect depends on where the bus connections are. It is expected that the CHF for the tall heater could increase if the power were connected at the sides instead of at the top and bottom. CHF for the tall heater is useful to estimate CHF as the asymptotic value for upper heaters of an array of microelectronic chips.

The height and width effects for CHF are plotted in Figure 16. CHF decreases as the height and width increase. CHF for a specific size of the heater can be estimated from this figure.

D. Conclusions

CHF data were obtained with variation of the width or height for vertical heaters with one side insulated. CHF increases as the height or width become small.
Figure 16. Composite plot for height and width effects for CHF
By statistical methods, nondimensionalized critical heat fluxes were correlated as a function of the nondimensionalized height or width. The maximum heat flux for microchips can be predicted by the correlations.
IV. ARRAYS OF SIMULATED MICROELECTRONIC CHIPS

A. Introduction

This section reports the experimental study of natural convection and boiling from arrays of small heating surfaces simulating microchips mounted on a circuit board. The interactions among microchips is of considerable importance in predicting the thermal behavior of the chips.

In reviewing the existing literature, no previous experimental studies of arrays of small heaters on a vertical surface have been found. Sparrow and Faghri [44] numerically solved boundary layer equations for the problem of two in-line wide vertical flush mounted heaters (assumed isothermal) for natural convection in air. Jaluria also numerically solved this problem for multiple wide heaters [45, 46] and experimentally and numerically studied two line sources [47]. He also solved the more general equations for two in-line wide vertical heaters using finite difference methods [48]. The difference between the solutions of boundary layer theory and the more general equations increases as Grashof number decreases.

Experimental studies on heater interactions were done with arrays of wires [49, 50] and small tubes [51, 52]. The general observation is that heat transfer at the upper heaters is enhanced by the buoyancy driven flow of the lower heaters; however, the enhancement is partially offset by the higher fluid temperature. Also of related interest, there are many analytical and experimental works on the plumes induced from single wide heat sources [12, 53-56].
The development and interaction of the plumes of small heat sources is very complicated; thus, it is not easy to solve the problem numerically. A practical first step is to get physical understanding by experimentation.

B. Experimental Apparatus and Procedure

Natural convection and saturated boiling were studied using the basic apparatus for single flush chips. The main difference was that multiple foil heaters (5 mm × 5 mm) were installed on strips of circuit board. These strips were then stacked with various combinations of plain circuit boards and sandwiched with copper bus bars and plexiglas retainers as shown in Figure 17. To avoid contact resistance problems, the heaters were joined at both ends with soldered shorting strips. The staggered surfaces were arranged simply by sliding the heater strips. Arrays of in-line and staggered flush surfaces are shown in Figure 18 (a, b, c).

Microelectronic chips normally protrude from the substrate. To better conform to this real situation, an experimental program was also developed to study single, protruding simulated chips and an array of such chips. Small pieces of the bare circuit board were added to simulate the shape of the microchips as shown in Figure 18 (d, e). The chip model (4.9 mm × 5.3 mm wide) was built out so that the face of heating foil was 1.1 mm from the substrate. This is comparable to actual chips studied previously [57, 58].
Figure 17. Test-section assembly for multiple heaters. (a) front view, b) view from left
Figure 18. Schematic layouts of flush heaters (a, b, c) and protruding heaters (d), and protruding heater details—top view (e).
Multiple heaters were tested individually to assess any bias resulting from thermocouple installation and other construction variables. The heat transfer coefficients fell within a band of ± 3.5% from average values for the flush heaters and ± 3% for the protruding heaters. This is due to the usual experimental uncertainty; no significant bias was observed. Also, the resistance of each heater was measured, since variations in resistance would affect the individual heater power during multiple heater operation. The variation in resistance about the mean was ± 3.5% for both types of heaters.

Natural convection tests were done at room temperature in R-113. Pool boiling experiments were done at saturated conditions in R-113. Once again, the Nusselt number and the Rayleigh number were determined at the heater midpoint.

C. Experimental Results and Discussion for Single-Phase Tests

1. Flush surfaces

An in-line array of two test sections was tested with the variation of distance between the heating surfaces ranging from G/H = 1.42 to 9.94. Typical results for G/H = 2.46 are presented in Figure 19. The top test section has a substantially lower heat transfer coefficient.

The ratios of \( \frac{\text{Nu}_x}{\text{Ra}_x^{1/5}} \) for the top and bottom test sections are plotted for all values of G/H in Figure 20. This ratio compensates for the slight differences in heat flux and film fluid properties between top and bottom heaters, due to the different substrate heat loss.
Figure 19. Typical data for two flush in-line heaters
Figure 20. Heat transfer coefficient ratios for two flush in-line heaters (numerical studies represent average values)
correction. The average values and the ranges for all ratios tested are given; however, the variations of these ratios are very small. As the distance between the test sections increases, the heat transfer coefficient for the top surface increases until \( G/H \) is about 3.5. The heat transfer coefficient ratios then remains constant at about 0.9 for higher values of \( G/H \).

The present results are compared with the analytical results for wide strips in air. Both Sparrow and Faghri [44] and Jaluria [45] found that the average heat transfer coefficient on the upper surface exceeded that for the lower surface at large \( G/H \). The results of Sparrow and Faghri [44] and Jaluria [45, 46] are quite different. The boundary conditions, uniform wall temperature and uniform heat flux, respectively, should not make a difference due to use of ratios. Furthermore, the use of the average values should not pose a problem as the average coefficient for small plates is close to the midpoint value. It is important to note that Jaluria's numerical results [48] obtained by solving the more general equations, are lower than results obtained by solving the boundary layer equations [45, 46]. The implication is that as the assumptions for numerical methods become closer to real conditions, the numerical results become closer to the experimental results.

Physically, the heat transfer at the top surface is influenced by the velocity and temperature profiles of the plume from the bottom heater. Apparently, this influence extends beyond the maximum \( G/H \)
considered in the present test. It is expected that the coefficient ratio will increase beyond \( \frac{G}{H} \approx 10 \) and eventually approach 1.0. We have no explanation for the intermediate asymptote of 0.9. Carey and Mollendorf [12] reported a monotonic decrease in surface temperature above a flush heat source; hence, no intermediate asymptote is to be expected.

An in-line array with three test sections was studied with variation of the distance between heaters. One set of data is shown in Figure 21. The data for the bottom surface are again similar to those for a single test section. The heat transfer coefficient clearly decreases with an increasing number of vertical heaters, but the decrease is less as the distance between heaters increases, as shown in Figure 22. The ratios of coefficients for the middle to the bottom heater are in approximate agreement with the above results for two heaters. The differences could be due to the small construction differences noted earlier. The ratios of coefficients for top and middle or middle and bottom surfaces are included in Figure 22. It is clear that for all cases considered, as the distance between heaters increases, the heat transfer coefficients for the upper surfaces increase.

The results for arrays of staggered surfaces are shown in Figure 23 (\( \frac{P}{W} = 0 \) represents the in-line case). In all tests, the bottom heater had about the same heat transfer coefficient while the top heater responded differently. When \( \frac{G}{H} \) is small and \( \frac{P}{W} \) is very large, the
Figure 21. Typical data for three flush in-line heaters
Figure 22. Heat transfer coefficient ratios for three flush in-line heaters: middle/bottom and top/middle
Figure 23. Heat transfer coefficient ratios for staggered arrays of two flush surfaces.
thermal plume from the bottom heater does not extend to the top offset heater, and the ordinate should approach unity. This suggests that if the data were extended to large P/W, the curves would rise and asymptotically approach 1.0.

At small G/H, the increase in the coefficient for the top heater could be due to removal of that heater from the high temperature of the plume. At large G/H, the plume temperature has decayed to the point where it has little effect; however, the beneficial transverse conduction heat transfer for the top heater is reduced by the lower plume. The heat transfer coefficient thus decreases.

2. **Protruding surfaces**

As shown in Figure 24, the natural convection data for the protruding surfaces are about 14% higher than those of the flush surfaces and about 120% higher than the predicted values. The increase in heat transfer coefficient above that for the flush surface is likely due in part to increased flow disturbance at the leading edge and trailing edge. The side flow hitting the heated surface could have a higher velocity for the protruding surface than for the flush surface. That would also increase the heat transfer coefficient.

Experimental results were obtained with two in-line protruding heaters with variation of spacing from G/H = 1.15 to 7.93. Typical results for G/H = 2.03 are presented in Figure 25. The data for the bottom heater agreed well with results for the single test section. Heat transfer coefficients for the top surface are higher than those for
Figure 24. Natural convection data for a single protruding heater
FOIL HEATER
R-113
$T_b \sim 27 \, ^\circ C$
$Pr = 6.9 - 8.3$

Figure 25. Typical data for two protruding in-line heaters
the bottom surface. The coefficient ratios for the top and bottom test sections are plotted for all values of G/H in Figure 26. The average values and the ranges are given; once again the fluctuation of these ratios is very small. As the space between test sections increases, the heat transfer coefficient of the top surface increases. It is quite significant that the top coefficient is always greater than the bottom coefficient; however, at large G/H it is expected that the ratio will return to unity. The enhancement is most likely due to the leading edge disturbance at the top heater.

D. Experimental Results and Discussion for Pool Boiling Tests

Boiling tests were carried out with a single protruding surface or with two in-line surfaces (flush or protruding) with fixed spacing. The boiling curve for the single protruding surface agreed with that for the single flush surface, as shown in Figure 27. There was a small difference in boiling with decreasing heat flux at low heat flux; this could be the result of small differences in the distribution of boiling sites.

Turning to the array data, boiling inception occurred earlier with the top surfaces than with the bottom surfaces, as shown in Figure 28 for the flush surfaces and Figure 29 for the protruding surfaces. Thus, the top heaters had smaller overshoots than the bottom heaters. This is apparently due to the thicker boundary layer for the top heater because of the plume from the bottom heater. The established boiling curves for
Figure 26. Heat transfer coefficient ratios for two protruding in-line heaters.
Figure 27. Boiling data for single protruding heater compared with single flush heater.

FOIL HEATER
H = 4.6 mm, W = 5.1 mm
1.1 mm PROTRUDING HEATER
R-113
T_s = 46.7 °C
○ INCREASING POWER
● DECREASING POWER

\( q'' \), W/m^2

\( \Delta T_s \), K
Figure 28. Boiling data for two flush in-line heaters
Figure 29. Boiling data for two protruding in-line heaters
either heater in an array were essentially the same for flush and smooth surfaces.

E. Conclusions

An experimental study of arrays of small (5 mm \times 5 mm) heaters was carried out in natural convection and boiling. The natural convection tests utilized two or three heaters in-line or staggered.

With in-line flush heaters, the heat transfer coefficients for the upper heaters are lower than those for the bottom heater. As the distance between the heaters increases, the heat transfer coefficients for the upper surfaces increase. In a staggered array, the heat transfer coefficient for the top heater increases as transverse distance between heaters increases at small G/H; however, the opposite effect is observed at large G/H.

In natural convection, the heat transfer coefficient for a single protruding heater is about 14\% higher than that for a flush surface. Heat transfer coefficients for the top heater in an array are higher than those for the bottom heater, in contrast to the results for the flush heaters, and increase as the distance between heaters increases.

In boiling from two in-line flush or protruding heaters, the inception of boiling for the top surfaces took place at lower superheat than for the bottom surfaces. There is very little difference in the established boiling behavior for the top and bottom surfaces, for either flush or protruding surfaces.
V. ULTRASONIC ENHANCEMENT OF CHIP COOLING

A. Introduction

Ultrasonic agitation of the pool is an active enhancement technique that can affect an entire module in the pool.

When acoustic vibrations are applied to liquids or gases, heat transfer may be improved by acoustic streaming. With liquids, it is possible to operate with ultrasonic frequencies due to favorable coupling between a solid and a liquid. At frequencies of the order of a megacycle, another type of streaming called crystal wind may be developed. In addition, since intensities are usually high enough to cause cavitation, that may become the dominant mechanism of heat transfer enhancement.

For natural convection, Seely [59] tested water, isopropyl alcohol, coolanol, transformer oil, and glycerol and found that nonboiling heat transfer to subcooled water was improved by as much as 160% by the application of 36 kHz ultrasonic vibrations. The improvement was attributed to cavitation, and appeared to be directly related to the pressure amplitude near the heated wire. Zhukauskas et al. [60] carried out studies with subcooled water and transformer oil for a variety of horizontal tubes and vertical plates. Ultrasonic vibrations ranging from 27 to 697 kHz increased heat-transfer coefficients by as much as 180% at low Grashof numbers, but at higher Grashof numbers the increase was substantially less. Crystal wind and cavitation were noted and
suggested as mechanisms. Larson and London [61] reported extensive studies of heat transfer from a sphere to water and toluene where the liquid was subjected to ultrasonic vibrations of 20 ~ 1000 kHz. As much as a 300% increase was observed in the heat-transfer coefficient in natural convection. At low frequencies, this increase was attributed to cavitation, while at higher frequencies the quartz wind streaming appeared to be important. The tests were run with higher Grashof numbers than those used by Zhukauskas and co-workers; however, no pronounced decrease in the effect of vibration was noted as the Grashof number was increased.

Fand [62] performed experiments where a heated cylinder was subjected to high intensity, 6.5 kHz vibrations in an acoustic water tunnel with no flow velocity. It was established that cavitation was responsible for the 33% maximum improvement in heat transfer; there was no detectable improvement unless cavitation occurred. Wong and Chon [63] reported increases in heat transfer coefficient of up to 700% in water with 21 ~ 306 kHz. Since cavitation was the primary factor in the enhancement, the liquid condition—especially gas solubility and temperature—was considered to be very important. Yashchenko [64] studied water, transformer oil, and glycerin at frequencies ranging from 22 to 600 kHz and 50 to 600 Hz for lower limit in field with strength of up to $5 \times 10^4$ W/m$^2$. The heat transfer coefficients with ultrasonic and sonic vibrations, which were increased up to several hundred percent, were correlated for both acoustic and ultrasonic fields.
More recently, Hoshino and co-workers reported a series of very careful studies with degassed water. Standing waves of 28 kHz were established at relatively low sound intensities and the heated section position was altered [65]. The maximum heat transfer coefficient occurred midway between the node and antinode, where the sound intensity is maximum. The improvements were only of the order of 10% since there was no cavitation. When nondegassed water was used [66], cavitation was induced at a flat plate heater at low sound intensities and improvements of several hundred percent were obtained. It is significant to note, however, that the lowest coefficients were obtained with the plate vertical and the transducer at the bottom of the pool. Local heat transfer coefficients were obtained with a 17.5 mm cylindrical test section [67].

Turning to subcooled pool boiling heat transfer, Isakoff [68] and Wong and Chon [69] found that the enhanced convective curve merged with the established boiling curve. Li and Parker [70] suggested that there was a small reduction in the superheat at established boiling with ultrasonics. Yashchenko [64] suggested that heat transfer coefficients were slightly increased at low heat fluxes, but ultrasonics were not effective at high heat fluxes since the ultrasonic energy was inhibited from reaching the surface; however, insufficient data are given to assess the effect of the ultrasonics on burnout fluxes. Wong and Chon [69] found negligible effect of ultrasonic vibrations on burnout heat flux for subcooled methanol. On the other hand, increases were reported
by Isakoff [68]: 60% increase in the heat flux at burnout for saturated water; Ornatskii and Shcherbakov [71]: 30 to 80% increase in burnout flux for water (above the effect of subcooling alone) as the subcooling was increased from 3 K to 80 K; and Markels et al. [72]: 50% increase in burnout flux for saturated isopropanol.

An exploratory study was undertaken to confirm the effects of ultrasonic vibrations on single-phase and boiling heat transfer. No tests of R-113 with ultrasonic vibrations appear to have been reported even though this fluid is widely used in ultrasonic degreasing systems.

B. Experimental Apparatus

A schematic diagram of apparatus is shown in Figure 30. The ultrasonic tank (Branson B-32H) had 3 transducers (lead-zirconate-titanate) attached to the bottom. The basic frequency of 55-kHz was modulated with 120 Hz. The combined output of the transducer was about 75 W, and the average intensity was 8000 W/m². The heater built into the side of tank and a cold water coil were used to control the pool temperature. The working fluid was R-113. A reflux condenser connected to a plexiglas cover minimized loss of the fluid.

An Electronic Measurement Co. AC-DC rectifier of 36 amp limit was used to supply power to small diameter stainless steel tubes. The tubes, 1.65 or 2.11 mm outside diameter, were chosen so that the rolled out vertical height, \( \pi D/2 \), corresponded approximately to the height of a
Figure 30. Apparatus for normal pool boiling and ultrasonic enhancement tests
chip. To extend testing to burnout, the laboratory AC motor-DC generator was used. The generator was rated at 125 V and 200 amp while running at 1755 rpm. A standard ammeter and a digital voltmeter were used to establish the test section amperage and voltage drop with the rectifier while a precision digital multimeter was used to measure the amperage (via a shunt) and the voltage drop across the test section when using the larger power supply.

Two copper-constantan thermocouples (30 gauge) with insulated beads were inserted into the tubes (at one third of the heater length from each end). Outside wall temperatures were inferred by subtracting the calculated tube wall temperature drop from the average inside tube temperature (Appendix). Two thermocouples were immersed near the test tube to measure the pool temperature. A thermocouple selector, another precision multimeter, and an ice bath completed the measuring circuit for the thermocouples.

A vinyl tube was connected to the bottom of tank to measure the liquid level. A mercury manometer was used to measure any pressure buildup in the tank. The saturation temperature was evaluated at the pool pressure at the level of the test section. Good correspondence between the inferred and measured pool temperatures was achieved under saturated conditions.

A video camera and recorder were used to record the voltage, amperage, and thermocouple EMF. Because of the rapid changes in temperature and power at burnout, the recorded values were displayed on a TV to find the burnout heat flux.
C. Experimental Results and Discussion

1. Free convection data

To verify the instrumentation and data reduction procedure, single-phase tests were run at the three pool temperatures established for the ultrasonic tests. These data are presented in Figure 31. The predicted values are shown to be in good agreement with the measurements.

The predictions were taken from the recommended curve of McAdams [73] for natural convection from single horizontal cylinders to gases and liquids. Over the range of interest, this prediction [73] can be expressed as

\[ \log(Nu) = 0.24071 \log(GrPr) - 0.23468 \]  
for \(4.98 \leq \log(GrPr) \leq 5.67\)

with properties evaluated at the film temperature.

2. Position of test section and pool level

After vibration intensity and pool temperature, the position of the test section and pool level are important variables. To explore these variables, a series of tests were run at the three standard pool temperatures (29.4 °C, 37.6 °C, 46.8 °C).

Following the study of Hoshino et al. [65], the best placement of the test section is at the maximum in force which occurs midway between the zero and maximum values of pressure and velocity. Furthermore, for
Figure 31. Comparison of free convection data with predictions
maximum transmission of ultrasonic energy, the free surface is at zero pressure. These distances are readily related to the wavelength of the ultrasound, which is 13 mm for R-113 at 20 °C and 55 kHz. Referring to the following table, Position 4 represents the optimum placement.

Table 1. Test section position and height of surface

<table>
<thead>
<tr>
<th>Position</th>
<th>Test section location</th>
<th>Height of surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.66 mm = 3.28 ( \lambda )</td>
<td>87.22 mm = 6.71 ( \lambda )</td>
</tr>
<tr>
<td>2</td>
<td>23.61 mm = 1.82 ( \lambda )</td>
<td>87.22 mm = 6.71 ( \lambda )</td>
</tr>
<tr>
<td>3</td>
<td>20.61 mm = 1.58 ( \lambda )</td>
<td>87.22 mm = 6.71 ( \lambda )</td>
</tr>
<tr>
<td>4</td>
<td>24.05 mm = 1.85 ( \lambda )</td>
<td>91.00 mm = 7.00 ( \lambda )</td>
</tr>
</tbody>
</table>

The results given in Figure 32 indicate that the position results in little change in thermal response to ultrasonics when the pool is saturated. With a moderate subcooling, Figure 33, the response for the optimum position is essentially the same as that for the other positions. The behavior is different as the pool temperature is further reduced.

As noted in Figure 34, it was impossible to get accurate data at low heat flux due to wall temperature fluctuations. This was attributed
Figure 32. Influence of test-section position and liquid level on saturated pool boiling heat transfer with ultrasonics.
Figure 33. Influence of test-section position and liquid level on subcooled pool boiling with ultrasonics, $T_b = 37.6 \, ^\circ C$
Figure 34. Influence of test-section position and liquid level on subcooled pool boiling with ultrasonics, $T_b = 29.4 \, ^\circ C$
to operation—in the temperature range where optimum transmission of ultrasonic energy is observed. For solvents such as R-113, the acoustic coupling is improved as the pool is heated, with a dramatic increase in the activity of the pool being evident at about 25 °C. The wall temperature fluctuations result from cavitation bubble implosion which heats both the test section and pool thermocouples. (The pool temperatures were recorded in the absence of ultrasonics.) With this enhanced acoustic energy transmission, it is not surprising that there is a difference between the stable data shown for positions 1 and 4. Position 4 is definitely superior for both increasing and decreasing heat flux.

Returning to the high pool temperature data presented in Figures 32 and 35, it is speculated that the lack of a pool level effect is due to poor transmission of ultrasonic energy due to cavitation at the transducer. Even though the thermal behavior was dependent on positioning only for the lowest temperature, all the subsequent high power tests were run at position 4.

3. **Effects of ultrasonics on boiling curves**

Complete boiling curves are presented in Figures 35-37. The general impression is that ultrasonics produces a significant shift in all regions of boiling curve. In all cases, nonboiling and partial boiling performance is improved. The improvement in single-phase heat transfer coefficient for the saturated pool is only 10%; for the 37.6 °C
Figure 35. Influence of ultrasonics on saturated pool boiling heat transfer and burnout.
Figure 36. Influence of ultrasonics on subcooled pool boiling heat transfer and burnout, $T_b = 37.6 \, ^\circ C$
Figure 37. Influence of ultrasonics on subcooled pool boiling heat transfer and burnout, $T_b = 29.4 \, ^\circ C$
pool an improvement of 50% is noted, and for the 29.4 °C pool the improvement is approximately 200%. These figures are quite in line with observations of other investigators noted in the introduction.

In all cases, the ultrasonic agitation greatly reduces the temperature overshoot preceding incipient boiling. The boiling curve shifts to higher superheat with the saturated pool, but the shift is to lower superheat with subcooled pools. The latter behavior seems more reasonable, if only as a result of the higher single-phase convective coefficients merging with established boiling at a higher superheat. The saturated pool behavior may be due to a variation in the ultrasound transmission pattern which causes cavitation heating or impedes bubble removal from surface.

The burnout fluxes as noted in Figures 35-37 are repeated in Figure 38. Saturated data without ultrasonics are in the range of the burnout flux envelope of Sun and Lienhard [38]. A best estimate of the effect of tube diameter and subcooling is given by combining the correlation of Zuber [36] with the Sun-Lienhard geometry correlation

\[
\frac{q''_{\text{chf}}}{q''_{\text{chf,z}}} = 0.89 + 2.27 \exp(-3.44/R')
\]

where

\[
R' = R(g(\rho_f - \rho_v)/\sigma)^{1/2}
\]

and the Ivey [74] subcooling correction

\[
\frac{q''_{\text{chf,sub}}}{q''_{\text{chf}}} = 1.0 + 0.1(\rho_v/\rho_f)^{1/4}[C_p\rho_f(T_s - T_b)/h_{fg}p_v]
\]

For the present conditions,

\[
q''_{\text{chf,z}} = 1.99 \times 10^5 \text{ W/m}^2
\]
Figure 38. Burnout data with and without ultrasonics, including a comparison with best available correlation.
\[ R' = 2.1 \text{ or } 1.64 \]

\[ \frac{q''_{\text{chf}}}{q''_{\text{chf, z}}} = 0.9 \]

\[ \frac{q''_{\text{chf, sub}}}{q''_{\text{chf}}} = 1.0 + 0.02047(T_s - T_b) \]

where \((T_s - T_b)\) is in K.

On the average basis the ultrasonics produces a slight increase in burnout flux for the saturated subcooled pool. The improvements are less than those reported in the literature.

D. Conclusions

Experiments were performed with a horizontal cylinder in ultrasonics at saturated and subcooled conditions. Data for natural convection without ultrasonics are in good agreement with the predicted values.

In an ultrasonic field, the effect of position of the test section for natural convection and boiling is small; however, this effect becomes larger as pool temperature decreases. The heat transfer coefficient is improved up to 200% for subcooled natural convection. The enhancement for natural convection and boiling decreases as the pool temperature and/or heat flux increase. Critical heat fluxes for the saturated and subcooled conditions are slightly increased by an ultrasonic field.
VI. HEAT SINKS WITH MICROHOLES AND MICROFINS

A. Introduction

The increasingly high performance of microelectronic chips has led to direct immersion cooling with boiling of fluorocarbon liquid coolants [3, 6]. Boiling enhancement now appears to be required to achieve the higher power levels required for normal operation conditions and testing. While it is possible to machine the surface of silicon chips to improve boiling performance [57, 58, 75], such surface modifications are not usually permissible. Accordingly, the emphasis of work on improved cooling of chips has been on heat sinks that are potentially detachable. The objectives of the enhancement are to reduce the temperature overshoot prior to incipient boiling (which is responsible for large boiling curve hysteresis), reduce wall superheat in established boiling, and increase the peak nucleate heat flux. Also, the early work on plain studs has given way to the use of heat sinks incorporating enhanced boiling surfaces.

Oktay and Schmeckenbecher [76] found that an irregular "dendritic" structure of nickel plated directly on the chips reduced the wall superheat in established boiling and eliminated the temperature overshoot with FC-88. Oktay [58] modified a 1 mm thick copper heat sink by drilling four 0.8 mm diameter vertical holes ("tunnels") in the copper plate. The heat sink was attached to the chip with thermal grease. The boiling performance for FC-86 was well above that for the
plain silicon surface, and the temperature overshoot with the tunnels vertically oriented was reduced from 20 K to 10 K.

Nakayama et al. [77] recently reported large enhancements with complex structured studs in FC-72. The temperature overshoot was reported to be small, probably due to the presence of many nucleation sites on the greatly enlarged surface area.

Heat sinks can be attached with an adhesive such as wax so that the heat sinks and wax can be readily removed after the tests. However, in order to accurately evaluate the heat transfer characteristics of the heat sinks, it was decided that the initial studies should avoid the thermal contact resistance introduced by the wax film. Detachable heat sinks will be discussed in the two subsequent chapters. A unique bar calorimeter was developed for this purpose.

B. Experimental Apparatus and Procedure

1. Test section

As shown in Figure 39, power is supplied at one end of the bar and the temperature gradient is measured at the other end to determine both heat flux and surface temperature. The tip of the bar can be enhanced by machining or coating according to the type of heat sink under consideration. The bar was made of brass 360. With this material, the accuracy of the heat flux depends primarily on the thermal conductivity. The thermal conductivity of brass was inferred from the
Figure 39. Typical test section for study of boiling heat sinks
electric resistivity with the correlation suggested by Smith and Palmer [78] and Powell [79]:

\[
k = 2.39 \times 10^8 \frac{T}{\rho} + 0.075 \quad \text{W/cmK}
\]

(16)

where \( T \) is the absolute temperature (K) and \( \rho \) is the electric resistivity (ohm-cm).

This Lorenz-type law is accurate for various types of brass to within about 1%. Brass 360 from two sources was used in this experiment; hence, it was necessary to obtain the electrical resistivity for each batch of bar stock. This was done at the ERI Electronics Shop (ISU) using a furnace and a circuit containing a precision resistor. The estimated accuracy of \( T \) and \( \rho \) as given in Tables 2 and 3 are 1 K and 1%, respectively. The predicted thermal conductivity is included in Tables 2 and 3. Most of the tests were run with test sections fabricated from brass batch No. 2.

The short test section shown in Figure 39 was accurate for the high heat flux tests, but had unacceptable error at heat fluxes in the natural convection range. A long test section was developed for saturated natural convection. In this region, the temperature gradient is very small; hence, to reduce temperature measurement error, the distance between thermocouples must be increased. The largest dimension shown in Figure 39 is increased from 30 to 110 mm. The exposed surface of the short or long plain bars was finished with 320 MX micromesh paper. Special tests were run with the bar protruding beyond the polyester casting.
Table 2. Properties of brass 1

<table>
<thead>
<tr>
<th>T (K)</th>
<th>$\rho$ (ohm cm)</th>
<th>$k$ (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>$8.08 \times 10^{-6}$</td>
<td>95.7</td>
</tr>
<tr>
<td>327</td>
<td>$8.23 \times 10^{-6}$</td>
<td>101.4</td>
</tr>
<tr>
<td>357</td>
<td>$8.29 \times 10^{-6}$</td>
<td>109.3</td>
</tr>
</tbody>
</table>

Table 3. Properties of brass 2

<table>
<thead>
<tr>
<th>T (K)</th>
<th>$\rho$ (ohm cm)</th>
<th>$k$ (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>$8.28 \times 10^{-6}$</td>
<td>93.5</td>
</tr>
<tr>
<td>325</td>
<td>$8.71 \times 10^{-6}$</td>
<td>96.7</td>
</tr>
<tr>
<td>353</td>
<td>$9.09 \times 10^{-6}$</td>
<td>100.3</td>
</tr>
</tbody>
</table>
Short test sections were the basis for the two heat sink configurations: a row of drilled holes near the outer edge normal to the axis and fins parallel to the axis. In both cases, the brass protrudes beyond the polyester coating. Pertinent dimensions are given in Table 4.

For study of the width effect, two additional test sections were constructed. They were of nominal height 5 mm and width 25.4 mm, long and short so that the full range of heat fluxes could be covered accurately.

Omega 36 gauge copper-constantan thermocouples were used to measure the test section and pool temperatures. A No. 68 drill (d = 0.079 mm) was used to make holes in the brass bar for the thermocouples. The holes were carefully placed to avoid distortion of the heat flow. In addition, the thermocouple depth and channeling were selected to minimize the conduction error. Insulation of the test section was by casting a cylinder of polyester casting resin (Silmar Division/Vistron Corp.) around the bar. Near the heater, fiberglass was used to insulate the test sections. Thus, the heat loss was minimized and the polyester protected from melting.

The heater in the test section was a Watlow Firerod cartridge heater (200 W, length = 31.8 mm, diameter = 6.35 mm; 150 W, length = 25.4 mm, diameter = 6.35 mm). A variac (0 ~ 130 V) was used to control the power level. (The power input could not be used to calculate heat flux because of heat loss.)
Table 4. Characteristics of enhanced surfaces tested

a) Microhole heat sinks

<table>
<thead>
<tr>
<th>Test section</th>
<th>Bar protuberance</th>
<th>End to hole center</th>
<th>Hole diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 holes</td>
<td>1.13 mm</td>
<td>0.62 mm</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>4 holes</td>
<td>1.39 mm</td>
<td>0.71 mm</td>
<td>0.6 mm</td>
</tr>
</tbody>
</table>

b) Microfin heat sinks

<table>
<thead>
<tr>
<th>Test section</th>
<th>Bar protuberance</th>
<th>Fin length</th>
<th>Slot width</th>
<th>Fin thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 slots</td>
<td>1.00 mm</td>
<td>0.76 mm</td>
<td>0.26 mm</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>4 slots</td>
<td>1.05 mm</td>
<td>0.81 mm</td>
<td>0.77 mm</td>
<td>0.36 mm</td>
</tr>
</tbody>
</table>
The general apparatus is described in Figure 39. The unpowered ultrasonic tank was again used. The working fluid (R-113), reflux condenser, manometer, and multimeter were the same as those used previously.

2. Procedure

Tests were carried out by setting the power level and recording the bar temperature once equilibrium was reached. The temperature profile was then fitted with a second-order curve which was extrapolated to the edge of the polyester resin. The flush surface served as a reference while the protruding bar resembled a chip (with inactive front side). The slots or holes then simulated heat sinks attached to a chip.

The heat flux followed from \( q'' = \frac{k\Delta T}{\Delta x} \) where \( k \) was taken for sample in question. The results were plotted as boiling curves: \( q'' \) vs. \( \Delta T_s \).

C. Experimental Results and Discussion

Figure 40 indicates that the free convection data are also well above the classical prediction for an isothermal heater [22].

\[
Nu_H = 0.68Pr^{1/2}Gr^{1/4}/(0.952 + Pr)^{1/4}
\]  

(17)

This is due to small test section size, as discussed earlier in Chapter II. Large scatter resulted when the short test section was operated at
Figure 40. Comparison of results for saturated boiling with flush short and long test sections
low heat flux. On the basis of these tests, the short test section was not used for heat fluxes below $10^4$ W/m$^2$. It is noteworthy that there is no discernible temperature overshoot prior to incipient boiling. There is, however, a substantial hysteresis in the boiling curve, with the decreasing power superheats being smaller.

Figure 41 presents subcooled results for repeated cycles of increasing and decreasing power. The fully established nucleate boiling curve lies slightly to the left of the curve for saturated conditions shown in Figure 40. It is evident that the increasing and decreasing data are quite consistent for the indicated variations.

Figure 42 depicts another subcooled test with increasing and decreasing power. There is a further apparent shift to the left of the saturated curve (Figure 40). It is interesting, however, that the temperature overshoot is clearly seen for the first time (arrow).

The effect of test section width is evident from a comparison of Figures 43 and 40. The former has a five-fold increase in width and lies much closer to the free convection prediction. Stepwise initiation of fully developed boiling is observed.

Although the bar did not permit definitive critical heat flux data, a lower bound of CHF was obtained by taking the departure from nucleate boiling, i.e., the departure of the fully developed nucleate boiling curve from log-linear behavior. An upper bound is given by the peak nucleate heat flux corresponding to the estimated zero slope point—if that point was clearly evident. Figure 44 presents the results of this
Figure 41. Results for intermediate subcooling with flush short test section
Figure 42. Results for high subcooling with flush short test section

R-113
1 atm, $T_s = 46.7^\circ$C
$T_f = 29.4^\circ$C
Figure 43. Results for saturated boiling with flush short and long test section of extra width
Figure 44. Subcooled critical heat flux for test sections of various size, as compared with standard correlation
exercise. The CHF data are all well above an accepted prediction for subcooled pool boiling. This seems to support the result of Chapter III that the smaller width test sections have higher CHFs.

Turning now to the protruding smooth test section, Figure 45 indicates that there is a small reduction in the superheat for both increasing and decreasing power. The increase in apparent heat transfer coefficient, however, is well below the surface area increase of 90%.

The boiling curves are shifted significantly to the left with the drilled test sections, Figures 46-48. Vigorous boiling was observed from the top of the vertical holes and sporadic boiling was noted from either side of the horizontal channels. The curves are quite similar, regardless of the number of holes or the orientation. It is particularly interesting that vertical and horizontal holes perform about the same. The improvement is quite modest considering the area increase (250%). In general, the enhancement is not what would be expected from the previous results for the Oktay "tunnels" [58].

The results for the slotted surfaces are given in Figures 49 and 50. The area increase of these surfaces relative to the flush surface is 220-300%. The 7 slots are quite similar to the data for 7 holes at higher heat fluxes. The main improvement seems to be with 4 slots, yet the superheat is never reduced by more than 5 K.
Figure 45. Results for saturated boiling with protruding (1 mm) short test section
Figure 46. Results for saturated boiling with 7 vertical holes—short test section.
Figure 47. Results for saturated boiling with 7 horizontal holes--short test section
Figure 48. Results for saturated boiling with 4 vertical holes—short test section

R-113
1 atm, $T_s = 46.3\, ^\circ C$

$\dot{q}', \text{ W/m}^2$

$\Delta T_s, \text{ K}$

$\Delta T_s, \text{ K}$

FIRST RUN

SECOND RUN
Figure 49. Results for saturated boiling with 7 vertical slots—short test section
Figure 50. Results for saturated boiling with 4 vertical slots—short test section.
D. Conclusions

The bar calorimeter test apparatus allows an accurate determination of the heat transfer characteristics with small test sections, especially those enhanced surfaces. After developing this technique, several sizes of heaters were tested in R-113. The size effect of the heater for natural convection and CHF agreed qualitatively with the results of Chapters II and III.

The typical large temperature overshoots prior to the inception of boiling were not observed, apparently due to the high thermal capacity of the heaters; however, small overshoots were observed one time. The enhancement of heat transfer for the plain and enhanced surfaces was smaller than the area increase because of the low thermal conductivity of brass. Modest enhancements in boiling were obtained with microfin or microhole heat sinks.
A. Introduction

The present study complements and extends the bar calorimeter experiments of the previous chapter where the protruding surface of the brass bar was modified with holes or fins. The purpose of this part of the experimental program is to follow up on that successful experience and check out the influence of geometrical variables such as number of holes, hole diameter, fin number, fin thickness, etc.

B. Experimental Apparatus

This experimental program was conducted using the basic apparatus mentioned earlier in connection with the flush heaters in R-113 (Chapter II). The dimensions of the holes and fins are shown in Figure 51. These dimensions, actual surface areas, and area ratios of actual to projected areas are tabulated in Table 5. Holes and fins were prepared with drills and saw cutters, respectively, on a milling machine. The 1 mm thick copper heat sinks were cemented with M610 epoxy resin to the 12.7 μm nichrome foil. The epoxy was cured at 130 °C for 3 to 4 hours. The electrical connections were made with brass foil transition pieces as shown in Figure 52.
Figure 51. Microhole and microfin heat sinks
Table 5. Dimensions of the microhole and microfin test sections

a) Microhole test sections

<table>
<thead>
<tr>
<th>Test section</th>
<th>Microhole diameter</th>
<th>Height</th>
<th>Width</th>
<th>Area</th>
<th>Area ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.34</td>
<td>4.34</td>
<td>4.67</td>
<td>56.3</td>
<td>2.78</td>
</tr>
<tr>
<td>2</td>
<td>0.51</td>
<td>4.42</td>
<td>4.62</td>
<td>63.4</td>
<td>3.19</td>
</tr>
<tr>
<td>3</td>
<td>0.71</td>
<td>4.52</td>
<td>4.34</td>
<td>73.9</td>
<td>3.89</td>
</tr>
<tr>
<td>4</td>
<td>0.89</td>
<td>4.67</td>
<td>4.47</td>
<td>80.7</td>
<td>4.03</td>
</tr>
</tbody>
</table>

b) Microfin test sections

<table>
<thead>
<tr>
<th>Test section</th>
<th>Fin Pitch</th>
<th>Fin gap</th>
<th>Height</th>
<th>Width</th>
<th>Area</th>
<th>Area Ratio</th>
</tr>
</thead>
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<td>4.62</td>
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<td>60.2</td>
<td>2.77</td>
</tr>
</tbody>
</table>

unit: mm or mm²
Figure 52. Construction details of heat sink test section
Visual-observations were also emphasized during these tests. The test section was inserted in plexiglas tank, 14 cm x 24 cm and 21 cm high as shown in Figure 2. The R-113 in the tank was kept at saturated conditions with a heating coil which circulated hot water from a constant temperature water bath. The front and one side of the tank was used to observe the boiling and take pictures. The other sides were insulated with fiberglass.

Correction of the heat flux was done using the same methods for plain heaters (Appendix), but the heat loss corrections were negligible.

C. Experimental Results and Discussion

1. **Saturated boiling**

A plain copper block heat sink was used to establish the reference for the enhanced surface heat sinks. In Figure 53, the data points represent heat fluxes calculated using base (nichrome heater) area, while dashed curves are based on total surface area. As expected, the data for the flush heater and the copper block (total area) are similar for natural convection. The small differences in established boiling curves are likely due to the differences in surface material and finish. The overshoot was 11 K with the plain heat sink, which is less than the overshoot of 19 K with the flush surface. This is probably due to the difference in surface conditions; specially, the highly polished foil is likely to contain fewer reentrant cavities. The enhancement of the
Figure 53. Boiling data for plain copper heat sink (nominal and total area basis) compared with flush heater.
plain copper block seems to be due to a simple increase of area, as would be expected since the block is nearly isothermal.

No allowance has been made for the thermal resistance of the epoxy; however, it is felt that this resistance is very small. The data as presented do represent the desired wall superheat, which embodies the boiling film coefficient, the copper block, and the epoxy bond.

It is of interest to note that a temperature fluctuation (up to about 1 K) was observed for the plain surface at high heat flux because the nichrome foil was very thin. However, the temperature fluctuations for the test sections with heat sinks were very small. This suggests excellent thermal coupling between the foil and the copper block.

The data for a typical microhole surface (holes of \( d = 0.71 \text{ mm} \) oriented vertically) are shown in Figure 54. A relatively small temperature overshoot took place with this test section. This could be due to the large area and rough surface produced by the drilling. It is apparent that the performance of the drilled heat sink is superior to that of the plain heat sink. The improvement exceeds the area increase as shown by the shift in the dashed curve relative to the curve for the flush heater. Over 10 W per chip was dissipated with less than 15 K superheat.

A uniform temperature distribution cannot be assumed in the copper block with holes because the temperature field is disturbed. This is suggested by the significant reduction in the natural convection heat transfer coefficient based on total surface area.
Figure 54. Boiling data for typical microhole heat sink (nominal and total area basis) compared with plain heat sink and flush heater
Bubbles were generated in the tunnels at low heat flux, but boiling occurred on the end surface only at high heat flux. In boiling at low heat flux, heat transfer was enhanced by bubble agitation and liquid pumping in the holes. The flow regime was probably bubbly or slug flow. At high heat flux, annular flow was expected even though the holes were very short (4.6 mm). Due to the increased velocity, the average heat transfer coefficient in the holes was likely to be higher than that for the exterior surfaces. This accounts for the superior performance of the drilled heat sink on a total area basis.

To pursue this further, tests were run with the holes oriented horizontally. As shown in Figure 55, the vertical data are slightly higher as would be expected due to augmentation of circulation by the chimney effect. Oktay [58] obtained a similar trend with FC-86.

The data obtained for heat sinks with different sizes of holes are plotted for decreasing power in Figure 56. The \( d = 0.71 \) mm configuration is clearly the best performer at the high heat fluxes of interest. Also included in Figure 56 are data from Chapter VI taken with a brass bar calorimeter. Due to the high thermal conductivity, the copper heat sinks are superior to brass heat sinks of comparable geometry.

Data for a typical vertical microfin surface are presented in Figure 57. There is considerable enhancement in free convection, as would be expected from the increased surface area; however, the boiling performance is quite similar to that for the plain heat sink.
Figure 55. Boiling data for typical microhole heat sinks; horizontal orientation compared with vertical orientation.
Figure 56. Composite boiling data (decreasing power) for microhole heat sinks
Figure 57. Boiling data for typical microfin heat sink (nominal and total area basis) compared with plain heat sink and flush heater.
Transformed data based on total surface area also shown in Figure 57 suggest that the fins are not isothermal in free convection. The reduction in average heat transfer coefficient could also be due to the restriction of flow between the fins. In boiling, the average performance is better than the plain surface. When the reduced fin efficiency due to higher heat transfer coefficient is considered, it is apparent that the boiling performance of the space between fins is better than that of a plain vertical surface.

The boiling performance did not vary much with fin thickness, as shown in Figure 58. In general, a larger gap between fins was more effective at high heat flux, while a smaller gap was more effective at low heat flux. The best performance was obtained with the heat sink (5 fins) having a fin thickness of 0.5 mm and a fin gap of 0.47 mm.

In general, microhole heat sinks have better performance than microfin heat sinks. The $d = 0.71$ mm hole heat sink has the best performance in boiling among the present heat sinks. One explanation for this is that the microfin heat sinks do not exhibit the beneficial two-phase channel flow noted in connection with the tunnel tests.

2. **Subcooled boiling**

The performance of heat sinks in subcooled pool boiling is of interest because of the higher heat fluxes that can be dissipated than with subcooled pools. Saturated boiling performance cannot, in general, be extrapolated to subcooled conditions [80].
Figure 58. Composite boiling data for microfin heat sinks
Subcooled boiling was investigated with several of the surfaces at a 30 °C pool temperature. The subcooled boiling curve for the plain heat sink is compared with that of the flush surface in Figure 59. The enhancement in subcooled boiling is less than in saturated boiling, perhaps due to the large temperature difference between base and pool.

The best performing tunnel surface was chosen for the subcooled boiling test. As shown in Figure 60, the enhancement relative to the plain heat sink is quite large.

The boiling curves for saturated and subcooled conditions are compared in Figure 61. The saturated and subcooled curves for each surface appear to merge at high heat flux. The asymptotes, however, are not the same due to the area increase. The general conclusion is that the relative performance of the heat sinks in subcooled boiling is about the same as that in saturated boiling. The general effectiveness of subcooled boiling is evident, as the heat fluxes are comfortably above 10 W/chip for heaters the size of typical microchips.

D. Conclusions

Copper heat sinks can be readily attached with epoxy to simulated microchips. Microhole heat sinks (four holes of various diameter) and microfin heat sinks (four fins of various thickness) provide superior saturated boiling performance compared to a plain heat sink. Holes of 0.71 mm diameter provide the best enhancement, about five times the wall
Figure 59. Subcooled boiling data for plain heat sink and flush heater
Figure 60. Subcooled boiling data for microhole heat sink compared with plain heat sink
Figure 61. Comparison of saturated and subcooled boiling data for microhole heat sink and flush heater.
superheat of the heater at the same heat flux; the improvement is
greater than the area increase. The performance improvement of this
surface is about the same for subcooled boiling as for saturated
boiling. A heat dissipation of 10 W/chip (4.5 mm x 4.5 mm) was achieved
with the d = 0.71 mm hole heat sink for both saturated and subcooled
boiling.
VIII. DETACHABLE HEAT SINKS WITH COMMERCIAL STRUCTURED POROUS SURFACES

A. Introduction

Some of the best performances in boiling are obtained with structured surfaces. Many experimental studies of commercial structured surfaces have been published.

Nakayama [81] and Nakayama et al. [82, 83] obtained data for Thermoexcel-E with variable pore diameter and several working fluids, and large enhancements were obtained. They suggested a semi-empirical model to describe the boiling process. Yilmaz and Westwater [84] compared boiling curves for these and other commercially available surfaces with isopropyl alcohol and p-xylene. The High Flux surface outperformed the Thermoexcel-E surface. Marto and Lepere [85] also compared boiling curves for Thermoexcel-E and High Flux surfaces in FC-72 and R-113. Heat transfer coefficients for FC-72 were lower than for R-113. The High Flux surface was again better than the Thermoexcel-E surface. Bergles and Chyu [86] got large enhancements with the High Flux surfaces in R-113. Temperature overshoots of 3 ~ 7 K were observed. They suggested a qualitative mechanism of vaporization in the pores.

The purpose of this chapter is to explore the performance of a variety of small enhanced heat sinks. The geometries include plates with the commercial surfaces mentioned above.
B. Experimental Apparatus and Procedure

The commercial heat sinks were made from special flat sections of test samples suitable for R-113 provided by the manufacturers. A section of Thermoexcel-E surface with pore diameters of 0.12 mm was used. The High Flux surface had a porous layer about 0.7 mm; the particle size ranged between 27 and 32 μm.

The procedures of assembling the test sections and conducting experiments were similar to the microhole test sections described in the previous section. It was easy to detach these heat sinks from the heater with a razor blade.

C. Experimental Results and Discussion

Experimental results for the Thermoexcel-E heat sink are shown in Figure 52. Data for the two runs agree well with decreasing power. During the first run, there was 7 K overshoot; however, the overshoot in the second run was only 3 K. The reduction in overshoot is attributed to the action of a single "rogue" nucleation site at the bottom of the heat sink. The resulting disturbance augmented the natural convection heat transfer coefficient for the rest of the heat sink. During fully established boiling, the bubbles generated at the pores were much larger than those previously observed from nucleation cavities on smooth surfaces. With decreasing power, the enhancement of heat transfer was larger at low heat flux than at high heat flux. The heat transfer
Figure 62. Boiling data for Thermoexcel-E heat sink compared with flush heater.
coefficient (at constant heat flux) is about six times that of the flush heaters at high heat flux. The rather shallow slope at high heat flux is a characteristic of Thermoexcel-E surfaces, e.g. [81, 85].

Boiling curves with decreasing power are compared in Figure 63 with available data for Thermoexcel-E surfaces in R-113. The data sets are quite different, probably due to the differences in pore diameter, experimental methods, and the test section configurations. Nakayama et al. [81, 82, 83] reported that pore diameter of the test section is very important. The enhancement of heat transfer appears to require larger pores at high heat flux and smaller pores at low heat flux. While the basic geometry of the heaters should not affect the boiling curves, the present Thermoexcel-E data reflect the extra surface area of the sides. Boiling took place at the sides at high heat flux.

A one-dimensional fin analysis was performed to find the optimum length and heat transfer characteristics of boiling heat sinks (Appendix). This analysis was applied to the present test of a short heat sink with the Thermoexcel-E surface. For the numerical example, data for the Thermoexcel-E surface of Nakayama [81] and correlation of Stephan and Abdelsalam [32] were used for boiling curves of the end and side surfaces, respectively. The optimum length of the heat sink does strongly depend on the constraint of the base temperature (Figure A12). The analysis also shows the importance of the sides of the heat sink. The overall boiling curve is expected to shift to lie between the curves for enhanced end surface and the plain side surface. As shown in Figure 64, the data are in good agreement with the predicted performance.
Figure 53. Composite of data for Thermoexcel-E surface with R-113
Figure 64. Comparison of predicted and observed performance of Thermoexcel-E heat sink
Figure 65. Composite of data for High Flux heat sink compared with flush heater
As shown in Figure 65, the overshoot for the High Flux surface was about 7 K, a value comparable to those of other surfaces. There was a sudden temperature increase at a heat flux of $4 \times 10^5$ W/m$^2$ which prompted decrease of the power. Data were in very good agreement between increasing and decreasing power except at very high heat flux, due to the temperature increase at the peak heat flux. The performance of the High Flux surface is excellent. Data of the present study are in good agreement with the published data sets [85, 86] as shown in Figure 65. Data of O'Neill [cited in 84] have somewhat higher heat transfer coefficients than other data sets including the present data; however, the surface specifications were not reported.

All boiling curves with heat sinks tested in the present program were collected to compare the performance with decreasing power, as shown in Figure 66. The High Flux surface exhibits the best performance.

Other data for heat sinks on actual or simulated microchips are depicted in Figure 67. It is difficult to compare the heat sinks tested due to different subcooling, working fluids, attachment methods, surface areas, and heat loss corrections. However, in fully developed boiling, subcooled boiling curves usually merge with saturated boiling curves. Even though the effects of working fluids are not clear, the tunnel heat sink of Oktay [58] and MP10 of Nakayama et al. [77] have similar boiling curves at high heat flux and are better than the other heat sinks.
Figure 66. Composite of data for heat sinks tested in the present program.
Figure 67. Data for heat sinks tested by other investigators
With R-113, the tunnels \((d = 0.8 \text{ mm})\) of Oktay [58] are in the range of diameters that give the best performance in the present study (the best performance was obtained with holes \(d = 0.71 \text{ mm}\)). Nakayama et al. [77] used a relatively large cylinder with cylindrical fins as the heat sink. The microfin heat sinks tested in the present study are inferior to the Thermoexcel-E or the High Flux surfaces.

D. Conclusions

Boiling data for Hitachi Thermoexcel-E and Linde High Flux heat sinks were obtained for R-113. In a comparison of all of data obtained with the present detachable heat sinks, the best performance was obtained with the High Flux surface.

Previous tests show that the tunnel heat sink of Oktay and the microfin and porous heat sink of Nakayama et al. have good transfer performance in FC-86 and FC-72, respectively. With R-113, however, the tunnels \((d = 0.8 \text{ mm})\) of Oktay and microfins of Nakayama et al. are not the best heat sinks according to the present tests.
IX. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

An experimental study of microelectronic chip cooling under normal and enhanced conditions is reported here. The major conclusions drawn from the individual investigations are as follows:

1. Natural convection for single flush heaters

   The heat transfer coefficient increases with decreasing width, with the effect greater in R-113 than in water. The data for the widest test section are about 20% above the classical prediction. This is attributed to leading edge effects.

2. Boiling heat transfer for single flush heaters

   While increasing power during boiling tests, the temperature overshoot depends strongly on the locations of the boiling sites on the heating surface. Three types of overshoot were observed. Reverse overshoot was observed with decreasing power at low heat flux as nucleation sites were deactivated. The boiling curves for different widths in R-113 agreed very well.

3. Critical heat flux for single flush heaters

   CHF data were obtained with variation of the width or height for vertical heaters with one side insulated. CHF increases as the height or width decreases.
4. Natural convection for arrays of the flush heaters

With in-line flush heaters, the heat transfer coefficients for the upper heaters are lower than those for the bottom heater. As the distance between the heaters increases, the heat transfer coefficients for the upper surfaces increase. In a staggered array, the heat transfer coefficient for the top heater increases as transverse distance between heaters increases at small G/H; however, the opposite effect is observed at large G/H.

5. Natural convection for the protruding heaters

The heat transfer coefficient for a single protruding heater is about 14% higher than that for a flush surface. Heat transfer coefficients for the top heater in an array are higher than those for the bottom heater in contrast to the results for the flush heaters, and increase as the distance between heaters increases.

6. Boiling heat transfer of arrays of the heaters

For boiling from two in-line flush or protruding heaters, the inception of boiling for the top surfaces took place at lower superheat than for the bottom surfaces. There is very little difference in the established boiling behavior for the top and bottom surfaces, for either flush or protruding surfaces.

7. Enhancement with ultrasonics

Experiments were carried out with a submerged horizontal cylinder subjected to ultrasonics at saturated and subcooled conditions. The effect of position of the test section for natural convection and
boiling is small; however, this effect becomes larger as pool temperature decreases. The heat transfer coefficient is improved up to 200% for subcooled natural convection. The enhancement for natural convection and boiling decreases as the pool temperature and/or heat flux increase. Critical heat fluxes for the saturated and subcooled conditions are slightly increased by an ultrasonic field.

8. Heat sinks with microholes and microfins

With the bar calorimeter test apparatus, the enhancement of heat transfer for the plain and enhanced surfaces was smaller than the area increase because of low thermal conductivity of brass. Modest enhancements in boiling were obtained with microfin or microhole heat sinks.

9. Detachable heat sinks with microholes and microfins

Copper heat sinks can be readily attached with epoxy to simulated microchips. Microhole and microfin heat sinks provided superior saturated boiling performance compared to a plain heat sink. Holes of 0.71 mm diameter provide the best enhancement, about six times the heat flux of the heater at same temperature difference; the improvement is greater than the area increase. The performance improvement of this surface is about the same for subcooled boiling as for saturated boiling. A heat dissipation of 10 W/chip (4.5 mm x 4.5 mm) was achieved with the d = 0.71 mm hole heat sink for both saturated and subcooled boiling.
10. Commercial heat sinks

Boiling data for Hitachi Thermoexcel-E and Linde High Flux heat sinks were obtained. Comparing all of the data obtained with detachable heat sinks, the best performance was obtained with the High Flux surface.

B. Recommendations

While this study has clarified many of the issues related to cooling of microelectronic chips, a number of areas remain for additional study.

With single flush heaters, important size effects were observed for natural convection and critical heat flux. The width effects should be studied with different working fluids and correlated with a single equation. Also, leading edge effects with conduction heat transfer to substrate and fluid should be incorporated into analytical or numerical studies of free convection. More detailed studies are required for width and leading edge effects with protruding heaters. Other, more expensive fluorocarbon fluids have been used for testing microelectronic chips in industry, and the first computers using direct immersion cooling have been announced. However, data for critical heat flux with those working fluids are not available. The width and leading edge effects are also not available. The correlations obtained in this study should, therefore, be tested with other fluorocarbon fluids.
Incipient boiling is also important in testing microelectronic chips. Three kind of boiling inception were observed in this study. Surface conditions and experimental procedures seem to affect the incipient boiling point. More work is required to clarify the conditions under which large temperature overshoots occur and to develop means to reduce or eliminate the overshoot. These tests should be carried out with actual silicon chips (thermal chips).

Heater interactions of simulated microelectronic chips for natural convection were studied, but more work with additional fluids is required to obtain a general correlation. Heater interactions for protruding heaters were different from those for the flush heaters. The protruding effect on these interactions should also be studied with other working fluids. Data for critical heat flux for an array of heaters would be useful.

Several kinds of heat sinks were tested in this study. Those heat sinks should be also tested in other fluorocarbon fluids and compared with each other. The optimum lengths of heat sinks should be obtained experimentally and compared with the present analytical predictions. More work is required to develop high performance heat sinks. The tests should include critical heat flux.


ACKNOWLEDGMENTS

This dissertation has been carried out under the supervision of Distinguished Professor Arthur E. Bergles. I wish to express my deepest gratitude to my major advisor for his guidance, assistance, and invaluable advice throughout the course of this study. This study could not have been undertaken without his direction and guidance. I deeply thank him for this honor.

I wish also to thank Dr. Richard H. Fletcher, Dr. George H. Junkhan, Dr. Bruce R. Munson, and Dr. James C. Hill, the members of my dissertation committee, for their assistance and guidance throughout my Ph.D. program.

Mr. Gaylord L. Scandrett and Mr. Robert D. Steed of the Department of Mechanical Engineering, and Mr. Edwin D. Gibson of Ames Laboratory provided technical assistance. Their aid is sincerely appreciated.

This study was supported by IBM Corporation, Data Systems Division, Poughkeepsie, New York. The assistance of Mr. Richard C. Chu and Dr. Paul W. Ing of IBM is appreciated. Dr. Patrick S. O'Neill (Union Carbide Corporation) and Dr. Wataru Nakayma (Hitachi Ltd., Japan) are also thanked for providing sample surfaces.

Special thanks are extended to my parents. I am especially grateful to my wife, Hye-Kyung, for her aid and to my son, Hong-Bum.
A. Heat Loss Corrections

Four effects were combined as possible corrections to the measured heater temperature and heater power. These corrections are first discussed as if they were independent of each other. Symbols and coordinates are given in Figure A1 to clarify the procedure of heat loss corrections.

**Figure A1. Sketch of the test section showing coordinate and heat loss**

1. **Temperature drop in the foil**

Assumptions:

1) Perfect insulation on back side.

2) Uniform heat generation.
The governing equation in the foil, as shown in Figure A2, is

$$\frac{d^2 T}{dy^2} + \frac{q''}{k_h} = 0 \quad (A1)$$

with boundary conditions:

1) at $y = 0$, $\frac{dT}{dy} = 0$

2) at $y = 0$, $T = T_w$

![Figure A2. Temperature profile in the foil](image)

The general solution is given by

$$T = C_1 + C_2 y - \frac{q''}{2k_h} y^2 \quad (A2)$$

where $C_1$ and $C_2$ are integral constants. From the boundary conditions, $C_1 = T_w$ and $C_2 = 0$. The solution then becomes

$$T = T_w - \frac{q''}{2k_h} y^2 \quad (A3)$$
The surface temperature of the foil is obtained from equation (A3) at \( y = t \). The temperature difference between inside and outside of the foil is \( \frac{q''}{2k_h} t \) from equation (A3) where \( q'' \) is the heat flux at the foil surface \((y = t)\) and \( q'' = q'' t \). The order of magnitude of this temperature drop should be known to consider or neglect it when reducing data.

For nichrome foil,

\[ k_h = 12.6 \text{ W/mK}, \quad t = 12.7 \times 10^{-6} \text{ m} \]

For \( q'' = 10^5 \text{ W/m}^2 \), the temperature drop is 0.05 K.

For steel foil,

\[ k_h = 43 \text{ W/mK}, \quad t = 25.4 \times 10^{-6} \text{ m} \]

For \( q'' = 10^5 \text{ W/m}^2 \), the temperature drop is 0.03 K.

The conclusion is that the temperature drop through the foil is negligible.

2. Heat loss through thermocouples

The heat loss through the thermocouple wires reduces the temperature and heat flux to the fluid at the center of the foil. In order to calculate the heat loss, it is necessary to know the "ambient" plexiglas temperature and the conductance coupling the wires to the plexiglas. A linear temperature profile in the plexiglas is assumed for simplicity. The expected temperature profiles of plexiglas and thermocouples are shown in Figure A3.
Figure A3. Expected temperature profiles of plexiglas and thermocouple wires

The governing differential equation is

\[
\frac{d^2T}{dy^2} + \frac{UP}{kA}(T - T_w) = 0
\]  

(A4)

with boundary conditions

1) at \( y = 0 \), \( T = T_w \)
2) as \( y \to -\infty \), \( T = T_b \)

where \( U, P, k, \) and \( A \) are overall heat transfer coefficient, perimeter, equivalent thermal conductivity of thermocouples, and equivalent area of thermocouples, respectively. The equivalent radii and thermal conductivities of thermocouples are defined by Eckert and Goldstein [87] as follows:

\[
kA = (k_{cu} + k_{cn})\pi r_w^2
\]
\( r_w \) = radius of single thermocouple wire

\( r_1 \) = equivalent radius of thermocouple wires, \( \sqrt{2r_w} \)

\( r_2 \) = equivalent radius of teflon insulation, \( (L_1 + L_2)/4 \)

\( P = 2\pi r_1 \)

The equivalent radii of thermocouples and teflon insulation are shown in Figure A4. The surrounding temperature \( T_{\infty} \) is given as

\[
T_{\infty} = \xi + (\xi - T_b)y/L \quad \text{for} \quad -L \leq y \leq 0
\]

and

\[
T_{\infty} = T_b \quad \text{for} \quad -\infty \leq y \leq -L
\]

where \( \xi \) = the temperature of the foil as \( r \to \infty \).

Figure A4. Thermocouple wires (a) and equivalent radii (b)

For \(-L \leq y \leq 0\), the general solution is given by

\[
T = \xi + (\xi - T_b)y/L + C_1 e^{my} + C_2 e^{-my}
\]  \hspace{1cm} (A5)
where \( m = \sqrt{U_1 P/kA} \) and \( U_1 = k_t/r_1 \ln(r_2/r_1) \). By applying boundary condition 1,

\[
C_1 = T_w - \xi - C_2
\]

For \(-\infty \leq y \leq -L\), the general solution is given by

\[
T = T_b + C_3 e^{m'y} + C_4 e^{-m'y}
\]  \hspace{1cm} \text{(A6)}

where \( m' = \sqrt{U_2 P/kA} \) and \( U_2 = 1/[r_1 \ln(r_2/r_1)/k_t + r_1/(r_2 h)] \). By applying the boundary condition 2,

\[
C_4 = 0
\]

Two solution should be continuous at \( y = -L \). The conditions for continuity are that equations (A5) and (A6) have same temperature and slope at \( y = -L \). The temperature profile of thermocouples for \(-L < y \leq 0\) is

\[
T = \xi + (\xi - T_b)y/L + (T_w - \xi - C_2)e^{my} + C_2 e^{-my}
\]  \hspace{1cm} \text{(A7)}

and for \(-\infty < y < -L\)

\[
T = T_b + C_3 e^{m'y}
\]  \hspace{1cm} \text{(A8)}

where

\[
C_2 = -[(T_w - \xi)e^{-mL(m'-m)} - (\xi - T_b)/L]/[e^{mL(m+m')} + e^{-mL(m-m')}] 
\]

and

\[
C_3 = e^{-m'L}(C_4 e^{mL} + (T_w - \xi - C_1)e^{-mL})
\]

The heat loss to the thermocouples is

\[
q_{tc} = - kA \frac{dT}{dy}igg|_{y=0}
\]

or

\[
q_{tc} = - kA[2mC_1 - m(T_w - \xi) - (\xi - T_b)/L]
\]  \hspace{1cm} \text{(A9)}

For the 36 gauge copper-constantan thermocouples,

\[
k_{cu} = 386 \text{ W/mK}, \quad k_{cn} = 22.7 \text{ W/mK}, \quad r_w = 6.35 \times 10^{-5} \text{ m}
\]
After these constants are substituted in above equations, the heat loss to thermocouples is obtained with equation (A9) as follows:

\[ q_{tc} = 1.29 \times 10^{-5} \left[ 1.21(\xi - T_w) + 0.6(T_w - T_b) \right] \text{ W} \quad \text{(A10)} \]

The unknown temperature \( \xi \) is evaluated from the energy balance at the bead of the thermocouple wires. The heat transfer rate from the foil to the thermocouple bead, as shown in Figure A5, is analyzed with the following assumptions:

1) The thermocouple bead is at uniform temperature \( T_w \)
2) No temperature gradient exists normal to the heater (proved in Section 1).

![Figure A5. Sketch of the thermocouple bead](image)
The differential equation for the foil region outside the bead is

\[ r^2 \frac{d^2 \theta}{dr^2} + r \frac{d \theta}{dr} - \varepsilon^2 \frac{d \theta}{dr} + \frac{\varepsilon^2}{k_n} r^2 \theta = 0 \]  

(A11)

where \( \theta = T - T_b \) and \( \varepsilon^2 = h/k_n t \). The boundary conditions are

1) as \( r \to \infty \), \( \theta = \xi - T_b \)
2) at \( r = r_b \), \( \theta = T_u - T_b \)

The general solution, a form of Bessel's zero-order equation, is

\[ \theta = C_1 \text{I}_0 (\varepsilon r) + C_2 \text{K}_0 (\varepsilon r) + \frac{q''}{k_n \varepsilon^2} \]

where the I_0 and K_0 are the modified Bessel functions of the first and second kind, respectively. The last term \( (q''/k_n \varepsilon^2) \) of this solution is \( (\xi - T_b) \). The integration constant \( C_1 \) must be zero to satisfy boundary condition 1.

The overall energy balance, as shown in Figure A5, is

\[ q_{tc} = q_1 + q_2 \]

where

\[ q_1 = k_n \frac{dT}{dr} r=r_b = -2\pi k_n \omega r_b \varepsilon K_1 (\varepsilon r_b) C_2 \]

and

\[ q_2 = h(\xi - T_u) r_b \varepsilon^2 \]: generation rate - convection to fluid

The integration constant \( C_2 \) is obtained from the energy balance equation:

\[ C_2 = \frac{q_{tc} - q_2}{2\pi k_n \omega r_b \varepsilon K_1 (\varepsilon r_b)} \]

The radial temperature distribution is thus given by

\[ \xi - T = \frac{q_{tc} - q_2}{2\pi k_n \omega r_b \varepsilon K_1 (\varepsilon r_b)} \frac{K_0 (\varepsilon r)}{K_1 (\varepsilon r_b)} \]  

(A12)
After substituting equation (A10) in (A12), \( \xi \) is evaluated by applying the second boundary condition. The heat flux loss at the section of the heater corresponding to the thermocouple bead is given by \( q'^2 = h(\xi - T_w) \).

As a numerical example for the 5 mm x 5 mm nichrome heater,

\[
\begin{align*}
    h &= 600 \text{ W/m}^2\text{K}, \quad k_h = 12.6 \text{ W/mK}, \\
    t &= 12.7 \times 10^{-6} \text{ m}, \quad r_b = 2 \times 10^{-4} \text{ m}, \\
    \varepsilon &= 1.94 \times 10^3 \text{ l/m}, \quad K_1(\varepsilon r_b) = 2.18, \quad K_0(\varepsilon r_b) = 1.12
\end{align*}
\]

For \( T_w - T_b = 10 \text{ K} \), \( \xi - T_w = 0.01 \text{ K} \) from equation (A12). The heat flux reduction is 0.1%.

There was no difference of experimental data between 40 gauge Iron-constantan and 36 gauge copper-constantan thermocouple wires. The heat loss through the wires is negligible as indicated by both the experimental and analytical results.

3. **Back-side heat loss correction**

The back side of the heater, as shown in Figure A6, is not insulated perfectly. Heat loss to the substrate is estimated with the following assumptions:

1) Uniform heat loss through the back side to the substrate.

2) Temperature is maximum at the midpoint.

3) Semi-infinite plate.
The analytical solution is given by Carslaw and Jaeger [88].

\[ T_w - T_b = \frac{q''_{bc}}{\pi k_b} \left[ W \sinh^{-1}(H/W) + H \sinh^{-1}(W/H) \right] \]

or

\[ q''_{bc} = \frac{\pi k_b (T_w - T_b)}{W \sinh^{-1}(H/W) + H \sinh^{-1}(W/H)} \quad \text{(A13)} \]

where \( q''_{bc} \) = heat loss to back side

A typical natural convection data point illustrates the magnitude of this correction:

\( q'' = 9492 \text{ W/m}^2 \) (nominal heat flux), \( T_w - T_b = \Delta T = 12 \text{ K} \)

\( H = W = 5 \times 10^{-3} \text{ m (5 mm)} \)

The thermal conductivity of circuit board and plastic is

\( k_b = 0.26 \text{ W/mK} \)
Heat loss to back side is calculated from equation (A13)

\[ q''_{bc} = 1123 \text{ W/m}^2 \ (12\%) \]

For a boiling case, the amount of correction is same with that for natural convection. The percentage of correction is only 4% (nominal heat flux = 30000 W/m² for 12 K superheat).

This correction is not accurate for arrays of the heaters; however, it seems to be good estimation because the closest distance between the heaters is half of the height of the heaters.

4. Heat loss to power connections

Assumptions:

1) One dimensional heat flow.

2) End temperature is the same as that of the working fluid.

3) Correction is considered only in the region of the thermocouple bead, a 0.4 mm strip at the center of the foil.

As real boundary condition at the end is unknown, the temperature profile calculated with assumption 2 is expected to be close to the actual temperature profile only near the center. The temperatures at the ends of the heaters are actually expected to be higher than that of the fluid; hence, the heat loss from the strip will be overestimated.

The governing equation for the heater, as shown in Figure A7, is obtained by heat balance:

\[ \frac{d^2T}{dx^2} - \frac{h}{k_h t} (T - T_b) + \frac{q''}{k_h t} = 0 \]  

(A14)

with boundary conditions:
1) at x = 0, \( \frac{dT}{dx} = 0 \)
2) at x = W/2, \( T = T_b \)

Figure A7. Sketch illustrating conduction through the foil

The temperature distribution is

\[ T - T_f = \frac{q''}{h} \left[ 1 - \frac{\cosh(mx)}{\cosh(mW/2)} \right] \quad (A15) \]

where \( m = (h/k_t)^{1/2} \). The nondimensional temperature profile is

\[ \frac{T - T_b}{T_w - T_b} = \frac{\cosh(mW/2) - \cosh(mx)}{\cosh(mW/2) - 1} \quad (A16) \]

This temperature profile is compared with a typical and experimental run in Figure A8. Hence, two thermocouples were placed on the heater.

As shown in Figure A8, assumption 2 results in significant error near the ends of the heater, but near the center of the heater (x = 0.2 mm), it should not make a significant error for estimating the conduction heat loss. Also, this heat loss is not the dominant factor,
Figure A8. Sketch illustrating temperature profile as shown later in a numerical example. The heat conduction along the foil at $x = x_1$ (0.2 mm) is

$$q_{fc} = -k_A \frac{dT}{dx} \bigg|_{x=x_1}$$

where $A_t = H t$. The average heat flux reduction in the central region is then

$$q''_{fc} = -k_h \frac{A_t}{A_f} \frac{dT}{dx} \bigg|_{x=x_1}$$

where $A_f = H x_1$. This equation reduces to

$$q''_{fc} = \frac{tk_h}{x_1 H} \frac{msinh(mx_1)}{cosh(mW/2)}$$

(A18)

A numerical example is given in the next section.
5. Numerical example

A typical natural convection data point will be used to show the procedure for the heat loss correction.

\[ q'' = 9492 \, \text{W/m}^2 \] (nominal heat flux), \( T_w - T_b = \Delta T = 12 \, \text{K} \)
\[ H = W = 5 \times 10^{-3} \, \text{m} \]

Heat loss to the back side is calculated from equation (A13) as calculated in section 3:

\[ q''_{bc} = 1123 \, \text{W/m}^2 \]

The heat loss to the back side as a percentage of the nominal heat flux is

\[ \frac{q''_{bc}}{q''} = \frac{1123}{9492} = 0.12 \, (12\%) \]

The heat transfer coefficient is calculated from \( (q'' - q''_{bc})/\Delta T \).

The heat loss along the foil is obtained when this heat transfer coefficient is inserted in equation (A18). Then, the heat transfer coefficient is again calculated from \( (q'' - q''_{bc} - q''_{fc})/\Delta T \). This new heat transfer coefficient is inserted in equation (A18) until the given convergence criteria of 5% is satisfied. The heat loss to the power connections, expressed as a surface heat flux correction in the central region, is calculated from equation (A18):

\[ q''_{fc} = 112 \, \text{W/m}^2 \]

The heat loss along the foil as a fraction of the nominal heat flux is

\[ \frac{q''_{fc}}{q''} = \frac{112}{9492} = 0.012 \, (1.2\%) \]

Superpositions for heat loss corrections are schematically shown in Figure A7. The nominal heat flux is reduced by \( q''_{bc} \) and \( q''_{fc} \).
calculated from equations (A13) and (A18), respectively. After heat losses to the back side and through the foil are corrected, the actual heat dissipation is expected as shown in Figure A9.

Figure A9. Expected heat flux profile
B. Propagation of Error

To put the various experimental systematic errors into perspective and gain appreciation for the accuracy of the experimental data, a propagation of error analysis was performed for several experimental runs. A sample calculation is presented here, using the data for natural convection in R-113 with a 5 mm x 5 mm heater.

The parameter of interest to this propagation of error calculation is the uncertainty of the experimental results, \( \text{Nu}_x \) and \( \text{Ra}_x \). \( \text{Nu}_x \) can be written in the following form:

\[
\text{Nu}_x = \frac{h_x}{k} = \frac{q''H/2}{\Delta T_k}
\]

or

\[
\text{Nu}_x = \frac{V_1}{2\Delta T_k} (1-LF)
\] (A19)

where \( LF \) = heat loss factor (heat loss/supplied power).

The experimental uncertainty, as used here, is the absolute value of maximum expected deviation from the reported experimental results.

The variation of \( \text{Nu}_x \) due to uncertainty in property data or measurements can be obtained by taking the logarithm and differentiating it:

\[
\frac{\Delta \text{Nu}_x}{\text{Nu}_x} = \frac{\Delta V}{V} + \frac{\Delta I}{I} - \frac{\Delta W}{W} - \frac{\Delta (\Delta T)}{\Delta T} - \frac{\Delta k}{k} + \frac{\Delta (LF)}{(1-LF)}
\] (A20)

The uncertainty of \( \text{Nu}_x \) is determined by taking the root of the sum of the squares of the terms in equation (A20):

\[
\frac{\Delta \text{Nu}_x}{\text{Nu}_x} = [\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta (\Delta T)}{\Delta T}\right)^2 + \left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta (LF)}{1-LF}\right)^2]^{1/2}
\] (A21)
Ra^*_x can also be written in following form:

\[ Ra^*_x = \frac{Nu_x Gr^*_x}{\Delta T k_v H W} \]  

or

\[ Ra^*_x = \frac{VI(1-LF) \beta g(H/2) 4}{\Delta T k_v H W} \]  

So, the uncertainty of Ra^*_x is given by

\[
\frac{\Delta Ra^*_x}{Ra^*_x} = \left[ \left( \frac{\Delta Nu_x}{Nu_x} \right)^2 + \left( \frac{\Delta V}{V} \right)^2 + \left( \frac{\Delta I}{I} \right)^2 + \left( \frac{\Delta H}{H} \right)^2 + \left( \frac{\Delta AT}{AT} \right)^2 + \left( \frac{\Delta \beta}{\beta} \right)^2 + 3\left( \frac{\Delta H}{H} \right)^2 + 2\left( \frac{\Delta V}{V} \right)^2 + \left( \frac{\Delta k_v}{k_v} \right)^2 + \left( \frac{\Delta (LF)}{1-LF} \right)^2 \right]^{1/2}
\]

A sample calculation for the uncertainties of Nu_x and Ra^*_x is presented for one set of experimental data:

\[ H = W = 5 \text{ mm}, \quad AT = 12.6 \text{ K} \]
\[ I = 1.966 \text{ A}, \quad V = 0.15 \text{ V} \]

The accuracies of measurement and instrument calibration are

\[ \Delta H = \Delta W = 0.1 \text{ mm}, \quad \Delta (AT) = 0.2 \text{ K} \]
\[ \Delta I = 0.014 \text{ A}, \quad \Delta V = 0.00007 \text{ V} \]

The uncertainties of H, W, AT, I, and V are

\[ \frac{\Delta H}{H} = \frac{\Delta W}{W} = 0.02 \text{ (2%)}, \quad \frac{\Delta (AT)}{AT} = 0.016 \text{ (1.6%)}, \]
\[ \frac{\Delta I}{I} = 0.007 \text{ (0.7%)}, \quad \frac{\Delta V}{V} = 0.005 \text{ (0.5%)}, \]

The heat loss correction (LF) was 13.2% and the uncertainty of this correction (\(\Delta LF\)) is expected to be less than 2%. So,

\[ \frac{\Delta LF}{I-LF} = 0.023 \text{ (2.3%)} \]
The uncertainties of properties of R-113 [13] are

\[ \frac{\Delta k}{k} = 0.0009 \ (0.09\%), \ \frac{\Delta \beta}{\beta} = 0.00008 \ (0.008\%), \ \frac{\Delta \nu}{\nu} = 0.02 \ (2\%) \]

The uncertainties of \( \text{Nu}_x \) and \( \text{Ra}_x^* \) for given experimental data are obtained by substituting the uncertainty of each term in equations (A21) and (A23), respectively.

\[ \frac{\Delta \text{Nu}_x}{\text{Nu}_x} = 0.0335 \ (3.35\%) \]
\[ \frac{\Delta \text{Ra}_x^*}{\text{Ra}_x^*} = 0.067 \ (6.7\%) \]
C. Critical Heat Flux Data for Vertical Flush Heaters with R-113 at 1 Atm.

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* N: Nichrome (12.7 μm), S: Steel (25.4 μm), C: Constantan (10 μm)
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D. Temperature Drop in the Cylindrical Heater

The differential equation in the cylinder with heat generation, as shown in Figure A10, is

$$\frac{1}{r} \frac{d}{dr}(r \frac{dT}{dr}) = -\frac{q''}{k_c}$$

with boundary conditions:

1) at $r = r_i$, $T = T_{wi}$

2) at $r = r_o$, $-k_c \frac{dT}{dr} = q''$

The general solution is

$$T = C_1 \ln(r) + C_2 - q'' r^2 / 4k_c \tag{A24}$$

The relation between $q''$ and $q''$ is given by the overall energy balance:

$$q'' \pi (r_o^2 - r_i^2) = q'' 2\pi r_o$$
or

\[ q'' = 2r_o q'' / (r_o^2 - r_i^2) \]

The integration constants \( C_1 \) and \( C_2 \) are obtained by the boundary conditions

\[ C_1 = r_i r_o^2 q'' / [k_c (r_o^2 - r_i^2)] \]

and

\[ C_2 = r_i r_o^2 q'' [0.5 - \ln(r_i)] / [k_c (r_o^2 - r_i^2)] \]

With these integration constants substituted in equation (A24), the outside wall temperature is given at \( r = r_o \) by

\[ T_{w_0} = T_{w_i} + r_i r_o^2 q'' [0.5 + \ln(r_o/r_i) - (r_o/r_i)^2] / [k_c (r_o^2 - r_i^2)] \]  \[ (A25) \]

The radius and thermal conductivity of the stainless steel cylinder (type 304) are

\[ r_o = 1.05 \text{ mm}, \quad r_i = 0.8 \text{ mm}, \quad k_c = 16.3 \text{ W/mK} \]

The temperature drop is

\[ T_{w_0} - T_{w_i} = -8.0 \times 10^{-6} q'' \text{ K} \]

where \( q'' \) is in \( W/m^2 \).

For \( q'' = 10^5 \text{ W/m}^2 \), temperature correction (drop) is 0.8 K.
E. One-dimensional Analysis for the Heat Sinks

As a heat sink or stud is attached to a microelectronic chip, it is of considerable importance to get the optimum length of the heat sink and the relation between heat flux and junction temperature. A one-dimensional fin analysis is conducted assuming that the boiling curves of the heat sinks are known and given by log-linear relations in the range of superheat considered:

1) for the side surface, \( q'' = F(T - T_b) = a(T - T_b)^n \)
2) for the end surface, \( q'' = G(T_e - T_b) = b(T_e - T_b)^\ell \)

Figure All. Sketch illustrating the heat sink

The governing differential equation for the heat sink, as shown in Figure All, is given as

\[
\frac{d^2T}{dx^2} - \frac{Pa}{k_sA} (T - T_b)^n = 0
\]

(A26)
with boundary conditions:

1) at \( x = 0 \), \( T = T_0 \)

2) at \( x = L \), \( -k \frac{dT}{dx} = b(T_e - T_b) \)

where \( P, A, \) and \( k_s \) are perimeter, cross sectional area, and thermal conductivity of the heat sink, respectively.

Let \( \theta = T - T_b \) and \( M = \frac{Pa}{k_s A} \)

Then, the governing differential equation becomes

\[
\frac{d^2 \theta}{dx^2} - M \theta^n = 0
\]

with boundary conditions:

1) at \( x = 0 \), \( \theta = \theta_0 \)

2) at \( x = L \), \( \frac{d\theta}{dx} = -\frac{b}{k_s} \theta^\ell \)

Let \( S = \frac{d\theta}{dx} \). Then,

\[
\frac{d^2 S}{dx^2} = \frac{dS}{dx} = S \frac{dS}{d\theta}
\]

So, the governing differential equation will be

\[
S \frac{dS}{d\theta} - M \theta^n = 0 \quad (A27)
\]

Now, separate variables and integrate the governing equation. Then,

\[
s^2 - \frac{2M}{n+1} \theta^{n+1} = C_1 \quad (A28)
\]
The constant $C_1$ is by boundary condition 2

$$C_1 = D\theta_e \frac{2z}{n+1}$$

where $D = b^2/k_s^2$. The solution is

$$S^2 = D\theta_e \frac{2z}{n+1} + \frac{2M}{n+1} (\theta^{n+1} - \theta_e^{n+1})$$  \hspace{1cm} \text{(A30)}

At $x = 0$,

$$S_0^2 = D\theta_e \frac{2z}{n+1} + \frac{2M}{n+1} (\theta_0^{n+1} - \theta_e^{n+1})$$

or, $S_0 = -\left[D\theta_e \frac{2z}{n+1} + \frac{2M}{n+1} (\theta_0^{n+1} - \theta_e^{n+1})\right]^{1/2}$  \hspace{1cm} \text{(A31)}

A negative sign is chosen to make the heat flux positive. Equation (A31) gives the relation between heat flux and junction temperature.

Equation (A31) is differentiated with respect to $\theta_e$ to find the maximum heat flux for a specified $\theta_o$:

$$D \frac{2z}{n} \theta^{2z-1} - 2M \theta^n = 0$$

or $\theta_e = \left(\frac{Dz}{M}\right)^{1/(n-2z+1)}$  \hspace{1cm} \text{(A32)}

Whether the value of $\theta_e$ given by equation (A32) gives a maximum or minimum heat flux is decided by

$$\frac{d^2S}{d\theta^2} = D \frac{2z(2z-1)}{n} \theta_e^{2z-2} - 2M \theta^{n-1}$$

$$= 2\theta_e^{2z-2} \frac{Dz}{nM^2} \left(\frac{2z-1}{n} - 1\right)$$

When $n > 2z - 1$, there is a maximum heat flux. But when $n < 2z - 1$, 
there is no maximum heat flux. The conditions for maximum heat flux are 

\[ n > 2l - 1 \text{ and } \theta_e > \left( \frac{Dl}{H} \right)^{1/(n-2l+1)} \]

As the end of heat sink is a good heat transfer surface (e.g., Thermoexcel-E or a porous surface), there is an optimum length of the heat sink.

Now, find \( L \) that gives the maximum \( S_\theta \) with the specified \( \theta_e \). The relation between \( L \) and \( \theta_e \) is obtained from equation (A30) with boundary condition 1. Equation (A30) is

\[
S = -\left[ D\theta_e^{2l} + \frac{2Ml}{n+1} (\theta_e^{n+1} - \theta_e) \right]^{1/2}
\]

This nonlinear differential equation can be integrated if it is linearized:

\[
\theta = \theta_e + \Delta \theta \\
\theta^{n+1} = (\theta_e + \Delta \theta)^{n+1} = \theta_e^{n+1} + (n+1)\Delta \theta \theta_e^n
\]

or, \( \theta^{n+1} = \theta_e^{n+1} + (n+1)(\theta - \theta_e)\theta_e^n \) (A34)

Substitute equation (A34) in equation (A33):

\[
\frac{d\theta}{dx} = -\left[ D\theta_e^{2l} + 2M\theta_e^n (\theta - \theta_e) \right]^{1/2}
\]

(A35)

and integrate the result by separation of variables:

\[
\frac{1}{2E}\left[ 2E(\theta - \theta_e) + D\theta_e^{2l} \right]^{1/2} = -x + C_2
\]

(A36)

where \( E = M\theta_e^n \). By the boundary condition 1, \( \theta = \theta_b \), at \( x = 0 \). Thus
$$C_2 = \frac{1}{2E} [2E(\theta_o - \theta_e) + D\theta_e^2]^1/2$$ \hspace{1cm} (A37)

After substituting equation (A34) in (A33) and applying the end condition, \( \theta = \theta_e \) at \( x = L \),

$$L^2 + 2BL - C = 0$$ \hspace{1cm} (A38)

where

\[
\begin{align*}
B &= \sqrt{D\theta_e^2/2E} \\
C &= (\theta_o - \theta_e)/2E
\end{align*}
\]

For a numerical example, it is appropriate to use the boiling curves of the smooth surface for the side surface and of the Thermoexcel-E surface for the end surface of the heat sink for R-113. The boiling curves for the foil heaters of the present study are in good agreement with correlation of Stephan and Abdelsalam [32]:

- smooth surface \( q'' = 1.45 \Delta T^{3.92} \)
- Thermoexcel-E [81] \( q'' = 10^4 \Delta T^{1.5} \)

Coefficient values of \( a, b, n, \) and \( \xi \) are

\[
\begin{align*}
a &= 1.45 \text{ W/m}^2\text{K}^{3.92} , & b &= 10^4 \text{ W/m}^2\text{K}^{1.5} \\
n &= 3.92 , & \xi &= 1.5
\end{align*}
\]

The criteria for a maximum heat flux \( (n > 2\xi - 1) \) is satisfied. The material and cross section of the heat sink are copper and 5 mm x 5 mm. The constants are

\[
\begin{align*}
k_s &= 380 \text{ W/mK}, & M &= 3.05 \text{ l/m}^2\text{K}^{2.92}
\end{align*}
\]
D = 692.5 1/m²K, θ_e = 20.84 K, E = 4.51 × 10⁵ K/m²

B = 2.78 × 10⁻³ m, C = 2.22 × 10⁻⁶ (θ_o - 20.84) m²

The optimum length of the heat sink is calculated from equation (A38) for θ_e = 30 K

L_opt = 1.45 mm

and the maximum heat flux from equation (A31) is

q''_{max} = 1.91 × 10⁶ W/m²

The optimum length of the heat sink and the associated maximum heat flux are plotted in Figure A12 as functions of the junction temperature difference. To get heat flux, the temperature of the end surface is calculated from equation (A38). Heat flux is then obtained by inserting this temperature in equation (A31). The length of the heat sink has little effect on the heat transfer rate when the base temperature difference is larger than 25 K, as shown in Figure A13. But when the junction temperature difference is low, the heat sink should be short. The relation between length and heat transfer rate of the heat sink is affected strongly by the base temperature difference and the boiling curves of the side and end surfaces of the heat sink.
Figure A12. Optimum length and maximum heat flux of heat sink
Figure A13. Relation of heat flux to length of heat sink