Determination of effective porosity of soil materials

Robert Horton
_Iowa State University_, rhorton@iastate.edu

Michael L. Thompson
_Iowa State University_, mlthomps@iastate.edu

John F. McBride
_Iowa State University_

Follow this and additional works at: [http://lib.dr.iastate.edu/agron_reports](http://lib.dr.iastate.edu/agron_reports)

Part of the [Agricultural Science Commons](http://lib.dr.iastate.edu/agron_reports), and the [Agronomy and Crop Sciences Commons](http://lib.dr.iastate.edu/agron_reports)

Recommended Citation
Horton, Robert; Thompson, Michael L.; and McBride, John F., "Determination of effective porosity of soil materials" (1988).
_Agronomy Reports_. 5.
[http://lib.dr.iastate.edu/agron_reports/5](http://lib.dr.iastate.edu/agron_reports/5)

This Report is brought to you for free and open access by the Agronomy at Iowa State University Digital Repository. It has been accepted for inclusion in Agronomy Reports by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Determination of effective porosity of soil materials

Abstract

The performance of a compacted soil liner is partly a function of the porosity, which is important because the transport of materials through the liner occurs via the pore space. This project studies the pore spaces of compacted soil materials to estimate the effective porosity, which is the portion of the pore space where the most rapid transport of leachate occurs. Pore space of three soil materials, till, loess, and paleosol, was studied by using mercury intrusion porosimetry, water absorption, and image analysis. These analyses provided cumulative porosity curves from which the pore size distribution of soil samples were estimated. Theory was developed to estimate the effective porosity of a compacted soil material based upon a model of its pore size distribution and pore continuity. The effective porosities of compacted till, loess, and paleosol materials are estimated to be 0.04, 0.08, and 0.09, respectively. These values are 10 to 20% of the total porosities. Comparisons between measured and predicted C1 travel times through compacted soil samples were made in order to verify the estimated effective porosities. The estimated effective porosities are reasonable because predicted C1 first breakthrough times are similar to the measured first breakthrough times in the soils studied. For these three soils predicted first breakthrough times are 5 to 10 times earlier when effective porosity is used in the Darcy-equation based calculations as compared to Darcy-equation-based calculations that utilize total porosity.

Disciplines

Agricultural Science | Agronomy and Crop Sciences

Rights

Works produced by employees of the U.S. Government as part of their official duties are not copyrighted within the U.S. The content of this document is not copyrighted.
DETERMINATION OF EFFECTIVE POROSITY
OF SOIL MATERIALS

by

Robert Horton
Michael L. Thompson
John F. McBride
Iowa State University
Ames, Iowa 50011

Cooperative Agreement No. CR-811093-01-0

Project Officer

Walter E. Grube, Jr.
Land Pollution Control Division
Hazardous Waste Engineering Research Laboratory
Cincinnati, Ohio 45268

HAZARDOUS WASTE ENGINEERING RESEARCH LABORATORY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
CINCINNATI, OHIO 45268

REPRODUCED BY,
U.S. DEPARTMENT OF COMMERCE
NATIONAL TECHNICAL
INFORMATION SERVICE
SPRINGFIELD, VA 22161
### Determination of Effective Porosity of Soil Materials

The performance of a compacted soil liner is partly a function of the porosity, which is important because the transport of materials through the liner occurs via the pore space. This project studies the pore spaces of compacted soil materials to estimate the effective porosity, which is the portion of the pore space where the most rapid transport of leachate occurs. Pore space of three soil materials, till, loess, and paleosol, was studied by using mercury intrusion porosimetry, water desorption, and image analysis. These analyses provided cumulative porosity curves from which the pore size distribution of soil samples were estimated. Theory was developed to estimate the effective porosity of a compacted soil material based upon a model of its pore size distribution and pore continuity. The effective porosities of compacted till, loess, and paleosol materials are estimated to be 0.04, 0.08, and 0.09, respectively. These values are 10 to 20% of the total porosities. Comparisons between measured and predicted Cl⁻ travel times through compacted soil samples were made in order to verify the estimated effective porosities. The estimated effective porosities are reasonable because predicted Cl⁻ first breakthrough times are similar to the measured first breakthrough times in the soils studied. For these three soils predicted first breakthrough times are 5 to 10 times earlier when effective porosity is used in the Darcy-equation based calculations as compared to Darcy-equation-based calculations that utilize total porosity.

<table>
<thead>
<tr>
<th>KEY WORDS AND DOCUMENT ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESCRIBITORS</td>
</tr>
<tr>
<td>Release to Public.</td>
</tr>
</tbody>
</table>

### Technical Report Data

1. REPORT NO. EPA/600/2-88/045
2. TITLE AND SUBTITLE Determination of Effective Porosity of Soil Materials
3. REPORT DATE August 1988
4. AUTHOR(S) Robert Horton, Michael L. Thompson, John F. McBride
5. PERFORMING ORGANIZATION NAME AND ADDRESS Iowa State University, Ames, Iowa 50011
6. SPONSORING AGENCY NAME AND ADDRESS Hazardous Waste Engineering Research Laboratory, Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, OH 45268
7. SUPPLEMENTARY NOTES Project Officer: Walter E. Grube, Jr. (513) 569-7798
8. ABSTRACT The performance of a compacted soil liner is partly a function of the porosity, which is important because the transport of materials through the liner occurs via the pore space. This project studies the pore spaces of compacted soil materials to estimate the effective porosity, which is the portion of the pore space where the most rapid transport of leachate occurs. Pore space of three soil materials, till, loess, and paleosol, was studied by using mercury intrusion porosimetry, water desorption, and image analysis. These analyses provided cumulative porosity curves from which the pore size distribution of soil samples were estimated. Theory was developed to estimate the effective porosity of a compacted soil material based upon a model of its pore size distribution and pore continuity. The effective porosities of compacted till, loess, and paleosol materials are estimated to be 0.04, 0.08, and 0.09, respectively. These values are 10 to 20% of the total porosities. Comparisons between measured and predicted Cl⁻ travel times through compacted soil samples were made in order to verify the estimated effective porosities. The estimated effective porosities are reasonable because predicted Cl⁻ first breakthrough times are similar to the measured first breakthrough times in the soils studied. For these three soils predicted first breakthrough times are 5 to 10 times earlier when effective porosity is used in the Darcy-equation based calculations as compared to Darcy-equation-based calculations that utilize total porosity.

### Distribution Statement

Release to Public.
NOTICE

The information in this document has been funded wholly or in part by the United States Environmental Protection Agency under assistance agreement CR-811093-01-0 to Iowa State University. It has been subjected to the Agency's peer and administrative review, and it has been approved for publication as an EPA document. Mention of trade names or commercial products does not constitute endorsement or recommendation for use.
Today's rapidly developing and changing technologies and industrial products and practices frequently carry with them the increased generation of solid and hazardous wastes. These materials, if improperly dealt with, can threaten both public health and the environment. Abandoned waste sites and accidental releases of toxic and hazardous substances to the environment also have important environmental and public health implications. The Hazardous Waste Engineering Research Laboratory assists in providing an authoritative and defensible engineering basis for assessing and solving these problems. Its products support the policies, programs and regulations of the Environmental Protection Agency, the permitting and other responsibilities of State and local governments and the needs of both large and small businesses in handling their wastes responsibly and economically.

This report describes the results of a study of the pore spaces of soil compacted to engineering specifications for a landfill liner. Effective porosity, which is that portion of the pore space where the most rapid transport of fluids takes place, is a parameter important to engineers designing earthen barrier structures. The data in this report will be useful to earth science and engineering professionals involved in design, construction, and testing of earthen barriers for waste management.

For information, please contact the Land Pollution Control Division of The Hazardous Waste Engineering Research Laboratory.

Thomas R. Hauser, Director
Hazardous Waste Engineering Research Laboratory
ABSTRACT

Hazardous waste disposal landfills require liners constructed of compacted soil material to help prevent the migration of hazardous wastes. The performance of a compacted soil liner is partly a function of the porosity. Porosity is important because the transport of materials through the liner will occur via the pore space. The main purpose of this project is to study the pore spaces of compacted soil materials and to estimate the effective porosity, which is the portion of the pore space where the most rapid transport of leachate occurs.

The pore space of three soil materials, till, loess, and paleosol, is studied by using mercury intrusion porosimetry, water desorption, and image analysis. These analyses provide cumulative porosity curves from which the pore size distribution of a soil sample may be estimated.

Theory is developed to estimate the effective porosity of a compacted soil material based upon a model of its pore size distribution and pore continuity. The effective porosities of the compacted till, loess, and paleosol materials are estimated to be 0.04, 0.08, and 0.09, respectively. These values are 10 to 20% of the total porosities.

Comparisons between measured and predicted Cl⁻ travel times through compacted soil samples are made in order to verify the estimated effective porosities. The estimated effective porosities are reasonable because predicted Cl⁻ first breakthrough times are similar to the measured first breakthrough times in compacted till, loess, and paleosol materials. For the three soil materials used in this study, predicted first breakthrough times are 5 to 10 times earlier when effective porosity is used in the Darcy-equation-based calculations as compared to Darcy-equation-based calculations that utilize total porosity.

This report was submitted in fulfillment of cooperative agreement CR-811093-01-0 by Iowa State University under the sponsorship of the U.S. Environmental Protection Agency. This report covers a period from 10/1/83 to 4/30/86, and work was completed as of 12/31/85.
CONTENTS

Abstract ........................................................................................................ iv
Figures ......................................................................................................... vi
Tables .......................................................................................................... x
Acknowledgments ......................................................................................... xii

1. Introduction .......................................................................................... 1
2. Conclusions ......................................................................................... 5
3. Recommendations ............................................................................... 7
4. Materials and Methods ....................................................................... 8
5. Theoretical .......................................................................................... 19
6. Results and Discussion ...................................................................... 36

References ................................................................................................. 113
FIGURES

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic diagram of the experimental set-up used for the measurement of soil permeability and for solute breakthrough curves.</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>Influence of n on effective porosity (E) at various values of $\Theta_s$ ($\Theta_s = 0.05$).</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>Secant CD corresponds to the pair $(r, \Theta)$. CD is a distance $r$ from the center of the circle, O. The normal to CD through O makes the angle $\Theta$ with respect to the angular origin OK.</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>Construction of a pattern inside a circle of radius 1 with: $\Theta = \pi/2$, $\Theta_2 = \pi/2$, $\lambda = 5/\pi$, $P(\text{pore}) = 0.3$. Draw a circle with radius $r = 1$ and choose an angular origin (1). Equation (13) shows that $N = 5$ for this case. Choose five $(r, \Theta)$ pairs using the method described in the text and draw the corresponding secants (2). Assign each region formed to the states &quot;P&quot; and &quot;M&quot; (pore and matrix) so that each region has a probability of 0.3 of being assigned to the pore state (3). Color the pore cells black and erase the lines dividing the matrix cells (4).</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>Segment GH represents all points along line $OH$ which have secants intersecting the line segment AB. Note that AB makes the angle $\Theta$ with respect to the angular origin.</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>Moisture density relations for (a) the till derived soil, (b) the loess derived soil, and (c) the paleosol soil material.</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>Mercury intrusion porosimetry determined pore size distributions for till samples using three drying techniques.</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>Mercury intrusion porosimetry determined pore size distributions for compacted loess samples using three drying techniques.</td>
<td>41</td>
</tr>
</tbody>
</table>
Number | FIGURES (continued)                                                                 | Page |
---     |----------------------------------------------------------------------------------------------------------------------------------------|------|
9       | Mercury intrusion porosimetry determined pore size distributions for compacted paleosol samples using three drying techniques.          | 46   |
10      | Mercury intrusion porosimetry determined pore size distributions for undisturbed till samples using three drying techniques.          | 47   |
11      | Mercury intrusion porosimetry determined pore size distributions for undisturbed loess samples using three drying techniques.          | 48   |
12      | Mercury intrusion determined pore size distributions for undisturbed paleosol samples using three drying techniques.                  | 49   |
13      | Mercury intrusion porosimetry determined cumulative porosity curve for compacted till samples using three drying techniques.          | 50   |
14      | Mercury intrusion porosimetry determined cumulative porosity curves for compacted loess samples using three drying techniques.         | 51   |
15      | Mercury intrusion porosimetry determined cumulative porosity curves for compacted paleosol samples using three drying techniques.      | 52   |
16      | Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed till samples using three drying techniques.          | 53   |
17      | Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed loess samples using three drying techniques.      | 54   |
18      | Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed paleosol samples using three drying techniques.    | 55   |
19      | Water desorption curves for compacted soil materials.                                                                               | 59   |
20      | Water desorption curves for undisturbed till material.                                                                               | 60   |
21      | Water desorption curves for undisturbed loess material.                                                                               | 61   |
22      | Permeability as a function of time for compacted till material.                                                                        | 63   |
<table>
<thead>
<tr>
<th>Number</th>
<th>FIGURES (continued)</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>Permeability as a function of time for compacted loess material.</td>
<td>64</td>
</tr>
<tr>
<td>24</td>
<td>Permeability as a function of time for compacted paleosol material.</td>
<td>65</td>
</tr>
<tr>
<td>25</td>
<td>Cl(^{-}) breakthrough curves for compacted till material.</td>
<td>67</td>
</tr>
<tr>
<td>26</td>
<td>Cl(^{-}) breakthrough curves for compacted loess material.</td>
<td>68</td>
</tr>
<tr>
<td>27</td>
<td>Cl(^{-}) breakthrough curves for compacted paleosol material.</td>
<td>69</td>
</tr>
<tr>
<td>28</td>
<td>Three replicates of measured and fitted (solution to convective-dispersive equation) Cl(^{-}) breakthrough data for compacted till materials.</td>
<td>70</td>
</tr>
<tr>
<td>29</td>
<td>Three replicates of measured and fitted (solution to convective-dispersive equation) Cl(^{-}) breakthrough data for compacted loess materials.</td>
<td>71</td>
</tr>
<tr>
<td>30</td>
<td>Three replicates of measured and fitted (solution to convective-dispersive equation) Cl(^{-}) breakthrough data for compacted paleosol materials.</td>
<td>72</td>
</tr>
<tr>
<td>31</td>
<td>Cl(^{-}) breakthrough curves for undisturbed till material.</td>
<td>73</td>
</tr>
<tr>
<td>32</td>
<td>Cl(^{-}) breakthrough curves for undisturbed loess material.</td>
<td>74</td>
</tr>
<tr>
<td>33</td>
<td>Cl(^{-}) breakthrough curves for undisturbed paleosol material.</td>
<td>75</td>
</tr>
<tr>
<td>34</td>
<td>Theoretical cumulative porosity curves as influenced by the n-parameter ((\alpha = 1.0, S = \theta/\theta_s)).</td>
<td>77</td>
</tr>
<tr>
<td>35</td>
<td>Measured and curvefitted cumulative porosity for compacted till material.</td>
<td>78</td>
</tr>
<tr>
<td>36</td>
<td>Measured and curvefitted cumulative porosity for compacted loess material.</td>
<td>79</td>
</tr>
<tr>
<td>37</td>
<td>Measured and curvefitted cumulative porosity for compacted paleosol material.</td>
<td>80</td>
</tr>
<tr>
<td>38</td>
<td>Theoretical relative permeability ((K_r)) curves as influenced by the n-parameter ((S = \theta/\theta_s)).</td>
<td>82</td>
</tr>
<tr>
<td>39</td>
<td>Porosity image obtained by photographing fluorescent dye in soil pores with a 35-mm camera and a macro lens. Diameter of soil sample = 5.5 cm.</td>
<td>88</td>
</tr>
<tr>
<td>Number</td>
<td>FIGURES (continued)</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>40</td>
<td>Example of an image of soil porosity obtained with circularly polarized light. Undisturbed Clarinda soil materials. Frame length = 6.6 mm.</td>
<td>90</td>
</tr>
<tr>
<td>41</td>
<td>Example of a backscattered electron image of a compacted Clarinda soil sample showing holes in the surface. Frame length = 100 µm.</td>
<td>91</td>
</tr>
<tr>
<td>42</td>
<td>Image analyzer determined pore size distribution histogram for compacted till material.</td>
<td>96</td>
</tr>
<tr>
<td>43</td>
<td>Image analyzer determined pore size distribution histogram for compacted loess material.</td>
<td>97</td>
</tr>
<tr>
<td>44</td>
<td>Image analyzer determined cumulative porosity curves for (a) till materials and (b) loess materials.</td>
<td>99</td>
</tr>
<tr>
<td>45</td>
<td>Original and Markov-generated pore patterns for soil material A.</td>
<td>102</td>
</tr>
<tr>
<td>46</td>
<td>Original and Markov-generated pore patterns for test image B.</td>
<td>103</td>
</tr>
<tr>
<td>47</td>
<td>Original and Markov-generated pore patterns for soil material C.</td>
<td>104</td>
</tr>
<tr>
<td>48</td>
<td>Original and Markov-generated pore patterns for soil material D.</td>
<td>105</td>
</tr>
<tr>
<td>49</td>
<td>Image analyzer determined pore size distribution histograms for the original and Markov generated pore patterns of soil material A.</td>
<td>106</td>
</tr>
<tr>
<td>50</td>
<td>Image analyzer determined pore size distribution histograms for the original and Markov generated pore patterns of soil material C.</td>
<td>107</td>
</tr>
<tr>
<td>51</td>
<td>Image analyzer determined pore size distribution histograms for the original and Markov generated pore patterns of soil material D.</td>
<td>108</td>
</tr>
<tr>
<td>52</td>
<td>Image analyzer determined cumulative porosity curves for the original and Markov-generated pore patterns of soil material D.</td>
<td>109</td>
</tr>
<tr>
<td>53</td>
<td>Original and Markov-generated pore patterns for test image E.</td>
<td>111</td>
</tr>
<tr>
<td>54</td>
<td>Original and Markov-generated pore patterns for test image F.</td>
<td>112</td>
</tr>
<tr>
<td>Number</td>
<td>Table Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>Listing of the values of $N(L,\theta)$ for the four possible cases.</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>Particle size distribution, liquid limit, plastic limit, and plasticity index of subsoil materials.</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>Classification of subsoil materials according to USDA, AASHTO, and Unified classifications.</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>Particle density, undisturbed bulk density, optimum bulk density and moisture content of subsoil materials.</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>Total porosity of soil materials before and after drying.</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>Volumetric soil water content at four tensions for compacted soil samples.</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
<td>Permeability of three replicates of each Proctor-type compacted subsoil material.</td>
<td>62</td>
</tr>
<tr>
<td>8</td>
<td>Permeabilities of compressed and extruded subsoil materials.</td>
<td>62</td>
</tr>
<tr>
<td>9</td>
<td>Permeabilities of undisturbed soil samples.</td>
<td>66</td>
</tr>
<tr>
<td>10</td>
<td>Total porosities of compacted soil materials before and after freeze drying.</td>
<td>76</td>
</tr>
<tr>
<td>11</td>
<td>Parameters used with equation (8) to describe the cumulative porosity of the soil materials.</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>Measured permeabilities and times of first chloride breakthrough and predicted effective porosities and noninteracting pollutant breakthrough times for compacted soil materials (permeameters were 11.64 cm long).</td>
<td>84</td>
</tr>
<tr>
<td>13</td>
<td>Results of porosity analyses obtained by Leitz TAS image analyzing computer.</td>
<td>94</td>
</tr>
<tr>
<td>Number</td>
<td>TABLES (continued)</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>14</td>
<td>Markov probability statistics for selected pore patterns. The transitional probabilities are for pore to pore (P+P), pore to matrix (P+M), matrix to pore (M+P), and matrix to matrix (M+M). The corresponding patterns are shown in Fig. 45, 46, 47, 48, 53, and 54.</td>
<td>100</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

The authors are indebted to many individuals for their contributions to this study. Major contributions were made by J. Barmettler, H. Bruce, L. Wolf, T. Meyer, and G. J. Kluitenberg in conducting laboratory experiments. We thank Dr. Darrell Norton and the National Soil Erosion Laboratory, West Lafayette, Indiana, for access to the Leitz image analysis system and for assistance with the image analysis data. P. Murphy and D. Kyle contributed with the clerical work of the project.

The study was supported by the U. S. Environmental Protection Agency under Cooperative Agreement CR-811093-01-1 to Iowa State University, by Iowa Agriculture and Home Economics Experiment Station Project No. 2556, and by an Iowa State University Research Grant.

We thank Dr. Walter E. Grube, Jr., the U. S. EPA Project Officer for providing helpful comments, discussions, and support during the course of the project.
SECTION 1
INTRODUCTION

Our technological society produces a variety of hazardous chemicals that must be disposed of. For the foreseeable future, landfills will be used for the disposal of these hazardous wastes. Ideally, the disposal of hazardous wastes in landfills should be done with minimal effects on the environment.

Compacted clay is frequently used to line hazardous waste disposal landfills. The purpose of the compacted clay liner is to restrict the movement of hazardous liquid materials out of the landfill. Currently, saturated hydraulic conductivity (usually referred to as soil permeability in the Environmental Protection Agency's design standards) is the most common measurement used to estimate the ability of liner material to contain wastes (USEPA, 1978). Soil saturated hydraulic conductivity, \( K \) (\( L^3/L^2/T \)), is defined from Darcy's equation:

\[
K = \frac{J}{I} \quad [1]
\]

where \( J \) is the fluid flux density (\( L^3/L^2/T \)) and \( I \) is the hydraulic gradient (\( L/L \)). Soil saturated hydraulic conductivity has dimensions of volume per unit area of liner per unit time, and thus, it is a bulk parameter related to the areal average fluid flow in the liner. Saturated hydraulic conductivity is often used to make preliminary estimates of solute transit times, but knowledge of average fluid flow alone is not adequate for accurate prediction of pollutant breakthrough or travel time. Nonuniform flow velocities through a cross-sectional area should be accounted for because faster-than-average fluid flow may be responsible for the first appearance of the pollutant below the liner.

The convective-dispersive equation is commonly used to model
miscible displacement processes in porous media. The diffusive-dispersive coefficient in this equation attempts to describe the nonuniform solute velocities (Van Genuchten and Wierenga, 1986). Because the nonuniform velocities are described, this model is an improvement over the areal average velocity approach. The disadvantage of the convective-dispersive equation is that empirical measurements of the diffusive-dispersive coefficient are required for each soil and set of flow conditions. The breakthrough curve is generally the basis of the determination (Nielsen and Biggar, 1961). With compacted clayey materials of low permeability, the measurement of a solute breakthrough curve may take months. Moreover, the solute breakthrough curve itself yields a direct measurement of travel time, thereby decreasing the need for the convective-dispersive equation for prediction of first breakthrough time.

Effective porosity, E, has been described as that portion of the total liner porosity that contributes significantly to fluid flow. Effective porosities are less than total porosities because some of the pore space is discontinuous (dead end) and some of the pore space is so narrow that fluids in these spaces are essentially immobile. Coats and Smith (1964), Skopp and Warrick (1974), and Van Genuchten and Wierenga (1976) all provide mathematical models that explicitly include effective pore space. They describe how to use numerical solutions and experimental breakthrough curve data to solve for effective porosity. This report will not address these approaches in detail, but rather, will develop a new technique based upon Darcy's equation for estimating effective porosity and solute travel time. Because the fluid flux density through a clay liner can be determined by using Darcy's equation, the mean effective fluid velocity, \( \bar{V}_E \), can be described by:

\[
\bar{V}_E = \frac{K}{E} \tag{2}
\]

With a constant hydraulic gradient and the liner K and E both known, the average pollutant travel distance per unit time can be estimated. Thus, the time of first breakthrough, T, of a noninteracting pollutant can be predicted by the equation:
\[ T = \frac{EL}{KI} \]

where \( L \) is the liner thickness (m).

The Environmental Protection Agency requires that a disposal unit liner prevent migration during the active life of a unit (USEPA, 1982). The active life of a unit containing noninteracting pollutants can be estimated by using Eq. (3). Knowledge of the liner permeability alone is not sufficient information to accurately estimate the length of time of liner effectiveness. When a disposal unit is constructed with known liner thickness, \( L \), and designed for a known hydraulic gradient, \( I \), measurement of both permeability and effective porosity are required to estimate the active life of the unit with Eq. (3). Although methods of measuring permeability of compacted clay materials have received much attention in the literature (Olson and Daniel, 1981), determination of effective porosity has received little attention.

This report presents a method for estimating the effective porosity of compacted clay materials. This method is based upon the soil pore size distribution. The estimates of effective porosity are used in Eq. (3) to predict the travel time of a noninteracting solute through samples of compacted materials, independent of solute breakthrough experiments. This work adds to the previous capillary-tube based efforts of modeling miscible displacement (Lindstrom and Boersma, 1971; Rao et al., 1976; Reddell and Smajstrla, 1983) by providing a simple method for predicting solute travel time through compacted soil material. The proposed method is tested by comparing predicted and measured travel times.

The specific project objectives are:

1. To determine saturated hydraulic conductivities and solute breakthrough curves for undisturbed and remoulded samples of till, loess, and paleosol soil materials.
2. To obtain morphometric measurements of porosity for the soil materials.
3. To correlate measured solute breakthrough curves with predicted values based on the morphometric porosity measurements.
4. To develop a model to predict the effect of changes in porosity on leachate breakthrough.

The study consisted of laboratory measurements to describe physical properties of the soil materials. Theory predicting solute travel time through the soil materials was developed and tested.
SECTION 2
CONCLUSIONS

1. The theory that utilized pore size distribution determined by mercury intrusion porosimetry provided reasonable estimates of effective porosity for compacted soil materials.

2. Predictions of Cl\textsuperscript{-} first breakthrough made by using effective porosity and permeability were reasonably close to measured Cl\textsuperscript{-} breakthrough for all three compacted soil materials.

3. The active lifetime of hazardous-waste-disposal units may be overestimated if liner effective porosity is not properly estimated and used in the prediction of pore-fluid velocity.

4. The permeabilities of till, loess, and paleosol samples compacted at water contents 1 to 2% higher than optimum on the moisture-density relation curve were less than $10^{-9}$ m/s.

5. The effective porosities of the compacted till, loess, and paleosol soil materials were estimated to be 0.04, 0.08, and 0.09, respectively.

6. The effective porosities of the three compacted soil materials studied were between 10 and 20% of the total porosities.

7. Undisturbed samples of the till, loess, and paleosol soil materials had permeabilities greater than $10^{-9}$ m/s.

8. Cl\textsuperscript{-} breakthrough curves of undisturbed cores were highly skewed, indicating the presence of macropores and thus a bimodal distribution of pore sizes.

9. Removal of water from large samples of soil materials containing smectitic clay was difficult to do without causing samples to shrink, causing a decrease in total porosity.

10. Shrinkage of soil samples occurred when standard acetone replacement techniques were used to remove water.
11. Removal of water without observable large changes in soil structure was possible by using methanol replacement prior to acetone exchange.

12. Compared to oven drying and acetone drying, freeze drying caused minimal loss of soil porosity.

13. Freeze dried samples of compacted soil materials seemed to have increased the volume of pores in the 0.2 to 2.0 μm range compared with the original porosity before drying.

14. Impregnating samples of compacted soil material with plastic without altering the porosity was very difficult.

15. Once soil samples were impregnated with plastic the task of discerning pores from matrix required considerable human judgment and could not be left to a computer alone.

16. Image analyzers had a difficult time describing soil porosity in reasonable ways.

17. Use of Markov statistics to describe soil pore patterns was found to be simple and promising for compacted materials with unimodal pore size distributions.

18. The Markov method was difficult to use effectively on patterns with bimodal pore size distributions and non-random spatial distributions of pores on an image.
SECTION 3

RECOMMENDATIONS

1. An estimate of effective porosity of the clay liner should be used to calculate the active life of a disposal unit.
2. Careful mixing and compacting of soil materials is necessary to remove the cracks and pores that occur in undisturbed soil materials.
3. The proposed theory for estimating effective porosity should be tested on other compacted soil materials by comparing predicted and measured solute breakthrough times.
4. The theory should be tested with a retardation term, \( R, (T = ELR/KI) \) that reflects positive or negative adsorption.
5. The theory should be tested by comparing predicted and measured breakthrough times for low hydraulic gradient (<25) flow conditions.
6. Further work should be done to determine if methanol exchange and acetone replacement methods will preserve soil porosity before resin impregnation.
7. Further work should be done with soil impregnation techniques to try to find a way to impregnate soil without first removing the water.
8. Further work should be done to account for pore shape and continuity in order to estimate the effective porosity.
9. Methods should be developed to quantify pore structure in three dimensions in order to better characterize the porosity.
SOIL MATERIALS

We collected subsurface samples from three soils developed in major Iowa parent materials: till, loess, and paleosol. The till-derived soil was Nicollet, a fine-loamy, mixed, mesic Aquic Hapludoll. The loess-derived soil was Fayette, a fine-silty, mixed, mesic Typic Hapludalf. Clarinda, an exhumed paleosol, is a fine, montmorillonitic, mesic, sloping Typic Argiaquoll. The clay fraction of each material was dominated by smectite, with small amounts of clay mica and kaolinite.

Undisturbed cores were collected with a hydraulically driven tube 0.065 m in inner diameter. The cores were cut into segments about 0.076 m long, placed in plastic bags, and stored at 4°C. Bulk samples of each soil material were collected by shovel excavation. The disturbed samples were air-dried, crushed, sieved, and stored in labeled containers.

General Physical Properties

After the bulk samples were air-dried and ground to pass a 2-mm sieve, selected physical properties were determined. The particle density of each subsoil material was determined with pycnometers according to Blake (1965a). Sand, silt, and clay contents were determined either by total fractionation (sedimentation and weighing) or by a pipette technique similar to that of Day (1965). Atterberg limits (plastic limit, liquid limit, plasticity index) were obtained for each subsoil material with standard methods of the American Society for Testing Materials, ASTM (1964).

The particle size distribution data and the Atterberg limits were used to classify each soil material according to the United States
Department of Agriculture (USDA) textural triangle, the American Association of State Highway and Transportation Officials (AASHTO) classification, and the Unified classification.

The bulk density of each subsurface material was determined with undisturbed samples. The paraffin coating technique described by Blake (1965b) was used on core fragments of 70 to 100 cm$^3$.

**Moisture-Density Relationship**

The standard Proctor moisture-density relations of each air-dried, ground subsoil material were determined. Several sub-samples of each subsoil material were re-wetted to a range of moisture contents and then compacted in a 0.102-m diameter mold by using a 2.49 kg rammer with a 0.305-m drop, according to the ASTM Test D-698-78, Method A (1982).

A measured mass of air-dried, sieved soil was spread thinly on a sheet of plastic. Distilled water was misted on by using a spray bottle until the soil at the surface was wetted. Then the soil was massed in the center of the sheet by moving the corners of the plastic towards the middle. The soil was further mixed by gently rubbing it between the hands in order to break any clods that may have formed. Next, the soil was spread out again and the above repeated until the predetermined amount of water was applied. A uniformly moist, clod-free, friable soil was thus obtained, which was then placed in a dishpan, sealed in a plastic bag, and stored in a constant temperature and humidity room for a minimum period of time as specified by ASTM standards. Finally, a standard moisture-density relation test was performed.

**Mercury Intrusion Porosimetry**

The pore size distribution of undisturbed and compacted samples of each subsoil material was determined by mercury porosimetry. Compacted samples were prepared at moisture contents 1 to 2% higher than "optimum", which was obtained from the moisture-density relations. First, the undisturbed and compacted samples were broken into fragments of ~2 cm$^3$. Second, the fragments were plunged into liquid nitrogen and then dried in a freeze dryer for 4 to 5 days (Zimmie and Almaleh, 1976).

An alternative method of removing water from the samples was also sought. We compared the effects of acetone drying to freeze drying and slow oven drying on subsamples about 2-6 cm$^3$ by using mercury
porosimetry. Fragments of compacted samples were dried in three ways. Some were dried for one week in an oven set at 40°C. Some were plunged into liquid nitrogen and then dried in a freeze dryer for 4 to 5 days (Zimmie and Almaleh, 1976). Finally, some fragments were dried by acetone vapor exchange by a method similar to that of FitzPatrick and Gudmundsson (1978). These fragments were exposed to acetone vapor in increasing concentrations of up to 100%. Then the fragments were placed in contact with liquid acetone for 24 hours (to be strictly comparable to resin-impregnation techniques). Finally, the acetone was allowed to evaporate before intrusion of Hg.

Porosities of triplicate fragments (~1 cm$^3$) of each subsoil material and each drying method were measured by using a Quantachrome SP-200 mercury porosimeter in four intrusion steps: 0 to 0.1 MPa, 0.1 to 8.3 MPa, 8.3 to 41.4 MPa, and 41.4 to 414 MPa (60,000 psi). Because the volume of mercury intruded was recorded continuously as a function of pressure, cumulative pore size could be calculated with the capillary rise equation: $r = - (1/P)(2\gamma \cos \phi)$, where $r$ is the equivalent pore radius (m), $P$ is the pressure (Pa), $\gamma$ is the surface tension of mercury (N/m), and $\phi$ is the wetting angle of mercury (assumed to be 140°C). Cumulative porosity was plotted as cumulative intruded volume versus equivalent pore radius.

The specific methodology used on our soil samples after drying was to carefully reduce samples to approximately a 1.0-cm length and 0.5-cm width; the sample chamber could hold about 3 such pieces which together weighed around 2.0 g. Then, the sample was weighed to the nearest one hundredth gram. Next, the sample was placed into the sample holder, which consisted of a sample chamber attached to a bored stem; the bore had a volume of 0.5 cm$^3$, which was the maximum volume of mercury that could be intruded. An electrode plate was placed over the opening of the sample chamber after coating with a thin layer of vacuum grease. Then, a plastic housing was fitted over the sample chamber and an endcap was screwed on, assuring a tight seal between electrode plate and sample holder. The system was weighed to the nearest one hundredth gram.

Next, the above system was inserted into the filling apparatus, consisting of a rotating jar (one-quarter-filled with mercury) and a
vacuum pump. Air was evacuated from the filling apparatus until -1 kg/cm$^2$ of pressure was reached. Then, the bell jar was rotated from a horizontal position to a vertical one until the stem of the sample holder was immersed in mercury. Next, air was allowed to enter the bell jar in small increments. Since the sample chamber was sealed by the pool of mercury, a pressure gradient developed. At -0.76 kg/cm$^2$ dials indicated that mercury had filled the stem and sample chamber. The bell jar was then rotated to the horizontal, the volume readout dial zeroed, the microprocessor initialized, and pressure allowed to go from -0.76 kg/cm$^3$ to ambient. Once at ambient, the microprocessor was switched to recall and the volume, percent volume, $D_v(r)$, and surface area versus pressure plots were drawn on graph paper.

To prepare for the high-pressure intrusion, the sample holder was removed from the filling apparatus. Upon visual inspection, one could see that mercury was not to the top of the stem since some had already intruded into the sample. At this step, the stem could be topped off either with mercury and weighed in order to calculate the bulk density or with hydraulic oil for high-pressure intrusion; there must be a 1-cm length of hydraulic oil along the end of the stem for this. Next the sample holder was encased in a thick metal sheath and subsequently dropped into the high-pressure chamber. Then, the chamber was sealed and hydraulic oil pumped in to bleed the system of air.

At this point, the pressure and volume dials were zeroed, the maximum pressure (psi) for the run set, the microprocessor initialized, and the pressurizing motor turned on. Pressurization continued until the maximum was reached; the rate of pressurization was adjustable. When the maximum pressure was reached, the pressure and volume were recorded. Then the porosimeter began its depressurizing cycle and shut off automatically at zero pressure; again, the pressure and volume were recorded. Next, the plots were drawn on the appropriately scaled graph paper. **NOTE:** By the time of the start of the next pressure intrusion, the volume readout had decreased some more; this lower number served as the zero for the next plot since the microprocessor zeroed the x-y scale every time it initialized.

The above steps were repeated for all of the desired pressure
intrusions except that the volume dial was not zeroed. At the end of
the 0-60,000 psi intrusion the sample holder was removed from the
pressure chamber and sheath and the stem was topped off with mercury;
the system was weighed in order to calculate the bulk density after
intrusion to 60,000 psi (414 MPa).

We calculated initial total porosity ($\varepsilon$) of the samples from the
relation $\varepsilon = 1 - (\rho_b / \rho_p)$, where $\rho_b =$ bulk density and $\rho_p =$ particle
density. Bulk density of compacted samples was known because a known
mass of soil material was compacted to a known volume. Particle density
for each soil material was determined with a pycnometer, according to
Blake (1965a).

Bulk density of freeze-dried samples at 0.1 MPa (i.e., ambient
pressure) in the porosimeter was calculated by using the relation:

$$\rho_b = \frac{m_1}{(v_1 + v_2) - \left[\frac{(m_3 - m_2)}{\rho_{Hg}}\right]}$$

where $m_1 =$ sample mass (Mg), $v_1 =$ volume of sample holder ($m^3$), $v_2 =$
volume of mercury intruded at 0.1 MPa ($m^3$), $m_2 =$ mass of sample holder
(Mg), $m_3 =$ mass of mercury-filled sample holder with sample after
intrusion to 0.1 MPa (Mg), and $\rho_{Hg} =$ density of mercury (13.541 Mg/m$^3$
at
20$^\circ$C). Total porosity calculated from these bulk density values and the
pycnometer-determined particle density was consistently very close
(within 0.01 to 0.02 cm$^3$ of porosity per cm$^3$ of soil) to total porosity
values calculated for freeze-dried samples after mercury intrusion to
414 MPa, suggesting that total porosities at ambient pressure and after
pressurization were essentially equal. Total porosity of oven-dried and
acetone-dried samples was calculated solely from mercury intrusion data.

Water Desorption

Standard soil-water characteristic data were obtained for compacted
samples by determining soil water contents of duplicate samples at 0.05,
0.1, 0.2, and 1.5 MPa. These pressures correspond to equivalent pore
radii of 3, 1.5, 0.75, and 0.1 $\mu$m, respectively, and were chosen to
measure pores in size ranges roughly comparable to those measured by the
mercury porosimeter. Samples were equilibrated at those pressures by
using a standard pressure plate apparatus (Bouma et al., 1974). The
water contents of undisturbed till and loess materials were determined
at 0.01, 0.025, 0.05, 0.075, 0.10, 0.20, 0.30, and 0.40 MPa.
Permeability

Permeability of compacted soil materials was measured in two ways. The first method used compressed and extruded samples, and the second method used Proctor-type, compacted samples.

From the moisture-density relation data, a point one or two percentage points above "optimum" moisture content was chosen. A calculation was made to determine the mass of moist soil at the given moisture content needed to form a 6.35 cm diameter by 2.54 cm high cylinder at the corresponding dry density. The loose, moist soil was placed into a mold consisting of a base-plate, cylindrical wall and plunger. A hydraulic press was used to force the plunger down until a mark was reached indicating that the plunger was 2.54 cm from the base plate; the hydraulic press did not indicate a load-pressure at this stage. Next, the compacted soil sample was extruded by forcing the plunger through. Then, the sample was placed in a properly labeled plastic bag and stored in a refrigerator.

The pressed soil samples described above were placed in 7.62-cm diameter acrylic permeameters. Paraffin, kept at 60°C, was poured between the wall of the permeameter and the sample in 0.5-cm increments until the top of the sample was reached; the paraffin was allowed to solidify between increments. In this manner, any gaps formed when the paraffin cooled were filled by the next increment.

Next, a solution of de-aired, saturated CaSO₄ with 0.06% formaldehyde to control microbial growth, was introduced into the permeameter and pressurized from an air-pressure source. The air was separated from the solution by means of a rubber membrane within the solution container (Figure 1 displays the physical set-up). Hydraulic gradients in the range of 100-200 were used. Because of the relatively small sample height, this required a hydraulic head of approximately 250-500 cm H₂O (3.5 - 7.5 psi). Once the hydraulic gradient was established, a minimum of 5 pore-volumes was passed to obtain a constant flux from which to calculate the hydraulic conductivity.

With the second method, the permeability of each compacted subsoil material was measured by using a 0.102-m diameter, 0.116-m long permeameter. Three replicates of each subsoil material were compacted
Figure 1. Schematic diagram of the experimental set-up used for the measurement of soil permeability and for solute-breakthrough curves.
at moisture contents ~1 to 2% above optimum, determined from the moisture-density relation. De-aired, saturated CaSO₄ solution (adjusted to 0.06% formaldehyde to control microbial growth) was introduced at 6.9 kPa pressure at the bottom of the permeameter to slowly saturate the compacted sample. The source of the pressure was compressed air. The air was separated from the saturated CaSO₄ solution by a rubber membrane within the solution container to help prevent the desaturation of the soil material. After saturating the material, the CaSO₄ solution was introduced at the top of the permeameter at hydraulic gradients of ~170 to 270, and the rate of solution movement through the sample was measured over time.

Permeability was determined on undisturbed soil samples for each soil material. The undisturbed materials were placed into permeameters and sealed with paraffin wax. Constant-head methods (Klute, 1965) were used to determine soil permeability.

**Solute Breakthrough**

Solute breakthrough measurements were made for compacted and undisturbed soil samples. The breakthrough measurements were made on the same soil samples used to determine permeability. Thus, breakthrough measurements were made on two sizes of disturbed samples and one size of undisturbed sample.

Solute breakthrough curves were obtained for each compacted and undisturbed subsoil sample. A de-aired, 0.05N CaCl₂ solution with 0.016% Acid Fuchsin (a red dye), and 0.06% formaldehyde was used as the tracer solution. The dye gave a visual test for any permeameter wall leakage. The tracer solution was exchanged for the saturated CaSO₄ solution and allowed to leach through the compacted soil sample under the same hydraulic gradient used in the permeability test. The leachate was collected in equal-volume increments with a fraction collector and the leachate was analyzed for chloride concentration with an automatic titrator.

**IMAGE ANALYSIS**

**Sample preparation techniques**

We "fixed" the pores of soil samples by filling them with a plastic resin (impregnation) and hardening the resin (curing) by heating it.
Because the resin was not miscible with water, it was necessary to remove all water from the samples before impregnation. The standard method among soil micromorphologists to remove water is by replacement with acetone (Miedema et al., 1974; FitzPatrick and Gudmundsson, 1978), and this was the first method tried.

Typically, a compacted sample about 50 cm$^3$ was placed either in contact with acetone or in 100% acetone vapor for a period of about one week. The acetone was changed every day or every other day until it contained less than 3% water. At that point, the sample was impregnated with a mixture of polyester resin and acetone (50/50 by volume) and placed under a vacuum for 2-3 hours. The sample was left to sit, covered, for a period of 2 to 5 days, then it was uncovered and left in a laboratory hood for another 2 to 5 days. Finally, the sample was gradually heated over a period of 2 to 5 days from 30°C to 60°C. This slow heat treatment normally caused the samples to harden well enough that they could be cut open with a diamond trim saw. Details of treatments similar to those described above may be found in FitzPatrick’s (1984) book.

We removed water from the high-clay soil samples (Fayette and Clarinda) by a two-step process that employed methanol. We immersed moist, compacted samples of about 50 cm$^3$ in 100% methanol in a covered container. Each day for a week the methanol was changed until the amount of water in the methanol was <5%. Then the samples were immersed in acetone in the same covered container. The acetone was changed each day until the level of methanol in the acetone was <5%. Following these treatments, the samples were impregnated with a resin/acetone mixture in the normal way.

Obtaining an image

The primary technique that we used to obtain an image of the porosity of impregnated soil samples was with a macro lens and a 35-mm camera. In the resin/acetone mixture used for impregnation we dissolved a small amount (~0.3%) of a dye (Uvitex-OB, CIBA-GEIGY Chemicals Corp.) that is sensitive to ultraviolet light. After the impregnated sample was cured, cut open, and polished by standard petrographic techniques, it was photographed under ultraviolet light by using Kodak Ectagraphic
HC (high contrast) film. Because only pores contained resin and dye, only pores were exposed on the film. Following printing, these images were typically about 2X the size of the original samples. This technique was originally developed by Murphy et al. (1977a).

We experimented with two other techniques to obtain porosity images. In the first instance, we prepared thin sections of the hardened soil blocks by standard petrographic techniques. These sections were about 30μm thick, as measured by the birefringence of quartz in the samples. When viewed in circularly polarized light, all mineral materials should have some birefringence unless they are opaque or the c-axis of the mineral is exactly perpendicular to the stage of the microscope (Pape, 1974). By using this principle, we made photographs in circularly polarized light with a modified petrographic microscope and high-contrast film. After printing, these images were typically about 20X the size of the original samples.

The final technique used to obtain images of soil porosity employed a scanning electron microscope and backscattered electron detectors. A hardened block of soil was cut into pieces measuring about 2.5 x 2.5 x 1.2 cm. One face of each piece was polished with polishing grits, gradually changing from 180 grit to 600 grit to 1-μm diamond grit. A 200-Å coating of gold was put on the polished surface with a vacuum sputter coater. The sample was then viewed with a JEOL U3 scanning electron microscope operated in backscatter mode at 25 kV. In theory, the difference in atomic number between resin in the pores and minerals in the soil matrix should provide significant enough contrast for the two phases to be distinguished. Images were typically magnified 1000X. This approach to obtain soil porosity images was first reported by Jongerius and Bisdom (1981). Problems that we identified with each of the imaging techniques we used are discussed in the Results and Discussion section.

Analyzing the images

Once obtained, images of soil porosity are normally very complex. While it is possible to digitize such images by hand, this would be an enormous task. Therefore, we sought image analyzing computers that would automatically digitize each image and then quantify the digitized
elements. We took three approaches.

First, we utilized a microcomputer, a digitizing board compatible with the microcomputer, and a video camera that captured the porosity image and sent it to the digitizing board. We adapted the algorithms of Lin and Harbaugh (1984) to perform a pattern analysis of each image.

Second, we employed an image analyzing computer manufactured by LeMont Scientific, Co. This computer is capable of digitizing an image into 4096 x 4096 pixels, but there is no way to edit the image manually. Algorithms used in measuring objects in the image are proprietary, but indexes of area, perimeter, length, width, and orientation for each object in the image are obtained. The accuracy of area and perimeter measurements depends upon the intensity of digitization chosen by the operator, the available computer memory, and the algorithm itself. "Length" and "width" measurements are actually projections of an object onto certain coordinate axes specified by the algorithm. Orientation is the angle made by the long axis of an object with respect to the horizontal.

Third, we employed a Leitz Texture Analysis System (TAS). This computer is capable of digitizing an image into 512 x 480 pixels. In addition, each image can be edited manually and complex operations such as erosion and dilation can be performed on the image by computer programs. Individual objects in the image are measured with respect to equivalent spherical diameter (derived from an area measurement), perimeter, orientation, Feret minimum, and Feret maximum. Feret dimensions are measurements made by projecting each object onto chosen coordinate axes. Although the specific algorithms used by the Leitz TAS are proprietary, their principles have been discussed by Serra (1980).

All three of our approaches to automatic image analysis had the same basic mechanics: a video camera, a digitizing board, and a computer. There were differences in the amount of computer memory available to store and manipulate each image and in the sophistication of the programs used in analysis. The LeMont image analyzer was also set up to obtain a digital image directly from a scanning electron microscope without the intermediary of a video camera.
SECTION 5
THEORETICAL

This section contains description of the theory developed to estimate the effective porosity of compacted soil materials. Also in this section is a description of the Markov analysis techniques used in this study.

In general, there are two ways to investigate soil porosity: behavioral measurements and morphological measurements. The behavior of fluids (both liquids and gases) in soils is largely determined by the morphology of the pore space. To make predictions about fluid behavior in soils under field conditions, one normally measures fluid behavior under laboratory conditions and then makes extrapolations. Alternatively, it is possible to make morphological measurements of the pores in a soil (e.g., size, shape, and continuity), and then make behavioral predictions based on fundamental theory of fluid behavior. Morphological measurements are also used to distinguish one soil from another or to identify the effects of soil treatments (e.g., compaction or tillage).

The simplest method of morphological measurements relies on the principle that the pressure (or tension) at which a fluid is held in a capillary tube depends on the radius of the tube and on the surface tension and wetting angle of the liquid. This principle is formalized in the capillary rise equation:
\[ r = -\left(\frac{1}{P}\right)(2\gamma \cos \phi) \]  [5]
where \( r \) = equivalent pore radius (m), \( P \) = pressure (Pa), \( \gamma \) = surface tension of the liquid (N/m), and \( \phi \) = wetting angle of the liquid.

Two common techniques are based upon this equation: water retention measurements and mercury intrusion porosimetry. In water desorption
measurements, water is gradually pulled out of a saturated soil sample stepwise by equilibration at different pressures. In mercury intrusion porosimetry, mercury is gradually pushed into the soil sample with increasing pressure. In both instances, the volume of fluid pulled out or pushed in at known pressures or tensions gives information about pore sizes in the sample. Typically, a continuous distribution of pore radii is obtained. Such a distribution assumes that all pores in the sample act like continuous, discrete capillary tubes. Even though these assumptions are an inaccurate description of soil porosity, the information gained from capillary-rise measurements can be used as a starting point for modeling soil porosity and for making predictions of fluid behavior based on porosity. In this regard, we have employed the approaches of Van Genuchten (1980) and Mualem (1976).

Another general approach to morphological measurements of soil porosity uses visual images of the pores in a soil sample. Of course, the image itself is a simplified model of the porosity, especially because the image is normally two-dimensional. But information different from that of capillary-rise techniques can be obtained from images. Besides pore size, characteristics such as pore shape and pore patterns can be studied.

There are three different conceptual approaches to image analysis of soil pores: measurement of individual pores, stochastic modeling, and stereology. In our work, we have employed the first two methods. The third method, which has its roots in analysis of biological micrographs (Weibel, 1979) has only recently been directed to soil porosity images (Ringrose-Voase and Nortcliff, 1987).

Measurements of individual pores on photographs can be made manually, but it is the image analyzing computer which has made this approach practical. For each pore on an image, indexes of size and shape are obtained by counting pixels (picture points) covered by the pore. Statistics for individual pores are summed and distributions of pore size and shape on the image are calculated. These techniques have been discussed by Bullock and Murphy (1980), among others.

Instead of measuring individual pores on an image, it is also possible to analyze the pattern of pores on an image, if one assumes
that the pores are randomly distributed in the two-dimensional space of the image. Dexter (1976) first developed this approach for soil porosity images, and a similar approach has been used by Lin and Harbaugh (1984) to model porosity in rocks. Briefly, a two-dimensional image is again digitized into pixels. The probabilities of transition from pore to pore or from pore to non-pore (i.e., soil matrix) are compiled. Probabilities so measured may be used to compare one image to another or to generate another, statistically identical, image. Although Lin and Harbaugh (1984) have extended the theoretical approach from two to three dimensions, serial-section images are required, and their preparation is very time-consuming.

Although considerable effort has been made to develop image analysis techniques for soil porosity studies, there have been relatively few predictions of soil behavior based on image analysis measurements. Bouma et al. (1979), for example, needed a matching factor to compare predicted saturated hydraulic conductivity with hydraulic conductivity measured in the laboratory. Walker and Trudgill (1983) attempted to correlate image analysis measurements with tracer breakthrough measurements in laboratory columns. Dexter (1978) found image analysis to be most useful as part of a computer simulation model of root movements in tilled soil. Image analysis has also been used to identify the effects of soil compaction on pore shape and orientation (Murphy et al., 1976).

**EFFECTIVE POROSITY**

To estimate the travel time of noninteracting pollutants through a liner by Eq. (3), the liner's effective porosity must be determined. Earlier, effective porosity was described as that portion of total porosity that contributes significantly to fluid flow. We now define "significantly" by defining effective porosity as that portion of total porosity that conducts fluid faster than the average pore-water velocity. Large continuous pores conduct fluid faster than do small continuous pores. Therefore, in a porous medium that exhibits a range of pore sizes, a range of pore-water velocities will exist as a fluid passes through. Higher-than-average pore-water velocities will be responsible for the first appearance of pollutants, whereas lower-than-
average pore-water velocities will not. Thus, the determination of the soil porosity versus soil pore-water velocity distribution is crucial to the evaluation of the soil's effective porosity.

The pore-water velocity distribution can be calculated from the unsaturated permeability, $K(\Theta)$, as a function of fluid-filled porosity, $\Theta$. Based upon theory developed by Mualem (1976), Van Genuchten (1980) showed that the unsaturated permeability could be predicted from the total porosity, the residual porosity (i.e., the volumetric water content when water films lose effective continuity), and a measure of the uniformity of pore sizes by the equation:

$$K(\Theta) = K \left[ \frac{\Theta - \Theta_r}{\Theta_s - \Theta_r} \right]^{m} \left\{ 1 - \left[ 1 - \frac{\Theta - \Theta_r}{\Theta_s - \Theta_r} \right]^{1/m} \right\}^{-m/2} \tag{6}$$

where $\Theta_s$ is the total porosity, $\Theta_r$ is the residual porosity, and $m = [1 - (1/n)]$ for $0 < m < 1$, where $n$ is an empirical parameter that describes the uniformity of pore size. $\Theta_s$, $\Theta_r$, and $n$ can be determined from a cumulative porosity curve.

Cumulative porosity curves may be obtained by either soil-water desorption techniques, image analysis, or mercury-intrusion porosimetry. For recompacted material, the latter method is generally preferred because of its speed and range. Both approaches are based on the capillary-rise equation, which for mercury porosimetry may be formulated:

$$r = -\frac{1}{P} \left( \frac{2 \gamma \cos \phi}{r} \right) \tag{7}$$

where $r$ = pore radius (m), $P$ = pressure (Pa), $\gamma$ = surface tension of mercury (N/m), and $\phi$ = wetting angle of mercury (140°). To obtain $\Theta_s$, $\Theta_r$, and $n$ from mercury-intrusion curves, we adapted an equation originally developed by Van Genuchten (1980) for soil-water desorption curves:

$$\Theta = \Theta_r + (\Theta_s - \Theta_r) \left[ 1 + \left( \frac{\Theta - \Theta_r}{\Theta_s - \Theta_r} \right)^n \right]^{(1/n) - 1} \tag{8}$$

where $r$ is the pore radius from Eq. (7) and $\alpha$ is an empirical parameter. To use Eq. (8) to model mercury-intrusion curves, we define $\Theta_s$ as equal to the total porosity of the freeze-dried samples, and we transform the mercury-intrusion cumulative porosity curve to a form similar to the soil-water characteristic curve as follows: $\Theta = \Theta_s - \Theta_{Hg}$.
where $\Theta_{\text{Hg}}$ = mercury-filled porosity. Once the cumulative porosity curves are transformed, the $\Theta_r$, $n$ and $\alpha$ parameters (Eq. 8) can be determined by using nonlinear regression techniques (Van Genuchten, 1978).

The $\Theta_r$ and $n$ parameters can also be estimated from a graphical display of the transformed data. The residual porosity, $\Theta_r$, is approximated as the value of $\Theta$ that the cumulative porosity curve asymptotically approaches as limiting pore size decreases. Mercury-intrusion cumulative porosity curves for compacted soil samples generally have $\Theta_r$-values approximately equal to their non-mercury-filled porosities after intrusion to 414 MPa (60,000 psi). For soil materials considered for hazardous waste disposal liners, $\Theta_r$ is generally less than 0.1. The $n$-parameter is estimated as follows: the cumulative porosity data is plotted as $\Theta$ vs log $r$; next, the absolute value of the slope, $s$, of the cumulative porosity data is calculated at a value of $\Theta$ approximately halfway between $\Theta_r$ and $\Theta_s$. The $n$-parameter is then determined by using the equation (following Van Genuchten, 1980):

$$n = \begin{cases} \exp (0.8 \text{Sp}) & (0<\text{Sp}<1) \\ 1/ \left( \frac{0.5755}{\text{Sp}} - \frac{0.1}{\text{Sp}^2} - \frac{0.025}{\text{Sp}^3} \right) & (\text{Sp}>1) \end{cases}$$

where $\text{Sp}$ is equal to $s/(\Theta_s - \Theta_r)$.

The pore fluid velocity distribution can be described by differentiating Eq. (6) and multiplying by the hydraulic gradient.

$$I(dK(\Theta)/d\Theta) = \{KI/[2S^{1/2}(\Theta_s - \Theta_r)]\} [1-(1-S^{1/m})^2 + 2S^{(1/m-1/2)}$$

$$[(1-S^{1/m})^{m-1} - (1-S^{1/m})^{2m-1}]$$

where $S = (\Theta - \Theta_r)/(\Theta_s - \Theta_r)$. Equation (10) is monotonic in that $(dK(\Theta)/d\Theta)$ continuously decreases as $\Theta$ decreases.

A design engineer determines soil permeability by measurement and controls the hydraulic gradient and total porosity, $\Theta_s$, by construction. This information allows the calculation of an average fluid velocity, $\bar{V}$, for a liner ($\bar{V} = KI/\Theta_s$). But some portion of the total porosity conducts fluid at a velocity greater than the average
velocity, and some portion conducts fluid at a velocity less than the average velocity. We defined effective porosity, $E$, as that portion of the total porosity that conducts fluid faster than the average fluid velocity. Therefore a minimum cutoff pore-fluid velocity, $V_{\text{min}}$, is set equal to $\bar{V}$. Once $V_{\text{min}}$ is selected, $I(dK(\theta)/d\theta)$ is set equal to $V_{\text{min}}$ and Eq. (10) is solved for $S$. The determined value of $S$ represents a porosity cutoff point, $S_c$, i.e., all values of $S$ larger than $S_c$ have an associated pore-fluid velocity greater than $V_{\text{min}}$, and all values of $S$ smaller than $S_c$ have an associated pore-fluid velocity less than $V_{\text{min}}$. Thus, the effective porosity equals that proportion of the pore space greater than or equal to $S_c$. Effective porosity, $E$, can be determined from the equation:

$$E = (1 - S_c) (\theta_s - \theta_r)$$

Once the effective porosity is estimated, the time required for noninteracting pollutants to first appear at the bottom of a clay liner can be predicted by Eq. (3).

Our method to estimate the effective porosity determines the portion of total porosity that conducts fluid faster than $V_{\text{min}}$. This portion is assumed responsible for the total Darcian fluid flux density. In reality, this assumption is not entirely correct because the portion of porosity that conducts at less than $V_{\text{min}}$ is responsible for a small part of the Darcian fluid flux density. The mean velocity of pore water in the range of pore space that we refer to as effective porosity is actually:

$$\bar{V}_E = \frac{[K - K(\theta_c)] I}{(\theta_s - \theta_c)}$$

where $\theta_c$ represents the cutoff value of water filled pore space.

Equation (2) provides a slightly elevated value of $\bar{V}_E$ because in Eq. (2) the $K(\theta_c)$ term is assumed to equal zero. In reality, $K(\theta_c) \neq 0$, but $K(\theta_c)$ is much less than $K$ so that it can be conveniently ignored.

Equation (11) shows that effective porosity is a function of $\theta_s$, $\theta_r$, and $S_c$. $S_c$ can be determined as the root of Eq. (10) once $V_{\text{min}}$, $K$, $I$, and $n$ are specified. We have proposed selecting $V_{\text{min}}$ by setting it equal to $\bar{V}$. Because $\bar{V}$ is directly proportional to $K$ and $I$, $V_{\text{min}}$ must also be directly proportional to $K$ and $I$. A result of this specified equality, $V_{\text{min}} = \bar{V}$, is that $S_c$ becomes independent of $K$ and $I$; i.e., the
Figure 2. Influence of $n$ on effective porosity ($E$) at various values of $\theta_s$ ($\Theta = 0.05$).
same value of $S_C$ will be calculated as the root of Eq. (10) for all values of $K$ and $I$ as long as the product of $K$ and $I$ is greater than zero. Therefore, the effective porosity, $E$, as defined in this paper, is a function only of $\Theta_s$, $\Theta_r$, and $n$. The relationship between $E$ and $n$ for a range of $\Theta_s$ with $\Theta_r = 0.05$ is shown in Fig. 2.

Once $\Theta_s$ and $n$ are estimated from the measured cumulative porosity data, the effective porosity can be estimated from Fig. 2. With an estimate of the effective porosity, the time required for the first appearance of noninteracting pollutants at the bottom of a clay liner can be predicted by using Eq. (3).

Markov Analysis

Much of the theory and terminology used in our Markov analysis comes from Markov statistics. Markov statistics are useful for analyzing systems that have two somewhat contradictory properties (1) randomness in time or space, and (2) relatedness in time or space. Relatedness is said to exist if all events are influenced in some way by preceding events. Any system that displays these properties can be called Markovian.

For example, the weather system is strongly Markovian in that weather conditions on one day influence conditions on the following day. Markov statistics allow us to analyze this kind of dependence and use it to model a system more accurately than if we had assumed the system to be totally random.

An ordered sequence of events is spoken of as a Markov chain. In our analysis we are interested in first-order, discrete Markov chains. First-order indicates that the influence of only the immediately preceding event will be analyzed, although a given event may be influenced by several preceding events. The word discrete indicates that each event can have only a finite number of states. The change of state value from one event to the next is called a transition (this is true even if the state value remains the same between events).

For example, consider a system in which I ask my friend to have dinner with me. The events correspond to replies. Each event has one of two states: 'yes' or 'no.' We assume that a sequence of responses are: no, no, no, yes, yes, yes, yes, yes, no, no, no, yes, yes.
The marginal probability of a state is defined as the ratio of the frequency of the state to the total number of events in the chain. In our example, the marginal probability of a 'yes' is 7/13.

Using this same example, we can see how the influence of events on each other can be determined. In the above chain there are four no->no transitions; five yes->yes transitions; two no->yes transitions; and one yes->no transition. Altogether there are 12 transitions. The probability of a given type of transition can be found by dividing its frequency by the total number of transitions.

This information can be arranged as follows:

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Final State = Yes</th>
<th>Final State = No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>5/12</td>
<td>1/12</td>
</tr>
<tr>
<td>No</td>
<td>2/12</td>
<td>4/12</td>
</tr>
</tbody>
</table>

At this point it is convenient to introduce the notation \( P(A|A) \), which we will interpret to mean the probability of an event with state A immediately following an event with state A in a Markov chain. By using only marginal probabilities to estimate \( P(\text{yes} | \text{no}) \), we arrive at \( P(\text{yes} | \text{no}) = 7/13 \). An estimate of \( P(\text{yes} | \text{yes}) \) is also 7/13 because marginal probabilities give us no information about how the preceding state affects the following state. However, using transitional probabilities we arrive at \( P(\text{yes} | \text{no}) = 2/12 \) and \( P(\text{yes} | \text{yes}) = 5/12 \). The motivation of computing transitional probabilities is that they allow us to more accurately predict the state of events following a given event and are better descriptors of the existing data than the marginal probabilities alone.

Many properties of soil, such as thermal diffusivity, water retention, compactability, hydraulic conductivity, and miscible displacement, depend largely on the spatial arrangement of pores in the soil matrix. Often the matrix-pore arrangement is modeled assuming that the matrix elements or the pores have a simple uniform geometry. For example, the capillary tube models and the packed sphere models are often assumed. Naturally these simplified geometrics do not reflect the range of complexity and irregularity of soil structure. One method of describing soil structure that puts fewer restrictions on the sizes and
shapes of the pore and matrix elements involves Markov statistics.

Markov statistics have been used in the consideration of transitions between pores and matrix elements along a straight line of a soil surface. A one-dimensional Markov analysis has been used to compare tillage effects on soil properties and to predict root growth (Dexter, 1976, 1978). Multidimensional Markov analyses may prove to be even more useful.

Multidimensional applications of Markov statistics in the literature are few. Pielou (1964) treated vegetation patterns in the plane, and Kretz (1969) produced a statistical study of mineral grains also in the plane. Lin and Harbaugh (1984) proposed an algorithm, intended for computer image analysis, to compute Markov statistics from two- and three-dimensional geological data. These statistics were used in combination with a random number generator to produce patterns with Markov statistics similar to the original data. Lin and Harbaugh empirically demonstrated the consistency between the statistics of the original patterns and of the patterns produced with the random number generator. Lin and Harbaugh did not establish whether the generated patterns shared properties beyond similar Markov statistics with the original patterns, or whether the algorithm was applicable for any given original pattern. We have attempted to determine whether the algorithm of Lin and Harbaugh is appropriate for use in soil porosity studies and to suggest the circumstances under which the algorithm used by Lin and Harbaugh does not apply.

Presented in this section is the theory behind the algorithm of Lin and Harbaugh (1984). A restricted case of this algorithm is considered in which planar patterns consist of only two states (pore and matrix).

By using this algorithm, pore-matrix patterns are obtained by assigning independently and randomly the states "pore" or "matrix" to cells that subdivide a circular region. The geometry of these cells is governed by three parameters: θ₁, θ₂, and λ. The parameters θ₁ and θ₂ limit the range of elongation directions for individual cells. The parameter λ affects both the total number and the sizes of the cells.

To understand how cells are determined, consider a circle of radius R whose center is the pole of a set of polar axes. The cells that
subdivide this circle are formed by the intersections of secants to the circle. Each secant corresponds to an ordered pair \((r, \theta)\) where \(r\) and \(\theta\) are restricted as follows:

\[
-\frac{\pi}{2} \leq \theta_1 < \theta < \theta_2 \leq \frac{\pi}{2}
\]

For an ordered pair \((r, \theta)\) the corresponding secant is the unique secant at distance \(r\) from the pole whose normal through the pole makes the angle \(\theta\) with respect to the angular origin (see Figure 3).

The pairs \((r, \theta)\) are chosen from a rectangular region in the Cartesian \(r-\theta\) plane. Letting \(R^*\) represent this region, then

\[
R^* = \{(r, \theta): -R < r < R, \theta_1 < \theta < \theta_2\}
\]

Each pair is chosen from \(R^*\) by using a planar Poisson point process with average density \(\lambda\) per unit area. (One can generate \((r, \theta)\) pairs corresponding to a planar Poisson process by choosing randomly and independently \(r\) and \(\theta\) and then pairing them, keeping in mind the restrictions of Equation 12.)

The number of secants comprising a pattern is assumed to be determined by a random variable with a Poisson distribution and mean \(N\) where:

\[
N = \lambda \iint_{R^*} r \, dr \, d\theta
\]

A complete set of parameters for generating a pattern consists of \(\theta_1, \theta_2, \lambda\) and \(P(\text{pore})\), i.e., probability that any given cell will be assigned to the pore state. When generating a pattern from a set of model parameters, the number of secants is best chosen to be \(N\) (rounded to an integer). Figure 4 shows the stepwise graphical construction of a pore-matrix pattern.

The parameters \(\theta_1\) and \(\theta_2\) can be estimated by simply noting the range of elongation directions of the pores in a given pattern. The smaller limit of elongation direction corresponds to \(\theta_1\) and the larger to \(\theta_2\). If there is no particular orientation to the pores then \(\theta_1 = -\frac{\pi}{2}\) and \(\theta_2 = \frac{\pi}{2}\).

Naturally, the pore-matrix patterns encountered in practice will not be composed of cells generated by random secants. However, to estimate \(\lambda\) it is assumed that the patterns of interest are generated by the procedure shown in Figure 4. (The validity of this assumption has
Figure 3. Secant $\overline{CD}$ correspond to the pair $(r, \theta)$. $\overline{CD}$ is a distance $r$ from the center of the circle, $O$. The normal to $\overline{CD}$ through $O$ makes the angle $\theta$ with respect to the angular origin $OK$. 
Figure 4. Construction of a pattern inside a circle of radius 1 with: $\theta = \pi/2$, $\theta_2 = \pi/2$, $\lambda = 5/\pi$, $P(\text{pore}) = 0.3$. Draw a circle with radius $r = 1$ and choose an angular origin (1). Equation (13) shows that $N = 5$ for this case. Choose five $(r, \theta)$ pairs using the method described in the text and draw the corresponding secants (2). Assign each region formed to the states "P" and "M" (pore and matrix) so that each region has a probability of 0.3 of being assigned to the pore state (3). Color the pore cells black and erase the lines dividing the matrix cells (4).
not yet been analyzed in the literature). To arrive at an estimate of $\lambda$, estimates of the expected number of secants intersecting a segment lying within a pore-matrix pattern are made using two different methods.

The value of $N(L, \beta)$, the expected number of secants intersecting a segment of length $L$ that makes an angle $\beta$ relative to the angular origin, does not depend upon the position of the segment within the pattern. Thus the general segment with length $L$ and orientation $\beta$ will be referred to as segment $AB$.

Let $S_i$ represent the state of the point with label $i$. For two points (A and B) in different cells, $P(S_A | S_B)$ represents the conditional probability that point A is in state $S_A$ given that point B is in the same state. $P(S_A)$ is the overall or marginal probability of state $S_A$ within the pattern. In an extension of a theorem by Switzer (1965), Lin and Harbaugh (1984) show:

$$P(S_A | S_B) = P(S_A) [1 - \exp(-N(L, \beta))] + \exp(-N(L, \beta))$$

where $N(L, \beta)$ is the expected number of secants intersecting AB.

Solving for $N(L, \beta)$ yields:

$$N(L, \beta) = \ln \left[ \frac{P(S_A | S_B) - P(S_A)}{1 - P(S_A)} \right]$$

$N(L, \beta)$ can also be written in terms of the sampling density, $\lambda$. This is done by determining which $(r, \theta)$ pairs correspond to secants intersecting AB. By letting $R^r$ symbolize the region of all $(r, \theta)$ that correspond to secants intersecting AB in a Cartesian $r, \theta$-plane, and from the definition of $\lambda$:

$$N(L, \beta) = \lambda \int_{R^r} dr \theta$$

($R^r$ depends on the position of AB, but the value of the integral does not.)

As an example, $N(L, \beta)$ is computed here for the case: $|\beta - \theta_1| < \pi/2$ and $|\beta - \theta_2| < \pi/2$.

For this case consider any fixed $\theta$ strictly between $\theta_1$ and $\theta_2$. There is a set of points, GH, along the line $(r, \theta)$ whose normals intersect AB. The length of GH can be found by drawing an extension line through point A parallel to the line GH. See Figure 5. Next the secant CD normal to the line GH and passing through point B is drawn. If the intersection of CD and the extension line is labeled point E, then AE has the same length as GH. Trigonometry shows the length of AE
Figure 5. Segment \( \overline{GH} \) represents all points along line \( OH \) which have secants intersecting the line segment \( \overline{AB} \). Note the \( \overline{AB} \) makes the angle \( \beta \) with respect to the angular origin.
is $L^\ast \cos (\theta - \Theta)$. By using this result, one can see that:

$$\int_{\theta_2}^{\theta_1} L^\ast \cos (\theta - \Theta) \, d\theta$$

By using equation 16 and integrating,

$$N(L, \Theta) = 2\lambda L \sin \left[ (\Theta_2 - \Theta_1)/2 \right] \cos \left[ \Theta - (\Theta_2 + \Theta_1)/2 \right]$$

The value of $N(L, \Theta)$ for the other three possible cases is computed by using an analogous technique. Table 1 shows a listing of all four cases and the corresponding values for the area of $R'$. Equating the values of $N(L, \Theta)$ in equations 15 and 16 and rearranging terms yields:

$$\lambda_b = \ln \left\{ \left[ P(S_A | S_B) - P(S_A) \right] / \left[ 1 - P(S_A) \right] \right\} \int_{R'} d\theta$$

The subscript on $\lambda$ indicates that the estimates of this parameter depend on the value of $\Theta$. This dependence on $\Theta$ comes from the fact that both $P(S_A | S_B)$ and the integral in equation 19 depend on $\Theta$.

To arrive at estimates for $P(S_A)$ and $P(S_A | S_B)$, the pattern can be sampled at equally spaced rectilinear grid points corresponding to a standard set of $x$- and $y$-axes. $P(S_A)$ is independent of direction. There will be two estimates for $P(S_A)$: one for each possible value of $S_A$ (pore and matrix). $P(S_A)$ can be found by dividing the number of sample points in state $S_A$ by the total number of points in the sample. $P(S_A | S_B)$ is estimated by assuming that all points in the sample are from different cells. The value for $P(S_A | S_B)$ is computed for two directions (the $x$ and $y$ directions) by Lin and Harbaugh (1984). Let $S(x,y)$ be the state of the sampling point with coordinates $(x,y)$. Let the function $F$ be defined for points $(x,y)$ within the pattern:

$$F(x) = \begin{cases} 0 & \text{if } S(x+1,y) \neq S(x,y), \text{ or if } (x+1,y) \text{ lies outside the pattern}. \\ 1 & \text{if } S(x+1,y) = S(x,y) \end{cases}$$

Then for the $x$ direction:

$$P(S_A | S_B) = \frac{\Sigma_x \Sigma_y F(x,y)}{\Sigma_x \Sigma_y 1}$$

$P(S_A | S_B)$ can be computed in a similar fashion for the $y$-direction. Notice that for each direction there are two values of $P(S_A | S_B)$; one for
each possible value of \( S \). Appropriate values of the terms of equation 19 for the two different states and two directions can be used to obtain four estimates of \( \lambda \). It is not known how to best combine these four estimates of \( \lambda \), therefore both arithmetic and geometric means will be shown for each pattern.

Table 1. Listing of the Values of \( N(L, \beta) \) for the Four Possible Cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \int f_R^1 , d\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\beta - \theta_1</td>
</tr>
<tr>
<td>(</td>
<td>\beta - \theta_1</td>
</tr>
<tr>
<td>(</td>
<td>\beta - \theta_1</td>
</tr>
<tr>
<td>(</td>
<td>\beta - \theta_1</td>
</tr>
</tbody>
</table>
SECTION 6
RESULTS AND DISCUSSION

GENERAL PHYSICAL PROPERTIES

Table 2 presents the particle size distribution, liquid limit, plastic limit, and plasticity index determined for subsoil materials of Nicollet, Fayette, and Clarinda. Nicollet material had the least clay and the most sand, whereas Clarinda material had the most clay and an intermediate sand content. Fayette material, derived from loess, was highest in silt. The percentage clay in each subsoil material is reflected by the values of the Atterberg limits. As percentage clay increases, the Atterberg limits increase.

Table 2. Particle size distribution, liquid limit, plastic limit, and plasticity index of subsoil materials.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Particle size distribution</th>
<th>Liquid limit</th>
<th>Plastic limit</th>
<th>Plasticity index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clay (2) Silt (2-50) Sand (50-2000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nicollet</td>
<td>21 25 54</td>
<td>28</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>Fayette</td>
<td>35 57 8</td>
<td>35</td>
<td>21</td>
<td>14</td>
</tr>
<tr>
<td>Clarinda</td>
<td>44 35 21</td>
<td>56</td>
<td>28</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 3 presents the classification of each subsoil material according to the USDA, AASHTO, and Unified classifications. According to the AASHTO classification, which classifies soil material according to its suitability for engineering uses (A-1 is the highest suitability class), the Clarinda subsoil material is the least suited for
engineering uses (A-7 is the next to lowest suitability class). In addition, according to the AASHTO and Unified classifications, Clarinda subsoil materials are subject to extremely high volume change.

Table 3. Classification of subsoil materials according to USDA, AASHTO, and Unified classifications.

<table>
<thead>
<tr>
<th>Soil</th>
<th>USDA</th>
<th>AASHTO</th>
<th>Unified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>sandy clay loam</td>
<td>A-6 (3)</td>
<td>CL</td>
</tr>
<tr>
<td>Fayette</td>
<td>silty clay loam</td>
<td>A-6 (10)</td>
<td>CL</td>
</tr>
<tr>
<td>Clarinda</td>
<td>clay</td>
<td>A-7-6 (18)</td>
<td>CH</td>
</tr>
</tbody>
</table>

Moisture-Density Relationships

Table 4 presents the particle density, undisturbed bulk density, and the optimum bulk density and moisture content from the moisture-density relations determination. Nicollet subsoil material compacts to the highest bulk density. Compacted Clarinda subsoil material has an optimum bulk density lower than undisturbed Clarinda subsoil material. Thus, the compactive force experienced in the past by the undisturbed Clarinda subsoil material was greater than the compactive force during the moisture-density relations test.

Table 4. Particle density, undisturbed bulk density, optimum bulk density and moisture content of subsoil materials.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Particle density (Mg/m³)</th>
<th>Undisturbed bulk density (Mg/m³)</th>
<th>Optimum bulk density (Mg/m³)</th>
<th>Optimum moisture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>2.59</td>
<td>1.66</td>
<td>1.84</td>
<td>14</td>
</tr>
<tr>
<td>Fayette</td>
<td>2.71</td>
<td>1.41</td>
<td>1.67</td>
<td>18</td>
</tr>
<tr>
<td>Clarinda</td>
<td>2.69</td>
<td>1.59</td>
<td>1.45</td>
<td>27</td>
</tr>
</tbody>
</table>

37
Figure 6 presents the moisture-density relations for Nicollet, Fayette, and Clarinda subsoil materials. From the figure, one can see that the compaction of the subsoil materials at higher-than-optimum moisture content provides bulk densities slightly lower than the maximum values. The subsoil materials were compacted at higher-than-optimum moisture content to obtain higher percentage water saturation.

**Mercury Intrusion Porosimetry**

The calculated total porosity of the samples after drying was always lower than the calculated initial total porosity (Table 5). For all compacted samples, oven drying resulted in the greatest absolute loss of porosity. For example, in Clarinda materials, total porosity dropped from 0.46 to 0.25 cm$^3$/cm$^3$, i.e., about 46% relative change. On the other hand, freeze-dried samples shrank the least; for example, total porosity of Fayette freeze-dried samples changed very little. For compacted samples, relative shrinkage was consistently greatest for Clarinda materials and least for Fayette materials. There was no discernible pattern related to particle size distribution, although the greatest shrinkage occurred in materials with the highest clay content.
Figure 6. Moisture density relations for (a) the till derived soil.
Figure 6. Moisture density relations for (b) the loess derived soil.

Gravimetric Moisture (%) vs. Unit Weight (KN/m^3) for FAYETTE.
Figure 6. Moisture density relations for (c) the paleosol soil material.
Table 5. Total porosity of soil materials before and after drying.

<table>
<thead>
<tr>
<th></th>
<th>Compacted Relative Porosity change from before drying</th>
<th>Undisturbed Relative Porosity change from before drying</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Porosity after drying</td>
<td>Porosity change from before drying</td>
</tr>
<tr>
<td>Nicollet</td>
<td>.29 cm³/cm³</td>
<td>.36 cm³/cm³</td>
</tr>
<tr>
<td>freeze dried</td>
<td>.25</td>
<td>14</td>
</tr>
<tr>
<td>acetone dried</td>
<td>.23</td>
<td>21</td>
</tr>
<tr>
<td>oven dried</td>
<td>.19</td>
<td>35</td>
</tr>
<tr>
<td>Fayette</td>
<td>.40 cm³/cm³</td>
<td>.48 cm³/cm³</td>
</tr>
<tr>
<td>freeze dried</td>
<td>.39</td>
<td>3</td>
</tr>
<tr>
<td>acetone dried</td>
<td>.37</td>
<td>8</td>
</tr>
<tr>
<td>oven dried</td>
<td>.30</td>
<td>20</td>
</tr>
<tr>
<td>Clarinda</td>
<td>.46 cm³/cm³</td>
<td>.41 cm³/cm³</td>
</tr>
<tr>
<td>freeze dried</td>
<td>.38</td>
<td>17</td>
</tr>
<tr>
<td>acetone dried</td>
<td>.31</td>
<td>30</td>
</tr>
<tr>
<td>oven dried</td>
<td>.25</td>
<td>46</td>
</tr>
</tbody>
</table>

Undisturbed samples also had less total porosity after drying than before (Table 5). Oven-dried samples of all three soil materials showed the greatest absolute and relative losses of total porosity compared with the initial total porosity. The least losses of total porosity occurred in the freeze-dried samples of Fayette and Clarinda soil materials and in the acetone-dried samples of Nicollet soil materials. It is unlikely that the total porosities of undisturbed Nicollet samples after freeze drying and acetone drying are significantly different. As
in the compacted samples, there appeared to be no clear relationship between particle size distribution and shrinkage.

Each of the drying techniques caused some change in total porosity. Oven drying caused the greatest loss of total porosity, and acetone drying generally caused an intermediate loss. Freeze drying caused minimal loss of total porosity, confirming what other workers have reported (e.g., Zimmie and Almaleh, 1976). Although the lower changes in total porosity by freeze drying may suggest that this technique alters soil pores the least, pore size distribution analysis provides additional insight concerning the three approaches to drying.

Figures 7-12 display size distributions for pores with equivalent pore radii (epr) greater than 0.002 μm in the soil materials investigated. The data are given as absolute porosities attributed to pores in a logarithmic epr range. Each number in the bar graph represents the mean of the replicates determined. Cumulative porosity curves are shown in Figs. 13-18.

In the compacted soil samples, there were marked differences in pore size distributions among the drying treatments. For example, most of the porosity in freeze-dried, compacted Nicollet soil materials was derived from pores in the 0.2-2 μm epr range (Fig. 7). In acetone-dried samples, however, porosity was more evenly distributed in the 0.02-0.2 μm and 0.2-2 μm epr ranges. In oven-dried samples, pores with epr between 2 and 20 μm contributed most to porosity.

Freeze drying of compacted Fayette soil materials resulted in porosity of nearly 0.27 cm³/cm³ contributed by pores in the 0.2-2 μm epr range (Fig. 8). Acetone-dried and oven-dried Fayette samples had essentially identical pore size distributions, except in the 0.02-2 μm epr range. In that range, acetone-dried samples had about 0.11 cm³/cm³, whereas oven-dried samples had about 0.05 cm³/cm³. The difference of 0.06 cm³/cm³ is comparable to the 0.07 cm³/cm³ difference in total porosity between the drying treatments (Table 5).

In compacted Clarinda materials, freeze drying also resulted in a significant volume of pores in the 0.2-2 μm epr range (Fig. 9). In both acetone-dried and oven-dried samples, pores in the 0.002-0.02 μm epr range contributed most to porosity. Acetone-dried samples had more
Figure 7. Mercury intrusion porosimetry determined pore size distributions for till samples using three drying techniques.
Figure 8. Mercury intrusion porosimetry determined pore size distributions for compacted loess samples using three drying techniques.
Figure 9. Mercury intrusion porosimetry determined pore size distributions for compacted paleosol samples using three drying techniques.
Figure 10. Mercury intrusion porosimetry determined pore size distributions for undisturbed till samples using three drying techniques.
Figure 11. Mercury intrusion porosimetry determined pore size distributions for undisturbed loess samples using three drying techniques.
Figure 12. Mercury intrusion determined pore size distributions for undisturbed paleosol samples using three drying techniques.
Figure 13. Mercury intrusion porosimetry determined cumulative porosity curve for compacted till samples using three drying techniques.
Figure 14. Mercury intrusion porosimetry determined cumulative porosity curves for compacted loess samples using three drying techniques.
Figure 15. Mercury intrusion porosimetry determined cumulative porosity curves for compacted paleosol samples using three drying techniques.
Figure 16. Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed till samples using three drying techniques.
Figure 17. Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed loess samples using three drying techniques.
Figure 18. Mercury intrusion porosimetry determined cumulative porosity curves for undisturbed paleosol samples using three drying techniques.
porosity than oven-dried samples in each epr range except the 20-70 μm epr range.

Pore size distributions of undisturbed samples showed basically the same effects of drying treatments as did those of compacted samples (Figs. 10-12). Specifically, freeze drying consistently resulted in high pore volumes coming from pores in the 0.2-2 μm epr range. In contrast, pores in the 0.02-0.2 μm epr range contributed significantly to porosity in the acetone-dried, undisturbed samples. In oven-dried, undisturbed samples porosity was greatest in the 2-20 μm epr range for Nicollet, in the 0.2-2 μm epr range for Fayette, and in the 0.002-0.02 μm epr range for Clarinda. In this regard, oven-dried, undisturbed samples were directly comparable to oven-dried, compacted samples.

The consistent dominance of pores in the 0.2-2 μm epr range in freeze-dried samples of all soils, both disturbed and compacted, is striking. A similar peak in pore size distribution of freeze-dried soil materials has been reported by Lawrence et al. (1979), Murray and Quirk (1980), and Delage and Pellerin (1984) for samples initially frozen in liquid nitrogen, Freon, or 2-methyl butane. We hoped that significant changes in porosity by growth of ice crystals might be prevented by freezing samples so quickly that large ice crystals could not form. We used samples on the order of 1 cm in diameter, for example, to ensure that heat could be withdrawn rapidly from the sample interior. Although we froze the samples in liquid N2 at 77 K, we were only careful to keep the samples at temperatures less than 273 K before sublimation. During sublimation, temperature of the freeze dryer's condenser was ~248 K. Even if water in the samples had frozen quickly into small crystals upon initial submersion in liquid N2, there may have been recrystallization of ice at temperature greater than 77 K yet less than 273 K. Burton and Oliver (1935), for example, found that even at low temperature ice could recrystallize and suggested that, at atmospheric pressure, ice crystals are stable only at temperatures less than 163 K. Meryman (1957) observed recrystallization at 203 K; crystals of 1 μm diameter formed in only 30 s. In other words, under the conditions employed in our study, recrystallization of ice may have increased the volume of pores in the 0.2-2 μm epr range compared to the original porosity before drying.
The effects of acetone drying on pore size distribution are difficult to ascertain. The similarity of porosity in the 0.002-0.02 μm epr range for acetone-dried and oven-dried soil samples suggests that either the techniques do not alter very small pores much or that pores are altered similarly by both techniques. The data further suggest that, in acetone drying, pores with epr in the 0.02-0.2 μm range are favored—for either preservation or production—compared to the other drying techniques. Finally, porosity due to pores in the 0.2-2 μm epr range of acetone-dried samples was generally intermediate between the porosities of oven-dried and freeze-dried samples in that range. There was no discernible pattern for pores >2 μm epr.

We note that direct comparison of our results with those of acetone exchange before resin impregnation is not possible because in the latter instance acetone does not need to be completely removed from the samples (i.e., polyester resins are miscible with acetone). Before mercury intrusion, however, acetone in the samples had to be evaporated. Total porosity values (Table 5) indicate that some shrinkage did occur, and it may have happened at the evaporation step.

Water Desorption

Porosity determined by oven drying, freeze drying, and acetone exchange of compacted samples may be compared with porosity calculated from soil water contents of compacted samples equilibrated at several pressures (Table 6). Pores filled with water at 0.05 MPa actually constituted a volume greater than or equal to the initial porosity calculated for the compacted samples. This is because some swelling of the samples occurred at saturation. At the other end of the pressure range, porosity determined by water content was comparable to that obtained by oven drying and acetone exchange for two of the soil materials. For example, pressures of 2.0 MPa and 1.5 MPa correspond to equivalent pore radii of 0.1 and 0.75 μm, respectively. For Nicollet and Clarinda soil materials, pores of that size range contributed about 0.03 cm³/cm³. This may be compared with porosities of 0.02 to 0.07 cm³/cm³ in the 0.2-2 μm epr range of Nicollet and Clarinda, as determined by oven drying and acetone exchange (Fig. 7 and 9). Porosity determined by water content of Fayette compacted materials was about
0.06 cm$^3$/cm$^3$ in the 0.1-0.75 μm epr range. This is not very similar to porosity in the 0.2-2 μm epr range determined by the three methods of this study for compacted Fayette materials (Fig. 8). In all cases, freeze drying resulted in more porosity in the 0.2-2 μm epr range than did any other drying methods or the water content method.

Table 6. Volumetric soil water content at four tensions for compacted soil samples.

<table>
<thead>
<tr>
<th>Soil</th>
<th>0.05 MPa</th>
<th>0.1 MPa</th>
<th>2.0 MPa</th>
<th>1.5 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>.31</td>
<td>.28</td>
<td>.24</td>
<td>.21</td>
</tr>
<tr>
<td>Fayette</td>
<td>.40</td>
<td>.36</td>
<td>.35</td>
<td>.29</td>
</tr>
<tr>
<td>Clarinda</td>
<td>.48</td>
<td>.45</td>
<td>.42</td>
<td>.39</td>
</tr>
</tbody>
</table>

Water desorption curves for compacted and undisturbed samples are presented in Figs. 19-21.

Permeability

Table 7 presents the permeabilities of three replicates of each subsoil material compacted in Proctor molds. The mean permeability of each subsoil material was less than 10$^{-9}$ m/s, the value currently used by the USEPA as the criterion for a material's suitability as a liner material. Fayette materials had the highest permeability of the three subsoil materials, although Clarinda materials had more porosity. Fayette materials must have had larger openings or better connections in the pore space than did Clarinda materials.
Figure 19. Water desorption curves for compacted soil materials.
Figure 20. Water desorption curves for undisturbed till material.
Figure 21. Water desorption curves for undisturbed loess material.
Table 7. Permeability of three replicates of each Proctor-type compacted subsoil material.

<table>
<thead>
<tr>
<th>Soil</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>$5.47 \times 10^{-11}$</td>
<td>$6.58 \times 10^{-11}$</td>
<td>$1.22 \times 10^{-10}$</td>
<td>$8.08 \times 10^{-11}$</td>
</tr>
<tr>
<td>Fayette</td>
<td>$2.84 \times 10^{-10}$</td>
<td>$2.67 \times 10^{-10}$</td>
<td>$2.61 \times 10^{-10}$</td>
<td>$2.71 \times 10^{-10}$</td>
</tr>
<tr>
<td>Clarinda</td>
<td>$4.18 \times 10^{-11}$</td>
<td>$2.97 \times 10^{-11}$</td>
<td>$5.05 \times 10^{-11}$</td>
<td>$4.07 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

Soil permeability as a function of time is shown in Figs. 22-24. The permeability of all three soil materials held somewhat constant during the periods of measurement.

Table 8 presents permeabilities of the smaller compressed and extruded soil samples. Larger variation was found in measuring permeability of these samples. The average permeabilities of the compressed soil samples were greater than those for the larger, compacted samples.

Table 8. Permeabilities of compressed and extruded subsoil materials.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>$6.4 \times 10^{-9}$</td>
<td>$7.9 \times 10^{-9}$</td>
</tr>
<tr>
<td>Fayette</td>
<td>$2.8 \times 10^{-8}$</td>
<td>$8.9 \times 10^{-9}$</td>
</tr>
<tr>
<td>Clarinda</td>
<td>$7.9 \times 10^{-9}$</td>
<td>$1.3 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

Table 9 presents permeabilities of the undisturbed soil samples. Both average and variance for the undisturbed samples are greater than for disturbed samples.
Figure 22. Permeability as a function of time for compacted till material.
Figure 23. Permeability as a function of time for compacted loess material.
Figure 24. Permeability as a function of time for compacted paleosol material.
Table 9. Permeabilities of undisturbed soil samples.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Permeability (m/s)</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>2.9 x 10^{-6}</td>
<td>2.4 x 10^{-6}</td>
<td></td>
</tr>
<tr>
<td>Fayette</td>
<td>2.2 x 10^{-5}</td>
<td>3.0 x 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>Clarinda</td>
<td>9.8 x 10^{-7}</td>
<td>1.0 x 10^{-6}</td>
<td></td>
</tr>
</tbody>
</table>

Solute Breakthrough Curves

Figures 25-30 present the solute breakthrough curves from the three replicates of each compacted soil material. The plots are relative concentration versus relative pore volume. One pore volume is equal to the total porosity of the bulk soil volume. In all cases, chloride appeared in the effluents when about 0.2 - 0.25 of one pore volume of solution had leached through the sample. With the same hydraulic gradient, the first appearance of chloride under liners of equal thickness of the compacted subsoil materials would occur first with Fayette, then Nicollet, and lastly Clarinda materials. All curves are somewhat symmetrical, implying that no large pores (not a bimodal pore size distribution) were contributing significantly to flow. The solid curves shown in the figures represent "best fit" values obtained from a solution of the convective-dispersive equation (Van Genuchten and Wierenga, 1986).

Figures 31-33 present solute breakthrough curves for undisturbed soil samples. All of the figures display asymmetrical shapes. The asymmetrical shapes are characteristic of a bimodal pore size distribution. A few large cracks or channels must have been responsible for most of the flow in the undisturbed cores. Thus, early breakthrough occurred with tailing toward $C/C_0 = 1$.

The breakthrough curves for the disturbed and undisturbed samples are distinctly different. All three soil materials as they occur in nature must be mixed and compacted in order to remove the presence of
Figure 25. Cl⁻ breakthrough curves for compacted till material.
Figure 26. Cl\(^-\) breakthrough curves for compacted loess material.
Figure 27. Cl$^-$ breakthrough curves for compacted paleosol material.
Figure 28. Three replicates of measured and fitted (solution to convective-dispersive equation) Cl\(^{-}\) breakthrough data for compacted till materials.
Figure 29. Three replicates for measured and fitted (solution to convective-dispersive equation) Cl⁻ breakthrough data for compacted loess materials.
Figure 30. Three replicates of measured and fitted (solution to convective-dispersive equation) Cl\(^-\) breakthrough data for compacted paleosol materials.
Figure 31. Cl⁻ breakthrough curves for undisturbed till material.
Figure 32. Cl− breakthrough curves for undisturbed loess material.
Figure 33. Cl⁻ breakthrough curves for undisturbed paleosol material.
cracks before they would be useful as hazardous-waste-disposal liners.

**Effective Porosities**

Our method of determining effective porosity depends on both the cumulative pore size distribution and the total porosity. Yet, to determine pore size distribution by mercury porosimetry, all water must be removed from the sample. Some of the total porosity in the samples was lost when water was removed by freeze drying (Table 10). Some loss of porosity occurred in the Nicollet and Clarinda samples during freeze drying. The greatest decrease in porosity was with Clarinda soil materials, the materials highest in clay content. Total porosity in Fayette soil materials was not diminished by freeze drying.

**Table 10.** Total porosities of compacted soil materials before and after freeze drying.

<table>
<thead>
<tr>
<th>Soil material</th>
<th>Before drying</th>
<th>After drying</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>Fayette</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Clarinda</td>
<td>0.46</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The function used to describe the cumulative porosity curve (Eq. 8) has two empirical parameters, $a$ and $n$. Only $n$ is required to describe the unsaturated permeability. Figure 34 presents a family of curves over a range in $n$ from 1.1 to 5. The curves show that the spread on a log scale of pore sizes decreases as $n$ increases. Thus, the theory provided for calculating unsaturated permeability considers the spread in pore size distribution as the predominant factor.

Figures 35, 36, and 37 show the cumulative porosities for Nicollet, Fayette, and Clarinda soil samples, respectively. The discrete, plotted symbols represent values determined by mercury porosimetry. The solid curves represent the least-squares, nonlinear regression fit of Eq. (8) to the measured data. In all cases the model does a reasonable job of matching the measured data. The $a$, $n$, $\theta_r$, and $\theta_s$ parameters for each
Figure 34. Theoretical cumulative porosity curves as influenced by the n-parameter ($\alpha = 1.0, S = \theta/\theta_s$).
Figure 35. Measured and curvefitted cumulative porosity for compacted till material.
Figure 36. Measured and curvefitted cumulative porosity for compacted loess material.
Figure 37. Measured and curvefitted cumulative porosity for compacted paleosol material.
soil are presented in Table 11.

Table 11. Parameters used with equation (8) to describe the cumulative porosity of the soil materials.

<table>
<thead>
<tr>
<th>Soil Material</th>
<th>$a^1$</th>
<th>$n^1$</th>
<th>$\theta_r^1$</th>
<th>$\theta_s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td>4.08</td>
<td>1.52</td>
<td>0.00</td>
<td>0.28</td>
</tr>
<tr>
<td>Fayette</td>
<td>0.88</td>
<td>2.11</td>
<td>0.05</td>
<td>0.40</td>
</tr>
<tr>
<td>Clarinda</td>
<td>1.22</td>
<td>1.98</td>
<td>0.08</td>
<td>0.40</td>
</tr>
</tbody>
</table>

1/ $a$, $n$, and $\theta_r$ were determined by least squares nonlinear regression of mercury intrusion porosimetry data and Eq. (8).

2/ $\theta_s$ was specified as the total porosity of the soil samples after freeze drying.

Equation (6) describes the unsaturated permeability of soil once the $n$-parameter is known. Figure 38 presents a family of curves displaying the influence of the $n$-parameter on unsaturated permeability. As $n$ increases, the relative permeability tends to decrease more gradually as the soil begins to desaturate. This indicates that, the larger the $n$-parameter, the larger is the fraction of porosity with relative importance for conducting fluids. As $n$ increases, one can expect the effective porosity to represent a larger portion of the total liner porosity.

Figure 2 presents a family of curves, over a range in $\theta_s$ from 0.3 to 0.5, displaying the influence of $n$ on the effective porosity when $\theta_r$ is constant at 0.05. If similar sets of curves were drawn for $\theta_r = 0.0$ and $\theta_r = 0.10$, only small differences from Fig. 2 would be observed; i.e., the maximum difference in $E$ would be 0.007. Thus, Fig. 2 can be used to obtain estimates of effective porosity of compacted clay liners with $\theta_s$ between 0.3 and 0.5, $\theta_r$ between 0.0 and 0.1, and $n$ between 0.1 and 3.0.
Figure 38. Theoretical relative permeability ($K_r$) curves as influenced by the $n$-parameter ($S = \theta/\theta_s$).
The method developed in this paper for estimating the effective porosity of compacted clay liners depends only on parameters describing liner pore structure; i.e., \( \theta_s \), \( \theta_r \), and the n-parameter. Cumulative porosity measurements provide the basis for the determination of the appropriate parameters. By using the pore structure parameters presented in Table 11, effective porosities for the Nicollet, Fayette, and Clarinda soil materials are estimated from Fig. 2 as 0.04, 0.08, and 0.08, respectively. Because the total porosity of the Clarinda sample decreased as a result of drying from 0.46 to 0.40, \( E \) was recalculated for \( \theta_s = 0.46 \). Thus, \( E \) was 0.09 for the Clarinda soil material. Once effective porosity is estimated, pore-fluid velocity and noninteracting pollutant travel time can be calculated for a compacted liner with a known permeability, hydraulic gradient, and thickness.

Table 12 presents measured values of permeability and times of first chloride breakthrough (relative effluent concentration equal to 0.01), estimated effective porosities, and predicted times of first breakthrough of noninteracting pollutants for compacted Nicollet, Fayette, and Clarinda samples. Each compacted soil material had permeability less than the EPA-preferred value of \( 1 \times 10^{-9} \) m\(^3\)/m\(^2\)/s for hazardous wastes disposal liners. The estimated effective porosities were between 10 and 20% of the total porosities of the compacted samples. The predicted breakthrough time was calculated based on the mean permeability. The measured chloride breakthrough time was adjusted to correspond to the breakthrough time of a permeameter with the mean permeability.
Table 12. Measured permeabilities and time of first chloride breakthrough and predicted effective porosities and non-interacting pollutant breakthrough times for the compacted soil materials (permeameters were 11.64 cm long).

<table>
<thead>
<tr>
<th>Soil Material</th>
<th>Permeability (m$^3$/m$^2$/s)</th>
<th>Measured Cl$^-$/breakthrough time (d)</th>
<th>Effective porosity</th>
<th>Predicted breakthrough time (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nicollet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 1</td>
<td>5.47 x 10$^{-11}$</td>
<td>3.0</td>
<td>0.04</td>
<td>2.6</td>
</tr>
<tr>
<td>Rep 2</td>
<td>6.58 x 10$^{-11}$</td>
<td>4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 3</td>
<td>1.22 x 10$^{-10}$</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>8.08 x 10$^{-11}$</td>
<td>3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fayette</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 1</td>
<td>2.84 x 10$^{-10}$</td>
<td>1.7</td>
<td>0.08</td>
<td>1.6</td>
</tr>
<tr>
<td>Rep 2</td>
<td>2.67 x 10$^{-10}$</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 3</td>
<td>2.61 x 10$^{-10}$</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.71 x 10$^{-10}$</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clarinda</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 1</td>
<td>3.38 x 10$^{-11}$</td>
<td>4.8</td>
<td>0.09</td>
<td>5.1</td>
</tr>
<tr>
<td>Rep 2</td>
<td>2.17 x 10$^{-10}$</td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep 3</td>
<td>1.11 x 10$^{-10}$</td>
<td>3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.21 x 10$^{-10}$</td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The predicted noninteracting pollutant breakthrough times agreed
well with the measured chloride breakthrough times. Predicted breakthrough values were about equal to measured values for Fayette. Predicted breakthrough was earlier than measured for Nicollet and later than measured for Clarinda. For one replicate of Nicollet and one replicate for Clarinda, the measured and predicted breakthrough times were nearly equal. Because the clay fractions of the compacted samples had a net negative charge and chloride is an anion, it is reasonable to assume that chloride could move through the samples faster than a noninteracting pollutant (i.e., tritiated water). Nicollet soil material had less clay than the other materials, and thus anion exclusion would be expected to be less.

The theory presented provides the basis for estimating effective porosity of compacted clay liners. To predict the travel time of noninteracting substances through a clay liner, the hydraulic gradient, liner permeability, liner thickness, and effective porosity are required. As an example of how the theory can be used, we will now make such a prediction. The information that we assume is: liner thickness = 1 meter, hydraulic gradient = 1.33, liner permeability = $10^{-9}$ m$^3$/m$^2$/s.

If we follow only the Darcy equation, Eq. (1), fluid flux density is $1.33 \times 10^{-9}$ m$^3$/m$^2$/s. However, the fluid flux density does not represent the fluid velocity within the liner. We know that a portion of the total porosity is responsible for conducting fluid at velocities higher than the average velocity and the remaining portion of the porosity conducts fluids at velocities lower than the average velocity. What is important in pollutant transport is the portion of porosity that conducts fluid at higher-than-average velocities. To determine the average effective fluid velocity, we must first estimate the effective porosity.

Effective porosity is determined based on theory previously presented. By using mercury porosimetry data for the Nicollet materials presented in Fig. 35 and Eq. (9), the n-parameter is determined as 1.5, $\theta_r$ as 0.00, and $\theta_g$ as 0.28. By using Fig. 2 effective porosity is estimated to be 0.04. Based upon Eq. (3) the travel time for noninteracting solutes through a Nicollet clay liner in this example is predicted to be 0.95 years. Therefore, under these conditions, we
predict that after about one year, noninteracting pollutants will reach the bottom of the clay liner. Although the concentration of the noninteracting pollutant may be low because of dilution and lateral mixing by diffusion, the initial fraction is predicted to appear after one year. In this example, if the product of liner permeability and hydraulic gradient (i.e., fluid flux density, \( J \)) is mistakenly used as the fluid velocity, the pollutant travel time is calculated to be 23.8 years. If the fluid velocity is mistakenly assumed to be equal in all soil pores, the noninteracting pollutant travel time is calculated as 6.7 years. Thus, travel time for noninteracting pollutants is significantly overestimated if effective porosity is not taken into account. The active lifetime of hazardous waste disposal units may be overestimated if liner effective porosity is not properly estimated and used in the prediction of pore-fluid velocity.

IMAGE ANALYSIS

**Sample preparation techniques**

Direct replacement of water in large (10.4 cm\(^3\)) soil samples by acetone was successful only with samples with the lowest clay content, i.e., Nicollet soil materials. Both Fayette and Clarinda soil samples developed large cracks and fell apart when placed in 100% acetone. We also experimented with a graded series of replacement fluids, increasing the acetone content in water gradually from 10% to 100% acetone, but these samples also shrank and disintegrated. Similar results were obtained when we placed the samples in acetone vapor. These results were unexpected because the acetone replacement technique has been reported by others to be simple and successful (Murphy, 1982; FitzPatrick, 1984). But in contrast to many soils, the clay minerals of the three Iowa soils are largely smectitic. This means that relatively large amounts of water can be held in clay interlayers. We believe that, when this interlayer water was withdrawn by the polar acetone molecules, the clay layers collapsed and sample shrinkage occurred.

On the other hand, replacement of water by methanol proved to be successful for large samples of Fayette and Clarinda soils. No shrinking or swelling of the soils was observed during methanol exchange, and the samples remained intact. Methanol was successfully
exchanged with acetone, and then acetone was exchanged with the polyester resin in which Uvitex-OB had been dissolved. After the resin was cured, porosity images were obtained from polished blocks.

**Obtaining an image**

Soil porosity images obtained by using a 35-mm camera and a macro lens were, in general, adequate for further analysis. Considerable experimentation was required to adjust the light intensity, the camera's aperture, and the time of exposure for optimum photographs. The main advantage to this approach for porosity imagery is that it is simple and relatively inexpensive. On the other hand, there are two reasons that images so obtained are inherently somewhat inaccurate. First, only pores greater than about 50-60 μm in equivalent spherical diameter can be adequately distinguished on the photograph. This means that, if smaller pores are of interest, they must be imaged by a different technique. Moreover, “total porosity” measured on the photograph is unlikely to be as high as true total porosity of the sample because small pores have been excluded from the analysis. Second, shallow pores at the surface of the polished sample may not contain enough dye to show up on the photograph. Or, alternatively, thin, transparent sand-size grains at the surface may allow florescence from an underlying pore to shine through. Figure 39 is an example of an image of soil porosity produced with a 35-mm camera and a macro lens.

Porosity images obtained from thin sections viewed in circularly polarized light also required a lot of experimentation regarding the best film, light intensity, and time of exposure. Best results were obtained with Kodak Ectagraphic HC film. In general, the longer the exposure time was, the less porosity was displayed on the (positive) photograph. In other words, just as with the macro lens technique, some judgment was required to determine which exposure corresponded the closest to porosity of the sample.

The advantage to obtaining soil porosity images by using circularly polarized light is that the thin sections can be studied in transmitted light for other characteristics, e.g., mineralogy, cutans, peds, etc. There are, however, several disadvantages. First, one should exclude from analysis any pores that are smaller in diameter than the thickness
Figure 39. Porosity image obtained by photographing fluorescent dye in soil pores with a 35-mm camera and a macro lens. Diameter of soil sample = 5.5 cm.
of the thin section, i.e., about 30 μm. This is because such pores cannot be distinguished from grains of the same size that do not extend through the thickness of the thin section. Second, it is difficult to choose a representative field for analysis at this scale. Porosity may constitute from 10% to more than 60% of a given field of view even in the same thin section. Thus many analyses may be required to statistically evaluate the porosity. Finally, at this scale, it is often difficult to distinguish individual pores. Pore space can be so interconnected that discrete pores do not exist on the image. In such instances, methods of image analysis that are based on measuring individual pores probably are not appropriate. Figure 40 is an example of a porosity image obtained with circularly polarized light.

Images of soil porosity were also made by using a scanning electron microscope in backscatter mode. This technique has the advantage of simple preparation of samples—blocks of soil in hardened resin need only be cut to size and finely polished. On the other hand, use of the electron microscope is expensive. Moreover, the problem of finding representative fields at high magnification is similar to that outlined for the circularly polarized light technique. The most severe problem we experienced was getting the resin to fill all micropores of the sample and to harden sufficiently. We found that even after polishing the sample, holes remained in the surface. Figure 41, for example, is a backscattered electron image of a compacted Clarinda sample, showing holes in the surface. These pits provided enough backscatter contrast that they could be measured by an image analyzer. But the pits on the sample may not be the original pores; thus further analysis of the backscatter image would provide doubtful information about sample porosity.

We used a number of variations in our laboratory procedure to cure the plastic resin: variations in the amount of catalyst, variations in the temperature and speed of the heat treatment, for example. But none was completely successful at the scale viewed with an electron microscope. As FitzPatrick (1984) has noted, there are still many unknown factors in the process of curing impregnated blocks of soil; it is often impossible to predict if a technique or variation will succeed.
Figure 40. Example of an image of soil porosity obtained with circularly polarized light. Undisturbed Clarinda soil materials. Frame length = 6.6 mm.
Figure 41. Example of a backscattered electron image of a compacted Clarinda soil sample showing holes in the surface. Frame length = 100 μm.
or fail, especially with very clayey soils.

Analyzing the images

Apart from reporting and discussing the actual results of the image analysis performed on porosity images of the Nicollet, Fayette, and Clarinda soil samples, it is appropriate to document some observations on the use of an image analyzing computer. These observations may alert other investigators to potential pitfalls or points of concern.

1-The "image" received by the computer depends upon the level of light falling on the photograph, the resolution of the video camera, and the resolution of the digitizing board. We found that it was especially important to use a video camera of high quality. The camera we used in conjunction with a microcomputer employed a 1-inch Newvicon tube and had 800-line resolution. This was sufficient to send high-quality images to the microcomputer's digitizing board. Video cameras associated with the LeMont and Leitz image analysis systems were also adequate for obtaining high-quality images.

2-Once the image is digitized, an adjustment of the gray level of the pixels allows the operator to match as well as possible the binarized image with the original porosity image. This procedure depends a lot on incident light level and angle, camera quality, and the patience of the operator. The gray level that is finally chosen by the operator is a compromise between that level which fills in all pores and adds unwanted "spots" on the image and that level which does not completely fill in all pores.

3-On image analyzing computers such as the Leitz TAS, it is possible to remove selectively pixels which otherwise would be counted as pores because they have gray levels above the set threshold. This can be done by automatic methods (erosion and dilation) or by manual editing with a light-pen touching the monitor or a digitizing tablet. Such editing is very useful to optimize the image to be analyzed. But it is also time-consuming, and it once again introduces an element of human judgment.

4-There is a theoretical problem with making measurements of object size on two-dimensional images. If an object touches the edge of an image, its true size cannot be observed and measured. Weibel (1979)
proved that statistically accurate measurements could be obtained nevertheless if two contiguous edges of the image (say, top edge and one side edge) were defined as "forbidden zones". Any objects that touch the forbidden zone, then, are to be excluded from analysis, whereas objects touching the remaining two sides are included. Such an approach assumes a random distribution of objects in the image and sufficient number of images to adequately represent the subject being analyzed. In our work, the Leitz TAS was programmed to exclude from analysis pores that touched the forbidden zone. The LeMont image analysis system was programmed to exclude pores that touched any of the four sides of the image. The image analysis program used with the microcomputer accepted all pores in the image.

A final observation is that in image analysis of soil porosity, the number of pores to be measured and the number of images to be analyzed has not been well established. We tried to obtain images of porosity from as many replicate samples as possible when using the 35-mm camera and macro lens. Measurements on each image were averaged with other replicates to produce the results discussed in the remainder of this section.

Of the three types of image analyzers tested in this study, we consider the Leitz TAS to be the most reliable. Data reported in Table 13 were obtained by using the Leitz TAS of the National Soil Erosion Laboratory, West Lafayette, Indiana. For each image analyzed, the following information is presented: number of pores counted, mean pore area (\( \mu m \)) and standard deviation about the mean, and measured porosity (%) of the image. Only pores greater than about 50 \( \mu m \) diameter could be recognized consistently, so the porosity value necessarily ignores pores smaller than that. About 11 cm\(^2\) of each image was analyzed, which corresponds to about 3.4 cm\(^2\) of each sample after magnification of the image is taken into account. Data for four replicate samples of Nicollet soil material and four replicate samples of Fayette soil material are presented. Two different images of each Fayette sample (obtained by polishing and photographing both ends of a compacted, impregnated sample) were analyzed. Only one image of compacted Clarinda soil material was available for analysis because of technical problems.
with water removal described earlier.

Table 13. Results of porosity analyses obtained by Leitz TAS image analyzing computer.

<table>
<thead>
<tr>
<th>Soil material and replicate</th>
<th>n</th>
<th>Mean pore area $\mu m^2$</th>
<th>Standard deviation $\mu m^2$</th>
<th>Porosity %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nicollet</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>430</td>
<td>236</td>
<td>154</td>
<td>1.31</td>
</tr>
<tr>
<td>B-1</td>
<td>1182</td>
<td>255</td>
<td>152</td>
<td>3.99</td>
</tr>
<tr>
<td>D</td>
<td>442</td>
<td>254</td>
<td>442</td>
<td>1.51</td>
</tr>
<tr>
<td>E</td>
<td>797</td>
<td>267</td>
<td>797</td>
<td>3.73</td>
</tr>
<tr>
<td>B-2</td>
<td>368</td>
<td>209</td>
<td>94</td>
<td>0.74</td>
</tr>
<tr>
<td><strong>Fayette</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>319</td>
<td>302</td>
<td>215</td>
<td>1.68</td>
</tr>
<tr>
<td>A2</td>
<td>374</td>
<td>260</td>
<td>183</td>
<td>1.45</td>
</tr>
<tr>
<td>B1-17</td>
<td>542</td>
<td>246</td>
<td>140</td>
<td>1.67</td>
</tr>
<tr>
<td>B2</td>
<td>265</td>
<td>275</td>
<td>204</td>
<td>1.18</td>
</tr>
<tr>
<td>C1</td>
<td>588</td>
<td>280</td>
<td>176</td>
<td>2.47</td>
</tr>
<tr>
<td>C2</td>
<td>489</td>
<td>282</td>
<td>244</td>
<td>2.56</td>
</tr>
<tr>
<td>D1</td>
<td>578</td>
<td>336</td>
<td>234</td>
<td>3.73</td>
</tr>
<tr>
<td>D2</td>
<td>182</td>
<td>370</td>
<td>261</td>
<td>1.43</td>
</tr>
<tr>
<td>B1-12</td>
<td>701</td>
<td>243</td>
<td>145</td>
<td>2.16</td>
</tr>
<tr>
<td><strong>Clarinda</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>344</td>
<td>322</td>
<td>247</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Variability of the porosity measurements is the most striking aspect of the data, and it occurs at every level of analysis. For example, the mean pore area for all analyzed samples is of the same order of magnitude and ranges from 236 to 370 $\mu m^2$. But standard deviations about the means are large, suggesting that the distributions
of pore area values are relatively spread out. Variability of results within replicates also occurred. For example, even though mean pore areas for the four Nicollet replicates are close to one another, the number of pores counted ranged from 430 to 1182 and the porosity values ranged from 1.31 to 3.99%. Even different images from the same replicate produced different results. For example, Fayette sample D had one end with 578 pores and 3.73% porosity, whereas the other end had 182 pores and 1.43% porosity.

As discussed in previous sections, variability in image analysis measurements can also be due to human judgment factors. The length of time allowed for exposure of the photograph under UV light influences the number of pores that are detected. For example, two photographs with different exposure times were taken of Nicollet sample B (B-1 and B-2 in Table 13). The image analyzer noted a significant difference in the number of pores counted and in the porosity between the two photographs of the same polished sample face. Similarly, human judgment is involved in setting the detection of gray levels on the image analyzer itself. Fayette sample B1 was analyzed at two detector settings (B1-17 and B1-12 in Table 13). This required only slight machine adjustment by the operator and, at the time of analysis, either detection level was considered adequate for analysis of the image. Yet there was again a significant difference between the analyses with respect to number of points counted and percent porosity.

On the assumption the pores were circular in cross-section, an equivalent radius was calculated for each pore and the results were plotted in a frequency diagram. (This assumption was reasonable for some pores and not for others, of course, but it allowed us to roughly compare all pores at the same time). Percent relative frequency was calculated to allow relative comparisons of pore size distributions because there was so much variability in the absolute pore data of the samples. Representative histograms are given in Fig. 42 and 43 for Nicollet and Fayette samples. Most pores had equivalent radii of 50-200 \( \mu m \). In general, Fayette samples had more pores larger than 300 \( \mu m \) equivalent pore radius than did Nicollet samples. Clarinda samples were not included in this comparison because only one image was analyzed.
Figure 42. Image analyzer determined pore size distribution histogram for compacted till material.
Figure 43. Image analyzer determined pore size distribution histogram for compacted loess material.
A cumulative area curve (analogous to cumulative relative porosity) was also plotted for each analyzed image. Figure 44 is representative for Nicollet and Fayette samples. (Clarinda materials are again omitted because only one image was available). Slopes of the cumulative area curves are similar to one another, and they are steep on a logarithmic scale. Each point plotted on the diagram represents an individual pore, so the clustering of equivalent radius values between 50 and 200 μm is clearly demonstrated. The influence of relatively large pores on total porosity can also be observed on these diagrams. For example, pores larger than 300 μm equivalent radius account for about 50% of the measured porosity of the Fayette sample, but only about 30% of the measured porosity of the Nicollet sample. This observation that large pores influence measured porosity more in the Fayette samples than in the Nicollet samples was consistent for nearly all the images.

In conclusion, the variability of image-analyzed porosity among replicates and the sensitivity of the measurements to human judgment limit what can be said about porosity in the compacted samples of this study. Analysis of data on a relative basis, however, does suggest that most pores in all samples were relatively small with respect to equivalent radius and area. Moreover, the few large pores that occurred had a significant impact on total porosity of the samples.

Markov Analysis

Twenty different patterns were analyzed using the Markov approach. We will report a series of original and generated patterns in order to show the strength and weaknesses of this method. It is important to note that the algorithm does not guarantee that the statistics for an original and its resulting generated patterns will be the same. An obvious reason for this disparity is that the generated image is the result of a series of random processes. Thus the generated patterns have a range of possibilities.

For all isotropic patterns investigated, we found that a pattern could be easily generated such that each of its Markov parameters matched within 10% of the values of the Markov parameters of the original pattern (see Table 14). While each generated pattern is based on one set of Markov parameters, these Markov parameters can correspond
Figure 44. Image analyzer determined cumulative porosity curves for (a) till materials and (b) loess materials.
to an infinite number of generated patterns. This is due to the random elements of the algorithm. And because of these random elements, it is sometimes necessary to generate several patterns before an acceptable generated pattern is obtained. The agreement of parameters to within 10% was obtained in three or fewer attempts in all but three cases.

Table 14. Markov probability statistics for selected pore patterns.

The transitional probabilities are for pore to pore (P→P), pore to matrix (P→M), matrix to pore (M→P), and matrix to matrix (M→M). The corresponding patterns are shown in Figs. 45, 46, 47, 48, 53, and 54.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Transitional Probabilities</th>
<th>Marginal Probabilities</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P→P</td>
<td>P→M</td>
<td>M→P</td>
</tr>
<tr>
<td>A original</td>
<td>.740</td>
<td>.250</td>
<td>.093</td>
</tr>
<tr>
<td>A generated</td>
<td>.740</td>
<td>.257</td>
<td>.098</td>
</tr>
<tr>
<td>B* original</td>
<td>.966</td>
<td>.034</td>
<td>.007</td>
</tr>
<tr>
<td>horizontal</td>
<td>B generated</td>
<td>.980</td>
<td>.020</td>
</tr>
<tr>
<td>vertical</td>
<td>B original</td>
<td>.857</td>
<td>.143</td>
</tr>
<tr>
<td>horizontal</td>
<td>B generated</td>
<td>.752</td>
<td>.247</td>
</tr>
<tr>
<td>C original</td>
<td>.869</td>
<td>.131</td>
<td>.057</td>
</tr>
<tr>
<td>C generated</td>
<td>.861</td>
<td>.139</td>
<td>.062</td>
</tr>
<tr>
<td>D original</td>
<td>.740</td>
<td>.260</td>
<td>.024</td>
</tr>
<tr>
<td>D generated</td>
<td>.745</td>
<td>.255</td>
<td>.023</td>
</tr>
<tr>
<td>E original</td>
<td>.948</td>
<td>.052</td>
<td>.036</td>
</tr>
<tr>
<td>F original</td>
<td>.948</td>
<td>.052</td>
<td>.032</td>
</tr>
</tbody>
</table>

* anisotropic pattern
The patterns that required generating more than three patterns to obtain Markov parameter agreement within 10% consisted of bi-modal pore size distributions. This indicates that the algorithm is inefficient in producing patterns with a bi-modal distribution. This deficiency in the algorithm can be predicted based on the fact that the areas of the polygons follow a unimodal distribution (Kendall and Moran, 1963), and that the assignment of states to adjacent polygons is random, making bi-modal distributions in the generated patterns improbable. In addition, it seems necessary that pores in the initial image have a random spatial distribution. Fortunately, the pore patterns for compacted materials are generally unimodal, with a random spatial distribution.

Figures 45-48 present examples of the original and generated pore patterns. Visual inspection reveals that the generated "pores" lack the rounded edges present in the original patterns. Yet the original and generated patterns are similar to one another. Unmistakable pairing of original and generated patterns may not be always possible for the wide range of soil porosity patterns possible; but these results indicate that for the unimodal pore distributions encountered, Markov statistics have potential as useful descriptors.

Histograms showing the frequency of various pore sizes for patterns A, C, and D (Figs. 49-51) indicate that the algorithm produces patterns that have pore size distributions somewhat similar to the original patterns. Figure 51 illustrates how patterns with similar model parameters can vary with respect to their distribution of large pores. This indicates that the algorithm may not be sensitive enough to be used in studies involving the prediction of hydraulic conductivity. Other methods to model soil porosity (capillary tube models) also fail in the prediction of hydraulic conductivity. Perhaps this Markov approach, like previous models, will prove to be a useful descriptor of relative hydraulic conductivity, and thus of the effective porosity.

Figure 52 displays the cumulative porosity curves for the original and generated pore patterns shown in Fig. 48. The curves appear to be similar in spread of pores and curve slope near their middles. This provides further evidence that the effective porosities of the original and generated pore patterns would be similar. The theory developed in
Figure 45. Original and Markov-generated pore patterns for soil material A.
Figure 46. Original and Markov-generated pore patterns for test image B.
Figure 47. Original and Markov-generated pore patterns for soil material C.
Figure 48. Original and Markov-generated pore patterns for soil material D.
Figure 49. Image analyzer determined pore size distribution histograms for the original and Markov-generated pore patterns of soil material A.
Figure 50. Image analyzer determined pore size distribution histograms for the original and Markov-generated pore patterns of soil material C.
Figure 51. Image analyzer determined pore size distribution histograms for the original and Markov-generated pore patterns of soil material D.
Figure 52. Image analyzer determined cumulative porosity curves for the original and Markov-generated pore patterns of soil material D.
this report uses the slope of the cumulative porosity curve in order to estimate effective porosity. Thus, Markov statistics seem to be useful for describing pore patterns such that effective porosity is reasonably estimated. This is applicable to the expected unimodal pore distributions in compacted soil materials.

Because a wide range of patterns can be successfully modeled, one might hope to characterize different soil patterns by using model parameters. But Figs. 53 and 54, which have similar model parameters, correspond to strikingly different patterns. Again the problems occur with the bi-modal distribution. Intuitively, one might guess that patterns with similar marginal probabilities and similar total pore perimeters will have similar transitional probabilities. Thus, in order for the Markov statistics to be useful in characterizing soil pore patterns the patterns should be unimodal in pore size. Such pore size distributions seem likely to occur in most compacted soil materials but not for many undisturbed soil materials.
Figure 53. Original and Markov-generated pore patterns for test image E.
Figure 54. Original and Markov-generated pore patterns for test image F.
REFERENCES


