Tilings with Singularity and Spiral Tilings Obtained from Archimedean Tilings

Angelica Rioga Sta. Ana  
*University of the Philippines Diliman, arstaana1@up.edu.ph*

Jonathan V. Caalim  
*University of the Philippines Diliman*

Manuel Joseph C. Loquias  
*University of the Philippines Diliman*

Follow this and additional works at: [https://lib.dr.iastate.edu/aperiodic2018](https://lib.dr.iastate.edu/aperiodic2018)  
Part of the [Chemistry Commons](https://lib.dr.iastate.edu/aperiodic2018), and the [Materials Science and Engineering Commons](https://lib.dr.iastate.edu/aperiodic2018)

Tilings with Singularity and Spiral Tilings

Obtained from Archimedean Tilings

Jonathan V. Caalim, Manuel Joseph C. Loquias, Angelica R. Sta. Ana

Institute of Mathematics, College of Science, University of the Philippines Diliman,
1101 Quezon City, Philippines
arstaana1@up.edu.ph

A singular point $P$ of a plane tiling is a point in the plane where every circular disk about $P$ meets infinitely many tiles of the tiling. Tilings with a singular point were obtained in [1] by applying the conformal map

$$\varphi_\alpha(z) = \exp\left(\frac{2\pi i}{\alpha} z\right)$$

(1)

to the three regular Archimedean tilings for certain values of $\alpha$. The symmetry group of each resulting tiling with singularity is isomorphic either to a finite cyclic or dihedral group. Moreover, for each positive integer $n$, it is possible to construct a tiling with singularity that has rotation symmetry of order $n$ by choosing a suitable $\alpha$.

This contribution aims to generate tilings with singularity by applying the same $\varphi_\alpha$ to the remaining eight semiregular Archimedean tilings. We obtain sufficient conditions for $\alpha$ so that the image of a semiregular Archimedean tiling under the map $\varphi_\alpha$ is a tiling with a singular point. In addition, we also identify the symmetry groups of the resulting tilings with singularity. For instance, if $\omega$ denotes $\exp(2\pi i/3)$, then the images of the 3.6.3.6 tiling under $\varphi_\alpha$ (where $\alpha = -10 + 2\omega$) and the 3.4.3.3.4 tiling under $\varphi_\alpha$ (where $\alpha = -4 + 4\omega$) are both tilings with singularity at the origin as shown in Fig. 1 and Fig. 2, respectively. Observe that the tiling in Fig. 1 has symmetry group isomorphic to the cyclic group $C_2$ while the tiling in Fig. 2 has symmetry group isomorphic to the dihedral group $D_4$.

Loosely speaking, tilings which give the viewer a psychological “spiral effect” are said to be spiral. In this case, the tiling in Fig. 1 is spiral. On the other hand, the tiling in Fig. 2 contains no “spiral effect”. There are several efforts in the literature to give a suitable mathematical definition for spiral tilings. Very recently,
Klaassen [2] proposed several criteria for a tiling to be called spiral. A tiling $\mathcal{Z}$ is said to be spiral-like if there is a partition of $\mathcal{Z}$ into classes (called arms) such that for each arm, there exists a simple unbounded non-intersecting curve (called thread) meeting each tile in an arm exactly once while spinning infinitely often around a point. Two tiles that belong to the same arm $A$ of a spiral-like tiling are referred to as direct neighbors if their intersection is cut by the thread of $A$ or contains more than a finite number of points. A spiral-like tiling with exactly one singular point is then called a spiral tiling if whenever a pair of tiles $T_1$ and $T_2$ that are direct neighbors are mapped by rotation, translation, and scaling onto another pair of tiles $T_3$ and $T_4$, then tiles $T_3$ and $T_4$ are also direct neighbors within an arm. We test these criteria by looking at whether the tiling $\varphi(\mathcal{Z})$ with singularity obtained from some Archimedean tiling $\mathcal{Z}$ is a spiral tiling according to the definition given in [2].

To illustrate, the image of the square tiling $4^4$ under the map $\varphi_\alpha$, where $\alpha = 6$, gives no “spiral effect” as shown in Fig. 3. However, this tiling can be partitioned into arms that satisfy the conditions of a spiral tiling. One such partition is given in Fig. 4 where one arm corresponds to one color and the threads are denoted by the broken logarithmic spirals.