Alternative Engel Curves. It is not uncommon to hear or to read that the income elasticity of demand for food in this country declines with rising consumer income. Common as the assertion is, there seems to be no published theoretical or empirical work which would verify or reject the assertion, with one recent exception.1

One place to turn in studying the relation between income level and income elasticity is the literature on Engel's Law. As I understand it, Engel's Law merely states

\[
\frac{X}{Y} = f(Y), \quad f(Y) > 0, \quad f'(Y) < 0,
\]

where \( X \) = food expenditure per capita or per household member, \( Y \) = disposable income similarly measured and \( f'(Y) = \frac{df(Y)}{dY} \). The relation between income elasticity of expenditure and Engel's Law is

\[
\frac{dE_{XY}}{dY} = \frac{Y^2}{X} \left[ f''(Y) + f'(Y) \left( \frac{Y}{X} - \frac{Yf''(Y)}{[f(Y)]^2} \right) \right],
\]

where \( E_{XY} \) = income elasticity of food expenditures and \( f''(Y) = \frac{d^2f(Y)}{dY^2} \). The relation between Engel's Law and marginal propensity to spend on food is

\[
\frac{d^2X}{dY^2} = Yf''(Y) + 2f'(Y).
\]

\[
\frac{dE_{XY}}{dY} < 0 \text{ if } f''(Y) < 0, \text{ or if } f''(Y) > 0 \text{ but }
\]

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X might be defined as \( X = P \cdot Q \) where \( P \) = average price paid and \( Q \) = quantity purchased. Now \( E_{XY} = E_{QY} + E_{PY} \) where \( E_{QY} \) = income elasticity of demand (income elasticity of quantity) and \( E_{PY} \) = income elasticity of price. And \( \frac{dE_{XY}}{dY} = \frac{dE_{QY}}{dY} + \frac{dE_{PY}}{dY} \). Hence, even if we know that \( \frac{dE_{XY}}{dY} \leq 0 \), this tells us little about \( \frac{dE_{QY}}{dY} \), the effect of income on income elasticity of demand.

In view of the importance for long-range agricultural policy of the question of the magnitudes of \( \frac{dE_{XY}}{dY}, \frac{dE_{QY}}{dY} \) and \( \frac{dE_{PY}}{dY} \) this is an area to which some research resources might profitably be allocated: What is the relation between income level and income elasticities or marginal propensities? Tornqvist, S. J. Prais and Aitchison and Brown have suggested procedures that would be relevant to such a study.

Tornqvist has proposed an Engel curve for necessities of the form

\[
(4) \quad X(\text{or } Q) = \frac{aY}{Y + b}.
\]

This question yields an income elasticity estimate of \( b/(Y + b) \); it declines with rising income. A corollary to the hypothesis \( \frac{dE_{QY}}{dY} \leq 0 \) is that \( E_{QY} \) becomes zero at some high level of income. Consumption at this point would represent the satiation level of consumption. In equation (4) this satiation level is at a consumption level of \( a \); that is, (4) has an upper limit of \( a \) (which is reached only at an infinitely large income).

---

2/ Referred to in Herman Wold, *Demand Analysis*, (New York: John Wiley and Sons), 1953.


Prais concluded that a typical Engel curve may be described by the semi-logarithmic equation:

\[ X = a + b \log Y, \]  

and that quality changes, as measured by price, are satisfactorily described by

\[ P = h + k \log Y. \]

Dividing (5) by (6),

\[ Q = \frac{a}{b} + \frac{\log Y}{k} = \frac{h}{k} + \frac{\log Y}{k}. \]

Q has a maximum value of \( b/k \).

Aitchison and Brown propose an equation of the form:

\[ Q = \int_{-\infty}^{z} \frac{-t^2}{2\sqrt{2\pi}e} \, dt \]

where \( K = \) a satiation level of consumption,

\[ z = a + b \log Y. \]

They fit the equation in the form

\[ Q = K A(aYb) + u \]

where \( u \) is the random residual term and \( A(aYb) \) is the standardized log-normal distribution function at \( aYb \) and is related to the standardized (zero mean and unit variance) normal distribution function by \( A(aYb) = N \log (aYb) \).

Graphic analysis indicated that \( b = 1 \) for the food products with which they were dealing.

The income elasticity = 0 for (8) or (9) when the saturation level of consumption is reached, at \( Q = K \), and income elasticity continuously increases as income declines. \( E_{QY} = 1 \) when \( Q = 0.38K \). Aitchison and Brown conclude

---

6/ Prais, op. cit.

7/ Aitchison and Brown, op. cit.
that (10) gives superior fits to semilogarithmic equations, which in turn give superior fits to logarithmic equations.

Published results for the United States. The nearest thing we have in this country to a study of the relation between income level and income elasticity is a recent study by Rockwell.\textsuperscript{8} Utilizing data from the 1955 Household Food Consumption Survey, he divided farm and nonfarm households into three income classes as follows:\textsuperscript{9}

\begin{tabular}{|l|c|c|}
\hline
Family income class & Nonfarm households & Farm households \\
\hline
Low & 0-$3,399 & 0-$1,499 \\
Medium & $3,400-$4,999 & $1,500-$3,499 \\
High & $5,000 and over & $3,500 and over \\
\hline
\end{tabular}

Approximately one-third of the farm households were in each farm income class, and one-third of the nonfarm households were in each nonfarm income class.

Within each of these six classes a linear regression was computed

\begin{equation}
X \text{ (or } Q) = a + bY + cN
\end{equation}

where \(X\), \(Q\) and \(Y\) are per person and \(N\) is household size. Elasticities were computed at the mean values of the variables for each of the six classes.

Some results from this study are presented in Tables 1 and 2. Table 3 presents some elasticities derived from Tables 1 and 2. Tables 4 and 5 present some comparisons of the income elasticities in Tables 1 and 2. In reading these latter three tables, keep in mind that there is no way of determining from published data how many of these differences are statistically significant. Many of the differences shown are probably not statistically significant.

In Table 3, \(E_{py} > 0\) in 72 cases and \(E_{py} < 0\) in 11 cases. If we assume the nonsignificant values in Tables 1 and 2 to be zero, \(E_{py} > 0\) in 48 cases and \(E_{py} < 0\) in 8 cases. This apparent tendency for consumers to pay somewhat

\textsuperscript{8} Rockwell, op. cit.

\textsuperscript{9} Ibid., pp. 42-43.
higher prices as incomes rise deserves further study for its possible implications for primary agricultural adjustment and for marketing firm behavior. How much of the higher price goes for quality variation over which the farmer has some control and from which he can receive higher prices, e.g., grade A over grade B eggs? How much goes for quality variation over which the processors or marketing firms have control and from which they receive higher prices?

The figures in Table 1 might be called income elasticities of value of consumption; this differs from income elasticity of expenditures since value includes gifts and home-produced food. The income elasticity of total value of consumption falls with rising incomes among nonfarm households; it rises and then falls among farm households. In dealing with smaller aggregates than all food, Table 4 shows that elasticity falls in about as many cases as it rises in moving from one income class to a higher income class. There would appear to be quite a few groups of commodities which are not going to encounter declines in the national average income elasticity of demand as consumer incomes rise over time, at least not for many years.

Temporal developments in the distribution of income are a relevant factor in making projections as to national average income elasticities of demand. This includes distribution by socio-economic class, as well as size distribution. For example, the data in Table 5 suggest that there are differences between the income elasticities of farm and of nonfarm families.

Relation Between Income Elasticity at Retail and at the Farm. It may be worthwhile to consider the relation between changes in income elasticity of demand at the retail level and changes in the derived income elasticity of demand at the farm level.

Assume a perishable commodity for which net imports are negligible. Then the same quantity will be sold by marketing firms as is sold by farmers. Assume the product is sold by farmers to marketing and processing firms and is sold by them in turn to consumers. Let the farmer's supply equation be

\[ S(Q, P_f, Z_s) = 0, \]

the marketing firm's behavior equation be

\[ M(Q, P_f, P_r, Z_M) = 0, \]

and the consumer demand function be

\[ D(Q, P_r, Y, Z_D) = 0. \]
Table 1. Income elasticities of demand, based on value of consumption at home per person, for all households, one week, spring 1955.\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>Nonfarm households by family income class</th>
<th></th>
<th>Farm households by family income class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>All foods and beverages</td>
<td>0.25</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>Milk and milk products, excluding butter(^1)</td>
<td>0.18</td>
<td>-0.04(^2)</td>
<td>0.09</td>
</tr>
<tr>
<td>Fats and oils, excluding bacon and salt pork(^1)</td>
<td>0.05(^2)</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Flour and other cereal products</td>
<td>-0.13</td>
<td>-0.08(^2)</td>
<td>-0.04(^2)</td>
</tr>
<tr>
<td>All meat</td>
<td>0.39</td>
<td>0.30</td>
<td>0.16</td>
</tr>
<tr>
<td>All poultry</td>
<td>0.29</td>
<td>0.53</td>
<td>0.17</td>
</tr>
<tr>
<td>All eggs</td>
<td>0.17</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>Sugars and sweets</td>
<td>0.00</td>
<td>0.15(^2)</td>
<td>0.06(^2)</td>
</tr>
<tr>
<td>Potatoes and sweet potatoes</td>
<td>0.08(^2)</td>
<td>-0.02(^2)</td>
<td>0.00</td>
</tr>
<tr>
<td>Other fresh vegetables(^3)</td>
<td>0.20</td>
<td>0.66</td>
<td>0.19</td>
</tr>
<tr>
<td>Fresh fruit(^4)</td>
<td>0.18</td>
<td>0.14(^2)</td>
<td>0.39</td>
</tr>
<tr>
<td>Frozen fruits and vegetables except frozen potatoes</td>
<td>0.69</td>
<td>0.75</td>
<td>0.36</td>
</tr>
<tr>
<td>Canned fruits and vegetables except potatoes and sweet potatoes</td>
<td>0.25</td>
<td>-0.08(^2)</td>
<td>0.04(^2)</td>
</tr>
<tr>
<td>Fruit and vegetable juices</td>
<td>0.27</td>
<td>0.30</td>
<td>0.13</td>
</tr>
<tr>
<td>Dried fruits and vegetables(^5)</td>
<td>0.15</td>
<td>0.19(^2)</td>
<td>0.02(^2)</td>
</tr>
<tr>
<td>All beverages</td>
<td>0.40</td>
<td>0.51</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Footnotes to Table 1.

1/ Approximately the quantity of fluid milk to which the dairy products are equivalent in calcium.

2/ Not significantly different from zero at the 10 percent level.

3/ Includes home-canned and -frozen vegetables that were brought into the home in fresh form.

4/ Includes home-canned and -frozen fruits that were brought into the home in fresh form.

5/ Includes both commercially and home dried products. Dried weight basis.
Table 2. Income elasticities of demand, based on quantity consumed at home per person, for all households, one week, spring 1955.1

<table>
<thead>
<tr>
<th>Nonfarm households by family income class</th>
<th>Farm households by family income class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Medium</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk and milk products, excluding butter</td>
<td>0.08</td>
<td>-0.072</td>
<td>0.032</td>
</tr>
<tr>
<td>Fats and oils, excluding bacon and salt pork</td>
<td>-0.022</td>
<td>0.082</td>
<td>0.06</td>
</tr>
<tr>
<td>Flour and other cereal products</td>
<td>-0.36</td>
<td>-0.142</td>
<td>-0.07</td>
</tr>
<tr>
<td>All meat</td>
<td>0.27</td>
<td>0.20</td>
<td>0.06</td>
</tr>
<tr>
<td>All poultry</td>
<td>0.17</td>
<td>0.52</td>
<td>0.102</td>
</tr>
<tr>
<td>All eggs</td>
<td>0.12</td>
<td>0.122</td>
<td>0.15</td>
</tr>
<tr>
<td>Sugars and sweets</td>
<td>-0.10</td>
<td>-0.012</td>
<td>0.022</td>
</tr>
<tr>
<td>Potatoes and sweet potatoes</td>
<td>0.012</td>
<td>0.062</td>
<td>0.07</td>
</tr>
<tr>
<td>Other fresh vegetables 3</td>
<td>0.16</td>
<td>0.30</td>
<td>0.11</td>
</tr>
<tr>
<td>Fresh fruit 4</td>
<td>0.23</td>
<td>0.052</td>
<td>0.27</td>
</tr>
<tr>
<td>Frozen fruits and vegetables except frozen potatoes</td>
<td>0.66</td>
<td>0.66</td>
<td>0.39</td>
</tr>
<tr>
<td>Canned fruits and vegetables except potatoes and sweet potatoes</td>
<td>0.20</td>
<td>-0.102</td>
<td>-0.022</td>
</tr>
<tr>
<td>Fruit and vegetable juices</td>
<td>0.21</td>
<td>0.34</td>
<td>0.13</td>
</tr>
<tr>
<td>Dried fruits and vegetables 5</td>
<td>-0.23</td>
<td>0.092</td>
<td>-0.012</td>
</tr>
</tbody>
</table>

Footnotes to Table 2.

1/ Approximately the quantity of fluid milk to which the dairy products are equivalent in calcium.

2/ Not significantly different from zero at the 10 percent level.

3/ Includes home-canned and -frozen vegetables that were brought into the home in fresh form.

4/ Includes home-canned and -frozen fruits that were brought into the home in fresh form.

5/ Includes both commercially and home dried products. Dried weight basis.
### Table 3. Income elasticities of price: income elasticity of value from Table 1 minus income elasticity of quantity from Table 2. \textsuperscript{a/}

<table>
<thead>
<tr>
<th></th>
<th>Nonfarm households by family income class</th>
<th>Farm households by family income class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Milk and milk products, excluding butter\textsuperscript{1/}</td>
<td>0.10</td>
<td>0.03\textsuperscript{2/}</td>
</tr>
<tr>
<td>Fats and oils excluding bacon and salt pork</td>
<td>0.07\textsuperscript{2/}</td>
<td>0.07\textsuperscript{2/}</td>
</tr>
<tr>
<td>Flour and other cereal products</td>
<td>0.23</td>
<td>0.06\textsuperscript{2/}</td>
</tr>
<tr>
<td>All meat</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>All poultry</td>
<td>0.12</td>
<td>0.01</td>
</tr>
<tr>
<td>All eggs</td>
<td>0.05</td>
<td>0.04\textsuperscript{2/}</td>
</tr>
<tr>
<td>Sugars and sweets</td>
<td>0.10</td>
<td>0.16\textsuperscript{2/}</td>
</tr>
<tr>
<td>Potatoes and sweet potatoes</td>
<td>0.07\textsuperscript{2/}</td>
<td>-0.08\textsuperscript{2/}</td>
</tr>
<tr>
<td>Other fresh vegetables\textsuperscript{3/}</td>
<td>0.04</td>
<td>0.36</td>
</tr>
<tr>
<td>Fresh fruit\textsuperscript{4/}</td>
<td>-0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>Frozen fruits and vegetables except frozen potatoes</td>
<td>0.03</td>
<td>0.09</td>
</tr>
<tr>
<td>Canned fruits and vegetables except potatoes and sweet potatoes</td>
<td>0.05</td>
<td>0.02\textsuperscript{2/}</td>
</tr>
<tr>
<td>Fruit and vegetable juices</td>
<td>0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td>Dried fruits and vegetables\textsuperscript{5/}</td>
<td>0.38</td>
<td>0.10\textsuperscript{2/}</td>
</tr>
</tbody>
</table>

Footnotes to Table 3.

1/ Approximately the quantity of fluid milk to which the dairy products are equivalent in calcium.

2/ $E_{XY}$ from Table 1 or $E_{QY}$ from Table 2 or both not significant at the 10 percent level.

3/ Includes home-canned and-frozen vegetables that were brought into the home in fresh form.

4/ Includes home-canned and-frozen fruits that were brought into the home in fresh form.

5/ Includes both commercially and home dried products. Dried weight basis.
Table 4. Comparisons of income elasticities by income groups. a/

<table>
<thead>
<tr>
<th></th>
<th>Farm households</th>
<th>Nonfarm households</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{XY}^{L} &gt; E_{XY}^{M}$</td>
<td>6</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>$E_{XY}^{M} &gt; E_{XY}^{H}$</td>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>$E_{XY}^{L} &gt; E_{XY}^{M}$</td>
<td>6</td>
<td>9</td>
<td>15 37</td>
</tr>
<tr>
<td>$E_{XY}^{L} &lt; E_{XY}^{M}$</td>
<td>5</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>$E_{XY}^{M} &lt; E_{XY}^{H}$</td>
<td>7</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>$E_{XY}^{L} &lt; E_{XY}^{H}$</td>
<td>6</td>
<td>4</td>
<td>10 32</td>
</tr>
<tr>
<td>$E_{QY}^{L} &gt; E_{QY}^{M}$</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>$E_{QY}^{M} &gt; E_{QY}^{H}$</td>
<td>5</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>$E_{QY}^{L} &gt; E_{QY}^{H}$</td>
<td>4</td>
<td>8</td>
<td>12 32</td>
</tr>
<tr>
<td>$E_{QY}^{L} &lt; E_{QY}^{M}$</td>
<td>5</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>$E_{QY}^{M} &lt; E_{QY}^{H}$</td>
<td>5</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>$E_{QY}^{L} &lt; E_{QY}^{H}$</td>
<td>5</td>
<td>6</td>
<td>11 31</td>
</tr>
</tbody>
</table>

a/ Excluding "all foods and beverages" value. $E_{XY}$ = income elasticity of value; $E_{QY}$ = income elasticity of quantity consumed. Superscripts L, M and H represent low, medium and high income families. Nonsignificant values in Tables 1 and 2 assumed to be zero.
Table 5. Comparison of farm and nonfarm income elasticities within comparable income groups. a/

<table>
<thead>
<tr>
<th></th>
<th>Low income families</th>
<th>Medium income families</th>
<th>High income families</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{QY}^N &gt; E_{QY}^F$</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>$E_{QY}^N &lt; E_{QY}^F$</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>$E_{XY}^N &gt; E_{XY}^F$</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>$E_{XY}^N &lt; E_{XY}^F$</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

a/ Excluding "all foods and beverages" value. $E_{XY}$ = income elasticity of value; $E_{QY}$ = income elasticity of quantity. Superscripts N and F refer to nonfarm and farm families. Nonsignificant values in Tables 1 and 2 assumed to be zero.
Q = quantity exchanged,

P_f = farm price,

Z_s = other factors affecting supply,

P_r = retail price,

Z_m = other factors affecting marketing firm's behavior,

Y = consumer income,

Z_D = other factors affecting consumer demand.

If M and D are of such form that it is possible to eliminate P_r, the derived demand equation can be obtained from them as m(Q, P_f, Y, Z_D, Z_M).

Consumer income elasticity of demand is \( E_{QY} = \frac{Y}{Q} \left( \frac{\delta Q}{\delta Y} \right) \). Derived income elasticity of demand is \( e_{QY} = \frac{Y}{Q} \left( \frac{\delta Q}{\delta Y} \right) \). The relation between the two is 10

\[
(14) \quad e_{QY} = \frac{E_{QY}}{1 - \left( \frac{\delta P_r}{\delta Q} \right) \left( \frac{\delta Q}{\delta P_r} \right) M D}
\]

We are interested in the relation between changes in \( E_{QY} \) and changes in \( e_{QY} \).

Define:

\[
(15) \quad E(e_{QY}/E_{QY}) = \frac{\partial e_{QY}}{\partial E_{QY}} \frac{E_{QY}}{e_{QY}}
\]

Then

\[
(16) \quad \frac{\partial e_{QY}}{\partial E_{QY}} = \left[ 1 - \left( \frac{\partial P_r}{\partial Q} \right) \left( \frac{\delta Q}{\delta P_r} \right) M D \right] + \left[ E_{QY} \left( \frac{\delta P_r}{\delta Q} \right) \frac{\delta Q}{\delta P_r} M \frac{\delta E_{QY}}{\delta Q} \right]
\]

\[
= (1 - AB) + \left( E_{QY} AC \right)
\]

\[
= \frac{(1 - AB) + \left( E_{QY} AC \right)}{(1 - AB)^2}
\]
If \( Q \) does not appear in \( M \), i.e., the retail price firms desire to charge (and the desired marketing margin) is unaffected by the quantity passing through the marketing sector,

\[
E(e_{QY}/E_{QY}) = \frac{1 + AC \cdot e_{QY}}{1 - AB}
\]

If \( Q \) appears in \( M \), we must consider what happens to \( C \), the slope with respect to price of the demand curve, as income rises and income elasticity falls. It seems reasonable to believe that \( C \) will be much less than one in absolute value, if not zero. There is evidence that \( e_{QY} \) is much less than one for most farm products. If \( A \) is also less than one, \( AC \cdot e_{QY} \) is the product of three fractions, all small, and will be almost zero. Generally \( B < 0 \) and \( A > 0 \). Hence, \( E(e_{QY}/E_{QY}) < 1 \) and derived income elasticity at the farm level declines at a slower rate than consumer income elasticity.

It is tempting to follow this discussion with a discussion of the effect of changes in \( Y \) and \( E_{QY} \) upon \( E_{QP} \), the price elasticity of demand. This question is probably more important to marketing firms than is the question of changes in \( E_{QY} \). I know of nothing in either our theoretical or empirical literature which casts any light on this question.

**Demand for Marketing Services**

The previous section was devoted to a discussion of one common presumption which has not received adequate investigation. This section is devoted to a brief look at another presumption which requires investigation. Actually, it deals with two related ideas which have significance for primary agricultural adjustment and for marketing and processing firms.

One is the hypothesis that we have reached sufficiently high levels of consumer income in this country that income elasticity of demand for food (or farm food products) is less than the income elasticity of demand for food marketing services. The other is the hypothesis that price elasticity of demand for services exceeds the price elasticity of demand for farm foods.
Daly,¹¹/ Burk¹²/ and Bunkers and Cochrane¹³/ have used national time series data to test the first hypothesis, and each has accepted it. Unfortunately, even though this hypothesis has great intuitive and introspective appeal to me, I cannot accept their conclusions since I cannot accept their methods. Briefly, my objections to their work are that: (1) the series they use to represent "quantity of marketing services" are not measures of quantity of marketing services and (2) their series on "price index for marketing services" does not measure the price of marketing services.¹⁴/

So far as I know, no work has been done to test the second hypothesis. It, too, has intuitive appeal for me.

Perhaps we can get some insight into these two hypotheses from some qualitative considerations. In the next few paragraphs, let us use food to mean a physical item produced on a farm and possessing nutritive value. It is evident that food as such has few if any closely competitive goods, although many foods are substitutes for each other. It is also a product for which humans have a saturation level of consumption beyond which they will not go.

Marketing services, on the other hand, have a number of competitors. An important function performed by marketing services, perhaps the most important one in the mind of the housewife, is the saving in time and effort made possible by their use. This is also the main function performed by many other products and services the housewife can buy. Consequently, marketing services are competitive with all sorts of things, ranging from a second car in the family to electrically operated swizzle sticks and including housemaids and most all electrical appliances. Marketing services are also competitive with leisure time products and services such as books or fishing


tackle and movies. Meal preparation and leisure time activities are alternative uses of the housewife's time. Further, most people in this country come nearer to achieving a satiation level of food consumption than a satiation level of leisure time. Related to the idea of satiation is the fact that food -- nutrition -- is a physical necessity, whereas marketing services performed by businessmen are not. (The qualifier "performed by businesses" is included to denote that the services may be necessary, but need not be done by firms.)

These considerations lead me to expect that there is a real consumer income level above which the income elasticity of demand for services exceeds the income elasticity of demand for food. The way this manifests itself at the retail level is in a higher income elasticity of demand for those products which combine larger amounts and/or more kinds of services with given amounts of food. What this income level is and what proportion of the families in this country have higher incomes, I do not even hazard a guess. These same considerations also lead me to expect that price elasticity is greater for those products containing more services than for those containing less.

If price and income elasticities of demand are higher for products containing a larger proportion of marketing services, the result might be to raise derived price and income elasticities of demand at the farm while reducing the level of derived demand.

Rewrite equation (14) as

\[ e_{QY} = \frac{E_{QY}}{1 - e_{QY} E_{Qr}} \]

where

\[ e_{Qr} = \frac{Q}{P_r} \left( \frac{\partial P_r}{\partial Q} \right)_M \]

and

\[ E_{Qr} = \frac{P_r}{Q} \left( \frac{\partial Q}{\partial P_r} \right)_D \]

The relation between the price elasticities of derived and consumer demand is

\[ e_{Qf} = \frac{E_{Qr} e_{rf}}{1 - e_{Qr} E_{Qr}} \]

\[ e_{rf} \] Hildreth and Jarrett, op. cit.
where \( e_{rf} = \frac{P_f}{P_r} \left( \frac{\partial P_f}{\partial P_r} \right)_M \).

\( e_{rf} \) has been referred to as the elasticity of price transmission. For a constant percentage margin \( e_{rf} = 1 \); for a constant dollar margin or a combined constant dollar and constant percentage margin, \( e_{rf} \ll 1 \).

Let us assume the combined case and let \( k \) = the constant dollar portion of the margin. As a growing portion of services is combined with a given amount of farm food products, \( k \) can be expected to increase. As \( k \) increases, \( e_{rf} \) falls. If it is assumed, as was suggested above, that increasing \( k \) also increases consumer price and income elasticities, the relation between \( k \) and \( e_{QY} \) is

\[
(20) \quad \frac{\partial e_{QY}}{\partial k} = \frac{D + E_{QY} e_{Q} \frac{\partial E_{QY}}{\partial k} + \frac{\partial E_{QY}}{\partial Q_{QY}}}{D^2} + \frac{D \frac{\partial E_{Qr}}{\partial Q_{QY}} + E_{QY} e_{Qr} \frac{\partial E_{Qr}}{\partial k}}{D^2} + \frac{E_{QY} E_{Qr}}{D^2} \frac{\partial e_{Qr}}{\partial k}.
\]

If \( \frac{\partial E_{QY}}{\partial E_{Qr}} = \frac{\partial E_{Qr}}{\partial k} = 0 \), this reduces to

\[
(21) \quad \frac{\partial e_{QY}}{\partial k} = \frac{1}{D} \frac{\partial E_{QY}}{\partial k} + \frac{E_{QY} e_{Qr} \frac{\partial E_{Qr}}{\partial k} + E_{QY} E_{Qr} \frac{\partial e_{Qr}}{\partial k}}{D^2}.
\]

where \( D = 1 - e_{Q} E_{Qr} \).

If we were dealing with a purely competitive marketing system, \( e_{Q} \) would represent the reciprocal of the elasticity of supply. Intuitively it seems like \( \frac{\partial e_{Qr}}{\partial k} \) \(<0 \) since an increase in \( k \) corresponds to an increase in marginal cost. On the above assumptions, the first and last terms in (21) are positive and the second one is negative, and the sum may be positive or negative.
The relation between \( k \) and \( e_Qf \) is

\[
\frac{\partial e_Qf}{\partial k} = \frac{1}{D^2} \frac{\partial E_{Qr}}{\partial k} + \frac{DE_{Qr}}{D^2} \frac{E_{Qr}^2 e_{rf}}{D} \frac{\partial e_{rf}}{\partial k} + \frac{DE_{Qr} e_{rf} \partial e_{rf}}{D^2} \frac{\partial e_{rf}}{\partial k} + \frac{DE_{Qr} e_{rf}}{D} \frac{\partial e_{rf}}{\partial k} \]

If \( \frac{\partial e_{rf}}{\partial e_{Qr}} = \frac{\partial e_{rf}}{\partial e_{Qr}} = 0 \), (22) reduces to

\[
\frac{\partial e_Qf}{\partial k} = \frac{1}{D^2} \frac{\partial E_{Qr}}{\partial k} + \frac{DE_{Qr} e_{rf}}{D} \frac{\partial e_{rf}}{\partial k} + \frac{1 + E_{Qr} e_{Qf}}{D} \frac{\partial e_{Qf}}{\partial k}
\]

On the above assumptions, the first and last terms are negative and the second is positive.

**Advertising**

The last subject I plan to deal with in this paper is the effect of commodity advertising on consumption of farm foods and on farm income. Recent work by Basmann gives some theoretical insight into the problem. He assumes a consumer utility function

\[
u = u(x_1, x_2, \ldots, x_n; \theta_1, \theta_2, \ldots, \theta_n)
\]

in which the \( x_i \) represent quantities of goods and services consumed during a given time period. The \( \theta_i \) are parameters which describe the form of the ordinal utility function; the values of the \( \theta_i \) are assumed to depend on the \( a_j \), where \( a_j \) denotes the advertising expenditures on commodity \( x_j \). Maximizing (24) subject to the budget restraint yields the usual first and second order conditions for individual consumer equilibrium. Differentiating the

equilibrium conditions with respect to $a_j$ and solving the resulting system of equations yields an expression for shifts in demand with respect to advertising expenditures.

$$\frac{dx_i}{da_j} = -\sum_{h=1}^{n} b_{hj} s_{hi} P_h$$

where $s_{hi}$ is the substitution term $s_{hi} = \frac{U_{hi}}{U}$, $P_h$ is the price of product $h$ and $b_{hj} = \frac{\partial u_h}{\partial a_j}$. $u_h$ is the proportionate change in the marginal utility of $x_h$ with respect to advertising expenditures on $x_j$.

If it is assumed that $a_j$ affects only the marginal utility of $x_j$, $b_{jj} > 0$ and $b_{hj} = 0$ for all $h \neq j$. Then

$$\frac{dx_i}{da_j} = -b_{jj} s_{ji} P_j$$

If $x_i$ and $x_j$ are substitutes, $s_{ji} > 0$ and $\frac{dx_i}{da_j} < 0$; if $x_i$ and $x_j$ are complements, $s_{ji} < 0$ and $\frac{dx_i}{da_j} > 0$. From equation (26) the change in market demand is obtained by summing over all individuals in the market $\sum_r \frac{dx_{ir}}{da_j} = dX_j$.

To illustrate how these results might be applied to the study of farm commodity advertising, let us take a simple case. Assume (26) and its underlying assumptions are appropriate. Assume $p_j$ a constant in order that we need not consider the elasticity of supply nor the partial derivative of (26) with respect to $p_j$. The results to be obtained would be applicable to this sort of a question: "Farm and retail prices of product $j$ are now satisfactory or profitable, but we know production is going to increase by about $dx_j$. How much will need to be spent on advertising in order to maintain farm and retail prices of product $j$ at their present level?" This is, we want

$$\sum_r \frac{dx_{ir}}{da_j} = -p_j \sum_r b_{jjr} s_{jjr} = dX_j$$

Assume

$$-p_j \sum_r b_{jjr} s_{jjr} \approx -Np_j b_{jj} s_{jj}$$

that is, the sum of the products can be closely approximated by the product of $N$ and the two means, $\bar{b}_{jj} = \frac{1}{N} \sum_r b_{jjr}$ and $\bar{s}_{jj} = \frac{1}{N} \sum_r s_{jjr}$. Equating

$$\frac{d \sum_r x_{jr}}{da_j} = dX_j$$

$$-Np_j \bar{b}_{jj} \bar{s}_{jj} \frac{dx_j}{da_j} = dX_j$$

(29)
or
\[ \partial (30) \quad a_j = - \frac{dX_j}{Np_j b_{jj} \bar{s}_{jj}}. \]

As would be intuitively reasonable, the larger that \( b_{jj} \) is, the smaller the advertising expenditure need be. (30) also shows that the larger the substitution term, the smaller the advertising expenditure need be.

One other question that would be relevant is this: "Will a net increase in farm income be obtained?" That is we want

\[ \partial (31) \quad \frac{\partial Y_F}{\partial a_j} \geq 0 \]

where \( Y_F = \sum_i (p_i^F - c_i - a_i) X_i \)

and \( p_i^F = \) farm price of product \( i \)

\( c_i = \) farm cost of producing one unit of product \( i \)

\( a_i = \) farm expenditures on advertising product \( i \) divided by the total quantity of \( i \).

\[ \partial (32) \quad \frac{\partial Y_F}{\partial a_j} = -Np_j b_{jj} \sum_i (p_i^F - c_i - a_i) s_{ji} - X_j \geq 0. \]

Solving for the break-even value of \( \bar{b}_{jj} \),

\[ \partial (33) \quad \bar{b}_{jj} = \frac{X_j}{Np_j \sum_i (p_i^F - c_i - a_i) \bar{s}_{ji}}. \]

This analysis is terribly oversimplified. It assumes infinite supply elasticities and constant marketing margins. The existence of variable margins can be taken into account fairly easily in the analysis, but the existence of non-infinite supply elasticities is more difficult to incorporate into the analysis.

At present we have no empirical information on \( \bar{b}_{jj} \) or on \( b_{jir} \). We can, however, obtain some information on the \( s_{ji} \) fairly readily. From the theory of consumer behavior,

\[ \partial (34) \quad \frac{\partial x_{ir}}{\partial p_j} = -x_{jr} \quad \frac{\partial x_{ir}}{\partial Y_r} + s_{jir}. \]
Aggregating over all individuals, dividing by population, and replacing the first term on the right hand side by an approximation,

\[ (35) \frac{\partial}{\partial p_j} \frac{\sum r x_{ir}}{N} = - \frac{\partial}{\partial Y} \frac{\sum r x_{ii}}{N} + \frac{\sum r s_{ji}}{N}. \]

From empirical demand analysis, estimates of \( \frac{1}{N} \frac{\partial}{\partial p_j} \sum r x_{ir} \) and \( \frac{\partial}{\partial Y} \sum r x_{ii} \) can be obtained and (34) can be used to estimate average substitution terms \( \bar{s}_{ji} \).

Having estimates of \( \bar{s}_{ji} \) and \( \bar{s}_{ji} \) and assuming \( dX_j \) is not so large as to result in a change in any of the substitution terms, equation (33) could be solved to obtain the break-even value of \( b_{jj} \). This value of \( b_{jj} \) and larger ones could be checked only on intuitive grounds. If the break-even value or larger values appeared "reasonable" they could be substituted into (30) to obtain an estimate of the necessary advertising expenditure and into (32) to find their effect on farm income.

A more relevant question with respect to income would be, "Will a net increase in income be obtained by the producers of \( X_j \)?" To answer this question would require a more complicated analysis. It would require classifying the producers of \( X_j \) into at least four groups: (1) producers of \( X_j \) only or of \( X_j \) and independent products (\( s_{ji} = 0 \)); (2) producers of \( X_j \) and of substitutes (\( s_{ji} > 0 \)) only; (3) producers of \( X_j \) and of complements (\( s_{ji} < 0 \)) only; (4) producers of \( X_j \), complements and substitutes.