INTRODUCTION

Current nondestructive evaluation techniques generally do not produce identical indications when applied to flaws of the same length. The chance of detecting a given crack length depends on many factors, such as the location, orientation and shape of the flaw, materials, inspectors, inspection environments, etc. As a result, the probability of detection (POD) for all cracks of a given length has been used in the literature to define the capability of a particular NDE system in a given environment. Some POD curves are shown in Fig. 1 for various laboratory inspection techniques. Many other POD curves can be found, for instance, in Refs. 1-3.

In practical applications, a nondestructive inspection limit, \( a_{\text{NDE}} \), is chosen, which is a crack length that usually corresponds to a high detection probability and a high confidence level. For instance, the damage tolerant specification \([4, 5]\) requires that \( a_{\text{NDE}} \) should be the crack length associated with a 90% detection probability and 95% confidence level. The fracture mechanics residual

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life, \( N_f \), is the life for the crack length \( a_{\text{NDE}} \) to propagate to the critical crack length \( a_c \) under expected usage environments as shown in Fig. 2(a). The return to service interval, denoted by \( N_R \) is equal to \( N_f \) divided by a safety factor \( S_f \), i.e., \( N_R = N_f / S_f \). If no crack is detected during inspection, the component is returned to service and the crack length in that component is reset to be equal to \( a_{\text{NDE}} \) as shown in Fig. 2(b). Hence the inspection limit \( a_{\text{NDE}} \) is also referred to as the reset crack length. In the damage tolerant analysis, a safety factor of 2.0 has been used.

It follows from Fig. 1 that the NDE reliability consists of two types of wrong indications; (i) failure to give a positive indication in the presence of a crack whose length is greater than \( a_{\text{NDE}} \), referred to as Type I error, and (ii) given a positive indication when the crack length is smaller than \( a_{\text{NDE}} \), referred to as Type II error. For safety critical components in airframe structures, Type I error is of primary concern. In the Retirement-For-Cause (RFC) analysis of gas turbine engine components, however, both Type I and Type II errors are important, because the criterion used in RFC analyses is the minimization of the life cycle cost (LCC) for engine components [e.g., 6-7]. As a result, the reduction for both types of error is one of the main objectives of the present study.

The Type I error allows the components containing a crack length longer than \( a_{\text{NDE}} \) to return to service, thus greatly increasing the potential safety hazard. For a given NDE system the Type I error can be made as small as possible by choosing a large value for the inspection limit \( a_{\text{NDE}} \). However, as the value of \( a_{\text{NDE}} \) increases the return to service interval \( N_R \) reduces thus increasing the frequency

![Fig. 1. POD curves for various NDE systems.](image-url)
or cost of inspections. Moreover, while the Type I error can be reduced by increasing the value of $a_{\text{NDE}}$, the Type II error increases accordingly as shown in Figs. 3(a) and 3(b).

The type II error rejects good components and hence has an adverse effect on the cost of replacement and the life cycle cost. For a given NDE system with a single inspection, it is impossible to reduce the Type II error without increasing the Type I error and vice versa. It is obvious then that the ideal inspection capability of an NDE system is a unit step function as shown in Fig. 3(c), in which both Type I and Type II errors are zero. Unfortunately, such an ideal NDE system may be far from reality.

There are situations where the critical crack size is small, such that the required $a_{\text{NDE}}$ value with high levels of detection probability and confidence may not be achieved by an NDE system. It is also conceivable that a given NDE system at depot level may
not be able to reduce the life cycle cost such that it may be more economical to just replace the components at overhaul without inspections. In order to circumvent those difficulties, possible applications of multiple inspections are studied herein.

The POD curve of an NDE system applies to the inspection of a critical location only. For an engine disk containing a large number of critical locations to be inspected, including bolt holes, cooling holes, rim holes, etc., the inspection reliability for a disk differs significantly from that for a hole, because a disk is retired if at least one hole is rejected by the NDE system. Applications of the present results to the inspection reliability of
engine disks were documented in Ref. 8 as will be discussed later.

The objectives of this paper are (i) to formulate and derive mathematically the resulting POD curve for components under multiple inspections, (ii) to investigate quantitatively the potential benefit of multiple inspection procedures, (iii) to establish the direction in which the capability of an NDE system should be improved, and (iv) to establish the strategy and sequence for multiple inspections in order to reduce either Type I error or Type II error or both. Examples are given to demonstrate the proposed multiple inspection approach. Applications of the present approach to the Retirement-For-Cause of gas turbine engine disks are also discussed.

THEORY OF MULTIPLE INSPECTIONS

In the literature, the POD curve for a particular NDE system has been established from repeated inspection data in which the inspector may or may not be the same. Then the inspection data set is analyzed using either the Binomial method [e.g., 2-3] or the method of regression analysis [9]. It should be emphasized, however, that both Binomial and regression methods assume explicitly that the result of each inspection using the same NDE system (whether the same inspector or not) is statistically independent of the other (i.e., independent sampling). Thus all the POD curves available in the literature are based on the premise that the results of multiple inspections with the same NDE system are statistically independent. Such an assumption may be subject to criticisms. However, in order to be consistent with current practice and to use available POD curves, the assumption will be employed first in the following formulation. The mathematical formulation in which the results of multiple inspections are not statistically independent is given in Appendix I. The issue of dependent and independent multiple inspections along with the application to engine or structural components will be discussed later.

The formulation and solution will be established in an appropriate perspective for multiple inspections. With the mathematical solution put into appropriate setting, one can manipulate various NDE capabilities (or POD curves) to achieve a most economical multiple inspection system in terms of Type I error, Type II error and both.

Let the following POD curves associated with m inspection systems be given;

\[
\text{POD}(a;1) = \text{probability of detecting the crack length, } a, \text{ under No. 1 NDE system;}
\]

\[
\text{POD}(a;j) = \text{probability of detecting the crack length, } a, \text{ under No. j NDE system; } j=1,2,\ldots,m.
\]
In general, POD(a;i) is different from POD(a;j). However, they may also be identical as a special case. Two basic inspection rules and their combinations are described in the following:

**Union Rule**

A component (or a critical location) is rejected if it is rejected by either one of the NDE systems employed as shown in Fig. 4.

Let POD(a;1U2) = probability that the crack length, a, is detected by either No. 1 NDE system or No. 2 NDE system or both. Then, based on the assumption that the inspections by both NDE systems are statistically independent, the resulting POD curve for the component is given by

\[
POD(a;1U2) = 1 - [1-POD(a;1)][1-POD(a;2)]
\]

or

\[
POD(a;1U2) = POD(a;1) + POD(a;2) - POD(a;1)POD(a;2)
\]

In a similar fashion, the probability of detection (POD) under m NDE systems, denoted by \( P(a; \bigcup_{j=1}^{m} j) \) is obtained as

\[
POD(a; \bigcup_{j=1}^{m} j) = 1 - \prod_{j=1}^{m} [1-POD(a;j)]
\]

In general, the application of multiple inspections using the union rule will reduce the Type I error and the inspection limit \( a_{NDE} \), thus improving the structural safety and reliability. However, it may also increase the Type II error; the extent of which depends on the particular functional form of POD(a;j); j=1,2,...,m. An exceptional case is that if POD(a;2) has a lower bound at \( a_{NDE} \), i.e., POD(a;2) is equal to zero for \( a\leq a_{NDE} \), then there is no increase in the Type II error under two inspections. A schematical flow chart for the inspection procedures using two NDE systems and the union rule is shown in Fig. 5.

![Fig. 4. Rejected components shown in shaded areas.](image)
Fig. 5. Inspection procedure for two NDE systems with union rule.

**Intersection Rule**

A component (or a critical location) is rejected if and only if it is rejected by all the NDE systems employed as shown in Fig. 4.

If \( \text{POD}(a;1\cap 2) \) denotes the probability of detecting the crack length, \( a \), by both No. 1 and No. 2 NDE systems, then we have

\[
\text{POD}(a;1\cap 2) = \text{POD}(a;1) \cdot \text{POD}(a;2)
\]

(4)

In a similar manner, the probability of detecting the crack length, \( a \), by every one of \( m \) NDE systems employed is given by

\[
\text{POD}(a; \cap_{j=1}^{m} j) = \prod_{j=1}^{m} \text{POD}(a;j)
\]

(5)

In general, the application of multiple inspections using the intersection rule alone will degrade the NDE capability. It is precisely due to such a property that the Type II error can be reduced. However, caution should be taken such that the degradation for the Type I error will be insignificant. For instance, using two inspection systems the condition for not having a serious adverse effect on the Type I error is that \( \text{POD}(a;2) \) should be very close to unity at \( a = a_{\text{NDE}} \) as will be described later. If \( \text{POD}(a;2) \) has an upper bound at \( a_{\text{NDE}} \), i.e., \( \text{POD}(a;2) = 1 \) for \( a \geq a_{\text{NDE}} \), then the Type I error
will not be effected. A flow chart for the inspection procedure using two NDE systems and the intersection rule is shown in Fig. 6.

Combination Rule for Three Inspections

As described previously, the application of either the union rule or the intersection rule alone can not reduce both Type I and Type II errors simultaneously. However, a combined use of both union and intersection rules along with an appropriate choice of POD(a;3) can result in a simultaneous improvement for both types of error. Because of practical limitations, such as the facility and inspection cost, we shall describe only a possible combination of union and intersection rules using three inspections as follows.

Let POD[a;(1U2)∩3] be the probability of detecting the crack length, a, under three inspections, where the union rule is applied to No. 1 and No. 2 NDE systems and the intersection rule is employed for the No. 3 NDE system. The resulting POD curve from the application of No. 1 and No. 2 NDE systems, POD(a;1U2), is given by Eq. 2. Hence, it follows from Eq. 4 that

\[
POD[a;(1U2)∩3] = POD(a;3)POD(a;1U2)
\]

Substituting Eq. 2 into Eq. 6, one obtains

\[
POD[a;(1U2)∩3] = POD(a;3)[POD(a;1) + POD(a;2) - POD(a;1)POD(a;2)]
\]

Fig. 6. Inspection procedure for two NDE systems with intersection rule.
Fig. 7. Inspection procedure for three NDE systems with union-intersection rule.

The inspection procedure for three NDE systems presented in Eq. 7 is shown by a flow chart in Fig. 7 and explained as follows. After the first inspection by the No. 1 NDE system, components are divided into two populations: an accepted one and a rejected one. The accepted population is further inspected by the No. 2 NDE system (to reduce Type I error) and the accepted components are returned to service. The components rejected by the No. 2 NDE system along with the components rejected by the No. 1 NDE system are then inspected by the No. 3 NDE system (to reduce Type II error). Then the rejected components (by No. 3 NDE system) are replaced or repaired and the accepted components are returned to service. It should be mentioned that POD(a;3) for the third NDE system should be very close to 1.0 (such as 0.99) at \( a=a_{NDE} \) so that the adverse effect on the Type I error is minimal. Likewise, the bandwidth of POD(a;3) should be as narrow as possible in order to reduce the Type II error effectively. Inspections using such an NDE system may be expensive and time
consuming, but the number of components to be inspected by the No. 3 NDE system may be small. This will be discussed later.

If \( m \) NDE systems are employed, then there are many different combinations of union and intersection rules that can be investigated. However, the basic mathematical approach to derive the resulting POD curve is the same as that described previously. For instance, for a special case in which \( \text{POD}(a;1) = \text{POD}(a;2) = \text{POD}(a;3) \), then the resulting POD curve using the union rule is given by

\[
\text{POD}(a; U_j) = 1 - [1 - \text{POD}(a;1)]^3, \quad j=1
\]

When the fourth NDE system with \( \text{POD}(a;4) \) is further applied to inspect those rejected components using the intersection rule, the resulting POD curve is given by

\[
\text{POD}[a;( U_j) \cap 4] = \text{POD}(a;4) \text{POD}(a; U_j), \quad j=1, \ldots, 3
\]

in which \( \text{POD}(a; U_j) \) is given by Eq. 8.

**Inspection Sequence for Minimum Number and Cost of Inspections**

The resulting POD curves under multiple inspections derived in the previous sections are independent of the sequence (or order) of applications of multiple NDE systems. However, the number of inspections required for components and hence the cost of inspection is indeed influenced by the sequence of inspections. For the case of two inspections with the union rule as shown in Fig. 5, \( \text{POD}(a;1) \) should be better than \( \text{POD}(a;2) \) such that fewer components will be accepted by the No. 1 NDE system. Hence the number of components to be inspected by the No. 2 NDE system is minimal. In general, for multiple inspections with the union rule, the NDE system with the highest resolution capability should be applied first in order to minimize the subsequent number of inspections. However the minimization of the number of inspections implies the minimization of the inspection cost only when the cost per inspection for both No. 1 and No. 2 NDE systems is almost the same. If there is a difference in the inspection cost for two NDE systems, then the system with the lower inspection cost should be the No. 1 NDE system. This is because the No. 1 NDE system has to inspect all the components, whereas only those components accepted by the No. 1 NDE system will be inspected by the No. 2 NDE system.

On the other hand, for the case of two inspections using the intersection rule as shown in Fig. 6, \( \text{POD}(a;2) \) should be better
than POD(a;1), so that the number of components rejected by the No. 1 system, which should be inspected by the No. 2 NDE system, will be minimal. For the case of three inspections using the union-intersection rule, the number of components to be inspected by the No. 3 NDE system is independent of the inspection sequence. However, for the first two inspections using the union rule, POD(a;1) should be better than POD(a;2) in order to achieve a minimum number of inspections.

Again, depending on the cost of inspection for each NDE system, the rationale described previously can be applied to minimize the inspection cost.

Correlated Multiple Inspections

The solutions obtained above for the resulting POD curves under multiple inspection procedures are based on the assumption that inspection results from multiple NDE systems are statistically independent. This assumption is consistent with the current practice for establishing the POD curve for each NDE system in which independent sampling has been assumed, i.e., each inspection result is assumed to be statistically independent.

It is more economical to perform multiple inspections using the same NDE system. However, the question of whether the results of multiple inspections using the same NDE system but under different inspection conditions will be independent or correlated has not yet been fully resolved. Take eddy-current inspection for instance. The following conditions may be different in multiple inspections; inspector, gain of NDE signal, scanning speed for inspection, position of probe, signal data processing, surface preparation of inspected parts, loading condition of parts, the same system at different locations or facilities, etc.

For a fully automated NDE system currently under development for the RFC system, it is anticipated that the error or uncertainty due to human operation and others will be greatly reduced, and the systematic error or uncertainty of the NDE system itself will prevail. Thus multiple inspections using such a fully automated NDE system alone for a single location may be highly correlated if the preparation for the surface condition of the location is identical.

The mathematical solutions for correlated inspections are presented in Appendix I. With correlated inspection systems, however, additional POD information is needed. For instance, the conditional probability of detecting the crack length, a, by the No. 1 NDE system under the condition that the crack has been detected by the No. 2 NDE system is required.
Application of Independent Inspections

One of the motivations of the present investigation is its application to the Retirement-For-Cause of gas turbine engine components, such as disks. A disk usually contains many holes, such as bolt holes, cooling holes and rim holes, in which cracks may occur. Since the crack in each hole may have a different length, orientation and geometry, the inspection of one hole can be assumed to be statistically independent of the inspection of another hole. Likewise, in order to reduce Type II error, the rejected holes may be cleaned, polished or even replicated, in which case a high resolution capability for the POD curve can be achieved. It is reasonable to assume that the inspection for a hole with replication is statistically independent of the inspection for the same hole without replication even when the same NDE system is used. The application of the present results to RFC of engine components is presented in Reference 8.

Type I and Type II Errors

The effect of multiple inspections on the POD curve, Type I and Type II errors, and the inspection limit $a_{\text{NDE}}$ will be demonstrated later. While the capability of a particular NDE system is defined by its POD curve, the Type I and Type II errors depend not only on the POD curve itself but also on the preinspection distribution of the flaw length in the component. For instance, if all the crack lengths in the component prior to inspection are smaller than $a_{\text{NDE}}$, then the Type I error is zero.

Let $F(a)$ and $f(a)$ be the distribution function and the probability density function, respectively, of the flaw length in the component prior to inspection. Then the Type I and Type II errors, denoted by $P_I$ and $P_{II}$, respectively, are given by

$$P_I = \int_{a_{\text{NDE}}}^{\infty} f(x)[1-\text{POD}(x)]dx \quad (10)$$
$$P_{II} = \int_0^{a_{\text{NDE}}} f(x)\text{POD}(x)dx \quad (11)$$

in which

$$P_I = \text{probability of missing (or accepting) a crack length longer than } a_{\text{NDE}},$$
$$P_{II} = \text{probability of detecting (or rejecting) a crack length smaller than } a_{\text{NDE}},$$

and POD(a) is the POD curve of a particular NDE system.

Both $P_I$ and $P_{II}$ given by Eqs. 10 and 11 are the quantitative measures of Type I and Type II errors. Two qualitative measures of
Type I and Type II errors which depend exclusively on the POD curve may also be appealing,

\[ A_I = \int_{a_{\text{NDE}}}^{\infty} [1 - \text{POD}(x)] dx \]  
\[ A_{II} = \int_0^{a_{\text{NDE}}} \text{POD}(x) dx \]

It is apparent from Eqs. 12 and 13 that \( A_I \) is the area above the POD curve from \( a_{\text{NDE}} \) to infinity and \( A_{II} \) is the area under the POD curve from zero to \( a_{\text{NDE}} \), as shown in the shaded areas of Fig. 3. While \( A_I \) and \( A_{II} \) are not the quantitative measures of Type I and Type II errors, they may serve for qualitative comparisons between the capability of various NDE systems when the value of the inspection limit \( a_{\text{NDE}} \) is fixed.

When multiple inspections are employed, the POD(a) function appearing in Eqs. 10-13 represents the resulting POD curve derived in Eqs. 1-9. Hence it should be replaced by the appropriately corresponding POD curve resulting from multiple inspections.

**Distribution of Preinspection Flaw Length**

Theoretically, the distribution of the flaw length for a component prior to inspection can be derived from the distributions of (i) time to crack initiation (or equivalent initial flaw length), (ii) crack growth rate in service, and (iii) service loads. Such an approach, however, is very complex. From the NDE standpoint, it is reasonable to assume the Weibull distribution for the preinspection flaw length for illustrative purposes. Available information on the statistical variabilities of the equivalent initial flaw length, the crack propagation rate and the service loads for engine disks indicates that the statistical dispersion (coefficient of variation) of the crack length in service is of the order of 100%. The Weibull distribution with a 100% coefficient of variation degenerates into a special case of the negative exponential distribution. As a result, the distribution of the preinspection flaw length will be assumed to follow the negative exponential distribution in the present study, i.e.,

\[ f(a) = \lambda e^{-\lambda a} \quad a \geq 0 \]
\[ F(a) = 1 - e^{-\lambda a} \quad a \geq 0 \]

in which \( f(a) \) and \( F(a) \) are, respectively, the probability density function and the distribution function of the preinspection flaw length. In Eq. 14, \( 1/\lambda \) represents the average flaw length.
NUMERICAL EXAMPLE

Eddy current inspection data for fastener holes in skin and stringer wing assembly, referred to as HAVE CRACK, were available in Ref. 3 and analyzed by the Binomial method. The same data set was further analyzed in Ref. 9 using the regression method with the assumed functional form for the POD curve as follows;

\[
POD(a;1) = \frac{\exp(\alpha + \beta/na)}{1 + \exp(\alpha + \beta/na)}
\]

in which \(\alpha = -2.9\) and \(\beta = 1.7\) [see Fig. 8]. This POD curve is replotted in Fig. 9(a) as Curve 1. The crack length associated with 90\% and 96\% detection probabilities are, respectively, 20.05 mm and 35.7 mm.

The data set used to establish the POD curve was generated using the same cracked specimens but inspected by different inspectors and NDE systems at different locations. If the components are inspected twice at different locations, i.e., POD(a;1)=POD(a;2), the resulting POD curve, i.e., POD(a;1U2), with the union rule is shown as Curve 2 (Eq. 2) in Fig. 9(a). One can further improve the Type I error by performing a third inspection using the union rule. With POD(a;1)=POD(a;2)=POD(a;3) the resulting POD(a;1U2U3), Eq. 3, is displayed in

Fig. 8. Eddy current inspections of skin and stringer wing assembly, 60 inspections per fastener hole [from Refs. 3 and 9].
Fig. 9(a) as Curve 3. If the inspection limit, $a_{NDE}$, is required to be 12.4 mm, i.e., $a_{NDE}=12.4$ mm, then $A_I$ and $A_{II}$ (Eqs. 12 and 13) are computed and shown in Table 1. It is observed from Table 1 that $A_I$ reduces and $A_{II}$ increases as the number of inspections increases.

The distribution of the flaw length prior to inspection is given by Eq. 14 in which the average flaw length is assumed to be 5 mm, i.e., $\lambda=0.2/\text{mm}$. The probability density function given by Eq. 14 is displayed in Fig. 9(a) as a dashed curve. With such a preinspection flaw length distribution, the probability that the crack length will exceed $a_{NDE}=12.4$ mm is 8.37%, i.e., on the average there are 8.37%
of the components that will have a crack length longer than 12.4 mm. Hence the average percentage of good components, \( P_G \), prior to inspection is 91.63\%, i.e., \( P_G = 91.63\% \).

The Type I and Type II errors, \( P_I \) and \( P_{II} \), are computed from Eqs. 10 and 11 in which \( POD(x) \) is replaced by \( POD(x;l) \), \( POD(x;L2) \) and \( POD(x;L2U3) \), respectively. The results are shown in Table 1. It is observed from Table 1 that the Type I error reduces drastically as the number of inspections with the union rule increases. However, the Type II error increases simultaneously.

To reduce the Type II error, an additional inspection with a high resolution capability is used in conjunction with the intersection rule. For illustrative purposes, the POD curve of the NDE system for the additional inspection using the intersection rule is assumed to resemble that of the x-ray system shown in Fig. 1, i.e.,

\[
POD(a;3^*) = \begin{cases} 
0 & \text{for } a < 9.6 \text{ mm} \\
\frac{a - 9.6}{2.8} & \text{for } 9.6 < a < 12.4 \\
1 & \text{for } a > 12.4 \text{ mm}
\end{cases}
\] (16)

It is obvious that the POD curve given above is a straight line between 9.6 mm and 12.4 mm (see Fig. 1).

The resulting POD curve, denoted by \( POD[a;(L2)\cap3^*] \) and \( POD[a;(L2U3)\cap3^*] \), are presented in Fig. 9(b) as solid and dashed curves, respectively. Note that \( POD[a;(L2)\cap3^*] \) is the resulting POD curve under two inspections with the union rule, i.e., \( POD(a;1) = POD(a;2) \), as well as the third inspection with the POD curve given by Eq. 16. Likewise \( POD[a;(L2U3)\cap3^*] \) is the resulting POD curve under three inspections with the union rule and an additional inspection with the intersection rule where the POD curve is given by Eq. 16. Both \( POD[a;(L2)\cap3^*] \) and \( POD[a;(L2U3)\cap3^*] \) are computed from Eqs. 7 and 9, respectively.

Table 1. Type I and Type II Errors; \( a_{NDE} = 12.4 \text{ mm}, P_G = 91.63\% \)

<table>
<thead>
<tr>
<th>No. of Inspections</th>
<th>Type</th>
<th>( A_I ) (mm)</th>
<th>( A_{II} ) (mm)</th>
<th>( P_I )</th>
<th>( P_{II} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>POD(a;1)</td>
<td>2.50</td>
<td>5.99</td>
<td>1.15\times10^{-2}</td>
<td>28.6%</td>
</tr>
<tr>
<td>2</td>
<td>POD(a;L2)</td>
<td>0.24</td>
<td>8.30</td>
<td>1.72\times10^{-3}</td>
<td>42.7%</td>
</tr>
<tr>
<td>3</td>
<td>POD[a;(L2)\cap3^*]</td>
<td>0.24</td>
<td>1.10</td>
<td>1.72\times10^{-3}</td>
<td>5.9%</td>
</tr>
<tr>
<td>3</td>
<td>POD(a;L2U3)</td>
<td>0.03</td>
<td>9.36</td>
<td>2.74\times10^{-4}</td>
<td>50.8%</td>
</tr>
<tr>
<td>4</td>
<td>POD[a;(L2U3)\cap3^*]</td>
<td>0.03</td>
<td>1.38</td>
<td>2.74\times10^{-4}</td>
<td>6.2%</td>
</tr>
</tbody>
</table>
The Type I and Type II errors, \( P_I \) and \( P_{II} \), as well as \( A_I \) and \( A_{II} \) are computed from Eqs. 10-13 and the results are shown in Table 1. It is observed from Table 1 and Fig. 9(b) that multiple inspections can reduce both Type I and Type II errors simultaneously.

CONCLUDING REMARKS

An exploratory study has been made of the possible application of multiple inspection procedures and its potential pay-offs. The solutions are obtained for both independent NDE systems and dependent NDE systems. The resulting POD curve under multiple inspections is derived quantitatively from the POD curves of individual NDE systems when the union rule, the intersection rule and the combination thereof are used. Thus, researchers and practicing engineers can play with different combinations of NDE systems and procedures to arrive at an optimum strategy for their particular purpose.

Numerical examples are given using available POD curves from certain NDE systems to illustrate the basic idea and the application of multiple inspection strategy. It is shown that multiple inspections using the union rule alone, in general, reduce Type I error but increase Type II error, and the effect is reversed if the intersection rule is employed alone. It is further shown that both Type I and Type II errors can be simultaneously reduced significantly by the combined use of union and intersection rules. However, caution should be exercised in selecting the third NDE system (or POD curve) in order to minimize possible adverse effects. The sequence of inspections to minimize the inspection cost has also been discussed.

For gas turbine engine components, such as disks, inspections are performed for each critical location, including bolt holes, cooling holes, rim holes, etc. A disk which consists of a large number of holes is normally retired if at least one hole is rejected by the NDE system. The POD curve for any one particular NDE system as well as the resulting POD curve under multiple inspections presented herein applies to one hole only. Thus the POD curve for a disk containing a large number of holes differs significantly from that of the NDE system. With the multiple inspection theory developed herein, an exploratory study has been made [Ref. 8] for the inspection reliability of engine disks containing many holes. The significant advantage of multiple inspections for engine disks has been demonstrated in Ref. 8.

ACKNOWLEDGEMENT

The idea of multiple inspection procedures and its potential pay-offs was proposed by Dr. Walter H. Reimann. The authors are most grateful to Dr. Reimann for his guidance and encouragement.
during the course of this investigation. Valuable and helpful discussions with D. M. Forney, T. Cooper, Drs. T. Nicholas and J. Moyzis are gratefully acknowledged.

REFERENCES


DISCUSSION

S. Bush (Battelle, Pacific Northwest Laboratory): This is a very powerful tool, but to put it in perspective, what you will see on multiple inspections is that you may or may not affect your type 2 error, but you never do as well as you think you will on type 1 errors. To cite an example: for three inspections, three different sets of equipment on the same samples, only 32% found all of the flaws. When I say all of the flaws I mean that there was a common finding of the flaw, and that means that a very high percentage of them did not find it. This is an inherent problem in almost every inspection mode that you get into.

J.N. Yang (Wright-Patterson AFB): Depending on different sets of rules; if you use your union rule, then you will reduce the type 1 error.

S. Bush: You are right. Your union intersection combination would obviously resolve the problem with correct projection.

J.N. Yang: Right.

S. Bush: That simply indicates there are basic limitations even in the multiple approach.

J.N. Yang: Yes.

C.A. Rau (Failure Analysis Associates): Depending on the cause of the type 2 errors in the automated system, the type 2 error might go away just in reinspecting the same part. I think just re-inspecting with the automated system, which may be a lot less expensive than a whole other system, may markedly reduce the type 2 errors.

J. N. Yang: In this paper we said we use different NDE systems. Actually, one of the special cases is that you use a certain NDE system. That's why in one of the viewgraphs we said we prefer to use a single NDE system which is a special case of our mathematical solution. We will work together with Mr. Rummel to analyze the independence and dependence situation when we use the same inspection system.

C.A. Rau: I just want to follow up very briefly. The key point is that, if you think in terms of a bolt hole, the fact that you got a false call in one does not mean that the whole bolt hole is defective. If you require that the false call come from precisely the same geometric position, you will find a combination of type 2's of bolt holes will go way down compared to just looking at the redundant probability of a false indication in the entire bolt hole.