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Capacity Expansion in the Integrated Supply Network for an Electricity Market

Shan Jin, *Student Member, IEEE*, Sarah M. Ryan, *Member, IEEE*

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I. INTRODUCTION

We consider an integrated supply network for an electricity market including fuel transportation, electricity transmission and individual generation companies. The decision makers in each level of the supply network have distinct objectives to optimize. The fuel suppliers, who deliver the fuel to the power generation companies, want to minimize their fuel transportation cost that includes both the fuel cost and fuel delivery cost. The independent system operator (ISO) who settles the locational marginal prices (LMPs) and dispatches the electricity through the transmission network, aims to maximize the total welfare of both the sellers and the buyers of electricity in a wholesale market. The individual generation companies, who buy the fuel, generate electricity and sell it at the LMPs, wish to maximize their profits. All of these decision makers optimize simultaneously in the electricity market subject to capacity constraints.

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The overall welfare of market participants could be increased by capacity expansions to relieve constraints. Expansions at different levels and locations in the supply network could increase the availability of low-cost fuel, enable higher utilization of efficient generation resources, and level out the LMPs. These decisions are the responsibility of the facility owners, who naturally determine capacity investments to achieve their own objectives. However, such decentralized expansion decisions may not be optimal for the whole integrated electricity supply system. In this paper, we examine the investment decisions that a leader would make on behalf of the overall system, to maximize the total welfare resulting from decentralized operational decisions less the total investment cost. The results of optimizing from this global perspective reveal the interactions among constraints at different levels and identify bottlenecks in the integrated supply network. They could be used as a target in the development of consistent incentives or regulations to encourage the lower level players to make individual decisions that more closely approximate the global optimum for the overall system.

Considering both the investment and operational levels of decision-making, we propose a mixed integer bilevel programming model. The upper level leader decides how to expand the capacity of the integrated supply system. Once the capacity expansion decisions are made, lower level decision makers including the fuel suppliers, the ISO and the generation companies simultaneously make their optimal operating decisions to realize their respective objectives. Thus, this model falls between a fully centralized setting, in which a single entity makes both investment and operational decisions, and a fully decentralized setting, where multiple entities make decisions at both levels. The fully centralized version is included as a special case (1-level MIQP; see Section III.B). We set up a simple two-period model in which capacity investments are made in the first period based on equivalent hourly costs, and the system is operated in the second period, which represents a typical hour in a future scenario. The model could be elaborated to account for sequential expansions over time and/or multiple future scenarios of demand, fuel costs, and other uncertain conditions.

Given the capacity decisions in the integrated supply network, the simultaneous optimization of the lower level sub-problems, which mutually interact, leads to an equilibrium problem. It has been thoroughly studied and solved as a mixed complementarity problem (MCP) [1] and validated by comparison with a computational agent simulation [2]. In the

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restructured wholesale electricity market, the prices submitted by the generator companies are determined according to their marginal costs, which depend mostly on the fuel costs [3]. Here we assume that the fuel suppliers, represented for simplicity by a single fictional fuel dispatcher, decide the quantities of the fuel shipped over various routes to minimize the cost of satisfying demands of all of the generator companies. The ISO manages the electricity wholesale market where the buyer (inverse) demand functions are linear functions of the LMPs, and makes the electricity dispatch decision. The individual generation companies, considered as Cournot competitors, pay attention to the price differentials among the electricity nodes and determine their electricity quantities to sell under a type of bounded rationality [4].

In general, mixed integer bilevel programming (MIBLP) problems are hard to solve. Moore and Bard [5] presented the challenge of the MIBLP problem and proposed a series of heuristics to efficiently find a good feasible solution. DeNegre et al. [6] further proposed a branch and cut method to improve the branch and bound algorithm in [5]. Colson et al. [7] reviewed methodologies and applications of bilevel programming problems and described their connections with mathematical programs with equilibrium constraints (MPECs).

The bilevel programming technique was introduced to model the integrated system of multiple participants usually with different objective functions. Many applications involving restructured electricity markets have formulated the ISO market clearing problem as the lower level. Hu and Ralph [8] modeled a game among the consumers and generators submitting the bids to the ISO in the upper level. There are also formulations considering the bidding strategy of the generation companies in the upper level [9, 10]. Generation or transmission expansion decisions by an individual participant can also be modeled as the upper level with the lower level representing the market outcomes [11, 12]. Including multiple decision-makers in the upper level constitutes an equilibrium problem with equilibrium constraints, which is much harder to solve.

In our model, the integer variables appear only in the upper level. Therefore, the Karush–Kuhn–Tucker (KKT) optimality conditions can still be applied to the lower level problems. Upon applying the KKT conditions to all the sub-problems, the bilevel problem becomes a mathematical program with complementarity constraints (MPCC). The MPCC is a special case of MPEC, which has attracted great attention for the past decade as more and more engineering and economic applications involve equilibrium modeling. Ferris and Pang [13] gave a comprehensive summary of the engineering and economic applications of MPCC and the available solution algorithms. Its solution requires an equivalent reformulation of complementarity constraints and global convergence of the solution can be guaranteed only under certain conditions. Hu et al. [14] presented a methodology to find the global optimal solution of a linear program with linear complementarity constraints by reformulating the constraints with binary variables.

The contributions of this paper are fourfold: (1) We investigate a fuel transportation, generation and transmission expansion problem of an integrated electricity supply system in which an equilibrium in a restructured market is reached by the fuel suppliers, generator companies and ISO solving simultaneous and interdependent optimization problems. Instead of letting each market player make his own expansion decision, we find the optimal expansion decision for the whole integrated system from the global perspective. (2) We incorporated discrete transmission expansion decisions by using binary variables in the direct current optimal power flow constraints. (3) The problem is formulated as a bilevel programming problem. The challenge posed by the discrete decision variables makes it difficult to achieve global optimality. We provided three problem reformulations to bound the objective and find a global optimum. (4) The model and solution procedure are illustrated by a small case study that shows how the global expansion decision affects the LMPs of each node in the transmission grid, the buyers' surplus, the sellers' surplus and the transmission rents.

II. Model Formulation

We formulate a bilevel capacity expansion problem of an integrated electricity supply network, where we also optimize the sub-problems of fuel suppliers, generators and the ISO in the lower level.

TABLE I
SETS OF NODES AND ARCS

Set	Description	Indices
N	Electricity nodes	i, j
F	Fuel supply nodes	G
L	Transmission lines	ij
A	Fuel supply lines	gj
N_g	Set of electricity nodes supplied by fuel supply node g	j
F_j	Set of fuel supply nodes supplying the electricity node j	g

TABLE II DECISION VARIABLES

Decision Variable		Description
Upper Level	$\boldsymbol{nU}$	Fuel transport capacities after expansion
	$\boldsymbol{nV}$	Generation capacities after expansion
	$\boldsymbol{z}$	Binary decision variables for new transmission lines
Lower Level	$\boldsymbol{x}$	Fuel delivered at fuel transportation arcs
	$\boldsymbol{q}$	Demand satisfied at electricity nodes
	$\boldsymbol{\theta}$	Voltage angles at electricity nodes
	$\boldsymbol{f}$	Electricity flows on transmission lines
	$\boldsymbol{y}$	Generation amounts at electricity nodes
	η	(Scalar) price at the reference electricity node

Table I indicates the sets of nodes and arcs of the integrated electricity supply network. Tables II and III respectively give the notation for both decision variables and parameters of the model. The variable η is a scalar and all the other variables and parameters are vectors. All fuel quantities are expressed in MWh equivalents. Appendix A shows how to incorporate heat

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rates and efficiencies of converting fuels to electricity.

TABLE III
PARAMETERS

Parameter	Description
a	Intercepts of electricity demand prices as linear functions of quantities
b	Slopes of electricity demand prices as linear functions of quantities
c	Costs per MWh-equivalent of fuel transported
fc	Investment cost for fuel transportation network expansion
gc	Investment costs for generation expansion
tc	Investment costs for transmission line expansion
θ^{max}	Maximum values for voltage angles
θ^{min}	Minimum values for voltage angles
φ	Nodal electricity price premia at electricity nodes
π	Nodal prices for fuel delivered to generators
V	Generation capacities at electricity nodes
U	Capacities of fuel supply arcs
K	Capacities of transmission lines
W	Quantities of fuel available at fuel supply nodes
B	Susceptances of transmission lines

A. *Mixed Integer Bilevel Program (MIBLP)*

The objective function of the upper level is to maximize the social welfare including the buyers' surplus, power producers' surplus and transmission rents, less the investment costs of expansions in fuel transportation, transmission network and power generation capacities. We expand capacity of the existing assets for both the fuel network and generation. For the transmission network, we assume the capacity expansions are realized by building new lines selected from a set of candidates. All investment costs are linear.

$$\max_{nU,nV,z} \sum_j \left(\frac{1}{2} b_j q_j^2 + a_j q_j\right) - \sum_{gj\in A} c_{gj} x_{gj} - \sum_{gj\in A} fc_{gj}\left(nU_{gj} - U_{gj}\right) - \sum_{j\in N} gc_j\left(nV_j - V_j\right) - \sum_{ij\in L} tc_{ij} K_{ij} z_{ij} \qquad (1)$$

The electricity demand at each electricity node is defined as a linear function of the nodal price. The intercept and slope of the inverse demand function are respectively a_j and b_j with $b_j <$ 0 at electricity node *j*. Fixed (inelastic) demands can also be incorporated in the lower level model [15].

For the lower level optimization problems, we consider the three major players in the electricity supply network consisting of fuel suppliers, ISO and generators, who all optimize under the same electricity market conditions. The fuel dispatcher minimizes the fuel transportation cost delivered to the generators; the ISO maximizes the social welfare of participants in the wholesale electricity market; and the parameters in the generators' problems. The dual variables φ in Eq. (16) of the ISO's problem are the electricity nodal price premium parameters in the generator problems. The electricity quantities determined by the generators are the fuel demand parameters in the fuel dispatcher's problem and the electricity supply quantities for the ISO's problem.

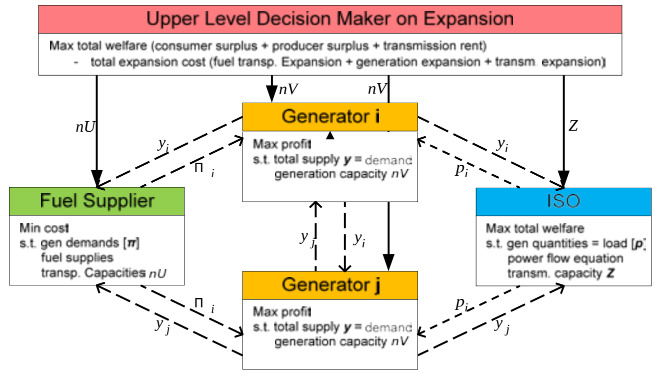

Fig. 1. The Bilevel Program with the Interacting Lower Level Optimization Problems

Given the generation amount y_j at each electricity node *j*, the fuel dispatcher's optimization problem is:

Fuel Dispatcher's decision problem

$$\min_{x\geq 0} \sum_{gj\in A} c_{gj} x_{gj} \qquad (2)$$

$$s.t. \ -\sum_{j\in N_g} x_{gj} \geq -W_g \quad \forall g\in F \quad [\varphi_g \geq 0] \qquad (3)$$

$$\sum_{g\in F_j} x_{gj} = y_j, \quad \forall j\in N \quad [\pi_j] \qquad (4)$$

$$-x_{gj} \geq -nU_{gj} \quad \forall gj\in A \quad [\rho_{gj} \geq 0] \qquad (5)$$

$$x_{gj} \geq 0 \qquad (6)$$

The fuel dispatcher aims to minimize the transportation cost subject to the constraints of fuel supply capacity (3), electricity demand (4) and the fuel transportation arc capacity (5).

The market operator, ISO, seeks to maximize the social welfare based on the direct current optimal power flow (DCOPF) model with the full-structured form [16, 17]. Given the generation amounts y_j at the electricity nodes, the ISO's decision problem is:

ISO's decision problem

$$\max_{q,\theta,f} \sum_j \left(\frac{1}{2} b_j q_j^2 + a_j q_j\right) \qquad (7)$$

$$s.t. \ q_j + \sum_i f_{ji} - \sum_i f_{ij} = y_j, \quad \forall j\in N \quad [p_j] \qquad (8)$$

$$\theta_j \leq \theta_j^{max} \quad \forall j\in N \quad [\alpha_j^+ \geq 0] \qquad (9)$$

$$-\theta_j \leq -\theta_j^{min}, \quad \forall j\in N \quad [\alpha_j^- \geq 0] \qquad (10)$$

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generators maximize their profits from selling the electricity.

However, these three different optimization problems interact with each other as shown in Fig. 1. The dual variables π in Eq. (4) of the fuel dispatcher's problem are the marginal cost

$$f_{ij} - B_{ij}(\theta_i - \theta_j) - \left(1 - z_{ij}\right)M_{ij} \leq 0, \quad \forall ij \in L \quad [\gamma_{ij} \geq 0] \tag{11}$$

$$B_{ij}(\theta_i - \theta_j) - f_{ij} - \left(1 - z_{ij}\right)M_{ij} \leq 0, \quad \forall ij \in L \quad [\gamma^{-}_{ij} \geq 0] \tag{12}$$

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$$f_{ij} \le z_{ij} K_{ij}, \ \forall ij \in L \qquad [\lambda_{ij}^{+} \ge 0] \quad (13)$$

$$-f_{ij} \le z_{ij} K_{ij}, \ \forall ij \in L \qquad [\lambda_{ij}^{-} \ge 0] \quad (14)$$

The ISO's decision problem, based on the full-structured DCOPF model of [17], is equivalent to the reduced structure DCOPF model in [1].

Condition (8) represents the flow balance at each electricity node. Constraints (9) and (10) give the bounds on each voltage angle. Equations (11) and (12) incorporate the physical characteristics of the transmission grid so that the (linearized) power flow equations will always be satisfied. The maximum capacity of each transmission line is enforced by equations (13) and (14).

Instead of the standard power flow equation $B_{ij}(\theta_i - \theta_j) - f_{ij} = 0$, binary decision variables z and large values M are used in (11) and (12) to represent discrete investment decisions on new transmission lines in disjunctive constraints similar to those in a transmission-switching model [16, 18]. The variable z_{ij} indicates the existence of the transmission line *ij*. If the candidate transmission line has been added, then z_{ij} equals 1, the value of M_{ij} does not matter at all, and the two inequalities are equivalent to the traditional power flow equation. On the other hand, if z_{ij} is 0, then the value of M_{ij} matters. The value of M_{ij} should be large enough to impose no additional constraint on $B_{ij}(\theta_i - \theta_j) - f_{ij}$. We fixed all the *z* variables corresponding to the lines which exist prior to the expansion decision to be 1. For the candidate transmission lines, their corresponding *z* variables can be either 1 or 0, to represent building those lines or not.

Regarding the parameters *M*, excessively large values might cause numerical difficulties when solving the problem. One of the assumptions of the DCOPF model is that the voltage angle difference of any transmission line is quite small [17]. Here we adopt the assumption in [16] with upper and lower bounds on θ of $\pm$ 0.6. Because the electricity flow *f* is also bounded by *K*, the quantity $\left|B_{ij}(\theta_i - \theta_j) - f_{ij}\right|$ is bounded by $1.2B_{ij} + K_{ij}$, which therefore represents a sufficiently large value of M_{ij}.

We assume that the electricity wholesale market takes the form of oligopolistic competition. The multiple generators are modeled as Cournot competitors in the electricity wholesale market. Each of them determines its electricity quantity to sell. Besides the Cournot model, there are also many other approaches available to model the generator competition in the electricity market [19]. A more realistic approach is the supply function equilibrium game that allows each firm to submit a bid function with different quantities offered at different prices. However, it suffers from computational inefficiency and multiplicity of equilibria due to its non-convexity. Another popular approach is a Bertrand game in which the producer makes the decision on selling price instead of quantity. Therefore it is more likely to give a similar result as in a competitive electricity market where the prices are all set to the marginal cost. This market behavior does not agree with the oligopolistic competition that we assumed. The Cournot model is not as realistic as the supply function equilibrium model. However, it is much easier to solve and in the long run the market behavior is close to Cournot result [19, 20]. Since the total amount of electricity generated affects the electricity market prices, here we assume that each generator also determines the LMP η at the reference electricity node [4].

The LMP at the reference electricity node is

$$\eta = p_{ref} \qquad (15)$$

and the price premium at each node is then defined as

$$\varphi_j = p_j - \eta \qquad (16)$$

where the LMPs *p* are dual variables of the energy conservation constraints (8).

Given the fuel price π_i and price premium φ at its node, which are derived from the dual variables of the fuel supplier's decision problem and ISO's decision problem, respectively, the decision problem of generator *i* is:

Generator's decision problem

$$\max_{y_i \ge 0, \eta} \ (\eta + \varphi_i - \pi_i)\, y_i \qquad (17)$$

$$\text{s.t. } y_i = \sum_j \frac{\varphi_j - a_j}{b_j} - \eta \sum_j \frac{1}{b_j} - \sum_{j \ne i} y_j, \ \forall i \in N \ [\beta_i] \qquad (18)$$

$$y_i \le nV_i, \ \forall i \in N \qquad [\mu_i \ge 0] \quad (19)$$

$$y_i \ge 0 \qquad (20)$$

Equation (18) represents $\sum_i q_i = \sum_i y_i$, the balance of total demand and total generation in terms of the residual demand seen by generator *i*. Constraint (19) indicates the maximum generation capacity. The generator's problem can also be extended to take carbon emission regulations into account, as described in Appendix B.

If we explore only the lower level optimization problems by

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simply ignoring the upper level objective function, the lower level is equivalent to the problem studied by Ryan et al. [1]. The only difference is the equivalent modification of the ISO's decision problem to incorporate the transmission expansion decision. We verified our equivalent new model with fixed capacities by comparing its numerical results with those in [2]. Because the existence of a Nash equilibrium has been proved in [1], it also holds for our lower level problems. To explore the potential multiplicity of equilibria, we solved both the maximization and minimization problems for multiple different objective functions in the numerical instance of Section IV with investment variables fixed, and they all returned with the same equilibrium solution, which suggests that the equilibrium is unique in that instance.

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B. Mathematical Program with Complementarity Constraints (MPCC)

The MPCC problem is to optimize an objective function subject to complementarity constraints that can be expressed with the standardized format $0 \le x \perp f(x) \ge 0$.

The MIBLP problem presented in Section II.A has integer decision variables only in the upper level. Thus, the lower level optimization problem can be reformulated in terms of complementarity constraints by applying the KKT conditions to each player's optimization problem. This transforms the original bilevel program into an equivalent MPCC with a mixed integer quadratic objective function. The objective function is given by equation (1) and the full set of constraints is:

$$0 \le x_{gj} \perp c_{gj} + \omega_g - \pi_j + \rho_{gj} \ge 0, \ \forall gj \in A \tag{21}$$

$$\sum_{g \in F_j} x_{gj} = y_j, \ \forall j \in N \tag{22}$$

$$0 \le \omega_g \perp W_g - \sum_{j \in N_g} x_{gj} \ge 0, \ \forall g \in F \tag{23}$$

$$0 \le \rho_{gj} \perp nU_{gj} - x_{gj} \ge 0, \ \forall gj \in A \tag{24}$$

$$a_j + b_j q_j - p_j = 0, \ \forall j \in N \tag{25}$$

$$\alpha_j^{-} - \alpha_j^{+} + \sum_{i,ji \in L} B_{ji}\left(\gamma_{ji}^{+} - \gamma_{ji}^{-}\right) - \sum_{i,ij \in L} B_{ij}\left(\gamma_{ij}^{+} - \gamma_{ij}^{+}\right) = 0, \forall j \in N \tag{26}$$

$$p_j - p_i - \gamma_{ij}^{+} + \gamma_{ij}^{-} - \lambda_{ij}^{+} + \lambda_{ij}^{-} = 0, \ \forall ij \in L \tag{27}$$

$$q_j + \sum_{ji \in L} f_{ji} - \sum_{ij \in L} f_{ij} = y_j, \ \forall j \in N \tag{28}$$

$$0 \le \theta_j^{max} - \theta_j \perp \alpha_j^{+} \ge 0, \ \forall j \in N \tag{29}$$

$$0 \le -\theta_j^{min} + \theta_j \perp \alpha_j^{-} \ge 0, \ \forall j \in N \tag{30}$$

$$0 \le B_{ij}\left(\theta_i - \theta_j\right) - f_{ij} + \left(1 - z_{ij}\right)M_{ij} \perp \gamma_{ij}^{+} \ge 0, \ \forall ij \in L \tag{31}$$

$$0 \le -B_{ij}\left(\theta_i - \theta_j\right) + f_{ij} + \left(1 - z_{ij}\right)M_{ij} \perp \gamma_{ij} \ge 0, \ \forall ij \in L \tag{32}$$

$$0 \le z_{ij}K_{ij} - f_{ij} \perp \lambda_{ij}^{+} \ge 0, \ \forall ij \in L \tag{33}$$

$$0 \le z_{ij}K_{ij} + f_{ij} \perp \lambda_j^{-} \ge 0, \ \forall ij \in L \tag{34}$$

$$0 \le y_j \perp -\eta - \varphi_j + \pi_j + \beta_j + \mu_j \ge 0, \ \forall j \in N$$

III. Reformulation and Solution

A. Nonlinear Programming Reformulation (MPCC-NLP)

To solve the MPCC, its complementarity constraints must be reformulated. One method is to transform them into nonlinear functions. Consider a generic complementarity constraint as:

$$0 \le r \perp s \ge 0 \tag{39}$$

The product reformulation replaces it with constraints that r and s are nonnegative and $r's = 0$ [21]. The complementarity constraint can also be expressed in terms of a nonlinear complementarity problem (NCP) function $\Phi(r,s)$ that satisfies $\Phi(r,s) = 0$ if and only if $r's = 0$ and $r,s \ge 0$. An example of $\Phi(r,s)$ is the Fischer-Burmeister function $\sqrt{r^2 + s^2} - r - s$ [22, 23]. Both of these methods maintain the reformulated complementarity conditions as constraints. A third method is to penalize positive values of the reformulated nonlinear function $r's$ in the objective function.

These three methods are available as options in the NLPEC solver in the General Algebraic Modeling System (GAMS) [24]. For the numerical instance in Section IV, no optimal solutions were found with either the Fischer-Burmeister function or the penalty reformulations. Local optimality was achieved by the product reformulation, which converted the MPCC to a mixed integer nonlinear programming problem. However, global optimality of the solution is not guaranteed. We can conclude only that the solution is feasible but not necessarily globally optimal for the MIBLP problem. Its objective value is therefore a lower bound on the optimal value.

To identify the global optimum of the MIBLP problem, we further explored two additional methods to solve the problem described in sections B and C respectively.

B. Single-Level Mixed Integer Quadratic Program (1-level MIQP)

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(35)

$$y_j + \sum_{i \in N} \frac{1}{b_i} \beta_j = 0, \ \forall j \in N \quad (36)$$

$$0 \le \mu_j \perp nV_j - y_j \ge 0, \ \forall j \in N \quad (37)$$

$$nU_{gj} \ge U_{gj}, \ nV_j \ge V_j, \ z_{ij} \in \{0,1\} \quad (38)$$

Constraints (21) – (24) are the KKT conditions for the fuel dispatcher's problem (2) – (6), while (25) – (34) are the KKT conditions for the ISO's decision problem (7) – (14), and (35) – (38) are the accumulated KKT conditions for all generation companies' problems (17) – (20).

Due to the non-convexity of the feasible region, the MPCC problem is difficult to solve. In the next section we outline three reformulations and describe how they help to identify and evaluate the global optimal solution.

The MIBLP model assumes that a central decision maker anticipates the lower level decision makers' reactions to his investment decision on the capacity expansion of all the facilities involved and makes the optimal decision to maximize the total benefit. All of the lower level decision makers will respectively make their optimal operational decisions, given the leader's decision.

In the 1-level MIQP relaxation, we assume that there is only one centralized decision maker in the market making all investment and operational decisions to optimize the benefit of the whole system, while satisfying all the physical constraints from each part of the integrated network. In this case, the generator companies are no longer able to make their own strategic decision to maximize their profit but simply accept the market optimal decisions.

This relaxed problem can be derived by removing all of the

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objective functions of the lower level optimization problems. The objective function of the 1-level MIQP problem is equation (1), and the constraints are equations (3) – (6), (8) – (14) and (18) – (20). Because the ISO and fuel dispatch objectives are already included in (1), only the generator strategic capability is removed. Since the problem is a relaxation of the original problem, its optimal solution provides an upper bound for the MIBLP problem, and therefore can be used to bound the optimality gap once a feasible solution is provided.

The difference of the optimal objective values derived from the MPCC-NLP and the 1-level MIQP gives a range in which the global optimal objective value must lie. If the gap between them is small enough, the global solution can be well approximated by the solution of MPCC-NLP problem.

C. Binary Variables Reformulated Mathematical Program with Complementarity Constraints (MPCC-BIN)

To solve the problem more efficiently and, more importantly, to obtain a global optimal solution, we converted MPCC problem into an equivalent mixed integer quadratic program by introducing a set of binary variables σ and the large parameters M [14]. For instance, the reformulation of equation (39) is:

$$0 \le r \le M\sigma \quad (40)$$

$$0 \le s \le M(1-\sigma) \quad (41)$$

These inequalities ensure that either r or s must be zero. The complementarity constraints have been converted into the mixed integer linear constraints, and the whole problem becomes an MIQP problem.

The constraints (22), (25) – (28), (36) and (38) of MPCC problem remain the same and the constraints (21), (23) – (24), (29) – (35) and (37) are reformulated by the binary variables. A large number of binary variables are introduced in the MPCC-BIN reformulation.

IV. Numerical Results

We studied a six node transmission network with four fuel suppliers illustrated in Fig. 2 [2].

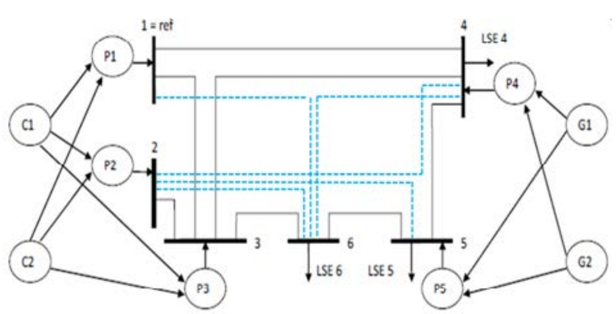

Fig. 2. Integrated Electricity Supply Network including the Fuel Suppliers, Transmission Grid and Generators

The generators P1, P2 and P3 are coal-fired plants, supplied by coal suppliers C1 and C2, and the generators P4 and P5 are natural gas-fired plants supplied by two natural gas suppliers G1 and G2. The LSEs 4, 5 and 6 represent the electricity loads. The solid lines are the existing transmission lines and the dashed lines are the candidate transmission lines for possible expansion.

The numerical results are based on a single hour. All of the expansion costs are also estimated on an hourly basis. The generation and transmission capital costs are derived from the Joint Coordinated System Plan report [25]. All of the other parameters in Table VI and VII are based on [2]. Tables IV, V, VI and VII give the parameters of the case study.

TABLE IV
INVESTMENT COSTS OF FUEL DELIVERY, GENERATION AND TRANSMISSION CAPACITY

Investment Cost	**Coal**	**Natural Gas**
fc ($/MWh)	1.5	4
gc ($/MWh)	10	6
tc ($/MWh)	4	

TABLE V
FUEL CAPACITY AND TRANSPORTATION COST

Fuel Suppliers		**Electricity Nodes**				
		1	2	3	4	5
$U_{g,j}$ (MWh equivalent)	C1	200	30	30	0	0
	C2	800	500	500	0	0
	G1	0	0	0	130	60
	G2	0	0	0	1200	800
c_{gj} ($/MWh)	C1	65	73	70	-	-
	C2	80	75	72	-	-
	G1	-	-	-	120	115
	G2	-	-	-	125	122

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TABLE VI
GENERATION CAPACITY AND PARAMETERS FOR INVERSE DEMAND FUNCTION

Electricity Nodes	V_j (MW)	b_j ($/MWh/MWh)	a_j ($/MWh)
1	600	-∞	0
2	400	-∞	0
3	400	-∞	0
4	1000	-0.08	250
5	600	-0.08	300
6	N/A	-0.08	350

TABLE VII
TRANSMISSION CAPACITY, SUSCEPTANCE OF THE NETWORK AND INITIAL STATUS OF THE TRANSMISSION LINES

Transmission Line	Transmission Capacity $K_{i,j}$ (MW)	Susceptance B_{ij} (Ω^{-1})	z_{ij}^0
(1, 3)	400	156.25	1
(1, 4)	240	33.67	1
(2, 3)	1000	∞	1
(3, 4)	150	32.89	1
(3, 6)	250	35.59	1
(4, 5)	240	33.67	1
(5, 6)	350	92.59	1
(2, 6)	400	32.89	Can
(2, 5)	400	35.59	Can
(1, 6)	400	32.89	Can
(1, 2)	400	156.25	Can
(2, 4)	400	32.89	Can
(4, 6)	400	33.67	Can

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TABLE VIII
NUMERICAL RESULTS OF ORIGINAL EQUILIBIRUM MPCC-NLP, 1-LEVEL MIQP AND MPCC-BIN PROBLEMS

Decision Variables		Problems: Original Equilibrium	MPCC-NLP	1-Level MIQP	MPCC-BIN
Objective Value		286191	559895	575362	564275
Social Welfare		286191	598973	618312	600173
Buyer Surplus		*77798*	*242068*	*333332*	*243554*
Generator Surplus		*119318*	*160840*	*52715*	*148060*
Transmission Rent		*89075*	*196065*	*232265*	*208559*
Fuel Expan. Cost		0	12174	14598	12173
Gen. Expan. Cost		0	17305	20353	15725
Trans. Expan. Cost		0	9600	8000	8000
Fuel Expansion	(C1,1)	0	1018	755	755
	(C1,2)	0	18	67	125
	(C1,3)	0	131	360	312
	(G1,4)	0	722	860	713
	(G1,5)	0	1883	2346	1883
Gen. Expansion	1	0	628	365	365
	5	0	1353	1816	1353
Trans. Expansion	(2,6)	0	1	1	1
	(2,5)	0	1	1	1
	(1,6)	0	1	1	1
	(1,2)	0	1	0	0
	(2,4)	0	1	1	1
	(4,6)	0	1	1	1
Fuel Delivered	(C1,1)	200	1218	955	955
	(C1,2)	30	48	97	155
	(C1,3)	30	161	390	342
	(C2,1)	157	0	0	0
	(C2,2)	30	0	0	0
	(C2,3)	142	0	10	0
	(G1,4)	130	852	990	843
	(G1,5)	60	1943	2406	1943
	(G2,4)	790	0	10	0
	(G2,5)	540	0	10	0
Amount generated by Generator	1	357	1218	955	955
	2	60	48	97	155
	3	172	161	400	342
	4	920	852	1000	843
	5	600	1943	2416	1943
Generation Consumed	4	1256	1341	1501	1344
	5	386	1665	2138	1665
	6	468	1218	1229	1229
Electricity Price	1	90	97	73	90
	2	77	76	75	79
	3	77	76	75	79
	4	150	143	130	142
	5	269	167	129	167
	6	313	253	252	252
Electricity Flow	(1,2)	0	344	0	0
	(1,3)	168	344	400	400
	(1,4)	190	228	240	240
	(1,6)	0	303	315	315
	(2,3)	60	-105	-400	-342
	(2,4)	0	150	150	150
	(2,5)	0	115	115	115
	(2,6)	0	231	231	231
	(3,4)	150	150	150	150
	(3,6)	250	250	250	250
	(4,5)	4	-44	-44	-44
	(4,6)	0	83	83	83
	(5,6)	218	350	350	350

We implemented all the problem formulations: MPCC-NLP, 1-level MIQP, and MPCC-BIN, via the modeling language of GAMS and called its inner solvers to solve the problems. The original equilibrium represents the solution to the lower level problem only and is found by the PATH solver [26]. The MPCC-NLP is solved by the DICOPT solver [27] which cannot guarantee global optimality. The 1-level MIQP and MPCC-BIN problems are both solved by the CPLEX solver [29] to global optimality. The numerical results are indicated in Table VIII. The "Original Equilibrium" solution in Table VIII gives the MPCC-NLP results without any expansion. The MPCC-BIN is solved with each value of *M* equal to 10000. The methodology to derive appropriate values for *M* is described in Appendix C.

The BARON NLP solver could find a global optimum for MPCC-NLP if all mathematical expressions have finite lower and upper bounds [28]. We did not pursue this avenue because finding upper bounds is equivalent to identifying large enough values of *M* for the binary reformulation and finding lower bounds requires additional effort.

Let the optimal objective values found by solving MPCC-NLP, 1-level MIQP and MPCC-BIN be ζ_1, ζ_2 and ζ_3, respectively, and the global optimal value of the MIBLP problem be ζ_{opt}. The optimality gap of the MPCC-NLP solution is $|\zeta_1 - \zeta_{opt}|$. Since the 1-level MIQP problem is a relaxation, ζ_2 is an upper bound of ζ_{opt}. Therefore, the optimality gap is bounded by $|\zeta_1 - \zeta_2|$, which indicates how far the obtained optimal solution might be from the global optimal solution. The optimality gap by percentage can also be defined as $|\zeta_1 - \zeta_{opt}|/\zeta_{opt} \times 100\%$, bounded from above by $|\zeta_1 - \zeta_2|/\zeta_1 \times 100\%$. In our numerical study, the optimality gap $|\zeta_1 - \zeta_2|$ is 15467 and the bound on the percentage optimality gap is 2.76%, which implies that the feasible solution solved by MPCC-NLP is within 2.76% of the global optimum.

The MPCC-BIN reformulation is also equivalent to the MIBLP problem. Moreover, it is also a maximization problem with a concave quadratic objective function. Therefore the global optimal solution is guaranteed. The CPLEX solver verifies convexity by checking that the Hessian matrix of both objective function and the constraints is positive semi-definite. This allows inclusion of a lower bound, but not an upper bound,

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constraint on the objective function value. We compared the computational time of the problem with and without the lower bound obtained from MPCC-NLP. The computation time in seconds for solving the original equilibrium, MPCC-NLP, and 1-level MIQP are respectively 0.484 and 0.125. It takes 0.953 seconds to solve MPCC-BIN without any bound, and 0.64 seconds with the lower bound provided by MPCC-NLP. That is, the bound improves the computational efficiency by nearly 33%.

The optimal solutions suggest that the decision maker should

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build the candidate transmission lines (2, 6), (2, 5), (1, 6), (2, 4) and (4, 6) to achieve the global optimum. It is obvious in Table IV that the coal-fired generators are much cheaper than the natural gas-fired generators. Thus the electricity is more likely to flow from the left to right in Figure 2, especially given that all of the loads are located on nodes 4, 5 and 6. Before making the expansion decision, transmission congestion exists on lines (3, 4) and (3, 6). Without the accessibility of the cheaper electricity, the LMPs on LSE nodes 4, 5 and 6 are much higher, which also suggests that more transmission lines are required to help deliver the electricity from left to right. The expansion made on the candidate transmission lines increases the transmission capacity to deliver more electricity from the coal-fired generators to the loads, and thus will certainly help to balance the electricity prices of the network.

As for the coal generators, the cheapest fuel source is C1. Therefore the decisions have been made to expand the arcs (C1, 1), (C1, 2) and (C1, 3). Similar decisions have been made to expand the natural gas transportation from the relatively cheaper source G1. Even though the natural gas costs are about twice as high as the coal costs, the gas transport capacity is expanded because of the electricity transmission capacity limits. The grid congestion makes it impossible to use all of the electricity available from the coal-fired plants.

The decision on the generation expansion matches the expansion on the fuel transportation and the transmission grid.

The expansions help generate and deliver more cheap electricity to satisfy the demand and thus improve the balance in the electricity prices on the nodes. It also leads to an increase of sellers' and buyers' surplus. Although the price differences among the nodes has been decreased, the transmission congestion still exists and the increasing number of transmission lines results in even more transmission rents in total. All of the effects achieve an increase in overall welfare of the integrated electricity supply system.

Also from Table VIII, by comparing the results of both MIQP and MPCC-BIN problems, we are able to see how the strategic decisions made by generation companies affect the performance of the electricity market, due to the fact that MIQP problem is a relaxation of the MIBLP problem obtained by only eliminating the strategic behavior of the generators. Without generator strategic operational behavior, more fuel supply and generation facilities are expanded so that the electricity prices are lower, which results in a large increase in buyer surplus and decrease in generator surplus.

V. Conclusion

In this paper, we investigated a capacity expansion bilevel programming problem. In the lower level, we take into account an integrated electricity supply system including the fuel transportation, generation and transmission, as well as the interactions among them in a restructured electricity market, where the buyer demand is modeled as a linear function of the electricity price. Capacity expansion decisions are made by an upper level decision maker from a global point of view.

In the absence of strategic operational decisions by generators, the total social welfare increases. Electricity buyers are better off while the generators are worse off. Fuel and generation facilities are expanded more, which leads to lower electricity prices. In our numerical study, generator strategic operational decisions reduce the welfare less investment cost by 2%.

We used two reformulations of the MPCC to efficiently identify a global optimum. The NLP reformulation takes less time to solve but its solution is not guaranteed to be globally optimal. It provides a feasible solution and lower bound on the optimal value. On the other hand, the binary formulation takes more time to solve, but it is able to identify the global optimal solution. Including the lower bound derived from MPCC-NLP significantly improves its computation time. However, it takes effort to find an appropriate M value as a tight bound for the mathematical expressions in the complementarity constraints to implement the MPCC-BIN reformulation. Small values of M could eliminate the optimal solution but excessively large ones increase the computation time. The relaxed 1-level MIQP provides upper bounds on the optimal value and optimality gap. It can be solved easily, but will not necessarily provide a feasible solution for the original problem.

A six bus case study is provided to illustrate the three methodologies and give the combined expansion results in fuel transportation, generation and transmission. We also analyze the effect of the global optimal expansion decision on the integrated electricity supply system.

For future research, the investment costs for the fuel transportation, generation and transmission subsystems can be further extended to nonlinear cost functions that incorporate economies of scale. A dynamic decision making process can also be represented to optimize investment decisions in multiple periods over a long term horizon. The major uncertainties including natural gas cost and electricity demand can also be taken into account in the model. To do so, we can first generate different future scenarios for the uncertainties and then incorporate them into the model as a stochastic MPCC problem. Furthermore, we can also compare the results of the model in our paper with the ones from a more realistic point of view in which every asset owner makes his own capacity expansion decisions. This comparison will show how much the optimal decisions identified from a global point of view could benefit the integrated electricity supply system and provide possible targets for policy design.

Appendix A
Elaboration of Fuel Dispatcher's Decision Problem

Two fuel resources, coal and natural gas, which normally have different units \$/ton and \$/thousand cubic feet, are considered in Section IV. To make the units of different fuel types match and most importantly compatible with the unit of energy (MWh), we converted the units of fuel into MWh equivalents and set their cost parameters $\boldsymbol{c}$, capacity limit parameters $\boldsymbol{W}$ and $\boldsymbol{nU}$, and investment costs $\boldsymbol{fc}$ accordingly.

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It is also possible to directly model the fuel dispatcher problem with original units of fuel resources by incorporating the heat rate ***H*** and efficiency ε conversion into the model.

The fuel cost ***c*** can also further be decomposed into two parts: fuel cost ***cl*** and the delivery cost ***del***. In this case, the objective function Eq. (1) is changed to:

$$\max_{nU,nV,z,q} \sum_j \left(\frac{1}{2} b_j q_j^2 + a_j q_j\right) - \sum_{gj \in A} \left(cl_{gj} + del_{gj}\right) x_{gj}$$
$$- \sum_{gj \in A} fc_{gj}\left(nU_{gj} - U_{gj}\right) - \sum_{j \in N} gc_j \left(nV_j - V_j\right)$$
$$- \sum_{ij \in L} tc_{ij} K_{ij} z_{ij}$$

And the fuel dispatcher's decision problem is revised as:

$$\min_{x \ge 0} \sum_{gj \in A} \left(cl_{gj} + del_{gj}\right) x_{gj}$$
$$s.t. \; -\sum_{j \in N_g} x_{gj} \ge -W_g \quad \forall g \in F \quad [\omega_g \ge 0],$$
$$\sum_{g \in F_j} x_{gj} e_g = \frac{y_j \varepsilon_j}{H_j}, \quad \forall j \in N \quad [\pi_j]$$
$$-x_{gj} \ge -nU_{gj} \quad \forall gj \in A \quad [\rho_{gj} \ge 0],$$
$$x_{gj} \ge 0$$

Since the units of ***x*** are \$/ton and \$/thousand cubic feet respectively, we have parameter ***e*** (BTU/ton or BTU/thousand cubic feet) to convert both of them into \$/BTU.

The revision affects the units of π, which also represent the marginal costs in the objective function of generator's problem. It can be expressed as:

$$\max_{y_i \ge 0,\eta} \left(\eta_i + \varphi_i - \frac{H_j}{\varepsilon_j}\pi_i\right) y_i$$

Appendix B
Incorporation of Carbon Emission Regulations

One of the ways to consider carbon emission concerns is simply to adopt a carbon emission cost p_{co2} with the unit \$/ton. The objective function of the generator's problem is restated as:

$$\max_{y_i \ge 0,\eta}$$

then changed to:

$$\max_{y_i \ge 0,\eta} \left(\eta + \varphi_i - \pi_i\right) y_i - p_{co2}\left(E_i y_i - N_i\right)$$

If the carbon emission $\boldsymbol{E_i y_i}$ exceeds its allowance, the generation company must buy allowances from others for the extra emissions. Otherwise, the generation company can make a profit by selling its unused allowances. In addition, a market clearing equilibrium constraint must be added to the upper level optimization problem [30]:

$$0 \le p_{co2} \perp \left(\sum_i E_i y_i - \sum_i N_i\right) \le 0 \qquad (42)$$

If the total carbon emission is less than the total amount of the allowance, the carbon allowance trade is free. Otherwise, there is price p_{co2}>0 for buying each ton of the carbon allowance.

Appendix C
Setting the Values for *M* in the Binary Reformulation

One way to set the *M* value is to roughly estimate the biggest possible values for all the *r* and *s* in the equilibrium constraints, which is equivalent to estimating the upper bounds of the dual and primal variables.

For the fuel dispatcher's decision problem, Eq (21), (23) and (24) are the relevant equilibrium constraints. The primal variables ***x*** are bounded by ***nU***, the new fuel transportation capacities after expansion. The expansion will not be infinite due to the limited electricity demand. The dual variables ω, ***π*** and ***ρ*** represent how much the objective function changes if the right-hand side (RHS) changes by 1 unit. For Eq. (3) and (5), the largest possible change in objective function happens when an extra unit of the cheapest fuel resource becomes available and substitutes one unit of the most expensive fuel resource, which is estimated as $\overline{c_{gj}} - \underline{c_{gj}} = 60$ in this specific instance. Likewise, we can also obtain the upper bound $\overline{c_{gj}}$ =125 for ***π*** which represents the marginal fuel cost.

For the generator's decision problem, we estimated the bound of the variables in the same manner so that ***y*** is bounded by ***nV***, and $\boldsymbol{\mu_g}$ and $\boldsymbol{\beta_j}$ are both bounded by $\overline{a_j} - \underline{\pi_j}$ =285.

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$(\eta + \varphi_i - \pi_i) y_i - p_{co2} E_i y_i$

The parameter ***E*** is the tons of carbon emitted per MWh of energy produced depending on different generation technologies.

Another way to incorporate the carbon emission is in a cap-and-trade system. Each of the generation companies is given a certain number of carbon emission allowances ***N***. Generation companies are allowed to trade the allowances as long as the total carbon emissions of the system are within the limit [30]. The objective function of the generator's problem is

For ISO's decision problem with equilibrium constraints (29) – (34), ***f***, ***θ*** and ***q*** are bounded by ***K***, $\boldsymbol{\theta_{max}}$ and ***nV***, respectively. The upper bound for ***p*** is ***a***, the intercepts of the inverse demand functions. The dual variables of the voltage angle constraints, $\boldsymbol{\alpha}^+$ and $\boldsymbol{\alpha}^-$, will not affect the objective function because the voltages are always within the bounds. The dual variables $\boldsymbol{\lambda}^+$ and $\boldsymbol{\lambda}^-$ indicate how much the welfare changes if the capacity limit changes by one unit. Since it is quite difficult to estimate the impact of a one unit flow change on ***q***, we can roughly evaluate the largest possible change in ***q*** and the welfare accordingly. Likewise, we obtain

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the bounds for γ^+ and γ^-.

According to the rough estimating result of the bounds, letting each value of *M* be 10,000 will be large enough.

Another way to give the *M* an appropriate guess is to take advantage of the MPCC-NLP solution. It provides a general idea of neighborhoods for the optimal values of the variables, which can be used to estimate a tighter and more realistic *M*. Based on our MPCC-NLP solution, we estimate that 5000 should be large enough for *M*.

In our case study of MPCC-BIN, we tried out different values for *M*, ranging from 2000 to 100000. The results indicate that all of these values are valid for *M* because they all result in the same optimal solution. Better computational performance could be achieved by setting different *M* values for each of the mathematical expressions in the complementarity constraints.

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