Quasicrystal Tilings in 3D and their Empires

Dugan Hammock
Quantum Gravity Research, dugan@quantumgravityresearch.org

Fang Fang
Quantum Gravity Research, fang@quantumgravityresearch.org

Klee Irwin
Quantum Gravity Research, klee@quantumgravityresearch.org

Follow this and additional works at: https://lib.dr.iastate.edu/aperiodic2018

Part of the Chemistry Commons, and the Materials Science and Engineering Commons

Hammock, Dugan; Fang, Fang; and Irwin, Klee, "Quasicrystal Tilings in 3D and their Empires" (2018). Aperiodic 2018 ("9th Conference on Aperiodic Crystals"). 62.
https://lib.dr.iastate.edu/aperiodic2018/2018/abstracts/62

This Poster Presentation is brought to you for free and open access by the Conferences and Symposia at Iowa State University Digital Repository. It has been accepted for inclusion in Aperiodic 2018 ("9th Conference on Aperiodic Crystals") by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Quasicrystal Tilings in 3-Dimensions and their Empires

Dugan Hammock, dugan@quantumgravityresearch.org
Fang Fang, fang@quantumgravityresearch.org
Klee Irwin, klee@quantumgravityresearch.org
April 10, 2018

Abstract

The cut-and-project method for computing quasicrystals is a robust algorithm which provides a mathematical framework for more detailed analysis of the tilings they generate. The method is characterized by a lattice $\Lambda \subset \mathbb{E}^N$ and its projections onto an affine subspace $\pi : \Lambda \rightarrow \mathbb{E}_\parallel \simeq \mathbb{R}^n$. The cut-window $W \subset \mathbb{E}_\perp$ inside the orthogonal complement of $\mathbb{E}_\parallel$ provides a filter for determining which points are incorporated into a particular tiling, $T \subset \mathbb{E}_\parallel$: a point $\lambda_\parallel$ is included in $T$ if and only if $\lambda_\perp$ falls within the cut-window $W$. The cut-window contains regions corresponding to individual tiles, a particular tile is attached to a point $\lambda_\parallel$ if and only if $\lambda_\perp$ falls within that tile’s corresponding region inside $W$. Taking the intersections of overlapping regions decomposes the cut-window into sectors which correspond to individual vertex configurations. Computing the relative volumes of these regions gives analytical values for the vertex frequencies. We also present an algorithm for defining a region in the cut-window which corresponds to the forced tiles, local configuration, and the empire given an arbitrary set of initial tiles. We focus on tilings of $\mathbb{R}^3$ and present constructions and analysis for the Ammann tiling (projection of $\Lambda = \mathbb{Z}_6 \rightarrow \mathbb{R}^3$) as well as a quasicrystal with 36 vertex types ($\Lambda = D_6 \rightarrow \mathbb{R}^3$) as studied extensively by Kramer.

Figure 1: Various sectors in the cut-window corresponding to three different vertex types in a quasicrystal defined by a cut-and-projection of the $D_6$ lattice onto $\mathbb{R}^3$. 