The factor bias of technical change and technology adoption under uncertainty

Tae-Kyun Kim
Iowa State University

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The factor bias of technical change and technology adoption under uncertainty

Kim, Tae-Kyun, Ph.D.
Iowa State University, 1989
The factor bias of technical change and technology adoption
under uncertainty

by

Tae-Kyun Kim

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DEDICATION

To my parents.
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1. PREVIEW

Technical and technological change\(^1\) has probably been one of the major influences on the nature of the human life and can have an impact on welfare, economic indicators, social structures, and society. Thus, this wide topic has been studied not only by economists but by sociologists, geographers, management scientists, and others.

The primary focus of the early economic studies on technical and technological change was simply to investigate the impact of technical change on output, growth, employment, and income distribution. That is, technical change itself was treated as an exogenous rather than an endogenous variable. Since the 1960s, however, economists have attempted to explore the economic determinants of technical change. This theory is referred to as induced innovation.

Despite the abundance of literature that is available in this area, previous studies may not have captured the full impact of uncertainty and risk. To date, the possibility of a link between uncertainty and the bias of technical change has not been examined.

\(^1\)Perhaps one ought to distinguish between technical change and technological change, the former being an improvement of an existing technique and the latter an addition of a new technique to the existing spectrum (Elster, 1983, p. 95). In most of the literature, however, the distinction between the two terms is not made. Thus, we do not distinguish between these two terminologies in this study.
thoroughly. Intuitively, it would appear that such a link should exist. Therefore, the first issue examined in this study is how uncertainty in markets or in production may affect the factor bias of technical change.

Adoption and diffusion of technological innovations have been topics of considerable interest among economists because an economy can not be affected in any material way by new technologies until the use or ownership of those technologies is widespread. In agriculture, this topic has attracted considerable attention because new technologies seem to offer an opportunity to substantially increase production and income.

There is a large body of literature on the effects of government price stabilization programs on producer and consumer welfare and on economic indicators in the agricultural sector. However, to our knowledge, no one has examined how those programs affect the type of technologies that are adopted or the rate of diffusion of new technologies. That there should be some relationship between price variability and technological adoption is clear. Thus, the second issue of this study is to discover the relationship between price variability (risk) and technology adoption (diffusion).

The dissertation is written with the objective of contributing to the development of the theory on technical change and technology adoption. These two issues are analyzed to derive the relationship between uncertainty (risk) and technical change and technology adoption. In addition, some policy implications related to risk and uncertainty are discussed.

The plan of this study is as follows. A brief review of the theory of technical change and the technology adoption models is given in Chapter 2. This chapter
begins with neoclassical theory to explain technical change as just another instance of maximization under constraints. Technology adoption models are also discussed.

Chapter 3 considers the factor bias of technical change under price uncertainty. The first section involves an introduction and a problem statement. An Itô control model is proposed, and the basic assumptions upon which the analysis is based are discussed. Subsequent sections examine the factor bias of technical change with uncertainty in input prices, output price, and output price and an input price. Concluding comments are given in the last section of Chapter 3.

The results of Chapter 3 indicate that, under input price uncertainty, if the elasticity of substitution is greater than one, technical change will be biased toward the input that has the more certain price. Output price uncertainty has no effect on the direction of technical change bias but has an effect on the degree of technical change bias. If the output price and an input price are assumed to be uncertain, the correlation between the two stochastic processes will play an important role in the factor bias of technical change. Assuming that the elasticity of substitution is greater than one, if input price and output price are highly and positively correlated, technical change will be biased toward the input that has an uncertain price. On the other hand, if they are insignificantly or negatively correlated, technical change will be biased toward the input that has a certain price. It is also shown that Hicks's induced innovation proposition may be derived for the deterministic case or the case of risk neutrality.

2There are some other theories that have been used to explain technical change. They are said to be Schumpeter's theories, evolutionary theories, and Marxist theories (see Elster, 1983, pp. 112-184).
In Chapter 4, the bias of technical change is examined in the presence of production uncertainty. It is shown that, if the elasticity of substitution is greater than one, technical progress will be biased toward risk-reducing inputs and against risk-increasing inputs. Another important result provided in this chapter is that the degree of technical change bias would be increased as riskiness increases or as the firm becomes more risk averse.

In Chapter 5, the model is extended to introduce hedging or forward contracts. It is assumed that firms may also participate in the forward markets to reduce risk from output price uncertainty. The results indicate that, under output price uncertainty, the existence of a forward market has no effect on the direction of technical change bias but has an effect on the degree of bias. If uncertainty exists both in output price and input price and the forward market is unbiased, technical progress will be biased toward the input which has a certain price. Concluding comments and some policy implications are discussed in the last section of Chapter 5.

Chapter 6 deals with the issue of price uncertainty and technology adoption. The first section contains an introduction and a motivation. In the subsequent sections, the definitions are given to categorize new technologies, the relationship between price variability and technology adoption and technological change is examined, and an adjustment cost is introduced to explain the dynamic path and the relationship between the speed of diffusion and price variability. The last section of Chapter 6 provides a summary and concluding comments.

The results of Chapter 6 show that a higher level of price variability will be one reason for variable-input-saving technological change. On the other hand, a lower
level of price variability will lead to variable-input-using technological change. It is also shown that a higher level of price variability would be associated with a lower (higher) speed of diffusion for yield-increasing (cost-reducing) technology. An important implication that can be drawn from this chapter is that the introduction of a price stabilization policy will encourage producers to adopt yield-increasing technologies, whereas the removal of one of these schemes will increase the development and adoption of cost-reducing technologies.

Chapter 7 presents a short summary of the general results and policy implications of this study. Itô differentiation and dynamic programming for a vector of state equations, and the relationship between the bias of technical change and factor augmentation are briefly discussed in the Appendix.
2. A REVIEW OF THE BIAS OF TECHNICAL CHANGE AND TECHNOLOGY ADOPTION

2.1 Neoclassical Theories of the Factor Bias of Technical Change

2.1.1 The production function approach

The fundamental neoclassical tool for the study of technical change is the notion of a production function that designates a quantitative relation between inputs and outputs. We might postulate production as a process with many inputs and one output. The standard neoclassical approach assumes that there are two inputs: labor and aggregate capital. Furthermore, all the information in the production function is assumed to be conveyed by the iso-quant, which is stated as the locus of factor combinations that give the same output.

The neoclassical theory of production accepts the concept that the firm adopts a factor combination that maximizes its net revenue or profit. This concept also implies cost minimization for a fixed output level. That is, under perfect competition, the firm chooses the point on the unit iso-quant that lies on the lowest iso-cost curve. Therefore, a change in the factor price ratio will force the firm to convert from one

\footnote{The production process may be understood in several ways because inputs and outputs can be regarded as points or flows.}
factor combination to the other. This conversion is called the substitution effect.

In this framework, technical progress might be defined as a shift of the unit isoquant toward the origin. The rate of technical change is defined as the relative change in total unit costs when the techniques in each period are those that would minimize unit costs when factor prices are constant. The biases of technical change can be measured by the relative change in the factor ratio when factor prices are constant (Salter, 1960, p. 30).

Neoclassical theories can illustrate technical change in terms of rational choice within constraints. This fact suggests that the rate and the bias of technical change should result from a considered choice by the firm.

2.1.2 The factor bias of technical change

2.1.2.1 Definitions The bias of technical change can be defined in terms of proportional change in the capital-labor ratio at constant factor prices (Salter, 1960).

\[
B_1 = \frac{\partial (K/L)}{\partial t} \frac{1}{K/L}.
\]

Technical change is labor saving if \(B_1 > 0\), neutral if \(B_1 = 0\), and capital saving if \(B_1 < 0\).

A labor-saving (capital-using) innovation makes labor more plentiful relative to capital than it was previously, with the result that the marginal product (MP) of labor should fall relative to that of capital. Because the equilibria require that the ratio of marginal products be equal to the ratio of factor prices, this change is equivalent to a rise in the price of capital relative to that of labor. According to this explanation, the bias of technical change may again be defined in terms of the proportional change in
the ratio of marginal products at a constant factor ratio (Thirtle and Ruttan, 1987, p. 15).

\[ B_2 = \frac{\partial(F_k/F_l)}{\partial t} \cdot \frac{1}{F_k/F_l}. \]

Binswanger (1978) showed that \( B_1 \) is actually equal to \( B_2 \) multiplied by the elasticity of substitution.

If, at constant factor prices, the ratio of capital to labor increases, the ratio of the capital share relative to that of labor must also increase. This result implies that technical change must be labor saving if the capital share increases, neutral if capital and labor shares remain constant, and capital saving if the relative capital share falls. Thus, the bias of technical change can be defined in terms of proportional change in the share at constant factor prices (Binswanger, 1974b, p. 964).

\[ B_3 = \frac{\partial \alpha_i}{\partial t} \cdot \frac{1}{\alpha_i}, \]

where \( \alpha_i \) is the share of factor \( i \). This definition has the advantage that it leads to a single measure of bias for each factor in the multi-factor case.

2.1.2.2 Source of technical change bias The orthodox explanation for the source of the factor bias of technical change is that the bias is the result of a change in the relative prices of the factors. A relatively low price of capital (labor) leads to capital-using (labor-using) innovations. The pioneer statement of this view is found in Hicks's proposition:

The real reason for the predominance of labour-saving inventions is surely that which was hinted at in our discussion of substitution. A change in the relative prices of the factors of production is itself a spur to invention, and
to invention of a particular kind—directed to economizing the use of a factor which has become relatively more expensive. The general tendency to a more rapid increase of capital than labour which has marked European history during the last few centuries has naturally provided a stimulus to labour-saving invention. (Hicks, 1963, p. 125)

Labor-saving (capital-saving) innovations seem to be the rational response of firms to rising (decreasing) wages. However, Elster (1983) has suggested that the Hicksian proposition might have "an easily committed logical fallacy." That is, if firms act collectively, the Hicksian proposition may explain the bias of technical change. But, of course, entrepreneurs act individually, not collectively, and so the proposed explanation fails. Assuming that a firm must pay a cost for innovations, it will face a "Prisoners' Dilemma"; i.e., it is better for all firms if all firms act collectively than if none does so, but it is tempting for the individual firm to detect and to benefit from inventions undertaken by the others without making a contribution itself to a public good (Elster, 1983, p. 102). Therefore, Hicks's proposition may not explain rational behavior of the individual firm because of this external effect.

Two models that would make individual firm behavior rational have been proposed by Fellner (1961). The first model drops the assumption of perfect competition. Assuming that imperfect competition exists, the firm can fully internalize the benefit of innovation. The second model is a learning mechanism. The firm may learn from past experience. For example, a past trend of rising wages will cause the firm to search for labor-saving innovations.
Another approach to explaining rational choice of factor bias in technical change was proposed by Kennedy (1964). He assumes that, at any given time, the firm faces an innovation possibility frontier that imposes constraints on technically possible innovations. And the frontier should relate the proportion of one factor that could be saved to the proportion saved of the other factor. However, this approach does not give us any reason to believe that the innovation possibility frontier has a reality for the firm. The theory assumes that the innovations will occur at the point of the frontier that, at the ruling factor prices, permits the greatest reduction in unit cost. However, this assumption does not tell how the firm is supposed to find the frontier and move along it. That is, this theory rests on dubious microeconomic foundations (Nordhaus, 1973, p. 218).

A pioneering attempt to have a microeconomic foundation was the work of Kamien and Schwartz (1969). In their analysis, the question of endogenous factor-augmenting technical change in the context of the profit-maximizing firm was addressed. They apply Kennedy's innovation possibility frontier and assume that the rate of impact of factors and the rate of output are instantaneously variable and factor augmentations are determined at the level at which the revenue is maximized. Under these assumptions, Kamien and Schwartz proved that the technical change of the firm will asymptotically approach Hicks neutral if the elasticity of factor substitution is less than one.

Binswanger (1974b) has developed an induced innovation model incorporating a research production frontier. By assuming decreasing marginal productivity of

---

2 This approach is called the Kennedy-Weiseker-Samuelson innovation possibility frontier.
resources in applied research and development, he constructed a model of induced factor-saving technical change based on the profit-maximizing behavior of the firm without the assumption of a fixed research budget (Hayami and Ruttan, 1985, p. 87).

More recently, Sato and Ramachandran (1987) have shown a profit-maximizing firm that, faced with differential increases in input prices, could develop compensating cost-reducing technologies. They have also proven the Hicksian proposition for a monopolistic firm with an infinite time horizon.

Another approach to explaining technical change is the theory of induced institutional change, which was developed by Hayami and Ruttan (1971). The importance of institutional change can be stressed because many technical changes, especially in agriculture, have been produced by public sector institutions.

2.2 The Measurement of Technical Change Biases

The measurement of technical change biases have generally proceeded in two directions. One is the parametric measures that are obtained from direct specification and estimation of production technology via production, cost, or profit function approaches. The other is nonparametric measures that attempt to investigate the consistency between theoretical constraints of production analysis and observed production behavior.

The pioneering works in the parametric approach were conducted by David and van de Klundert (1965), Sato (1970), and Lianos (1971). All these studies used value-added functions in two-factor models to measure technical change biases.

Binswanger (1974a) proposed a method of measuring the bias of technical change
with multiple factors of production. Technical change biases were measured by estimating a system of cost share equations derived from a homothetic transcendental logarithmic cost function. The changes in cost shares not explained by price changes were interpreted as measurements of the technological change biases. Even though this method was applied to a number of analyses, a severe restriction is homothetic technology. If the technology is nonhomothetic, changes in factor cost shares stem from output changes, scale changes, input price changes, or biased technical changes (Berndt and Khaled, 1979).

A model designed to test for technological advancement, factor-input, and scale bias was developed by Stevenson (1980). He asserted that technological change may be biased with regard both to the factor inputs and to the scale characteristics of the production process (Stevenson, 1980, p. 162).

Weaver (1983) developed an estimation method that was particularly attractive for study of multiple-output, multiple-input technologies. Technological change biases were measured by using the transcendental logarithmic form of the expected profit function.

Antle (1984) estimated profit share equations derived from transcendental logarithmic profit function by using aggregate time-series data for pre-war and post-World War II periods. A time trend was used to represent technological change, and the effects of the time trend on equilibrium factor cost shares was interpreted as a measure of technological change biases. This work found different average biases in the pre- and post-World War II periods and concluded that the differences were consistent with the induced innovation hypothesis.
A problem for the analysis of technical change was suggested by Antle (1986). He showed that the structures of microeconomic and aggregate production models are different. The process that generates aggregate output depends on the information that firms use to form expectations, whereas the process that generates individual firm's output does not. Therefore, aggregate data can be used to measure and explain technological change only if sufficient identifying restrictions are imposed on the econometric model (Antle, 1986).

A common method used in the parametric approach is to specify a cost function or profit function. This method, however, can be valid if the true cost function or profit function is a member of the parametric class considered; otherwise the inference may be biased. Gallant (1981, 1982) and Chalfant and Gallant (1985) addressed the question of functional form and found that Fourier series approximations that may be globally flexible are appropriate. Thus, the use of Fourier form may merit further study in the parametric approach.

The second direction of the measurement of technical change biases is the nonparametric analysis that has the advantage of not being dependent on a particular form of production technology. Early works by Hanoch and Rothschild (1972) provided a foundation for examining the productive efficiency exhibited by observed behavior prior to estimation of parametric models. And, recently, Varian (1984) extended this work.

Chavas and Cox (1988a) extended the nonparametric approach of production decisions developed by Hanoch and Rothschild (1972) and Varian (1984) to incorporate output-augmenting technical change. This approach was empirically implemented
with a standard linear programming algorithm. Following the analysis, Chavas and Cox (1988b) extended the nonparametric production analysis so that it incorporates both Hicks-neutral and biased technical change.

An important point in the measurement of technical change biases was mentioned by Diamond, McFadden, and Rodriguez (1978). They showed the nonidentifiability of the elasticities of substitution and the bias of technical change. That is, one may not be able to identify simultaneously the elasticities of factor substitution and the bias of technical change in the absence of prior information. Depending on the data and a priori hypothesis about the structure of technical change, the measurement of technical change biases may be exactly identified, identified up to a range of indetermining, or not identified at all (Diamond, McFadden, and Rodriguez, 1978). Neither the parametric approach nor the nonparametric approach solves the nonidentifiability problem directly. However, this problem could be circumvented by using prior information.

2.3 Technology Adoption and Diffusion

2.3.1 Theoretical development

2.3.1.1 Probit model approach The adoption of new technology is viewed as a function of the characteristics of firms at one point in time in the static probit and logit approaches. If the exogenous "stimulus" variables that explain adoption behavior change over time, then an increasing proportion of the population will cross the "threshold" and adopt new technology (Thirtle and Ruttan, 1987, p. 108). A diffusion curve can be derived from these dynamic probit models.
Even though this probit model approach has been used in durable good demand analyses, David (1969) rationalized and applied this approach to production process innovations. More recently, a major advance was made by Davies (1979). His model has the advantage of explaining post-innovation technological improvements that are attributed to learning by doing.

2.3.1.2 Learning by doing approach This approach has been extended since Mansfield’s seminal work (1966), which did not present any real theoretical justification for the inclusion of variables such as the expected profitability of a change in technology, the size of a firm, and a firm’s liquidity.

In both Stoneman (1981) and Linder, Fischer, and Pardey (1979), a model of technology diffusion was constructed in which firms learn in a Bayesian fashion from their experience (Stoneman, 1983, p. 77). Stoneman (1981) showed that the intrafirm rate of diffusion of the new technology may follow the sigmoid pattern, assuming that the new technology entails adjustment cost. Feder and O’Mara (1982) formulated an aggregate innovation diffusion model based on the assumption that individual farmers revise their beliefs in a Bayesian fashion.

Jensen (1982) developed a decision-theoretic model of individual firm adoption behavior, which might be used to derive an expected diffusion curve. The approach used was to view adoption as a problem of decision making under uncertainty when learning might occur. The conclusion is that if firms do not know whether an innovation is profitable they may delay adoption in order to gather information and reduce this uncertainty.

Jensen (1983) assumed that firms face a choice between two innovations without
knowing which of the two is better and that firms have *a priori* information that one of the innovations is better, which is updated by Bayes’ rule as trials with the innovations are made. An expected diffusion curve for the better innovation could be derived as an ogive shape.

### 2.3.1.3 Game theoretic approach

Recently, the problem of the timing of adoption in duopoly models has attracted renewed attention. Reinganum (1981a, 1981b) showed that strategic behavior alone can lead to a Nash equilibrium of different adoption dates and can have a diffusion curve. This concept is demonstrated in a duopoly model (Reinganum, 1981a) and an oligopoly model (Reinganum, 1981b). The results also indicated that an increase in the number of firms can delay adoption of new technology.

### 2.3.2 Agricultural adoption studies

In the agricultural economic literature, land allocation between technologies and the input-land ratio of modern inputs under different circumstances have been analyzed. Risk and farm size are the most important variables in land allocation between technologies.

Hiebert (1974) examined the effect of uncertainty resulting from imperfect information on the decision to adopt fertilizer-responsive seed varieties. The results indicated that additional information and the enhanced ability to decode information increase the likelihood of technology adoption. This analysis is consistent with the

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*A good summary on the adoption of agricultural innovations is provided in Feder, Just, and Zilberman (1985).*
analyses of Nelson and Phelps (1966), Welch (1970), and Kislev and Shchori-Bachrach (1973). Feder (1980) assumed that uncertainty is associated only with a new crop (technology) and showed that the land-allocation decision between the old and new crops depends on the relationship between relative risk aversion and income. This analysis was also consistent with Hiebert's (1974) findings.

More recently, Just and Zilberman (1983) extended the land-allocation decision with all inputs. Their results demonstrated that the levels of modern inputs used depends on whether the modern inputs are risk reducing or risk increasing and whether relative risk aversion is increasing or decreasing. In addition, the results also indicated that the stochastic relationship of returns per hectare under the traditional and modern technologies plays a large role in determining the role of farm size in technology adoption.

Another approach of the static analysis is the "safety-first" type of models that assume that the utility of income is zero below a specific level and one above that level. Bell (1972) showed that this specific level is positively related to the farmer's wealth. Roumasset (1976) demonstrated that nonadoption of new high-yielding varieties may be the result of a higher disaster-level yield probabilities associated with high-yielding varieties in rain-fed crops.

The adoption models that have been surveyed thus far consider adoption of a single innovation. However, modern technologies may be introduced as a package with several components. Feder (1982) examined adoption decisions involving two interrelated agricultural technology innovations: One was neutral to scale, the other was a lumpy innovation with capital. The results indicated that adoption behavior
depends on the degree of complementarity between two technological innovations and on a binding constraint. Byerlee and de Polanco (1986) examined the stepwise adoption of technological packages empirically by using on-farm experimental and survey data.

2.3.2.1 Limitations Even though the adoption models in the agricultural economic literature might well be useful, there are some limitations that must be solved. First, yields are the only random variables in most analytical models of adoption behavior under uncertainty. In reality, output and input prices also may be random variables, and their uncertainty may affect technological choices and adoption. Some of the implications of output-price uncertainty on adoption behavior can be deduced from models with yield uncertainties by interpreting yield functions as revenue functions (Feder, Just, and Zilberman, 1985).

Second, the nonexistence of government agricultural policies in most adoption models is troublesome. For example, price stabilization schemes will affect the type of technologies that are adopted or the rate of diffusion of new technologies because these schemes may remove the risk that is related to the adoption of a technology. Price support schemes, food taxes and subsidies, and input and output quotas may also affect diffusion processes.

Third, most adoption researches have thus far viewed the adoption decision in dichotomous terms (adoption or nonadoption). However, an interesting question might be related to the intensity of use (e.g., the percentage of land that is planted to high-yielding varieties).
Finally, most analytical models consider yield-increasing\(^4\) technologies such as high-yielding varieties. However, these models may not explain the technology innovation that reduces the average variable cost per hectare (cost-reducing technologies) because these are different concepts for a risk-averse firm whose utility function is non-linear in wealth (concave). Further studies should rectify this problem by properly differentiating technologies.

\(^4\)The definitions of yield-increasing and cost-reducing technologies are presented in Chapter 6.
3. THE FACTOR BIAS OF TECHNICAL CHANGE UNDER PRICE UNCERTAINTY

3.1 Introduction

Economists have long believed that relative prices play an important role in economic progress. Hicks (1963) proposed that technological or technical innovations are the result of a change in the relative prices of the production factors. This proposal is referred to as the induced innovation theory. The theory has been modeled in microeconomic terms and tested empirically.¹

Although models of this type may be useful descriptively and empirically, they may not capture the essential features of uncertainty that can affect the rate and bias of technical change. To date, the possibility of a link between uncertainty and the bias of technical change has not been examined. Intuitively, it seems that such a link should exist. The adoption of any particular innovation involves risk. One would imagine that the decision to develop and adopt an innovation would depend on the degree of uncertainty and risk to the producer. It has not always been possible to

explain why certain technologies have been developed and adopted in one country or region and not in another. Relative prices of factors are, of course, important in this process but may not be the only determining factor.

The purpose of this chapter is to investigate the effects of price uncertainty on the bias of technical change. The results will show that, under input price uncertainty, assuming the same rates of growth of input prices and assuming a risk-averse firm, technical change will be biased toward the inputs that have the less variable prices if the elasticity of substitution is greater than one. The direction of technical change bias will not be affected by output price uncertainty, but the degree of bias will be influenced. The correlation between input prices and output price will affect the bias of technical change. Assuming input and output price uncertainty, technical change will be biased toward inputs whose prices are negatively or slightly positively related to output price if the elasticity of substitution is greater than one.

Because technical change is assumed to be based on the stock of accumulated technical knowledge, a dynamic model is developed in the following section. This model must, of necessity, be stochastic; consequently, we use Itô calculus to develop a stochastic control model that contains endogenous factor-augmenting technical change.

In the next section, an Itô control model is proposed and the basic assumptions upon which the analysis will be based are described. In subsequent sections, the effect of uncertainty on technical change in only the input prices, in only the output price, and in both the output price and one input price are analyzed. The last section provides concluding remarks.
3.2 The Basic Model

Assuming the presence of factor-augmenting technical change, the production function can be written as

\[ Y(t) = F[A(t)X_1(t), B(t)X_2(t)], \]

where \( F[.] \) is assumed to be a differentiable quasi-concave function, \( X_1(t) \) and \( X_2(t) \) are the levels of two inputs at time \( t \), and \( A(t) \) and \( B(t) \) represent the levels of factor augmentation at time \( t \).

It is assumed that a representative firm can fully internalize the benefits from its research.\(^2\) This assumption limits the analysis to those firms that can obtain patents or to government-sponsored research programs that conduct research on problems specific to the domestic environment. A less formal interpretation would allow us to view the research that ensues as that which would be desired by those firms that ultimately use the proceeds of the research. If these firms signal researchers via market forces or the political process, then research programs should reflect the desires of these end users in a manner that is consistent with the assumption.

We also assume that the firm finances all its research internally and that the levels of research expenditure as a proportion of total revenue, \( e_i PY \), determine the rate of

\(^2\)If we do not assume full internalization, there will be an economic externality; that is, an individual firm can do nothing to capture the benefits of technical change through that firm's own behavior. A very common assumption in the industrial organization literature on endogenous technical progress is a noncompetitive market structure (Sato and Ramachandran, 1987). However, for simplicity, this study does not assume a noncompetitive market.
increase of $A(t)$ and $B(t)$ according to the following technical progress functions:  

$$dA = Ah_1(e_1PY)dt,$$

$$dB = Bh_2(e_2PY)dt,$$

where $h_1(.)$ and $h_2(.)$ are assumed to be continuous and twice differentiable, $h_1' > 0 > h_1''$, $P$ is the output price, and $e_i$ is the endogenously determined rate of research investment.

Suppose that the firm maximizes the expected discounted utility of the terminal state variables ($W$, $A$, and $B$) such that

$$J(S_0) = \max E[e^{-\gamma T}V(S_T)|S_0 = S_0],$$

where $J$ represents the indirect utility function resulting from the optimization, $S$ denotes a vector of state variables ($W$, $A$, and $B$), $r$ is the rate of firm’s time preference, $T$ is the terminal time, and $V(.)$ represents the utility function of terminal state variables. Because we assume that the firm may sell not only its wealth but also the factor augmentation or accumulated information ($A$ and $B$) at the terminal time, the firm’s indirect utility function depends on all state variables.

The firm’s wealth is assumed to consist of an inventory of risk-free assets, $L$, valued at the market price, $P_L$ (negative $L$ denotes liabilities), so that

$$W = P_LL.$$  

---

This function represents the impact of research expenditure on the rate of change of factor augmentation. This model does not fully take into account the delayed or lag effects of the stocks of basic and applied knowledge on productivity gains. For a detailed discussion on the influences of basic and applied knowledge, see Sato and Suzawa (1983).
Assuming an inventory of risk-free assets whose market prices change over time, the change in wealth can be calculated by Itô's lemma as

\[ dW = LdP_L + P_L dL + dP_L dL. \]  \hspace{1cm} (3.3)

The change in wealth equals net income from all sources. The term \( LdP_L \) represents capital gains on the current inventory of assets. The last two terms on the right-hand side of (3.3) are the net value of additions to wealth from sources other than capital gains (Merton, 1971). Furthermore, for simplicity, we assume that there are no risky assets and no consumption.\(^5\) Thus, these two terms represent income or losses from production. This specification can be presented as

\[ (P_L + dP_L)dL = \pi dt + d\pi, \]

where \( \pi dt + d\pi \) is the realization of production income, \( P_L + dP_L \) is the price at which risk-free assets are purchased in the immediate future, and \( dL \) is the quantity of assets purchased.

Therefore, the change in wealth equals capital gains or losses on the inventory of risk-free assets plus the realization of production income such that

\[ dW = LdP_L + \pi dt + d\pi. \]  \hspace{1cm} (3.4)

The price of risk-free assets is assumed to change as follows:

\[ \frac{dP_L}{P_L} = \delta_L dt, \]  \hspace{1cm} (3.5)

\(^4\)See (8.2) in the Appendix.

\(^5\)For a detailed discussion on the portfolio and consumption, see Merton (1971).
where $\delta_L$ is the known growth rate for the price of risk-free assets. Therefore, the change in the firm's wealth can be represented as

$$dW = (\delta_L W + \pi)dt + d\pi.$$  \hspace{1cm} (3.6)

Equations (3.1) through (3.6) comprise the basic framework. To analyze the effects of price uncertainty on the bias of technical change, some additional assumptions are required. These assumptions are specified in the sections that follow.

### 3.3 Input Price Uncertainty

We assume, that at the time of decision making, the output price and production level are known; however, the two input prices, $P_1$ and $P_2$, are uncertain, and the firm expects these prices to be log-normally distributed as

$$dP_1 = P_1 \delta_1 dt + P_1 \sigma_1 dz_1, \text{ and}$$

$$dP_2 = P_2 \delta_2 dt + P_2 \sigma_2 dz_2,$$

where $\delta_i$ is the forecast for the growth rate of the price of the $i^{th}$ input, $\sigma_i$ is the standard deviation of the forecast, $z_i$ is a Wiener process in which $E[dz_i(t)] = 0$, $E[dz_i(t)]^2 = dt$, and $E[dz_1 dz_2] = \gamma_{12}$, and $\gamma_{12}$ is the contemporaneous correlation coefficient between two stochastic processes.

The firm chooses the levels of input use, $X_1$ and $X_2$, and the research expenditures, $\epsilon_1$ and $\epsilon_2$, at the beginning of each production period. The firm receives the known price per unit, $P$, at the end of each period. Expected production income at the initial time can, therefore, be written as

$$\pi = (1 - \epsilon_1 - \epsilon_2)PY - P_1 X_1 - P_2 X_2.$$ \hspace{1cm} (3.7)
If we stochastically differentiate this equation by using Itô’s lemma [see (8.2) in the Appendix], we get

\[ d\pi = -X_1 dP_1 - X_2 dP_2 \]

\[ = \left[ -X_1 P_1 \delta_1 - X_2 P_2 \delta_2 \right] dt - \left[ X_1 P_1 \sigma_1 ; X_2 P_2 \sigma_2 \right] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}. \]

The stochastic production income in the immediate future can be obtained by adding an expression representing \( \pi dt \) and by using the above equation such that

\[ \pi dt + d\pi = \left[ (1 - \epsilon_1 - \epsilon_2) P_Y - P_1 X_1 (1 + \delta_1) - P_2 X_2 (1 + \delta_2) \right] dt \quad (3.8) \]

\[ -\left[ X_1 P_1 \sigma_1 ; X_2 P_2 \sigma_2 \right] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}. \]

By substituting (3.8) into (3.6), the accounting identity that is the change in the firm’s wealth can be expressed as

\[ dW = \left[ \delta W + (1 - \epsilon_1 - \epsilon_2) P_Y - P_1 X_1 (1 + \delta_1) - P_2 X_2 (1 + \delta_2) \right] dt \quad (3.9) \]

\[ -\left[ X_1 P_1 \sigma_1 ; X_2 P_2 \sigma_2 \right] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}. \]

The model specifies that the firm’s objective is to maximize the expected discounted utility of terminal stock of state variables, (3.2), subject to the accounting identity, (3.9), and technical progress constraints, (3.1), where the levels of input use, \( X_1 \) and \( X_2 \), and the rates of research expenditures, \( \epsilon_1 \) and \( \epsilon_2 \), are control and decision variables, and wealth, \( W \), and factor augmentation, \( A \) and \( B \), are the stock and state variables, respectively.
The Itô version of the Bellman equation\(^6\) associated with this model specification may be derived as follows.

\[
-J_t = \text{Max} \left[ J_w \{ \delta L W + (1 - \epsilon_1 - \epsilon_2)PY - P_1 X_1 (1 + \delta_1) \right. \\
- \left. P_2 X_2 (1 + \delta_2) \} + J_A \{ Ah_1 (\epsilon_1 PY) \} + J_B \{ Bh_2 (\epsilon_2 PY) \} \\
+ \frac{1}{2} J_{ww} (-P_1 X_1 \sigma_1 ; -P_2 X_2 \sigma_2) \begin{pmatrix} 1 & \gamma_{12} \\ \gamma_{12} & 1 \end{pmatrix} \begin{pmatrix} -P_1 X_1 \sigma_1 \\ -P_2 X_2 \sigma_2 \end{pmatrix},
\]

where \( J_w \) and \( J_{ww} \) are, respectively, the first- and second-order derivatives with respect to wealth, and \( J_A \) and \( J_B \) are the first-order derivatives with respect to A and B, respectively.

The expression to be maximized is a dynamic certainty equivalent denominated in utils. For convenience, we can call this expression a stochastic Hamiltonian.\(^7\) The stochastic Hamiltonian and first-order conditions\(^8\) for \( X_1, X_2, \epsilon_1, \) and \( \epsilon_2 \) can be written as

\[
H = J_w \{ \delta L W + (1 - \epsilon_1 - \epsilon_2)PY - P_1 X_1 (1 + \delta_1) - P_2 X_2 (1 + \delta_2) \}
+ J_A \{ Ah_1 (\epsilon_1 PY) \} + J_B \{ Bh_2 (\epsilon_2 PY) \}
+ \frac{1}{2} J_{ww} (-P_1 X_1 \sigma_1 ; -P_2 X_2 \sigma_2) \begin{pmatrix} 1 & \gamma_{12} \\ \gamma_{12} & 1 \end{pmatrix} \begin{pmatrix} -P_1 X_1 \sigma_1 \\ -P_2 X_2 \sigma_2 \end{pmatrix},
\]

\[
\frac{\partial H}{\partial X_1} = J_w \{ (1 - \epsilon_1 - \epsilon_2)PF_1 A - P_1 (1 + \delta_1) \} + J_A \{ A^2 h_1^t \epsilon_1 PF_1 \} 
+ J_B \{ ABh_2^t \epsilon_2 PF_1 \} + J_{ww} \{ P_1^2 X_1 \sigma_1^2 + P_1 \sigma_1 P_2 X_2 \sigma_2 \gamma_{12} \} = 0,
\]

\(^6\)See the Itô version of the Bellman equation in the Appendix.

\(^7\)Malliaris and Brock (1982).

\(^8\)See (8.3) and (8.4) in the Appendix
\[ \frac{\partial H}{\partial X_2} = J_w\{(1 - \epsilon_1 - \epsilon_2)PF_2B - P_2(1 + \delta_2)\} + J_A\{ABh_1^I\epsilon_1PF_2\} \]
\[ + J_B\{B^2h_2^I\epsilon_2PF_2\} + J_{ww}\{P_2^2X_2\sigma_2^2 + P_1\sigma_1X_1P_2\sigma_2\gamma_{12}\} = 0, \]
\[ \frac{\partial H}{\partial \epsilon_1} = J_w\{-PF\} + J_A\{Ah_1^IPF\} = 0, \quad \text{and} \quad (3.13) \]
\[ \frac{\partial H}{\partial \epsilon_2} = J_w\{-PF\} + J_B\{Bh_2^IPF\} = 0, \quad \text{and} \quad (3.14) \]

where \( F_1 \) and \( F_2 \) are the first-order derivatives with respect to \( AX_1 \) and \( BX_2 \), and \( h_1^I \) and \( h_2^I \) are the first-order derivatives with respect to \( \epsilon_1 \) and \( \epsilon_2 \), respectively.

Substituting (3.13) and (3.14) into (3.11) and (3.12) yields the following input decision rules:

\[ PF_1A = P_1(1 + \delta_1) - \frac{J_{ww}}{J_w}\{P_1^2X_1\sigma_1^2 + P_1\sigma_1P_2X_2\sigma_2\gamma_{12}\}, \quad \text{and} \quad (3.15) \]
\[ PF_2B = P_2(1 + \delta_2) - \frac{J_{ww}}{J_w}\{P_2^2X_2\sigma_2^2 + P_1\sigma_1P_2\sigma_2\gamma_{12}\}. \quad \text{(3.16)} \]

If there is no uncertainty, i.e., \( \sigma_1 = 0 \) and \( \sigma_2 = 0 \), then (3.15) and (3.16) indicate that the firm will apply inputs to the point at which their marginal-value-products equal their purchase prices. Because \( J \) denotes the indirect utility function resulting from optimization, \( -\frac{J_{ww}}{J_w} \) will be the Arrow-Pratt measure of absolute risk aversion. For a risk-neutral firm, \( J \) is a linear function of wealth \( (J_{ww} = 0) \); therefore, (3.15) and (3.16) will be identical to the deterministic case. For a risk-averse firm, \( \frac{J_{ww}}{J_w} \) is negative and the marginal risk premium is positive. In this case, (3.15) and (3.16) indicate that, if there is no correlation between input prices \( (\gamma_{12} = 0) \), a risk-averse firm will apply less inputs than will a risk-neutral firm. With a negative correlation
between $dz_1$ and $dz_2$ ($\gamma_{12} < 0$), the marginal risk premium will be smaller and a risk-averse firm will, therefore, apply more inputs than it would in the case where no correlation exists between input prices.

Combining (3.13) and (3.14) yields

$$J_w = J_A h_1^t = J_B h_2^t.$$  

(3.17)

This equation indicates that the firm will allocate research expenditures, $\epsilon_1$ and $\epsilon_2$, to the point at which the marginal utility of wealth, $J_w$, equals the marginal utility of $A$ and $B$, times the stock level of $A$ or $B$, times the first derivative of the technical progress function with respect to research expenditures.

Partially differentiating (3.17) with respect to $W$, $A$, and $B$ and Itô differentiating (3.17) yield

$$J_{ww} = J_{Aw} h_1^t = J_{Bw} h_2^t,$$  

and

$$Ah_1^t dJ_A = Bh_2^t dJ_B.$$  

(3.18)

(3.19)

The changes in marginal indirect utility over time [see (8.5) in the Appendix], which are also first-order conditions for the stochastic control model, can be written as

$$dJ_w = -J_w \delta L dt - J_{ww} [P_1 X_1^* \sigma_1 ; P_2 X_2^* \sigma_2] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix},$$  

(3.20)

$$dJ_A = -[J_w \{(1 - \epsilon_1^* - \epsilon_2^*)P F_1^* X_1^*\} + J_A \{h_1^* + A h_1^* P F_1^* X_1^*\} + J_B \{B h_2^* \epsilon_2^* P F_1^* X_1^*\}] dt$$

$$-J_{Aw} [P_1 X_1^* \sigma_1 ; P_2 X_2^* \sigma_2] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix},$$

and

\[ dJ_B = -[J_w(1 - \epsilon_1^* - \epsilon_2^*)PF^*_2X^*_2) + J_A\{Ah'^*_1PF^*_2X^*_2}\] 
\[ +J_B\{h^*_2 + Bh'^*_2PF^*_2X^*_2\}]dt \]
\[ -J_{Bw}[P_1X^*_1\sigma_1; P_2X^*_2\sigma_2] \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix}, \]

where * represents the optimal value. Substituting (3.17), (3.18), (3.21), and (3.22) into (3.19) and simplifying yields

\[ [J_wPF^*_1X^*_1Ah'^*_1 + J_wh'_1]dt = [J_wPF^*_2X^*_2Bh'^*_2 + J_wh'_2]dt. \] (3.23)

Combining (3.1) and (3.23) gives

\[ \dot{A} - \frac{\dot{B}}{B} = h'^*_1 - h'^*_2 = PF^*_2BX'^*_2h'^*_2 - PF^*_1AX'^*_1h'^*_1, \] (3.24)

where \( \dot{A} \) and \( \dot{B} \) are the time rates of change, \( \dot{A} = \frac{dA}{dt} \) and \( \dot{B} = \frac{dB}{dt} \).

Substituting (3.15) and (3.16) into (3.24) yields an explicit equation that explains the relationship between technical change and input price uncertainty.

\[ \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_2(1 + \delta_2)X^*_2h'^*_2 - P_1(1 + \delta_1)X^*_1h'^*_1 \]
\[ -J_{ww}\frac{P_2^2X^*_2\sigma_2^2 + P_1X^*_1\sigma_1P_2\gamma_12\sigma_2^2}{J_{ww}}X^*_2h'^*_2 \]
\[ + J_{ww}\frac{P_1^2X^*_1\sigma_1^2 + P_1\sigma_1P_2X^*_2\sigma_2\gamma_12}{J_{ww}}X^*_1h'^*_1. \] (3.25)

This equation implies that factor augmentations are determined by the expected cost share, \( P_i(1 + \delta_i)X^*_i \), the marginal productivity of the technical progress function, \( h'^*_i \), the Arrow-Pratt measure of absolute risk aversion, \( -\frac{J_{ww}}{J_{ww}} \), the standard deviation of the forecast, \( \sigma_i \), and the contemporaneous correlation coefficient, \( \gamma_{12} \).

In the deterministic case (\( \sigma_1 = 0 \) and \( \sigma_2 = 0 \)) or in the case of a risk-neutral firm \( (J_{ww} = 0) \), and assuming, without loss of generality, that \( P_1X^*_1 = P_2X^*_2 = P_iX^*_i \),
and that $h_1^{t*} = h_2^{t*} = h_i^{t*}$, the equation will be.

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_i X_i^{t*} h_i^{t*} (\delta_2 - \delta_1).$$

By substituting this equation into (8.13) in the Appendix, we can derive the bias of technical change as

$$\Omega = \frac{(1 - e)}{e} [P_i X_i^{t*} h_i^{t*} (\delta_2 - \delta_1)],$$

where $\Omega$ denotes the Hicks definition of technical change bias and $e$ represents the elasticity of substitution. This equation leads directly to the Hicksian proposition and corollary.

**Proposition 1 (Hicksian):** Assuming no uncertainty or for a risk-neutral firm and assuming that the elasticity of substitution is greater than one, if the expected growth rate of the price of input 1, $\delta_1$, is greater (less) than that of input 2, $\delta_2$, then technical change will be biased toward input 2 (input 1).10

**Corollary 1:** The Hicksian proposition can be completely identified in the deterministic case or for a risk-neutral firm.

For a risk-averse firm, however, input price variability will also influence the bias of technical change. Assuming that $P_1 X_1^{t*} = P_2 X_2^{t*} = P_i X_i^{t*}$ and that $h_1^{t*} = h_2^{t*} = h_i^{t*}$, then (3.25) can be simplified to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_i X_i^{t*} h_i^{t*} (\delta_2 - \delta_1) - \frac{J_{ww}}{J_w} P_i^2 X_i^{2*} h_i^{t*} (\sigma_2^2 - \sigma_1^2).$$

10Sato and Ramachandran (1987) have also proved this Hicksian proposition.
If the firm is risk averse, \( J_{ww} \) is negative and (3.26) implies that factor augmentations will depend on \( \delta_1, \delta_2, \sigma_1^2, \) and \( \sigma_2^2. \)

Substituting (3.26) into (8.13) in the Appendix yields

\[
\Omega = \frac{(1 - e)}{e} \left[ P_i X_i^* h_i^* (\delta_2 - \delta_1) - \frac{J_{ww}}{J_w} P_i^2 X_i^2 h_i^* (\sigma_2^2 - \sigma_1^2) \right].
\]

Note that, even if there is no difference in the expected growth rates of input prices, technical change will be biased so long as input prices are uncertain and \( \sigma_1^2 \) does not equal \( \sigma_2^2. \) This equation also suggests that, if the elasticity of substitution is greater than one, endogenous technical change will be biased toward the input that has the more certain price. These results are summarized in the following proposition and corollary.

**Proposition 2:** For a risk-averse firm, even if there is no difference in the expected growth rates of input prices, the rate of technical change will be biased because of the existence of uncertainty in input prices. Assuming that \( \delta_1 = \delta_2 \) and \( e > 1, \) if \( \sigma_2^2 > \sigma_1^2, \) then technical change will be biased toward input 1 (\( \Omega < 0 \)). If \( \sigma_2^2 < \sigma_1^2, \) then technical change will be biased toward input 2 (\( \Omega > 0 \)).

**Corollary 2:** For a risk-averse firm facing different expected growth rates in input prices and input price uncertainty, the possibility of Hicks-neutral technical change (\( \Omega = 0 \)) may not be excluded because of the existence of input price uncertainty and risk.
3.4 Output Price Uncertainty

Assume that, at the time of decision making, the two input prices are known but that the output price is uncertain and that the firm expects the input and output prices to be log-normally distributed as follows:

\[ dP_1 = P_1 \delta_1 dt, \]
\[ dP_2 = P_2 \delta_2 dt, \] and
\[ dP = P \delta dt + P \sigma dz_p. \]

Without the loss of generality, we can assume that research expenditures are chosen as expenditure levels rather than as a proportion of total revenue. Expected production income at the current moment may be expressed as

\[ \pi = PY - P_1 X_1 - P_2 X_2 - E_1 - E_2, \]

where \( E_1 \) and \( E_2 \) are the research expenditures on \( A \) and \( B \), respectively. In addition, the technical progress functions can be written as

\[ dA = Ah_1(E_1) dt, \] and \( dB = Bh_2(E_2) dt. \] (3.27)

The change in production income and the change in the firm's wealth can be derived by Itô differentiation [see (8.2) in the Appendix] as

\[ d\pi = YdP - X_1 dP_1 - X_2 dP_2 \]
\[ = [YP - X_1 P_1 \delta_1 - X_2 P_2 \delta_2] dt + YP \sigma dz_p, \] and
\[ dW = [\delta L W + PY(1 + \delta_p) - P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2) \]
\[ - E_1 - E_2] dt + PY \sigma dz_p. \]
The stochastic Hamiltonian and first-order conditions for control variables can be written as [see (8.3) and (8.4) in the Appendix]

\[
H = J_w[\delta LW + P(1 + \delta_p)F(AX_1, BX_2) - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) - E_1 - E_2] + J_A[Ah_1(E_1)] + J_B[Bh_2(E_2)] + \frac{1}{2}J_{ww}P^2F^2\sigma_p^2
\]

\[
\frac{\partial H}{\partial X_1} = J_w[P(1 + \delta_p)F_1A - P_1(1 + \delta_1)] + J_{ww}[P^2\sigma_p^2FF_1A] = 0, \quad (3.28)
\]

\[
\frac{\partial H}{\partial X_2} = J_w[P(1 + \delta_p)F_2B - P_2(1 + \delta_2)] + J_{ww}[P^2\sigma_p^2FF_2B] = 0, \quad (3.29)
\]

\[
\frac{\partial H}{\partial E_1} = -J_w + J_AAh'_1 = 0, \quad (3.30)
\]

\[
\frac{\partial H}{\partial E_2} = -J_w + J_BBh'_2 = 0. \quad (3.31)
\]

By combining the provided first-order conditions, input decision rules under output price uncertainty can be depicted as

\[
P(1 + \delta_p)F_1A = P_1(1 + \delta_1) - \frac{J_{ww}}{J_w}[P^2\sigma_p^2FF_1A], \quad \text{and}
\]

\[
P(1 + \delta_p)F_2B = P_2(1 + \delta_2) - \frac{J_{ww}}{J_w}[P^2\sigma_p^2FF_2B].
\]

In the deterministic case or for the case of a risk-neutral firm, the firm applies inputs to the point at which the marginal-value product equals input price. For a risk-averse firm, inputs will be utilized to the point at which the marginal-value product equals input price plus a marginal risk premium. Thus, a risk-averse firm will apply less inputs than will a risk-neutral firm.
Combining first-order conditions (3.30) and (3.31) yields

\[ J_w = J_A h_1'(E_1) = J_B h_2'(E_2). \]  

(3.32)

\( E_1 \) and \( E_2 \) will be determined at the point at which the marginal utility of wealth equals the marginal utility of \( A \) or \( B \), times the level of factor augmentation, times the marginal productivity of the technical progress function with respect to research expenditures.

To derive a relationship between output price uncertainty and the bias of technical change, we differentiate (3.32) and find the change in marginal utility over time as [see (8.5) in the Appendix]

\[
J_{ww} = J_A h_1' = J_B h_2', \tag{3.33}
\]

(3.33)

\[
A h_1' dJ_A = B h_2' dJ_B, \tag{3.34}
\]

(3.34)

\[
dJ_w = -J_w \delta_L dt + J_{ww} P F^* \sigma_p dzp, \tag{3.35}
\]

(3.35)

\[
dJ_A = -[J_w P(1 + \delta_p) F_1^* X_1^* + J_A h_1^* + J_{ww} P^2 F^* F_1^* X_1^* \sigma_p^2] dt + J_{Aw} P F^* \sigma_p dzp, \tag{3.36}
\]

(3.36)

\[
dJ_B = -[J_w P(1 + \delta_p) F_2^* X_2^* + J_B h_2^* + J_{ww} P^2 F^* F_2^* X_2^* \sigma_p^2] dt + J_{Bw} P F^* \sigma_p dzp. \tag{3.37}
\]

(3.37)

Substituting (3.33), (3.36), and (3.37) into (3.34) and clarifying yields

\[
[J_w P(1 + \delta_p) F_1^* X_1^* A h_1'^* + J_w h_1'^* + J_{ww} P^2 F^* F_1^* X_1^* \sigma_p^2 A h_1'^*] dt \tag{3.38}
\]

\[
= [J_w P(1 + \delta_p) F_2^* X_2^* B h_2'^* + J_w h_2'^* + J_{ww} P^2 F^* F_2^* X_2^* \sigma_p^2 B h_2'^*] dt.
\]

Combining (3.27) and (3.38) yields

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = h_1'(E_1^*) - h_2'(E_2^*) = P(1 + \delta_p) F_2^* X_2^* B h_2'^* \tag{3.39}
\]
Substituting the first-order conditions (3.28) and (3.29) into (3.39) yields
\[
\frac{A}{B} - \frac{\hat{A}}{\hat{B}} = X_2^* h_2^{**} P_2 (1 + \delta_2) - X_1^* h_1^{**} P_1 (1 + \delta_1).
\]
Assuming that \( P_1 X_1^* = P_2 X_2^* = P_i X_i^* \) and \( h_1^{**} = h_2^{**} = h_i^{**} \), the equation can be simplified as
\[
\frac{A}{B} - \frac{\hat{A}}{\hat{B}} = P_i X_i^* h_i^{**} (\delta_2 - \delta_1).
\]
Substituting this equation into (8.13) in the Appendix yields
\[
\Omega = \frac{1 - e}{e} [P_i X_i^* h_i^{**} (\delta_2 - \delta_1)].
\]

This equation indicates that output price uncertainty has no impact on the direction of technical change bias. However, note that the optimal input use by a risk-averse firm is less than that of a risk-neutral firm. Moreover, the optimal input use decreases as \( -\frac{J_{ww}}{J_w} \) increases or as \( \sigma_p^2 \) increases. These relationships lead directly to the following proposition and corollaries.

**Proposition 3:** Assuming that the output price is uncertain and that \( e > 1 \), if the expected growth rate of the price of input 1, \( \delta_1 \), is greater (less) than that of input 2, \( \delta_2 \), then the technical change will be biased toward input 2 (input 1).

**Corollary 3:** The degree of technical change bias for a risk-averse firm is less than that for a risk-neutral firm or for the deterministic case.
Corollary 4: If the firms are risk averse, then the degree of technical change bias would decrease as the firm becomes more risk averse or as the riskiness increases.

### 3.5 Input and Output Price Uncertainty

In this section, we assume that the output price, $P$, and an input price, $P_1$, are unknown at the moment of decision making and that the firm expects all prices to be log-normally distributed. The other input price, $P_2$, is assumed to be certain.

\[
\begin{align*}
    dP &= P\delta_p dt + P\sigma_p dz_p, \\
    dP_1 &= P_1\delta_1 dt + P_1\sigma_1 dz_1, \text{ and} \\
    dP_2 &= P_2\delta_2 dt.
\end{align*}
\]

Under these assumptions, expected production income can be defined as

\[
\pi = PF(AX_1, BX_2) - P_1X_1 - P_2X_2 - E_1 - E_2.
\]

By differentiating stochastically, using (8.2) in the Appendix, and manipulating, the stochastic production income in the immediate future and the change in the firm’s wealth can be written as

\[
\pi dt + d\pi = [P(1 + \delta_p)F - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) - E_1 - E_2]dt \\
+ [PF\sigma_p ; -P_1X_1\sigma_1] \begin{bmatrix} dz_p \\ dz_1 \end{bmatrix},
\]

\(^{11}\)It is possible to derive similar results in the situation in which all prices, $P$, $P_1$, and $P_2$, are uncertain. The model, however, will be more complex.
and

\[ dW = [\delta L W + P(1 - \delta p)F - P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2)] \]
\[ -E_1 - E_2]dt + [PF \sigma_p^2 - P_1 X_1 \sigma_1^2] \begin{bmatrix} \frac{d\gamma_1}{d\gamma_1} \\ \frac{d\gamma_2}{d\gamma_2} \end{bmatrix}. \]

The firm’s objective is to maximize the expected discounted utility of terminal state variables, (3.2), subject to the accounting identity, (3.40), and technical progress constraints, (3.27). The stochastic Hamiltonian and first-order conditions for \( X_1, X_2, E_1, \) and \( E_2 \) can be written as [see (8.3) and (8.4) in the Appendix.]

\[ H = J_W[\delta L W + P(1 + \delta p)F(A X_1, B X_2) - P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2)] \]
\[ -E_1 - E_2] + J_A[A h_1(E_1)] + J_B[B h_2(E_2)] + \frac{1}{2} J_{\varphi w}[PF \sigma_p] - P_1 X_1 \sigma_1^2 \begin{bmatrix} 1 & \gamma_1 \sigma_1^2 & \sigma_p^2 \end{bmatrix} \begin{bmatrix} -P_1 X_1 \sigma_1^2 \end{bmatrix}, \]

\[ \frac{\partial H}{\partial X_1} = J_W[P(1 + \delta p)F_1 A - P_1(1 + \delta_1)] + J_{\varphi W}[P\sigma_1^2 \sigma_p F F_1 A] \]
\[ -PP_1 \sigma_1 \gamma_1 F \sigma_p - P_1 X_1 \sigma_1 \sigma_p \sigma_p F_1 A + P_1^2 X_1 \sigma_1^2_1 = 0, \]

\[ \frac{\partial H}{\partial X_2} = J_W[P(1 + \delta p)F_2 B - P_2(1 + \delta_2)] + J_{\varphi W}[P^2 \sigma_2^2 \sigma_p F F_2 B] \]
\[ -P_1 X_1 \sigma_1 \gamma_1 \sigma_p F_2 B = 0, \]

\[ \frac{\partial H}{\partial E_1} = -J_W + J_A A h_1^f = 0, \]

\[ \frac{\partial H}{\partial E_2} = -J_W + J_B B h_2^f = 0, \]
where $\gamma_{P1}$ denotes the contemporaneous correlation coefficient between two stochastic processes, $P$ and $P_1$.

In the deterministic case ($\sigma_P = 0$ and $\sigma_1 = 0$) or for the case of a risk-neutral firm ($J_{ww} = 0$), the firm applies inputs to the point at which the expected marginal-value product equals the respective expected input price. For a risk-averse firm ($J_{ww} < 0$), inputs will be utilized to the point at which the expected marginal-value product equals the expected input price plus a risk premium. If the contemporaneous correlation coefficient between $P$ and $P_1$ is negative ($\gamma_{P1} < 0$), then the risk premium is greater, causing the firm to apply less input. On the other hand, if $\gamma_{P1}$ is positive, the risk premium is lower and $X_1$ and $X_2$ will be increased.

Combining the first-order conditions (3.43) and (3.44) yields

$$J_w = J_A Ah_1(E_1) = J_B Bh_2(E_2).$$

(3.45)

$E_1$ and $E_2$ will be determined at the point at which the marginal utility of wealth equals the marginal utility of $A$ or $B$ times the level of factor augmentation and marginal productivity of the technical progress function.

To retrieve a relationship between uncertainty and the bias of technical change, we can differentiate (3.45) and find the change in marginal utility over time as [see (8.5) in the Appendix.]

$$J_{ww} = J_{A_w} Ah_1' = J_{B_w} Bh_2',$$

(3.46)

$$Ah_1'dJ_A = Bh_2'dJ_B,$$

(3.47)

$$dJ_w = -J_w \delta_L dt + J_{ww}[PF^*\sigma_P ; -P_1X_1^*\sigma_1]$$

$$+ \begin{bmatrix} dz_p \\ dz_1 \end{bmatrix},$$

(3.48)
\[ \frac{dJ_A}{dJ_B} = \frac{h_1(E_1^*) - h_2(E_2^*)}{h_1(E_1^*) - h_2(E_2^*)} = -[P(1 + \delta_p)F_1^*X_1^* Ah_1^*]
+ \frac{J_{ww}}{J_w} P^2 \sigma_p^2 F_1^* X_1^* h_1^* - \frac{J_{ww}}{J_w} PP_1 X_1^* \sigma_p \sigma_1 \gamma_1 F_1^* X_1^* Ah_1^* \]
+ \left[ P(1 + \delta_p)F_2^* X_2^* Bh_2^* \right] + \frac{J_{ww}}{J_w} P^2 \sigma_p^2 F_2^* X_2^* Bh_2^* 
- \frac{J_{ww}}{J_w} PP_1 X_1^* \sigma_p \sigma_1 \gamma_1 F_2^* X_2^* Bh_2^* \]

Substituting the first-order conditions (3.41) and (3.42) into (3.52) yields
\[ \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = X_2^* h_2^* [P_2(1 + \delta_2)] - X_1^* h_1^* [P_1(1 + \delta_1)] 
+ \frac{J_{ww}}{J_w} P^2 X_1^* \sigma_1^2 - PP_1 \sigma_1 \gamma_1 F^* \sigma_p X_1^* h_1^*. \]
By assuming that $P_1 X_1^* = P_2 X_2^* = P_i X_i^*$ and that $h_i' = h_i'^* = h_i^*$, we can further simplify the equation to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_i X_i^* h_i'^*[(\delta_2 - \delta_1) - \frac{J_{ww}}{J_w}(P P_1 \sigma_1 \gamma p_1 F^* \sigma_p - P_1^2 X_1^* \sigma_1^2)].$$

Substituting this equation into (8.13) in the Appendix yields

$$\Omega = \frac{(1 - e)}{\epsilon} P_i X_i^* h_i'^*[(\delta_2 - \delta_1) - \frac{J_{ww}}{J_w}(P P_1 \sigma_1 \gamma p_1 F^* \sigma_p - P_1^2 X_1^* \sigma_1^2)].$$

In the deterministic case ($\sigma_p = 0$ and $\sigma_1 = 0$) or in the case of the risk-neutral firm ($J_{ww} = 0$), the Hicksian proposition may be derived. For a risk-averse firm ($J_{ww} < 0$), even if there is no difference in the growth rates of input prices, the technical change will be biased if the output price and an input price are assumed to be uncertain. If the correlation coefficient between the two processes of $P$ and $P_1$ is zero ($\gamma_{p1} = 0$), we can derive proposition 2. If $\gamma_{p1}$ is less than zero and $\epsilon > 1$, then technical change will be biased toward $X_2$. However, if $\gamma_{p1}$ is greater than zero, the direction of technical change will not be determined. That is, assuming that the elasticity of substitution is greater than one, if two processes of $P_1$ and $P$ are highly and positively correlated, the technical change may be biased toward $X_1$, and if they are insignificantly and positively or negatively correlated, the technical change may be biased toward $X_2$.

Proposition 4: For a risk-averse firm, if the output price and an input price are assumed to be uncertain, even if there is no difference in the growth rates of input prices, the rate of bias of endogenous technical progress will be different. Assuming that $\delta_1 = \delta_2$ and that $\epsilon > 1$, if $\gamma_{p1} < \frac{P_1 \sigma_{1X_i^*}}{P F^* \sigma_p}$, the technical change will be biased
toward input 2 ($\Omega > 0$). If $\gamma_{p1} > \frac{p_1 \sigma_1 x^*}{P F^* \sigma_p}$, then technical change will be biased toward input 1 ($\Omega < 1$).

3.6 Concluding Remarks

The theory of endogenous technical progress assumes that a change in the relative price of inputs will lead to the invention of factor-saving technologies. Such inventions may be undertaken by a public institution or by an individual firm. In this chapter, an Itô control model has been developed to investigate the effects of uncertainty on the bias of technical change. In addition, uncertainty has been modeled as a set of Wiener processes in which the level of variance and covariance of these processes implies the level of uncertainty.

We show that uncertainty in input prices will influence the rate of bias of technical progress. It is shown that the Hicksian proposition can be justified in both the deterministic case and in the case of a risk-neutral firm. For a risk-averse firm, the rate of bias of technical progress under input price uncertainty will not conform to the Hicksian proposition. By assuming that there is no difference in the growth rates of input prices, we conclude that endogenous technical change will be biased toward the input that has the more certain price if the elasticity of substitution is greater than one. This conclusion implies that, under uncertainty, even if the growth rates are different among input prices, the possibility of Hicks-neutral technical change exists.

Under output price uncertainty, the direction of technical change bias is not affected by uncertainty but the degree increases as the firm becomes more risk averse or as riskiness increases. If uncertainty exists in an input price and in the output
price, the contemporaneous correlation coefficient between these two processes will play an important role in the rate of technical change bias. If two processes, $P$ and $P_1$, are significantly and positively correlated and the elasticity of substitution is greater than one, then technical change may be biased toward the input that has an uncertain price. On the other hand, if $P$ and $P_1$ are insignificantly and positively or negatively correlated, technical change may be biased against the input that has an uncertain price.

The most important result in this chapter concerns the influence of input price uncertainty on the bias of technical change. We show that a firm will conduct research to increase the productivity of the inputs whose prices are less variable. This testable proposition is intuitive and has some important implications for economic policy.

Governments can and often do influence the prices of inputs. This influence occurs via market intervention or through the legislative process. Examples of the former method include government stabilization of interest rates, currency, wages, and other input prices (i.e., feed grains). Examples of the latter method include laws designed to influence labor contracting, interest rates, and imports. To the extent that our assumptions are valid, the model shows that all these policies will influence technical progress. Governments that wish to maximize labor productivity may find that the relaxation of interest rates and currency stabilization policies work in their favor. Governments that wish to increase the productivity of land or capital may find it in their long-term interests to stabilize prices in the land rental and capital markets, respectively.

This chapter presents no results about the magnitude of the possibly offsetting
effects of changes in relative prices of inputs versus changes in the variability of these prices. It may well be true that relative prices completely dominate the investment process. It would be unwise to base policy decisions on the results presented here until the relative magnitude of these two effects can be estimated.
4. THE FACTOR BIAS OF TECHNICAL CHANGE UNDER PRODUCTION UNCERTAINTY

4.1 Introduction

Chapter 3 has generalized the theory of the bias of technical change to incorporate price uncertainty and risk aversion. Considerable attention should be placed on the effects of price uncertainty. Production uncertainty, however, may have a relatively greater impact than that of market uncertainty for many sectors of an economy, particularly those involving biological growth (Pope and Kramer, 1979, p. 489). In practice, a number of instruments have been developed to reduce risk from price uncertainty (for example, hedging and forward contracting), but few instruments exist that reduce risk from production uncertainty (e.g., crop insurance is not widespread in the agricultural sector). Further, production uncertainty has been used as an important analytical tool for explaining techniques of production and adoption of technologies in development economics.\(^1\)

The purpose of this chapter is to show how the incorporation of production uncertainty alters the Hicksian proposition presented in Chapter 2. The inputs that increase the variance of output are separated from those that reduce the variance of

\(^1\)See Feder (1980), Hiebert (1974), Just and Zilberman (1983), and Stoneman (1981) for detailed discussions on technology adoption.
output. The results show that, under production uncertainty, firms will deviate from the Hicksian proposition to economize on the use of inputs that increase the variability of production. This divergence will depend on the level of production uncertainty and on the degree of risk aversion of the decision maker; that is, the degree of technical change would be increased as the riskiness increases or as the firm becomes more risk averse.

The model depends heavily on the specified assumptions; consequently, the results may not have immediate use in policy analysis. Nevertheless, the results are important in that they might help to explain some of the deviations from the Hicksian proposition that have been found in the empirical studies.\(^2\) Should it be possible to validate the theoretical results that follow, the concepts should help governments or researchers develop innovations with a greater likelihood of acceptance in a target audience.

This chapter employs the stochastic control model developed in Chapter 3. The model allows us to derive some preparations regarding the impact of production uncertainty on factor-augmenting technical change, presented in the next section. The results are presented in terms of the implied deviations from the induced innovation theory. The final section contains concluding remarks and some tentative implications for policy.

4.2 The Model

The stochastic microeconomic model described in Chapter 3 will be modified according to assumptions on production uncertainty. In this chapter, we assume that the firm can observe the uncertain production prospects at the time of decision making and that the firm expects the production level, \( Y \), and input prices, \( P_1 \) and \( P_2 \), to be log-normally distributed as

\[
\begin{align*}
    dY &= Y \sigma_y d\widetilde{zy}, \\
    dP_1 &= P_1 \delta_1 dt, \text{ and} \\
    dP_2 &= P_2 \delta_2 dt,
\end{align*}
\]

where \( \sigma_y \) is the standard deviation of the percentage change in production, \( \delta_i \) is the forecast for the growth rate of the price of \( i^{th} \) input, and \( \widetilde{zy} \) is a Wiener process in which \( E[d\widetilde{zy}(t)] = 0 \) and \( E[d\widetilde{zy}(t)]^2 = dt \).

The definition that is derived from arguments of the standard deviation function\(^3\) is used in this chapter. If the first derivative of standard deviation of the percentage change in production with respect to input \( i \) is positive \( (\sigma_y \alpha_i > 0) \), then \( X_i \) is risk-increasing input. On the other hand, if the derivative is negative \( (\sigma_y \alpha_i < 0) \), then \( X_i \) is risk-reducing input. For example, fertilizer or yield-increasing varieties might be termed risk increasing and irrigation or disease-resistant varieties might be termed risk reducing.

The firm chooses to apply two inputs, \( X_1 \) and \( X_2 \), and two levels of research

\(^3\)The static uncertainty literature contains some other definitions of risk-reducing and risk-increasing inputs (Hertzler, 1987). See Pope and Kramer (1979) or Just and Pope (1979) for a detailed discussion.
expenditure, $E_1$ and $E_2$, at the time of decision making. Expected production income at the current time may be expressed as

$$\pi = PY - P_1X_1 - P_2X_2 - E_1 - E_2.$$ 

By using Itô's lemma [see (8.2) in the Appendix], we can stochastically differentiate for the change in production income to get

$$d\pi = PdY - X_1dP_1 - X_2dP_2 = PY\sigma_ydz_y - X_1P_1\delta_1dt - X_2P_2\delta_2dt.$$ 

The stochastic production income in the immediate future, $\pi dt + d\pi$, can be obtained as

$$\pi dt + d\pi = [PY - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) - E_1 - E_2]dt + PY\sigma_ydz_y.$$ 

By using (3.6), the change in the firm's wealth that is consistent with the accounting identity can be expressed as

$$dW = [\delta_LW + PY - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) - E_1 - E_2]dt + PY\sigma_ydz_y.$$ 

The model specifies that the objective of the firm is to maximize the expected discounted utility of terminal stock of state variables (3.2) subject to the accounting identity (4.1) and the technical progress constraints (3.27). $X_1, X_2, E_1,$ and $E_2$ are the control and choice variables, whereas $W, A,$ and $B$ are the stock and state variables respectively.
The Itô version of the Bellman equation associated with this model specification can be derived as follows.

\[-J_t = \text{Max} \left\{ J_w \{ \delta^L + PF - P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2) \} - E_1 - E_2 \} + J_A(Ah_1(E_1)) + J_B(Bh_2(E_2)) + \frac{1}{2} J_{ww} \{ P^2 F^2 \sigma^2_y \}. \]

By using (8.3) in the Appendix, the stochastic Hamiltonian can be written as

\[ H = J_w [\delta^L W + PF(A X_1 + BX_2) - P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2) - E_1 - E_2] + J_A[Ah_1(E_1)] + J_B[Bh_2(E_2)] + \frac{1}{2} J_{ww} P^2 F^2 \sigma^2_y. \]

First-order conditions for \( X_1, X_2, E_1, \) and \( E_2 \) may be obtained as [see (8.4) in the Appendix]

\[ \frac{\partial H}{\partial X_1} = J_w [PF_1 A - P_1 (1 + \delta_1)] + J_{ww} [P^2 \sigma^2_y F F_1 A + \sigma^2_y \sigma^2_y x_1] = 0, \]

\[ \frac{\partial H}{\partial X_2} = J_w [PF_2 B - P_2 (1 + \delta_2)] + J_{ww} [P^2 \sigma^2_y F F_2 B + \sigma^2_y \sigma^2_y x_2] = 0, \]

\[ \frac{\partial H}{\partial E_1} = -J_w + J_A Ah'_1 = 0, \text{ and} \]

\[ \frac{\partial H}{\partial E_2} = -J_w + J_B Bh'_2 = 0. \]

From the first-order conditions in (4.2) and (4.3), input decision rules under production uncertainty can be depicted as

\[ PF_1 A = P_1 (1 + \delta_1) - \frac{J_{ww}}{J_w} [P^2 \sigma^2_y F F_1 A + \sigma^2_y \sigma^2_y x_1], \text{ and} \]

\[ PF_2 B = P_2 (1 + \delta_2) - \frac{J_{ww}}{J_w} [P^2 \sigma^2_y F F_2 B + \sigma^2_y \sigma^2_y x_2]. \]
These relationships have an intuitive interpretation. In the deterministic case with no uncertainty, \( \sigma_y = 0 \), the firm applies inputs to the point at which the marginal-value product equals the expected input price. For a risk-neutral firm, where \( J \) is a linear function of wealth \( (J_{w,w} = 0) \), we get the same result as in the deterministic case. For a risk-averse firm, where \( J_{w,w} \) is negative and the marginal risk premium is positive, inputs will be utilized to the point at which their marginal-value product equals input price plus a marginal risk premium. If an input is risk reducing, the marginal risk premium will decrease. On the other hand, if an input is risk increasing, the marginal risk premium will increase. Therefore, if \( X_i \) is risk reducing \( (\sigma_{yx_i} < 0) \), a risk-averse firm will apply more \( X_i \)'s than will a risk-neutral firm. On the contrary, if \( X_i \) is risk increasing \( (\sigma_{yx_i} > 0) \), a risk-averse firm will apply less \( X_i \)'s than will a risk-neutral firm.

Combining first-order conditions (4.4) and (4.5) yields

\[
J_w = J_A Ah'_1(E_1) = J_B Bh'_2(E_2). \tag{4.6}
\]

This equation implies that \( E_1 \) and \( E_2 \) will be determined at the point at which the marginal utility of wealth equals the marginal utility of \( A \) or \( B \), times the stock level of factor augmentation, times the first derivative of the technical progress function.

To retrieve the relationship between production uncertainty and the bias of technical change, we must differentiate (4.6) partially with respect to \( W, A, \) and \( B \) and also differentiate (4.6) by using Itô's lemma to get

\[
J_{w,w} = J_A w Ah'_1 = J_B w Bh'_2, \quad \text{and} \tag{4.7}
\]

\[
Ah'_1 dJ_A = Bh'_2 dJ_B. \tag{4.8}
\]
The change in marginal utility over time may be expressed as [see (8.5) in the Appendix]

\[ dJ_w = -J_w \delta \beta \sigma_{y} d\alpha, \] (4.9)

\[ dJ_A = -[J_w \rho F \sigma_{y} + J_A h + J_w P^2 F \sigma_{y}^2] dt \] (4.10)

\[ dJ_B = -[J_w \rho F \sigma_{y} + J_B h + J_w P^2 F \sigma_{y}^2] dt \] (4.11)

By substituting (4.6), (4.7), (4.10), and (4.11) into (4.8) and simplifying, we may obtain

\[ [J_w \rho F \sigma_{y} + J_w h + J_w P^2 F \sigma_{y}^2] dt = [J_w \rho F \sigma_{y} + J_B h + J_w P^2 F \sigma_{y}^2] dt. \]

Combining (3.27) and (4.12) yields

\[ \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = h_1(E_1^*) - h_2(E_2^*), \] (4.13)

\[ = \rho F \sigma_{y} + \frac{J_w \rho F \sigma_{y}^2}{J_w} \]

This set of equations comprises the model. In the next section, these conditions will be combined with the propositions discussed earlier.

4.3 The Results

First, substituting (4.2) and (4.3) into (4.13) derives an explicit equation that explains the relationship between production uncertainty and the factor bias of tech-
Equation (4.14) summarizes the important results of this chapter. This equation states that the difference in the optimum rate of factor augmentation \( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \) will depend on the difference between the net benefit of research devoted to reducing reliance on input 1 and the net benefit of research devoted to reducing reliance on input 2. The term \( \frac{J_{yw}}{J_w} \) will be negative for the individual's concave utility functions. The terms \( P_i(1 + \delta_i) \) represents the expected costs of input \( i \) caused by an expected increase in its price. The term \( -\frac{J_{yw}}{J_w} P^2 F^2 \sigma_y \sigma_y x_i \) will be positive for as long as the \( i^{th} \) input increases the variability of output; i.e., risk increasing. Notice that if the input is risk reducing, this second term may offset the expected growth in input price.

To further simplify, assume, without loss of generality, that both inputs have similar costs, \( P_1X_1^* = P_2X_2^* = P_iX_i^* \), and that optimum factor-augmenting research expenditures have similar returns for both inputs; i.e., \( h_i^t = h_2^t = h_i^t \). This result simplifies (4.14) to

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_iX_i^* h_i^t ((\delta_2 - \delta_1) - \frac{J_{yw}}{J_w} P^2 F^2 \sigma_y (\sigma_y x_2 - \sigma_y x_1)).
\] (4.15)

Substituting (4.15) into (8.13) in the Appendix yields

\[
\Omega = \frac{(1 - e)}{e} P_iX_i^* h_i^t ((\delta_2 - \delta_1) - \frac{J_{yw}}{J_w} P^2 F^2 \sigma_y (\sigma_y x_2 - \sigma_y x_1)).
\]

If there is no uncertainty, \( \sigma_y \) will equal zero and \( \delta_1 \) and \( \delta_2 \) will be the known input prices. This result leads directly to Proposition 1 (Hicksian): When there is no
uncertainty or when firms are risk neutral, relative input prices will determine the bias of technical change.

In situations in which firms are risk averse and where the production process is uncertain, the bias of technical change will be different from that predicted by Hicks. The magnitude of this difference will depend on the difference between $\delta_1$ and $\delta_2$ and on $\sigma y_x_1$ and $\sigma y_x_2$ and $\frac{J_{yw}}{J_w}$.

If $X_1$ is risk increasing and $X_2$ is risk reducing, then technical change will be biased toward $X_2$ in a manner that is independent of relative input prices if the elasticity of substitution is greater than one. This result can best be seen by assuming that $\delta_1 = \delta_2$; i.e., the expected growth rates of input prices are similar. If this assumption is the case, (4.15) can be written as

$$\Omega = \frac{1 - e}{e} P_t X_t^* h_t^* [\frac{-1}{\frac{J_{yw}}{J_w}} P^2 T^{2*} \sigma y (\sigma y_{x_2} - \sigma y_{x_1})].$$

This equation leads directly to the following proposition.

**Proposition 5:** For a risk-averse firm facing production uncertainty, the rate of endogenous factor-augmenting technical change will be biased even for the case in which there is no difference in the expected growth rates of input prices. If $e > 1$ and $\sigma y_{x_2} > \sigma y_{x_1}$, then the technical change will be biased toward input 1 ($\Omega < 0$). If $\sigma y_{x_2} < \sigma y_{x_1}$, then the technical change will be biased toward input 2 ($\Omega > 0$).

One interesting, though unlikely, possibility is that the effect of uncertainty might offset the changes in relative input prices, which happen if $(\delta_1 - \frac{J_{yw}}{J_w} P^2 T^{2*} \sigma y \sigma y_{x_1})$ is equal to $(\delta_2 - \frac{J_{yw}}{J_w} P^2 T^{2*} \sigma y \sigma y_{x_2})$. This result implies that $\frac{A}{A}$ can be equal to $\frac{B}{B}$,
even if the firm faces different expected growth rates in input prices. This implication leads to the following corollary.

**Corollary 5:** For a risk-averse firm facing different expected growth rates in input prices and production uncertainty, the Hicks-neutral technical change can not be excluded ($\Omega = 0$) because of the existence of production uncertainty and risk.

We might investigate the impacts of increasing risk on the degree of bias of technical change by using (4.15) or (4.16). For a risk-averse firm, the term ($\Omega$) is positively related to $\sigma_y$ if $\sigma_{yx_2}$ is greater than $\sigma_{yx_1}$. And it is negatively related to $\sigma_y$ if $\sigma_{yx_2}$ is less than $\sigma_{yx_1}$. Define $\sigma_y$ as increasing risk. The degree of technical change bias toward $X_1$ increases as $\sigma_y$ increases if $\sigma_{yx_2}$ is greater than $\sigma_{yx_1}$. And the degree toward $X_2$ will increase as $\sigma_y$ increases if $\sigma_{yx_2}$ is less than $\sigma_{yx_1}$. This relationship is summarized in the following proposition.

**Proposition 6:** The degree of technical change bias would increase as riskiness increases.

A similar relationship between the degree of technical change bias and the degree of a firm's risk aversion can be derived by using (4.15). $\Omega$ will increase as $-\frac{J_{yw}}{J_w}$ increases if $\sigma_{yx_2}$ is greater than $\sigma_{yx_1}$. This result implies that the degree of factor

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4 Increasing risk can be defined as a mean-preserving spread of the distribution, following Rothschild and Stiglitz (1971). Here, it is simply defined as a standard deviation function, $\sigma_y$. 
bias toward $X_1$ increases as the firm becomes more risk averse. If $\sigma_{yx_2}$ is less than $\sigma_{yx_1}$, the degree of factor bias toward $X_2$ increases as the firm becomes more risk averse. This result leads to Proposition 7.

**Proposition 7:** The degree of technical change bias would increase as the firm became more risk averse.

This proposition is intuitively appealing. We expect the risk-averse firm to be willing to pay more for research expenditures to reduce the use of risk-increasing inputs and to develop and adopt the technologies that use risk-reducing inputs. And the more research expenditure that the firm is willing to pay, the more risk averse the firm is.

### 4.4 Concluding Remarks

This chapter investigates how production uncertainty affects the bias of technical change. The similar model used in Chapter 3 incorporates production uncertainty. Inputs are characterized from arguments of the standard deviation function. The results show that, under production uncertainty, technical progress for the risk-averse firm will be affected by the characteristics of inputs (i.e., risk increasing or risk reducing). Technical change will be biased toward risk-reducing inputs and against risk-increasing inputs if the elasticity of substitution is greater than one. It is shown that the degree of bias of technical change would be increased as the riskiness increases or as the firm becomes more risk averse. It is also shown that the Hicksian
proposition is a special case in which there is no uncertainty or in which firms are risk neutral.

From a policy perspective, the principal conclusion that can be drawn from this chapter is that the degree of output uncertainty will influence the types of technology that are developed and adopted. For example, new hybrid varieties of corn might be more favorably received in parts of the United States where weather conditions are more stable, whereas risk-reducing innovations would be more useful in areas where weather patterns are more volatile. Internationally, one would expect the risk-increasing technologies of the Green Revolution to be successfully adopted in countries or regions with stable output patterns. Also, the results suggest that research should concentrate on technologies that reduce risk in regions where production uncertainty is high.

Finally, the overall conclusion of Chapters 3 and 4 is that uncertainty is an important factor in determining the bias of technical change. These chapters, however, do not assume the existence of instruments that reduce uncertainty and risk. Hedging and forward contracting reduce risk from output price uncertainty. Crop insurance deals with production uncertainty. Therefore, one obvious extension of the model is to introduce these instruments.
5. HEDGING AND THE BIAS OF TECHNICAL CHANGE UNDER UNCERTAINTY

5.1 Introduction

Previous models have explored the effects of uncertainty on the bias of technical change. These models all assume that there is no management of risk or uncertainty. In practice, however, futures markets exist for many agricultural products and some metals (Holthausen, 1979). That is, firms may also use hedging or forward contracts that are designed to reduce risk from uncertainty. On the other hand, the theory of the competitive firm under uncertainty and considering futures markets has been developed extensively in the literature.¹

This chapter attempts to integrate these two approaches. The model used in Chapter 3 is extended to incorporate forward markets in the theory of the bias of technical change under price uncertainty. The results show that, when only the output price is uncertain, the existence of a forward market has no effect on the direction of technical change bias but has an effect on the degree of bias. In Chapter 3, we showed that, assuming input and output price uncertainty, the sign and size

of the correlation coefficient between output price and input price would play an important role in the bias of technical change. However, if the firm can use forward contracts, this correlation coefficient does not affect the direction of technical change bias. That is, if the forward market is unbiased and the elasticity is greater than one, the technical change will be biased toward the input that has a certain price.

In the next section, only output price uncertainty and forward markets are considered. Input price uncertainty is added in the following section. Concluding remarks are presented in the final section of this chapter.

5.2 Hedging and Output Price Uncertainty

The representative firm is assumed to face a random output price and to expect all prices to be log-normally distributed. Output can either be sold in the future at the random price $P$ or sold forward at the certain price $b$. Thus, we must add the process of certain price, which is shown below, to the model specification of output price uncertainty in Chapter 3.

$$db = \delta_b dt,$$

where $\delta_b$ denotes the forecast for the growth rate of the certain price.

Under the specified assumptions, expected income from production at the current time can be written as

$$\pi = P(F - G) - P_1X_1 - P_2X_2 + bG - E_1 - E_2,$$

where $G$ represents the amount of output hedged in the forward market. If we stochas-
tically differentiate this equation by using (8.2) in the Appendix,

\[
d\pi = (F - G)dP - X_1dP_1 - X_2dP_2 + Gdb
\]

\[
= [(F - G)P_\delta - X_1P_1\delta_1 - X_2P_2\delta_2 + Gb\delta_b]dt + (F - G)P\sigma_pd\nu.
\]

By using equations (3.6) and (5.1) and manipulating, the accounting identity, which is the change in the firm's wealth, can be obtained as

\[
dW = \delta LW + (F - G)P(1 + \delta_p) - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2)
\]

\[
+ Gb(1 + \delta_b) - E_1 - E_2]dt + (F - G)P\sigma_pd\nu.
\]

Note that the amount of output hedged in the forward market is the control and choice variable. With the objective function, (3.2), and constraints, (3.27) and (5.2), the stochastic Hamiltonian and first-order conditions for control variables can be written as [see (8.3) and (8.4) in the Appendix]

\[
H = J_w[\delta LW + (F - G)P(1 + \delta_p) - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2)
\]

\[
+ Gb(1 + \delta_b) - E_1 - E_2] + J_A[Ah_1(E_1)]
\]

\[
+ J_B[Bh_2(E_2)] + \frac{1}{2}J_{ww}(F - G)^2P^2\sigma^2_p,
\]

\[
\frac{\partial H}{\partial X_1} = J_w[P(1 + \delta_p)F_1A - P_1(1 + \delta_1)] + J_{ww}[(F - G)P\sigma^2_pF_1A] = 0, \quad (5.4)
\]

\[
\frac{\partial H}{\partial X_2} = J_w[P(1 + \delta_p)F_2B - P_2(1 + \delta_2)] + J_{ww}[(F - G)P\sigma^2_pF_2B] = 0, \quad (5.5)
\]

\[
\frac{\partial H}{\partial E_1} = -J_w + J_AAh_1^I = 0, \quad (5.6)
\]
\[
\frac{\partial H}{\partial E_2} = -J_w + J_B B h_2' = 0, \text{ and} \tag{5.7}
\]

\[
\frac{\partial H}{\partial G} = J_w[-P(1 + \delta_p) + b(1 + \delta_b)] + J_{ww}[-(F - G)P^2\sigma_p^2] = 0. \tag{5.8}
\]

Rewriting (5.8) yields

\[
b(1 + \delta_b) = P(1 + \delta_p) - \frac{J_{ww}}{J_w}(G - F)P^2\sigma_p^2.
\]

This equation indicates that, if the firm is assumed to be risk neutral \((J_{ww} = 0)\), the term \(J_w[-P(1 + \delta_p) + b(1 + \delta_b)]\) is independent of \(G\). This result implies that no solution exists if the forward market is biased, \(P(1 + \delta_p) \neq b(1 + \delta_b)\), and \(G\) is indeterminate if the forward market is unbiased, \(P(1 + \delta_p) = b(1 + \delta_b)\). That is, the forward market will not operate if firms are risk neutral. For a risk-averse firm, if the forward market is unbiased, the firm will hedge its entire output, \(G = F\). If \(P(1 + \delta_p)\) is less than \(b(1 + \delta_b)\), the firm might speculate by selling forward an amount that is greater than its output, \(G > F\). If \(P(1 + \delta_p)\) is greater than \(b(1 + \delta_b)\), the firm will hedge an amount that is less than its output, \(G < F\).

Substituting (5.8) into (5.4) and (5.5) gives

\[
b(1 + \delta_b)F_1A = P_1(1 + \delta_1), \text{ and} \tag{5.9}
\]

\[
b(1 + \delta_b)F_2B = P_2(1 + \delta_2). \tag{5.10}
\]

These equations explain input decision rules under output price uncertainty and considering the forward contracts. That is, the firm will apply inputs to the point at which their marginal-value products equal input prices. Therefore, all risk-averse
firms in the market will key their input decisions to the forward price and input prices.

Combining first-order conditions (5.6) and (5.7) yields

\[ J_w = J_A h_1'(E_1) = J_B h_2'(E_2). \]  

(5.11)

This equation implies that the control variables, \( E_1 \) and \( E_2 \), will be applied to the point at which the marginal utility of wealth equals the marginal utility of \( A \) and \( B \) times the level of factor augmentation and marginal productivity of the technical progress function.

To derive the relationship between hedging and the bias of technical change, we should differentiate (5.11) partially with respect to \( W, A, \) and \( B \) and also differentiate (5.11) totally to get

\[ J_{ww} = J_{Aw} h_1' = J_{Bw} h_2', \]  

(5.12)

\[ A h_1' dJ_A = B h_2' dJ_B. \]  

(5.13)

The change in marginal utility over time, which are also first-order conditions, can be obtained as [see (8.5) in the Appendix]

\[
\begin{align*}
    dJ_w &= -J_w \delta_L dt + J_{ww}(F^* - G^*) \rho_p dz_p, \\
    dJ_A &= -[J_w P(1 + \delta_p) F_1^* X_1^* + J_A h_1^*] \\
    &\quad + J_{ww}(F^* - G^*) P^2 \sigma_p^2 F_1^* X_1^* dt + J_{Aw}(F^* - G^*) \rho_p dz_p, \text{ and} \\
    dJ_B &= -[J_w P(1 + \delta_p) F_2^* X_2^* + J_B h_2^*] \\
    &\quad + J_{ww}(F^* - G^*) P^2 \sigma_p^2 F_2^* X_2^* dt + J_{Bw}(F^* - G^*) \rho_p dz_p. 
\end{align*}
\]
Substituting (5.11), (5.12), (5.15), and (5.16) into (5.13) and simplifying yields

\[ J_w(1 + \delta_p)F_1^*X_1^*Ah^*_1 + J_wh_1^* + J_ww(F^* - G^*)P^2 \sigma^2_F F_1^*X_1^*Ah^*_1 \]

\[ = J_w(1 + \delta_p)F_2^*X_2^*Bh_2^* + J_ww(F^* - G^*)P^2 \sigma^2_F F_2^*X_2^*Bh_2^*. \]  

(5.17)

Combining (3.27) and (5.17) yields

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = h_1(E_1^*) - h_2(E_2^*) = P(1 + \delta_p)F_1^*X_1^*Ah^*_1
\]

\[
+ \frac{J_ww(F^* - G^*)P^2 \sigma^2_F F_2^*X_2^*Bh_2^*}{J_w} - (1 + \delta_p)F_1^*X_1^*Ah^*_1
\]

\[
- \frac{J_ww(F^* - G^*)P^2 \sigma^2_F F_2^*X_2^*Bh_2^*}{J_w}.
\]  

(5.18)

Substituting the first-order condition, (5.4) and (5.5) into (5.18) yields

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = h_2^*(E_2^*) - X_2^*h_2^*(1 + \delta_2).
\]

By assuming \( P_1X_1^* = P_2X_2^* = P^*_iX_i^* \) and \( h_1^* = h_2^* = h_i^* \), the equation can be simplified to

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P^*_iX_i^*h_i^*(\delta_2 - \delta_1).
\]

By substituting this equation into (8.13) in the Appendix, we may obtain

\[
\Omega = \frac{(1 - e)}{e}P^*_iX_i^*h_i^*(\delta_2 - \delta_1).
\]

This equation indicates that the direction of technical change bias is not affected by the existence of forward markets. However, note that the input decision is based on the forward price and not on the concept that the firm expects to prevail and that the degree of risk aversion will have no effect on the production decision. These relationships lead to following proposition.
**Proposition 8:** Assuming output price uncertainty, the existence of hedging or forward markets has no impact on the direction of technical change bias.

By comparing the first-order conditions of output price uncertainty in Chapter 3 with (5.9) and (5.10), it is clear that if \( P(1 + \delta_p) \) equals or is less than \( b(1 + \delta_b) \), and optimal input use under the assumption of the existence of a forward market is greater than that of nonexistence. This leads directly to following corollary.

**Corollary 6:** Under output price uncertainty, if \( P(1 + \delta_p) \) equals or is less than \( b(1 + \delta_b) \), the existence of hedging would increase the degree of technical change bias.

### 5.3 Hedging and Input and Output Price Uncertainty

In Chapter 3, we showed that, assuming output price uncertainty and one input price uncertainty, the contemporaneous correlation coefficient between output price and input price would play an important role in the bias of technical change. In this section, we investigate the influence of hedging or forward contracts on the results that are derived in Chapter 3.

Expected production income will be the same as that in the previous section. By differentiating stochastically, using (8.2) in the Appendix, and manipulating, the stochastic production income in the immediate future and the change in the firm's wealth can be obtained as

\[
\pi dt + d\pi = [P(1 + \delta_p)(F - G) + b(1 + \delta_b)G - P_1X_1(1 + \delta_1) - P_2X_2(1 + \delta_2) - E_1 - E_2]dt \tag{5.19}
\]
\[ +[(F - G)P \sigma_p ; -X_1 P_1 \sigma_1] \begin{bmatrix} dz_p \\ dz_1 \end{bmatrix}, \]

and

\[ dW = \left[ \delta_L W + P(1 + \delta_p)(F - G) + b(1 + \delta_b)G \\ -P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2) - E_1 - E_2 \right] dt 
+ [(F - G)P \sigma_p ; -X_1 P_1 \sigma_1] \begin{bmatrix} dz_p \\ dz_1 \end{bmatrix}. \]

The stochastic Hamiltonian and first-order conditions for the control variables 
\((X_1, X_2, E_1, E_2, G)\) can be written as [see (8.3) and (8.4) in the Appendix]

\[ H = J_w[\delta_L W + P(1 + \delta_p)(F - G) + b(1 + \delta_b)G \\ -P_1 X_1(1 + \delta_1) - P_2 X_2(1 + \delta_2) - E_1 - E_2] 
+ J_A[Ah_1(E_1)] + J_B[Bh_2(E_2)] 
+ \frac{1}{2} J_{ww}[(F - G)P \sigma_p ; -X_1 P_1 \sigma_1] \begin{bmatrix} 1 & \gamma_{p1} \\ \gamma_{p1} & 1 \end{bmatrix} \begin{bmatrix} (F - G)P \sigma_p \\ -X_1 P_1 \sigma_1 \end{bmatrix} \]

\[
\frac{\partial H}{\partial X_1} = J_w[P(1 + \delta_p)F_1 A - P_1(1 + \delta_1)] + J_{wW}[(F - G)P^2 \sigma_p^2 F_1 A \\
-P_1 X_1 \sigma_1 \gamma_{p1} P \sigma_p F_1 A - (F - G)P_1 \sigma_1 \gamma_{p1} P \sigma_p + X_1 F_1^2 \sigma_1^2] = 0, \]

\[
\frac{\partial H}{\partial X_2} = J_w[P(1 + \delta_p)F_2 B - P_2(1 + \delta_2)] + J_{wW}[(F - G)P^2 \sigma_p^2 F_2 B \\
-P_1 X_1 \sigma_1 \gamma_{p1} P \sigma_p F_2 B] = 0, \]

\(\gamma_{p1} = 1\) and \(\gamma_{p1} = 0\) respectively.
\[
\frac{\partial H}{\partial E_1} = -J_w + J_A Ah'_1 = 0, \quad (5.23)
\]

\[
\frac{\partial H}{\partial E_2} = -J_w + J_B Bh'_2 = 0, \quad \text{and} \quad (5.24)
\]

\[
\frac{\partial H}{\partial G} = J_w[b(1 + \delta_b) - P(1 + \delta_p)] + \frac{J_{ww}}{J_w}[-(F - G)P^2\sigma_p^2 + P_1X_1\gamma_{p1}P\sigma_p] = 0. \quad (5.25)
\]

If we assume that the forward market is unbiased, \(P(1 + \delta_b) = P(1 + \delta_p)\), equation (5.25) yields

\[
(F - G)P\sigma_p = P_1X_1\gamma_{p1}. \quad (5.26)
\]

Equation (5.26) implies that, if the forward market is unbiased and the contemporaneous correlation coefficient is zero, the risk-averse firm will hedge its entire output. If \(\gamma_{p1}\) is greater than zero and \(b(1 + \delta_b) = P(1 + \delta_p)\), the risk-averse firm will hedge less than its output \((G < F)\). If \(\gamma_{p1}\) is less than zero and \(b(1 + \delta_b) = P(1 + \delta_p)\), the risk-averse firm might speculate by selling forward an amount greater than its output \((G > F)\).

Substituting (5.25) into (5.21) and (5.22) yields

\[
b(1 + \delta_b)F_1A = P_1(1 + \delta_1) - \frac{J_{ww}}{J_w}[-(F - G)P_1\gamma_{p1}P\sigma_p + X_1P_1^2\sigma_1^2], \quad \text{and} \quad (5.27)
\]

\[
b(1 + \delta_b)F_2B = P_2(1 + \delta_2). \quad (5.28)
\]

These equations indicate that, for a risk-averse firm, the optimum input level is determined by the forward price, input prices, and marginal risk premium. An important
result is that risk-averse firms will key their input decisions not to the random price but to the forward price.

By combining first-order conditions, (5.23) and (5.24), and differentiating the combined equation, we can obtain the same equations with (5.11), (5.12), and (5.13). To derive the explicit relationship, the change in marginal utility over time can be written as [see (8.5) in the Appendix]

\[
dJ_w = -J_w \delta_L dt + J_{ww}[(F^* - G^*)P \sigma_p ; -X_1 P_1 \sigma_1] \left[ \frac{dz_p}{dz_1} \right], \quad (5.29)
\]
\[
dJ_A = -[J_w P(1 + \delta_p)F_1^* X_1^* + J_A h_1^*]
+ J_{ww}(F^* - G^*)P^2 \sigma_p^2 F_1^* X_1^* - P_1 X_1^* \gamma_p \sigma_1 P \sigma_p F_1^* X_1^*] dt
+ J_{Aw}[(F^* - G^*)P \sigma_p ; -X_1 P_1 \sigma_1] \left[ \frac{dz_p}{dz_1} \right], \quad (5.30)
\]
\[
dJ_B = -[J_w P(1 + \delta_p)F_2^* X_2^* + J_B h_2^*]
+ J_{ww}(F^* - G^*)P^2 \sigma_p^2 F_2^* X_2^* - P_1 X_1^* \gamma_p \sigma_1 P \sigma_p F_2^* X_2^*] dt
+ J_{Bw}[(F^* - G^*)P \sigma_p ; -X_1 P_1 \sigma_1] \left[ \frac{dz_p}{dz_1} \right]. \quad (5.31)
\]

Substituting (5.11), (5.12), (5.30), and (5.31) into (5.13) and combining equation (3.27) yields

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = h_1(E_1^*) - h_2(E_2^*) = -[P(1 + \delta_p)F_1^* X_1^* Ah_1^*]
+ J_{ww} P^2 \sigma_p^2 (F^* - G^*)F_1^* X_1^* Ah_1^* - \frac{J_{ww}}{J_w} P_1 X_1^* \sigma_p \sigma_1 \gamma_p F_1^* X_1^* Ah_1^*]
+ [P(1 + \delta_p)F_2^* X_2^* Bh_2^* + \frac{J_{ww}}{J_w} P^2 \sigma_p^2 (F^* - G^*)F_2^* X_2^* Bh_2^*]
- \frac{J_{ww}}{J_w} P_1 X_1^* \sigma_p \sigma_1 \gamma_p F_2^* X_2^* Bh_2^*]. \quad (5.32)
\]
By substituting (5.25), (5.26), (5.27), and (5.28) into (5.32), and by assuming that \( P_1X_1^* = P_2X_2^* = P_iX_i^* \) and \( h_1^* = h_2^* = h_i^* \), the equation can be simplified to

\[
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = P_iX_i^* h_i^*[\gamma_p 2 X_1^* \sigma_1^2(\gamma_p^2 - 1)].
\]

Note that the above equation is derived by assuming unbiased forward market, \( P(1 + \delta_p) = b(1 + \delta_b) \). We can derive the technical change bias by substituting the above equation into (8.13) in the Appendix.

\[
\Omega = \frac{(1 - e)}{e} P_iX_i^* h_i^*[\gamma_p 2 X_1^* \sigma_1^2(\gamma_p^2 - 1)].
\]

Because \( \gamma_p^2 \) is less than 1, \( \Omega \) is always positive. This equation indicates that the technical change will be biased toward input 2 if the elasticity of substitution is greater than one and if there is no difference in the expected growth rates of input prices.

This result leads directly to the following proposition.

**Proposition 9:** Assuming that \( \delta_1 = \delta_2 \) and a risk-averse firm facing an unbiased forward market, if the elasticity of substitution is greater than one, then technical progress will be biased toward input 2, which has a certain price (\( \Omega > 0 \)).

### 5.4 Concluding Remarks

In this chapter, the model is extended to incorporate forward markets in the theory of the technical change bias under uncertainty. The results show that, under output price uncertainty, the direction of technical change is not affected by the existence of forward markets. However, if the forward markets are unbiased, the degree of technical change would increase as the forward markets are activated. It is
also shown that, if uncertainty exists in both the output price and input price and the forward market is unbiased, technical progress will more likely be biased toward the input that has a certain price.

By comparing these results with those in Chapter 3, we might obtain some important policy implications. The conclusion can be drawn from Chapter 3 that price stabilization programs would increase the rate and degree of technical change bias. This chapter, however, implies that activation of forward markets may be an alternative way to influence technical progress. If uncertainty exists in both the output price and input price and the correlation coefficient is negative or insignificantly positive, the activation of forward markets has no effect on the direction of technical change bias. On the other hand, in Chapter 3, we show that, if output price is significantly and positively correlated with input price, the technical change will be biased toward the input that has an uncertain price. The activation of forward markets, however, might alter the direction of technical change bias. That is, technical change will be biased toward the input that has a certain price.
6. TECHNOLOGY ADOPTION UNDER UNCERTAINTY

6.1 Introduction

There is a large body of literature on the effects of government price stabilization programs on producer and consumer welfare and on economic indicators.\(^1\) On the other hand, technology adoption models have been analyzed to determine the adoption behavior and to derive the diffusion curve.\(^2\) To date, however, no one has examined how price stabilization programs affect the types of technologies that are adopted or the rate of diffusion of new technologies. That there should be some relationship between price stability and technology adoption is clear. The adoption of new technologies involves risk, and price stabilization programs are designed to remove risk. Also, some technologies are riskier than others. It seems reasonable to hypothesize that producers will rank new technologies differently under different price regimes.

Figure 6.1 plots the wheat yield per acre for selected countries for the past three decades. Wheat yields in France and the United Kingdom have been increasing at a


Figure 6.1: Wheat yield per harvested hectare for various countries
faster rate than have those in Canada and Australia. In fact, the data indicate that wheat yields in Australia have not improved. It is interesting to note that the rate of growth of wheat yields in the United Kingdom picked up after its entry into the European Community (EC) in 1973 and that the rate of growth for France, which has been a member of the EC for a much longer period, is more stable.

Without further information, it is difficult to determine whether these changes are the result of movements along the supply curve or whether they reflect differences in technology adoption. It is possible, for example, that Australian researchers have improved yields at a rate similar to those in France and that the lack of yield improvement reflects lower fertilizer use or the use of lower-quality land. If one accepts the view that the EC has successfully stabilized prices via its intervention system and that producers in Australia and Canada have been open to the world market, then it also seems possible that these divergent rates of growth result from differences in output price variation.

Figure 6.2 presents information on the milk yield per cow in selected countries. Because the United Kingdom and Ireland joined the EC simultaneously and milk producers in The Netherlands and in the United States have had relatively stable output prices, we can see that these data lend support to the hypothesis that price-stabilization programs tend to increase "yields."

6.2 Definitions

To examine these concepts more formally, we must define yield-increasing and cost-reducing technological changes. We define technological change in a manner
Figure 6.2: Milk yield per cow for various countries

- Ireland
- Netherlands
- New Zealand
- U. K.
- U. S.
that is tractable and yet that will produce results that are directly interpretable. From this perspective, one might group together those technologies that are biased against land or other fixed assets. A useful alternative grouping would include those technologies that reduce the variable costs of production per unit of output. These we call cost-reducing technologies.

The concepts of yield-increasing and cost-reducing technologies, while frequently discussed by lay groups, are often poorly defined. This chapter uses definitions that are intuitive, have applications in most agricultural enterprises, and are consistent with economic theory. Several intuitive examples of the difference between these types of technological change are discussed next.

In the area of livestock, for example, research activities that focus on increasing hog litter size or on the development of expensive confinement facilities to increase the rate of gain or of survival might be termed yield increasing. Research that focuses on feed efficiency or on genetic selection for animals that can thrive outdoors or on males whose offspring create fewer birthing difficulties might be termed cost reducing. In horticulture, research on glass-house production can be compared with research on the development of frost- or disease-resistant varieties that can be grown without the protection of glass houses. For crops, research on winter wheat production that requires nitrogen top-dressings and several applications of pesticides and herbicides can be contrasted with the development of spring wheat strains that can grow without fertilizer. Other comparisons might be made between irrigation and summer fallow research or between research on soil drainage and on soil tilth aimed at increasing the number of acres tilled per man hour.
The concepts of yield-increasing and cost-reducing technologies have several immediate applications in policy analysis. For example, governments wishing to enhance the international competitiveness of their agricultural sectors should encourage cost-reducing technologies. Those governments with fears about food security should encourage yield-increasing research. In developing economies, it might sometimes be beneficial to increase food production via the yield-increasing technologies of the Green Revolution, whereas, in other situations, technologies that result in cheap food and high wages for those employed in the agricultural sector might be more desirable.

The concepts also have implications for environmental economics. Many of the additional inputs on a fixed land base required for yield-increasing technologies can be harmful to the environment. Examples include nitrogen and pesticide contamination of water, salinity problems with irrigated soils, and waste-disposal problems with intensive livestock confinement operations. Cost-reducing technologies will, in general, require less input use and will involve the development of varieties of animals and plants that are native to a particular area. As such, cost-reducing technologies might reduce some environmental problems.

Farm labor can be regarded as a fixed cost when family members are involved or as a variable cost when wages are paid. Cost-reducing technologies increase the marginal-value product of hired labor, and yield-increasing technologies increase the marginal-value product of family labor. The choice of yield-increasing or cost-reducing technologies will therefore have implications for both farm size and the institutional structure of agricultural firms.

The dichotomy between cost-reducing and yield-increasing technologies is ob-
viously a simplistic way to categorize technological change. Many changes exhibit aspects of both. The more formal analysis presented here uses only the two simplistic definitions. The inclusion of technologies that have both yield-increasing and cost-reducing effects would needlessly complicate the derivations.

6.2.1 Definition of yield-increasing and cost-reducing technologies

A technological innovation is said to be yield increasing if it produces a higher yield per acre (animal) and does not reduce optimal variable costs per acre (animal).

A technological innovation is said to be cost reducing if it reduces optimal variable costs per acre (animal) but does not increase yields per acre (animal).

The adoption of yield-increasing technologies implies variable-inputs-using technical change, whereas the adoption of cost-reducing technologies implies variable-inputs-saving technical change.

These two types of technological change have similar effects on the profit function: Both reduce average total costs and therefore increase profits, but the source of these cost reductions is different. Yield-increasing technologies reduce the average fixed cost, whereas cost-reducing technologies reduce the average variable cost. As we shall see, these effects lead to different supply responses under price uncertainty for risk-averse producers. The intuition here can be developed with the following analogy. Canadian wheat farmers are endowed with land and some labor. The purchase of fertilizer in this case is similar to the purchase of a lottery ticket with which the payoff is related to total revenue. The number of “tickets” purchased will depend on the expected variability of returns and on the individual’s risk aversion. The
variance of returns is directly related to the number of tickets purchased because return variance is equal to the number of tickets squared times the variance on an individual purchase. The decision about the number of acres to plant is a less risky proposition than a lottery, especially if the opportunity cost of land and labor is low. Apart from the seed, producers will not regret incurring fixed costs, even in bad years. In bad years, however, the producer may wish that he or she had used less fertilizer or had simply planted the seed without fertilizer (i.e., purchased fewer tickets). The greater the price risk, the less willing producers will be to adopt technologies that require more variable costs and the more willing they will be to adopt technologies that reduce variable costs. This is because yield-increasing technologies force them to purchase more “lottery tickets,” whereas cost-reducing technologies reduce this number.

6.3 The Model

This study employs and modifies the microeconomic model developed by Just and Zilberman (1983). Consider the decisions made by the owner of a single farm with fixed landholdings, \( L \), with a sale price of \( P_l \) who uses a traditional technology, \( T_0 \). Assume that the farmer discovers two technological innovations; one which is yield increasing, \( T_1 \), and the other which is cost-reducing, \( T_2 \). Also assume that the farmer must allocate the landholdings between technology 1 and technology 2. [The traditional technology \( (T_0) \) will never be preferred.]

Let \( \pi_{it} \) represent net returns per acre for technology \( i \) at time \( t \). This equation
can be represented as

\[ \pi_{it} = P_t Y_{it} - C_i[t (Y_{it}, w_t)], \text{ for } i = 0, 1, \text{ and } 2, \]

where \( P \) is unit price of output, \( Y \) is yield per acre, \( C[.] \) represents the variable cost function, and \( w \) is a vector of variable input prices. By definition, \( Y_{1t} > Y_{2t} = Y_{0t} \) and \( C_{1t} \geq C_{0t} > C_{2t} \). Also assume that output price is the only random variable with known mean, \( \bar{P} \), and variance, \( \sigma^2_{P_t} \).

Now assume that the producer has a von Neumann-Morgenstern utility function, \( U[. ] \), defined on wealth, in which \( U' > 0 \) and \( U'' < 0 \). End-of-harvest wealth, \( W_t \), can be written as the sum of the land value and the profitability from farming.

\[ W_t = P_t \bar{L} + L_{1t} \pi_{1t} + L_{2t} \pi_{2t}, \]

where \( L_{it} \) is the amount of land allocated to technology \( i \) at time \( t \). Note that if adoption costs are zero, the farmer will not use the old technology.

The a priori land allocation decision is determined by maximizing the following objective function with respect to \( L_{1t}, L_{2t} \).

\[ \text{Max } V(L_{1t}, L_{2t}) = EU[P_t \bar{L} + L_{1t} \pi_{1t} + L_{2t} \pi_{2t}], \quad (6.1) \]

subject to \( L_{1t} + L_{2t} = \bar{L}, \)

\[ L_{1t}, L_{2t} \geq 0. \]

The first-order condition for land allocation is

\[ \frac{\partial V}{\partial L_{1t}} = E[U'(\pi_{1t} - \pi_{2t})] = E[U'(P_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}))] = 0. \quad (6.2) \]

Specifying a first-order Taylor series approximation to \( U'(W) \) yields

\[ U'(W) = U'(\bar{W}) + U''(\bar{W})[(P_t - \bar{P}_t)(Y_{2t} \bar{L} + (Y_{1t} - Y_{2t})L_{1t})], \quad (6.3) \]
and using this approximation in (6.2), the following expression is obtained.

$$\frac{1}{U'(W)} \frac{\partial V}{\partial L_{1t}} = \begin{align*} &P_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) \
&- \phi \sigma_{pt}^2 (Y_{1t} - Y_{2t}) \bar{L} + (Y_{1t} - Y_{2t})^2 L_{1t} = 0, \end{align*} \tag{6.4}$$

where $\phi = \frac{-U''(W)}{U'(W)}$ is the Arrow-Pratt measure of absolute risk aversion at mean wealth.

The approximation of the first-order condition, (6.4), can be solved for the optimal level of $L_{1t}^*$ and $L_{2t}^*$.

$$L_{1t}^* = \frac{P_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) - \phi \sigma_{pt}^2 (Y_{1t} - Y_{2t}) \bar{L}}{\phi \sigma_{pt}^2 (Y_{1t} - Y_{2t})^2}, \text{ and} \tag{6.5}$$

$$L_{2t}^* = \bar{L} - L_{1t}^*. \tag{6.6}$$

Note that as $\sigma_{pt}^2$ or $\phi$ tends to zero, $L_{1t}^*$ will tend to $\bar{L}$ for as long as $P_t(Y_{1t} - Y_{2t}) > (C_{1t} - C_{2t})$. Also, if $P_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t})$ is less than $\phi \sigma_{pt}^2 (Y_{1t} - Y_{2t}) \bar{L}$, then the producer will allocate all of his land to the cost-reducing technology. Assuming that neither of these conditions is true, we can see that the proportion of the land allocated to the yield-increasing technology is inversely proportional to the degree of price variability, $\sigma_{pt}^2$. The opposite is true for cost-reducing technologies. This relationship suggests the following proposition.

**Proposition 10:** Under the assumptions specified, as price variability decreases, ceteris paribus, the proportion of land devoted to fixed-inputs-saving (yield-increasing) technologies will increase. Consequently, the proportion of land devoted to variable-inputs-saving (cost-reducing) technologies will decrease.
Corollary 7: As price variability increases, the proportion of land devoted to variable-input-saving technologies will increase, whereas the proportion devoted to fixed-input-saving technologies will decrease.

This proposition tells us that, under the conditions outlined, increases in price variability will encourage the adoption of cost-reducing technologies or variable-inputs-saving technologies. Reductions in the level of price variability will lead producers to favor yield-increasing or fixed-factor-saving technological change. Note that this result follows even when the two technologies have similar yield-risk effects. The result depends not on yield risk but on the heteroscedastic nature of return risk based on the yield per fixed acre.

6.4 The Introduction of Adjustment Costs

One obvious extension of the outlined model is to introduce positive adjustment costs. Stoneman (1981) developed an intrafirm diffusion model under uncertainty that can be readily adopted to this purpose. Assume that the cost of adjustment, $A_{it}$, of technology $i$ at time $t$ is related to the rate of change in $L_{it}$ according to

$$A_{it} = \Theta_i \frac{(L_{it} - L_{it-1})^2}{2L_{it-1}}.$$

(6.7)

Assuming that adjustment costs for the second alternative technology are zero, the first-order condition (6.4) can be rewritten as

$$\frac{1}{U'(W)} \frac{\partial V}{\partial L_{1t}} = \bar{P}_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) - \frac{\partial A_{1t}}{\partial L_{1t}}$$

$$-\phi_\sigma^2 p_t [Y_{2t}(Y_{1t} - Y_{2t})L + (Y_{1t} - Y_{2t})^2 L_{1t}] = 0.$$

(6.8)
If the firm chooses \( L_{it} \) given \( L_{it-1} \), the rate of adoption can be derived from (6.7) and (6.8).

\[
\frac{dL_{1t}}{dt} \frac{1}{L_{1t}} = \frac{1}{\Theta_1} [P(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) - \phi \sigma_{pt}^2(Y_{1t} - Y_{2t}L)(1 - \frac{L_{1t}^*}{L_{1t}})].
\]  

(6.9)

Solving (6.9) for \( L_{1t} \) gives

\[
L_{1t} = \frac{L_{1t}^*}{1 + \text{EXP}(-\alpha_1 t - \beta_1)},
\]  

(6.10)

where

\[
\alpha_1 = \frac{1}{\Theta_1} [P(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) - \phi \sigma_{pt}^2(Y_{1t} - Y_{2t}L)].
\]  

(6.11)

and \( \beta_1 \) equals the log of \( L_1 \) at time zero; that is, when the producer discovers the technology divided by the difference between \( L_{1t}^* \) and the initial level:

\[
\beta_1 = \log \frac{L_{10}}{L_{1t}^* - L_{10}}.
\]  

(6.12)

Note that if \( \alpha_1 > 0 \), the intrafirm diffusion process will be logistic, even though no learning mechanism is incorporated. This result is in agreement with most studies of this process, which have found that new technologies tend to be adopted in a sigmoid pattern through time. The result is also consistent with Stoneman's (1981) result in which he generates a logistic diffusion curve in the case in which only one new technology is available.

By using an adjustment cost function, Stoneman's (1981) model, however, considers the diffusion path of a single technology. It is therefore useful to see whether
the result can be repeated when the producer might choose between two technologies.

If we repeat the above analysis for the second technology, we get

$$L_{2t} = \frac{L_{2t}^*}{1 + EXP(\alpha_2 t - \beta_2)},$$

(6.13)

where

$$\alpha_2 = \frac{1}{\Theta_2} [P_t(Y_{1t} - Y_{2t}) - (C_{1t} - C_{2t}) - \phi \sigma_{pt}^2 (Y_{1t} - Y_{2t})],$$

(6.14)

and

$$\beta_2 = \log \frac{L_{20}}{L_{2t}^* - L_{20}}.$$  

(6.15)

Note that to motivate the existence of a standard logistic diffusion process, $\alpha_2$ must be negative. The variable $\alpha_1$ in logistic curve (6.10) is directly proportional to the speed of diffusion.\(^3\) Whereas the variable $\alpha_2$ in (6.13) is the inverse of the speed of diffusion, obviously $\sigma_p^2$ is negatively related to $\alpha_1$ and $\alpha_2$. This result leads to the following proposition and corollary.

**Proposition 11:** A lower level of price variability will increase the speed of diffusion for the yield-increasing technology.

**Corollary 8:** A higher level of price variability will increase the speed of diffusion for the cost-reducing technology.

This proposition implies that price stabilization policies increase the diffusion rate of yield-increasing technologies and reduce the speed of diffusion of cost-reducing technologies.\(^3\) See Mansfield (1966) and Stoneman (1981).
technologies.

The above propositions and corollaries can be explained by Figures 6.3 and 6.4. Assuming that only yield-increasing technologies need adjustment costs and that $\alpha_1 > 0$, Figure 6.3 shows that a logistic curve for yield-increasing technology diffusion can be drawn. $L_{2t}^*$ will be a declining curve, and a lower level of $\sigma_{pt}^2$ will raise $L_{1t}^*$ and the speed of diffusion of yield-increasing technologies. Thus, variable-input-using technological change occurs.

Figure 6.4 shows that, if only cost-reducing technologies need adjustment costs and if $\alpha_2$ is negative, $L_{2t}^*$ and the speed of diffusion for cost-reducing technologies increase and $L_{1t}^*$ decreases as $\sigma_{pt}^2$ goes up. This result implies variable-input-saving technological change.

6.5 Concluding Remarks

This chapter provides some results concerning technology adoption patterns and technological change for two innovations that are assumed to be introduced simultaneously: one yield increasing, the other cost reducing. The analysis assumes that the output price is the only random variable and that the farmer is risk averse. The results show that a higher level of price variability will be one of the reasons for variable-inputs-saving technological change. On the other hand, a lower level of price variability will lead to variable-inputs-using technological change.

The relationship between price variability and the speed of technology diffusion can also be determined by this model. The result is that a higher level of price variability will be associated with a low (high) speed of diffusion for yield-increasing
Figure 6.3: Dynamic path with $\alpha_1 > 0$ and adjustment costs in yield-increasing technology adoption [$\sigma_{pt}^2(a) > \sigma_{pt}^2(b)$]
Figure 6.4: Dynamic path with $\alpha_2 < 0$ and adjustment costs in cost-reducing technology adoption $[\sigma_{pt}^2(c) < \sigma_{pt}^2(d)]$
(cost-reducing) technology.

An important implication of this chapter is the linkage between government price stabilization policies and technology adoption and technological change. A tentative conclusion is that the introduction of a price-stabilization policy will encourage producers to adopt yield-increasing technologies and that the discontinuation of a price-stabilization program will increase the development and adoption of cost-reducing technologies. This conclusion implies that countries wishing to develop or encourage the adoption of technologies that increase the productivities of the fixed factors of production (yield increasing) may find that price stabilization programs work for them. On the other hand, countries that wish to encourage the adoption of technologies that reduce average variable costs and purchased-input use (cost reducing) may find that price stabilization programs work against them.

It may also be noted that the framework of this chapter also suggests some interesting empirical possibilities. The survey data for individual farms may be useful to undertake meaningful empirical work.
7. SUMMARY

In economics, issues associated with technical and technological change are so manifold that still numerous questions remain to be answered. As a result, further explorations in this topic are possible and desirable to increase the economic welfare of society. The main issues of this dissertation are those problems that are related to technical and technological change under uncertainty; one issue is the factor bias of technical change under uncertainty, and the other is the technology adoption under price uncertainty.

Chapter 2 provides a brief review of the theory of technical change and technology adoption. The review of literature indicated that the effect of uncertainty on the bias of technical change has not been investigated. Thus, the first issue examined in this dissertation is how uncertainty may affect the factor bias of technical change. This issue is examined in Chapters 3, 4, and 5. The literature review also indicated that there have been no analyses of how government price stabilization programs influence the type of technologies that are adopted or the rate of diffusion of new technologies. The second issue, therefore, is to discover the relationship between price variability and technology adoption and diffusion. This issue is examined in Chapter 6.

Uncertainty might exist in the market or in the production process itself. Assuming price uncertainty, the theory of endogenous technical progress is extended.
Chapter 3 shows that uncertainty in input prices will affect the bias of technical change. It is shown that the Hicksian proposition can be justified in both the deterministic case and in the case of a risk-neutral firm. Assuming that the elasticity of substitution is greater than one, if a risk-averse firm faces input price uncertainty, endogenous technical change will be biased toward the input that has the more certain price. This result implies that, even if the growth rates are different among input prices, Hicks-neutral technical change is possible.

Output price uncertainty does not affect the direction of technical change bias but does affect the degree of bias. If uncertainty exists in an input and output price, the contemporaneous correlation coefficient plays an important role in the rate of technical change bias. If the elasticity of substitution is greater than one and the output price and input price are insignificantly and positively correlated or negatively correlated, technical change may be biased toward the input that has a certain price. On the contrary, if the prices are significantly and positively correlated, technical change may be biased toward the input that has an uncertain price.

An important result of this chapter is that a firm will conduct research to increase the productivity of the inputs whose prices are less variable. This intuitive proposition is testable and has some policy implications. Governments can and often do influence the prices of inputs. This influence occurs via market intervention or through the legislative process. To the extent that our assumptions are valid, these policies will affect technical progress. Governments that wish to maximize labor productivity may find that relaxation of interest rates and currency stabilization policies works in their favor.
Chapter 4 investigates the impact of production uncertainty on the bias of technical change. Inputs are characterized from arguments of the standard deviation function. The results show that, assuming that the elasticity of substitution is greater than one, technical change will be biased toward risk-reducing inputs and against risk-increasing inputs. It is also shown that the degree of bias would be increased as the riskiness increases or as the firm becomes more risk averse. From a policy perspective, the principal conclusion that can be drawn from this chapter is that production uncertainty will influence the types of technology that are developed and adopted. Also, the results suggest that research should concentrate on technologies that reduce risk in regions where production uncertainty is high.

Chapter 5 attempts to incorporate hedging or forward contracts into the model. The results show that, under output price uncertainty, the existence of forward markets has no effect on the direction of technical change bias but it affects the degree of bias. It is also shown that, under input and output price uncertainty, if the forward market is unbiased and the elasticity of substitution is greater than one, then technical change will be biased toward the input that has a certain price. By comparing these results with the results of Chapter 3, we can conclude that the activation of forward markets influences technical progress.

Chapter 6 provides some results concerning the relationship between price uncertainty and technology adoption patterns and the relationship between price uncertainty and technological change for two innovations that are simultaneously introduced. It is concluded that a higher level of price variability will be one of the reasons for variable-input-saving technological change. On the other hand, a lower
level of price variability will lead to variable-input-using technological change. It is also shown that a higher level of price variability will decrease the speed of diffusion for a yield-increasing technology and increase that for a cost-reducing technology. These results imply that the introduction of a price stabilization program will encourage producers to adopt yield-increasing technologies and that the discontinuation of a price stabilization program will increase the development and adoption of cost-reducing technologies.

The primary purpose of this dissertation is to contribute to the development of the theories of technical change and technology adoption. Some implications for policy makers and researchers are included. An overall conclusion is that some relationships between uncertainty and technical or technological change should exist. Also, the results might explain regional differences for technical change bias and the technology diffusion process. Thus, policy makers and researchers should recognize these relationships and endeavor to find regional uncertainty factors.
8. APPENDIX

8.1 Itô Control Model

In economics, the Itô control model is becoming popular for theoretical modeling in continuous time because little realism is sacrificed for a gain in analytical power. In this section, Itô differentiation and dynamic programming for a vector of state equations\(^1\) are briefly sketched for convenience in following the derivations in Chapters 3, 4, and 5.

Suppose that a vector of state variables is assumed to evolve in a continuous stochastic manner according to

\[
dS = \delta(t, S, C)dt + \sigma(t, S, C)dz,
\]

(8.1)

where \(S\) is a vector of state variables (m×1), \(\delta\) is a vector of the expected change function in the state (m×1), \(\sigma\) is a matrix of the standard deviation function (m×n), \(C\) represents a vector of control variables (p×1), and \(z\) denotes a vector of Wiener processes (n×1).

\(^1\)Refer to Arnold (1974), Hertzler (1987), Kamien and Schwartz (1981, Chapter 21), Malliaris and Brock (1982), and Mangel (1985) for a detailed discussion of Itô differentiation and dynamic programming.
8.1.1 Itô differentiation

Suppose that the variable \( D \) is a function of time and the stochastic state variables, \( D = G(t, S) \), then the vector formula of Itô differentiation is obtained as

\[
dD = G_t dt + \sum_{i=1}^{m} G_{S_i} dS_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} G_{S_iS_j} dS_i dS_j
\]

(8.2)

\[
= \left[ G_t + \sum_{i=1}^{m} G_{S_i}(\delta)_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} G_{S_iS_j}(\sigma\Lambda\sigma')_{ij} \right] dt
\]

\[
+ \sum_{i=1}^{m} G_{S_i}(\sigma dz)_i,
\]

where \( S_i \) is the \( i^{th} \) element of the \( S \) vector, \( G_{S_i} \) and \( G_{S_iS_j} \) are first and second partial derivatives of \( G \) with respect to elements of the \( S \) vector, \( (\delta)_i \) represents the \( i^{th} \) element of the \( \delta \) vector, \( \Lambda \) is the contemporaneous correlation matrix, \( (\sigma\Lambda\sigma')_{ij} \) is the \( ij^{th} \) element of the \( m \times m \) covariance matrix, \( \sigma\Lambda\sigma' \), and \( (\sigma dz)_i \) denotes the \( i^{th} \) element of the \( m \times 1 \) vector, \( \sigma dz \).

8.1.2 Dynamic programming

Assume that a firm chooses a vector of control variables to maximize its expected discounted utility, subject to (8.1), which represents the change in the state variables. The objective function may be written as

\[
J(S_0) = \text{Max} \ E[\int_0^T e^{-rt} U(S_t, C_t) \ dt + e^{-rT} V(S_T) | S_0 = s_0].
\]

The Itô version of the Bellman equation will be

\[
0 = J_t + \text{Max} \left[ e^{-rt} U + \sum_{i=1}^{m} J_{S_i}(\delta)_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} J_{S_iS_j}(\sigma\Lambda\sigma')_{ij} \right],
\]
where $J_{s_i}$ and $J_{s_i s_j}$ are partial derivatives of $J$ with respect to elements of the $S$ vector.

The stochastic Hamiltonian for a vector of state variables is

$$H = e^{-rt} U + \sum_{i=1}^{m} J_{s_i}(\delta)_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} J_{s_i s_j}(\sigma \Lambda \sigma')_{ij}.$$  \hspace{1cm} (8.3)

To choose the controls, $C$, partially differentiate $H$.

$$H_C = 0 = e^{-rt} U_C + \sum_{i=1}^{m} J_{s_i}(\delta C)_i$$

$$+ \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} J_{s_i s_j}(\sigma \Lambda \sigma')_{ij}; \ldots; \sum_{i=1}^{m} J_{s_i s_j}(\sigma \Lambda \sigma')_{ii} (\sigma')_i,$$

where $U_C$ is the partial derivative of utility with respect to the control variables and $(\delta C)_i$ and $(\sigma')_i$ are the partial derivatives.

By substituting the optimal controls into the Bellman equation, it can be converted to the Hamilton-Jacobi equation.

$$0 = J_t + \text{Max} \left[ e^{-rt} U^* + \sum_{i=1}^{m} J_{s_i}(\delta^*)_i \right.$$

$$+ \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} J_{s_i s_j}(\sigma^* \Lambda \sigma^*)],$$

where $^*$ represents the optimum value.

By Itô differentiating the marginal indirect utility, $J_\theta(t, S)$, and substituting the partial differentiation of the Hamilton-Jacobi equation with respect to $S$ into the Itô differentiation of the marginal indirect utility, another group of first-order conditions, that is, the evolution of the marginal utility of the state variables, may be obtained.

$$dJ_{s_k} = -[e^{-rt} U_{s_k} + \sum_{i=1}^{m} J_{s_i}(\delta s_k)_i]$$  \hspace{1cm} (8.5)
\begin{align*}
+ \sum_{i=1}^{m} \sum_{j=1}^{m} J_{s_i s_j}(\sigma \lambda)_j dt 
+ \sum_{j=1}^{m} J_{s_i s_j}(\sigma \lambda)_j (\sigma'_{s_k})_i dt 
+ \sum_{i=1}^{m} J_{s_k s_i}(\sigma dz)_i,
\end{align*}

where \( J_{s_k}, U_{s_k}, \delta_{s_k}, \) and \( \sigma_{s_k} \) represent derivatives with respect to \( S_k \).

Therefore, a complete set of first-order conditions characterizes the optimal controls (8.4), the evolution of the state variables (8.1), and the evolution of the marginal utility of the state variables (8.5).

### 8.2 The Bias of Technical Change

The relationship between the Hicks definition of technical change bias and factor augmentation was derived by Kamien and Schwartz (1969). In this section, the result is summarized for convenience.

The absolute value of the slope of iso-quant can be derived as

\[ \frac{F_2 B}{F_1 A} = Z(A, B, X_1, X_2), \]  

(8.6)

where \( Z \) is defined by (8.6). The change in the slope of the iso-quant through a given point due to factor augmentation may be obtained as

\[ \frac{d}{dt} \left( \frac{F_2 B}{F_1 A} \right) = \dot{A} \frac{\partial Z}{\partial A} + \dot{B} \frac{\partial Z}{\partial B}. \]  

(8.7)

Differentiating (8.6) with respect to \( A \) yields

\[ \frac{\partial Z}{\partial A} = \frac{B}{(F_1 A)^2} (AX_1 F_1 F_2 - AX_1 F_2 F_{11} - F_1 F_2) \]  

(8.8)

\[ = \frac{B}{(F_1 A)^2} (AX_1 F_1 F_2 + BX_2 F_2 F_{12} - F_1 F_2) \]  

(8.9)
where \( e \) is the elasticity of substitution \((e = \frac{F_1 F_2}{F_1 F_2})\). Equation (8.9) can be obtained by making use of the fact that \( F_1 \) is homogeneous of degree zero. Also, the linear homogeneity of \( F \) is utilized to get (8.10).

By using the same procedure, it can be established that

\[
\frac{\partial Z}{\partial B} = -\frac{F_2}{AF_1} \left( \frac{1-e}{e} \right). \tag{8.12}
\]

An explicit equation may be obtained by substituting (8.11) and (8.12) into (8.7).

\[
\Omega = \frac{\frac{d}{dt} (F_2 B / F_1 A)}{F_2 B / F_1 A} = \frac{1-e}{e} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right), \tag{8.13}
\]

where \( \Omega \) is technical change bias. Technical change is \( X_1 \)-using \((X_2\text{-saving})\) if \( \Omega < 0 \) and \( X_1 \)-saving \((X_2\text{-using})\) if \( \Omega > 0 \). Hicks-neutral technical change may be defined as \( \Omega = 0 \).
9. BIBLIOGRAPHY


Binswanger, H. P. "A Microeconomic Approach to Induced Innovation." Economic


Pope, R. D. and Kramer, R. A. "Production Uncertainty and Factor Demands for


