7-2004

Leak Location in Plates Using Spatial Fourier Transform Based Analysis

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Abstract
The location of air leaks in plate-like structures is examined using a spatial Fourier transform based analysis. Noise data is collected over 2-D spatial arrays at sensor locations, from which mean cross-correlations are compiled. Propagation properties, transit times, and energy distribution among modes are extracted through spatial Fourier transformation of these data. A simple algorithm to determine source location using a reduced set of transform data is demonstrated experimentally, based upon extraction of energy propagation direction.

Keywords
leak detection, sensors, Fourier transforms, nondestructive testing, nondestructive evaluation

Disciplines
Aerospace Engineering

Comments

This article appeared in *AIP Conference Proceedings*, 760 (2005): 962–969 and may be found at http://dx.doi.org/10.1063/1.1916777.

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Citation: AIP Conf. Proc. **760**, 962 (2005); doi: 10.1063/1.1916777

View online: http://dx.doi.org/10.1063/1.1916777

View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=760&Issue=1

Published by the American Institute of Physics.

LEAK LOCATION IN PLATES USING SPATIAL FOURIER TRANSFORM BASED ANALYSIS

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ABSTRACT. The location of air leaks in plate-like structures is examined using a spatial Fourier transform based analysis. Noise data is collected over 2-D spatial arrays at sensor locations, from which mean cross-correlations are compiled. Propagation properties, transit times, and energy distribution among modes are extracted through spatial Fourier transformation of these data. A simple algorithm to determine source location using a reduced set of transform data is demonstrated experimentally, based upon extraction of energy propagation direction.

INTRODUCTION

This paper addresses acoustic leak location in plate-like structures, in which the acoustic signals are carried by multiple dispersive modes of plate wave propagation. The motivating application is the location of air leaks in the outer skin of spacecraft, through the detection of plate waves generated by the leak at distributed sensor locations. Current methods of acoustic leak location applied in this configuration are by-and-large based on an over-simplified view of the manner in which acoustic energy is transported, by assuming that signals propagate in a single mode of propagation with a non-dispersive (frequency independent) wave velocity. The consequence of this oversimplification is a less-than-robust performance, depending upon the extent to which multi-mode dispersion is evident in the measured signals. The objective of this work is to identify the measurements and data processing necessary to fully account for multi-mode dispersion when performing acoustic leak location. This is accomplished by examining an appropriately rigorous mathematical expression of signal propagation via plate waves, and determining the operations needed to extract source position. From this determination follows a prescription of a required data collection configuration and signal processing steps. The scope of the funded project includes identifying sufficient data collection and processing steps, and experimentally verifying the identified procedures through a set of laboratory experiments. In the fundamental studies reported in this work, consideration focuses primarily on establishing what is needed to robustly determine source location, with the allowance that the needed measurements may not be practical with existing acoustic emission instrumentation. However, once the requirements for a

CP760, *Review of Quantitative Nondestructive Evaluation Vol. 24,* ed. by D. O. Thompson and D. E. Chimenti

completely rigorous analysis are established, the scope of the project additionally includes a determination of the extent to which the measurements can be simplified without serious loss of performance robustness. The following sections of this paper describe the background, development, and testing of measurements required to determine leak location in a general two dimensional measurement configuration. A resulting simplified analysis based on determination of the direction of signal propagation is presented and assessed experimentally.

BACKGROUND

Work reported last year considered the simplified canonical problem of leak location in a plate in which two sensors were arranged to bracket the acoustic source in a collinear fashion.[1] In this configuration, the source location was determined by a time-of-flight analysis, in which the difference in travel time to the two sensors was translated into distance from the mid-sensor position. This analysis entailed the isolation of the contributing modes of acoustic propagation, and the determination and compensation for the frequency dependence of the contributing mode wave velocities. It was noted in that work that the modes of propagation can be fully isolated by Fourier transforming the sensor signals in both time and space. Fourier transformation of the signals in space requires the collection of data over a range of spatial position, hence data was collected on an linear array of discreet points at each sensor location. Were it possible to collect data at all spatial positions simultaneously, a straightforward 2-D Fourier transformation in time and space yields the desired mode isolation. However, because the leak is generating random noise, an additional step of accumulating the mean power spectrum is necessary to extract the coherent phase information indicating the distance of propagation as a plate wave. This step utilizes the cross-correlation of signals acquired at different sensor locations. The isolation of cross-correlated modes in turn requires a three-dimensional Fourier transformation of the cross-correlated signals, over time and position at the two sensor sites. It was noted that the cross-correlated signals between the various discreet array positions at the two measurement sites need not be obtained simultaneously, owing to the stationary property of the mean power spectrum. This observation enabled demonstration of the procedure using a simple two channel acquisition system, with mechanical scanning of point sensors over the array of sensing positions at the two sensor sites. Experiments were reported last year which demonstrated the robustness of the leak location procedure using this mode of data acquisition.

Work reported this year generalizes the experimental configuration from a collinear leak and sensor arrangement, to the case of a leak and sensors more randomly positioned on a plate. The straight-forward extension of the procedure reported last year requires the collection of cross-correlated signals over two-dimensional spatial arrays at each of three sensor locations, followed by a 5-dimensional Fourier transformation in time and two spatial dimensions at pairs of sensor locations. The experimental data collection hardware was appropriately modified to perform this ambitious data acquisition and processing task. In the course of implementing the data analysis, however, it was noted that a substantial simplification in the data collection and analysis is possible by basing the source location on the determination of *propagation direction*, rather than transit time. The use of propagation direction is not an option when the sensors and source are collinear, hence it is seen that the case examined last year is a special case of the more general problem, to which the methodology developed this year does not apply.

ALGORITHM AND MEASUREMENT DEVELOPMENT

The canonical measurement upon which data analysis is based is the detection of the acoustic signal generated by turbulence at an air leak, using a sensor positioned at a distance from the acoustic source. An appropriate mathematical expression for the sensor output voltage is

$$v(r,\omega) = N(\omega) \sum_n A_n(\omega)\, r^{-1/2} \exp(i\, k^n(\omega)\, r) \tag{1}$$

Eq.(1) expresses the output voltage v as a function of distance from the leak r and time harmonic frequency ω. It is assumed that the sensor is sufficiently far from the source so as to represent the geometric decay of the plate wave field as having a $r^{-1/2}$ dependence. It is assumed the leak generates a random signal $N(\omega)$ with a stationary mean power spectra $P(\omega)$, expressed

$$< N(\omega)\, N^*(\omega) >= P(\omega) \tag{2}$$

where $P(\omega)$ is a broadband function having a δ-like response in the time domain adequate for indicating the occurrence of an event. The signal is carried by a number of modes denoted by summation index n. The amplitude of the mode n in the recorded signal is denoted by the coefficient $A_n(\omega)$, which encompasses the strength of coupling of the acoustic turbulence into a particular plate wave mode, and the sensitivity of the sensor to the displacements generated by that mode as it passes beneath the sensor. It is assumed that coupling into the mode is uniform with respect to direction. The wave number associated with each mode is denoted $k^n(\omega)$, which embodies the dispersive properties of the mode of propagation.

It is assumed that the distance between source and sensor is sufficiently large to allow the assumption of a linear wave front within a neighborhood of the sensor position. The distance r is thereby expressed

$$r \sim r^a + \underline{d}^a \bullet \underline{x}^a \tag{3}$$

where r^a is the distance to a fixed reference position at sensor site "a", $\underline{d}^a$ is a 2-D unit vector in the direction of propagation, and $\underline{x}^a$ is a 2-D vector connecting the fixed reference position to the sensor position. The terminology "sensor site" will refer to the neighborhood about the fixed reference point over which the sensor position is varied to collect local spatial dependence of the signal.

Using eq.(3), eq.(1) is re-expressed

$$v^a(\underline{x}^a,\omega) = N(\omega)\, (r^a)^{-1/2} \sum_n A_n(\omega) \exp(i\, k^n(\omega)\, (r^a + \underline{d}^a \bullet \underline{x}^a)) \tag{4}$$

It is assumed in writing eq.(4) that signal amplitude variation over the sensor site due to geometric decay of the wave field is negligible. To demonstrate mode isolation, consider a window function $W(\underline{x}^a)$ prescribed over sensor site "a", which is zero outside the identified neighborhood. The simplest example has a constant non-zero value over a square area about the fixed reference point. Multiplying eq.(4) by the window, and Fourier transforming the product in $\underline{x}^a$ yields

$$\hat{v}^a(\underline{k}^a,\omega) = N(\omega)\,(r^a)^{-1/2} \sum_n A_n(\omega) \exp(i\,k^n(\omega)\,r^a)\,\hat{W}(\underline{k}^a - k^n(\omega)\,\underline{d}^a) \quad (5)$$

where $\hat{v}^a(\underline{k}^a,\omega)$ denotes the 2-D windowed Fourier transform in space, parameterized by the spatial frequency vector $\underline{k}^a$. It is seen that if the Fourier transform of the window $\hat{W}^a(\underline{k}^a)$ is sufficiently δ-like (i.e. if the window is sufficiently wide), the value of eq.(5) when evaluated at $\underline{k}^a = k^n(\omega)\,\underline{d}^a$ is

$$\hat{v}^a(k^n(\omega)\,\underline{d}^a,\omega) = N(\omega)\,(r^a)^{-1/2}\,A_n(\omega)\exp(i\,k^n(\omega)\,r^a) \quad (6)$$

where it was assumed that

$$\hat{W}((k^n(\omega) - k^m(\omega))\,\underline{d}^a) \approx \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \quad (7)$$

Eq.(6) expresses the isolation of the wave mode n. Stated verbally, the spectral energy carried by a wave mode is localized in the three-dimensional transform space along a curve in the direction of propagation with radial distance from the ω-axis expressed by the wave number $k^n(\omega)$.

The goal of a time-of-flight analysis is to extract information on the distance r^a between source and sensor. Since the noise spectrum $N(\omega)$ is random, it is not possible to extract time-of-flight information from eq.(6) alone. This problem is treated by exploiting the stationary property of mean power spectrum, expressed by eq.(2). Assuming 3-D transformations of data are obtainable at sensor sites "a" and "b", a mean temporal cross-correlation of isolated mode types could be formed by averaging the product of transformed data obtained at site "a' and the conjugate of the transformed data collected at site "b"

$$\begin{aligned} \hat{C}^{ab}(\underline{k}^a,\underline{k}^b,\omega) &= \langle \hat{v}(\underline{k}^a,\omega)\,\hat{v}^{b*}(\underline{k}^b,\omega) \rangle \\ &= P(\omega)\frac{1}{\sqrt{r^a r^b}} \sum_n \sum_m A_n(\omega)\,A_m^*(\omega)\exp(i\,k^n(\omega)\,r^a - i\,k^m(\omega)\,r^b) \\ &\qquad \hat{W}(\underline{k}^a - k^n(\omega)\,\underline{d}^a)\,\hat{W}^*(\underline{k}^b - k^m(\omega)\,\underline{d}^b) \end{aligned} \quad (8)$$

The contribution of individual mode pairs can be isolated by evaluating eq.(8) on the curves $\underline{k}^a = k^n(\omega)\,\underline{d}^a$ and $\underline{k}^b = k^m(\omega)\,\underline{d}^b$

$$\hat{C}^{ab}(k^n(\omega)\,\underline{d}^a, k^m(\omega)\,\underline{d}^b,\omega) = < \hat{v}^a(k^n(\omega)\,\underline{d}^a,\omega)\,\hat{v}^{b*}(k^m(\omega)\,\underline{d}^b,\omega) >$$

$$\approx P(\omega)\frac{1}{\sqrt{r^a r^b}} A_n(\omega)\,A_m^*(\omega)\exp(i k^n(\omega)\,r^a - i k^m(\omega)\,r^b) \qquad (9)$$

where again the δ-like property of the window function, eq.(7), is imposed. Unlike eq.(6), the power spectrum P(ω) is not random, but rather a well-behaved function that will allow the extraction of time-of-flight information.

The methodology described in obtaining eq.(8) assumes that data appropriate for 3-D transformation is available from sites "a" and "b". In practice, this would require the simultaneous recording of noise waveforms at all spatial positions at sites "a" and "b". For example, using two 16x16 arrays of data collection points, this would require instrumentation having 512 input channels. As an alternative, it was observed last year that the spatial Fourier transforms could be equivalently applied *after* the compilation of the mean temporal convolution between pairs of sensor positions at sites "a" and "b". In this approach, mean temporal cross-correlations are recorded between all sensor position pairs ($\underline{x}^a$, $\underline{x}^b$), expressed in the temporal frequency domain as

$$C^{ab}(\underline{x}^a, \underline{x}^b,\omega) = < v^a(\underline{x}^a,\omega)\,v^{b*}(\underline{x}^b,\omega) > \qquad (10)$$

$$= P(\omega)\frac{1}{\sqrt{r^a r^b}}\sum_n\sum_m A_n(\omega)\,A_m^*(\omega)\exp(i k^n(\omega)\,(r^a + \underline{d}^a \bullet \underline{x}^a) - i k^m(\omega)\,(r^b + \underline{d}^b \bullet \underline{x}^b))$$

The mean temporal cross-correlation expressed by eq.(8) is obtained from eq.(10) by performing a windowed 5-D Fourier transformation in time, $\underline{x}^a$, and $\underline{x}^b$. Although equivalent in the end, the amount of data collected and the associated data processing times are two orders of magnitude greater than when first Fourier transforming the data at the sensor sites individually. As will be shown, it was realized that this large-scale task is probably not necessary for source location.

An algorithm is described to perform leak location by extracting time-of-flight information from eq.(8). Consider the following expression, which operates on like mode types (n = m) detected at sensor sites "a" and "b"

$$G_n^{ab}(s,\underline{p}^a,\underline{p}^b) = \int \hat{C}^{ab}(k^n(\omega)\,\underline{p}^a, k^n(\omega)\,\underline{p}^b,\omega)\exp(-i k^n(\omega)\,s)\,d\omega \qquad (11)$$

where $\underline{p}^a$ and $\underline{p}^b$ denote unit direction vectors. The source location can be determined by finding the value of s, $\underline{p}^a$ and $\underline{p}^b$ which maximizes $G_n^{ab}(s,\underline{p}^a,\underline{p}^b)$. Assuming that the received signals at sensor sites "a" and "b" obey the assumptions described in connection with eq.(3), it is readily seen that eq.(11) will be effectively zero for directions $\underline{p}^a$ and $\underline{p}^b$ other than $\underline{p}^a = \underline{d}^a$ and $\underline{p}^b = \underline{d}^b$. The directions $\underline{d}^a$ and $\underline{d}^b$ can therefore be determined directly by simply locating the 5-D position of the maximum spectral amplitude in the Fourier transformed data expressed by eq.(8). When evaluated using these directions, it is seen from eq.(11) that $G_n^{ab}(s,\underline{d}^a,\underline{d}^b)$ takes on the value

$$G_n^{ab}(s,\underline{d}^a,\underline{d}^b) = \frac{1}{\sqrt{r^a r^b}}\int P(\omega)\,|A_n(\omega)|^2\exp(i k^n(\omega)\,(r^a - r^b - s))\,d\omega \qquad (12)$$

For the well-behaved broad-band power spectra and wave mode amplitudes encountered in this work, eq.(12) will be maximum when $s = s_{max} = r^a - r^b$, i.e. the difference in source-to-sensor distances for sensor sites "a" and "b". The source therefore must lie in the directions $\underline{d}^a$ from sensor site "a", and $\underline{d}^b$ from sensor site "b", with distance r^b differing from distance r^a by s_{max}. Eq.(12) is referred to as a generalized cross-correlation, since it is seen that the expression reduces to the familiar cross-correlation of signals when the propagation is non-dispersive, i.e., when the wave number $k^n(\omega) = \omega/c_n$, where velocity c_n is a constant.

It is immediately evident that once directions $\underline{d}^a$ and $\underline{d}^b$ are known, no further analysis is required to determine the source location, provided the source and sensor locations are not collinear. This follows from the obvious fact that the source must lie at the intersection of lines through sensor sites "a" and "b" passing in directions $\underline{d}^a$ and $\underline{d}^b$, respectively. Hence, time-of-flight analysis is only required to determine the leak location in the case of collinear sources. This special case was demonstrated last year, which effectively evaluated eq.(12) for the special case of $\underline{d}^a = - \underline{d}^b$. In that case, the analysis assumed that all detected signals were propagating collinearly, therefore allowing the 5-D transformation to be reduced to a 3-D transformation.

Further examination of the non-collinear problem indicated that determination of the directions $\underline{d}^a$ and $\underline{d}^b$ does not require the full 5-D data collection and Fourier transformation. Consider the mean cross-correlated signal expressed by eq.(10), written now as

$$\begin{aligned} C^{ab}(\underline{x}^a, \underline{x}^b, \omega) &= \langle v^a(\underline{x}^a, \omega)\, v^{b*}(\underline{x}^b, \omega) \rangle \\ &= P(\omega)\;\; h^{b*}(\underline{x}^b, \omega)\;\; \frac{1}{\sqrt{r^a}} \sum_n A_n(\omega) \exp(i\,k^n(\omega)\,(r^a + \underline{d}^a \bullet \underline{x}^a)) \end{aligned} \tag{13}$$

where

$$h^{b*}(\underline{x}^b, \omega) = \frac{1}{\sqrt{r^b}} \sum_m A_m^*(\omega) \exp(-i\,k^m(\omega)\,(r^b + \underline{d}^b \bullet \underline{x}^b)) \tag{14}$$

The quantity $h^{b*}(\underline{x}^b,\omega)$ represents the signal transport response of the multi-mode plate wave propagation from the source to sensor "b". Fourier transformation of eq.(13) in $\underline{x}^a$ yields

$$\begin{aligned} \tilde{C}^{ab}(\underline{k}^a, \underline{x}^b, \omega) &= \langle \hat{v}(\underline{k}^a, \omega)\, v^{b*}(\underline{x}^b, \omega) \rangle \\ &= P(\omega) \frac{1}{\sqrt{r^a}} h^{b*}(\underline{x}^b, \omega) \sum_n A_n(\omega) \exp(i\,k^n(\omega)\, r^a - i\,k^m(\omega)\, r^b)\, \hat{W}(\underline{k}^a - k^n(\omega)\, \underline{d}^a) \end{aligned} \tag{15}$$

Considering the 3-dimensional ($\underline{k}^a$, ω) spectrum, it is evident from the δ-like property of the window function that the spectral energy will lie in the direction of propagation $\underline{d}^a$. It is seen that Fourier transformation in x^a only will isolate the modes detected at site "a", but that the signal contribution detected at site "b" will remain a complicated superposition of signals, as represented by the signal transport function $h^b(\underline{x}^b,\omega)$. Therefore, the result is sufficient for determining the direction of propagation, but insufficient for extracting time-of-flight information.

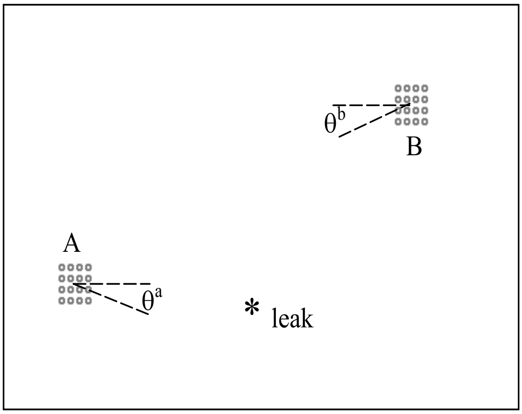

FIGURE 1. Measurement configuration.

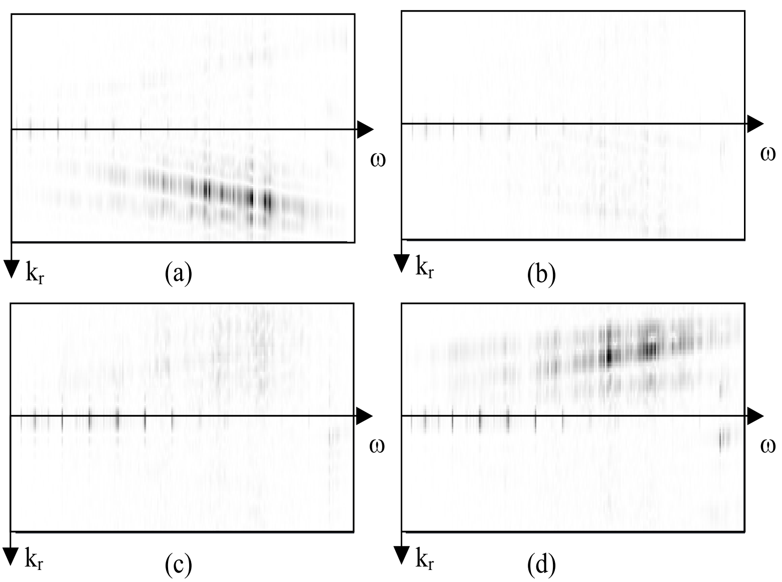

FIGURE 2. Measured spectra: a) site "a" 10^0, b) site "a" 46^0, c) site "b" 10^0, d) site "b" 46^0.

DIRECTION-BASED LEAK LOCATION

A location algorithm was formulated based on the evaluation of Fourier transformations at two sensor sites, as summarized by eq.(12). The algorithm is demonstrated here using experimental data collected from a 3/16th inch aluminum plate, on which an air leak was induced by drawing air through a .028 inch hole using a vacuum pump. Data was collected at two sensor sites, oriented in relation to the hole as indicated in fig.1. Sensor site "b" was positioned 37.4 cm to the right and 19.3 cm above sensor site "a". The leak was positioned 15.4 cm to the right and 2.8 cm below sensor site "a".

At each site, data was collected on a 16x16 grid with 2 mm point spacing. The spacing and grid dimension was selected to provide sufficient spatial frequency bandwidth for prevention of mode aliasing, and to provide sufficient mode isolation, via. eq.(7). Signals were recorded using a pair of transducers having an element diameter of 1.3 mm. One transducer was mechanically scanned over points at site "a', the other over points at site "b". Mean cross-correlations of signals from the two transducers were acquired and stored, using a bandpass of 10 – 500 kHz. Using one fixed position at site "b", the mean cross-correlations with signals at site "a" were Fourier transformed in time and in the two spatial dimensions of the acquisition grid, using a simple rectangular window. It was noted that the maximum spectral amplitude in the 3-D transform was received at a direction of 10 degrees in the $\underline{k}^a$ plane. A similar 3-D Fourier transform was applied to data collected at all points at site "b", correlated against a fixed position at site "a". It was noted that the maximum spectral amplitude in this case was received at a direction of 46 degrees in the $\underline{k}^b$ plane. Slices through these 3-D transform results are presented in fig.(2), taken at angles of 10 and 46 degrees in the $\underline{k}^a$ and $\underline{k}^b$ planes. Three modes are seen to be contributing to the signals, with one mode noticeably stronger than the other two. Based on the noted directions of propagation, the position of the leak source was computed to be 16.0 cm to the right and 2.8 cm below sensor site "a". The error in location measured by the spatial Fourier transform analysis was 0.6 cm, well within the acceptable margin.

SUMMARY

The use of spatial Fourier transformation of plate wave signals to isolate propagating modes and extract source location information was extended this year to address the 2-D problem of leak location in plate-like structures. It was determined that although a time-of-flight analysis analogous to that applied last year in the collinear source-sensor problem could be extended to 2-D, it is more expedient to perform the location using a directional analysis, in which the Fourier transformations are used to determine only the direction of energy transport at the receiver positions. The use of a directional analysis was demonstrated experimentally. Based on the observed robust performance of the spatial Fourier transformed-based leak location, ongoing work is examining the fabrication of simple array devices that will collect the required cross-correlation data in an application-compatible fashion.

ACKNOWLEDGEMENT

This material is based on work supported by NASA under award NAG-1-029-98.

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