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Abstract
The ultrasonic detection of cracks in the interior of an elastic solid by the use of surface transducers is a fundamental NDE problem. The presence of cracks may be detected either by observing the back scattered elastic waves using the launching transducer as a receiver or by observing obliquely scattered waves with a separate receiving transducer located elsewhere on the surface. Unfortunately, most of the theoretical work on the scattering of elastic waves from cracks has been confined to the case of a crack in an unbounded elastic solid, a situation far different from the experimental one. Even in that case, exact results are available only for the crack occupying a half plane. Exact results for cracks having finite surface area, such as a penny shaped cracks, are not available in detail, although many approximate calculations have been published, particularly in the low frequency limit.

Disciplines
Materials Science and Engineering | Structures and Materials
REVIEW OF THEORIES OF SCATTERING OF ELASTIC WAVES BY CRACKS*

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The ultrasonic detection of cracks in the interior of an elastic solid by the use of surface transducers is a fundamental NDE problem. The presence of cracks may be detected either by observing the back scattered elastic waves using the launching transducer as a receiver or by observing obliquely scattered waves with a separate receiving transducer located elsewhere on the surface. Unfortunately, most of the theoretical work on the scattering of elastic waves from cracks has been confined to the case of a crack in an unbounded elastic solid, a situation far different from the experimental one. Even in that case, exact results are available only for the crack occupying a half plane. Exact results for cracks having finite surface area, such as a penny shaped cracks, are not available in detail, although many approximate calculations have been published, particularly in the low frequency limit.

Before getting into some of the mathematical details of scattering from cracks, it is important to recognize that idealized cracks and real cracks may differ substantially in their behavior. For example, a cracked specimen may show different ultrasonic scattering characteristics depending on whether or not it is loaded. Such differences may be attributable to the closing of cracks under compression or the opening of cracks under tension. Important as such considerations are, they have received little attention from theorists. Consequently, until a better description of the boundary conditions at a real crack becomes available, the idealized theory must be employed.

From the theoretical point of view, a crack is a two dimensional surface of finite or infinite area located in the interior of an elastic solid. For example, a penny shaped crack can be thought of as the result of removing a thin disk shaped section of material from the interior of a solid. Boundary conditions are now applied at the surfaces of the void which has been created. In the case of a weak crack, the surfaces of the void are taken as free surfaces where the stress must vanish. In the case of a rigid crack, the void is imagined to be filled with a completely rigid material which pins the walls of the crack so that the displacement is zero everywhere. In each case the finite thickness of the disk is neglected and both faces are thought of as occupying the same plane. This approach, while mathematically convenient, avoids the question of how the faces of a crack interact with one another and whether the crack is open or closed. Typical example of such two dimensional cracks are shown in the first two figures (Figs. 1 and 2).

There are two features of a crack that are essential in determining its behavior as a scatterer of elastic waves. The first is that a crack represents a two dimensional surface across which the stress or displacement

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Fig. 1. Geometry of the Scattering Problem for a Penny Shaped Crack.
Fig. 2. Geometry of the Scattering Problem for a Crack Occupying a Quarter Plane.
or both can be discontinuous. The second is that cracks have edges which generate diffracted waves. The scattered waves produced when an incident compressional wave strikes a half plane crack is shown in Fig. 3. This Figure is an indication of the degree of complexity of the interaction of a crack with an elastic wave. Here we have an incident compressional wave, labeled I, which strikes the edge of a half plane crack viewed edge on. A diffracted compressional wave will be generated by this incident plane wave. A diffracted shear pulse also will be generated. In addition, there is going to be a reflected compressional wave and there will be a mode converted reflected shear wave. In addition to that, there will be a pair of conical waves or head waves, as they are called, which arise from mode conversion, and which travel out at the critical angle defined by the arcsine of the ratio of the shear velocity divided by the compressional velocity. In addition, if this is a free crack or a weak crack, there will be a Rayleigh wave which will be excited at the edge and which will propagate down the surface of the crack exponentially attenuated with depth away from the crack.

This is the type of problem which has been studied and solved in great detail; first in 1948 by Fridman for a rigid half plane crack; later in 1953 by Maue for a weak half plane crack; in great detail by Adrian De Hoop in his doctoral dissertation in 1958; and then repeated by many people after that. The physical situation is this. Imagine a crack like we have been discussing, but in a bounded elastic solid, or at least a semi-infinite one, so that there is a free surface of the solid. Until the reflected waves get back to the free surface on which the launching transducer is located, the theory applies. As soon as the reflected waves and the diffracted waves start to interact back with the free surface, then all of the conclusions based on a mathematical analysis of a crack in an unbounded solid no longer apply.

The half plane crack is a particularly simple situation. The reason it is so simple is because the displacement components parallel to the edge of the half plane are decoupled making the remaining problem two dimensional. Since this two dimensional problem has an exact solution, which is well known, the half plane problem provides a convenient means of testing various approximate solutions such as the Kirchhoff approximation. The important thing about the half plane problem is that it can be easily modeled experimentally. If you take a thin plate and you cut a notch in it and put a transmitter and a receiver on the edge of the plate, in that case the solution is described by a half plane problem with appropriately renormalized elastic constants which take into account that this is really a plate and not an infinite solid. This situation is indicated in Fig. 4. If you want a completely rigid crack, then you place a completely rigid filler material in the notch, much more rigid than the plate material, and you have a simple model of the rigid crack.

The solutions to the half plane problem and the experimental situation described here are directly related to one another, and they offer an interesting theoretical and experimental model in which one can look at various approaches to scattering from cracks by approximate methods. Consider the scattering of an arbitrary incident wave from a flat crack (see Fig. 1). In this situation,
Fig. 3. Diffracted and Reflected Wavefronts Produced an Incident Compressional Wave on a Half Plane Crack.
Fig. 4. Experimental Models for Scattering and Diffraction From Stress Free and Rigid Half Plane Cracks.
we're now talking about diffraction by an arbitrary crack occupying a two
dimensional surface. By an application of Green's theorem (see Fig. 5), one
obtains a representation for the amplitude of the scattered wave in terms
of the discontinuity or the jump in stress across the plane of the crack, \( \Sigma \),
and in terms of the discontinuity in displacement across the crack \( \Sigma \).

For a rigid crack, the displacement would be 0 on both surfaces, and you
would apply the boundary conditions on the crack so that the incident wave
and the scattered wave cancel one another. That gives an integral equation
which has been solved exactly for the half plane problem.

The Green's function approach to the problem involves the Green's function
for the elastic wave equation. The basic idea which Kirchhoff had in trying
to obtain approximate solutions is the following. Instead of calculating
jumps by solving the integral equation, Kirchhoff had the idea that on the
illuminated part of the scatterer, in the sense of geometrical optics, the
wave function and its normal derivative should be equal to their corresponding
values if the scatterer were absent, that is, if there were no crack there at
all. On the dark part of the scatterer, the wave functions and its normal
derivatives are 0. This would correspond to the assumption for the scattering
of elastic waves, which amounts to assuming that the stress and displacements
jump across a crack by an amount which is numerically equal to the corresponding
values of the incident wave at the illuminated surface of the crack. When
this assumption is made, it is found that all of the reflected waves are lost
in the solution.

The critical angle head wave is also lost in the solution. Only the
incident wave and the diffracted waves generated by the edge of the crack are
obtained. This explains why the Kirchhoff assumptions are supposed to solve
diffraction by a perfectly absorbing scatterer or in optical terms, by a black
screen.

The Kirchoff approximation can be modified in such a way that the correct
reflected waves are obtained. And this has been discussed at great length by
Bouwkamp and by De Hoop and others. However, the critical angle head waves are
always lost. One can think of the Kirchoff approximation as a method of
specifying the physical properties of a crack in terms of the jumps in displace­
ment and stress across it. If these jumps are numerically equal to the corre­sponding
values of the incident wave at the crack, the crack is perfectly
absorbing or black.

In general, however, the theories of Kirchoff and modifications of it are
poor substitutes for rigorous diffraction theory, which means wave equation
plus boundary conditions, because they do not correctly describe the field
in the vicinity of the scatterer, and in the long wavelength limit because
they entirely fail to predict the correct order of magnitude of the field far
from the scatterer, as has been pointed out in detail by Bouwkamp.

Now, if we turn to the scattering from a penny shaped crack (see Fig. 1),
some of the earliest work on that problem is due to Filipczynski in 1961.
Filipczynski treated this problem by setting up the elastic wave equation in
oblate spheroidal coordinates.
\[ c_{ijpq} \left( \frac{\partial^2 u_p}{\partial x_j \partial x_q} \right) - \rho \frac{\partial^2 u_i}{\partial t^2} = 0 \]

\[ u_i^s(x_1, x_2, x_3, \omega) e^{i \omega t} \]

\[ u_i^s(x_1, x_2, x_3, \omega) = \sum c_{jkpq} G_{ij} \left[ \frac{\partial u_p}{\partial x_j} \right]_+ n_k^+ dS \]

\[ + \left( \frac{\partial}{\partial x_q} \right) \sum c_{jkpq} G_{ip} \left[ u_j \right]_+ n_k^+ dS \]

\[ G_{ij}(x-x', \omega) = \frac{1}{4\pi \rho} \left\{ \frac{1}{\omega^2} \frac{\partial^2}{\partial x_i \partial x_j} \left\{ \frac{\exp(-ik_pr)}{r} - \frac{\exp(-ik_pr)}{r} \right\} + \frac{1}{\nu_s^2} \frac{\exp(-ik_pr)}{r} \delta_{ij} \right\} \]

where \( r = \left\{ (x_1-x'_1)^2 + (x_2-x'_2)^2 + (x_3-x'_3)^2 \right\}^{1/2} \), \( k_p = \omega/\nu_s \), \( k_s = \omega/\nu_s \), \( \rho \nu_p^2 = \lambda + 2\mu \) and \( \rho \nu_s^2 = \mu \).

\[ [u_i]^+ = u_i^+ - u_i^- \]

\[ [\tau_{ij}]^+ n_j^+ = ([\tau_{ij}]^+ - \tau_{ij}^-) n_j^+ \]

\[ \tau_{ij} = c_{ijpq} \frac{\partial u_p}{\partial x_q} \]

Fig. 5. Radiation Field of the Diffracted Compressional and Shear Waves Produced by a Normally Incident Compressional Wave on a Penny Shaped Crack. (From Mal Ref. 11)
The results of Filipczynski's calculations are outlined in Fig. 6. If we introduce a usual spherical coordinate system, we have a penny shaped crack in the $x_1 - x_2$ plane, then, in that case in the far field long wavelength limit, the displacement can be written as the gradient of the scalar and the curl of a vector potential with a single component. The result of a great deal of analysis reduces simply to this: According to Filipczynski's results, the scalar potential varies as the cosine of the angle with respect to the normal to the plane of the penny shaped crack, and the vector potential varies as the sine of that angle.

The coefficients, $\phi_0$ and $A_\phi$, are given by simple expressions in terms of the longitudinal wave number, $k_l$, and transverse wave number, $k_t$. If we take the derivatives of the potentials, then, to first order in $\frac{1}{k}$, the radial and the angular components of the displacement field have a $R \cos \theta$ and a $\sin \theta$ dependence. This is supposed to be for a free penny shaped crack in the long wavelength far field limit.

Many others have looked at the scattered field from a penny shaped crack, and I would like to briefly say a few words about what results they have obtained. Different expressions for the far field displacements due to diffraction of elastic waves by rigid and weak circular disks in unbounded elastic solids have been obtained by Mal'cev and formulas for the corresponding scattering cross sections appear in the works of Robertson and Filipczynski. Details of the computation of the scattering cross section when plane time harmonic compressional or shear waves are incident on two or three dimensional obstacles in an infinite elastic solid have been discussed by several authors. However, except for Filipczynski's work, all the other cited results have been obtained in cylindrical coordinates by iteratively solving integral equations for the scattered field in the long wavelength limit.

The work of Robertson is particularly interesting. He assumed that the scattered field due to a plane compressional wave normally incident on a penny shaped crack could be modeled by a harmonically oscillating piston on the surface of a semi-infinite elastic solid. Thus, Robertson replaced the problem of calculating the scattered field from a disc shaped flaw in an unbounded elastic medium by the problem of calculating the radiation field of a disc shaped transducer on the surface of an elastic half space. Robertson considered the case where the time harmonic normal stress is prescribed at the disc surface and the displacements are zero elsewhere on the boundary. He also treated the complementary case where the displacement is prescribed directly underneath the disc surface and the stresses are zero elsewhere on the boundary. Both types of problems are two part boundary value problems, leading to integral equations which have only been solved iteratively in the long wavelength limit.

There is another kind of disc shaped transducer problem that has been solved exactly by Miller and Pursey. It's the problem of the field due to an oscillating normal stress applied over a disc shaped region on an otherwise free surface of a semi-infinite elastic solid. Since, in this case, the stress alone is specified on the boundary, the problem can be solved exactly, and the solution obtained by Miller and Pursey has the form shown in Fig. 7. Here the scalar potential is $\phi$, the vector potential is $\Psi$, the components of displacement are $u_r$ and $u_\theta$, and $t_{rr}$ and $t_{r\theta}$ are the usual stresses. The boundary conditions satisfied by this solution are that the normal stress is unity for a radius less than "a" and vanishes for $r$ greater than "a", and the
\[ x_3 = R \cos \theta \]

\[ x_2 = R \sin \theta \sin \phi \]

\[ x_1 = R \sin \theta \cos \phi \]

\[ U = 2 \Phi + \nu(x_1, A) \]

\[ \phi = \frac{e^{-ik_L R}}{R} \cos \theta \]

\[ A_\theta = \frac{e^{-ik_L R}}{R} \sin \theta \]

\[ \Phi_0 = \frac{2a}{3\pi} \left( \frac{k_L a}{1 + \frac{1}{4} \left( \frac{k_I}{k_L} \right)^2} \right)^2 \]

\[ A_\Phi = \frac{2a}{3\pi} \left( \frac{k_L a}{1 + \frac{1}{4} \left( \frac{k_I}{k_L} \right)^2} \right)^2 \]

\[ u_R = -ik_L \Phi_0 \frac{e^{-ik_L R}}{R} \cos \theta \]

\[ u_\theta = ik_T A_0 \frac{e^{-ik_T R}}{R} \sin \theta \]

\[ u_\Phi = 0 \]

Fig. 6 Amplitude Factors for the Far Field Radiation pattern Produced by Scattering from a Penny Shaped Crack. (From Mal Ref. 11)
\[
\begin{align*}
\mathbf{u}_r &= \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z} \\
\mathbf{u}_z &= \frac{\partial \psi}{\partial z} + \left( \frac{\partial^2 \psi}{\partial z^2} + k_p^2 \right) \\
T_{rz} &= \mu \left( \frac{\partial \mathbf{u}_r}{\partial z} + \frac{\partial \mathbf{u}_z}{\partial r} \right) \\
T_{zz} &= -\lambda \frac{k_p^2 \psi}{\partial z} + 2\mu \frac{\partial \mathbf{u}_z}{\partial z}
\end{align*}
\]

\[
\begin{align*}
[T_{zz}]_{z=0} &= 1 \quad \text{for } r \leq a \\
[T_{zz}]_{z=0} &= 0 \quad \text{for } r > a \\
[T_{rz}]_{z=0} &= 0 \quad \text{for } 0 \leq r \leq a
\end{align*}
\]

\[
\begin{align*}
\psi(r, z) &= \frac{a}{\mu} \int_0^\infty \frac{(2k^2-k_p^2)}{F(k)} \exp(-\nu z) J_1(ka) J_0(kr) dk \\
\psi(r, z) &= \frac{a}{\mu} \int_0^\infty \frac{2\nu}{F(k)} \exp(-\nu z) J_1(ka) J_0(kr) dk
\end{align*}
\]

\[
\begin{align*}
\nu &= \sqrt{k^2 - k_p^2} \\
\nu' &= \sqrt{k^2 - k_B^2} \\
F(k) &= (2k^2-k_p^2)^2 - 4k^2\nu\nu'
\end{align*}
\]

Fig. 7. Field radiated from a circular disc.
The solution is very simple. It can be written down in terms of the scalar potential $\phi$ and the vector potential, $\psi$, satisfied by some combination of Bessel functions. The solution can be expanded asymptotically to get the far field result. The result obtained from the Miller and Pursey expression for the radial and angular components of the field are shown in Fig. 8. There is a $\sin 2\theta$ dependence in the angular displacement component and a $\cos \theta$ dependence in the $u_R$ which is the radial displacement component.

This should be compared with the other asymptotic expression shown in Fig. 8. Consider a pair of formulas for the asymptotic field due to scattering from a weak disc given by Mal. Again, you see the $\sin 2\theta$ dependence and a different angular dependence in this expression. These amplitude factors are given by the iterative solution of the integral equations. Filipczynski's result doesn't show any $\sin 2\theta$ dependence.

On the other hand, if we come to the case of a rigid disc, then Mal gives for the asymptotic displacement field a $\cos \theta$ dependence in $u_R$ and $\sin \theta$ dependence in $u_\theta$, which agree with the results of Filipczynski for the weak disc. Of course, that doesn't take into account the fact that these amplitude factors are still numerically prescribed, and they can vary. So, actually, what appears to be an analytic form may in fact be something different.

By comparison, Filipczynski's results are shown at the bottom of Fig. 8. From the work of Mal angular dependence for the scattered field when a plane wave is normally incident on a penny shaped crack in shown in Fig. 9. The $P(k)$ factors which appear in the equations in the middle of Fig. 8 are shown in Fig. 10 and are amplitude factors which are obtained by the numerical solution by iteration of the integral equation. These factors must be folded into the angular dependence in order to get the actual behavior of the radiated field.

What this all amounts to is that there are a number of different expressions in the literature which deal with the same problem, and they all appear, at least superficially, to have somewhat different analytical forms, and that's going to require some further looking into before we decide whether everybody has got the same answer or not.

I'd like to point out that the most recent work on the scattering by a penny shaped crack was done in 1973 by Knopoff and Garbin, and I think that that's a very thorough treatment of the problem, although it is still an approximate solution. It was published in the Quarterly of Applied Mathematics 30, 453 (1973), and has a good collection of references to earlier work.

Now, in closing I'd like to make some general remarks about other diffraction problems and approaches. The diffraction of plane elastic waves by two dimensional straight strips or cracks of finite width has been treated by Ang and Knopoff and by Loeber and Sih. More recently, Keer and Luong have considered the diffraction of waves and stress intensity factors in cracked layered composites. Non-axi-symmetric scattering of plane compressional elastic waves by a rigid disc has also been examined by Datta. Related numerical work using the finite-element approach to acoustic scattering from elastic and rigid discs immersed in water has recently been published by Hunt et al.
\[ u_R = -\frac{\alpha^2}{2\mu} e^{-1R} \frac{\cos(\xi^2 - 2\sin^2 \theta)}{F_0(\sin \theta)} \]

\[ u_\theta = -\frac{\operatorname{id} \xi^3}{2\mu} e^{-1\xi R} \frac{\sin 2\theta (\xi^2 \sin^2 \theta - 1)^{1/2}}{F_0(\xi, \sin \theta)} \]

\[ \xi = \frac{k_\beta}{k_\alpha}. \]

\[ F_0(\xi) = (2\xi^2 - \xi^2)^2 - 4\xi^2 (\xi^2 - 1)^{1/2} (\xi^2 - \xi^2)^{1/2}. \]

\[ u_R = \text{const.} \frac{ik \alpha R}{R} \frac{P(k_\alpha \sin \theta)}{(k^2 - 2 \sin^2 \theta) \frac{\sin \theta}{\sin k_\alpha \theta}} \]

\[ u_\theta = \text{const.} \frac{e}{R} \frac{k_\beta \sin k_\beta \theta}{\sin \theta} \frac{P(k_\beta \sin \theta)}{R} \]

\[ u_R = -\frac{i \cos \phi}{\rho \alpha^2} \frac{P(k_\alpha \sin \theta)}{R} \frac{-ik \alpha R}{R} \]

\[ u_\theta = \frac{i \sin \phi}{\rho \beta^2} \frac{P(k_\beta \sin \theta)}{R} \frac{-ik \beta R}{R} \]

\[ u_R = -ik \varphi_0 \frac{e}{R} \frac{\cos \theta}{R} \]

\[ u_\theta = ik \cos \theta \]

\[ u_\phi = 0 \]

Fig. 8. Asymptotic field scattered by penny-shaped cracks.
Fig. 9. Radiation field of the diffracted P and S waves for incident P waves.
Fig. 10. Amplitudes of $P(k)$ (solid curves) and $Q(k)$ (dashed curves). For $k_2 = 1$, the amplitudes of $P(k)$ and $Q(k)$ are almost identical.
That's only within the last several months, and it's an excellent paper, directly related to the type of NDE work that has been discussed in the previous papers. Their results complement those of Ermolov and Cohen based on analytical approximations. A comprehensive review of acoustic and electromagnetic scattering from discs and other simple shapes has been compiled by Bowman, Senior, and Uslenhhi, and some interesting information on the diffraction of elastic waves is contained in the excellent review by Pao and Mau.

In addition to the problems of scattering of elastic waves from half planes, discs, and strips, another two dimensional scattering surface of interest is a crack occupying a quarter plane. That type of problem has still not been solved exactly, and it appears that it is not likely to receive an exact solution in the elastic wave case.

In a brief review such as this, a great many topics of current interest in elastic wave propagation have to be omitted. These include recent advances in finite difference techniques, by Alterman and her students in Israel; finite element methods, particularly work done by Boore and by people at Berkeley; and applications of Keller's geometrical theory of diffraction. A reference to the application of Keller's work in the case of elasticity is a paper by Karal and Keller entitled "Elastic Wave Propagation in Homogeneous and Inhomogeneous Media", Journal of Acoustical Society 31, 694 (1959), in which he developed the geometrical theory for the elastic wave equation.

Many problems in NDE involve scattering from cracks and flat bottom holes in bounded or semi-infinite elastic solids as opposed to unbounded solids. The presence, in addition to a crack, of one or more extra free surfaces greatly complicates the mathematics of the scattering problem. The development of effective approximate methods to solve such problems and comparison of the results obtained with experiment is expected to contribute significantly to the progress of NDE in the months ahead.

References


References Continued


DR. TOM WOLFRAM: Questions?

PROF. GORDON KINO (Stanford University): Would you repeat the Keller reference?


PROF. JAMES KRUMHANSL (Cornell University): May I ask a question in clarification? I think I know about this reference and sort of went through it. I think it is more or less concerned with propagation of relatively smooth varying media.

DR. KRAUT: That's true.

PROF. KRUMHANSL: So, it does not directly apply to the problem.

DR. KRAUT: That's correct, but it does discuss the method of expansion.

PROF. KRUMHANSL: Oh, I see. Yes, I think he sets forth a philosophy which can be deployed but has yet to be deployed in the singular cases.

DR. KRAUT: That's true.

PROF. KRUMHANSL: The second thing I'm very interested in is your reference to Bouwkamp, because this is something that I don't know about. I understand the classic problem of the half plane, Kirchoff's solution and its physical limitations as you put forth. Now, the point is that Sommerfield gave an exact solution to this.

DR. KRAUT: Correct.

PROF. KRUMHANSL: There has been a good deal of recent work in the applied mathematical interpretation of the electromagnetic application using the Wiener-Hopf integral technique in order to carry out these solutions.

DR. KRAUT: That's correct.

PROF. KRUMHANSL: Now, are you saying -- I should ask it in either case -- has it been done for the anisotropic electromagnetic medium or has it been done for a two mode problem as the Wiener-Hopf integral equation technique, which is necessary to give the exact solution for this half plane in the elastic cases? Is that what Bouwkamp has done?

DR. KRAUT: Bouwkamp didn't deal with the anisotropic electromagnetic problem at all.

PROF. KRUMHANSL: In elastic cases or--
DR. KRAUT: Bouwkamp did not deal with the elastic case at all. De Hoop, Adrian De Hoop, who was a student of Bouwkamp's dealt with the general theory of scattering from a half plane, including both shear and compressional modes, in his thesis, in that thesis that I mentioned.

PROF. KINO: They were able to treat a two mode situation?

DR. KRAUT: Well, he wrote down the general equations, then he specialized it to isotropy, and he dealt with the two mode situation, yes. It's the incident compressional wave on a weak and rigid half plane, an incident shear wave on a weak and rigid half plane, and both wave numbers are present in the solution.

PROF. KINO: Is it set out by Wiener-Hopf theory?

DR. KRAUT: Yes, indeed. There are some very interesting questions. I mentioned the quarter plane problem, because I happened to have worked on it, and it leads to a question of whether or not you can generalize the Wiener-Hopf technique from one complex variable to more than one complex variable.

PROF. KRUMHANSL: One important question. In this case, does he take the two mode converted components as far as the scattering is concerned?

DR. KRAUT: Yes.

PROF. KRUMHANSL: That's De Hoop?

DR. KRAUT: De Hoop.

DR. MIKE BUCKLEY (Wright Patterson AFB): I get the feeling, at least for myself, that I'm looking at a lot of trees and I'm missing the forest, and perhaps I'm going to pick on you, Ed, but your job, or part of it, is to integrate and show how the program is tied together, so, I'm going to ask this question. Where are we right now on being able to determine the geometry of defects, and what can we hope for in the near term future?

DR. KRAUT: That's an important question to answer. I'm not sure that I can answer it yet.

DR. Y. H. PAO (Cornell University): In answer to Dr. Buckley's question, I came in here on Thursday from San Francisco. I would like to ask the Air Force--

DR. BUCKLEY: That's unfair.

DR. PAO: Wait a minute. Let me finish my question first. Now, we've had a problem radar tracking airplanes since 1940. Now I don't know how much money or how many years that we've spent trying to detect the size of the airplane and the shape of the airplane from radar scattering cross sections. Up to this date we have not heard a concrete answer from the radar scattering signal. Can you or can you not tell me the
size of the airplane which is coming? If you can answer that question, I think the elastic wave question is four times as complicated as that.

DR. KRAUT: I agree.

PROF. PAO: Can you answer that question?

PROF. KINO: In the acoustic case you are in the near field, and that's different. And you can get a lot more angular information in the acoustic case than you can get in the radar case.

PROF. PAO: I'm asking in the radar case. We don't have any--

PROF. KINO: In the radar case you don't have all that angular information. It moves in one direction and returns in one direction. In this case you can have a wider range and get different directions.

DR. WOLFRAM: I think it's clear to someone not involved in the field, such as myself, that it's very difficult, but I think the question which is being thrown out to the audience in general is, "Where are we?" Does anyone want to try to bite on that one?

PROF. HARRY F. TIERSTEN (Rensselaer Polytechnic Institute): You have many more people aware of problems than you have that weren't aware of them sometime ago, and it's just the beginning of your effort. So, I think you are asking the question somewhat, possibly prematurely.

DR. BUCKLEY: Maybe what I'm asking for is a technical summary of the state-of-the-art today and what we can expect. And maybe it's unfair to ask that on this short of a notice, but I would like to see an answer at this time.

PROF. TIERSTEN: I have one other question. Didn't Joe Keller do some more recent work than the one you mentioned, where he extended what he started at that time, I think with Karal also?

DR. KRAUT: That's quite possible.

PROF. TIERSTEN: Some, I think in JASA and some I think in the Journal of Mathematical Physics in the past five or six years.

PROF. KRMHANSL: That's correct. Heterogeneous media, statistical properties.

DR. WOLFRAM: I think that's a good question for Don Thompson to answer. Where are we, Don?

DR. DON THOMPSON (Rockwell International Science Center): It is a very tough question; a very important question. I look at it this way, though, that this year we've put together a number of building blocks. I think the next step is to start integrating these building blocks and focusing on a particular technique and take the particular problem and see how well we can do with it. Now, I don't know whether that's next year or
the year after that, but I do think it is important that we do have
now a number of techniques being explored, a number of problems being
solved, and certainly, it is much more hopeful at this time than it
was a year ago. And I think in that context it is closer.

DR. BUCKLEY: So, the answer is we have to look at this in more detail.

DR. DON THOMPSON: I think the answer is we've explored a number of techniques
and a number of problem bases. Now, I think we need to spend some time
and some thought on how you can start taking these blocks and putting
them together into a system that answers this specific question; and
then to choose a particular problem and try to evaluate these various
techniques, building blocks and bases that we have started. Whether or
not they are all finished—it's clear, we're not finished on all these—but
at least to select those that are most promising with respect to
a particular problem application and go from there. But, certainly, I
feel very pleased with the general progress in assembling some of the
information from the building blocks, from a good fundamental standpoint,
so that we know when we do have it, and that it's on correct and proper
grounds.

MR. JIM MEECHAN (Rockwell International): Since nobody wants to take a
rack at it, and since I don't know anything about the field—
I have interfaced an awful lot with the user community both in government
and in industry. If I go back about two years and then back beyond
that, there was a great philosophy being promulgated, mainly from the
engineering and manufacturing engineering fields in both the government
and industry, that what we want is zero defects in our materials. The
crack size, for example, that we want is infinitely small. There was
a lot of that philosophy and there was a lot of things done in industry and
coming out of government in specifications that perpetrated that
philosophy. I see very little of that now. There is still some of it
around, but I think there has been a swing over that says you have to
tolerate certain size flaws in your structures, and now, we're trying to
get at just what those flaws are and how big they are, but they are
not zero. That is a big step forward to me at least. So, I'll contribute
that.

COL. RON NOKES (Kelly Air Force Base): There is more to this problem than
spectrum analysis and all of what we've seen today. We don't get the
same response from an ultrasonic signal in supposedly the same type of
materials. We heard one metallurgist today. It seems to me that there's
got to be a lot more research done on why there are differences in
response in materials, the same types of materials. Also, there
are differences in response of transducers. Not all transducers give
the same signal. So, really, all the spectrum analysis is fine, but
you equate that to a guy going up in an airplane and applying a trans­
ducer to a 7075 forging-- it doesn't make any sense.

DR. DON THOMPSON: May I try and answer that? We certainly all recognize
that what you say is very true. Tomorrow you'll hear something of work
in transducer areas. I think as Bruce Thompson pointed out this
morning, we recognize that there are various elements that are necessary in order to make an ultrasonic system quantitative. Certainly the characterization of the defect is one of those aspects and that's what we have concentrated on this afternoon.

Tomorrow we'll talk about transducers in a more limited fashion. But, certainly, I think that when you get down to the problem of addressing differences in materials, this is a problem of interest, of course. But, in a way, it's an easier problem in the sense that attenuation can be understood, grain boundary scattering can be understood, and ideally, what you would like to do in an inspection mode is, somehow or another, transform the material properties out of it. Because what one is really interested in is in the signatures or the characterizations of the defect, and in that way the mud from the material itself is only a confusing issue. So, that's not an answer, but I just want to assure you that we are well aware of the concern and certainly hope to be able to do something about that.

DR. WOLFRAM: Any further comments?

PROF. TIERSTEN: I have a question. It seems to me that all the discussion of the various disturbances in solids were confined to the semi-infinite or infinite solid block, essentially a semi-infinite solid so you can get a transducer on it. Yet I think some of the problems are concerned with thin things, such as skins, and nothing was addressed to the transmission of waves along those skins.

DR. WOLFRAM: Does someone wish to reply to that?

DR. BRUCE THOMPSON (Rockwell International Science Center): I'll reply to that. I think that's a very fair statement. I think from the complexity of the problems that we've seen in the bulk, the philosophy that has been adopted has been to try to get ahold on that problem and that geometry. In other words, it should be as little complicated by geometry as possible. It's certainly not the final problem, but it is one we have a chance of making the best start on. So, I think your statement is right, and those are the reasons.

PROF. TIERSTEN: But the problems could be very, very different.

DR. BRUCE THOMPSON: Absolutely. That's absolutely right, but we feel we can identify the steps that we need to take to get the solution, and then the details will be considerably different, as you say, in the other problems, but we'll know how to get there.

DR. WOLFRAM: We have a short film, and that will then, I think, conclude this session. The film will be on wave pictures using Schlieren photography. This film will be introduced by Jerry Posakony of Battelle Northwest.
MR. POSAKONY: You've seen an awful lot today on equations, and we thought, as a closeout, we might show you what wave propagation looks like. At Battelle Northwest we have produced an ultrasonic Schlieren movie to demonstrate the fundamental wave propagation phenomena occurring at various reflecting, refracting and diffracting interfaces. We have used a pulsed neon laser with stop action photography to provide the slow motion analysis of the pulsed ultrasonic waves, and are able to observe the constructive and destructive interactions of these waves at these various intervals. The movie is sort of self describing. The work itself is by Larry Becker of the Battelle staff.