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NONLINEAR INCOME EFFECTS IN RANDOM UTILITY MODELS

Joseph A. Herriges and Catherine L. Kling*

Abstract—Random utility models (RUMs) are used in the literature to model consumer choices from among a discrete set of alternatives, and they typically impose a constant marginal utility of income on individual preferences. This assumption is driven partially by the difficulty of constructing welfare estimates in models with nonlinear income effects. Recently, McFadden (1995) developed an algorithm for computing these welfare impacts using a Monte Carlo Markov chain simulator for generalized extreme-value variates. This paper investigates the empirical consequences of nonlinear RUMs in the case of sportfishing modal choice, while refining and contrasting the available methods for welfare estimation.

I. Introduction

The constant marginal utility of income is viewed in most microeconomic applications to be a restrictive, special case of the more plausible scenario in which marginal utilities vary with respect to both prices and incomes. Yet, in the context of modeling discrete choices made by consumers (e.g., the selection of travel mode or which recreation site to visit), analysts have relied almost exclusively on random utility models (RUMs) that are linear in income, directly imposing a constant marginal utility of income.[1] This assumption is common even in cases in which the estimation of welfare measures is the primary goal of the empirical work where nonlinear income effects are likely to be important (e.g., Just et al., 1982).

The imposition of linear income effects has been accepted in part because of the inconvenience of estimating nonlinear models, but more importantly because of the difficulty of computing welfare estimates under these circumstances.[2] In fact, methods for computing welfare estimates using nested-logit models that allow for nonlinear income effects have only recently been devised (McFadden, 1995) and have not previously been implemented using actual data. Unfortunately for the practitioner, the procedures outlined by McFadden are computationally intensive, requiring repeated draws from a random sampler for the generalized extreme-value (GEV) distribution and an iterative algorithm to implicitly solve for individual welfare impacts. As an alternative, McFadden derives theoretical bounds on these welfare impacts that are computationally simpler than computing point estimates and which, for some applications, may provide sufficient information for policymakers. These recent developments raise the empirical question as to whether nonlinear income effects are important in practice and worth the additional computational burdens that they entail.

The purpose of this paper is both to investigate the empirical consequences of nonlinear income effects in RUMs and to extend and refine the available methods for obtaining welfare estimates in this context. We begin, in section II by reviewing the basic theory of welfare measurement in RUMs, including results specific to the standard linear model. Section III then identifies the three alternative approaches to computing welfare measures (once nonlinear income effects are permitted) and discusses the merits of each. We first review McFadden's algorithm for computing willingness to pay in a nested logit model with nonlinear income effects and discuss some technical issues related to his proposed resampling scheme. Second, we discuss the alternative of computing welfare measures based upon a representative consumer. This is the approach employed by Morey et al. (1993) and Shaw and Ozog (1997). Third, we consider in detail the suggestion by McFadden (1995) that bounds alone be computed on the welfare measures of interest. Specifically, we present a modification to his algorithm that increases its accuracy, provide an empirically tractable method for implementing his bounds when there are nonlinear income effects, and identify scenarios in which the welfare bounds are uninformative.

The empirical portion of this work, beginning with sections IV and V, is aimed at carefully comparing and contrasting the three alternative strategies for estimating welfare from RUMs. Data from the 1989 Southern California Sportsfishing Survey are used to estimate models of recreational angling that are nonlinear in both income and other arguments of the indirect utility function. Both Generalized Leontief and Translog functional forms are used in modeling the deterministic portion of the utility function. This use of flexible functional forms to approximate the indirect utility function in a RUM is apparently novel. The results are compared to measures constructed from linear models. In addition, several maintained hypotheses about the underlying error distribution are employed, including the extreme-value (EV) and several generalized extreme-value (GEV) distributions.

In section IV of the paper, the estimated models are used to construct welfare estimates for changes in the price of

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[1] Important exceptions are the papers by Morey et al. (1993) and Shaw and Ozog (1997) on recreation demand and Gertler and Glewwe (1990) modeling the demand for schooling.

[2] Another consequence of assuming a constant marginal utility of income, as noted by McFadden (1995, p. 10), is that it is then "possible to aggregate preferences into a social preference that generates the market demand functions using Roy's Identity." See Chipman and Moore (1980, 1990).

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angling, for changes in angling quality, and for the elimination of entire angling sites (due perhaps to the closure of a fishery). We follow each of the strategies for the welfare measurement just identified, obtaining both point estimates for the welfare changes and confidence bounds around these estimates. The final section is used to summarize our findings.

II. The Theory of Welfare Measurement in RUM and the Linear Model

The basic theory and structure of discrete-choice RUMs were developed by McFadden (1973, 1974, 1981), Domencich and McFadden (1975), and Diamond and McFadden (1974) to analyze consumer selections from among a set of discrete alternatives and to measure the welfare implications of changes to the available choice set. Early applications focused on transportation choices (e.g., Domencich & McFadden, 1975; Ben-Akiva & Lerman, 1985), though subsequent studies have used this modeling framework to consider issues in education (Gertler & Glewwe, 1990), housing demand (Börsch-Supan, 1987), and energy conservation (Cameron, 1985). More recently, there has been considerable interest in applying RUMs to recreational choices with the primary purpose of computing the welfare implications of changing environmental quality or the loss of access to a recreation area (due, for example, to an oil spill or other environmental disaster) (e.g., Yen & Adamowicz, 1994; Hausman et al., 1995; Morey et al., 1993).

In discrete-choice models, the utility that an individual consumer associates with a particular alternative j ($j = 1, \ldots, J$) is assumed to take the form: $U_j = U(z, \mathbf{q}_j, \epsilon_j)$, where z is the amount of a numeraire good consumed by the individual, $\mathbf{q}_j$ is a vector of characteristics associated with alternative j, and ϵ_j denotes heterogeneity in consumer preferences and unobserved factors associated with alternative j.[3] The consumer is assumed to choose that alternative yielding the highest utility subject to meeting his/her budget constraint; i.e., $y = p_j + z$ for the selected alternative, where p_j denotes the price of alternative j. Imposing the budget constraint yields the conditional indirect utility functions:[4]

$$U_j = U(y - p_j, \mathbf{q}_j, \epsilon_j), j = 1, \ldots, J. \tag{1}$$

The consumer's problem is then to select the alternative that yields the highest utility.

Using equation (1), the probability of choosing alternative j can be written as

$$P_j(y, \mathbf{p}, \mathbf{q}, \epsilon) = \text{Prob}\,[U(y - p_j, \mathbf{q}_j, \epsilon_j) \geq U(y - p_k, \mathbf{q}_k, \epsilon_k)\ \forall\, k \neq j], \tag{2}$$

where $\mathbf{p} = (p_1, \ldots, p_J)'$ and $\mathbf{q} = (\mathbf{q}_1', \ldots, \mathbf{q}_J')'$. The exact form that these choice probabilities will take depends on the assumed underlying distribution for $\epsilon = (\epsilon_1, \ldots, \epsilon_J)'$. If the ϵ_j's are i.i.d. variates drawn from an EV distribution, then the familiar multinomial specification results, whereas if ϵ is drawn from a GEV distribution then the nested-logit model results.

As noted above, RUMs are often estimated with the goal of measuring the welfare implications of changing the choice set, either the set of alternatives themselves or characteristics of the available alternatives. The compensating variation (cv) for such changes can be implicitly defined by[5]

$$\underset{j \in \mathbf{J}^0}{Max}\, U(y - p_j^0, \mathbf{q}_j^0, \epsilon_j) = \underset{j \in \mathbf{J}^1}{Max}\, U(y - p_j^1 - cv, \mathbf{q}_j^1, \epsilon_j), \tag{3}$$

where $\mathbf{J}$ denotes the choice set and the superscripts "0" and "1" are respectively used to distinguish the original versus the new conditions that are associated with the choice set. The resulting compensating variation is a random variable with the general form:

$$cv = cv(y, \mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, \epsilon). \tag{4}$$

It is the expected value of this random variable that is typically of interest for policy purposes.[6] Unfortunately, there is no general closed-form solution for E(cv), since cv can depend upon the ϵ_j's in a nonlinear fashion. The standard approach in the literature is to resolve this problem by making the following set of assumptions:

A.1 *Additive disturbances; i.e.,* $U_j = V(y - p_j, \mathbf{q}_j) + \epsilon_j$,
A.2 *GEV disturbances,* and
A.3 *Constant Marginal Utility of Income; i.e.,* $V(y - p_j, \mathbf{q}_j) = \alpha(y - p_j) + f(\mathbf{q}_j)$.

It can be shown that under these conditions a closed-form solution exists for E(cv), one which is independent of income (e.g., Hanemann, 1982; Small & Rosen, 1981; Morey, 1998; McFadden, 1995). While the first two assumptions may be of concern in some applications, the focus in this paper is on relaxing assumption A.3.

III. Relaxing the Linearity Assumption: Implications for Welfare Measurement

The difficulties with relaxing the assumption of linear income effects appear in the computation of welfare measures. One can no longer rely on a closed-form solution to compute welfare. If one wishes to allow nonlinear income

[3] Although we assume that the consumer is constrained to choose a single unit of the discrete good, the model can be generalized to allow multiple units.

[4] The indirect utility function is "conditional" on the choice of alternative j.

[5] As equation (3) implies, we are assuming that $\epsilon^0 = \epsilon^1$ (i.e., that the preference heterogeneity is invariant with respect to the policy scenario). As McFadden (1995, p. 4) notes, without this assumption the welfare impact of policy changes would no longer be well defined and identifiable.

[6] Of course, depending on the application, the analyst may be interested in other moments of the distribution of compensating variations.

effects, there are currently three alternative approaches from which to choose.[7] The first alternative requires resampling from the underlying error distribution and employing a numerical algorithm to solve for the implicitly defined compensation variation. The second is to adopt a representative consumer strategy and compute the welfare for this consumer. The third approach is to employ McFadden's bounds on the welfare estimates as applied to models with nonlinear income effects. We discuss each option in turn below.

A. *Alternative 1: Simulation*

The compensating variation defined in equation (3) is an implicit function of the characteristics of the choice set and distribution of preferences in the population, as captured by the functional form of U(·) and distributions of both (y, p, q) and the ϵ_j's. Suppose U(·) has been specified to be nonlinear in income, and econometric estimates of the parameters have been obtained for a given data set. One approach to computing an estimate of E(cv) is to begin with the simulation procedure suggested in McFadden (1995). The procedure is best understood as a series of steps, conducted first for each observation in the sample:

Step 1: At iteration t ($t = 1, \ldots, T$), a pseudorandom number generator is used to draw the vector $\hat{\epsilon}^t$ *from the estimated distribution of* ϵ.

Step 2: A numerical routine is then used to search iteratively for the cv^t *implicitly defined by*[8]

$$\underset{j \in \mathbf{J}^0}{Max}\, U(y - p_j^0, \mathbf{q}_j^0, \hat{\epsilon}_j^t) = \underset{j \in \mathbf{J}^1}{Max}\, U(y - p_j^1 - cv^t, \mathbf{q}_j^1, \hat{\epsilon}_j^t). \tag{5}$$

Step 3: The mean of the cv^t *over the T iterations provides a consistent estimate of* $E(cv|y, \mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1)$; *i.e., the mean value of cv for individuals with the set of observed characteristics* $(y, \mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1)$. *The resulting collection of* cv^t*'s likewise provides a simulated distribution of cv for individuals with the same set of observed characteristics.*

If the sample available to the analyst is representative of the target population, these three steps can be repeated for each observation and averaged to obtain an estimate of E(cv) for the population. Otherwise, a weighted average may be needed to correct for differences between the sample and target populations.

This procedure, while conceptually simple, requires the ability to resample from the assumed error distribution used to estimate the model. In this regard, two appealing choices for the distribution of ϵ are the extreme value and multivariate normal distributions, as pseudorandom number generators are easy to devise in these cases. However, the EV distribution yields the multinomial logit specification, which is known to suffer from the much discussed and maligned independence of irrelevant alternatives (IIA) assumption. If one is attempting to generalize the RUM by incorporating nonlinear income effects, it is not likely to be desirable to impose such a restrictive assumption on the disturbance terms. The multivariate normal probit (MNP) model, while certainly less restrictive than the multinomial logit model, is problematic for a different reason. Although recent advances in econometrics suggest that the MNP model may be feasible to estimate (e.g., McFadden, 1989; Börsch-Supan & Hajivassiliou, 1993), the computational burdens of obtaining parameter estimates for such models remains substantial.

The most common distributional assumption employed with RUMs is that the errors, ϵ, are drawn from a GEV distribution, resulting in the nested-logit model. This specification yields choice probability equations that are easy to construct (thus simplifying estimation) without imposing the IIA assumption that haunts the multinomial logit model. However, approximating a sample from a GEV distribution is not a trivial exercise. In fact, only recently has McFadden (1995) developed a Monte Carlo Markov chain method for constructing a sequence of random vectors, $\hat{\epsilon}'$, whose empirical distribution asymptotically approximates a GEV cumulative distribution. The approach to constructing an estimate of E(cv) is the same as above, except that Step 1 is replaced with the following GEV sampler routine:

Step 1A: *At iteration t($t = 1, \ldots, T$), a pseudorandom number generator is used to draw J + l independent (0, 1) uniform random variables,* ζ_j^t ($j = 1, \ldots, J$) and η^t. *J extreme value random variates are then formed using the transformation* $\tilde{\epsilon}_j^t = -\log(-\log(\zeta_l^t))$. *Finally, the following Markov chain is used to construct:*

$$\hat{\epsilon}^t = \begin{cases} \tilde{\epsilon}^t & \text{if } \eta^t \leq \dfrac{f(\tilde{\epsilon}^t)/g(\tilde{\epsilon}^t)}{f(\hat{\epsilon}^{t-1})/g(\hat{\epsilon}^{t-1})}, \\ \hat{\epsilon}^{t-1} & \text{otherwise} \end{cases} \tag{6}$$

where f(·) and g(·) denote the GEV and EV probability density functions, respectively.

The right-hand side of the inequality term in equation (6) can be interpreted loosely by noting that $f(\cdot)/g(\cdot)$ corresponds to the weights used in importance sampling (e.g., Geweke, 1989). Thus, the Markov chain replaces an earlier draw if the new draw has greater weight than the previous observation in the chain. McFadden (1995) proves that the mean

[7] Shonkwiler and Shaw (1997) have recently suggested a fourth alternative, using a finite-mixture model to estimate consumer preferences that is piecewise linear in income.

[8] In our application below, numerical bisection was used to solve for cv'.

compensating variation computed using Steps 1A, B, and C converges almost surely to E(cv) as $T \rightarrow \infty$.

Several potential difficulties are associated with the simulation estimator outlined above. First, the procedure is computationally intensive. As McFadden (1995) demonstrates in a Monte Carlo experiment, the number of iterations (**T**) required to achieve a given level of precision increases substantially as the GEV model departs from the EV distribution. In his experiment, the number of iterations required to obtain a 5% root mean squared error ranges from 755 (when ϵ is EV) to nearly 19,000 as the dissimilarity coefficient becomes 0.1.[9] The computational burden is all the more severe when it is recognized that the parameters underlying these cv calculations are themselves estimates. If confidence bounds on E(cv) are to be constructed recognizing the uncertainty of these estimates, the three-step simulation procedure will need to be repeated for a series of draws from the distribution of the estimated parameters.[10]

Finally, Step 2 of the simulation process assumes that cv' exists that implicitly solves equation (5). This need not be the case when a model with nonlinear income effects is used to approximate underlying preferences. The problem is akin to the difficulties found in continuous-demand systems, when estimated models yield preferences that are locally consistent with utility theory, but fail to have well-behaved global properties. In the current problem (while estimated nonlinear models may yield a positive marginal utility of income at the mean of the sample), the marginal utility of income can become negative at extremes of the sample or when substantial price or quality changes are considered. In these cases, there may not exist a cv' that solves equation (5).

B. *Alternative 2: A Representative Consumer Approach*

A second approach is to approximate E(cv) by computing the income compensation required to equate *expected* utility before and after a given price and/or quality change. This is the approach suggested and implemented by Morey et al. (1993). Formally, this corresponds to calculating the $\overline{\text{cv}}$ implicitly defined by

$$E\left[\underset{j \in \mathbf{J}^0}{Max}\, U(y - p_j^0, \mathbf{q}_j^0, \epsilon_j)\right] = E\left[\underset{j \in \mathbf{J}^1}{Max}\, U(y - p_j^1 - \overline{cv}, \mathbf{q}_j^1, \epsilon_j)\right]. \quad (7)$$

Under this alternative, the expected utility function is interpreted as the utility function of a representative consumer. When preferences satisfy assumptions A.1 and A.2 of the previous section (i.e., ϵ enters preferences additively and is assumed to be drawn from a GEV distribution), the expected utilities on the left and right sides of equation (7) are closed-form functions of the $V(y - p_j^0, \mathbf{q}_j^0)$'s and $V(y - p_j^1 - \overline{\text{cv}}, \mathbf{q}_j^1)$'s, respectively.[11] An iterative procedure can then be employed to solve this implicit equation without the resampling step that is required under the simulation approach. The appeal of this alternative is that it is simple to implement, while still allowing the analyst to relax the constant marginal utility of income assumption (i.e., A.3). However, McFadden (1995) notes that $\overline{\text{cv}}$ will generally be a biased estimator of mean compensating variation and, in his Monte Carlo results, finds that the percentage bias from using this approach increases as the size of the welfare change increases.[12] We include estimates based upon this approach in our empirical section below in order to investigate the extent of the bias in an applied setting.[13]

C. *Alternative 3: Theoretical Bounds*

Recognizing the computational difficulty of the GEV simulation approach, McFadden (1995) suggests that it may be easier to bound the welfare impacts of a policy change and that, for some applications, these bounds may provide sufficient information to decision makers. Towards this end, he proposes theoretical bounds on E(cv). In particular, let cv_{jk} denote the income reduction that is required to equate the utility from consuming alternative, j, before the quality/price change with the utility from consuming alternative, k, after the change, i.e., cv_{jk} is implicitly defined by

$$U(y - p_j^0, \mathbf{q}_j^0, \epsilon_j) = U(y - p_k^1 - cv_{jk}, \mathbf{q}_k^1, \epsilon_k)\ \forall j, k. \quad (8)$$

The cv_{jk}'s can be viewed as conditional compensating variations, as they are defined as conditional on the event (B^{ik}) that the individual selects alternative j prior to the attribute changes and selects alternative k after these changes and compensation cv. McFadden demonstrates that, given the event B^{jk}, these conditional compensating variations bound the true cv, with[14]

$$cv_{jj} \leq cv(y, \mathbf{p}^0, \mathbf{q}^0, \mathbf{p}^1, \mathbf{q}^1, \epsilon) \leq cv_{kk}. \quad (9)$$

[9] The dissimilarity coefficient, denoted θ below, corresponds to the inverse of McFadden's (1995) "s".

[10] A procedure for constructing confidence bounds on the mean compensating variation estimates is outlined in section VI.

[11] The exact form of the expected utility function depends upon the nesting structure assumed in the GEV distribution and are excluded here for the sake of space. See, for example, Morey (1998) for detailed expressions.

[12] Hanemann (1996) makes a similar observation.

[13] A second rationale for considering this representative-consumer approach emerges if one considers the ϵ's as capturing individual uncertainty rather than heterogeneity of preferences across individuals. In this situation, equation (7) reflects the compensation calculation that would be undertaken by a risk-neutral individual. This line of reasoning parallels the arguments put forth by Bockstael and Strand (1987) in the context of continuous-demand systems.

[14] McFadden provides the intuition for this result by noting that, if the consumer can move from the previously chosen alternative after a quality/price change, then the compensation needed to maintain the original level of utility may well be smaller than it would be if forced to stay with the original choice. Likewise, the flexibility of having chosen another alternative prior to the price/quality change means that the consumer might need a smaller compensation than the one needed if they are forced to the finally chosen alternative.

Taking expectations of this inequality yields McFadden's theoretical bounds on E(cv), with

$$\sum_{j \in J^0} P_j^0(y, \mathbf{p}^0, \mathbf{q}^0) cv_{jj} \leq E(cv) \leq \sum_{k \in J^1} P_k^1(y - cv, \mathbf{p}^1, \mathbf{q}^1) cv_{kk}, \tag{10}$$

where

$$P_j^0(y, \mathbf{p}^0, \mathbf{q}^0) = \text{Prob}[U(y - p_j^0, \mathbf{q}_j^0, \epsilon_j) \geq U(y - p_k^0, \mathbf{q}_k^0, \epsilon_k) \; \forall \; k \neq j] \tag{11}$$

denotes the choice probabilities prior to the attribute changes, and

$$P_j^1(y - cv, \mathbf{p}^1, \mathbf{q}^1) = \text{Prob}[U(y - p_j^1 - cv, \mathbf{q}_j^1, \epsilon_j) \geq U(y - p_k^1 - cv, \mathbf{q}_k^1, \epsilon_k) \forall \; k \neq j] \tag{12}$$

denotes the choice probabilities after the attribute changes and compensation cv. The key to employing the welfare bounds in equation (10) is to note that, when the error terms are assumed to enter the utility function in an additive manner (as is typically the case), then the cv_{ij}'s are dependent of the error distribution and need not be simulated. Given estimates of the choice probabilities in equation (11) and (12), the computational burden of simulating GEV errors can then be avoided entirely.

A number of issues arise in constructing the theoretical bounds detailed in equation (10). First, while the initial choice probabilities (i.e., $P_j^0(y, \mathbf{p}^0, \mathbf{q}^0)$) follow directly from the estimated model, the new choice probabilities $P_j^1(y - cv, \mathbf{p}^1, \mathbf{q}^1)$ depend upon the unknown compensating variation, cv.[15] Thus, the upper theoretical bound in equation (10) cannot be directly computed. One approach would be to approximate cv using a linear model (in which case cv has a closed-form solution) and to use this approximation in computing the upper bound choice probabilities. However, the resulting bounds are no longer guaranteed to truly bound E(cv). Alternatively, the theoretical bounds can be simulated just as E(cv) can be simulated. In particular, a consistent estimate of $P_j^1(y - cv, \mathbf{p}^1, \mathbf{q}^1)$ can be obtained using McFadden's GEV simulator, which provides a consistent estimator of any real-valued function that is integrable with respect to the distribution function of ϵ. In this case, that real-valued function is an indicator function for the selected alternative, given the new alternative characteristics and the implicitly solved for cv. Of course, in practice, practitioners would not bother with the theoretical bounds once they had available point estimates of the compensating variation itself.

A third approach to constructing the theoretical bounds is to note that equation (9) implies that

$$cv^L \equiv \underset{j}{Min}\; cv_{jj} \leq cv(y, \mathbf{p}^0, \mathbf{p}^1, \mathbf{q}^1, \epsilon) \leq cv^H \equiv \underset{j}{Max}\; \text{cv}_{jj}. \tag{13}$$

Using this result, we can bound the new choice probabilities, since

$$\begin{aligned} P_j^1(y - cv, \mathbf{p}^1, \mathbf{q}^1) &= \text{Prob}[U(y - p_j^1 - cv, \mathbf{q}_j^1, \epsilon_j) \geq U(y - p_k^1 - cv, \mathbf{q}_k^1, \epsilon_k) \; \forall \; k \neq j] \\ &\leq \text{Prob}[U(y - p_j^1 - cv^L, \mathbf{q}_j^1, \epsilon_j) \geq U(y - p_k^1 - cv^H, \mathbf{q}_k^1, \epsilon_k) \; \forall \; k \neq j] \\ &\equiv \tilde{P}_j^1(y, cv^L, cv^H, \mathbf{p}^1, \mathbf{q}^1). \end{aligned} \tag{14}$$

Notice that while

$$\sum_{j \in J'} \tilde{P}_j^1 \geq 1,$$

it need not sum exactly to unity. Substituting the results of equation (14) into equation (10) yields the following computable bounds on E(cv):[16]

$$\sum_{j \in J^0} P_j^0(y, \mathbf{p}^0, \mathbf{q}^0)\, cv_{jj} \leq E(cv) \leq \sum_{k \in J^1} \tilde{P}_k^1(y, cv^L, cv^H, \mathbf{p}^1, \mathbf{q}^1)\, cv_{kk}. \tag{15}$$

A second issue in the computation of the theoretical bounds is how best to estimate the cv_{jj} terms. McFadden suggests that the equivalent of a linear approximation be employed. This entails computing the difference in utility before and after the quality/price change and dividing by an intermediate value of the marginal utility of income over this change. The accuracy of this approximation is an empirical question, but it will undoubtedly decrease as the size of the welfare change increases. However, an exact calculation of the cv_{jj} can be recovered by applying a standard numerical routine (such as numerical bisection) to equation (8).

A third issue regarding these bounds is the type of welfare changes to which they can be meaningfully applied. McFadden writes the bounds in terms of quality changes, corresponding to changes in the level of the q's. We have generalized these in our formulations to consider changes in the prices as well. This is a minor extension, in and of itself,

[15] McFadden notes this point, but does not elaborate on it nor suggest a solution.

[16] Note that the computable lower bound and McFadden's theoretical lower bound are the same.

except that it highlights a case in which the bounds will be uninformative. Specifically, if the analyst is interested in computing a welfare change associated with the elimination of one or more alternatives (due, perhaps, to toxic contamination of several fishing sites or a large oil spill affecting recreation areas), the application of the bounds can be equivalent to considering a price change from its current level to an infinite price for the lost alternatives. For these alternatives, however, cv_{jj} is negative and infinite, since there is no finite level of compensation that can make the consumer as well off with an infinite price compared to the initial finite price, if they are unable to switch to another alternative. Thus, the (absolute value of) the lower bound is infinite. Likewise, the upper bound is zero since it depends on only the alternatives that remain after the site closings, and there is no change in the remaining alternatives' prices and/or qualities. In fact, recovery of welfare estimates associated with the entire elimination of one or more alternatives is a common goal of empirical analysis. In these instances, the theoretical bounds will provide no information to policymakers.

A final point concerning the empirical relevance of theoretical bounds in equation (10) and (14) is the recognition that these bounds will need to be computed using parameter estimates, and, as such, will themselves be random variables. Confidence intervals on the theoretical bounds can be computed using simulation techniques. Thus, even if the point estimates of the bounds are fairly tight, the width of the bounds when their statistical imprecision is accounted for may be wider than an analyst is comfortable with.

IV Data

The data used in our application were drawn from the Southern California Sportfishing Recreation Survey conducted in 1989. A complete description of the data can be found in Thomson and Crooke (1991) and Kling and Thomson (1996). Random telephone interviews were conducted in Southern California to identify recreational anglers, who were then requested to complete a follow-up mail questionnaire. Respondents provided a variety of information about their angling experiences including extensive information on their most recent saltwater fishing trip. This data included the month of their fishing trip, the species they targeted, the time it took to travel to and from the fishing site, the travel distance, and other expenditures associated with the trip. In addition, they reported whether they fished from the beach, a pier, a private boat, or a charter boat. These four alternatives constitute the possible modes of fishing from which anglers choose in our empirical models.

Respondents also reported their annual income and their household's ZIP code. The ZIP code data were used to compute roundtrip travel costs to their most recently visited fishing site. An opportunity cost of travel time (based on their reported wage rate) and any boat fees or fuel costs were added to these roundtrip costs to construct the final variable for the price of fishing.

Since the anticipated success of fishing is likely to be an important determinant of the decision to engage in angling as well as the choice of which mode of fishing to select, we include catch rates as an explanatory variable. Specifically, exogenous data on catch rates were provided by the Marine Recreational Fishery Statistics Survey, which is sponsored annually by the National Marine Fisheries Service. These catch rates are defined on a per-hour-fished basis for each major species by fishing mode. In the mail survey, anglers were questioned as to their targeted species. A catch rate variable was then constructed by summing the per hour catch rates associated with each angler's targeted species. Since these data were collected independently from the mail survey, the catch rate associated with each mode is exogenous to the angler. A total of 1,182 observations with complete data on income, prices, and catch rates were available for use in our analysis.

V. Model Specification

Our application focuses on modeling the mode choice (i.e., beach, pier, private boat, or charter boat) of recreational saltwater anglers. The model specification involves assumptions regarding the functional form of the indirect utility and the distribution of preferences in the population. We begin by using the standard assumption in the literature (*A.1* above) that the error terms enter the indirect utility function additively; i.e.,

$$U_j = V(y - p_j, \mathbf{q}_j) + \epsilon_j, \tag{16}$$

where y is now defined as monthly income.

Three alternative functional forms are considered for the deterministic portion of the indirect utility function $V(y - p_j, \mathbf{q}_j)$. To provide a basis of comparison, we begin by estimating the parameters of a simple, linear, indirect utility functional form. We also estimate Generalized Leontief (GL) and Translog (TL) models. Thus, the three specifications considered for the deterministic portion of the indirect utility function are

- *Linear:*

$$V_j(y - p_j, q_j) = \beta_{11}(y - p_j) + \beta_{22}(q_j) \tag{17}$$

- *Generalized Leontief (GL):*

$$\begin{aligned} V_j(y - p_j, q_j) = {} & \beta_{10}(y - p_j)^{1/2} + \beta_{20} q_j^{1/2} \\ & + \beta_{11}(y - p_j) + \beta_{22} q_j \\ & + \beta_{12}(y - p_j)^{1/2} q_j^{1/2} \end{aligned} \tag{18}$$

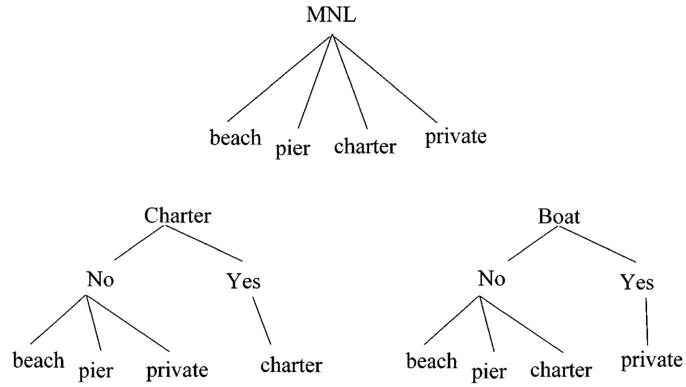
• Translog (TL)

$$V_j(y - p_j, q_j) = \beta_{10}\ln(y - p_j) + \beta_{20}\ln(q_j) + \beta_{11}\ln(y - p_j)^2 + \beta_{22}\ln(q_j)^2 + \beta_{12}\ln(y - p_j)\ln(q_j), \quad (19)$$

where q_j denotes the catch rate at site j. Notice that the linear specification represents a constrained version of the GL model, with $\beta_{10} = \beta_{20} = \beta_{12} = 0$.

In addition to identifying the functional form for $V(\cdot)$, the model specification requires distributional assumptions regarding ϵ. We estimate each model under three assumptions for the distribution of preferences (captured by ϵ): an extreme value distribution and two GEV distributions corresponding to two different correlation patterns among the alternatives. The extreme value assumption yields the multinomial logit (MNL) model, whereas the GEV assumptions yield alternative nested-logit models, with different nesting structures. The first GEV distribution groups pier, beach, and private boat into a single nest (assuming greater substitution possibilities among these three alternatives than between any one of these and charter boating). This is referred to as the ''charter'' model. A second GEV distribution is investigated wherein pier, beach, and charter boat enter a single nest and private boat is in its own nest. This is referred to as the ''private'' model. The tree structures typically presented for the nested-logit models corresponding to these two correlation patterns, along with the MNL model, are provided in figure 1.[17]

FIGURE 1.—ALTERNATIVE NESTING STRUCTURE



VI. Results

A. Parameter Estimates

Table 1 contains the parameter estimates from the linear, GL, and TL functional forms. Coefficient estimates for each these models are reported using the three nesting structures: the MNL model, the nested-logit charter model, and the nested-logit private model. While our primary interest is with the welfare predictions implied by each of these models, several useful insights emerge from table 1.

Focusing first on the coefficients that are associated with the deterministic portion of the indirect utility function (i.e., the β_{ij}'s), we see that most of these parameter estimates differ significantly from zero using either a 1% or 5% significance level. While interpreting the β_{ij}'s directly in the nonlinear models is difficult, β_{11} and β_{22} have natural interpretations for the linear models. The coefficient β_{11} corresponds to marginal utility of income and, as expected, is estimated to be positive, ranging between 0.01 and 0.02. Similarly, β_{22} indicates the marginal utility of catch rate (as a quality attribute of fishing mode) and is also estimated to be positive, ranging from 0.41 to 0.95. While these marginal utilities are nonlinear functions in the GL and TL models, their estimates at the sample means were found to correspond closely to those predicted by the linear specification.

The other parameter estimate presented in table 1 is the dissimilarity coefficient θ. The dissimilarity coefficient indicates the degree of correlation among alternatives within a nest of the assumed nesting structure. A well-known condition for consistency of a RUM model with stochastic utility maximization is that θ lies within the unit interval (Daly & Zachary, 1979; McFadden, 1978). When $\theta = 1$, the alternatives are uncorrelated and the multinomial logit specification results. On the other hand, as θ declines towards zero, alternatives within a nest become increasingly closer substitutes. Thus, one test of the multinomial logit specification (and the implied independence of irrelevant alternatives assumption) is whether the parameter θ differs significantly from 1. Clearly, the multinomial logit specification is rejected in this application, since θ is statistically different from 1 using a 1% level for each of the nested-logit models. Likelihood ratio tests of this restriction yield the same conclusion. Choosing between the charter and private models is more difficult, since one is not nested in the other. However, since both models have the same number of parameters, application of Pollak and Wales' (1991) likelihood dominance criterion suggests choosing the charter model as the best representation of preferences since (for each functional form specification) it yields a log-likelihood value above the value obtained for the private model.

Finally, the results in table 1 provide evidence on the statistical validity of the linear model that is typically employed in the literature. In comparing the flexible forms to the linear model, it is most direct to compare the GL and linear models, since the linear model is nested within the GL. For all three error structures, the linear model is rejected as a restriction on the GL specification using a likelihood ratio test statistic and a 5% significance level.[18] Thus, in general, we find that the more complex GL (and TL) model

[17] We investigated a third GEV distribution that grouped the beach and pier alternatives and the charter and private boat alternatives. However, the empirical results associated with the two structures reported in the paper dominated this structure based on both goodness-of-fit tests and consistency with utility maximization criteria.

[18] Note, however, that for any given error assumption, the TL models provide slightly higher likelihood values than those from the GL model.

TABLE 1.—PARAMETER ESTIMATES

Functional Form	Nesting Structure	β_{10}	β_{20}	β_{11}	β_{22}	β_{12}	θ	Log Likilihood
	MNL	—	—	0.02**	0.95**	—	1.00	−1,311.98
				(0.00)	(0.09)			
Linear	Charter	—	—	0.01**	0.41**	—	0.34[a]	−1,235.17
				(0.00)	(0.10)		(0.04)	
	Private	—	—	0.01**	0.85**	—	0.62[a]	−1,300.59
				(0.00)	(0.08)		(0.06)	
	MNL	1.38**	1.99**	0.01*	0.47*	−0.02*	1.00	−1,303.91
		(0.45)	(0.62)	(0.00)	(0.23)	(0.01)		
Generalized Leontief	Charter	1.07**	−0.00	0.00	0.70**	−0.01	0.31[a]	−1,223.24
		(0.26)	(0.44)	(0.00)	(0.17)	(0.01)	(0.04)	
	Private	1.05*	0.96*	0.01*	0.67**	−0.01	0.66[a]	−1,295.28
		(0.42)	(0.49)	(0.00)	(0.17)	(0.01)	(0.06)	
	MNL	−40.90**	2.05**	6.71**	0.14**	−0.14**	1.00	−1,297.47
		(2.52)	(0.37)	(0.39)	(0.01)	(0.044)		
Translog	Charter	−77.70**	0.44	7.37**	0.05**	−0.02	0.34[a]	−1,222.23
		(16.54)	(0.25)	(1.24)	(0.01)	(0.03)	(0.04)	
	Private	−33.24*	1.27**	5.05**	0.11**	−0.06	0.64[a]	−1,285.01
		(14.25)	(0.29)	(0.99)	(0.01)	(0.03)	(0.06)	

Notes: * Statistically different from 0 at a 5% significance level.
** Statistically different from 0 at a 1% significance level.
[a] Statistically different from 1 at a 1% significance level.

using either of the nested-logit error structures provides a statistically better fit of fishing mode choice when compared to the linear MNL model. The question from a policy perspective, however, is whether the more complex models yield substantially different welfare predictions, i.e., differences that are worth the increased cost of computing welfare impacts in these models. Towards this end, we turn now to a comparison of the welfare predictions using each model specification.

B. Welfare Estimates Using the GEV Sampler

Since the primary purpose of estimating RUMs for recreational angling is to compute welfare measures, we choose three different changes for which to compute welfare impacts. First, we estimate the compensating variation that is associated with a doubling of the price of each alternative fishing mode. Second, we consider the compensating variation that is associated with the doubling of the catch rate at all sites, and, third, we estimate the compensating variation that is associated with eliminating two of the modes (pier and beach). The latter change is also a price change, namely one that changes the price of two of the models from their current finite levels to infinity.

Table 2 provides estimates of the mean compensating variation that is associated with the three changes. The per trip welfare estimates reported in table 2 were computed using McFadden's GEV sampler and a search algorithm to solve for the implicitly defined *cv* in equation (3). Welfare impacts for each observation in the sample were constructed by averaging estimated *cv*'s computed using T = 1,000 iterations.[19] These individual welfare impacts were then averaged over the 1,182 observations in the sample.

TABLE 2.—POINT ESTIMATES OF WELFARE IMPACTS USING GEV SAMPLER

Nesting Structure	Linear	Generalized Leontief	Translog
a. Doubling Prices			
MNL	−47.71	−47.53	−49.56
Charter	−48.79	−48.80	−48.90
Private	−48.42	−48.20	−49.74
b. Doubling Catch Rates			
MNL	20.33	17.41	15.89
Charter	14.15	16.95	7.95
Private	26.79	23.72	18.83
c. Loss of Shore Modes			
MNL	−35.89	−35.24	−35.27
Charter	−22.91	−21.79	−22.49
Private	−30.73	−30.84	−28.83

Several points from table 2 are worth noting. First, the estimated welfare effects of doubling the price of each mode (table 2a) is relatively insensitive to the choice of functional form and nesting structure. The welfare loss estimates range only from −$47.53 to −$49.74 (the negative indicates that a reduction in welfare occurs). Interestingly, the average price of the four fishing modes, weighted by the original choice probabilities, is just under $52. The proximity of this average price to the estimated welfare changes in table 2a suggests that the uniform doubling of prices leads to few changes in mode choice in which case the uniformity of the welfare estimates is not surprising. If no mode changes were to occur, the appropriate compensating variation would simply be this average price.

[19] The choice of T = 1,000 was selected on the basis of a Monte Carlo experiment in which the process of estimating E(*cv*) using T iterations and the linear charter model was repeated 100 times. This exercise was conducted using various choices of T. The simulation results indicated that the estimated mean compensated variation changed little over the 100 trials once T exceeded 500, with the standard deviation of E(*cv*) over the 100 trials reduced to less than $0.05 by the time T = 1,000. Since the charter model represents the extreme specification in terms of its departure from the MNL model, McFadden's simulation results suggest that the GEV simulator would yield even more-accurate welfare predictions for the private and MNL alternatives.

Turning to table 2b, we find that quite a different result emerges when a doubling of catch rates is considered for each of the modes. Substantial disparities emerge in the welfare estimates, varying both by the functional form that is used for the indirect utility function and by the assumed error structure. These estimates ranges from a low of $7.95 in the case of the TL charter model to a high of $26.79 when the linear private model is used—more than a three-fold increase in the estimated E(*cv*). It is worth noting, however, that there is at least as much variability in the welfare predictions due to the choice of nesting structure as there is in the form of the indirect utility function.

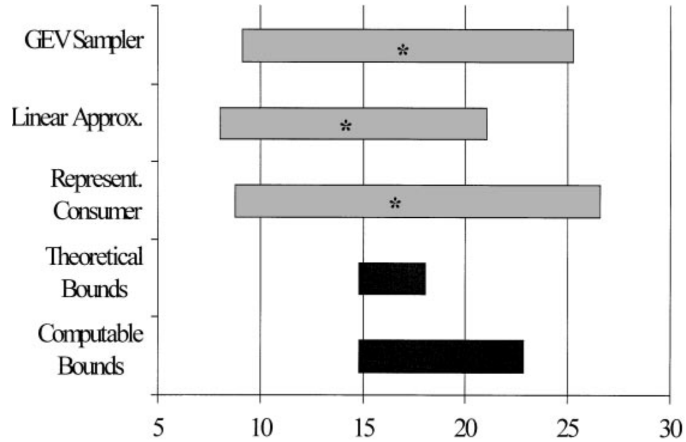
Finally, in the case of the loss of the shore modes (table 2c), there is very little difference among the alternative functional forms (reading across the rows of the table), but, again, notable differences among the error structures. For example, given the GL functional form, the welfare impact from losing both shore modes ranges from −$21.79 to −$35.24 over the alternative nesting specifications. In contrast, given the charter nesting structure, the welfare loss varies by little more than 5% over the alternative functional forms, from −$21.79 to −$22.91.

C. *Alternative Welfare Predictions*

The welfare estimates for the nonlinear models presented in table 2 were constructed using McFadden's GEV sampler and, hence, provide consistent estimates of the E(*cv*) associated with each policy scenario, given the selected functional form and nesting structure. However, these computations are quite costly, both in terms of the time required for an analyst to code the algorithms and in terms of computer time.[20] In contrast, the representative-consumer approach suggested by Morey et al. (1993) and the computable bounds in equation (14), based on McFadden's (1995) theoretical bounds, are much easier to obtain. Likewise, as noted previously, the linear model avoids the simulation problem entirely, yielding a closed-form equation for E(*cv*). Given the ease with which the linear model is estimated and the corresponding ease with which welfare measures (and standard errors) are computed, it may also be reasonable to treat the estimates from the linear model as yet another "second-best" approach to welfare measurement. An interesting empirical question is whether these simpler approaches yield substantially different welfare predictions from those obtained in table 2. Furthermore, these simpler approaches may be deemed even more palatable when the statistical precision of the point estimates are considered. Thus, if confidence intervals about the point estimates in table 2 typically encompass the linear or representative consumer approximations, it may be reasonable to compute the simpler measures.

[20] The calculations reported in this paper were conducted using GAUSS, version 3.11, on a 200 MHz Pentium Pro IBM-compatible PC with 32M of RAM. While the calculation of *each* point estimate in table 2 required only 15 minutes on this system, the confidence bounds reported in figures 2 and 3 *each* required approximately 48 hours to construct.

FIGURE 2.—ALTERNATIVE WELFARE FROM DOUBLING CATCH RATES—CHARTER MODEL



In order to shed light on this issue, figure 2 provides a comparison of five alternative estimators of the welfare loss due to a doubling of the catch rate.[21] The first alternative is the GEV sampler's estimate of $16.95, indicated by the asterisk in the top bar of the graph. The shade bar around the asterisk represents a 95% confidence bound around the GEV estimate, reflecting the fact that the underlying parameters used in constructing the welfare predictions are themselves random variables.[22] Similar point estimates and confidence bounds are provided when the welfare impacts are computed using the linear specification and when the nonlinear model is used, but a representative consumer approach is used to compute the welfare changes. These represent the second and third alternatives in figure 2. The fourth and fifth alternative estimators correspond to the theoretical and computable bounds given by equations (10) and (15) above.

Several results emerge from figure 2. First, in this application, the uncertainty regarding the GEV estimate of welfare is substantial, with the 95% confidence bound ranging from $9 to $25, encompassing both point estimates using the linear model and representative-consumer approaches. Second, the representative-consumer approach closely approximates the GEV sampler estimates. Both the point estimate of welfare ($16.51) and the confidence bounds are within 6% of the corresponding GEV sampler estimates. Finally, it is clear that the difficulties in computing the upper end of McFadden's theoretical bounds (specifically, P_k^1 of equation (9)) significantly reduces their information content. The theoretical bounds are relatively tight

[21] In constructing this figure, we assume that the correct model is the GL functional form with charter nested-logit structure. The GL was chosen for further study since it nests the linear model. An alternative to the charter nesting structure is considered in figure 3.

[22] These confidence bounds were constructed by means of simulation, using the asymptotic distribution of the maximum likelihood parameter estimates to reflect the uncertainty in the model coefficients. Five hundred coefficient vectors were randomly drawn from the asymptotic distribution of the estimates of (β,θ). For each of these parameter draws, the GEV welfare estimate of E(*cv*) was constructed. The 95% confidence bounds in figure 2 reflect the middle 95% of the resulting estimates, dropping the smallest and largest 2.5% of the values.

FIGURE 3.—ALTERNATIVE WELFARE IMPACTS FROM DOUBLING CATCH RATES—MNL MODEL

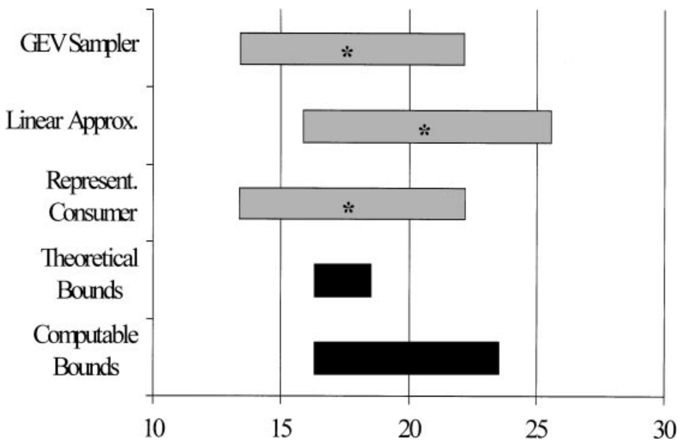
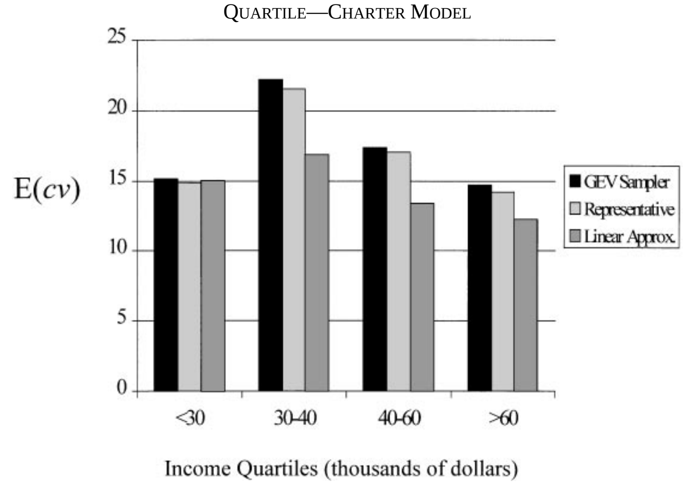


FIGURE 4.—ALTERNATIVE WELFARE PREDICTIONS BY INCOME QUARTILE—CHARTER MODEL



($15 to $18), while the alternative computable bounds are considerably wider, only narrowing the compensating variation to lie in the range from $15 to $23. The appeal of these computable bounds is further reduced when it is recognized that the bounds themselves are uncertain, depending upon the estimated parameters of the model. Confidence intervals around the computable bounds are likely to blanket all of the previous alternatives.

Figure 3 provides a comparable set of results when the MNL error structure is used. The results parallel those in figure 2. The point estimate of E(*cv*) for the representative consumer approach ($17.41) is virtually identical to the corresponding GEV sampler estimate, while the linear model's estimate is almost 17% larger. However, we again find that the difference in estimating the welfare gains by using the linear specification is swamped by the size of the confidence intervals surrounding each of the welfare estimates.

Finally, one limitation of our analysis thus far is that it has focused attention on E(*cv*) averaged over the entire sample. This narrowed focus creates two related problems. First, to the extent that our sample is not representative of the population of interest, this estimate of E(*cv*) will be misleading. At a minimum, a weighting scheme would be required. It is not immediately clear how the alternative welfare measures would perform in this case. Second, policy analysts are often interested not only in the aggregate welfare impact of a program, but also in how it affects specific segments of the population (e.g., low-income households). The comparisons in figures 2 and 3 may mask potentially important differences in the welfare measures predicted for these subpopulations.

Figure 4 addresses these concerns by providing E(*cv*) by income quartiles using McFadden's GEV sampler, the representative-consumer approximation, and the linear approximation. Here we again consider a doubling of the catch rate for all the modes using the GL model and the charter-nesting error structure, paralleling the results in figure 3. Three results emerge. First, as one might expect, E(*cv*) does vary by income level, starting out low at roughly $15 per choice occasion, rising at first as income increases, and then falling back below $15.[23] Second, the representative-consumer approximation to E(*cv*) continues to track closely the GEV sample estimate, even when we focus on specific income levels. Third, the bias in restricting preferences to be linear in income does vary by income level. For the lowest income quartile, all three methods yield roughly the same welfare estimates. For the remaining quartiles, however, the linear approximation yields welfare estimates that are 17% to 24% smaller than the corresponding GEV sampler estimates.

VII. Conclusions

This study has investigated the importance of nonlinear income effects in RUMs, with particular attention to welfare measurement. In addition to specifying a nonlinear structure for the deterministic portion of consumer preferences, using GL and TL models to provide flexible approximations to any nonlinear utility function, three distinct errors structures were considered. The resulting models were used to study mode choice among California anglers and to compare and contrast the available approaches for computing (or approximating) welfare changes when nonlinear income effects exist. These approaches include a resampling scheme based upon McFadden's GEV sampler, a linear model, a representative-consumer approach, and the computation of bounds on the welfare changes of interest. The approaches trade off computational ease for potential bias in the resulting welfare measures or uncertainty regarding their exact values.

Our analysis of California sportfishing represents, to our knowledge, the first application of McFadden's GEV sampler. Several key empirical results emerge. First, our findings

[23] One might, at first, expect that the linear model would yield the same estimate for E(*cv*) for each income quartile, given that this model assumes a constant marginal utility for both catch rates and income. However, as income changes, so do the travel costs associated with visiting a given site (since they depend in part on wage rates).

(highlighted in table 2) suggest that, in this application, there are more differences in the point estimates of welfare due to changes in assumed error distribution (e.g., multinomial logit versus nested logit) than there are due to the introduction of nonlinear income effects. Second, the consistent welfare estimates provided by the GEV sampler are not substantially different from the simpler linear and representative-consumer approximations, particularly when the stochastic nature of the underlying parameter estimates is considered. Finally, while the computable bounds are both readily constructed and allow for nonlinear income effects, they do not provide tight bounds on the welfare estimates, even when one ignores the uncertainty of the underlying parameter estimates. Clearly, analysts must be cautious in drawing too strong of inferences from the results of this one data set. First, additional empirical examples are needed to determine the robustness of our findings. Alternatively, a Monte Carlo analysis, investigating those characteristics of consumer preferences that would widen the gap between the alternative welfare estimators, would be a natural direction for future research. However, we believe these results provide a useful point of departure. Second, while the differences among the welfare estimates with and without nonlinear income effects are generally small, they may represent a significant sum of money in actual policy settings, making the additional effort required to employ nonlinear specifications worthwhile in some circumstances.

In addition to providing an empirical comparison of alternative functional forms and error structures, we have also advanced the understanding of welfare measurement in discrete-choice models by providing computable bounds based on McFadden's theoretical bounds, identifying cases in which those bounds are uninformative, and refining the procedures for computing the bounds themselves.

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