8-2010

Constraint-Based Synthesis of Shape-Morphing Structures in Virtual Reality

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Recommended Citation

Vance, Judy M. and Dorozhkin, Denis, "Constraint-Based Synthesis of Shape-Morphing Structures in Virtual Reality" (2010). *Mechanical Engineering Conference Presentations, Papers, and Proceedings*. 10.
http://lib.dr.iastate.edu/me_conf/10

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Abstract

This manuscript outlines a novel approach to the design of compliant shape-morphing structures using constraint-based design method. Development of robust methods for designing shape-morphing structures is the focus of multiple current research projects, since the ability to modify geometric shapes of the individual system components, such as aircraft wings and antenna reflectors, provides the means to affect the performance of the corresponding mechanical systems. Of particular interest is the utilization of compliant mechanisms to achieve the desired adaptive shape change characteristics. Compliant mechanisms, as opposed to the traditional rigid link mechanisms, achieve motion guidance via the compliance and deformation of the mechanism's members. The goal is to design a single-piece flexible structure capable of morphing a given curve or profile into a target curve or profile while utilizing the minimum number of actuators. The two primary methods prevalent in the design community at this time are the pseudo-rigid body method (PRBM) and the topological synthesis. Unfortunately these methods either tend to suffer from a poor ability to generate potential solutions (being more suitable for the analysis of existing structures) or are susceptible to overly-complex solutions. By utilizing the constraint-based design method (CBDM) we aim to address those shortcomings. The concept of CBDM has generally been confined to the Precision Engineering community and is based on the fundamental premise that all motions of a rigid body are determined by the position and orientation of the constraints (constraint topology) which are placed upon the body. Any mechanism motion path may then be defined by the proper combination of constraints. In order to apply the CBDM concepts to the design and analysis of shape-morphing compliant structures we propose a tiered design method that relies on kinematics, finite element analysis, and optimization. By discretizing the flexible element that comprises the active shape surface at multiple points in both the initial and the target configurations and treating the resulting individual elements as rigid bodies that undergo a planar or general spatial displacement we are able to apply the traditional kinematics theory to rapidly generate sets of potential solutions. The final design is then established via an FEA-augmented optimization sequence. Coupled with a virtual reality interface and a force-feedback device this approach provides the ability to quickly specify and evaluate multiple design problems in order to arrive at the desired solution.

Keywords

Shapes, Virtual reality

Disciplines

Mechanical Engineering

Proceedings of the ASME 2010 International Design Engineering Technical Conferences &
Computers and Information in Engineering Conference
IDETC/CIE 2010
August 15-18, 2010, Montreal, Quebec, Canada

DETC2010-29017

CONSTRAINT-BASED SYNTHESIS OF SHAPE-MORPHING STRUCTURES IN VIRTUAL REALITY

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This manuscript outlines a novel approach to the design of compliant shape-morphing structures using constraint-based design method. Development of robust methods for designing shape-morphing structures is the focus of multiple current research projects, since the ability to modify geometric shapes of the individual system components, such as aircraft wings and antenna reflectors, provides the means to affect the performance of the corresponding mechanical systems. Of particular interest is the utilization of compliant mechanisms to achieve the desired adaptive shape change characteristics. Compliant mechanisms, as opposed to the traditional rigid link mechanisms, achieve motion guidance via the compliance and deformation of the mechanism's members. The goal is to design a single-piece flexible structure capable of morphing a given curve or profile into a target curve or profile while utilizing the minimum number of actuators. The two primary methods prevalent in the design community at this time are the pseudo-rigid body method (PRBM) and the topological synthesis. Unfortunately these methods either tend to suffer from a poor ability to generate potential solutions (being more suitable for the analysis of existing structures) or are susceptible to overly-complex solutions. By utilizing the constraint-based design method (CBDM) we aim to address those shortcomings. The concept of CBDM has generally been confined to the Precision Engineering community and is based on the fundamental premise that all motions of a rigid body are determined by the position and orientation of the constraints (constraint topology) which are placed upon the body. Any mechanism motion path may then be defined by the proper combination of constraints. In order to apply the CBDM concepts to the design and analysis of shape-morphing compliant structures we propose a tiered design method that relies on kinematics, finite element analysis, and optimization. By discretizing the flexible element that comprises the active shape surface at multiple points in both the initial and the target configurations and treating the resulting individual elements as rigid bodies that undergo a planar or general spatial displacement we are able to apply the traditional kinematics theory to rapidly generate sets of potential solutions. The final design is then established via an FEA-augmented optimization sequence. Coupled with a virtual reality interface and a force-feedback device this approach provides the ability to quickly specify and evaluate multiple design problems in order to arrive at the desired solution.

INTRODUCTION

The concept of constraint-based compliant mechanism design has generally been confined to the Precision Engineering community and learned via apprenticeship. This method has been used to create elastic mechanisms which form the foundation of many precision instruments, compliant manipulators and consumer products. Although this method has been published in the literature [1, 2] these publications and their application to compliant mechanism design are not well known outside the Precision Engineering community. In addition, proficiency in using constraint-based methods for designing compliant mechanisms requires (1) commitment to a moderately steep learning curve (hence the reason for apprenticeship) and (2) “hands-on” experience to understand the stiffness characteristics of alternate designs. In this work, a generalized constraint-based concept design process and the supporting optimization/probability-based engineering decision making tools required for concept selection have been created.

These components have been integrated with VR so as to provide an experience which reduces the need for apprenticeship-based learning. This is particularly important in fields of application in which it is difficult to obtain hands-on experience/intuition. For instance, micro-scale and nano-scale compliant mechanisms are often difficult to design due to the difficulty in (1) obtaining a “feel” for how these devices operate and (2) visualizing how these devices function.

A decade of research into using VR as an engineering design tool has resulted in an understanding of the characteristics of VR that can be used to improve engineering design. Stereo viewing, position tracking and haptic force feedback provide a computer interface that allows participants to move and interact with digital objects as if they were real three-dimensional objects. These principles are implemented in a haptically-enabled design framework that provides a working space required to optimally view, assemble components and interact with constraint-based shape-morphing compliant mechanism design concepts. The framework is particularly useful when designing objects that require a large number of iterations and extensive physical prototyping in order to obtain a valid solution candidate.

BACKGROUND

Motivation

Development of robust methods for designing shape-morphing structures is the focus of several current research projects, both in the academic and the military communities. Geometric shapes of the individual system components, such as aircraft wings and antenna reflectors, directly affect the performance of the corresponding mechanical systems [3]. Of particular interest is the utilization of compliant mechanisms to achieve the desired adaptive shape change characteristics. Compliant mechanisms, as opposed to the traditional rigid link mechanisms, achieve motion guidance via the compliance and deformation of the mechanism’s members. The goal is to design a single-piece flexible structure capable of morphing a given curve or profile into a target curve or profile while utilizing the minimum number of actuators (ideally, just one) [4].

The synergy of constraint-based design theory methods and virtual reality working/ learning design space forms a basis for engineers and scientists to learn and effectively use the constraint-based design approach. This provides them with (1) a new perspective on how to perform synthesis and analysis of compliant mechanisms, (2) a generalized, well-disseminated design theory of mechanism design, (3) a means to rapidly master design for compliance/compliant mechanisms in fields which are difficult to build competence via hands-on experience, and (4) a fully immersive, collaborative, interactive design environment. This in turn has the potential to bring the field of compliant mechanism design to a broader audience which will be capable of better understanding how/why compliant mechanisms work, how to synthesize them, how to characterize them with general design metrics and how to best fabricate/integrate them into practical applications.

Virtual reality background

The term Virtual Reality (VR) refers to computer-generated three-dimensional (3-D) environments created by virtual environment (VE) systems, which can be interactively experienced and manipulated by the participants [5]. Stuart [6] defines a VE system as a human-computer interface capable of providing “interactive immersive multisensory 3-D synthetic environments.” In such systems the user’s motions are tracked with position sensors and used to update the visual and auditory displays in real-time. This creates the illusion for the participants of being inside of the environment [6]. In addition to providing the ability to explore a design problem in three-dimensional space, VR environments often allow users to manipulate the objects in the environment in an intuitive way using a variety of instrumented gloves, wands, and force-feedback devices

The scientific and engineering communities have embraced virtual reality as a valuable tool because it offers a unique way to investigate data. Benefits of the VR systems are especially evident in the area of engineering product development, where these systems are used throughout the whole range of the product development cycle: from modeling and evaluation of the first prototypes, to providing training opportunities for end-product users ([7], [8], [9]) .

VR-augmented compliant system design

The successful design of compliant mechanisms has been approached differently from two different design communities: mechanism design and precision machinery. The mechanism design community has adapted well-known rigid link design methods to the design of compliant mechanisms. As is the case in traditional rigid link mechanism design, this mathematical modeling approach sometimes results in theoretical mechanism configurations that are difficult to manufacture and suffer from fatigue failures. The precision machinery community takes an apprenticeship approach to the design of compliant mechanisms, which relies heavily on an individual’s experience and knowledge of the motion of compliant members. The result is that the optimal design achieved is highly dependent on the individual performing the design. In both approaches, a large number of design iterations and extensive physical prototyping are often required in order to arrive at a final solution.

The use of a well-designed virtual environment can support the integration of both approaches. Mathematical modeling based on methods from the precision machinery community can be combined and implemented in a three-dimensional virtual environment. The ability to define the design problem using three-dimensional input devices and the ability to visually verify the final solution aid in understanding both the design process and the result. In addition, interference avoidance, collision avoidance, aesthetics and ergonomics can all be prototyped in the virtual environment to test the final design.

EXISTING COMPLIANT MECHANISM DESIGN METHODS

There are three common design methods used to design compliant mechanisms: pseudo-rigid body modeling, topological synthesis, and constraint-based design. Each method presents advantages and challenges in the design process.

Pseudo-rigid body modeling

Pseudo-rigid body modeling (PRBM) [10, 11] is utilized as an alternative to rigorous large-deflection analysis methods in order to provide a more efficient method to arrive at and improve these initial designs. The pseudo-rigid-body approach models the deflection of flexible members using rigid-body components that have equivalent force-deflection characteristics (Fig. 1). The rigid analog of the compliant structure is then analyzed using traditional mechanism design methods and the principle of virtual work to ascertain its kinematic and elastomechanic properties. The PRB model has been used to design precision elastic mechanisms [12, 13] and many consumer products, but its primary aim remains to model rather than synthesize.

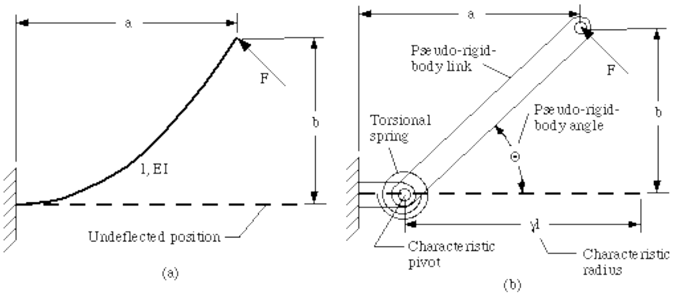


Figure 1. Flexible element (a) and its PRBM analog (b) [10].

Topological synthesis

Topological synthesis (TS) is a concept synthesis method that is based upon computer algorithms that examine a starting shape for a compliant mechanism and then determine how to add/subtract material that in order to create concepts that satisfy performance specifications [14, 15]. In this method the results of the mathematical model determine the layout of rigid and flexible elements. The final topology of the design is determined mathematically, without input from the designer. Solutions are often produced that have overly-complex topologies that would be difficult to manufacture (Fig. 2).

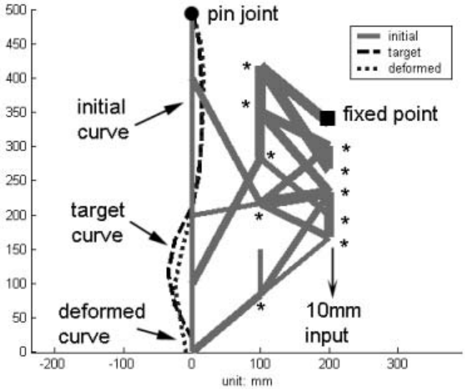


Figure 2. TS-generated compliant lumbar support [16].

Constraint-based design method

Constraint-based design is commonly used in the precision engineering community [1, 2]. The fundamental premise of the constraint-based method is that all motions of a rigid body are determined by the position and orientation of the constraints (constraint topology) which are placed upon the body. Any mechanism motion path may then be defined by the proper combination of constraints and non-constraints (Fig. 3). An unconstrained 3D rigid object has six degrees-of-freedom (DOF). Proper application of non-redundant constraints eliminates DOFs in one-to-one fashion.

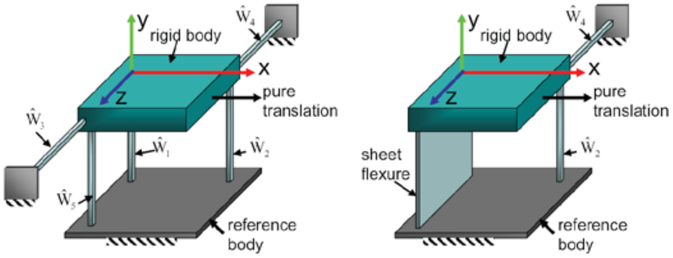


Figure 3. Examples of CBDM functionality.

METHODOLOGY

Currently, constraint-based design method (CBDM) is primarily utilized in the design of compliant mechanisms intended for movement of rigid stages (e.g., mirror in fiberoptic switch, probe positioning in a scanning-probe microscopy applications, etc.). The research presented here expands the scope of CBDM to the design of shape-morphing structures. The goal is to identify the number and topology of the constraints that will produce the desired shape. The method consists of two distinct steps: modeling the entire desired shape by a series of rigid four-bar linkages to identify candidate constraint anchor point regions, then refining the structure using finite element method to identify the location of the desired constraint anchor points. The suitability of fit of the final design shape is determined by a least squares error between the target shape and the achieved shape.

Rigid-body four-bar approximation

The method begins by dividing the source (initial) shape into a number of discrete segments. The endpoints of these segments are also located and identified on the target shape curve (Fig. 4).

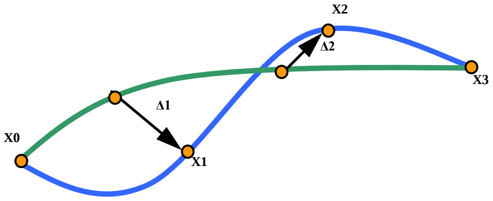


Figure 4. Discretization of the source and target curves.

A series of four-bar mechanisms is created to fit the source curve. Figure 5 depicts a single four-bar cell of the discretized compliant structure, which spans two neighboring anchor points and the corresponding two points on the deformable surface. The starting locations of the anchor points are chosen along the perpendicular bisectors associated with displacements **Δ** of the curve points (Fig. 4). Note that the coupler link of each four-bar is a segment of the source curve.

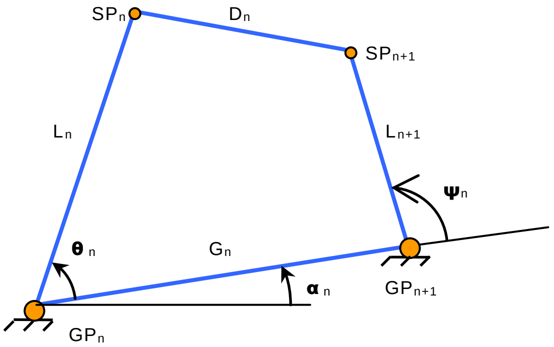


Figure 5. Single four-bar cell in the discretized deformable structure.

Here, $\mathbf{SP_n}$ and $\mathbf{SP_{n+1}}$ are the two neighboring points on the discretized source surface, and $\mathbf{GP_n}$ and $\mathbf{GP_{n+1}}$ are the anchor points. Given the four points, the individual link lengths in this 4-bar mechanism and, ultimately, the expression for $\boldsymbol{\psi_n}$ angle which relates it to the input $\boldsymbol{\theta_n}$ angle can be computed using traditional planar kinematics [17]. The $\boldsymbol{\psi_n}$ angle can then be used to determine the $\boldsymbol{\theta_{n+1}}$ angle, which can then be used to determine the configuration of the next four-bar cell in the structure. Since the cells are connected in series this process is repeated until all four-bar cell positions have been computed, providing the locations of all points in the structure. These sets of equations represent the configuration of the discretized structure, defined by the collection of anchor and surface points, and how it will deform with the given input angle $\boldsymbol{\theta_0}$ (Fig. 6):

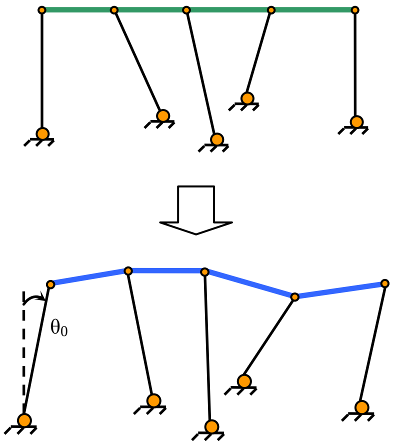


Figure 6. Deformation of a structure due to θ_0.

The next step is to optimize the structure to obtain the locations for each anchor point. The objective function follows the method proposed by Kota and Lu [4], which minimizes the difference between the target and the achieved profiles of the active surface. The locations of the anchor points must lie within a defined region R^C.

$$\text{minimize} \quad \frac{1}{n}\sum_{i=1}^{n}\sqrt{(X^D{}_i - X^T{}_i)^2 + (Y^D{}_i - Y^T{}_i)^2} \qquad (1)$$

$$\text{where} \quad X^D = G(m, X^S, Y^S, X^C, Y^C, \theta_0) \qquad (2)$$

$$Y^D = H(m, X^S, Y^S, X^C, Y^C, \theta_0) \qquad (3)$$

$$\text{subject to} \quad (X,Y)^C{}_j \in R^C, \quad j = 1,m \qquad (4)$$

$$\frac{\pi}{4} \le \theta_0 \le \frac{3\pi}{4} \qquad (5)$$

where (X^S, Y^S), (X^T, Y^T), (X^D, Y^D), and (X^C, Y^C) are the location of the points on the source curve, location of the points on the target curve, location of the points on the actual deflected curve, and location of the anchor points respectively. n is the total number of points on the curve, and m is the number of constraints. The coordinates of the segmented points on the deflected curve, (X^D, Y^D), are computed based on standard kinematic analysis of a four-bar linkage, outlined in the preceding section.

The results of the optimization are a set of potential locations of the anchor points based on the rigid four-bar linkage analysis. Next, each generated anchor point is used to define a potential anchor region to be used in the finite element analysis step (Fig. 7)

Finite element analysis model

During this step, the rigid body approximation is replaced with a flexible body model. The locations of the anchor points and the segment points are retained and the optimization is repeated as follows:

$$\text{minimize} \quad \frac{1}{n}\sum_{i=1}^{n}\sqrt{(X^{D}{}_{i}-X^{T}{}_{i})^{2}+(Y^{D}{}_{i}-Y^{T}{}_{i})^{2}} \qquad (6)$$

$$\text{where} \quad X^{D}=J(m,X^{S},Y^{S},X^{C},Y^{C},F) \qquad (7)$$

$$Y^{D}=K(m,X^{S},Y^{S},X^{C},Y^{C},F) \qquad (8)$$

$$\text{subject to} \quad (X,Y)^{C}{}_{j}\in P^{C}{}_{j}, \quad j=1,m \qquad (9)$$

where the deflected curve shape, (X^D, Y^D), is computed using finite element analysis, F is the actuation force, and P^C are the new potential anchor regions associated with anchor points generated by the rigid-body four-bar solution step. Figure 8 illustrates the procedure for this step.

Solution candidate generation sequence summary

The following procedure takes place during this design process (Figures 7 and 8):

a) Given the values of the anchor points and the input angle (initial or intermediate), and the location of the discretized vertices on the source curve (there has to be an equal number of both), determine theoretical response of the discretized rigid deformable structure to variations of the input angle.

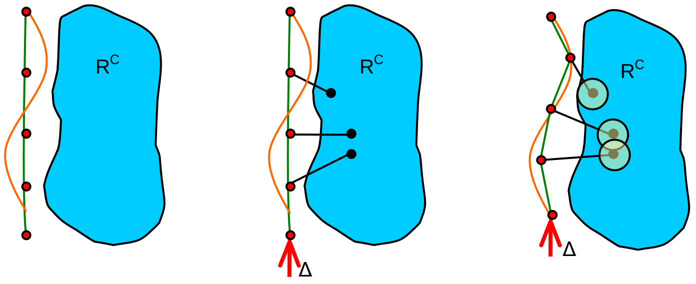
b) Vary the location of the anchor points within the available anchor region, and change the value of the input angle within the specified bounds, while computing the cumulative difference (Least-Squares Error) between the attained surface point locations and the desired locations of those points on the discretized target curve.

c) Stop once the lowest value of LSE is found – use the corresponding anchor positions as the bases for a refined set of the available anchor regions to be used in the evaluation of the compliant model.

d) Discard the rigid-body approximation, and model the compliant structure using finite element methods

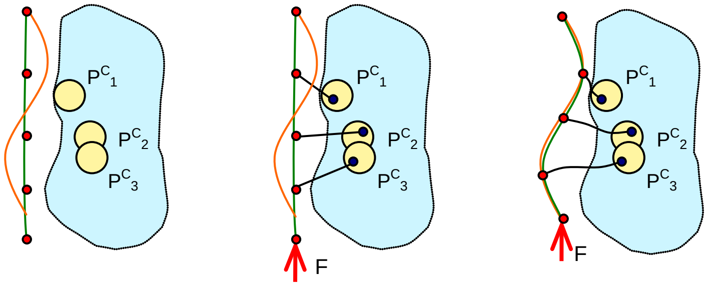
e) Repeat optimization using the refined anchor regions and computing compliant structure response

f) Generate the final solution



(a) (b) (c)

Figure 7. Design sequence (rigid structure).



(d) (e) (f)

Figure 8. Design sequence (compliant structure).

RESULTS

Utilizing the aforementioned theory a scalable virtual reality design framework has been developed. This framework can be used on a desktop VR system, consisting of a computer workstation equipped with a set of stereo glasses and a haptic interface device, as well as within fully immersive multi-screen projection environments (Figure 9). The VR interface allows designers to define the problem, generate candidate solutions and evaluate the solutions using an assortment of virtual tools. Design is assisted by the force feedback from the haptic interface, which allows precise positioning of the elements via ‘snapping’ to the already-defined features. Users have the ability to modify the material properties of the constructed compliant system, change the geometrical configuration of the components (e.g., beam cross-section), and to investigate the elastic response the generated structure in real time. An evolved set of haptically-assisted menus enables effective control over the design framework’s functionality.

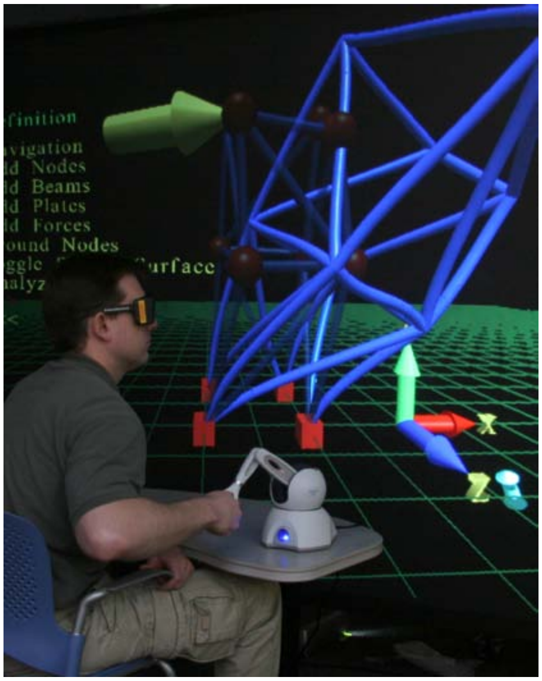


Figure 9. Virtual reality design environment.

During the design sequence the user utilizes the virtual environment to specify the design problems, including the geometrical configuration of the source and the target curves, as well as the material properties. Figure 10 depicts the virtual design environment with a sample problem - a compliant lumbar support, similar to that in the Figure 2, displayed on a 1 inch design grid. The material used in the example is Delrin 2700, with the individual element cross-section of 1/4 [in] x 1/16 [in].

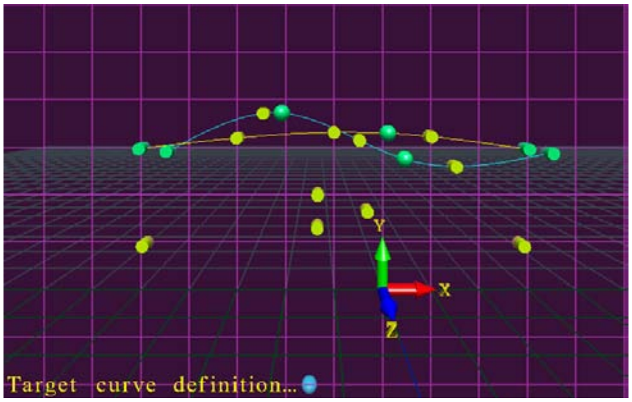


Figure 10. Design problem in VR.

The two distinct profile configurations of the compliant structure (yellow – source, cyan – target) have been specified. The continuous curves are cubic B-splines that pass through the available control points. Users have the ability to specify an arbitrary number of control points for both the source and the target profiles, as well as the ability to modify any existing points. This allows for specification of any potential profiles. Spheres on each of the curves are the control points, while cylinders are the initial estimated locations for the pivots of the discretized rigid-body representation of the compliant structure that will be used in the kinematic motion analysis. At this stage the user can also define the specific region that is available for placing the anchor candidates.

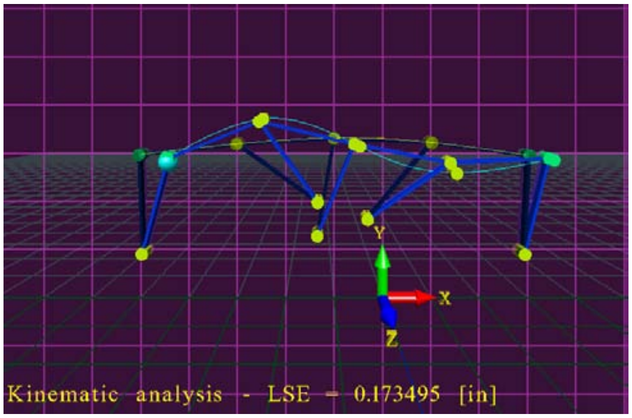


Figure 11. Rigid structure approximation.

Once the user is satisfied with the problem definition, he or she can proceed with the kinematic analysis of the discretized structure. Figure 11 depicts the chain of the individual four-bar linkages/cells responding to the motion of the driving link of the first cell in the chain (on the left). The Least-Squares Error is computed and its value is provided to the user. Following the optimization sequence the new anchor positions are generated. At this point in the design sequence the user has the option of manually deflecting the discretized rigid structure by grabbing the first node on the deformable surface (utilizing the haptic interface) and moving the node in space.

Following the rigid-body analysis the structure is re-modeled as a compliant entity. A built-in FEA solver generates the structure's response to actuation loads of various magnitudes, while varying the anchor locations and computing the corresponding LSE values. This iteration sequence is terminated once the lowest value of the LSE is obtained and the results are displayed to the user. The response of the compliant structure can be investigated by the user by grabbing one of the nodes and applying a load with the haptic device – essentially "feeling" in real time how the structure responds to forces of various magnitudes and directions (Figure 12).

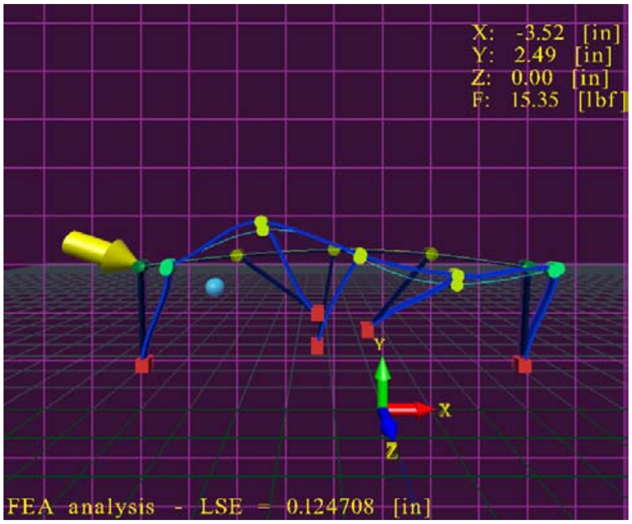


Figure 12. Compliant structure response.

CONCLUSIONS AND FUTURE WORK

A novel approach to the design of compliant shape-morphing structures using constraint-based design method has been developed as an alternative to the two primary methods prevalent in the design community at this time - the pseudo-rigid body method (PRBM) and the topological synthesis (which tend to suffer from either a poor potential solution synthesis capabilities or from susceptibility to overly-complex solutions). A tiered design method that relies on kinematics, finite element analysis, and optimization in order to apply the CBDM concepts to the design and analysis of shape-morphing compliant structures is presented. By discretizing the flexible element that comprises the active shape surface at multiple points in both the initial and the target configurations and treating the resulting individual elements as rigid bodies that undergo a planar or general spatial displacement we are able to apply the traditional kinematics theory to rapidly generate sets of potential solutions. An FEA-augmented optimization sequence establishes the final compliant design candidate. Coupled with a virtual reality interface and a force-feedback device this approach provides the ability to quickly specify and evaluate multiple design problems in order to arrive at the desired solution without an excessive number of design iterations and a heavy dependence on the intermediate physical prototypes.

In the subsequent work we plan to expand the design framework to include the ability to analyze general 3D response of compliant shape-morphing structures (out-of-plane deformations), to generate methods addressing the secondary design criteria (interference avoidance, collision avoidance, aesthetics, and ergonomics), as well as to continue improving the design framework interface (e.g., a better method for entering numerical data during the problem specification phase of the design process, which can be addressed by combining virtual menus and voice recognition),

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