

2010

Welfare Effects of Expansions in Equilibrium Models of an Electricity Market with Fuel Network

Sarah M. Ryan

Iowa State University, smryan@iastate.edu

Anthony Downward

University of Auckland

Andrew Philpott

University of Auckland

Golbon Zakeri

University of Auckland

Follow this and additional works at: http://lib.dr.iastate.edu/imse_pubs



Part of the [Industrial Engineering Commons](#), and the [Systems Engineering Commons](#)

The complete bibliographic information for this item can be found at http://lib.dr.iastate.edu/imse_pubs/29. For information on how to cite this item, please visit <http://lib.dr.iastate.edu/howtocite.html>.

Welfare Effects of Expansions in Equilibrium Models of an Electricity Market with Fuel Network

Abstract

The welfare of electricity producers and consumers depends on congestion in the transmission grid, generation costs that consist mainly of fuel costs, and strategic behavior. We formulate a game theoretic model of an oligopolistic electricity market where generation costs are derived from a fuel supply network. The game consists of a fuel dispatcher that transports fuels at minimum cost to meet generator demands, generators that maximize profit in Cournot competition, and an independent system operator (ISO) that sets nodal prices to balance electricity supply with linear demand functions. We prove the existence of an equilibrium. If fuel supplies are unlimited, the same equilibria hold in a simplified version of the game in which each generator optimizes its fuel acquisition from the network. In some very simple examples under different assumptions about the rationality of generators with respect to ISO decisions, paradoxical effects on total welfare can occur from expanding either electricity transmission capacity or the transportation capacity of low-cost fuel. We find some instances in which the paradox occurs only under bounded rationality of the generators, others where it occurs only if the generators are fully rational, and still others where it occurs to different degrees under the two rationality assumptions.

Keywords

capacity expansion, game theory, linear complementarity problem, market models

Disciplines

Industrial Engineering | Systems Engineering

Comments

This is a manuscript of an article from IEEE Transactions on Power Systems 25 (2010): 1337, doi: [10.1109/TPWRS.2009.2039587](https://doi.org/10.1109/TPWRS.2009.2039587). Posted with permission.

Welfare Effects of Expansions in Equilibrium Models of an Electricity Market with Fuel Network

Sarah M. Ryan, *Member, IEEE*, Anthony Downward, Andrew Philpott, *Member, IEEE*, and Golbon Zakeri

Abstract—The welfare of electricity producers and consumers depends on congestion in the transmission grid, generation costs that consist mainly of fuel costs, and strategic behavior. We formulate a game theoretic model of an oligopolistic electricity market where generation costs are derived from a fuel supply network. The game consists of a fuel dispatcher that transports fuels at minimum cost to meet generator demands, generators that maximize profit in Cournot competition, and an independent system operator (ISO) that sets nodal prices to balance electricity supply with linear demand functions. We prove the existence of an equilibrium. If fuel supplies are unlimited, the same equilibria hold in a simplified version of the game in which each generator optimizes its fuel acquisition from the network. In some very simple examples under different assumptions about the rationality of generators with respect to ISO decisions, paradoxical effects on total welfare can occur from expanding either electricity transmission capacity or the transportation capacity of low-cost fuel. We find some instances in which the paradox occurs only under bounded rationality of the generators, others where it occurs only if the generators are fully rational, and still others where it occurs to different degrees under the two rationality assumptions.

Index Terms—Game theory, market models, capacity expansion, linear complementarity problem.

I. INTRODUCTION

INCREASING demand for electricity combined with tighter supplies of fuels for generating plants have raised awareness of the interaction between electricity markets and the infrastructure for delivering fuel to generators. In restructured electricity markets, generating companies submit bids to supply electricity at prices based on their marginal costs, which are driven largely by fuel costs. In each regional wholesale power market, an independent system operator (ISO) manages electricity transmission and sets locational marginal prices (LMPs) to match supplies with demands at each location on the constrained grid. While considerable attention has been focused on expanding transmission capacity to reduce congestion in the electricity network, less effort has been spent on understanding how constraints in the fuel delivery networks interact with those in the electricity transmission network to affect the total social welfare.

An integrated model of fuel and electricity supply and delivery can help cultivate this understanding. In this paper, we develop a game theoretic model that accounts for (1) costs of extracting and transporting finite supplies of fuels across routes with limited capacity, (2) strategic decisions of generators at

different locations across a congested electricity transmission network, (3) price-sensitive demands for electricity, and (4) matching of electricity supply and demand across the network to maximize total social welfare subject to physical transmission constraints. Given the fuel and electricity network topology and capacities, fuel supplies and costs, transmission line reactances, and demand functions, the model produces LMPs and quantities of electricity generated and consumed at each location on the grid. These represent a static view of, say, a particular hour in some scenario of cost, capacity and demand. Social welfare measures derived from the electricity prices and quantities in different scenarios indicate the benefit of improving the fuel delivery or electricity transmission infrastructure.

The size and complexity of the electricity generation and transmission systems pose modeling challenges that are exacerbated by the need to account for strategic behavior by generators in the wholesale power markets. In recent years, a consensus seems to have emerged that considering generators as Cournot competitors, who at any given time decide on quantities of electricity to inject into the market, achieves a reasonable tradeoff between realism and tractability. In these models, the ISO's role is to determine transmission quantities that satisfy physical constraints and prices that match quantities supplied at each location with amounts customers are willing to buy. Such a model does not capture temporal dependencies such as ramping constraints, the actual structure of supply function bids, nor the potential for generators to learn from previous market outcomes. However for planning purposes, its results can provide a plausible snapshot of generation and transmission quantities along with nodal electricity prices that may be expected from a given set of generation capacities, transmission capacities and fuel costs. Recent comparisons of existing models for competition in electricity markets show that model assumptions concerning the timing of decisions and the rationality of the competing generators have significant impacts on the results [1]. In a game theoretic model that attributes full rationality to the generators, there may be multiple equilibria or none at all. In either case, it is impossible to draw any conclusions about the social welfare effects of changing model parameters such as costs or capacities. Various bounded rationality assumptions may result in easily computed equilibria that are more or less competitive than those observed in actual markets.

Accounting for the sources of fuel costs requires additional simplifications to maintain tractability. In the U.S. in 2007, 49% of net generation was from coal, which is transported from several coal-producing regions over a complex net-

S. Ryan is with the Department of Industrial and Manufacturing Systems Engineering, Iowa State University, Ames, IA 50011-2164 USA.

A. Downward, A. Philpott and G. Zakeri are with the Department of Engineering Science, University of Auckland, New Zealand.

work of railways, rivers and roads; 22% from natural gas, which moves from domestic wells and import points primarily through pipelines; and 19% from nuclear fuel, which carries severe safety restrictions in transport. Fuel costs accounted for 78% of the operating costs of fossil steam and 91% of the costs of the operating costs of gas turbine and small scale plants run by major investor-owned utilities [2]. The prices generators pay for these fuels, which account for both extraction and transportation, are set in bilateral contracts and commodities markets. These prices, which represent costs to the generators, also could be influenced by strategic competitive behavior among suppliers, transporters, and brokers, as well as the generators competing in these upstream fuel markets. In this paper, we model the entire fuel supply system as though it were managed by a benign fuel dispatcher, whose goal is to minimize the total cost of supplying the fuel required by the generators to produce their chosen quantities of electricity. A minimum cost network flow formulation includes available supplies as well as costs and capacities of transportation links between fuel supply locations and the generators. Quelhas et al. [3]; [4] recently formulated and validated such a model for the coal- and natural gas-based portion of the U.S. national electric energy system, while treating electricity demand as fixed.

Network models that integrate fuel supply markets with electricity markets have been developed, assuming those markets are perfectly competitive [5], [6]. But, due to concerns of generator market power partly caused by transmission limits, most recent models of electricity markets have incorporated some form of oligopolistic competition leading to a Nash equilibrium. The challenge is to find a balance between realism of the model assumptions and analytical tractability for large, complex systems. In particular, existence and uniqueness of equilibria are not always guaranteed. Linear mixed complementarity models have been developed in which generators set quantities as Cournot competitors [7] or submit supply functions with conjectures about competitor responses [8], [9]. Special computational procedures have been developed to compute equilibria in multi-leader games [10] or Cournot competition with anticipation of congestion effects [11]. Computational tests have shown how assumptions in Cournot models about market design, timing of decisions and generator rationality can affect the results significantly [1]. Recently, Yao et al. [12] introduced an electricity market model with a bounded rationality assumption that allows an equilibrium to be computed by solving a linear complementarity problem based on quadratic generation costs. Because the fuel transportation problem also can be expressed in terms of linear complementarity constraints, it appears promising to combine the constraints from the electricity network model with those from the fuel network model, effectively substituting piecewise-linear fuel costs for the quadratic generation costs. In numerical tests [13], electricity prices and transmission quantities from the resulting combined model show reasonable fidelity to results of a detailed agent-based simulation in which generators submit supply-function bids with learning.

The goal of this paper is to carefully examine whether an equilibrium is guaranteed to exist for such a model, and if

so, what effects on social welfare may result from increasing capacities of either fuel transportation links or electricity transmission lines, according to the model. We find some instances in which improvements in these infrastructures actually harm the total social welfare. In an application of the electricity market model of [12] to value transmission assets, Sauma and Oren [14] observed at least one instance where transmission expansion decreased the total social welfare predicted by the model. We explore a set of small instances in which similar effects are seen under either bounded rationality or full rationality assumptions. Paradoxical effects of capacity increases in transportation networks under competition also are known to exist [15] (English translation in [16]); [17].

A special case of our model is equivalent to one in which each fuel supply arc represents a different generation unit with a constant marginal cost. Therefore, increasing capacity in a fuel supply arc is akin to increasing the capacity of a unit. Only a few studies have addressed interactions between generation expansion and transmission expansion in a market context. In [18] and [19], an iterative simulation of coordinated investment decisions over a long time horizon was devised, in which the ISO sends market-based price signals to independent GENCOs and TRANSCOs. In [14], an equilibrium model similar to ours was the basis for studying the social welfare impacts of transmission expansions when generation expansion responses are taken into account, but the welfare impacts of generation expansions were not examined explicitly.

In Section II, we formulate equilibrium models for an electricity market combined with a fuel network under varying assumptions about generator rationality. In Section III, we show that an equilibrium exists under the bounded rationality assumption of [12]. In the case of unconstrained fuel supplies, we also formulate a simplified version in Section IV in which each generator optimizes its fuel acquisition from the network. The equilibria from the original formulation are preserved in this case. In Section V, we study small examples in the simplified formulation with two or three electricity nodes, which possess unique equilibria and also can be studied under full rationality. We show cases where expanding fuel transportation or electricity transmission capacity is detrimental to social welfare under either or both rationality assumptions. Section VI concludes.

II. MODEL AND NOTATION

TABLE I
SETS OF NODES AND ARCS

Set	Description	Indices	Cardinality
N	Electricity nodes	i, j	n_E
B	Electricity transmission lines	l	m_E
F	Fuel supply nodes	g	n_F
A	Fuel supply lines	gj	m_F

We model the electricity system along with its fuel supply as a network of electricity nodes and fuel supply nodes with transmission lines as directed arcs connecting the electricity nodes and transportation routes as directed arcs from the fuel nodes to the electricity nodes. Table I specifies the

TABLE II
 MODEL PARAMETERS

Parameter	Description	Dimension
a	Intercepts of electricity demand prices as linear functions of quantities	$n_E \times 1$
b	Slopes of electricity demand prices as linear functions of quantities	$n_E \times 1$
$M = \text{diag}(b)$	Diagonal matrix of demand function slopes	$n_E \times n_E$
V	Generation capacities	$n_E \times 1$
D	Power transfer distribution factors based on a fixed reference node $r \in N$	$m_E \times n_E$
K	Transmission line capacities	$m_E \times 1$
Z	Quantities of fuel available	$n_F \times 1$
c	Costs per MWh-equivalent of fuel transported	$m_F \times 1$
U	Capacities of fuel supply lines	$m_F \times 1$

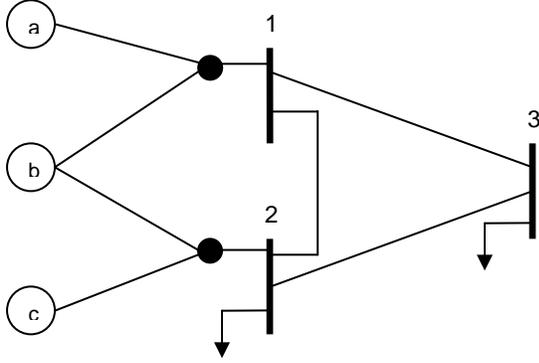


Fig. 1. Illustration with three fuel nodes $\{a, b, c\}$ and three electricity nodes $\{1, 2, 3\}$. Here, $V_3 = 0 = a_1$ and $b_1 = -\infty$.

sets of nodes and arcs. Assume each electricity node in $N = \{1, \dots, n_E\}$ has exactly one load-serving entity (LSE) and one generator. We allow for nodes without generators by setting their generation capacities to zero, while multiple generators at the same node can be modeled by connecting dummy nodes with infinite-capacity transmission lines. Figure 1 illustrates the model of a small system.

The data for the model include parameters of linear demand functions for the LSEs; capacities of generators, transmission lines, and fuel supply routes; and quantities of fuels available together with their costs. The model parameters are described in Table II. Assume each demand function slope $b_i < 0$, $i \in N$. Any node i without a load has $a_i = 0$, $b_i = -\infty$. Flows on the transmission lines are modeled in terms of a lossless direct current approximation of Kirchhoff's laws using power transfer distribution factors (PTDFs). The element $D_{l,j}$ specifies the change in flow on line l that results from a one-unit injection of electricity at node j accompanied by a corresponding one-unit withdrawal at the fixed reference node. It is the negative of the generation shift factor computed as in [20] from the transmission line reactances. For simplicity, we assume that generation costs are due solely to fuel. Multiple fuel types may be included in the fuel supply network, where all flows are in MWh of energy content. The cost per unit of flow on a fuel arc includes all costs for procuring the fuel at the origin, transporting it, and converting it to electricity at the destination. A given generator may obtain fuels from multiple supply nodes at different costs and each fuel node may supply

multiple generators. The fuel network in this paper ignores the different time horizons that arise from fuel storage capabilities and assumes costs and capacities are on an hourly basis. A non-bipartite network incorporating storage and multiple time scales as in [3] could be substituted.

The model centers on decisions of the generators. We assume that each is an independent Cournot competitor that decides its own generation quantity, given the corresponding quantities of other generators. In the Nash-Cournot equilibrium model, generators make these decisions simultaneously. We combine these decisions with downstream decisions of an independent system operator (ISO) that determines quantities of electricity supplied and consumed at each node, given the generation quantities, by specifying nodal injections. Similar game theoretic models of electricity markets have been analyzed in, e.g., [1] and [7] – [11] under varying assumptions about the extent to which each player anticipates the actions of the others. Typically in these papers, the generation cost is assumed to be a known function, such as linear or quadratic, of the generation quantity. In this paper, we explicitly model costs that result from a fuel supply network by including the fuel flows upstream of the generators. We assume these flows are decided by a non-strategic fuel dispatcher that minimizes the total cost of delivering the fuel required by the generators given their decisions. Primal and dual decision variables are summarized in Tables III and IV, respectively. The dual variable η represents the price of electricity at the reference node while p is the vector of nodal electricity prices. Also define the nodal price premia, $\phi \equiv p - \eta e$, where e is a vector of ones.

Let $F_j \subseteq F$ be the set of fuel supply nodes g such that there exists an arc from g to j and N_g be the set of electricity nodes j supplied by fuel node g . The fuel dispatcher's (primal) decision problem is a linear transportation problem:

$$\begin{aligned}
 \mathbf{FDP} \quad & \min_{x \geq 0} \sum_{gj \in A} c_{gj} x_{gj} \\
 \text{s.t.} \quad & - \sum_{j \in N_g} x_{gj} \geq -Z_g, \quad \forall g \in F \quad [\omega_g \geq 0] \\
 & \sum_{g \in F_j} x_{gj} = y_j, \quad \forall j \in N \quad [\pi_j] \\
 & -x_{gj} \geq -U_{gj}, \quad \forall gj \in A \quad [\rho_{gj} \geq 0]
 \end{aligned}$$

Here, the first set of constraints are on the fuel supplies, the second set ensure that the fuel delivery to each generator meets the demand created by electricity production, and the third enforce the fuel transportation capacities. We assume without loss of generality that $Z_g > 0$, $\forall g \in F$, and $U_{gj} \geq 0$, $\forall gj \in$

TABLE III
PRIMAL DECISION VARIABLES

Variable	Description	Player	Dimension
q	Demand satisfied at electricity nodes	ISO	$n_E \times 1$
r	Net injections of electricity at electricity nodes	ISO	$n_E \times 1$
x	Quantities of fuel delivered	Fuel Dispatcher	$m_F \times 1$
y	Generation amounts	Generators	$n_E \times 1$

TABLE IV
DUAL DECISION VARIABLES

Variable	Corresponding primal constraint	Player	Dimension
ω	Supplies at fuel nodes	Fuel Dispatcher	$n_F \times 1$
π	Demands for fuel by generators	Fuel Dispatcher	$n_E \times 1$
ρ	Fuel line capacities	Fuel Dispatcher	$m_F \times 1$
η	Total generation equal to satisfied demand	ISO	Scalar
p	Electricity market clearing conditions at nodes	ISO	$n_E \times 1$
λ^+	Upper bound on transmission from lower- to higher-numbered node	ISO	$m_E \times 1$
λ^-	Upper bound on transmission from higher- to lower-numbered node	ISO	$m_E \times 1$
β	Aggregated electricity demand for network	Generators	$n_E \times 1$
μ	Generation capacities	Generators	$n_E \times 1$

A.

The dual of the fuel dispatcher's problem is:

$$\begin{aligned} \text{FDD} \quad \max_{\omega, \rho \geq 0, \pi} \quad & - \sum_{g \in F} Z_g \omega_g + \sum_{j \in N} y_j \pi_j - \sum_{gj \in A} U_{gj} \rho_{gj} \\ \text{s.t.} \quad & -\omega_g + \pi_j - \rho_{gj} \leq c_{gj}, \quad \forall gj \in A \end{aligned}$$

Note that the generators' decisions, y , are treated as constants in the dual objective function.

Also given the decisions of the generators, the ISO's decision problem is to maximize social welfare, which is the total consumer willingness-to-pay less the sum of all the generation costs. It is equivalent to the sum of consumers' surplus, producers' surplus, and transmission rents, where the transmission rent from a flow on a line is the amount of flow multiplied by the difference in the nodal prices at either end. The consumer willingness-to-pay can be evaluated as the total area under the demand functions up to the quantities supplied. On the other hand, the total generation cost is constant with respect to the ISO's decisions and, therefore, can be omitted from the primal formulation as a quadratic program:

$$\begin{aligned} \text{ISOP} \quad \max_{q, r} \quad & \frac{1}{2} q^T M q + a^T q \\ \text{s.t.} \quad & e^T r = 0 \quad [\eta] \\ & q - r = y \quad [p] \\ & D r \leq K \quad [\lambda^+ \geq 0] \\ & -D r \leq K \quad [\lambda^- \geq 0] \end{aligned}$$

The four sets of constraints represent, respectively, requirements that there is no net injection by the ISO, conservation of energy at each node, and thermal capacity constraints on the transmission flows in either direction. The Dorn dual for this quadratic program (see page 233 of [21]) is given by:

$$\begin{aligned} \text{ISOD} \quad \min_{\eta, p, q, \lambda^+, \lambda^-} \quad & y^T p + K^T (\lambda^+ + \lambda^-) - \frac{1}{2} q^T M q \\ \text{s.t.} \quad & -M q + p = a \\ & \eta e - p + D^T \lambda^+ - D^T \lambda^- = 0 \\ & \lambda^+, \lambda^- \geq 0 \end{aligned}$$

The above two problems follow the usual duality relationship in the sense that if one is unbounded the other is infeasible and if they are both feasible then they both have optimal solutions having the same objective value. Note that the ISO's problem

will always have an optimal solution; moreover, observe that the ISO's dual problem is a convex (quadratic) optimization problem that is linear (therefore concave) in p and concave in q , and the objective function is continuous in the decisions, y , of the generators. Any solution to the ISO problem must satisfy the Karush-Kuhn-Tucker (KKT) conditions, where $\phi = D^T (\lambda^+ - \lambda^-)$:

$$a + M q - p = 0 \quad (1)$$

$$-\eta e + p - \phi = 0 \quad (2)$$

$$e^T r = 0 \quad (3)$$

$$q - r = y \quad (4)$$

$$0 \leq \lambda^+ \perp K - D r \geq 0 \quad (5)$$

$$0 \leq \lambda^- \perp K + D r \geq 0 \quad (6)$$

Next we examine generator i 's problem of maximizing profit, which is the difference of revenue and generation cost, subject to its capacity constraint. Revenue depends on the ISO decisions while cost is determined by the fuel dispatch. We will set the fuel charges such that each generator pays for just the fuel that they use (there is no congestion charge); this means that generator i would be charged

$$\sum_{g|gi \in A} c_{gi} x_{gi},$$

determined from the optimal solution to FDP. In section IV we prove that in the case of unlimited fuel supply ($Z = \infty$), generators seeing only the marginal costs of fuel associated with their current generation levels arrive at the same equilibrium as those who can anticipate their effect on the total fuel costs.

Generator i sells its production at the price p_i determined by the ISO. We consider two versions of the generator decision problem that differ according to their level of rationality concerning the ISO decisions. In the "fully rational" case, each generator anticipates the ISO injections, transmission flows and nodal prices that will result from its own generation decision, given the other generation amounts. In effect, the

generators are Stackelberg leaders with the ISO as follower. This case is modeled by including the full set of ISO KKT conditions as constraints in each generator's decision problem. Given $y_j, j \neq i$, generator i solves a mathematical program with equilibrium constraints (MPEC):

$$\begin{aligned} \text{GiP-F} \quad & \max_{y_i \geq 0} \quad (p_i - \pi_i)y_i \\ \text{s.t.} \quad & y_i \leq V_i \quad [\mu_i \geq 0] \\ & \text{Equations (1) - (6)} \end{aligned}$$

The game consisting of all generators simultaneously solving the above problem is often modeled as an equilibrium problem with equilibrium constraints (EPEC). These problems are non-convex due to congestion in the transmission network. Solving the joint-KKT conditions for all players simultaneously therefore yields *local Nash equilibria*. At these points some players may be locally, rather than globally optimal. This type of model is considered by Hu and Ralph in [22]. Furthermore, as discussed in [12], a game in which generators simultaneously solve these MPECs may have multiple equilibria or none at all. Also, a pure-strategy equilibrium that exists may satisfy some transmission constraint complementarity conditions as equalities so that a line is barely congested with equal prices at both ends.

Therefore, for tractability, we adopt the bounded rationality assumption of [12], in which each generator observes the price premia at each node relative to the reference node, and does not anticipate the effect of its decisions on the transmission quantities. Specifically, the ISO's dual decision variables, ϕ , are constants in each generator's decision problem (as are the fuel dispatcher's dual decision variables, π). In this formulation, generator i 's decision problem is:

$$\begin{aligned} \text{GiP-B} \quad & \max_{y_i \geq 0, \eta} \quad (\eta + \phi_i - \pi_i)y_i \\ \text{s.t.} \quad & y_i - \eta \sum_{j=1}^{n_E} \frac{1}{b_j} = \\ & \sum_{j=1}^{n_E} \frac{\phi_j - a_j}{b_j} - \sum_{j \neq i} y_j \quad [\beta_i] \\ & y_i \leq V_i \quad [\mu_i \geq 0] \end{aligned}$$

The first constraint expresses generator i 's knowledge of the market-clearing condition that total supply equals total demand in the network. Note that ISO constraints (3) and (4) imply $\sum_i q_i = \sum_i y_i$. Furthermore, for $i \in N$, conditions (1) and (2) imply $q_i = \frac{\eta + \phi_i - a_i}{b_i}$. Although the reference node price, η , actually is an ISO decision, it is treated as a decision variable in the optimization problem for each generator i because otherwise, the nodal prices and decisions of generators $j \neq i$ would uniquely determine y_i . In other words, given all of the other generation quantities and the nodal price premia, each generator anticipates the effect of its own generation quantity on the reference node price that clears the market. Finally, observe in the first constraint that, as the slopes b_j are negative, η is a decreasing linear function of y_i . Therefore each generator's problem has an objective that is concave in the player's strategy y_i .

III. EXISTENCE AND COMPUTATION OF EQUILIBRIA

Here we discuss the existence of an equilibrium to the game consisting of a fuel dispatcher choosing fuel prices, electricity generators injecting quantities of electricity and

an ISO determining price premia between nodes. Note that these decision variables are derived from the generator primal problems and the fuel dispatcher and ISO dual problems. Rosen's Theorem in [23] asserts that there always exists an equilibrium to a concave n -person game; this is defined as a game where the players' decisions are in a compact and convex set and their payoffs are concave in their own decision and continuous in the decisions of the other players. In our model, the feasible region for each of the players is defined by a set of linear constraints; moreover, each player's objective function is concave in its own decision variables and continuous in the decision variables of the other players. Unfortunately, however, in the current formulation of the game, many of the ISO and fuel dispatcher decisions are not in a bounded set. To correct this, we will introduce artificial bounds on those decision variables. The introduction of these bounds enables us to apply the proof in [23] which guarantees the existence of an equilibrium. However, it is not obvious that in imposing these bounds we do not introduce spurious equilibria. Therefore, we must verify that the equilibrium, which we now know exists, is in fact a bona fide equilibrium; i.e., it has not been created through the imposition of these bounds. To do this we show that an equilibrium to this game will not exist at any of the imposed bounds, and that any equilibrium would still exist if the bounds were removed.

Theorem 1: A Nash equilibrium exists for the game consisting of FDD, ISOD and $\{\text{GiP-B}, i \in N\}$.

Proof: See Appendix A. ■

In the game addressed by Theorem 1, all players are assumed to act at once. An equilibrium may be identified by simultaneously solving the KKT conditions for all players. That is, to equations (1) - (6), we append the KKT conditions for the fuel dispatcher and all the generator optimization problems:

$$0 \leq x_{gj} \perp c_{gj} + \omega_g - \pi_j + \rho_{gj} \geq 0, \quad \forall gj \in A \quad (7)$$

$$\sum_{g \in F_j} x_{gj} - y_j = 0, \quad \forall j \in N \quad (8)$$

$$0 \leq \omega_g \perp Z_g - \sum_{j \in N_g} x_{gj} \geq 0, \quad \forall g \in F \quad (9)$$

$$0 \leq \rho_{gj} \perp U_{gj} - x_{gj} \geq 0, \quad \forall gj \in A \quad (10)$$

$$0 \leq y_j \perp -\eta - \phi_j + \pi_j + \beta_j + \mu_j \geq 0, \quad \forall j \in N \quad (11)$$

$$y_j + \left(\sum_{i \in N} \frac{1}{b_i} \right) \beta_j = 0, \quad \forall j \in N \quad (12)$$

$$0 \leq \mu_j \perp V_j - y_j \geq 0, \quad \forall j \in N \quad (13)$$

The whole set of conditions comprises a linear mixed complementarity problem (MCP; see [24]). Note that the aggregate demand constraint in each generator's set of constraints is redundant given the ISO constraints. Along with the inequality conditions and corresponding nonnegative variables, there are $5n_E + 1$ equations that can be matched with the same number of unrestricted variables ($\pi, p, r, \beta, q, \eta$). This system can be solved efficiently, for instance by the PATH solver in GAMS [25], [26]. Existence of an equilibrium has been established in other game models of electricity markets solved as complementarity problems (e.g., [7]) by showing that the

coefficient matrix is positive definite, but this condition does not necessarily hold for our model.

IV. UNCONSTRAINED FUEL SUPPLIES

For the case where fuel supplies, Z , are ample to meet all generator demands, we also formulate a version of each generator's optimization problem that incorporates the acquisition of fuel at minimum cost. In this case, the model reduces to one in which each generator is supplied with fuel at a piecewise-constant increasing marginal cost, independent of the actions of the other generators.

For large Z , the corresponding dual multipliers $\omega = 0$ at optimality. Recall that for generator i , $F_i \subseteq F$ is the set of fuel supply nodes g such that there exists an arc from g to i and let $m_i = |F_i|$. The fuel dispatcher's cost minimization problem can be decomposed into a set of such problems, one for each generator. The primal problem for generator i is:

$$\begin{aligned} \text{FDPI} \quad & \min_{\{x_{gi}:g \in F_i\}} \sum_{g \in F_i} c_{gi} x_{gi} \\ \text{s.t.} \quad & \sum_{g \in F_i} x_{gi} = y_i \quad [\pi_i] \\ & -x_{gi} \geq -U_{gi}, \forall g \in F_i \quad [\rho_{gi} \geq 0] \\ & x_{gi} \geq 0, \forall g \in F_i \end{aligned}$$

The corresponding dual problem is:

$$\begin{aligned} \text{FDDi} \quad & \max_{\pi_i, \{\rho_{gi}:g \in F_i\}} y_i \pi_i - \sum_{g \in F_i} U_{gi} \rho_{gi} \\ \text{s.t.} \quad & \pi_i - \rho_{gi} \leq c_{gi}, \forall g \in F_i \\ & \rho_{gi} \geq 0, \forall g \in F_i \end{aligned}$$

From duality and complementary slackness, at optimality the values of the primal and dual objective functions are equal and both $x_{gi} \perp (c_{gi} + \rho_{gi} - \pi_i)$ and $\rho_{gi} \perp (U_{gi} - x_{gi})$ hold for all $g \in F_i$. Let the arcs in $\{g_i | g \in F_i\}$ be arranged in order of increasing cost, indexed by h and, given y_i , let k_i be the smallest index h such that $\sum_{h=1}^{k_i} U_h > y_i$. Then at optimality, $x_h = U_h$ for $h < k_i$ and

$$y_i \pi_i = \sum_{h=1}^{k_i} c_h x_h + \sum_{h=1}^{m_i} \rho_h U_h. \quad (14)$$

Equation (14) includes not only the transportation costs but also a congestion charge. If generators were to be charged this amount, whenever a fuel supply line would become saturated, the fuel price for the entire demand steps up to the price of the next cheapest available fuel line. This would cause the generators' cost functions to be discontinuous. In the simultaneous game this issue is not encountered as the generators only consider a fixed marginal cost of fuel; they would not anticipate how their actions may affect their marginal cost. However, if the game were structured such that the fuel dispatch were acting as a Stackelberg follower, the conjectural variations would be different and some generator might have incentive to influence their marginal cost. Hence, any equilibrium found here would not necessarily be the same as in the simultaneous game.

This situation, where the structure of the game can affect the equilibrium, can be avoided by using the cost mechanism introduced in section II. In the cost structure that we use,

treat the fuel network problem as though it were a pay-as-bid supply function; i.e., the cost for a generator is:

$$\sum_{h=1}^{k_i} c_h x_h = y_i \pi_i - \sum_{h=1}^{m_i} \rho_h U_h. \quad (15)$$

Under this cost structure, the cost of fuel is a continuous increasing function of the generation quantity.

For the game where the fuel dispatcher and generators act simultaneously, we define a new generator profit maximization problem, GiP-B', which is identical to GiP-B except that the cost portion of the objective is replaced by equation (15). Note that in this new objective, the only non-constant term is $y_i \pi_i$, hence the conjectural variations of the game consisting of FDD, ISOD and $\{\text{GiP-B}, i \in N\}$ are identical to those of the game consisting of FDD, ISOD and $\{\text{GiP-B}', i \in N\}$. Therefore the result of Theorem 1 is also valid for this modified game.

For the situation where the fuel dispatcher is treated as a Stackelberg follower, the minimum cost fuel dispatch is incorporated into the generators' problems. We let γ_{ik} be the cost of the k^{th} least expensive fuel arc directed into node i and Υ_{ik} be its capacity. Let w_{ik} be the amount of electricity that generator i produces from fuel obtained via the k^{th} cheapest arc and $w^i = (w_{i1}, \dots, w_{i,m_i})^T$. Then the profit maximization problem for generator i including fuel acquisition (assuming full rationality with respect to the ISO decisions) is:

$$\begin{aligned} \text{FGiP-F} \quad & \max_{w^i} \sum_{k=1}^{m_i} (p_i - \gamma_{ik}) w_{ik} \\ \text{s.t.} \quad & \text{Equations (1) - (6)} \\ & e^T w^i \leq V_i \quad [\mu_i \geq 0] \\ & -w_{ik} \leq 0, k = 1, \dots, m_i \quad [\rho_{ik}^- \geq 0] \\ & w_{ik} \leq \Upsilon_{ik}, k = 1, \dots, m_i \quad [\rho_{ik}^+ \geq 0] \end{aligned}$$

The corresponding problem assuming bounded rationality is:

$$\begin{aligned} \text{FGiP-B} \quad & \max_{w^i, \eta} \sum_{k=1}^{m_i} (\eta + \phi_i - \gamma_{ik}) w_{ik} \\ \text{s.t.} \quad & e^T w^i - \eta \sum_{j \in E} \frac{1}{b_j} = \\ & \sum_{j \in E} \frac{\phi_j - a_j}{b_j} - \sum_{j \neq i} y_j \quad [\beta_i] \\ & e^T w^i \leq V_i \quad [\mu_i \geq 0] \\ & -w_{ik} \leq 0, k = 1, \dots, m_i \quad [\rho_{ik}^- \geq 0] \\ & w_{ik} \leq \Upsilon_{ik}, k = 1, \dots, m_i \quad [\rho_{ik}^+ \geq 0] \end{aligned}$$

In the case of unconstrained fuel supplies, to analyze small instances it is more convenient to consider the game consisting of $\{\text{FGiP-B}, i \in N\}$ and ISOD. The following result in combination with Theorem 1 shows that an equilibrium exists for this game as well. Moreover, it follows that if the equilibrium for the FGiP-B game is unique, as is the case in the instances studied in Section V, then the equilibrium for the GiP-B game is unique also.

Theorem 2: Suppose $\mathcal{E} = \{x^*, y^*, \eta^*, \pi^*, \phi^*\}$ is an equilibrium in the game consisting of FDD, ISOD and $\{\text{GiP-B}', i \in N\}$. Then setting $w = x^*$ and retaining $\{y^*, \eta^*, \phi^*\}$ from \mathcal{E} constitutes an equilibrium in the game consisting of ISOD and $\{\text{FGiP-B}, i \in N\}$.

Proof: See Appendix B. ■

In the game that consists of ISOD and $\{\text{FGiP-B}, i \in N\}$, where the upstream fuel decisions are included in the generator optimization problems, conditions (7) – (13) are replaced by the corresponding conditions derived from problems $\{\text{FGiP-B}, i \in N\}$. The equivalent set of KKT conditions is given by:

$$\eta + \phi_i - \gamma_{ik} - \beta_i - \mu_i + \rho_{ik}^- - \rho_{ik}^+ = 0, i \in N; \quad k = 1, \dots, m_i \quad (16)$$

$$\sum_{k=1}^{m_i} w_{ik} + \beta_i \left(\sum_{j \in N} \frac{1}{b_j} \right) = 0, i \in N \quad (17)$$

$$0 \leq \rho_{ik}^- \perp w_{ik} \geq 0, i \in N; \quad k = 1, \dots, m_i \quad (18)$$

$$0 \leq \rho_{ik}^+ \perp \Upsilon_{ik} - w_{ik} \geq 0, i \in N; \quad k = 1, \dots, m_i \quad (19)$$

$$0 \leq \mu_i \perp V_i - \sum_{k=1}^{m_i} w_{ik} \geq 0, i \in N \quad (20)$$

In certain small instances, it may be possible to identify an equilibrium for the games where the generators have full rationality, but only by an *ad hoc* procedure.

V. PARADOXICAL EXAMPLES

In this section we assume fuel supplies are unconstrained and compare equilibria under the two rationality assumptions when the generators optimize their procurement of fuel. To focus on the total welfare impacts of fuel transportation and electricity transmission capacities, we also assume the generation capacities, V_i , are large. To examine the possible effects of increasing fuel transportation or electricity transmission capacities, we analyze the generator best responses under bounded rationality in a two-node system.

Suppose $N = \{1, 2\}$, $m_1 = m_2 = 2$, and the electricity nodes are connected by a single line. Let $f = -r_1 = r_2$. The surplus of generator i is

$$PS_i = (\eta + \phi_i)(w_{i1} + w_{i2}) - \gamma_{i1}w_{i1} - \gamma_{i2}w_{i2},$$

while the consumer surpluses at the two nodes are

$$CS_1 = -\frac{b_1}{2}(w_{11} + w_{12} - f)^2, \quad CS_2 = -\frac{b_2}{2}(w_{21} + w_{22} + f)^2.$$

The transmission rents are

$$TR = (\phi_2 - \phi_1)f$$

and the total welfare is

$$TW \equiv PS_1 + PS_2 + CS_1 + CS_2 + TR.$$

Suppose that both Υ_{12} and Υ_{21} are large numbers, so that the cost for generator 1 is $\gamma_{11}w_{11} + \gamma_{12}w_{12}$ and that for generator 2 is $\gamma_{21}w_{21}$. For simplicity, drop the constraints that $w_{i1} \geq 0, i = 1, 2$; and let $\rho_{i1} \equiv \rho_{i1}^+$ and $\rho_{i2} \equiv \rho_{i2}^-$. Let node 1 be the reference. Then the simplified set of necessary

conditions for an equilibrium to the game consisting of ISOD, FG1P-B and FG2P-B is:

$$a_1 + b_1(w_{11} + w_{12} - f) - \eta = 0 \quad (21)$$

$$a_2 + b_2(w_{21} + t) - \eta - \phi_2 = 0 \quad (22)$$

$$-\lambda_1^+ + \lambda_1^- = 0 \quad (23)$$

$$\phi_2 - \lambda_2^+ + \lambda_2^- = 0 \quad (24)$$

$$0 \leq \lambda_1^+ \perp K + f \geq 0 \quad (25)$$

$$0 \leq \lambda_1^- \perp K - f \geq 0 \quad (26)$$

$$0 \leq \lambda_2^+ \perp K - f \geq 0 \quad (27)$$

$$0 \leq \lambda_2^- \perp K + f \geq 0 \quad (28)$$

$$\eta - \gamma_{11} + \frac{b_1 b_2}{b_1 + b_2}(w_{11} + w_{12}) - \rho_{11} = 0 \quad (29)$$

$$\eta - \gamma_{12} + \frac{b_1 b_2}{b_1 + b_2}(w_{11} + w_{12}) + \rho_{12} = 0 \quad (30)$$

$$\eta + \phi_2 - \gamma_{21} + \frac{b_1 b_2}{b_1 + b_2}w_{21} = 0 \quad (31)$$

$$0 \leq \rho_{11} \perp \Upsilon_{11} - w_{11} \geq 0 \quad (32)$$

$$0 \leq \rho_{12} \perp w_{12} \geq 0 \quad (33)$$

The optimization problem of generator 2 is:

$$\begin{aligned} \text{FG2P-B} \quad & \max_{w_{21} \geq 0, \eta} (\eta + \phi_2 - \gamma_{21})w_{21} \\ \text{s.t.} \quad & w_{21} - \eta \frac{b_1 + b_2}{b_1 b_2} = \frac{-a_1}{b_1} + \frac{\phi_2 - a_2}{b_2} - y_1 \end{aligned}$$

Upon substitution of the constraint, the objective function is a quadratic function of $y_2 = w_{21}$ that is maximized by

$$y_2 = -\frac{y_1}{2} - \frac{a_1 b_2 + a_2 b_1 + b_2 \phi_2}{2b_1 b_2} + \frac{b_1 + b_2}{2b_1 b_2} \gamma_{21}.$$

That is, the best response of generator 2 given y_1 is linear in y_1 with slope $-\frac{1}{2}$.

Similarly, the optimal generation quantity for FG1P-B satisfies

$$y_1 = -\frac{y_2}{2} - \frac{a_1 b_2 + a_2 b_1 - b_1 \phi_2}{2b_1 b_2} + \frac{b_1 + b_2}{2b_1 b_2} \gamma_{11},$$

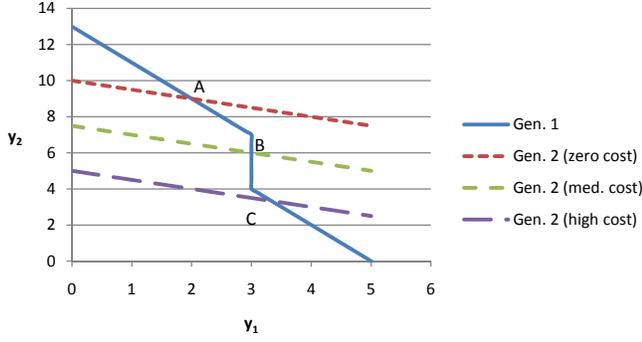
if this quantity is less than Υ_{11} ;

$$y_1 = -\frac{y_2}{2} - \frac{a_1 b_2 + a_2 b_1 - b_1 \phi_2}{2b_1 b_2} + \frac{b_1 + b_2}{2b_1 b_2} \gamma_{12},$$

if this quantity is greater than Υ_{11} ; and $y_1 = \Upsilon_{11}$ otherwise. That is, when plotted with y_2 on the vertical axis, the best response of generator 1 has slope -2 to the left and right of Υ_{11} and a vertical segment at $y_1 = \Upsilon_{11}$. From the slopes of these best response functions, it is clear that the equilibrium in (y_1, y_2) , where the best responses intersect, is unique.

Example 1. Let $p_1 = 20 - 20q_1$, $p_2 = 20 - \frac{20}{19}q_2$, $\gamma_{11} = 7$, $\Upsilon_{11} = 3$, $\gamma_{12} = 10$. Figure 2 illustrates the best responses for $\gamma_{21} = 0, 5$ or 10, with K large.

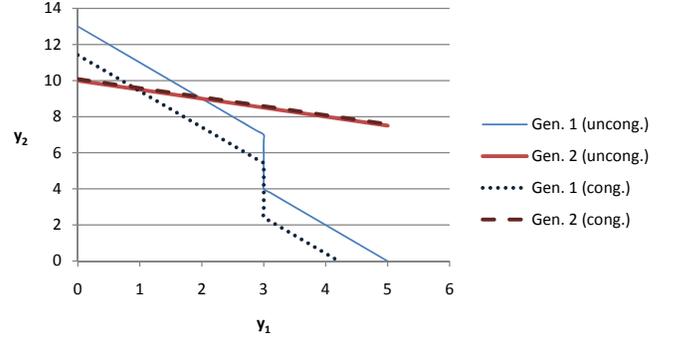
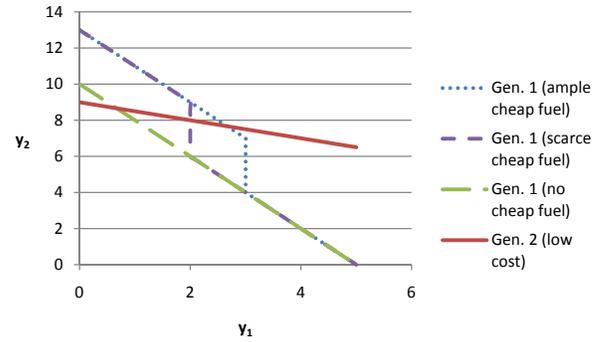
The equilibrium prices and transmission quantity, as well as the generation quantities, can be identified in any instance by solving equations (21), (22), (29) – (31) for five unknowns. If K is large, then all the dual variables for the transmission constraints (and therefore, the price premium, ϕ_2) equal zero. The uncongested equilibrium has $y_1 < \Upsilon_{11}$, such as point A in Figure 2, if the solution to the system with $w_{12} = \rho_{11} = 0$ has $w_{11} < \Upsilon_{11}$. It has $y_1 = \Upsilon_{11}$, as with point B in Figure

Fig. 2. Bounded rationality best responses for varying γ_{21} for Example 1.

2, if the solution $w_{11} = \Upsilon_{11}$ and $w_{12} = 0$ has nonnegative values for ρ_{11} and ρ_{12} . Or, $y_1 > \Upsilon_{11}$ in equilibrium, such as point C in the same figure, if setting $w_{11} = \Upsilon_{11}$ and $\rho_{12} = 0$ yields a solution with $w_{12} > 0$. For smaller values of K , the congested equilibrium falls into one of the three above cases for y_1 versus Υ_{11} with either $f = K$ and $\phi_2 = \lambda_2^+$ or $f = -K$ and $\phi_2 = -\lambda_2^-$. With K fixed, the welfare measures are quadratic in Υ_{11} over ranges where $y_1 = \Upsilon_{11}$ in equilibrium; with Υ_{11} fixed, the welfare measures are piecewise quadratic in K .

A. Effect of Transmission Capacity Increase

If power is flowing from node 1 to node 2 and the transmission capacity, K , is reduced, then the generator best response functions shift as shown in Figure 3 for Example 1. For this instance, Table V shows the unique equilibria that occur under the two rationality assumptions for different values of K . Under congestion, the shift in generation from the lower cost generator 2 to the higher cost generator 1 as K increases results in a reduction in the overall total welfare despite consumers being better off. Total welfare decreases from 149.12 when $K = 0$ to 145.5 when the transmission line is uncongested. In the full rationality version of the model, equilibria also are unique for large K and $K = 0$, and the single-node equilibrium (for large K) coincides with the bounded rationality equilibrium. When $K = 0$, the fully rational generators act as monopolists rather than duopolists as in the bounded rationality version, and the total welfare is 145.67, i.e., larger than the total welfare under full rationality when K is large. Thus, in this instance, increasing transmission capacity reduces the total welfare under both rationality assumptions. A recent study of the Belgian electricity market [27] tentatively concluded that increasing transmission capacity between that country and Germany or the UK, both of which have higher generation costs, could decrease market concentration in Belgium. Our example suggests that, although consumers' surplus in Belgium would increase, this might be outweighed by a loss in producer welfare incurred by replacing inexpensive local generation by more expensive imported power. Moreover, the combined total welfare of both Belgium and the exporting country (either Germany or the UK) could decrease with the added capacity.

Fig. 3. Bounded rationality best responses for $K \geq 1.45$ (uncongested) and $K = 1.0$ (congested) in Example 1.Fig. 4. Bounded rationality best responses for varying Υ_{11} .

Example 2. Paradoxical effects of increasing transmission capacity also can occur when demand is symmetric. Table VI illustrates an instance with $a_1 = a_2 = 100$, $b_1 = b_2 = -2$, $\gamma_{11} = 25$, $\gamma_{12} = 40$, $\Upsilon_{11} = 25$, $\gamma_{21} = 30$. Total welfare decreases in the bounded rationality model from 2338.88 when $K = 0$ to 2321.88 when K is large. Here, the duopoly equilibrium for $K = 0$ has $y_1 = \Upsilon_{11}$. As K increases, the incentive for the lower cost generator 1 to increase production is not enough to outweigh the jump in its marginal cost.

B. Effect of Fuel Transportation Capacity Increase

Again, suppose Υ_{12} and Υ_{21} are large numbers. In Example 1 with $\gamma_{21} = 0$, reducing Υ_{11} shifts the uncongested best response of generator 1 as shown in Figure 4. Restricting the availability of low-cost fuel to generator 1 reduces its generation and increases production by generator 2. Table VII shows that with $K \geq 1.45$, under full rationality, the total welfare equals 150 when $\Upsilon_{11} = 0$ and drops to 145.5 if $\Upsilon_{11} \geq 2$. The full rationality equilibrium is unique in both cases and each coincides with the corresponding equilibrium under bounded rationality. Thus, in this instance, increasing capacity of low-cost fuel transportation reduces total welfare under both assumptions about rationality with respect to the ISO decisions.

Example 3. The impact on total welfare of increasing availability of low-cost fuel also may differ depending on

TABLE V

EQUILIBRIA FOR EXAMPLE 1 WITH $\gamma_{21} = 0$ UNDER DIFFERENT GENERATOR RATIONALITY ASSUMPTIONS. FOR INTERMEDIATE VALUES OF K , WELFARE MEASURES ARE GIVEN AS COEFFICIENTS (a, b, c) OF $aK^2 + bK + c$.

K	Gen. Rat.	y_1	y_2	f	PS	CS	TR	TW
0	Full	0.25	9.50	0.00	97.11	48.56	0.00	145.67
0	Bdd.	0.62	9.74	0.00	95.32	53.80	0.00	149.12
$(0, 1.45)$	Bdd.	$0.62 + 0.95K$	$9.74 - 0.51K$	K	1.17, -8.81, 95.32	0.15, 4.41, 53.80	-1.47, 2.12, 0.00	-0.15, -2.28, 149.12
$[1.45, \infty)$	Bdd., Full	2.00	9.00	1.45	85.00	60.50	0.00	145.50

TABLE VI

EQUILIBRIA FOR EXAMPLE 2 UNDER BOUNDED RATIONALITY. FOR INTERMEDIATE VALUES OF K , WELFARE MEASURES ARE GIVEN AS COEFFICIENTS (a, b, c) OF $aK^2 + bK + c$.

K	Gen. Rat.	y_1	y_2	f	PS	CS	TR	TW
0	Bdd.	25.00	23.33	0.00	1169.44	1169.44	0.00	2338.89
$(0, 1.25)$	Bdd.	25.00	$23.33 - 0.67K$	K	0.44, 18.89, 1169.44	1.11, -34.44, 1169.44	-2.67, 3.33, 0	-1.11, -12.22, 2338.89
$[1.25, \infty)$	Bdd.	25.00	22.50	1.25	1193.75	1128.13	0.00	2321.88

TABLE VII

EQUILIBRIA FOR EXAMPLE 1 WITH $\gamma_{21} = 0$, $K \geq 1.45$ UNDER DIFFERENT GENERATOR RATIONALITY ASSUMPTIONS. FOR INTERMEDIATE VALUES OF Υ_{11} , WELFARE MEASURES ARE GIVEN AS COEFFICIENTS (a, b, c) OF $a\Upsilon_{11}^2 + b\Upsilon_{11} + c$.

Υ_{11}	Gen. Rat.	y_1	y_2	f	PS	CS	TR	TW
0	Full, Bdd.	0.00	10.00	-0.50	100.00	50.00	0.00	150.00
$(0, 2)$	Bdd.	Υ_{11}	$10.00 - 0.5\Upsilon_{11}$	$-0.5 + 0.975\Upsilon_{11}$	-0.25, -7, 100	0.125, 5, 50	0.00	-0.125, -2, 150
$[2, \infty)$	Full, Bdd.	2.00	9.00	1.45	85.00	60.50	0.00	145.50

generator rationality with respect to the ISO decisions. In the instance where $a_1 = 2, a_2 = 5, b_1 = b_2 = -1, \gamma_{11} = 0.5, \gamma_{12} = 1.24, \gamma_{21} = 0.5$, unique equilibria are found under both rationality assumptions for $\Upsilon_{11} = 0$ and $\Upsilon_{11} \geq \frac{5}{3}$. With transmission capacity $K = 1$ under bounded rationality, this increase in Υ_{11} results in a move from an uncongested equilibrium with total welfare 8.946 to a duopoly equilibrium under congestion with total welfare 10.222. But with $K = 1$ in the full rationality version, the same increase in Υ_{11} shifts the equilibrium from the same uncongested one as under bounded rationality to a new point with separate monopolies at the two nodes and total welfare reduced to 8.937. See Table VIII. This occurs because the fully rational generators act strategically with respect to the transmission line capacity.

Example 4. Paradoxical effects of increasing low cost fuel are not restricted to two-node examples. We also have found a three-node instance where under bounded rationality, increasing the availability of low-cost fuel causes a shift from one congested equilibrium to another with lower total welfare. As in Figure 1, generators are located at nodes 1 and 2 while demands exist at nodes 2 and 3. Transmission lines having the same reactance connect each pair of nodes. The data are $a_2 = 100, b_2 = -1000, a_3 = 100, b_3 = -1, \gamma_{21} = 22, \gamma_{22} = 25$, and $\gamma_{11} = 20$ with Υ_{11} large. Transmission capacity is ample between node 1 and the other nodes but limited to 24.9 between nodes 2 and 3. Increasing Υ_{21} from 23 to 24 in the bounded rationality model causes a shift from a congested equilibrium with total welfare 2750.5 to another with total welfare 2735.8 (see Table IX). Increasing the supply of low-cost fuel to generator 2 causes an increase in production there that is offset by a drop in price, so that its surplus as well as that of generator 1 and the consumers at node 3 all decrease. The biggest beneficiary of the drop in the fuel price is the transmission owner, who collects higher rents due to the wider

price spreads. In this instance, an infinite number of equilibria occur under full rationality for both values of Υ_{21} .

In these examples, the concave quadratic form for total welfare provides insight into the effect of capacity uncertainty on the total welfare. Let X represent either K or Υ_{11} and suppose it has mean μ_X and variance σ_X^2 . Then $TW(X) = aX^2 + bX + c$ implies $E[TW(X)] = a\sigma_X^2 + \mu_X^2 + b\mu_X + c$, so that when $a, b < 0$, both $\frac{\partial E[TW(X)]}{\partial \mu_X}$ and $\frac{\partial E[TW(X)]}{\partial \sigma_X}$ are negative; i.e., the expected total welfare decreases with both the mean and variance of the arc capacity. This suggests that unreliability of the added or expanded link could exacerbate the paradoxical behavior.

VI. CONCLUSION

We developed a mathematical equilibrium model that integrates the supply and transportation of fuels with the strategic decisions of generators and the market-balancing function of the ISO over a congested electricity network with price-sensitive demands. Although it approximates generators as Cournot competitors having bounded rationality with respect to ISO decisions, a previous study [13] showed that it yields electricity prices and quantities that are similar to results obtained from a more detailed agent-based simulation in which generators submit supply function bids with learning. In this paper, we proved the existence of an equilibrium under two formulations: one in which a fuel dispatcher delivers fuel at minimum cost and its dual prices act as cost signals to the generators; and another for the case of unlimited fuel supplies in which the generators make fuel acquisitions in light of the true costs. However, we found some instances in which expanding capacity of either electricity transmission or the transportation of low-cost fuel can harm the total welfare under bounded or full generator rationality or both. While the model appears useful for planning purposes, more

TABLE VIII
EQUILIBRIA FOR EXAMPLE 3 WITH $K = 1$ UNDER DIFFERENT GENERATOR RATIONALITY ASSUMPTIONS. FOR INTERMEDIATE VALUES OF Υ_{11} , WELFARE MEASURES ARE GIVEN AS COEFFICIENTS (a, b, c) OF $a\Upsilon_{11}^2 + b\Upsilon_{11} + c$.

Υ_{11}	Gen. Rat.	y_1	y_2	f	PS	CS	TR	TW
0	Full, Bdd.	1.013	2.493	0.760	3.622	5.324	0.000	8.946
(0, 1.013)	Bdd.	1.013	2.493	0.760	0, 0.740, 3.622	0, 0, 5.324	0.000	0, 0.740, 8.946
[1.013, 1.333]	Bdd.	Υ_{11}	$3.000 - 0.500\Upsilon_{11}$	$0.750\Upsilon_{11}$	-0.125, 0, 4.5	0.0625, 0.75, 4.5	0.000	-0.0625, 0.75, 9.0
[1.333, 1.667]	Bdd.	Υ_{11}	2.333	1.000	-1, 2.5, 2.722	0.5, -1, 6.056	0, 1, -1.333	-0.5, 2.5, 7.444
[1.667, ∞)	Bdd.	1.667	2.333	1.000	4.111	5.778	0.333	10.220
[1.250, ∞)	Full	1.250	1.750	1.000	4.625	3.812	0.500	8.937

TABLE IX
EQUILIBRIUM QUANTITIES AND PRICES UNDER BOUNDED RATIONALITY FOR EXAMPLE 4.

Υ_{21}	y_1	y_2	f_{12}	f_{13}	f_{23}	p_1	p_2	p_3	PS ₁	PS ₂	CS ₂	CS ₃	TR	TW
23	28.362	23.221	1.731	26.631	24.900	48.333	48.198	48.469	803.6	607.7	1.3	1327.7	10.1	2750.5
24	27.362	23.723	1.231	26.131	24.900	47.334	45.700	48.969	747.9	562.2	1.5	1302.1	122.1	2735.8

investigation into the reasons behind paradoxical effects of infrastructure expansions is warranted.

APPENDIX A EXISTENCE OF EQUILIBRIA

First we show that, in the GiP-B problem, the decision variables lie in a closed and bounded set. The problem can be restated as:

$$\begin{aligned} \text{GiP-B} \quad & \max_{y_i \geq 0} (\eta(y) + \phi_i - \pi_i)y_i \\ \text{s.t.} \quad & y_i \leq V_i \quad [\mu_i \geq 0] \end{aligned}$$

where

$$\eta(y) = \frac{y_i + \sum_{j \neq i} y_j - \sum_{j=1}^{n_E} \frac{\phi_j - a_j}{b_j}}{\sum_{j=1}^{n_E} \frac{1}{b_j}}$$

Let $\mathcal{V} \equiv \prod_{i \in N} [0, V_i]$ be the Cartesian product of all generators' feasible injections.

Lemma 3: The decision variable of generator i in GiP-B lies in a compact set.

Proof: As y_i is bounded from above and below, it clearly lies in a compact set. ■

Next, consider the ISO optimization problem. First, note that both ISOP and its Dorn dual, ISOD, are feasible for any y . Also note that, as M is negative definite and r is not in the objective of ISOP, the pair ISOP and ISOD possess strong duality. This means that they both have optimal solutions of the same objective value. First we will show that any q in an optimal solution to ISOP lies in a compact set independent of y .

Lemma 4: Consider ISOP for some vector of injections $y \in \mathcal{V}$. For any optimal solution, $\{\bar{q}, \bar{r}\}$, \bar{q} must lie in a compact set, \mathcal{Q} , which is independent of y .

Proof: For some vector $y \in \mathcal{V}$, ISOP has a feasible solution $\{q, r\} = \{y, 0\}$. As ISOP is a maximization problem, if an optimal solution to ISOP is $\{q, r\} = \{\bar{q}, \bar{r}\}$, this yields the following inequality:

$$\begin{aligned} \frac{1}{2}\bar{q}^T M \bar{q} + a^T \bar{q} & \geq \frac{1}{2}y^T M y + a^T y \\ & \geq \inf_{y \in \mathcal{V}} \left\{ \frac{1}{2}y^T M y + a^T y \right\}. \end{aligned}$$

Because M is negative definite, this inequality defines a compact set, $\mathcal{Q} \equiv$

$$\left\{ q : \frac{1}{2}q^T M q + a^T q \geq \inf_{y \in \mathcal{V}} \left\{ \frac{1}{2}y^T M y + a^T y \right\} \right\},$$

independent of y . ■

In the game, the generators make their decisions with knowledge of the ISO dual variables. Thus, we show that the prices as well as quantities found in ISOD also lie in a compact set that is independent of the generator decisions.

Lemma 5: Consider ISOD for some vector of injections $y \in \mathcal{V}$. For any optimal solution, $\{\bar{q}, \bar{p}, \bar{\eta}, \bar{\lambda}^+, \bar{\lambda}^-\}$, $\bar{q}, \bar{p}, \bar{\eta}, \bar{\lambda}^+$ and $\bar{\lambda}^-$ must lie in a compact set, which is independent of y .

Proof: As ISOD is the Dorn dual of ISOP, an optimal solution to ISOD defines an optimal solution for ISOP; hence, we know from Lemma 4 that imposing the constraint $\bar{q} \in \mathcal{Q}$ will not affect the solution set of ISOD. From the first constraint of ISOD, we have that p is a linear function of q ; hence, \bar{p} lies in a compact set independent of y .

For any $y \in \mathcal{V}$, ISOD has a feasible solution $\{q, p, \eta, \lambda^+, \lambda^-\} = \{-M^{-1}a, 0, 0, 0, 0\}$. As ISOD is a minimization problem, if an optimal solution to ISOD is $\{\bar{q}, \bar{p}, \bar{\eta}, \bar{\lambda}^+, \bar{\lambda}^-\}$, then the following inequality is valid:

$$y^T \bar{p} + K^T (\bar{\lambda}^+ + \bar{\lambda}^-) - \frac{1}{2}\bar{q}^T M \bar{q} \leq -\frac{1}{2}a^T M^{-1}a;$$

thus,

$$K^T (\bar{\lambda}^+ + \bar{\lambda}^-) \leq \frac{1}{2}\bar{q}^T M \bar{q} - y^T (a + M\bar{q}) - \frac{1}{2}a^T M^{-1}a.$$

As M is negative definite, we know that for a fixed y ,

$$\max_q \left\{ \frac{1}{2}q^T M q - y^T (a + Mq) \right\} = \frac{1}{2}y^T M y - y^T (a + My).$$

Hence,

$$\begin{aligned} K^T (\bar{\lambda}^+ + \bar{\lambda}^-) & \leq \frac{1}{2}y^T M y - y^T (a + My) - \frac{1}{2}a^T M^{-1}a \\ & \leq \sup_{y \in \mathcal{V}} \left\{ \frac{1}{2}y^T M y - y^T (a + My) - \frac{1}{2}a^T M^{-1}a \right\} = \theta. \end{aligned}$$

As $K > 0$ and ISOD requires that $\lambda^+, \lambda^- \geq 0$, we therefore have an upper bound on every element of λ^+ and λ^- :

$$0e \leq \bar{\lambda}^+, \bar{\lambda}^- \leq \frac{\theta}{\min_{l \in B} K_l} e.$$

These lie in compact sets independent of y . Finally, as η is a linear function of λ^+ , λ^- and p , it follows that $\bar{\eta}$ must lie in a compact set, independent of y . ■

Finally we consider the fuel dispatch problem and introduce the following bounds on the fuel dispatcher's (FDD) decision variables:

$$\omega_g, \rho_{gj}, \pi_j \leq \max_{gj \in A} \{c_{gj}\} + \max_{i \in N} \{a_i\} + \epsilon, \quad (34)$$

where $\epsilon > 0$. As all players' decision variables now lie in a compact set, we can invoke Rosen's theorem in [23] which proves that there exists an equilibrium for this game. Denoting equilibrium quantities by $*$, we will show that

$$\omega_g^*, \rho_{gj}^*, \pi_j^* < \max_{gj \in A} \{c_{gj}\} + \max_{i \in N} \{a_i\} + \epsilon.$$

Lemma 6: Consider the fuel dispatch problem FDP and its dual FDD; if x_{gj}^* is strictly positive at equilibrium then $\omega_g^*, \rho_{gj}^*, \pi_j^* \leq \max_{i \in N} \{a_i\}$.

Proof: Suppose $x_{gj}^* > 0$. Then as the FDD constraint is orthogonal to x we have

$$\pi_j^* = c_{gj} + \rho_{gj}^* + \omega_g^*. \quad (35)$$

For this to be an equilibrium point, FDP must be optimal; hence,

$$y_j^* = \sum_{g \in F} x_{gj}^* > 0. \quad (36)$$

Note that from GiP-B we have that

$$y_j (\pi_j) |_{\pi_j = \max_{i \in N} \{a_i\}} = 0, \quad (37)$$

where $\max_{i \in N} \{a_i\}$ is the maximum possible nodal price. We also note that the optimal y_i is non-increasing in π_i . This together with (36) and (37) imply

$$\pi_j^* \leq \max_{i \in N} \{a_i\}. \quad (38)$$

Finally, because c , ω and ρ are non-negative, (35) and (38) imply

$$\omega_g^* \leq \max_{i \in N} \{a_i\} \quad \text{and} \quad \rho_{gj}^* \leq \max_{i \in N} \{a_i\},$$

as required. ■

In the following theorem, we show that no equilibrium can exist at the imposed bounds.

Theorem 7: Suppose we have an equilibrium to the game consisting of ISOD, FDD and GiP-B, $i \in N$. At the equilibrium ω_g^* , ρ_{gj}^* , and π_j^* are all strictly less than the imposed bounds given by (34).

Proof: Here we will find true bounds on the equilibrium quantities on the fuel dispatcher.

The bound on ω_g^* is found as follows: either node g is such that $\exists j | x_{gj}^* > 0$, or $x_{gj}^* = 0 (\forall j)$; in the former case, from Lemma 6 we have

$$\omega_g^* \leq \max_{i \in N} \{a_i\},$$

while in the latter, as $0 \leq \omega_g \perp Z_g - \sum_j x_{gj} \geq 0$ and $Z_g > 0$, it follows that

$$\omega_g^* = 0.$$

Hence, $\omega_g^* \leq \max_{i \in N} \{a_i\} (\forall g)$.

The bound on ρ_{gj}^* is found as follows: either $x_{gj}^* > 0$ or $x_{gj}^* = 0$; if $x_{gj}^* > 0$ then from Lemma 6

$$\rho_{gj}^* \leq \max_{i \in N} \{a_i\},$$

or if $x_{gj}^* = 0$, as $0 \leq \rho_{gj} \perp U_{gj} - x_{gj} \geq 0$ and $U_{gj} > 0$ then

$$\rho_{gj}^* = 0.$$

Therefore, $\rho_{gj}^* \leq \max_{i \in N} \{a_i\} (\forall gj)$.

To find a bound on π_j , we know that either $y_j^* > 0$ or $y_j^* = 0$. In the former case, from Lemma 6,

$$\pi_j^* \leq \max_{i \in N} \{a_i\};$$

in the latter, as $\rho_{gj} = 0 (\forall g)$ and from FDD we know $\pi_j^* \leq c_{gj} + \rho_{gj}^* + \omega_g^*$; therefore,

$$\pi_j^* \leq \max_i \{c_{gj}\} + \max_{i \in N} \{a_i\},$$

and hence, $\pi_j^* \leq \max_{g \in F} \{c_{gj}\} + \max_{i \in N} \{a_i\} (\forall j)$. Therefore, at equilibrium we have,

$$\begin{aligned} \omega_g^* &\leq \max_{i \in N} \{a_i\}, \\ \rho_{gj}^* &\leq \max_{i \in N} \{a_i\}, \\ \pi_j^* &\leq \max_{g \in F} \{c_{gj}\} + \max_{i \in N} \{a_i\}. \end{aligned}$$

These inequalities imply

$$\omega_g^*, \rho_{gj}^*, \pi_j^* < \max_{gj \in A} \{c_{gj}\} + \max_{i \in N} \{a_i\} + \epsilon,$$

as required. ■

Here we have shown that there exist bounds for the fuel dispatcher's decision which would never be met at equilibrium. Following is the proof of Theorem 1:

Proof: The feasible region in each problem is defined by a set of linear constraints and, therefore, is convex. Moreover, each player's objective function is concave in its own decision variables and continuous in the other players' decision variables. Lemma 3 establishes that each generator's decision variables lie in a compact set, independent of ϕ and π . By Lemma 5, the optimal solution to ISOD is contained within some compact set independent of y . Therefore, artificial lower and upper bounds can be imposed on the ISOD variables whereby these bounds will never be active at an optimal solution for any $y \in \mathcal{V}$. This augmented problem has the same optimal solution as the original problem, but now satisfies the condition that its decision variables lie in a compact set. Theorem 7 states that artificial bounds on the fuel dispatcher decision variables render those decision sets compact but do not interfere with the equilibrium. The result follows from Theorem 1 in [23]. ■

APPENDIX B

UNLIMITED FUEL SUPPLIES

The proof of Theorem 2 is as follows:

Proof: As the ISOD problem is affected only by the value of y , it is clear that since \mathcal{E} is an equilibrium to the ISOD, FDPi, GiP-B' game, the ISOD problem will remain optimal.

Here we show that for each i , $\{w_{ik} = x_{ik}^*, k = 1, \dots, m_i\}$ and η^* are optimal in FGiP-B. Without loss of optimality, for a given value of y_i in FGiP-B, assign $w_{ik} = \Upsilon_{ik}$ for $k < s$, $w_{is} = y_i - \sum_{k=1}^{s-1} \Upsilon_{ik}$, and $w_{ik} = 0$ for $k > s$, where s is such that $\sum_{k=1}^{s-1} \Upsilon_{ik} \leq y_i < \sum_{k=1}^s \Upsilon_{ik}$; at the equilibrium point, t is the value of s that corresponds to y_i^* . From FDD it can be shown that if $w_{it}^* = 0$ then $\gamma_{i,t-1} \leq \pi_i^* \leq \gamma_{it}$; otherwise, $\pi_i^* = \gamma_{it}$.

The profit function for generator i in GiP-B', given $\pi_i = \pi_i^*$, can be written as

$$P_G(y) = (\eta(y) + \phi_i - \pi_i^*) y_i + \sum_{h=1}^{m_i} \rho_h U_h$$

We can substitute ρ_h out of the above equation as we know that $\rho_h = 0$ for $h \geq t$ and $\rho_h = \pi_i^* - c_h$ for $h < t$. This gives the following equation:

$$\begin{aligned} P_G(y) &= (\eta(y) + \phi_i - \pi_i^*) y_i \\ &\quad + \sum_{k=1}^{t-1} (\pi_i^* - c_h) U_h \\ &= (\eta(y) + \phi_i - \pi_i^*) \sum_{k=1}^{m_i} w_{ik} \\ &\quad + \sum_{k=1}^{t-1} (\pi_i^* - \gamma_{ik}) \Upsilon_{ik}. \end{aligned}$$

As \mathcal{E} is an equilibrium, this function is maximized at $y_i = y_i^*$. Now the profit function for generator i in FGiP-B is

$$P_F(y) = \sum_{k=1}^{m_i} (\eta(y) + \phi_i - \gamma_{ik}) w_{ik}.$$

We can see that both profit functions have the same value at the equilibrium point, \mathcal{E} .

$$\begin{aligned} P_G(y^*) &= (\eta(y^*) + \phi_i - \pi_i^*) \sum_{k=1}^{m_i} w_{ik}^* \\ &\quad + \sum_{k=1}^{t-1} (\pi_i^* - \gamma_{ik}) \Upsilon_{ik} \\ &= (\eta(y^*) + \phi_i - \pi_i^*) \sum_{k=1}^{m_i} w_{ik}^* \\ &\quad + \sum_{k=1}^{t-1} (\pi_i^* - \gamma_{ik}) w_{ik}^* \\ &= \sum_{k=1}^{m_i} (\eta(y^*) + \phi_i - \gamma_{ik}) w_{ik}^* \\ &= P_F(y^*). \end{aligned}$$

Now we will show that for any deviation from $w = x^*$ the profit from the GiP-B' problem will be greater than or equal to the profit from the FGiP-B problem. As the GiP-B problem is at a global maximum, by assumption, then FGiP-B must also be maximized at the same point.

Taking the difference of the two profit functions gives

$$P_G(y) - P_F(y) = \sum_{k=1}^{m_i} (\gamma_{ik} - \pi_i^*) w_{ik} + \sum_{k=1}^{t-1} (\pi_i^* - \gamma_{ik}) \Upsilon_{ik} \geq 0$$

Therefore for any ϵ ,

$$\begin{aligned} P_F(y^*) - P_F(y^* + \epsilon) &= \\ P_F(y^*) - P_G(y^* + \epsilon) + P_G(y^* + \epsilon) - P_F(y^* + \epsilon) &\geq \\ P_F(y^*) - P_G(y^*) + P_G(y^* + \epsilon) - P_F(y^* + \epsilon) &\geq 0 \end{aligned}$$

As all the players in the ISOD, FGiP-B game are optimal, we are at an equilibrium point. ■

ACKNOWLEDGMENT

This work was supported by the National Science Foundation under an International Research and Education in Engineering supplement to award 0527460. The first author thanks the Electric Power Optimization Centre in the Department of Engineering Science of the University of Auckland for its hospitality during an extended visit.

REFERENCES

- [1] K. Neuhoff, J. Barquin, M. G. Boots, A. Ehrenmann, B. F. Hobbs, F. A. M. Rijkers, and M. Vazquez, "Network-constrained models of liberalized electricity markets: The devil is in the details," *Energy Economics*, vol. 27, pp. 495–525, 2005.
- [2] Energy Information Administration, "Electric power annual 2007," January 2009, viewed online February, 2009, at http://www.eia.doe.gov/cneaf/electricity/epa/epa_sum.html.
- [3] A. Quelhas, E. Gil, J. D. McCalley, and S. M. Ryan, "A multiperiod generalized network flow model of the U.S. integrated energy system: Part I – model description," *IEEE Transactions on Power Systems*, vol. 22, no. 2, pp. 829–836, 2007.
- [4] A. Quelhas, E. Gil, and J. D. McCalley, "A multiperiod generalized network flow model of the U.S. integrated energy system: Part II – simulation results," *IEEE Transactions on Power Systems*, vol. 22, no. 2, pp. 837–844, 2007.
- [5] D. Matsypura, A. Nagurney, and Z. Liu, "Modeling of electric power supply chain networks with fuel suppliers via variational inequalities," *International Journal of Emerging Electric Power Systems*, vol. 8, no. 1, 2007, article 5.
- [6] Z. Liu and A. Nagurney, "An integrated electric power supply chain and fuel market network framework: Theoretical modeling with empirical analysis for New England," *Naval Research Logistics*, 2009, in press.
- [7] B. F. Hobbs, "Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets," *IEEE Transactions on Power Systems*, vol. 16, no. 2, pp. 194–202, 2001.
- [8] C. J. Day, B. F. Hobbs, and J.-S. Pang, "Oligopolistic competition in power networks: A conjectured supply function approach," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 597–607, 2002.
- [9] B. F. Hobbs and R. A. M. Rijkers, "Strategic generation with conjectured transmission price responses in a mixed transmission pricing system – part I: Formulation," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 707–717, 2004.
- [10] A. Ehrenmann, "Manifolds of multi-leader Cournot equilibria," *Operational Research Letters*, vol. 32, no. 2, pp. 121–125, 2004.
- [11] J. Barquín and M. Vázquez, "Cournot equilibrium calculation in power networks: An optimization approach with price response computation," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 317–326, 2008.
- [12] J. Yao, I. Adler, and S. S. Oren, "Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network," *Operations Research*, vol. 56, no. 1, pp. 34–47, 2008.
- [13] S. M. Ryan, "Market outcomes in a congested electricity system with fuel supply network," in *Proceedings of IEEE Power Engineering Society General Meeting*, July 2009.
- [14] E. E. Sauma and S. S. Oren, "Proactive planning and valuation of transmission investments in restructured electricity markets," *Journal of Regulatory Economics*, vol. 30, pp. 261–290, 2006.
- [15] D. Braess, "Über ein paradoxen der verkehrsplanung," *Unternehmensforschung*, vol. 12, pp. 258–268, 1968.
- [16] D. Braess, A. Nagurney, and T. Wakolbinger, "On a paradox of traffic planning," *Transportation Science*, vol. 39, no. 4, pp. 446–450, 2005.
- [17] H. Yang and M. G. H. Bell, "A capacity paradox in network design and how to avoid it," *Transportation Research, Part A*, vol. 32, no. 7, pp. 539–545, 1998.

- [18] J. H. Roh, M. Shahidepour, and Y. Fu, "Market-based coordination of transmission and generation capacity planning," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1406–1419, 2007.
- [19] J. H. Roh, M. Shahidepour, and L. Wu, "Market-based generation and transmission planning with uncertainties," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1587–1598, 2009.
- [20] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*. John Wiley & Sons, 1984.
- [21] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, *Nonlinear Programming Theory and Algorithms*, 2nd ed. John Wiley & Sons, 1993.
- [22] X. Hu and D. Ralph, "Using EPECs to model bilevel games in restructured electricity markets with locational prices," *Operations Research*, vol. 55, no. 5, pp. 809–827, 2007.
- [23] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n -person games," *Econometrica*, vol. 33, pp. 520 – 534, 1965.
- [24] M. C. Ferris and C. Kanzow, "Complementarity and related problems: A survey," November 1998, available in the archive of <http://www.cs.wisc.edu/cpnet/>.
- [25] S. P. Dirkse and M. C. Ferris, "The PATH solver: A non-monotone stabilization scheme for mixed complementarity problems," *Optimization Methods and Software*, vol. 5, pp. 123–156, 1995.
- [26] M. C. Ferris and T. S. Munson, "GAMS/PATH user guide," March 2000.
- [27] G. Küpper, E. Delarue, B. Delvaux, L. Meeus, D. Bekaert, B. Willems, S. Proost, W. D'haeseleer, K. Deketelaere, and R. Belmans, "Does more international transmission capacity increase competition in the Belgian electricity market?" *The Electricity Journal*, vol. 22, no. 1, pp. 21–36, 2009.

Sarah M. Ryan received her Ph.D. degree from The University of Michigan. She is currently Professor in the Department of Industrial and Manufacturing Systems Engineering at Iowa State University. Her research applies stochastic modeling and optimization to the planning and operation of service and manufacturing systems. Her work has been published in *IEEE Transactions on Power Systems*, *Operations Research*, *Management Science* and several other journals.

Anthony Downward is a Ph.D. candidate in the Department of Engineering Science at the University of Auckland. His main research interest is in the analysis of electricity markets, particularly the impact that transmission capacity has on the competitiveness of market participants.

Andy Philpott is a Professor in the Department of Engineering Science and Director of the Electric Power Optimization Centre at the University of Auckland. His research interests encompass optimization under uncertainty and game theory with particular application to electricity markets.

Golbon Zakeri received her Ph.D. from the University of Wisconsin - Madison. She is the co-founder of the Electric Power Optimization Centre (EPOC) and a Senior Lecturer in the Department of Engineering Science at the University of Auckland, Auckland, New Zealand. Her research interest is on analysis and optimization of problems that arise in the electricity sector, particularly those related to electricity markets. Her works have been published in *Operations Research*, *Mathematics of Operations Research*, *Mathematical Programming* and other journals.