2007

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One of the most important decisions regarding reverse logistics (RL) is whether to outsource such functions or not, due to the fact that RL does not represent a production or distribution firm's core activity. To explore the hypothesis that outsourcing RL functions is more suitable when returns are more variable, we formulate and analyse a Markov decision model of the outsourcing decision. The reward function includes capacity and operating costs of either performing RL functions internally or outsourcing them and the transitions among states reflect both the sequence of decisions taken and a simple characterization of the random pattern of returns over time. We identify sufficient conditions on the cost parameters and the return fraction that guarantee the existence of an optimal threshold policy for outsourcing. Under mild assumptions, this threshold is more likely to be crossed, the higher the uncertainty in returns. A numerical example illustrates the existence of an optimal threshold policy even when the sufficient conditions are not satisfied and shows how the threshold for outsourcing decreases while the probability of crossing any fixed threshold increases with the return fraction.

Keywords
Markov decision model, monotone policy, outsourcing, product life cycle, reverse logistics

Disciplines
Industrial Engineering | Systems Engineering

Comments
The Version of Record of this manuscript has been published and is available in International Journal of Production Research 2007, http://www.tandfonline.com/10.1080/00207540701450161. Posted with permission.

A Markov Decision Model to Evaluate Outsourcing in Reverse Logistics

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Abstract.

One of the most important decisions regarding reverse logistics (RL) is whether to outsource such functions or not, due to the fact that RL does not represent a production or distribution firm's core activity. To explore the hypothesis that outsourcing RL functions is more suitable when returns are more variable, we formulate and analyze a Markov decision model of the outsourcing decision. The reward function includes capacity and operating costs of either performing RL functions internally or outsourcing them, and the transitions among states reflect both the sequence of decisions taken and a simple characterization of the random pattern of returns over time. We identify sufficient conditions on the cost parameters and the return fraction that guarantee the existence of an optimal threshold policy for outsourcing. Under mild assumptions, this threshold is more likely to be crossed, the higher the uncertainty in returns. A numerical example illustrates the existence of an optimal threshold policy even when the sufficient conditions are not satisfied and shows how the threshold for outsourcing decreases while the probability of crossing any fixed threshold increases with the return fraction.

Keywords

Reverse Logistics, Outsourcing, Markov Decision Model, Monotone Policy, Product Life Cycle.

1. Introduction.

The flow of products does not stop upon their distribution to retailers and consumers: a substantial return flow of products may result from either generous return policies or extended producer responsibility legislation. We consider reverse logistics (RL) to include all activities associated with collecting, inspecting, reprocessing, redistributing, and disposing of items after they were originally sold (see Figure 1). Although it has long been perceived as a nuisance, recently it has been recognized as an area for improvement. Irrespective of their sizes, product types or geographic locations, most manufacturing, distribution and sales firms can benefit by improving the planning and control of RL activities. Unfortunately, not enough analytical models that assist in RL management decisions currently exist.

There are many reasons why products are returned, either by consumers or by the companies involved in the distribution chain. Retailers may return products for such reasons as damage in transit, expired date code, the model being discontinued or replaced, seasonality, excessive retailer inventories, or going out of business. Consumers can return products because of quality problems or failure to meet the consumer's needs; or for remanufacturing or proper disposal. Some of the products returned before or soon after sale can be resold profitably. In addition, after products have reached the end of their useful life, they may be able to be remanufactured, refurbished or repaired. These options can provide significant environmental and economic benefits in some instances, especially for products (e.g., electronic equipment) that have modular components that can be replaced, upgraded or refurbished. The value of remanufactured items typically will be lower than that of the same items produced for the first time, but substantially higher than that of items being sold for scrap, salvage or recycling (Stock, 1998).

Given that RL is not the firm's core activity, one of the most important decisions to be taken by any producer is whether or not to outsource such functions to a third-party reverse logistics provider (3PRLP). This typically is an irreversible decision, because the chosen strategy, once adopted, will not be changed frequently. The management of returns is complicated by the substantial uncertainties associated with their timing, volume and condition. This paper focuses on how the uncertainty in the amount of units returned each period affects the decision of whether or not to outsource their RL management.

Our central hypothesis is that outsourcing RL is more suitable when there is greater uncertainty about how many units may be returned. This hypothesis arose from a qualitative analysis of the published literature on outsourcing of RL, which is overviewed briefly in the next section. In Section 3, we formulate and analyze a Markov decision model of the outsourcing decision. The reward function includes the most significant components of the cost of either performing RL functions internally or outsourcing them, and the transitions among states reflect both the sequence of decisions taken and a simple characterization of the random pattern of returns over time. We assume that RL functions initially are performed internally. In order to focus simply on the outsourcing decision, we limit our attention to two possible actions in each period: either adjust internal capacity to match the expected number of returns in the next period, or switch permanently to outsourcing. By analyzing the cost and transition probability functions, in Section 4 we identify sufficient conditions for the existence of an optimal monotone policy over the partially ordered state space, which reduces to a threshold of cumulative returns for any given capacity level, beyond which outsourcing is optimal. The conditions are relationships among the cost parameters and the product return fraction that could be verified easily. Finally, we show that under mild assumptions, this threshold is more likely to be crossed when the uncertainty in returns is higher. Section 5 contains a numerical illustration of the existence of an optimal threshold policy even when the sufficient conditions are not met. It also illustrates how the threshold for outsourcing decreases while the probability of crossing any fixed threshold increases with the variability in the return volume. Finally, in Section 6, we draw conclusions and outline future research that can be developed based on this work.

2. Literature Review on Outsourcing RL Functions

The perceived importance of RL has increased lately. A recent estimate of annual sales of remanufactured products exceeds $50 billion in the United States alone (Guide and van Wassenhove, 2003). There are no worldwide estimates of the economic scope of reuse activities, but the number of firms engaged in this sector is growing rapidly in response to the opportunities to create additional wealth and the enactment of extended producer responsibility legislation in several countries. In a survey of current literature, Dowlatshahi (2005) identified the present state of theory in RL.

A number of researchers have addressed problems and opportunities in RL management. Such management of RL systems is complicated by factors that are less prevalent in the forward supply chain, such as the uncertainty in product returns. Recent work by Nakashima et al. (2004) illustrated how this uncertainty, which they characterized in terms of a virtual inventory level, can be modeled in a Markov decision process for controlling a remanufacturing system. Multiple criteria may require consideration, such as in the selection of alternatives for product end-of-life disposition (Bufardi et al., 2004).

As in the forward supply chain, some firms may opt to outsource logistical functions. In general, outsourcing can be defined as acquiring services from external service providers (Grover et al., 1994). This practice is increasingly pursued by organizations looking for cost benefits, operational efficiency, improved customer service and a better competitive position (Lieb and Randall, 1996; Boyson et al., 1999, Arroyo and Gaytán, 2007). Several streams of literature explain the bases of outsourcing decisions. Examples of these strands are Transactional Cost Theory (Williamson, 1979), Resource-based View (RBV) of the firm

(Wernerfelt, 1984), and evolutionary economics (Mahnke, 2001). Outsourcing research has focused extensively on the elaboration of the outsourcing process – identification of the outsourcing need, pros and cons of outsourcing, third party selection, establishment of the relation, and control and revision – however, few authors propose practical frameworks for guiding managers through the process of deciding when to outsource.

Van Laarhoven et al. (2000) define the outsourcing of logistics activities as "…activities carried out by a logistics service provider on behalf of a shipper and consisting of at least management and execution of transportation and warehousing. In addition, other activities can be included, for example, inventory management, information related activities … or even supply chain management." Razzaque and Sheng (1998) surveyed the literature related to outsourcing logistics functions. Most models on outsourcing logistics (Bagchi and Virum, 1998; Vining and Globerman, 1999; Tayles and Drury, 2001; Sink and Langley, 1997) consider the following steps: (1) definition of core competencies and strategy; (2) assessment of integral costs; and (3) analysis of suppliers and competitors. Several such studies have identified important considerations and emphasized the strategic nature of the decision. In a survey of logistics managers in US manufacturing firms, Daugherty and Dröge (1997) found that organizational structure had a significant effect on whether and to what extent logistical functions such as warehousing and transportation were outsourced. They noted that the basic economic justification of outsourcing logistics rests on economies of scale gained by specialization. Boyson et al. (1999) surveyed logistics managers and found that the decision to outsource was driven by profit growth and increased focus on core competencies. They concluded that, because significantly greater cost savings occurred when multiple logistical functions were outsourced, the outsourcing decision should be made strategically rather than to remedy specific deficiencies. Insinga and Werle (2000) also emphasized the strategic nature of outsourcing decisions and suggested that firms should outsource activities for which internal capability is weak and the potential for gaining competitive advantage is low. An annual survey of large US manufacturers revealed that outsourcing of logistical functions had reached a record level in the most recent results reported (Lieb and Bentz, 2005). Eighty percent of respondents outsourced at least one logistical function, most of whom outsourced several, and 37% reported contracting out reverse logistics (up from 26% in the previous year). Initiation of logistics outsourcing contracts was based primarily on cost, and significant impacts on cost reduction were reported. Recently, a prescriptive outsourcing model based on the satisficing principle was proposed by De Boer et al. (2006) for guiding outsourcing decision processes.

For the particular case of RL functions, outsourcing to a 3PRLP has been identified as one of the most important management strategies in recent years. Meade and Sarkis (2002) noted the three different choices available: to do nothing, to develop an internal RL function, or to find a 3PRLP and partner with them. They developed a model for selecting and evaluating 3PRLP once the choice to outsource had been made. Krumwiede and Sheu (2002) considered a model for market entry by a 3PRLP, but Dowlatshahi (2000) warned that some potential 3PRLPs lack the required knowledge of RL networks. One of the most important issues is to define whether the firm considers RL activities as part of its core functions. When this is not the case, outsourcing might represent a good alternative in order to allow the firm to focus on its core activities (Wu et al., 2005).

In a detailed qualitative analysis, Serrato (2006) found that some of the most important 3PRLPs are found in industry sectors with high return variability and a short product life cycle. High variability in returns reduces the economic feasibility of maintaining a firm's own RL facilities because the required capacity will be changing constantly. A faster response can be achieved by involving a 3PRLP, which specializes in these activities, and can take advantage of the economies of scale to convert RL functions into a profit-creating activity. On the other hand, not many 3PRLPs are active in industry sectors with lower return variability and longer product life cycles, because it is easier for the producer to develop its own facilities to deal with the return flow, even though RL may not be part of its core activities. Serrato (2006) developed a detailed analysis of these conclusions regarding outsourcing RL functions.

However, a qualitative analysis based on observation does not establish that observed practices are effective, nor does it explain how a specific characteristic such as return variability should influence the outsourcing decision. In general, outsourcing decisions are based on a variety of qualitative as well as quantitative considerations (Daugherty and Dröge, 1997). This paper's goal is to quantitatively examine the major characteristics that can be quantified so that the impact of return variability can be better understood as one input into a complex decision. To our knowledge, it is the first quantitative examination of the reverse logistics outsourcing decision. The cost relationships under which a threshold policy is shown to be optimal describe conditions under which return variability is an important consideration. The specific form of the threshold policy and the situations when the threshold is likely to be crossed identify the type of circumstance that should trigger a comprehensive study of possible outsourcing alternatives.

3. Markov Decision Model.

The model is designed to represent the major cost drivers in the outsourcing decision, the uncertainty in the return volume, temporal variability in sales, and the impracticality of multiple transitions between performing RL functions internally and outsourcing them. To focus on return volume variability, we assume that sales can be estimated accurately from relevant historical data and therefore are known. Returns depend on the amount of units previously sold and the fraction of them that will be returned through the firm's RL system. Define the following notation:

L = Length of the product life cycle, which depends on the particular RL scenario considered.

W = Time length defined by the firm to continue managing the returns for the product analyzed, after the last sale was made.

T = Length of the study horizon, $T=L+W$.

t = Decision epoch, $t = 1, \ldots, T - 1$, where decision epoch t represents the end of period t. Time T corresponds to the end of the problem horizon, where no decision is taken.

s_t = Amount of units sold by the firm during period t.

S_t = Cumulative sales experienced by the firm from period 1 through the end of period t, $S_t = \sum_{i=1}^{t} s_i$.

r = Return fraction, i.e., the expected fraction of units previously sold but not yet returned that will be returned in the next period.

x_t = Number of units returned in period t.

w_t =Cumulative number of units returned from period 1 to the end of period t, $w_t = \sum_{i=1}^{t} x_i$.

k_t = RL capacity held by the firm at the beginning of period t, which represents the number of units that can be processed in a single period.

n_t= Number of units sold and not returned at the end of period t, $n_t = S_t - w_t$.

The following assumptions underlie the Markov decision model (MDM):

Assumption 1: The sales in each period of the study horizon are known, for instance, they can be estimated reliably from the sales history of similar products (Tibben-Lembke, 2002).

Assumption 2: Each item that has been sold and not returned has a fixed probability, r, of being returned in the next period, independent of all other items. This is consistent with Toktay et al. (2003), where the number of periods between when a product was sold and when it was returned was modeled as a geometrically distributed random variable. It follows that given n_t at time t, the number of returns in period t+1 has a binomial distribution with parameters n_t and r, such that $E[x_{t+1}] = n_t r$ and $\mathrm{Var}[x_{t+1}] = n_t r(1-r)$. Note that the variance of return volume increases as n_t increases, for fixed r. It also increases as r approaches 0.5 from below. However, as Rogers and Tibben-Lembke (1999) observe, the return fraction in most industries is between zero and 0.3, approaching 0.5 only in some specific industry sectors.

Assumption 3: The firm's RL capacity is continuous; i.e., it can be added or subtracted in any quantity. However, to simplify the model and focus on the strategic nature of the outsourcing decision, we assume that if reverse logistics functions are carried out internally, then the capacity will be adjusted to equal the expected number of returns in the next period.

Assumption 4: If the number of returns in a period exceeds the RL capacity, the firm pays a shortage penalty, which represents the cost of either disposal or outsourcing the return processing on a temporary, emergency basis. No returns are carried over to a future period to be processed later. This assumption is relevant when returns are economically perishable, so that the positive value to be gained from handling them promptly is lost or greatly diminished by delay; or when storage is not physically feasible.

Several of these assumptions represent approximations of reality; for example, many firms hold inventories of returned products, capacity is frequently acquired in discrete chunks, and dependencies in whether or not products are returned may exist, especially if they are produced in the same facility and returned due to defect. They streamline the model to focus on the central binary decision of whether to outsource reverse logistics functions. Modifications to the model that relax or alter them are an important subject for further research, which we revisit in the Section 5. Additional assumptions concerning costs are stated later in this section.

3.1. Model Definition.

3.1.1. States.

The system state at each decision epoch t is defined as (k_t, w_t) for $t = 1, 2, \ldots, T$, where k_t represents the RL capacity owned by the firm during period t, measured in units per period, and w_t is the cumulative number of returns through the end of period t. As described below, the system states are partially ordered according to w_t. At decision epoch 0, the system state is $(k_0, 0)$.

3.1.2. Actions.

Given that the purpose of the MDM is to determine whether and when to outsource, we assume that at the end of any period t, two actions are available:

$a = 0$: Continue performing the RL activities internally by updating the firm's capacity to the expected amount of returns in the next period, i.e., $k_{t+1} = E[x_{t+1}] = n_t r$.

$a=1$: Adopt an outsourcing strategy for the RL activities by having a 3PRLP perform such activities and taking the firm's RL capacity to zero; i.e., $k_{t+1} = 0$. Given that RL does not represent a core activity for the firm, it is also assumed that once the outsourcing decision is taken, it remains in place for the rest of the problem horizon. This assumption is consistent with a survey result that, whereas 62% of respondents either outsourced handling of product returns or expected to do so in the near future, only 4% reported having outsourced them previously but no longer (Boyson et al., 1999).

Because n_t is an integer for all *t*, the problem has a discrete state space.

3.1.3. Transition Probabilities.

As the returns in each period follow a binomial distribution derived from the system state, and given that the sales function is also known, the transition probabilities among states are defined as $p_{t+1}\left[\left(k_{t+1}, w_{t+1}\right)\middle|\left(k_t, w_t\right), a\right]$ for $a \in \{0,\ 1\}$, where for *a* = 0 we have:

$$p_{t+1}\left[\left(n_t r, w_t + j\right)\middle|\left(k_t, w_t\right), 0\right] = \begin{cases} \binom{n_t}{j} r^j \left(1-r\right)^{n_t - j} & \text{for } j = 0,1,\ldots,n_t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and for *a* = 1 we have:

$$p_{t+1}\left[\left(0, w_t + j\right)\middle|\left(k_t, w_t\right), 1\right] = \begin{cases} \binom{n_t}{j} r^j \left(1-r\right)^{n_t - j} & \text{for } j = 0,1,\ldots,n_t \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

That is, the action taken determines the next period's capacity, but the second state variable w_{t+1} depends only on $n_t = S_t - w_t$ according to the binomial distribution for the returns.

3.1.4. Rewards.

Define the following set of costs, where a capacity unit represents firm's ability to process one returned item during a single period:

c_1: Unit investment cost for increasing the firm's capacity ($/capacity unit).

c_2: Unit capacity disinvestment cost ($/capacity unit).

c_3: Fixed internal capacity maintenance cost ($/capacity unit/period).

c_4: Unit internal labor cost ($/unit).

c_5: Unit shortage cost ($/unit).

c_6: Unit capacity salvage value ($/capacity unit).

c_7: Unit outsourcing cost ($/unit).

We assume c_1, c_3, c_4, c_5, $c_7 > 0$ because they represent costs for the firm, while c_2 and c_6 are unrestricted in sign, which models the net cost of contracting capacity and salvaging equipment as positive or negative. Figure 1 shows where these costs are located in the RL chain.

Figure 1. Relationship between RL chain and costs considered in the MDM.

Given that RL does not represent a core activity for the firm, profits from remanufacturing are not considered. We assume the following relationships between the cost parameters:

$$c_1 \geq |c_2| \tag{3}$$
$$c_3 \geq c_2 \tag{4}$$
$$c_4 < c_7 \tag{5}$$
$$c_7 < c_5 \tag{6}$$
$$c_5 \geq c_1 + c_3 + c_4 \tag{7}$$

First, (3) implies that what is gained when capacity is contracted is less than what was invested to expand it; i.e., there can be no profit from simply expanding and later contracting capacity. Inequality (4) states that the cost of decreasing the firm's capacity is no greater than the cost of maintaining it for an additional period. Also, (5) is reasonable because c_7 must cover both fixed and variable costs for the 3PRLP, whereas c_4 consists only of the variable cost for the firm. Because of the economies of scale expected to exist because RL is a core activity for the 3PRLP, fixed costs per unit for the 3PRLP are lower than fixed costs per unit for the firm. Note that c_7 represents the unit price charged by the 3PRLP, which it offers based on its own internal cost structure. In practice, contracts may include fixed fees and/or volume discounts, but we do not consider them in this paper. Also, (6) is appropriate because otherwise, all the 3PRLP's potential clients could keep their own capacity low and just pay the shortage cost rather than following an outsourcing option. On the other hand, inequality (7) represents a motivation to develop internal capacity, as the total internal cost of maintaining the capacity for one additional period and then processing one additional unit is less than the shortage cost for that unit.

With these cost parameters, the following reward structure is defined for actions a=0 or 1. Let $R_{t+1}\left[\left(k_t, w_t\right), a\right]$ be the expected reward at time $t+1$ when the system is in state $\left(k_t, w_t\right)$ and action a is taken. For a = 0, we have:

$$\begin{aligned} R_{t+1}\left[\left(k_t, w_t\right), 0\right] = & -c_1\left(n_t r - k_t\right)^+ - c_2\left(k_t - n_t r\right)^+ - c_3 n_t r \\ & -c_4 E\left[\min\left(x_{t+1}, n_t r\right)\right] - c_5 E\left[\left(x_{t+1} - n_t r\right)^+\right], \end{aligned} \tag{8}$$

where $(\cdot)^+$ denotes $\max(\cdot, 0)$. It is assumed that any unit that was not managed through the RL system in the period it was returned is lost and will not be remanufactured later. For a = 1,

$$R_{t+1}\left[\left(k_t, w_t\right), 1\right] = c_6 k_t - c_7\left(n_t - \sum_{l=t+1}^{L} s_l\left(1-\left(1-r\right)^{T-l}\right)\right), \text{ where } \sum_{l=t+1}^{L} = 0 \;\; if \;\; t+1 > L. \tag{9}$$

Here, c_7 represents the payment made to the 3PRLP for all expected returns from period t+1 until the end of the horizon. Recall that, given that RL is not a core activity for the firm, it is assumed that the outsourcing option will remain in effect for the remainder of the planning horizon, once that option is taken. Recall also from assumption 1 that the future sales can also be estimated accurately. This function also implies that the 3PRLP has infinite capacity, given the fact that RL does represent a core activity for it.

The terminal reward in period T is:

$$R_{T+1}\left[\left(k_T, w_T\right), a\right] = c_6 k_T - c_5 n_T, \quad \text{for} \quad a \in \{0,1\} \quad \text{and} \quad k_T > 0, \tag{10}$$

because the RL capacity defined by the firm is taken to zero in the last period, incurring the corresponding salvage value. Also, this function reflects the cost incurred by not being able to remanufacture any expected returned unit during period T or later.

3.2. System dynamics.

At the end of each period t the system:

1. Has capacity in the amount of k_t. A cumulative number, w_t, of units have been returned, and there are n_t units that are still in the market (were already sold and have not been returned);
2. Computes the expected number of returns in the next period and applies a control $\delta_{t+1}(k_t,w_t)$= 0 or 1. If $\delta_{t+1}(k_t,w_t) = 0$, k_{t+1} is set equal to $n_t r$ and the firm incurs either an investment cost $c_1(n_t r - k_t)^+$, or a disinvestment cost $c_2(k_t - n_t r)^+$ from adjusting the capacity, as well as a fixed cost $c_3 n_t r$; If $\delta_{t+1}(k_t,w_t) = 1$, k_{t+1} is set equal to zero and the firm incurs a salvage value $c_6 k_t$;
3. Experiences a random amount of returns x_{t+1}, which determines the new system state $(k_{t+1}, w_{t+1} = w_t + x_{t+1})$, and an amount of sales s_{t+1}, which determines the new cumulative sales level for the firm ($S_{t+1} = S_t + s_{t+1}$);
4. Incurs either an internal or a shortage cost, $c_4 \min(x_{t+1}, n_t r)$ or $c_5(x_{t+1} - n_t r)^+$, respectively, if $\delta_{t+1}(k_t,w_t) = 1$ and outsourcing cost $c_7\left(n_t - \sum_{l=t+1}^{L} s_l\left(1-(1-r)^{T-l}\right)\right)$, otherwise.

Given an initial system state $(k_0, 0)$, the problem is to find a sequence of decision functions $\{\delta_1^*(k_0,0), \delta_2^*(k_1,w_1), \ldots, \delta_T^*(k_{T-1},w_{T-1})\}$ that maximizes the total expected reward. The optimal policy can be obtained by solving recursively:

$$u_t\left(k_t, w_t\right) = \max\left\{\begin{array}{l} R_{t+1}\left[\left(k_t, w_t\right), 0\right] + \sum_{j=0}^{n_t} p_{t+1}\left(\left(n_t r, w_t + j\right) \middle| \left(k_t, w_t\right), 0\right) u_{t+1}\left(n_t r, w_t + j\right), \\ R_{t+1}\left[\left(k_t, w_t\right), 1\right] \end{array}\right\} \quad (11)$$

where $u_t(k_t, w_t)$ represents the maximum expected reward earned by continuing optimally from state (k_t, w_t) onwards.

4. A Threshold Policy

In principle, equation (11) can be solved recursively backwards from period T to identify an optimal action for each possible state. However, depending on the length of the study horizon, the sales volumes, and the granularity of the state space (i.e., the definition of a "unit" sold or processed), the number of states to be evaluated could grow very large. As Puterman (1994) observes, in order to reduce the amount of computation and increase appeal to decision-makers and managerial insight, it is desirable to identify a simple form for an optimal policy. Based on a partial ordering of the state space, below we establish the existence of an optimal monotone policy that corresponds to a threshold (in terms of the cumulative returns given a particular capacity level), beyond which the outsourcing action a=1 is optimal. Such a form also facilitates exploration of conditions under which outsourcing is more likely to be optimal. Below this threshold, the firm should continue performing the RL activities internally (a=0). Next, conditions for the existence of an optimal deterministic nondecreasing policy are defined.

4.1 General conditions for the existence of a optimal monotone policy.

Sets of conditions exist that ensure that optimal policies are monotone in the system state (Puterman, 1994). For such a concept to be meaningful, it is required that the state have a physical interpretation and some natural ordering. The expression "monotone policy" refers to a monotone deterministic Markovian policy.

For the MDM proposed, the states are partially ordered in terms of the cumulative returned units w_t. Specifically, for each t, let the states (k_t, w_t) be strictly partially ordered as $(k^1, w^1) \prec (k^2, w^2) \Leftrightarrow k^1 = k^2 \; and \; w^1 < w^2$, which is illustrated in Figure 2.

Figure 2. Illustration of the strict partial state ordering.

In addition to the partial ordering defined, a cumulative probability is needed to identify the conditions for a monotone nondecreasing policy:

$$q_{t+1}\left[(k_{t+1}, w_l)\middle|(k_t, w_t), a\right] = \sum_{w_t = w_l}^{n_t} p_{t+1}\left[(k_{t+1}, w_{t+1})\middle|(k_t, w_t), a\right],$$

where from (1):

$$q_{t+1}\left[(n_t r, w_l)\middle|(k_t, w_t), 0\right] = \begin{cases} \sum_{j=w_l-w_t}^{n_t} \binom{n_t}{j} r^j (1-r)^{n_t - j} & for \;\; w_l \geq w_t \\ 1 & for \;\; w_l < w_{t-1} \end{cases}$$

and from (2):

$$q_{t+1}\left[(0, w_l)\middle|(k_t, w_t), 1\right] = \begin{cases} \sum_{j=w_l-w_t}^{n_t} \binom{n_t}{j} r^j (1-r)^{n_t - j} & for \, w_l \geq w_t \\ 1 & for \, w_l < w_t \end{cases}$$

Finally, recall the definition of a superadditive function. Let X and Y be partially ordered sets and $g(x,y)$ a real-valued function on $X \times Y$. It is said that g is superadditive if for $x^- \leq x^+$ in X and $y^- \leq y^+$ in Y,

$$g(x^+, y^+) + g(x^-, y^-) \geq g(x^+, y^-) + g(x^-, y^+).$$

One set of conditions stated by Puterman (1994) for the existence of a monotone optimal policy are:

1. $R_{t+1}[(k_t,w_t),a]$ is nondecreasing in (k_t,w_t) for $a \in \{0,1\}$,

2. $q_{t+1}[(k_{t+1},w_{t+1}=w_l) \mid (k_t,w_t),a]$ is nondecreasing in (k_t,w_t) for all w_l and $a \in \{0,1\}$,

3. $R_{t+1}[(k_t,w_t),a]$ is a superadditive function on $(k_t,w_t) \times a$,

4. $q_{t+1}[k_{t+1},w_{t+1}=w_l) \mid (k_t,w_t),a]$ is a superadditive function on $(k_t,w_t) \times a$, and

5. $R_{T+1}[(k_T,w_T),a]$ is nondecreasing in (k_T,w_T).

When all of these conditions are satisfied, there exists a monotone nondecreasing policy that is optimal.

4.2. Specific conditions for an optimal threshold policy

In order to prove that these five conditions are satisfied, the following lemma will be used:

Lemma 1.

Suppose X_n is binomial with parameters n and r, where n = 2, 3, …, and $0 < r < 1$. Let $\mu_n = E[X_n] = nr$. Then for any n and l = 1, 2, …, n-1,

(*a*) $E\left[\left(X_n-\mu_n\right)^+\right] \geq E\left[\left(X_l-\mu_l\right)^+\right]$

(*b*) $E[\min(X_n,\mu_n)] \geq E[\min(X_l,\mu_l)]$

(*c*) For any integer m such that $0 \leq m \leq l$, $P[X_n > m] \geq P[X_l > m]$.

Proof: In the Appendix.

Theorem 1 applies Puterman's conditions to the context of this paper. The managerial implication is that for any fixed capacity level, there exists a threshold number of cumulative returns such that outsourcing is optimal for any greater or equal number of cumulative returns.

Theorem 1

If $\left(c_5-c_7\right)/\left(c_5-c_1-c_3-c_4\right) \leq r \leq \left(c_7-c_4\right)/\left(c_1+c_3\right)$, *then there exist optimal decision rules* $\delta_{t+1}^*(k_t,w_t)$ *for* t = 0, …, T-1 *that are nondecreasing in* (k_t,w_t) *according to the partial order defined.*

The proof is presented in the following five subsections:

4.2.1. Condition 1.

This condition holds when the cost of either action does not increase with the number of items sold but not yet returned. For a=1, it requires that $R_{t+1}\left[\left(k_t, w_t\right),1\right] \le R_{t+1}\left[\left(k_t, w_t+i\right),1\right]$ for $1 \le i \le n_t$, which follows immediately from $i > 0$ and $c_7 > 0$.

For a=0, the condition $R_{t+1}\left[(k_t, w_t),0\right] \le R_{t+1}\left[(k_t, w_t+i),0\right]$ for $1 \le i \le n_t$, i.e., that the expected internal RL reward increases with the cumulative amount of returned units w_t, is equivalent to:

$$\begin{aligned}&-c_1\left((n_t r-k_t)^+-((n_t-i)r-k_t)^+\right)-c_2\left((k_t-n_t r)^+-(k_t-(n_t-i)r)^+\right)-c_3 ri-\\&c_4\left(E[\min(X,n_t r)]-E[\min(Y,(n_t-i)r)]\right)-c_5\left(E\left[(X-n_t r)^+\right]-E\left[(Y-(n_t-i)r)^+\right]\right)\le 0, \qquad 1\le i\le n_t\end{aligned}$$

where c_1, c_3, c_4, c_5>0 and X (Y) is binomially distributed with parameters n_t ($n_t - i$), respectively, and r. From parts (a) and (b) of Lemma 1, the elements that multiply c_4 and c_5 are nonnegative.

This inequality can be analyzed in three cases. If $n_t r < k_t$, it is equivalent to:

$$\begin{aligned}&-ir(c_3-c_2)-c_4\left(E[\min(X,n_t r)]-E[\min(Y,(n_t-i)r)]\right)\\&-c_5\left(E\left[(X-n_t r)^+\right]-E\left[(Y-(n_t-i)r)^+\right]\right)\le 0,\quad 1\le i\le n_t\end{aligned}$$

which is satisfied, given inequality (4) and Lemma 1. If $k_t \le \left(n_t - i\right) r$, it can be reduced to:

$$\begin{aligned}&-ir(c_1+c_3)-c_4\left(E[\min(X,n_t r)]-E[\min(Y,(n_t-i)r)]\right)\\&-c_5\left(E\left[(X-n_t r)^+\right]-E\left[(Y-(n_t-i)r)^+\right]\right)\le 0,\quad 1\le i\le n_t\end{aligned}$$

which follows from the assumption of positive cost coefficients. Finally, if $\left(n_t - i\right) r \le k_t \le n_t r$, it is:

$$\begin{aligned}&(k_t-n_t r)(c_1+c_2)-c_4\left(E[\min(X,n_t r)]-E[\min(Y,(n_t-i)r)]\right)\\&-c_5\left(E\left[(X-n_t r)^+\right]-E\left[(Y-(n_t-i)r)^+\right]\right)\le 0,\quad 1\le i\le n_t\end{aligned}$$

which holds under inequalities (3) and (4).

4.2.2. Condition 2.

This condition holds when it is more likely to meet or exceed a given number of cumulative returns in the next period, if a higher number of returns have been experienced up to the current period. It requires that for a fixed $w_t = w_l$:

$$q_{t+1}\left[\left(k_{t+1} w_l\right)\left|\left(k_t, w_t\right), a\right.\right] \leq q_{t+1}\left[\left(k_{t+1}, w_l\right)\left|\left(k_t, w_t + i\right), a\right.\right], \; 1 \leq i \leq n_t,$$

which can be analyzed under the three cases: $w_l \leq w_t$, $w_t < w_l < w_t + i$ and $w_t + i < w_l$.

In the first case, the cumulative returns in period t are greater than or equal to w_l. Then, the probability that such cumulative returns will equal or exceed w_l in the next period is 1; i.e., the condition is satisfied as an equality. In the second case, the cumulative returns are already greater than w_l in the right hand side of the inequality ($w_l \leq w_t + i$). This implies that the probability on the right hand side equals 1, so that the inequality holds regardless of the probability on the left hand side. Finally, for the third case, this condition can be rewritten as:

$$\sum_{j=w_l-w_t}^{n_t} \binom{n_t}{j} r^j (1-r)^{n_t-j} \leq \sum_{j=w_l-w_t-i}^{n_t-i} \binom{n_t-i}{j} r^j (1-r)^{n_t-i-j}, \; w_t < w_t + i < w_l,,$$

which is equivalent to:

$$P\left[X \leq w_l - w_t - 1\right] \geq P\left[Y \leq w_l - w_t - 1 - i\right].$$

This follows directly from Lemma 1(c).

4.2.3. Condition 3.

$$R_{t+1}\left[\left(k_t, w_t\right), 1\right] - R_{t+1}\left[\left(k_t, w_t\right), 0\right] \leq R_{t+1}\left[\left(k_t, w_t + i\right), 1\right] - R_{t+1}\left[\left(k_t, w_t + i\right), 0\right].$$

This inequality holds when for a fixed capacity k_t, the incremental effect on the reward of switching to an outsourcing strategy increases with the cumulative number of returned units w_t. This condition can be rewritten as:

$$c_1\left(\left((n_t - i)r - k_t\right)^+ - \left(n_t r - k_t\right)^+\right) + c_2\left(\left(k_t - (n_t - i)r\right)^+ - \left(k_t - n_t r\right)^+\right) + ic_7 - irc_3 \geq$$
$$c_4\left(E\left[\min\left(X, n_t r\right)\right] - E\left[\min\left(Y, (n_t - i)r\right)\right]\right) + c_5\left(E\left[\left(X - n_t r\right)^+\right] - E\left[\left(Y - (n_t - i)r\right)^+\right]\right) \text{ for } 1 \leq i \leq n_t$$

where, as in Condition 1, X (Y) is binomially distributed with parameters n_t ($n_t - i$), respectively, and r, and from Lemma 1 (a) and (b), the expressions that multiply c_4 and c_5 are nonnegative.

Consider the same three cases as for Condition 1. If $n_t r < k_t$, the inequality is:

$$c_7 i - (c_3 - c_2) ir \geq c_4 \left(E[\min(X, n_t r)] - E[\min(Y, (n_t - i) r)] \right) + c_5 \left(E\left[(X - n_t r)^+\right] - E\left[(Y - (n_t - i) r)^+\right] \right) \tag{12}$$

Considering that $X = Y + \sum_{j=n_t-i+1}^{n_t} U_j$, where each U_j independently equals 1 with probability r and 0 otherwise, this inequality can be analyzed under the four possible cases shown in Table 1.

Table 1. Worst case analysis for Condition 3.

If $k_t \leq (n_t - i) r$, the inequality is:

$$c_7 i - (c_1 + c_3) ir \geq c_4 \left(E[\min(X, n_t r)] - E[\min(Y, (n_t - i) r)] \right) + c_5 \left(E\left[(X - n_t r)^+\right] - E\left[(Y - (n_t - i) r)^+\right] \right)$$

Following the same analysis, the resulting inequalities in the worst cases are shown in the last column of Table 1.

Finally, if $(n_t - i) r \leq k_t \leq n_t r$, the inequality is:

$$c_7 i - c_3 ir + c_1 (n_t r - k_t) + c_2 (k_t - (n_t - i) r) \geq c_4 \left(E[\min(X, n_t r)] - E[\min(Y, (n_t - i) r)] \right) + c_5 \left(E\left[(X - n_t r)^+\right] - E\left[(Y - (n_t - i) r)^+\right] \right)$$

By comparison with inequality (12), this inequality will be satisfied as long as:

$$(c_1 - c_2)(n_t r - k_t) \geq 0$$

which is true given (3).

Then, the following relationships are sufficient to satisfy Condition 3:

$$c_5 - c_7 \le (c_5 - c_4) r - (c_3 - c_2) r \tag{13}$$

$$c_7 - c_4 \ge (c_3 - c_2) r \tag{14}$$

$$c_5 - c_7 \le (c_5 - c_4) r - (c_1 + c_3) r \tag{15}$$

$$c_7 - c_4 \ge (c_1 + c_3) r \tag{16}$$

However given (3), inequalities (13) and (14) are redundant. Inequality (15) is equivalent to a lower bound on *r*:

$$r \ge \frac{c_5 - c_7}{c_5 - c_1 - c_3 - c_4} \tag{17}$$

while inequality (16) can be stated as an upper bound:

$$r \le \frac{c_7 - c_4}{c_1 + c_3} \tag{18}$$

These bounds represent sufficient conditions on the return fraction in terms of the cost parameters to guarantee superadditivity of the reward function. Because they were obtained by a worst case analysis that ignores the probability distributions of *X* and *Y*, it is possible that less restrictive conditions could be found.

4.2.4. Condition 4.

This condition implies that the difference between the cumulative probability that returns exceed a given number when taking the outsourcing option and when performing RL activities internally, does not decrease with the cumulative number of returns. This condition can be written as:

$$q_{t+1}\left[(k_{t+1}, w_l) \middle| (k_t, w_t^+), 1\right] - q_{t+1}\left[(k_{t+1}, w_l) \middle| (k_t, w_t^+), 0\right] \ge$$
$$q_{t+1}\left[(k_{t+1}, w_l) \middle| (k_t, w_t^-), 1\right] - q_{t+1}\left[(k_{t+1}, w_l) \middle| (k_t, w_t^-), 0\right]$$

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where $w_t^+ > w_t^-$. It is satisfied as an equality because the action determines the next capacity level while random events affect only the cumulative returns.

4.2.5. Condition 5.

This condition implies that the terminal reward increases with the number of cumulative returns. The inequality $R_{T+1}\left[\left(k_T, w_T\right), a\right] \le R_{T+1}\left[\left(k_T, w_T + i\right), a\right]$ for $1 \le i \le n_T$ can be written as $c_6 k_T - c_5 n_T r \le c_6 k_T - c_5\left(n_T - i\right) r$, which follows from $c_5 > 0$.

4.2.6 Implications.

Inequality (17) states a lower bound on the return fraction as a ratio of two quantities: the opportunity (regret) cost $c_5 - c_7$ of not taking the outsourcing option and incurring the corresponding shortage for a particular unit, and the difference between the shortage cost and the total cost of internal processing. The magnitude of the lower bound depends on the relationship between internal and outsourced costs of processing returns. Inequality (18) is an upper bound on the return fraction expressed as another ratio, where the numerator is the difference between the outsourcing cost and the internal variable cost and the denominator is the total internal investment and capacity cost. The magnitude of the ratio depends on the economies of scale achieved by the 3PRLP. Counter-intuitively, the bounds are wider when the unit outsourcing cost is large relative to the internal costs. It should be emphasized, however, that these inequalities represent sufficient rather than necessary conditions for the existence of an optimal nondecreasing policy. The numerical illustration in Section 5 shows that such a policy can exist even under severe violations of these conditions.

4.3. Impact of return uncertainty on crossing the threshold

The result of Theorem 1 implies that, in any period *t*:

$$k_t^1 = k_t^2 \quad and \quad w_t^1 < w_t^2 \Rightarrow a_t^*\left(k_t^1, w_t^1\right) \le a_t^*\left(k_t^2, w_t^2\right).$$

In the remainder of this section, the subscript t is suppressed for simplicity. Define the outsourcing threshold for each capacity level as:

$$\theta[k] = \begin{cases} \min\{w : a^*(k,w) = 1\} & \text{if } \theta[k] \text{ exists} \\ \infty & \text{otherwise.} \end{cases}$$

Lemma 2:

Let $\theta[k;r]$ be the value of $\theta[k]$ when the return fraction is r. If $0 \le r \le r + \Delta_r \le 0.5$ and

$$c_5 \le 4.375 c_4, \tag{19}$$

then

$$\theta[k;r] \ge \theta[k; r + \Delta_r] \; . \tag{20}$$

Proof: In the Appendix.

Note that the maximum value for the ratio c_5/c_4 in (19) is a lower bound on the requirement that holds for states with any number of outstanding items, $n \ge 2$. In a practical situation, when n would be much larger, the inequality imposes no significant constraint on c_5/c_4.

Lemma 3:

Let $q\left[(nr, w_l)\middle|(k,w), 0; r\right]$ be the value of $q\left[(nr, w_l)\middle|(k,w), 0\right]$ when the return fraction is r.

If $0 \le r + \Delta_r \le 0.5$ then:

$$q\left[\left(n(r+\Delta_r), w_l\right)\middle|(k,w), 0; r+\Delta_r\right] \ge q\left[(nr, w_l)\middle|(k,w), 0; r\right] \tag{21}$$

Proof: In the Appendix.

Theorem 2:

Suppose an optimal nondecreasing policy exists and the conditions of Lemma 2 are satisfied. Then

$$q\left[\left(n\left(r+\Delta_r\right),\theta\left[k;r+\Delta_r\right]\right)\middle|\left(k,w\right),0;r+\Delta_r\right]\ge q\left[\left(nr,\theta\left[k;r\right]\right)\middle|\left(k,w\right),0;r\right].$$

Proof:

From Lemma 2 we have $\theta\left[k;r\right]\ge\theta\left[k;r+\Delta_r\right]$, which implies that:

$$q\left[\left(nr,\theta\left[k,r+\Delta_r\right]\right)\middle|\left(k,w\right),0;r\right]\ge q\left[\left(nr,\theta\left[k;r\right]\right)\middle|\left(k,w\right),r\right]. \tag{22}$$

By Lemma 3,

$$q\left[\left(n\left(r+\Delta_r\right),\theta\left[k,r+\Delta_r\right]\right)\middle|\left(k,w\right),0;r+\Delta_r\right]\ge q\left[\left(nr,\theta\left[k;r+\Delta_r\right]\right)\middle|\left(k,w\right),r\right]. \tag{23}$$

Then, we have by transitivity that:

$$q\left[\left(n\left(r+\Delta_r\right),\theta\left[k,r+\Delta_r\right]\right)\middle|\left(k,w\right),0;r+\Delta_r\right]\ge q\left[\left(nr,\theta\left[k;r\right]\right)\middle|\left(k,w\right),r\right],$$

which completes the proof.

Theorem 2 shows that the suitability of the outsourcing option increases when the return fraction increases because the threshold value does not increase and the probability of crossing any given level of cumulative returns does not decrease. Because the variance of the number of returns increases with $r\le 0.5$, this result supports the hypothesis that greater variability in the return volume motivates outsourcing.

5. Numerical Illustration

To demonstrate the influence of higher uncertainty in the return volume on the suitability of an outsourcing option, consider a particular scenario defined by the parameters:

$$\begin{array}{ll} L=4 & c_3=3 \\ W=1 & c_4=8 \\ & c_5=24 \\ c_1=1 & c_6=2 \\ c_2=1 & c_7=13 \end{array} \tag{24}$$

and the sales function:

$$s_t = \begin{cases} \frac{2M}{L}t, \quad t = 1, 2, ..., L/2 \\ M - \frac{2M}{L}\left(t - L/2 - 1\right), t = L/2 + 1, ..., L \end{cases} \tag{25}$$

where M represents the maximum sales level experienced by the firm during the life cycle, and $M = 3$ in this numerical example. The values for the cost parameters satisfy conditions (3) to (7) but not the sufficient conditions (17) and (18) for the existence of an optimal nondecreasing policy. The lower bound on r from (17) is 11/12 and the upper bound from (18) is 5/4. However, solving by backward induction identifies such an optimal policy for values of the return fraction as small as 0.2.

Table 2 shows the values for the threshold in each set of states, for $r = \{0.2, 0.3, 0.4, 0.5\}$, as well as the probability $q_{t+1}\left[\left(k_{t+1}, w_l\right)\middle|\left(k_t, w_t\right), a\right]$ that the threshold (labeled as w_l) is crossed in each case. These values were obtained by creating a Matlab program, whose inputs are $r, L, W, c_1, c_2, c_3, c_4, c_5, c_6, c_7$, as well as the sales volume s_t during the analysis horizon. Based on this information, the program computes the possible states and orders them according to the criteria defined. The program also computes the amount of units n_t outstanding in the market for each state, as well as the corresponding transition probabilities and expected costs for $a = 0$ and $a = 1$. The terminal costs are also obtained. Based on this, the program solves the MDM by using backward induction, and shows the optimal action to take at each decision epoch.

Table 2. Value of the threshold and the probability of crossing it for $r = \{0.2, 0.3, 0.4, 0.5\}$

As can be seen in Table 2, a greater uncertainty in the return volume (greater r) increases the probability of crossing the corresponding threshold in each set of states; i.e., there is a greater probability that outsourcing ($a = 1$) will be the optimal action to take. For example for $t = 4$, if $r = 0.2$ then for $k_4 = 8r = 1.6$ there is no outsourcing threshold, but if $r = 0.4$ then for $k_4 = 4r = 1.6$ a finite threshold of 8 units does exist. Also, if $r = 0.3$ then for $k_4 = 8r = 2.4$ the threshold value is 8 and the crossing probability is 0.001. If $r = 0.4$ then for the same capacity $k_4 = 6r = 2.4$, the threshold value drops to 7 while the crossing probability rises to 0.04. This example confirms that greater variability in the return volume increases the uncertainty about the volume of units put into the corresponding RL system, which motivates the firm to follow an outsourcing strategy and take advantage of the economies of scale by involving a 3PRLP in managing the returned items.

6. Conclusions and Future Work

A Markov Decision Model (MDM) for evaluating the decision to outsource RL is developed in this research. It considers several elements that are critical in defining the characteristics of a RL network, such as the uncertainty in the return volume, the length of the product life cycle, the sales behavior, the particular RL costs incurred, and the length of time defined for the existence of that RL system. In particular, the uncertainty implied in the MDM is represented by the number of returned units, which depends on the number of units outstanding in the market and the return fraction.

Some sufficient conditions for the existence of an optimal monotone nondecreasing policy are derived as bounds on the return fraction defined by the cost parameters. The existence of an optimal monotone nondecreasing policy implies the presence of a threshold above which it is optimal to follow an outsourcing strategy for the RL system; otherwise, to continue performing the RL activities internally. This threshold is defined in terms of a partial ordering for the system states, where given a fixed capacity at a decision epoch, the states are ordered according to the cumulative returned units, such that if that volume goes above a particular level, then it is optimal to follow an outsourcing strategy and take advantage of the economies of scale implied by involving a 3PRLP, which has RL as its core function. The value of the threshold and probability it is crossed characterize the prevalence of outsourcing.

While the existence of an optimal monotone policy appeals to intuition and simplifies computation, its chief value in this paper is to assess conditions under which outsourcing is more likely to be optimal. The main result is that as the return fraction increases the outsourcing threshold is more likely to be crossed. Because the variance of the binomially distributed number of returns increases with reasonable values of the return fraction, this result supports the hypothesis that variability in return volumes motivates outsourcing RL.

A numerical illustration shows that an optimal threshold policy exists even when our sufficient conditions are not met. It confirms that, when the return fraction is higher, outsourcing thresholds are smaller and the probability of crossing them is higher.

Because the model presented here is the first to our knowledge to quantitatively examine the decision of whether or not to outsource RL, ample opportunity exists for further research that could relax the assumptions of the model, identify less restrictive conditions for existence of a monotone policy, or study the effect of other drivers for outsourcing.

By reconsidering the model assumptions, additional research could model more complex capacity and inventory management policies than the one considered here, either because the firm cannot adjust its capacity each period, or a different adjustment policy is found to result in better performance. In practice, firms do keep inventories of returned products even though their value frequently is declining rapidly. The irreversibility of the outsourcing decision assumed in this paper also could be relaxed, in view of the fact that the 3PRLP selected may fail to perform adequately. On the economic side, the potential for profits from reprocessing and selling returned items may be considered as a benefit of maintaining RL capacity internally. The model for contracting arrangements with the 3PRLP could consider a tiered pricing structure, capacity reservations, or other means by which the 3PRLP could cover its capacity investment costs and reduce risk.

Several problem parameters, such as the return fraction and/or the RL costs may not be constant during the product's life cycle. Nonstationary costs will be easy to incorporate, but variation in the return fraction will require more elaborate modifications to the analysis. More generally, another area of research would identify the requirements for the existence of an optimal monotone nondecreasing policy, when the returns follow a probability distribution different than the one described in this paper. A different stochastic model for returns could allow investigation of the effects of separate changes in the mean and variance of returns per period.

Regarding the analysis, the sufficient conditions for the existence of a monotone policy found are not necessary, and it may be possible to find less restrictive ones by an average- rather than worst-case analysis.

Finally, this paper focuses on how the return fraction influences the outsourcing decision. Many other important characteristics of the sales and return functions could be examined. The life cycle length has been identified in empirical studies as another differentiator between industry sectors where outsourcing RL is common and those where it is not (Serrato, 2006). Analysis of the influence of the life cycle length on outsourcing suitability represents a future research area to consider. The main challenge for this analysis in the MDM state space is the difference in the cardinality of the sets of ordered states obtained for each case. The size of the state space at each decision epoch is determined by the sales function of the product analyzed, as well as the length of the lifecycle.

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APPENDIX

Proof of Lemma 1.

(a) Suppose $l = n - 1$ and consider

$$E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)\right)^+\right] = E\left[E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)\right)^+ \middle| X_{n-1}\right]\right]$$
$$where\ X_n = X_{n-1} + U$$

where U = 1 with probability r and 0 otherwise. There are three possible cases for $X_{n-1} = m$.

If $m < nr - r$, then $E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)^+\right) \middle| X_{n-1} = m\right] = r\left(m + 1 - nr\right)^+$ given that $nr>m$. Then, this case yields a nonnegative result. If $nr - r \le m < nr$, then

$$E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)\right)^+ \middle| X_{n-1} = m\right] = \left(nr - m\right)\left(1 - r\right)$$

where, given that $nr>m$, the expression is also nonnegative. Finally, if $nr \le m$, then $E\left[\left(X_n - nr\right)^+\right] - E\left[\left(X_l - \left(nr - r\right)\right)^+\right] = 0$. Then, to complete the conditioning argument:

$$\begin{aligned}
&E\left[\left(X_n - nr\right)^+\right] - E\left[\left(X_{n-1} - \left(nr - r\right)\right)^+\right] \\
&= E\left[E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)\right)^+ \middle| X_{n-1}\right]\right] \\
&= \sum_{m=0}^{n-1} E\left[\left(X_n - nr\right)^+ - \left(X_{n-1} - \left(nr - r\right)\right)^+ \middle| X_{n-1} = m\right] P\left(X_{n-1} = m\right) \ge 0.
\end{aligned}$$

Then, considering that $E\left[\left(X_n-\mu_n\right)^+\right]-E\left[\left(X_{n-1}-\mu_{n-1}\right)^+\right]\geq 0$ and, since n is arbitrary, we have, by the transitive property:

$$E\left[\left(X_n-\mu_n\right)^+\right]-E\left[\left(X_l-\mu_l\right)^+\right]\geq 0 \quad for \quad any \quad l=1,2,\ldots n-1$$

which completes the proof.

(b) Suppose $l=n-1$ and consider

$$E[\min(X_n, nr)-\min(X_{n-1},(nr-r))]=E[E[\min(X_n, nr)-\min(X_{n-1},(nr-r))|X_{n-1}]]$$
$$where \ X_n = X_{n-1}+U$$

where U = 1 with probability r and 0 otherwise. Suppose $X_{n-1}=m<nr-r$. Then

$$E\left[\min\left(X_n, nr\right)-\min\left(X_{n-1},\left(nr-r\right)\right)\middle|X_{n-1}=m\right]=r\min\left(m+1,nr\right)+\left(1-r\right)m-m$$

since $m<nr-r \Rightarrow m<nr$. Also, $nr>m+r \Rightarrow \min(m+1,nr)\geq m+r$, so $r\min(m+1,nr)+(1-r)m-m\geq r(m+r)+(1-r)m-m=r^2$.

On the other hand, suppose $X_{n-1}=m\geq nr-r$. Then

$$E\left[\min\left(X_n, nr\right)-\min\left(X_{n-1},\left(nr-r\right)\right)\middle|X_{n-1}=m\right]=rnr+\left(1-r\right)\min\left(m,nr\right)-nr+r$$

because $m\geq nr-r \Rightarrow m+1\geq nr$. Also, $\min(m,nr)\geq nr-r$, which implies that $rnr+(1-r)\min(m,nr)-nr+r\geq nr^2+(1-r)(nr-r)-nr+r=r^2$.

To complete the conditioning argument:

$$\begin{aligned}
&E\left[\min\left(X_n,nr\right)-\min\left(X_l,nr-r\right)\right]\\
&=E\left[E\left[\min\left(X_n,nr\right)-\min\left(X_l,nr-r\right)\middle|X_l\right]\right]\\
&=\sum_{m=0}^{n-1}E\left[\min\left(X_n,nr\right)-\min\left(X_l,nr-r\right)\middle|X_l=m\right]P\left(X_l=m\right)\geq 0
\end{aligned}$$

Then:

$$E[\min(X_n,\mu_n)]-E[\min(X_{n-1},\mu_{n-1})]\geq 0.$$

and since n is arbitrary, we have, by the transitive property:

$$E[\min(X_n, \mu_n)] - E[\min(X_l, \mu_l)] \geq 0 \;\; \textit{for any}\; l = 1,2,...,n-1$$

which completes the proof.

(c) Suppose $l = n-1$ and consider

$$P[X_n > m] - P[X_{n-1} > m] = (P[X_{n-1} > m] + P[X_{n-1} = m]P[U = 1]) - P[X_{n-1} > m]$$
$$= \binom{n-1}{m} r^{m+1}(1-r)^{n-1-m} > 0, \qquad \textit{where}\; X_n = X_{n-1} + U$$

which implies that the probability of experiencing more than m successes is greater when one additional trial is added to the sequence of Bernoulli trials. Then, since n is arbitrary, we have, by transitivity:

$$P[X_n > k] - P[X_l > k] > 0 \qquad \textit{for any}\; l = 1,2,...,n-1$$

which completes the proof.

Proof of Lemma 2:

Let $w \equiv \theta[k;r]$, and let $R[(k,w),a;r]$ be the value of $R[(k,w),a]$ when the return fraction is r. In order to prove (20), the following inequalities must be satisfied, given the relationships for r and Δ_r defined in terms of the cost parameters:

$$R[(k,w),0;r] \leq R[(k,w),1;r] \tag{26}$$

$$R[(k,w),0;r+\Delta_r] \leq R[(k,w),1;r+\Delta_r] \tag{27}$$

where (26) comes from the definition of w. Inequality (27) implies that the threshold is not greater than w when the return fraction increases by Δ_r; i.e., the threshold does not increase when the return fraction increases, as stated in (20).

Given that by definition of w, inequality (26) is satisfied, and that the right-hand sides in both inequalities are equal (they do not depend on r), inequality (27) will be satisfied as long as:

$$R\left[(k,w),0;r+\Delta_r\right] \le R\left[(k,w),0;r\right], \tag{28}$$

where (28) can be rewritten as:

$$\begin{aligned} &c_1\left(\left(n\left(r+\Delta_r\right)-k\right)^+ -\left(nr-k\right)^+\right)+c_2\left(\left(k-n\left(r+\Delta_r\right)\right)^+ -\left(k-nr\right)^+\right)+c_3 n\Delta_r \\ &\quad +c_4\left(E\left[\min\left(x_1,n\left(r+\Delta_r\right)\right)\right]-E\left[\min\left(x_2,nr\right)\right]\right) \\ &\quad +c_5\left(E\left[\left(x_1-n\left(r+\Delta_r\right)\right)^+\right]-E\left[\left(x_2-nr\right)^+\right]\right)>0 \qquad \text{where } x_1 \sim Bin\left(n,r+\Delta_r\right), \\ &\qquad\qquad x_2 \sim Bin\left(n,r\right) \end{aligned} \tag{29}$$

Given (3) and (4), the following part of (29) is nonnegative:

$$c_1\left(\left(n\left(r+\Delta_r\right)-k\right)^+ -\left(nr-k\right)^+\right)+c_2\left(\left(k-n\left(r+\Delta_r\right)\right)^+ -\left(k-nr\right)^+\right)+c_3 n\Delta_r \ge 0$$

Consider the remaining elements of (29):

$$\begin{aligned} &c_4\left(E\left[\min\left(x_1,n\left(r+\Delta_r\right)\right)\right]-E\left[\min\left(x_2,nr\right)\right]\right)+ \\ &\quad c_5\left(E\left[\left(x_1-n\left(r+\Delta_r\right)\right)^+\right]-E\left[\left(x_2-nr\right)^+\right]\right) \end{aligned} \tag{30}$$

Let $g_n(r)=E\left[\min\left(X,nr\right)\right]$ and $f_n(r)=E\left[\max\left(X-nr,0\right)\right]$ where $X \sim \text{binomial}(n,r)$ and $n \ge 2$. Then, (30) will be true if the combined cost function $c_4 g_n(r)+c_5 f_n(r)$ is an increasing function of *r*.

The derivatives of f_n and g_n are discontinuous at $r = j/n$, $j = 1, \ldots, n-1$, which implies that a separate expression is needed for the derivatives in each interval for *r.* For $\frac{j}{n} < r \le \frac{j+1}{n}$, or equivalently $j < nr \le j+1$, where $2j < n$:

$$f_n(r)=\sum_{k=j+1}^{n}(k-nr)\,p(k)=\sum_{k=0}^{j}(nr-k)\,p(k)$$

and

$$g_n(r)=\sum_{k=1}^{j}kp(k)+nr\sum_{k=j+1}^{n}p(k)=nr-\sum_{k=0}^{j}(nr-k)\,p(k)$$

where $p(k)=\binom{n}{k}r^k(1-r)^{n-k}$. Then

$$c_4 g_n(r) + c_5 f_n(r) = c_4\left(nr - \sum_{k=0}^{j}(nr-k)\,p(k)\right) + c_5\left(\sum_{k=0}^{j}(nr-k)\,p(k)\right)$$
$$= c_4 nr + (c_5 - c_4)\sum_{k=0}^{j}(nr-k)\,p(k).$$

We wish to show that $\frac{d}{dr}\left(c_4 g_n(r) + c_5 f_n(r)\right) \geq 0$ for reasonable values of $c_4 < c_5$.

(a) $\frac{d}{dr}\sum_{k=0}^{j}(nr-k)\binom{n}{k}r^k(1-r)^{n-k} = \binom{n}{j}(n-j)r^j(1-r)^{n-j-1}\left[j+1-(n+1)r\right]$

The proof is inductive, using integration by parts. First, for $j = 0$, or equivalently, $0 < nr \leq 1$,

integrating by parts with $u = 1-(n+1)r$ and $dv = n(1-r)^{n-1}\,dr$ we get:

$$\int n(1-r)^{n-1}\left[1-(n+1)r\right]dr = -\left[1-(n+1)r\right](1-r)^n - \int (n+1)(1-r)^n\,dr$$
$$= -\left[1-(n+1)r\right](1-r)^n + (1-r)^{n+1} = nr(1-r)^n.$$

And for $j = 1$, or equivalently, $1 < nr \leq 2$,

$$n\int (n-1)(1-r)^{n-2}\left[2r-(n+1)r^2\right]dr = -n\left[2r-(n+1)r^2\right](1-r)^{n-1} + 2\int n(1-r)^{n-1}\left[1-(n+1)r\right]dr$$
$$= -nr\left[1-nr+1-r\right](1-r)^{n-1} + 2nr(1-r)^n$$
$$= nr(1-r)^n + (nr-1)nr(1-r)^{n-1},$$

where the second equality results from substituting for the $j = 0$ integral.

Now, for $j > 1$, assume:

$$\binom{n}{j-1}\int (n-j+1)(1-r)^{n-j}\left[jr^{j-1}-(n+1)r^j\right]dr = \sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{n-k}.$$

Then:

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$$\binom{n}{j}\int (n-j)(1-r)^{n-j-1}\left[(j+1)r^j-(n+1)r^{j+1}\right]dr$$
$$=-\binom{n}{j}r^j\left[(j+1)-(n+1)r\right](1-r)^{n-j}+(j+1)\binom{n}{j}\int (1-r)^{n-j}\left[jr^{j-1}-(n+1)r^j\right]dr$$
$$=-\binom{n}{j}r^j\left[j-nr+1-r\right](1-r)^{n-j}+\frac{j+1}{j}\binom{n}{j-1}\int (n-j+1)(1-r)^{n-j}\left[jr^{j-1}-(n+1)r^j\right]dr$$
$$=\binom{n}{j}(nr-j)r^j(1-r)^{n-j}-\binom{n}{j}r^j(1-r)^{n-j+1}+\frac{j+1}{j}\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{n-k}$$
$$=\sum_{k=0}^{j}(nr-k)\binom{n}{k}r^k(1-r)^{n-k}-\binom{n}{j}r^j(1-r)^{n-j+1}+\frac{1}{j}\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{n-k}$$

$$=\sum_{k=0}^{j}(nr-k)\binom{n}{k}r^k(1-r)^{n-k},\ \text{if}\ j\binom{n}{j}r^j=\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{j-1-k}.$$

(b) To show: $j\binom{n}{j}r^j=\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{j-1-k}$. This is equivalent to:

$$n(n-1)\cdots(n-j+1)r^j=(j-1)!\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{j-1-k}$$

and can be verified for $j = 1$ and $j = 2$. Then for $j > 2$, assume the equality is true for $j - 1$ as above. For j,

$$j!\sum_{k=0}^{j}(nr-k)\binom{n}{k}r^k(1-r)^{j-k}=j!\left[\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{j-k}+(nr-j)\binom{n}{j}r^j\right]$$
$$=j!(1-r)\sum_{k=0}^{j-1}(nr-k)\binom{n}{k}r^k(1-r)^{j-1-k}+(nr-j)n(n-1)\cdots(n-j+1)r^j$$
$$=j(1-r)n(n-1)\cdots(n-j+1)r^j+(nr-j)n(n-1)\cdots(n-j+1)r^j$$
$$=n(n-1)\cdots(n-j)r^{j+1}$$

This completes the proof of (a).

(c) Using (a), for $\frac{j}{n}<r\le\frac{j+1}{n}$, or equivalently $j<nr\le j+1$,

$$\frac{d}{dr}\left[c_4g_n(r)+c_5f_n(r)\right]=\frac{d}{dr}\left[c_4nr+(c_5-c_4)\sum_{k=0}^{j}(nr-k)p(k)\right]$$
$$=c_4n+(c_5-c_4)\binom{n}{j}(n-j)r^j(1-r)^{n-j-1}\left[j+1-(n+1)r\right]\equiv c_4n+(c_5-c_4)\phi(j,n,r).$$

Now, $\phi(j,n,r) < 0$ if $r > \frac{j+1}{n+1}$. Consider

$$\frac{d}{dr}(1-r)^{n-j-1}\left[(j+1)r^{j}-(n+1)r^{j+1}\right]=r^{j-1}(1-r)^{n-j-2}\left[n(n+1)r^{2}-2n(j+1)r+(j+1)j\right].$$

This quantity is negative between the two roots $r=\frac{j+1}{n+1}\pm\frac{\sqrt{n(j+1)(n-j)}}{n(n+1)}$. Clearly, the lower root is less than $\frac{j+1}{n+1}$, and it can be verified that the upper root is greater than or equal to $\frac{j+1}{n}$ (which is the upper endpoint of the interval for *r* where this expression for the combined cost function is valid) since $n \geq 2$. Therefore, $\phi(j,n,r)$ takes its most negative value at $r=\frac{j+1}{n}$, where it equals:

$$\begin{aligned}&\binom{n}{j}(n-j)\left(\frac{j+1}{n}\right)^{j}\left(1-\frac{j+1}{n}\right)^{n-j-1}\left[j+1-(n+1)\frac{j+1}{n}\right]\\&=-\binom{n}{j}(n-j)\frac{(n-j-1)^{n-j-1}(j+1)^{j+1}}{n^{n}}\end{aligned}$$

(d) Finally,

$$\begin{aligned}&\frac{d}{dr}\left[c_4 g_n(r)+c_5 f_n(r)\right]\geq 0 \text{ if } c_4 n \geq (c_5-c_4)-\binom{n}{j}(n-j)\frac{(n-j-1)^{n-j-1}(j+1)^{j+1}}{n^{n}}\\&\text{or } c_5 \leq c_4\frac{n^{n+1}+\binom{n}{j}(n-j)(n-j-1)^{n-j-1}(j+1)^{j+1}}{\binom{n}{j}(n-j)(n-j-1)^{n-j-1}(j+1)^{j+1}}\\&=c_4\left\{1+n^{n+1}\Big/\left[\binom{n}{j}(n-j)(n-j-1)^{n-j-1}(j+1)^{j+1}\right]\right\}=c_4\left[1+h(j,n)\right]\end{aligned}$$

Because $h(j,n)$ is an increasing function of *n*, then for a fixed value of *j*, $c_5 \leq c_4\left[1+h(j,2j+1)\right]$ suffices for all $n > 2j$. In turn, $h(j,2j+1)\geq h(1,3)=3.375$. Therefore, the expression in (30) is nonnegative as long as $c_5 \leq 4.375c_4$.

This completes the proof.

Proof of Lemma 3:

Inequality (21) can be rewritten as:

$$\sum_{j=w_l}^{n}\binom{n}{j}(r+\Delta_r)^j(1-r-\Delta_r)^{n-j} \geq \sum_{j=w_l}^{n}\binom{n}{j}r^j(1-r)^{n-j}$$

which is equivalent to:

$$P[x_1 > w_l - 1] \geq P[x_2 > w_l - 1] \; for \; w_l = \{1,2,...,n\} \quad (31)$$

where x_1 is binomial with parameters *n* and $r+\Delta_r$ and x_2 is binomial with parameters *n* and r. The result follows from the fact that the family of binomial distributions for fixed *n* is stochastically increasing in *r* (Shaked and Shanthikumar, 1994).

Figure 1.

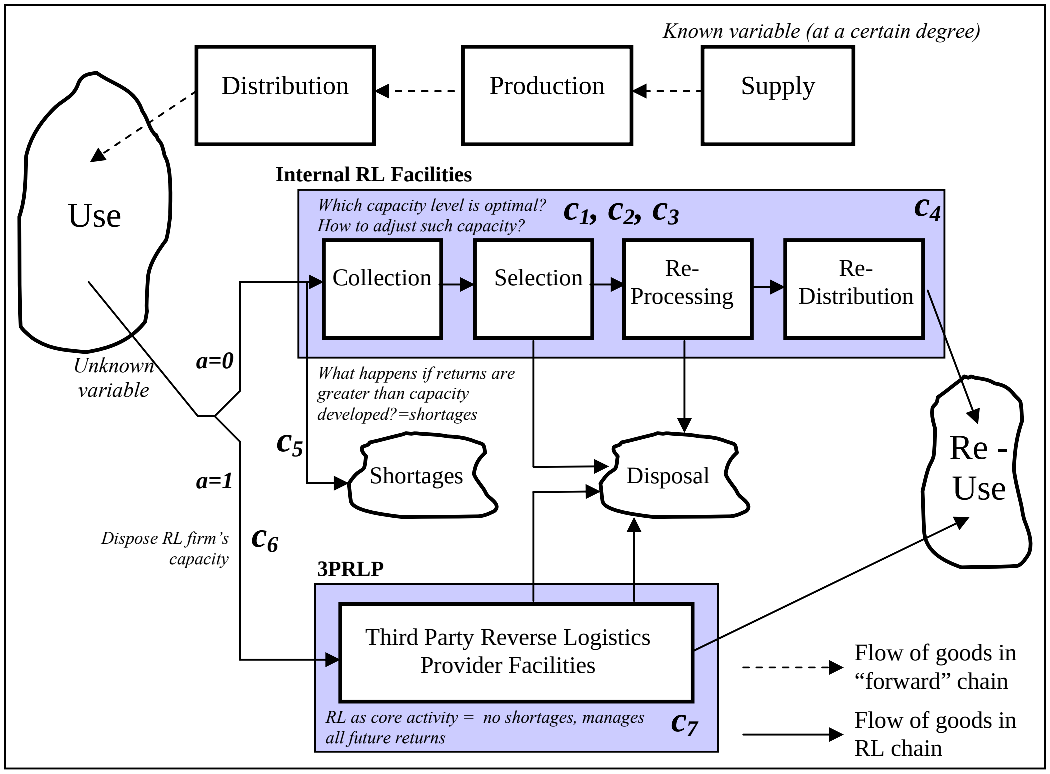
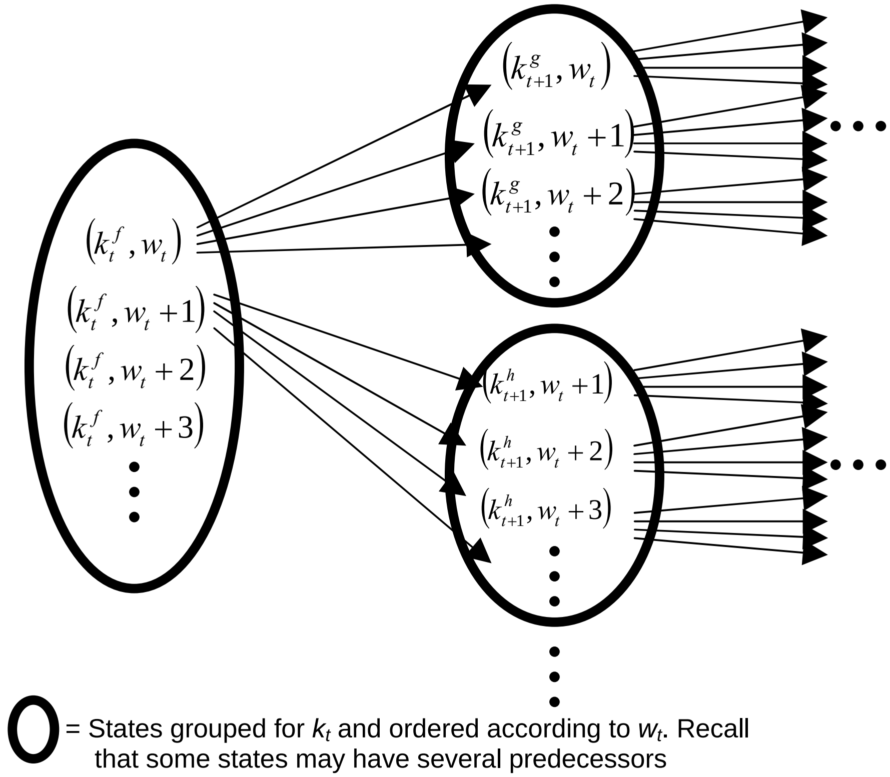

Known variable (at a certain degree)
Distribution
Production
Supply
Use
Internal RL Facilities
Which capacity level is optimal?
How to adjust such capacity?
c_1, c_2, c_3
c_4
Collection
Selection
Re-Processing
Re-Distribution
Unknown variable
a=0
What happens if returns are greater than capacity developed?=shortages
c_5
Shortages
Disposal
Re - Use
a=1
Dispose RL firm's capacity
c_6
3PRLP
Third Party Reverse Logistics Provider Facilities
RL as core activity = no shortages, manages all future returns
c_7
Flow of goods in "forward" chain
Flow of goods in RL chain

Figure 2.



$\left(k_t^f, w_t\right)$
$\left(k_t^f, w_t+1\right)$
$\left(k_t^f, w_t+2\right)$
$\left(k_t^f, w_t+3\right)$
$\left(k_{t+1}^g, w_t\right)$
$\left(k_{t+1}^g, w_t+1\right)$
$\left(k_{t+1}^g, w_t+2\right)$
$\left(k_{t+1}^h, w_t+1\right)$
$\left(k_{t+1}^h, w_t+2\right)$
$\left(k_{t+1}^h, w_t+3\right)$
= States grouped for k_t and ordered according to w_t. Recall that some states may have several predecessors

Table 1.

Case	Value on the right-hand side of the inequality	Worst case inequality for $n_t r < k_t$:	Worst case inequality for $k_t \le (n_t - i)r$
1) $X > n_t r$, $Y > (n_t - i)r$	$c_4 ir + c_5 (X - Y - ir)$ Worst case: $X - Y = i$	$c_7 - c_5 \ge$ $(c_3 - c_2)r - (c_5 - c_4)r$	$c_7 - c_5 \ge$ $(c_1 + c_3)r - (c_5 - c_4)r$
2) $X > n_t r$, $Y \le (n_t - i)r$	$n_t r(c_4 - c_5) + c_5 X - c_4 Y$ Worst case: $Y = nr - ir,$ $X = nr - ir + i$ because $c_5 \ge c_4$	$c_7 - c_5 \ge$ $(c_3 - c_2)r - (c_5 - c_4)r$	$c_7 - c_5 \ge$ $(c_1 + c_3)r - (c_5 - c_4)r$
3) $X \le n_t r$, $Y \le (n_t - i)r$	$c_4 (X - Y)$ Worst case: $X - Y = i$	$c_7 - c_4 \ge (c_3 - c_2)r$	$c_7 - c_4 \ge (c_1 + c_3)r$
4) $X \le n_t r$, $Y > (n_t - i)r$	$n_t r(c_5 - c_4) + c_4 X - c_5 Y + ir(c_4 - c_5)$ Worst case: $Y = (n_t - i)r,$ $X = n_t r$	$c_7 - c_4 r \ge (c_3 - c_2)r$	$c_7 - c_4 r \ge (c_1 + c_3)r$

Table 2.

	$r = 0.2$			$r = 0.3$			$r = 0.4$			$r = 0.5$		
t	k_t	w_l (1)	q_{t+1}	k_t	w_l	q_{t+1}	k_t	w_l	q_{t+1}	k_t	w_l	q_{t+1}
1	0	∞	0	0	∞	0	0	∞	0	0	0	1
2	0.4	∞	0	0.6	∞	0	0.8	∞	0	1.0	0	1
3	1.0	∞	0	1.5	∞	0	2.0	5	0.012	2.5	1	0.922
3	0.8	∞	0	1.2	∞	0	1.6	∞	0	2.0	3	0.524
3	0.6	∞	0	0.9	∞	0	1.2	∞	0	1.5	3	0.784
4	1.6	∞	0	2.4	8	0.001	3.2	6	0.049	4.0	4	0.405
4	1.4	∞	0	2.1	∞	0	2.8	7	0.188	3.5	4	0.580
4	1.2	∞	0	1.8	∞	0	2.4	7	0.040	3.0	5	0.455
4	1.0	∞	0	1.5	∞	0	2.0	8	0.010	2.5	5	0.663
4	0.8	∞	0	1.2	∞	0	1.6	8	0.025	2.0	6	0.524
4	0.6	∞	0	0.9	∞	0	1.2	8	0.064	1.5	7	0.352

(1) w_l = Threshold above which outsourcing is optimal ("∞"means there is no threshold; i.e., *a=0* is optimal for all states in that group). q_t = Probability that the threshold is crossed.

Captions for Figures and Tables:

Figure 1. Relationship between RL chain and costs considered in the MDM.

Figure 2. Illustration of the strict partial state ordering.

Table 1. Worst case analysis for Condition 3

Table 2. Value of the threshold and the probability of crossing it for $r = \{0.2, 0.3, 0.4, 0.5\}$