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Testing futures market efficiency: an empirical study

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by

Atcharawan Ngarmyarn

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1 INTRODUCTION

1.1 Statement of the Problem

Fama (1970) has defined an efficient market to be a market in which prices fully reflect all available information. Efficient market tests have a long history in the study of capital markets and futures markets. These tests on market efficiency are actually joint tests on efficiency itself and on the specified model of equilibrium expected prices or returns. There are numerous ways that equilibrium expected price or return can be specified. Some of the models that have been used in the literature are the random walk model, martingale model, submartingale model, market model, portfolio model and ARIMA model.

The models used in testing efficient futures market have mostly been borrowed from the literature on efficient capital markets. Because a futures market has different characteristics than a capital market, a test of a random walk model that is successful in capital market has been proved to be unacceptable in the futures market (Stevenson and Bear, 1970; Cargill and Rausser, 1975; Irwin and Uhrig, 1983). This is because both futures prices and cash prices are important in determining the equilibrium in futures markets. Therefore, tests that utilize either one alone seem to be inappropriate. One test that is often used to test if a futures market is efficient is to test whether futures prices are unbiased predictors of future
cash prices (Kofi, 1973; Bigman, Goldfarb and Schechtman, 1983). If the test is rejected, market efficiency cannot be concluded. However, these tests rely upon the assumption of risk neutrality of the participants. This is opposed to the normal backwardation hypothesis proposed by Keynes (1930) that there is risk transferring between hedgers and speculators in a way such that futures prices are biased downward predictors of future cash prices. The question is, if futures prices are biased predictors of future cash prices but the bias is forecastable, can we conclude that the futures market is inefficient? And if we think that the bias is caused from the risk averse behavior of economic agents, how can we extract that risk measure?\(^1\)

If we correctly believe that economic agents are risk averse rather than risk neutral, by incorporating this behavior as part of the predictive process together with futures prices, the bias predictors, though forecastable, will yield an equilibrium expected price model. This testable model can be used as a candidate of model specification on equilibrium expected prices or returns. And if actual prices vary randomly from these expected prices, the efficient market hypothesis is supported. Unfortunately, if the test is rejected, we still cannot conclude that the futures market is inefficient because of the nature of the joint test of efficient market. What we can conclude in this case is only that the model on equilibrium expected prices or

---

\(^1\)Some researchers have studied the existence of risk premium in futures markets. One method is to obtain risk premium in the context of capital asset pricing model (Dusak, 1973; Bodie and Rosansky, 1980). The risk measure in this study is the relationship between the variations in prices and the variations in the return on portfolio. Though the results show that returns and portfolio risks are close to zero, the variability of prices in the Keynesian sense is high. Kawai (1983), Turnovsky (1983) and Sarris (1984) have taken the risk averse behavior into the maximizing process of expected utility under uncertainty. The risk measure is part of the coefficient of the decision making process. However, these studies are theoretical studies and none of them has obtained the risk premium empirically.
returns is incorrect or that market is inefficient.

1.2 Purposes of the Study

The purposes of the study can be summarized as follows:

1. To obtain risk measures of producers and speculators empirically based on the theoretical model done by Turnovsky (1983).

2. To propose an explicit specification of the equilibrium expected cash prices by incorporating the risk averse coefficients obtained from (1) into the model.

3. To test if corn and wheat futures markets are efficient by testing the specified model in (2).

1.3 Organization

The organization of this study is as follows:

Chapter 2 contains the literature review on both theoretical studies and empirical studies on futures market efficiency tests. The efficient market hypothesis is reviewed first. Then, the general theory concerning the behavior of futures prices is reviewed. Finally, the empirical tests on the efficiency of futures markets are reviewed.

Chapter 3 presents a theoretical model based on a model developed by Turnovsky (1983). The derived model will include coefficients that characterize the degree of risk aversion of traders. The equilibrium expected price model that will be used as a guideline in efficiency test is also specified in this Chapter.
Chapter 4 summarizes the empirical estimation of the coefficients that will be used in testing the efficiency of futures market in the next chapter. The ones that are of particular interest are the coefficients that characterize the risk aversion of traders in corn and wheat futures market.

Chapter 5 contains the empirical tests on the efficiency of the corn and wheat futures market based on the equilibrium expected price model proposed in Chapter 3.

Chapter 6 gives a summary of and the conclusions drawn from the empirical results in the previous chapters.
2 LITERATURE REVIEW

This literature review will be separated into three parts. The first part is on the efficient market hypothesis. The second part is about the theories concerning futures markets. The last part looks at the empirical tests on the efficiency of futures markets.

2.1 On Efficient Market Hypothesis

Fama (1970) has defined an efficient capital market to be a capital market whose price fully reflects all available information. To determine empirically if price fully reflects available information, this definition has to be clarified. Fama (1976a) gave a more concrete definition of the testable implications of efficient market by asserting that in order for the markets to be efficient the market equilibrium can be characterized in terms of equilibrium expected returns, expectations are formed "rationally", and the information set that the market used must be the same as the information set that is truly available to the market.

To be more specific the efficient market test is normally done in terms of equilibrium expected price or return. The market is said to be efficient if the market's forecast of the price or return conditional on the information set that the market assesses is the same as the corresponding conditional expectations on all available
information. Equivalently, ¹

\[ E^m(P_t|\phi_{t-1}^m) = E(P_t|\phi_{t-1}), \]

or

\[ E^m(R_t|\phi_{t-1}^m) = E(R_t|\phi_{t-1}), \]

where

\[ R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \]

where \( P_t \) is an asset's price at time \( t \), \( R_t \) is the rate of return on the asset at time \( t \), \( \phi_{t-1} \) is the information set that is available at time \( t - 1 \), \( \phi_{t-1}^m \) is the information set at time \( t - 1 \) that the market assesses, \( E^m(.) \) is the subjective expectation of the market and \( E(.) \) is the objective expectation or mathematical expectation.

Hence the market is efficient if the actual prices or returns deviate randomly from the equilibrium expected prices or returns. However, since the efficient market test is the joint test on efficiency and any assumptions on how the equilibrium expected return is determined, if the result of the test turns out to reject the hypothesis on the randomness of prices around their equilibrium expected returns, it is difficult to conclude that the market is not efficient.

Fama (1970) also gave guidelines for the data that should be included in the information set. He classified the data into three information subsets. The weak form test is based on only the historical data of the price series itself. The semi-

¹The efficient markets hypothesis actually restricts the whole distribution of prices or returns, that is the joint probability density function of prices conditional on the information set that the market assesses is the same as the true joint probability density function conditional on the information set that is all available. That is \( f(P_{t1}, \ldots, P_{nt}|\phi_{t-1}^m) = f(P_{t1}, \ldots, P_{nt}|\phi_{t-1}) \) Hence, for the market to be efficient in a strong sense, not only the first moment but all moments of these two conditional distributions must be the same.
strong form test includes all publicly available information. And the strong form test involves all publicly available information as well as inside information that is available only to a particular group. These subclasses of information are used as a standard rule to decide what kind of data should be included in the information set.

The information set that is going to be used in an efficient market test is closely related to the selected equilibrium expected return model. The possible equilibrium expected return models are numerous, depending on the nature of the market that we are considering. Fama (1976a) has suggested four basic models for testing the efficiency of a capital market. Those equilibrium expected return models are: expected returns are constant, expected returns are positive, returns conform to the market model, and returns conform to the risk-return portfolio model.

The first model where expected returns are constant is the model where prices follow a random walk. If prices follow a random walk, the current period price is equal to the previous period price plus a white noise random disturbance. That is

\[ P_t = P_{t-1} + \epsilon_t \]

where \( E(\epsilon_t | \phi_{t-1}) = 0 \).

Therefore the expected price conditional on information set is \( E(P_t | \phi_{t-1}) = P_{t-1} \). Thus, the price process is a martingale. Under the random walk assumption the conditional probability density function of returns is the same as the unconditional one. That is

\[ f(R_t | \phi_{t-1}) = f(R_t), \]
which means the returns are serially independent. If returns are independent, their serial covariances are zero and their conditional mean is equal to the unconditional mean which is constant for the returns through time. Thus,

\[ E(R_t | \phi_{t-1}) = E(R_t). \]

And if the market is efficient,

\[ E^m(R_t | \phi_{t-1}^m) = E(R_t | \phi_{t-1}) = E(R_t). \]

To test if prices follow a random walk or martingale, it is sufficient to examine whether current actual prices deviate randomly from past period prices. If they do, then prices follow the random walk model.

The second model that is used in testing efficient market is the submartingale model. If prices follow a submartingale, the expected equilibrium price conditional on the available information set will be greater than or equal to the previous period’s price, i.e.,

\[ E(P_t | \phi_{t-1}) \geq P_{t-1}. \]

Since the rate of return is just the percentage change in prices, if \( E(P_t | \phi_{t-1}) \geq P_{t-1} \), it means that \( E(R_t | \phi_{t-1}) \geq 0 \) or equilibrium expected returns are positive. If this specified model is true, there will not be any trading rule that can beat buy and hold.

If prices follow the submartingale, the efficient market holds if the actual prices deviate randomly from the equilibrium expected prices, or equivalently

\[ E^m[P_t - E^m(P_t | \phi_{t-1}^m) | \phi_{t-1}^m] = E[P_t - E(P_t | \phi_{t-1}) | \phi_{t-1}] = 0 \]
The empirical test on this model is normally done by constructing some filter rules\(^2\) to see if there are some profits involved. Efficient market is concluded under this model if there are no filter rules that can beat buy and hold.

The third model is the market model. Let \( R_{jt} \) and \( R_{mt} \) represent the rate of return for the security \( j \) and the market rate of return\(^3\) at time \( t \), where \( R_{jt} \) and \( R_{mt} \) are stationary. The market model states that the security rate of return can be expressed as a linear function of the market portfolio return. That is,

\[
R_{jt} = \alpha_j + \beta_j R_{mt} + \epsilon_t,
\]

where

\[
E(\epsilon_t) = E(\epsilon_t R_{mt}) = 0 \quad \text{for all } t.
\]

Then

\[
\beta_j = \frac{\text{cov}(R_{jt} R_{mt})}{\text{var}(R_{mt})},
\]

and

\[
\alpha_j = E(R_{jt}) - \beta_j E(R_{mt}).
\]

Therefore, consistent estimates of \( \beta_j \) and \( \alpha_j \) are the OLS estimates, \( \hat{\beta}_j \) and \( \hat{\alpha}_j \) respectively. If an efficient market holds,

\[
E^m(R_{jt} | \phi_{t-1}^m, R_{mt}) = E(R_{jt} | \phi_{t-1}, R_{mt}),
\]

or

\[
\alpha_j^m + \beta_j^m R_{mt} = \alpha_j + \beta_j R_{mt}.
\]

\(^2\)The x% filter rule suggests that buying assets when their prices are x% higher than the previous low and selling the assets when their prices come down x% from the previous high.

\(^3\)The market rate of return is the weighted average of the returns of all stocks in the market. Normally, the Standard and Poor 500 Index is used as market rate of return.
Under this model, the expected return is obtained as

\[ \hat{R}_{jt} = \hat{\alpha}_j + \hat{\beta}_j R_{mt}. \]

If the actual returns deviate randomly from the predicted values of returns, an efficient market is concluded.

The last specification according to Fama is one in which returns conform to a risk-return relationship. This specification is based on the two-parameter portfolio model introduced by Markowitz (1959). If prices are drawn from a normal distribution then the entire distribution of prices can be characterized by two parameters, mean and variance. The minimum variance portfolio can be obtained by minimizing variance subject to a certain expected return. Under this model the hypothesized behavior of returns is

\[ R_{jt} = R_{ft} + \beta_{jm}(R_{mt} - R_{ft}) + \epsilon_t, \]

or

\[ R_{jt} - R_{ft} = \beta_{jm}(R_{mt} - R_{ft}) + \epsilon_t, \]

where \( R_{jt} \) and \( R_{mt} \) are stationary, \( R_{ft} \) is the riskfree rate of return\(^4\) which is assumed to be uncorrelated with \( R_{mt} \), and \( \epsilon_t \) is a random disturbance which represents the unsystematic risk that is hypothesized to be uncorrelated with \( R_{ft} \) and \( R_{mt} \).

Then

\[ \beta_{jm} = \frac{cov(R_{jt}, R_{mt})}{var(R_{mt})}, \]

\(^4\)The riskfree rate of return is supposed to be rate of return on the asset with no risk. Normally the treasury bill rate is used as the riskfree rate of return.
where $\beta_{jm}$ is known as the beta-coefficient which is interpreted as systematic risk representing the risk premium of asset $j$. When the beta-coefficient is multiplied by the difference between market rate of return and riskless rate of return, it represents the additional return that asset $j$ should yield to compensate for the risk premium, $\beta_{jm}$. The efficient market hypothesis states that

$$E^m(R_{jt}|\phi^m_{t-1}) = E(R_{jt}|\phi_{t-1}).$$

The predicted rate of return from this model is

$$\hat{R}_{jt} = \hat{R}_{ft} + \beta_{jm}(R_{mt} - R_{ft}).$$

As before, the randomness of actual $R_{jt}$ from $\hat{R}_{jt}$ supports efficiency of the market.

The survey done by Fama (1970) showed that the efficient market hypothesis cannot be rejected for many capital markets when the random walk or martingale model is used to calculate equilibrium expected returns.

Levich (1979) and Begg (1982) reviewed the survey of the efficient markets literature done by Fama (1970, 1976a). They asserted that tests of market efficiency are actually joint tests on the model of how equilibrium expected prices or returns are determined and on the randomness of prices around this equilibrium expected prices or returns. Correct specifications on the equilibrium expected returns are needed in order to determine if economic agents assess information optimally. One cannot automatically conclude that the market is inefficient by empirically rejecting the hypothesis of unsystematic forecast errors unless one is convinced that the model used to explain price determination is correct.
2.2 On Futures Markets

The appropriate equilibrium expected return model depends on the particular market under consideration. For example, Levich (1979) has done a study on foreign exchange markets in which he separates the efficient market test into a test on spot market efficiency and a test on forward market efficiency. However, testing the efficiency of these two markets separately may not be appropriate since they are closely related. If there are two markets in which prices are simultaneously determined, the nature of both markets should be taken into account in modelling either market.

Futures markets form another distinct kind of market whose existence depends upon the cash market. Commodity futures markets have a different nature from stock markets although the trading procedures may be similar. For example, commodities generally have shorter lives than stocks. Stocks are considered to be perpetuities while commodity lives usually do not last much more than a year. The events that determine their prices can be quite different. Drought and flood could affect prices in futures markets, while not doing much to the stock markets. These factors may lead to an equilibrium expected return model which is quite different than the models used to study capital markets.

Returns that an economic agent could make from futures markets depend on the difference between the realized cash prices when the contracts are expired and the futures prices when the contracts are made. In case of no actual delivery, the returns are the difference between the futures price when the contracts are released and the futures prices when the contracts are made. The futures prices when the contracts are released in the delivery month should not be much different from the
realized cash prices when the contracts mature because agents could obtain the information about the commodity up to the delivery month.

If the futures market is efficient, futures prices reflect all available information to traders. How well the futures prices actually serve as an information carrier to traders is debatable. Some claim that futures prices should be unbiased predictors of future cash prices if these markets are efficient. Some argue that risk attitudes can cause a bias even if the markets are efficient. If the bias is predictable and has been taken into account in the formation of expected future cash prices the futures market is efficient. Therefore, the biased prediction of future cash prices and the inefficiency of futures market should not be treated as the same thing.

Studies on the theory of futures markets behavior have been done by numerous economists. Two major theories, which need not be mutually exclusive, have been proposed. The first one is the theory of normal backwardation by John Maynard Keynes in 1930, and another one is the theory of price of storage by Holbrook Working in 1949.

According to the theory of normal backwardation which was proposed by Keynes (1930) and Hicks (1946), futures prices are downward biased estimates of the cash prices expected to prevail at the time the futures contracts are going to mature. Underlying this hypothesis is the hypothesized behavior of hedgers and speculators. Hedgers can avoid price risk by transferring risk to speculators. The speculators who step in to take this risk get some benefit called the risk premium. The risk premium that hedgers have to pay causes the downward bias in futures prices. The further the distance in time of the futures prices from the expected cash prices, the lower the futures prices are going to be, compared to expected cash
prices. Hence, under this hypothesis the hedgers on the average will be net short in futures market while the speculators will be net long.\(^5\)

Holbrook Working (1949) has proposed the theory of price of storage as another alternative to explain the relationship between cash prices and futures prices. Working had done empirical studies on the wheat futures market and could not find evidence supporting Keynes' backwardation hypothesis. Since wheat is a storable commodity, Working looks at the storable nature of the commodity to find an explanation for the price behavior. He stated: "...relationships between prices for delivery at the two different dates are commonly regarded as depending on the "cost" of carrying the stocks." With the existence of a futures market the hedgers can anticipate the return for storage by the difference between futures prices for two delivery dates at a given time. This difference, known as the spread, can be positive or negative. If the return for storage is positive the futures prices for later delivery date will be higher than the futures prices which mature earlier. Therefore, under Working's hypothesis, the incentive for holding stock is the expectation on positive return that the stock holders may get. And in the presence of futures market, the expected returns for storage can be approximated through futures prices. The stock holders can be assured of the returns on their storage by locking themselves through hedging.

Though Working had proposed his theory to challenge Keynes' backwardation theory based upon his empirical findings in the wheat futures market, the fact is

\(^5\)However, if the expected cash prices are lower than futures prices, the situation is called "contango", and the speculators are supposed to be net short if this concept of risk transferring does exist. Overall, if futures prices are biased predictors of future cash prices, either biased downward under normal backwardation or biased upward under contango, they have been known as Keynes' normal backwardation.
that futures markets do not always show a positive return to storage. Sometimes the returns are negative. These negative returns are what Working called "inverse carrying charges". Working explained that an inverse carrying charge arises when there is a shortage of stocks. If stocks are abundant, the returns for storage should be positive which is the pattern that contradicts Keynes' normal backwardation. When the stocks are scarce there is an inverse carrying charge which implies the same pattern of futures prices as Keynes' normal backwardation. There is the question that if the expected return is negative why do stockholders still store the commodities? One reason is given by Kaldor's concept of "convenience yield". Kaldor (1939-40) stated that stocks of all goods possess a yield which is the convenience to the stockholders. If the stockholders do not have stocks available on hand to use at all time, they may lose some benefit caused from an unexpected event. For example, if the stocks are raw materials, the unexpected demand in final goods will cause a derived demand in raw materials. Having raw materials on hand will smooth the production process. This convenience yield is a compensation to the stockholders and this should be deducted from the carrying charges, which are warehousing costs, insurance and interest costs. If the stockholders value the convenience yield more, the normal carrying charge may have the reverse sign.

Blau (1944-45) thought that this convenience yield was small and still preferred the concept of risk premium by Keynes as an explanation for the downward bias of futures prices. Though Blau agreed with the concept of carrying charges for storable commodities, she did not agree on an inverse carrying charge as Working had proposed. She accepted that in the absence of uncertainty, the differences between cash prices and futures prices were net carrying costs. However, with
uncertainty, the risk premium should be taken into account. She set the rule for speculators that futures prices should be equal to expected cash prices minus risk premiums for the buying limit and plus risk premiums for the selling limit. And for hedgers, futures prices should be equal to cash prices plus carrying costs and risk premiums for buying limit and equal to cash prices plus carrying costs minus risk premiums for selling limit. The traders should not pay more than the buying limit and should not get less than the selling limit. Actually what Blau did was to combine Working's price of storage concept and Keynes' risk premium concept together.

Working's theory was supported by Telser (1958). Telser indicated that futures prices display no trend based upon his empirical study on cotton and wheat futures markets. His finding is opposed to the theory advanced by Keynes and Hicks that futures prices display an upward trend as they approach maturity. He found out that the seasonal pattern of stocks determined the spread. However, Cootner (1960) argued that the empirical test done by Telser did not necessarily contradict the Keynes and Hicks argument because Telser ignored the return to capital.

There are still many researchers on futures markets who follow these two main theories and apply their models to different commodities. Those who tried to test if futures markets are efficient usually incorporated either one of these two theories or both.

Keynes focussed on the stabilizing role of futures market, while Working focussed on its allocative role. If only the allocative function is relevant, futures

---

6 Even though the price of storage theory of Working sounds relevant for storable commodities, it does not sound reasonable for the commodities that could not be stored. Because carrying charges do not exist for the unstorable commodities or
prices should be unbiased predictors of future cash prices. We can conclude that Working's theory supports both the random walk model and the model on the unbiased predictors of futures prices. When the hedgers or speculators close their positions before the contracts mature, their profits are the difference between two future prices quoted in different dates for the same delivery date. However, when they wait until the contracts mature, the profits are the differences between cash prices at the delivery point on the maturity date and futures prices quoted in the past. If returns are based on this later issue, on the average, there should not be above normal profit if the differences between actual cash prices and futures prices when the contracts are opened are random. With this argument, the futures prices are about the same as expected cash prices or futures prices are unbiased predictors of future cash prices. But if Keynes is right, no matter whether Working's carrying charges are present or not, the futures prices will be biased predictors of future cash prices.

Generally the model that is used to test if futures prices are unbiased predictors of future cash prices is as follow:

\[ P_t = \alpha + \beta \cdot F_t - t_i + \epsilon_t, \]

where \( P_t \) = cash prices at time \( t \).

\( t_i \cdot F_t \) = future prices at time \( t - i \) for the delivery at time \( t \).

\( \epsilon_t \) = independent disturbances, which are uncorrelated with \( t_i \cdot F_t \).

are minimal for the commodities which cannot be continuously stored. Thus this theory can be applied to only certain kinds of commodities. On the other hand, the insurance premium concept of Keynes can be applied to either one.
\[ \alpha, \beta = \text{intercept and slope parameters.} \]

\[ E(\epsilon_t) = E(t-1F_t, \epsilon_t) = 0. \]

Mostly, the researchers who use this hypothesized model test if \( \alpha \) is not significantly different from zero and \( \beta \) is not significantly different from unity in order to confirm that the futures market is efficient. This specified model is based on the assumption of risk neutrality.

However, if Keynes is correct, \( \beta \) need not be unity, since futures prices can be biased predictors of future cash prices. Some interpret the existence of risk premiums as the existence of inefficient markets, however, this may not be true. If the risk premiums are forecastable, i.e., the value of \( \beta \) is known, the futures prices still summarize all the relevant information in forecasting future cash prices.

Kawai (1983), Turnovsky (1983) and Sarris (1984) have studied the stabilizing role of futures markets theoretically. Their studies are basically alike. The risk averse behaviors are imputed in the utility maximizing of hedgers and speculators. The Arrow-Pratt absolute risk aversion is used in all three papers. The cost functions are approximated by quadratic costs which represent increasing marginal cost. The individual demand and supply in each group are assumed to be homogeneous for the sake of aggregation, i.e., individuals in each group have identical cost functions, identical risk averse coefficients and identical carrying costs. The studies confirm the risk premium concept of Keynes. The simultaneous determination of futures prices and cash prices show that futures prices are biased predictors of future cash prices.

Kawai derived the conditional variance of prices and reported that cash prices are stabilized by futures markets if price uncertainty is caused by disturbances
to consumer demand, are destabilized if price uncertainty is caused by inventory demand shocks, and are ambiguous under production disturbances.

Turnovsky concluded that if producers or speculators are risk averse, the introduction of futures market will change the price variances and slopes of the demand and supply such that the existence of futures market will stabilize cash prices. If both are risk neutral, the introduction of futures market does not change the long run mean or variance of cash prices.

Sarris also reported that futures markets tend to stabilize the period to period fluctuations in cash prices in both the short run and long run if storage speculators do not change their risk attitudes and if futures speculators are risk averse. If the producers use futures prices in making decisions, cash prices will be stabilized by futures markets.

The theoretical studies by Kawai, Turnovsky and Sarris found the equilibrium prices by using supply and demand functions derived from well-specified optimization problems. The assumption on rational expectation which is used in all three studies is equivalent to imposing the condition that the utilization of information by the market is efficient. Although these models are highly nonlinear, the estimation of the coefficients empirically is possible using nonlinear techniques. Therefore, the risk aversion coefficients can be estimated. And if we believe that the futures price is a biased predictor of the future cash price where the risk averse behavior is the only factor that causes the bias, once the bias is computed, the equilibrium expected price based on the futures prices and their bias can be estimated.
2.3 On Efficient Futures Market Tests

In this section, the empirical literature on the efficiency of futures markets will be reviewed. The empirical tests on this issue have been based upon different hypotheses on the determination of equilibrium expected prices or returns. Almost all of the tests that were used in testing the efficiency of futures markets were initially used to test the efficiency of capital markets.

The first one that was used in the literature is the random walk model. Larson (1960) used time series analysis to test the randomness of corn futures prices for two periods, from 1922 to 1931 and from 1949 to 1958. The moving average stochastic process generating the series suggests that there is no excessive fluctuation in corn futures prices and prices follow a random walk.

Stevenson and Bear (1970) used the random walk model to test the efficiency of corn and soybean futures markets. They performed three kinds of tests which are: testing the zero autocovariance of price changes, the analysis of runs\(^7\) and the filter rules. The serial covariances showed negative biases for one-day and two-day lags where the bias was larger in the soybean market than in the corn market. A positive bias was shown for five-day lags where the bias was larger in corn market. The run tests also yielded results supporting the serial covariance tests. The filter rule tests were constructed for the data from 1957 to 1968 and indicated that a five percent filter outperforms the buy and hold. Therefore, the random walk model was rejected in all tests according to Stevenson and Bear.

\(^7\)The analysis of runs is to observe if futures price for a particular length of time go up and down with approximately 50% chance for each to determine if futures prices follow the random walk.
Though Stevenson and Bear obtained different results than Larson, this may due to the different period of the data selected and the length of the lags used.

Cargill and Rausser (1975) have studied the random walk behavior of corn, oats, soybean, wheat, copper, live beef cattle and pork bellies futures markets. The tests are performed using time series methods in both the time domain and frequency domain. The tests, which are weak form tests, reject the random walk model, although Cargill and Rausser also stated that these results do not mean that the market efficiency is necessarily rejected.

Barnhart (1984) has studied the nonmartingale behavior of futures prices using daily closing futures prices. He used the equilibrium solutions that Turnovsky (1983) derived by simplifying them to:

\[ P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 e_t, \quad 0 < \beta_1 < 1 \]
\[ F_t = a_0 + a_1 P_{t-1} + a_2 e_t, \quad |a_1| < 1, \]

where \( P_t \) is current cash price, \( P_{t-1} \) is previous period cash price, \( F_t \) is futures price quoted at time \( t \) for the delivery at time \( t+1 \), and \( e_t \) is the disturbance of the supply of and demand for the cash commodity.

The tests are performed using time series methods in both the time domain and frequency domain. The study was done on copper, oats, plywood, lumber, wheat, feeder cattle, sugar, corn, gold, soybean, cocoa, frozen orange juice, coffee and barley. The tests, which are weak form tests, reject the martingale behavior of futures prices and the efficient market is rejected. The study continued by specifying an autoregressive-moving average for the futures price series. The coefficient on the first order autoregressive term was found to be close to unity. The nonmartingale
behavior of futures price was found to be influenced by the variance of the supply of and demand for the cash commodity which was measured by the coefficients of variation from the first and second order daily cash autoregressions.

Formal researchers always viewed the random walk model or martingale model as equivalent to an efficient market model by using the implied specification that rates of returns are constant over time. This specification may not hold true in futures market. Since the random walk model is typically rejected for futures market, especially for later periods, other alternatives have been considered.

The random walk model can be considered as a special case of an ARIMA model. Some researchers who specify their equilibrium expected return models using ARIMA models may come up with the tests that accept or reject the random walk model. And if the random walk model is rejected the specified ARIMA model is offered for a better alternative. Gupta and Mayor (1981) used the ARIMA model to test tin, copper, sugar and coffee futures markets using weekly data from 1976 to 1979. They conclude that these futures markets are efficient.

Apart from the random walk and ARIMA models, the next attempt on testing the efficiency of futures markets leaned on the fact that not only are futures prices important in determining the efficiency of futures markets, but also cash prices. One important procedure is to test if futures prices are unbiased predictors of future cash prices. This test emphasizes that cash prices when contracts mature determine the returns to hedgers and speculators.

Tomek and Gray (1970) have done the empirical test on corn, soybean and potatoes by using annual data from 1952 to 1968. The model is

\[ P_c = \alpha + \beta P_f, \]
where $P_c$ is the cash price at harvest time and $P_f$ is the spring time futures price. If futures prices are unbiased predictors of future cash prices, $\alpha$ is not significantly different from zero and $\beta$ is not significantly different from unity. The hypothesis could not be rejected for corn and soybean and was rejected for potatoes. This might be due to the underlying theory of Working's price of storage, because corn and soybean can be continuously stored, while potatoes cannot be. Therefore the price of storage theory cannot be applied successfully with potatoes.

This argument is supported by Leuthold (1972). He has done a semi-strong form test of efficiency of livestock futures markets. The results showed that live-hog futures cannot be relied upon in predicting future cash prices. His study is another example which shows that Working's theory does not work well with unstorable commodities like livestock. Other support comes from Martin and Garcia (1981). Their results from tests on live cattle, live hog and hog futures markets indicate that the futures prices of these nonstorable commodities provide poor forecasts of future cash prices.

Kofi (1973) had also conducted the test on wheat and potatoes futures markets for the year 1953 to 1969. However, he did not use futures prices at planting time alone. The futures prices used are varied, ranging from one month intervals to eleven month intervals. His results showed that the shorter the time, the closer the slope coefficient to one. The one month interval is the best predictor for wheat and the two month interval is the best predictor for potatoes. Comparing wheat which is a representative of continuous inventory futures markets, and potatoes which is a

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8The selected time they use concerned the assumption that producers are hedgers who enter futures market in the planting time to avoid price risk which will be realized during the harvest time.
representative of discontinuous inventory futures markets, the results indicate that wheat futures prices are more reliable predictors of future cash prices than potato futures prices. This is another verification that Working's theory of storage price does not hold for a discontinuous inventory futures markets.

Leuthold (1974) compared the predictive performance of cash prices and futures prices for live cattle futures market. The results showed that about 15 to 36 weeks before delivery, current cash prices are better predictors of future cash prices. Cash prices are more reliable and more stable predictors than futures prices for distant futures.

Bigman, Goldfarb and Schechtman (1983) tested the predictive performance of future prices on future cash prices in wheat, corn and soybean. The results showed limited support where an efficient market can be confirmed only for the short term futures contract (6 weeks or less). For long term futures contracts, the efficient market hypothesis is rejected. This result is contradictory to what is obtained by Tomek and Gray in their study of corn and soybean futures markets.

Epps and Kukanza (1985) have tested the same concept on corn, wheat and oats futures markets. Their results show a conditional bias which they conclude is caused from risk averse behavior.

The previous tests on the predictive performance of futures prices mostly are done by OLS. The restriction of OLS is that in order to get the consistent estimates, the futures prices and the error terms must be uncorrelated. However, if we use futures prices as proxies for expectations on future cash prices, such futures prices must be a function of the error terms in the same period. That means the estimated coefficients obtained by OLS will not be consistent since the futures prices appear
as explanatory variables in the model.

Canarella and Pollard (1985) recognized this estimation problem. They used the vector autoregressive method of Sims (1980) to determine the relationship between cash prices and futures prices. The study is done on corn, wheat, soybean oil, soybean meal and soybeans. By assuming futures prices and cash prices form covariance stationary stochastic process, the vector autoregression is used to determine the equilibrium expected prices. They concluded that there would not be long run profits in commodity futures markets. Though the problem of correlation between the explanatory variables and the error terms is solved in this model, the model is still ad hoc and does not rely on any theoretical background.

The results from some of the papers preceding indicate that Working's theory of price of storage does not work well for nonstorables commodities or discontinuous commodities. Even for storable commodities, the theory does not hold for all time periods. It leads back to the cause of biasedness. The major candidate is the risk premium concept of Keynes. Other reasons as cited in the literature are transaction costs and market thinness. The reason on risk transferring can be applied to any commodities, storable or nonstorable, continuous or discontinuous storable, and for the active market or thin market. Therefore the later development in establishing the equilibrium expected price or return to test efficient markets is on finding the risk premium. As Levich (1979) specified: "Test for unusual profits in spot and forward speculation must include a risk measure for the speculative activity".

The former studies on risk premium have been done under the framework of the capital asset pricing model (CAPM). Dusak (1973) estimated the risk premium under the context of CAPM for wheat, corn and soybean futures market from
1952 to 1967. The beta-coefficients were obtained from running regressions of the difference between the returns on commodity futures and riskless rate of returns on the difference between the market returns and the riskless rate of return. This risk premium does not measure the variability in prices per se, but measures how a particular return of an asset or a portfolio relates to market return in addition to riskfree rate. It is known as systematic risk under CAPM. Dusak used a 2 week holding period to compute return, 15 day treasury bill rate as the riskfree rate and the Standard and Poor 500 Index as market rate of return. She found that the systematic risks during the period of her study are close to zero on the average for all three commodities.

Bodie and Rosansky (1980) also used CAPM to obtain the risk premium of commodity futures from 1950 to 1976. They used a three month holding period to compute rate of returns, the 90 day treasury bill rate to compute the riskfree rate and the Standard and Poor 500 Index to represent market portfolio. The estimated beta-coefficients were negative. They verified Dusak's conclusion by computing the systematic risks for the same commodities during the same period as Dusak's. The estimated beta-coefficients are close to zero as Dusak has found. This is not surprising since the risk premiums should not stay constant over time.

One doubt is how reliable the CAPM is in explaining the risk premium in a futures market? And, if CAPM does work in futures markets, is it appropriate to use Standard and Poor Index as market portfolio? We know that the Standard and Poor Index is a pool for common stocks, and if commodity futures have an opposite performance to common stock, the rate of returns to commodity futures are not going to yield a positive correlation with Standard and Poor Index. Isn't it
better to compute the market portfolio for commodity futures specifically instead of using the portfolio of common stocks? And since the stabilizing function of futures market is to reduce uncertainty in price, the important argument is that how can this systematic risk be used as a risk premium to reduce price risk since this risk measure is different from Keynes' price variability (Dusak, 1973).

Chang (1985) has studied returns to speculators and the theory of normal backwardation in the corn, wheat and soybean futures markets for three time intervals, July 15, 1951 through June 30, 1962, July 15, 1962 through June 30, 1972, and December 31, 1972 through December 31, 1980. He used nonparametric procedures to study the speculators who were considered to have forecasting power if they were long when there is positive price change or short when there was negative price change. His study showed that large speculators had superior forecasting ability. The normal backwardation was supported where the risk premiums were more in later periods than in earlier periods. The profit that large speculators had were the combination of risk premiums and their forecasting ability.

Fama (1987) tried reconcile the theory of storage and the theory of normal backwardation. He found that the theory of storage had better statistical support when high storage cost commodities showed a positive relationship with high basis standard deviation. However, for the risk premiums aspect, the study showed that the average changes in future prices for 21 commodities as a portfolio produced marginally reliable normal backwardation which was more supported in agricultural products than in other products. However, the result was not strong enough to conclude nonzero premiums.

The studies on the risk premiums seem to be mixed and do not lend an entirely
satisfactory answer to the predictive performance of futures prices, and the search for a better risk premium should continue. One way is to look back at the issue of the underlying microeconomic foundation to see how risk averse behaviors are reflected in market outcomes. Such studies have been done by several researchers as mentioned in the second part of this chapter. However, the risk premiums that they found are only on the theoretical point of view. None has really obtained the risk premiums empirically. Therefore, the main purpose of this paper is to estimate these risk measures explicitly to verify if futures market is efficient according to the specified model.
3 MODELLING FUTURES MARKETS' EQUILIBRIA

3.1 Introduction

The empirical study of this paper is an attempt to offer another model on equilibrium expected price to test the efficiency of futures market. The model will consider the existence of the risk averse behavior by incorporating the risk measures into the agent's utility function. By estimating all the decisions of economic agents together as a system, the risk premiums can be obtained. And these risk premiums will be used to augment the futures price in order to determine the equilibrium expected future cash price. This equilibrium expected price model can then be used to test if the futures market is efficient.

The risk premiums that are going to be estimated in this study are based on the theoretical model derived by Turnovsky (1983).\textsuperscript{1} Though the main purpose of his study is to investigate if futures markets stabilize cash prices, the derived model, if it is empirically estimated, will yield the average risk premiums of traders in the long-run. These risk premiums are known as Arrow-Pratt coefficients of absolute risk aversion.

\textsuperscript{1}In fact, there are other researchers who have developed similar models on the simultaneous determination of cash prices and futures prices (e.g., Kawai, 1983; Sarris, 1984).
behaviors of traders according to Keynes' definition better than the ones obtained from CAPM. The risk premiums obtained from CAPM only indicate the relationship between the returns on a particular commodity and the market returns, not the traders' behaviors per se. Therefore, they are not going to give any explanation to the normal backwardation theory according to Keynes'. On the other hand, this proposed model developed by Turnovsky offers an alternative risk premium which directly represents the traders' behavior corresponding to Keynes' hypothesis.

Despite being a better specification on risk premium this method of finding risk premium is rather restrictive. The fact is that the traders are assumed to have constant absolute risk aversion, which may not be true. The constant absolute risk aversion behavior could not explain the behavior of economic agents who become less risk averse as they become richer. Such individuals' utility functions should exhibit decreasing absolute risk aversion. However, the specified constant absolute risk aversion coefficient has a nice property. It can be represented by only one parameter. Therefore it is easier to handle and easier to estimate than decreasing absolute risk aversion.

3.2 The Model

The model that is going to be presented in this paper can be separated into two parts. The first one concerns finding the risk averse coefficients of producers and inventory holders. The other one concerns testing the efficiency of futures market by incorporating the risk averse coefficients of producers and inventory holders that are obtained from the first part.
3.2.1 On risk averse coefficients

From the maximization of expected utility of producers and inventory holders, the decision rules of producers and inventory holders are derived in both cash market and futures market. Since consumers will not be allowed to trade in futures market in this model, the risk attitude of the consumer will not be our main concern and will be ignored in this study. Therefore, consumer demand is simply assumed to have a conventional form.\(^2\)

The detailed derivation of the model is presented in Appendix A, and the variable list is presented in Appendix B. What will be shown in this chapter is only an overview of the model to maintain a focus on the big picture.

3.2.2 Producers or hedgers

Producers are assumed to be hedgers who always go short in the futures market.\(^3\) The representative producer makes his commodity production decision at time \(t - 1\) for output to be realized in time \(t\). At the same time he also makes his decision on the amount of commodity that he is going to hedge in the futures market. The representative producer is assumed to make these decisions in a way that maximizes his expected utility in period \(t\). The utility function is assumed to be a member of the class of negative exponential utility functions. As a result, max-

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\(^2\)Even though it is not obtained by utility maximization explicitly, the same argument is also assumed as underlying behaviour of consumers.

\(^3\)Turnovsky's model allows the traders to be either long or short in futures market in each period. However, when we are dealing with empirical data, there will not be enough data to estimate the relationship under his specified model. Therefore I placed a more restrictive assumption that the hedgers always go short and speculators always go long.
imizing expected utility is equivalent to maximizing a linear function in the mean and variance of the producer’s profits. More specifically, we express the producer’s problem as,

$$\text{Max } E_{t-1}(u_t^h) = E_{t-1}(\pi_t^h) - \frac{1}{2} \alpha \sigma_p^2$$

$$= (q_t - x_{t-1}(t)) E_{t-1}(P_t) + P_{t-1}^f x_{t-1}(t)$$

$$- C_t q_t - \frac{c}{2} q_t^2 - G_{t-1} x_{t-1}(t)$$

$$- \frac{1}{2} \alpha (q_t - x_{t-1}(t))^2 \sigma_p^2$$

(1)

Maximizing this utility function yields the commodity supply of the representative firm together with the amount to be hedged in the futures market. The results are the following decision rules of a producer or hedger:

**j**\textsuperscript{th} firm’s supply:

$$q_{tj} = \frac{1}{c} P_{t-1}^f - \frac{1}{c} C_t - \frac{1}{c} G_{t-1}$$

(2)

**j**\textsuperscript{th} firm’s hedging:

$$x_{t-1}(t)_j = \left(\frac{1}{a} + \frac{1}{c}\right) P_{t-1}^f - \frac{1}{a} E_{t-1}(P_t)$$

$$- \frac{1}{c} C_t - \left(\frac{1}{a} + \frac{1}{c}\right) G_{t-1}$$

(3)

where

$$a = \alpha \sigma_p^2$$

By assuming that the producers are homogeneous with the same risk premium, \(\alpha\), the aggregation of individual firms’ production and hedging constitutes aggregate

\[\text{4The negative exponential utility functions exhibit constant absolute risk aversion where the risk averse coefficient is indifferent to the size of wealth. With the assumption that the distribution of wealth is normal, this kind of utility function can be transformed into a linear function in the mean and variance by the moment generating function method.}\]
supply and aggregate hedging. That is

Aggregate supply:

\[ S_t = \sum_{j=1}^{n} q_{tj} = \frac{n}{c} P_{t-1}^f(t) - \frac{n}{c} C_t - \frac{n}{c} G_{t-1} + v_1 t \]

Aggregate hedging:

\[ X_{t-1}(t) = \sum_{j=1}^{n} x_{t-1}(t)_j = \left( \frac{a}{c} + \frac{n}{c} \right) P_{t-1}^f(t) - \frac{n}{c} E_{t-1}(P_t) - \frac{n}{c} C_t - \left( \frac{a}{c} + \frac{n}{c} \right) G_{t-1} + v_2 t-1 \]

3.2.3 Inventory holders or speculators

Inventory holders are also assumed to be speculators who always go long in futures market.\(^5\) The representative speculator makes his decision to invest in stock in period \( t - 1 \) for distributing in period \( t \). At the same time he also speculates to buy some of the commodity in futures market by buying futures contracts at time \( t - 1 \) for delivery at time \( t \), expecting to sell cash at time \( t \). His profit or loss will be partly from his storage and partly from his speculation. Inventory holders, like producers, have preferences which can be described by a negative exponential utility function. Therefore, his decision problem can be shown as follows.

\[
\text{Max} \quad E_{t-1}(u^s_t) = E_{t-1}(\pi^s_t) - \frac{1}{2} \beta \sigma_\pi^2(s)
\]

\[ = i_{t-1} [E_{t-1}(P_t) - P_{t-1}] + y_{t-1}(t) [E_{t-1}(P_t) - P_{t-1}^f(t)] + K_t i_{t-1} - \frac{k}{2} i_{t-1}^2 - G_{t-1} y_{t-1}(t) - \frac{1}{2} \beta [i_{t-1} + y_{t-1}(t)]^2 \sigma^2_p \]

\(^5\)This is again a more restrictive assumption than Turnovsky’s. Turnovsky allows them to be either long or short in futures market.
Maximizing the above utility function gives us the inventory demand of the representative inventory holder and the amount of the speculative commodity in the futures market.

\( j^{th} \) firm’s inventory demand:

\[
i_{t-1,j} = \frac{1}{\kappa} P^f_{t-1}(t) - \frac{1}{\kappa} P_{t-1} - \frac{1}{\kappa} K_t + \frac{1}{\kappa} G_{t-1}
\] (7)

\( j^{th} \) firm’s speculation:

\[
y_{t-1}(t)_j = \frac{1}{b} E_t - \frac{1}{\kappa} P^f_{t-1}(t) + \frac{1}{\kappa} P_{t-1}
\] (8)

where \( b = \beta \sigma_p^2 \)

And again, assuming that the inventory holders are homogeneous with the same risk premium, \( \beta \), the aggregate inventory demand and speculation can be found as,

Aggregate inventory demand:

\[
I_{t-1} = \sum_{j=1}^{n} i_{t-1,j} = \frac{n}{\kappa} P^f_{t-1}(t) - \frac{n}{\kappa} P_{t-1} - \frac{n}{\kappa} K_t + \frac{n}{\kappa} G_{t-1} + v_{3t-1}
\] (9)

Aggregate speculation:

\[
Y_{t-1}(t) = \sum_{j=1}^{n} y_{t-1}(t)_j = \frac{n}{b} E_t - \frac{n}{\kappa} P^f_{t-1}(t) + \frac{n}{\kappa} P_{t-1} + \frac{n}{\kappa} K_t - \frac{n}{b} + \frac{n}{\kappa} G_{t-1} + v_{4t-1}
\] (10)

3.2.4 Consumer demand

The consumers’ demand function for the commodity is not derived from utility maximization explicitly. It is assumed to have a conventional form. It depends upon current cash price, the exogenous variables corresponding to the demand equation
of that particular commodity and also the demand disturbance. That is, the aggregate demand for the commodity is

$$D_t = n[d_0 - d_1 P_t + d_2 XD_t + v5_t]$$  \hspace{1cm} (11)$$

If we assume that $n$ is large, though the number of individual in each group may not be equal; however, as each $n$ becomes larger, their ratio approaches unity. Therefore $n$ can be cancelled out. And the number of the firms will be ignored in here. The equilibrium in each market can be found by treating $n$ equal to unity.

3.2.5 Market equilibrium

The equilibrium in cash market and futures market can be found to be following:

**Cash Market**

Supply:

$$S_t = \frac{1}{c} F^f_{t-1}(t) - \frac{1}{c} C_t - \frac{1}{c} G_{t-1} + v1_t$$  \hspace{1cm} (12)$$

Beginning Inventory:

$$I_{t-1} = \frac{1}{k} P^f_{t-1}(t) - \frac{1}{k} P_{t-1} - \frac{1}{k} K_t - \frac{1}{k} G_{t-1} + v3_{t-1}$$  \hspace{1cm} (13)$$

Ending Inventory:

$$I_t = \frac{1}{k} P^f_t (t+1) - \frac{1}{k} P_t - \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v3_t$$  \hspace{1cm} (14)$$

Consumer Demand:

$$D_t = d_0 - d_1 P_t + d_2 XD_t + v5_t$$  \hspace{1cm} (15)$$

Market Clearing:

$$S_t + I_{t-1} = D_t + I_t$$  \hspace{1cm} (16)$$
Futures Market

Hedging:

\[
X_{t-1}(t) = \left(\frac{1}{a} + \frac{1}{c}\right)P_{t-1}^f(t) - \frac{1}{a}E_{t-1}(P_t) - \frac{1}{c}C_t
- \left(\frac{1}{a} + \frac{1}{c}\right)G_{t-1} + v_{2t-1}
\]  
(17)

Speculations:

\[
Y_{t-1}(t) = \frac{1}{b}E_{t-1}(P_t) - \left(\frac{1}{b} + \frac{1}{k}\right)P_{t-1}^f(t) + \frac{1}{k}P_{t-1}
+ \frac{1}{k}K_t - \left(\frac{1}{b} + \frac{1}{k}\right)G_{t-1} + v_{4t-1}
\]  
(18)

Market Clearing:

\[
X_{t-1}(t) = Y_{t-1}(t)
\]

where \(v_{1t}, v_{2t}, v_{3t}, v_{4t}, \text{ and } v_{5t}\) are zero-mean and serially uncorrelated stochastic processes.

Let \(\delta_1^2 = E(v_{1t}^2), \ldots, \delta_5^2 = E(v_{5t}^2)\).

Because of the connection between cash market and futures market, the equilibrium cash price and futures price have to be solved simultaneously via nonlinear procedures. In Appendix A it is shown that these prices are given by the following equations:

### 3.2.6 Current cash price

\[
P_t = \theta P_{t-1} + \frac{1}{k\left(\frac{1}{a} + \frac{1}{b}\right)\theta - d_1\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) - \frac{1}{k\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b}\right)}}
\]
\[
(\ell + q \Delta X)^{-1} \mathcal{G} p + 0_p \left( \frac{q}{1} + \frac{2}{1} + \frac{v}{1} \right) \int_0^{\alpha} \theta \left( \frac{\theta}{1} \frac{\frac{q}{1} + \frac{2}{1} + \frac{v}{1}}{1} \right) \left( \frac{\theta}{1} \frac{\frac{q}{1} + \frac{2}{1} + \frac{v}{1}}{1} \right) \left( \frac{\theta}{1} \frac{\frac{q}{1} + \frac{2}{1} + \frac{v}{1}}{1} \right) \\
= (1 + \ell)^{1} f_d
\]

Current Futures Price

(61)
These results show that the current cash and futures prices are functions of past period prices, the structural parameters, the exogenous variables and the autoregressive parameters of all the exogenous variables. Utilizing these results we can derive (see Appendix A) the following reduced-form aggregate decision-rules:
3.2.8 Inventory demand

\[
I_t = \frac{1}{k} \left[ \left( \frac{1}{a} + \frac{1}{b} \right) \theta + \frac{1}{k} \right] \left( \frac{1}{a} \right) P_{t-1} + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \theta + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) - 1
\]

\[
\frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \theta - d_1 \left( \frac{1}{a} \right) \left( \frac{1}{b} \right) \left( \frac{1}{c} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right)
\]

\[
\frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \theta - d_1 \left( \frac{1}{a} \right) \left( \frac{1}{b} \right) \left( \frac{1}{c} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right)
\]

\[
\left[ \left( \frac{1}{a} + \frac{1}{b} \right) \theta \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{b} \right) - \frac{1}{k} \right] \right] d_0 + d_2 E_{t-1}(X D_{t+j})
\]

\[
+d_2 E_{t+1}(C_{t+j}) + \frac{1}{k} E_{t+1}(K_{t+j}) - \frac{1}{k} E_{t+1}(K_{t+j+1})
\]

\[
+ \frac{1}{k} E_{t+1}(C_{t+j+1}) + \frac{1}{k} E_{t+1}(K_{t+j+1}) - \frac{1}{k} E_{t+1}(K_{t+j+2})
\]

\[
+ \frac{1}{k} E_{t+1}(K_{t+j+2}) - 2 \left( \frac{1}{k} \theta \right) \left( G_{t+j+1} - \frac{1}{k} \theta \right)
\]

\[
+ \frac{1}{k} E_{t+1}(K_{t+j+1}) + \frac{1}{k} E_{t+1}(K_{t+j+2}) - \frac{1}{k} E_{t+1}(K_{t+j+2})
\]

\[
+ \frac{1}{k} E_{t+1}(K_{t+j+1}) + \frac{1}{k} E_{t+1}(K_{t+j+2}) - \frac{1}{k} E_{t+1}(K_{t+j+2})
\]
\[ -2\left(\frac{1}{k} - \frac{1}{c} \right) E_t(G_{t+j}) \right) + \frac{k}{\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right)} \left[ \frac{1}{c} C_{t+1} \right] \\
+ \frac{1}{k} K_{t+1} \left[ (\frac{1}{a} + \frac{1}{c} \right) - (\frac{1}{a} - \frac{1}{k}\right) (G_t + v4_t - v2_t) \\
- \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v3_t \right] \]  
(21)

### 3.2.9 Consumer demand

\[ D_t = d_0 - d_1 \theta P_{t-1} + d_2 XD_t \]

\[ -\frac{1}{k} \left(\frac{1}{a} + \frac{1}{b}\right) \theta - d_1 (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{k} (\frac{1}{a} + \frac{1}{c} + \frac{1}{b}) \]

\[ \left[ \frac{1}{c} \right] \theta \sum_{j=0}^{\infty} \theta j \left[ \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) d_0 + d_2 E_{t-1}(X D_{t+j}) \right] \]

\[ + \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) \]

\[ + \frac{1}{k} E_{t-1}(C_{t+j+1}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{c} E_{t-1}(C_{t+j}) \right] \]

\[ + \frac{1}{k} E_{t-1}(K_{t+j}) + 2 \left[ \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c}\right) E_{t-1}(G_{t+j}) - 2 \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c}\right) \right) \right] \]

\[ - \frac{1}{k} \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) \theta \sum_{j=0}^{\infty} \theta j \left[ \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) d_0 \right] \]

\[ + d_2 E_t(X D_{t+j}) + \frac{1}{k} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) \]

\[ + \frac{1}{k} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{c} E_t(C_{t+j}) \right] \]

\[ + \frac{1}{k} E_t(K_{t+j}) + 2 \left[ \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c}\right) E_t(G_{t+j}) - 2 \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c}\right) \right) \right] \]

\[ - \frac{1}{k} \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) \theta \sum_{j=0}^{\infty} \theta j \left[ \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) d_0 \right] \]
\[ X_t(t+1) = \left[ \frac{1}{a} + \frac{1}{c} \right] \left[ \frac{(\frac{1}{a} + \frac{1}{b})\theta + \frac{1}{k}}{\frac{1}{a} - \frac{1}{c} + \frac{1}{b} + \frac{1}{k}} \right] \frac{-\theta}{a} \theta P_{t-1} + \frac{1}{k} (\frac{1}{a} + \frac{1}{c}) \left[ \frac{1}{k} - \frac{1}{a} \right] \right] - \frac{1}{a} \theta P_{t-1} + \]

\[ \frac{(\frac{1}{a} + \frac{1}{b})\theta + \frac{1}{k} - \frac{1}{a} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)\theta}{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) \left[ \frac{1}{k} (\frac{1}{a} + \frac{1}{b})\theta - d_1 (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) \right] - \frac{1}{k} (\frac{1}{a} + \frac{1}{c} + \frac{1}{b})} \]

\[ \left[ \frac{1}{a} + \frac{1}{c} \right] \sum_{j=0}^{\infty} \theta_j \left[ (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) d_0 + d_2 E_{t-1}(X D_{t+j}) + \frac{1}{c} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \frac{1}{c} \right] \]

\[ E_{t-1}(C_{t+j+1}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{k} E_{t-1}(C_{t+j}) \right] + \frac{1}{k} E_{t-1}(K_{t+j}) + 2 \left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right] E_{t-1}(G_{t+j}) - 2 \left( \frac{1}{k} \frac{1}{c} - \frac{1}{c} \frac{1}{b} \right) E_{t-1}(G_{t+j}) - 1 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_0 + d_2 E_t(X D_{t+j}) \]

\[ + \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) + \frac{1}{c} \frac{1}{k} E_t(C_{t+j+1}) \]

\[ + \frac{1}{k} E_t(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{k} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) \right] + \frac{2}{k} \left( \frac{1}{a} + \frac{1}{c} \right) E_t(G_{t+j}) - 2 \left( \frac{1}{k} \frac{1}{c} - \frac{1}{c} \frac{1}{b} \right) E_t(G_{t+j-1}) \]

\[ - \left[ \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right] \epsilon_t + \frac{1}{k} (v^4 t - v^2 t) \]

\[ - \left( \frac{1}{c} + \frac{1}{k} \right) (v^4 t - v^2 t - 1) \right] + \frac{\theta}{k (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} \]
\[
\begin{align*}
(23) \quad & 1 + \mathcal{C} \left( \frac{q}{1} + \frac{a}{1} \right) - 1 + \mathcal{C} \frac{a}{1} - \\
& \left[ 1 + \mathcal{C} \left( \frac{q}{1} + \frac{a}{1} \right) - \left( \frac{q}{1} + \frac{a}{1} \right) \right] + 1 + \mathcal{X} \frac{q}{1} + \\
& 1 + \mathcal{C} \frac{a}{1} \left( \frac{q}{1} + \frac{a}{1} \right) + \left[ \left( 1 + \mathcal{C} \frac{a}{1} \right) \mathcal{G} \left( \frac{q}{1} + \frac{a}{1} \right) \right] + \\
& (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \left( \frac{q}{1} + \frac{a}{1} \right) - (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + \\
& \left[ (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} - (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + \\
& (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + 0p (\frac{q}{1} + \frac{q}{1} + \frac{a}{1} + \frac{a}{1}) \right] \\
& \mathcal{G} \frac{q}{1} \leq \infty \left[ \left( \frac{q}{1} + \frac{q}{1} \right) \frac{q}{1} - \left( \frac{q}{1} + \frac{a}{1} \right) \right] + \\
& \left[ 1 + \mathcal{C} \frac{a}{1} - \left( \frac{q}{1} + \frac{a}{1} \right) - \left( \frac{q}{1} + \frac{a}{1} \right) \right] + \\
& 1 + \mathcal{C} \frac{a}{1} \left( \frac{q}{1} + \frac{a}{1} \right) + \left[ \left( 1 + \mathcal{C} \frac{a}{1} \right) \mathcal{G} \left( \frac{q}{1} + \frac{a}{1} \right) \right] + \\
& (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \left( \frac{q}{1} + \frac{a}{1} \right) - (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + \\
& \left[ (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} - (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + \\
& (1 + \mathcal{C} \frac{a}{1}) \mathcal{G} \frac{q}{1} + 0p (\frac{q}{1} + \frac{q}{1} + \frac{a}{1} + \frac{a}{1}) \right] \mathcal{G} \frac{q}{1} \leq \infty
\end{align*}
\]
3.2.11 Aggregate speculation

\[ Y_t(t+1) = \left[ \frac{1}{k} + \frac{1}{b} \theta \right] \left\{ \frac{1}{k} + \frac{1}{b} \right\} \left( \frac{1}{a} \right) \theta P_{t-1} + \]

\[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{k} \right) \theta P_{t-1} - \left( \frac{1}{a} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ \sum_{j=0}^{\infty} \theta^j \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{c} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \left( \frac{1}{c} \right) \]

\[ + \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) - \left( \frac{1}{a} \right) \left( \frac{1}{b} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{k} E_{t-1}(K_{t+j}) + 2\left( \frac{1}{k} \left( \frac{1}{a} \right) \right) \theta P_{t-1}(X \text{ and } t+j) \]

\[ + \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) + \frac{1}{k} \left( \frac{1}{c} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{k} E_t(K_{t+j+1}) - \left( \frac{1}{a} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{k} E_t(K_{t+j+1}) - \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{k} E_t(C_{t+j+2}) + \frac{1}{k} E_t(K_{t+j+2}) - \left( \frac{1}{a} \right) \theta \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \theta P_{t-1} \]

\[ + \frac{1}{k} E_t(K_{t+j+1}) + 2\left( \frac{1}{k} \left( \frac{1}{a} \right) \right) \theta P_{t-1}(X \text{ and } t+j+1) \]
The only equation that is already a reduced form equation is the supply equation, $S_t$.

$$S_t = \frac{1}{c} P_{t-1}(t) - \frac{1}{c} C_t - \frac{1}{c} G_{t-1} + \nu_1 t$$  \hspace{1cm} (25)$$

By estimating either one of equation (19) through (24), the consistent estimates can be obtained. However, to get the efficient estimates, the contemporaneous correlation of the disturbances should be taken into account, and all reduced form equations should be estimated as a system. The method that will be used is an iterative seemingly unrelated regressions estimator (Harvey, 1981a) which will take into account the contemporaneous correlation of the reduced-form disturbances.

Though all the reduced form equations above are highly nonlinear, there are
only nine structural parameters together to be estimated. Those are $\alpha$, $\beta$, $\sigma^2_p$, $\theta$, $c$, $k$, $d_0$, $d_1$ and $d_2$. These parameters are restricted to be positive. All the ones that are hypothesized to be negative are already assigned the negative signs in the derivation.

By employing the nonlinear estimation technique, these parameters can be estimated. The ones that we are especially interested in are $\alpha$, $\beta$, $\sigma^2_p$, $c$ and $k$. $\alpha$, and $\beta$ are the risk premiums of producers and speculators; $\sigma^2_p$ is the long-run average one-period variance of price; and, $c$ and $k$ are costs of producer firms and inventory holders respectively.

3.3 On Testing the Efficiency of Futures Markets

From equation (19) in the Appendix A, the futures price is expressed as a function of expectation of future cash price, current cash price, exogenous variables and other disturbances as follows:

$$P_t^f (t+1) = \frac{1}{(\frac{1}{a} + \frac{1}{c} - \frac{1}{b} - \frac{1}{k})} \left[ (\frac{1}{a} + \frac{1}{b})E_t(P_{t+1}) - \frac{1}{k}P_t ight. $$

$$+ \frac{1}{c}C_{t+1} + \frac{1}{k}K_{t+1} + \left[ (\frac{1}{a} + \frac{1}{c})$$

$$- (\frac{1}{b} + \frac{1}{k}) \right] G_t + v4_t - v2_t \right] $$

(26)

Since we know that current futures price and current cash price are simultaneously determined, if we think of this expected cash price as equilibrium expected price, this equilibrium expected cash price surely depends upon current futures price, current cash price and also other exogenous variables.
Even though we know that the expected cash price has to be predetermined before the equilibrium value of current futures price and current cash price, however, if the market is efficient, rational expectations should hold. That means we can solve the expected price endogenously. It does not matter what kind of expectation that each individual forms, rational expectation hypothesis just tells us about the average consensus of expectation of all participants in determining current futures prices and current cash prices. Though this expectation cannot be observed directly, the results of this consensus are reflected in the equilibrium value of current futures prices and current cash prices. Hence, by observing these prices and other exogenous variables, we should be able to trace back to what the participants of these two markets agree upon the average expectation.

This hypothesized equilibrium expectation can be found through the observed value of current futures price, current cash price and other exogenous variables. And equation (26) previously becomes:

\[
\hat{P}_{t-1} = \frac{\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right)}{\left(\frac{1}{a} - \frac{1}{b}\right)} P_{t+1} - \frac{1}{k} P_t \\
- \frac{\frac{1}{c}}{\left(\frac{1}{a} + \frac{1}{b}\right)} C_{t+1} - \frac{\frac{1}{k}}{\left(\frac{1}{a} + \frac{1}{b}\right)} K_{t+1} \\
- \frac{\left(\frac{1}{a} + \frac{1}{c}\right) - \left(\frac{1}{b} + \frac{1}{k}\right)}{\left(\frac{1}{a} + \frac{1}{b}\right)} G_t
\]  

(27)

where \( \hat{\sigma}_p^2 = \hat{a} \hat{\sigma}_p^2 \) and \( \hat{\sigma}_p^2 = \hat{\beta} \hat{\sigma}_p^2 \).

This expectation is not just the expectation that is used to set current futures price but it is also the expectation that is used to set current cash price. On the
other hand, if we want to find the signal that tells us about the average expectation of economic agents, not only current futures prices will tell us, but also current cash prices, production cost and carrying cost. All jointly give us the information on equilibrium expected cash price. How heavy each signal is, depends on the risk averse coefficients of producers and inventory holders, indirect production cost, indirect carrying cost and also the variance of cash price itself.

We know that if the coefficients in (27) are going to be solved from this equation by substituting actual cash price at time $t - 1$ on the left hand side, not even the consistent estimates could be obtained. However, if we want to find the value of expected price directly from equation (27), it can be done simply. Because all those coefficients are already obtained via the system of nonlinear estimation as mentioned in before. And if we are not interested only in obtaining the consistent estimates that we can get, but also the asymptotically efficient estimates, we can get the estimates by estimating the whole system together via iterative seemingly unrelated method. Plugging those estimates and the actual value of current futures price, current cash price and exogeneous variables into equation (27), the expected cash price could be obtained.

And if actual price at time $t-1$ deviate randomly from this equilibrium expected cash price, efficient market is concluded. However, as already mentioned, if the hypothesis is rejected we still cannot say that the futures market is inefficient.

With this proposed model for testing efficient futures market, the empirical test based on the data on corn and wheat futures market can be performed. The details of the estimation used will be discussed in the next chapter.
4 ON FINDING RISK AVERSE COEFFICIENTS

4.1 Estimation Strategy

In this chapter the estimation of the parameters from the model specified in Chapter Three will be presented. The particular parameters that are of our interest are the parameters that characterize the degree of risk aversion of traders, the indirect cost of production and the indirect carrying cost. These parameters will be used in testing the efficiency of futures markets in the next chapter.

Because the reduced-form equations involve the infinite sums of the conditional expectations of the future value of exogenous variables, some additional structure must be imposed on these variables in order to express these infinite sums as closed forms. If the vector of the exogenous variables follows a finite order vector autoregression, the problem of infinite sums can be overcoming by following a procedure described by Hansen and Sargent (1980).

The reduced-form prediction formula of Hansen and Sargent can be traced back to Wald's theorem. If $X_t$ forms a jointly covariance stationary stochastic process where $X_t = (x_{1t}, x_{2t}, \ldots, x_{pt})'$, then there exists the vector moving average (VMA) representation of $X_t$ as:

$$X_t = B(L)\epsilon_t$$  \hspace{1cm} (28)
where $B(L) = \begin{bmatrix} B_{11}(L) & \cdots & B_{1p}(L) \\ \vdots & \ddots & \vdots \\ B_{p1}(L) & \cdots & B_{pp}(L) \end{bmatrix}$ is an $p \times p$ matrix of infinite order polynomials in the lag operator $L$.

$L$ is the lag operator.

$B^{ij}(L)$ is an infinite order polynomial in the lag operator $L$.

The elements of $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{pt})'$ are jointly fundamental white noise for $X_t$. If there exists $B(L)^{-1}$ such that it is one sided convergent in nonnegative powers of $L$, which is when all the roots of $|B(z)| = 0$ lie outside the unit circle, then $X_t$ has a vector autoregressive (VAR) representation, i.e.,

$$A(L)X_t = \epsilon_t \quad (29)$$

where $A(L) = B(L)^{-1}$, and

$$A(L) = I - A_1L - A_2L^2 \cdots - A_rL^r.$$

According to Hansen and Sargent (1980), if $r$ is finite, then the infinite sums of the conditional expectation of the future value of exogenous variables can be transformed into a finite order vector autoregression. That is,
\[
\sum_{k=0}^{\infty} \theta^k E_t X_{t+k} = A(\theta)^{-1}[I + \sum_{j=1}^{r-1} \sum_{k=j+1}^{r} \theta^{k-j} A_k]L^j X_t
\]
where \( E_t X_{t+k} \) = predicted value of the vector of exogenous variables for time \( t + k \), predicted at time \( t \).
\( X_t \) = vector of current value of exogenous variables
\( A_k \) = matrix of the \( k^{th} \) order VAR coefficients.
\( I \) = identity matrix.
\( L_j \) = \( j^{th} \) power lag operator.

Although equation (30) can be used to estimate the infinite sums in equations (19), it requires that the set of parameters to be estimated be expanded to include the VAR coefficients as will be shown in equation (31). The most desirable estimator of the risk averse coefficient is to estimate the structural coefficients and the VAR coefficients jointly. However, this method would involve jointly estimating too many parameters by numerical methods to be feasible.

Another method is to do two step estimation. First, estimate the VAR coefficients by applying OLS to equation (29). Second, estimate the structural coefficients of the original reduced-form equations conditional on the estimates of the VAR parameters. By doing so, the validity of the estimates of the structural parameters will depend on the validity of the estimates of the VAR coefficients. Therefore the standard errors of the estimates will be underestimated. Only if all the VAR parameters are estimated without error will the parameter estimates obtained be efficient.
Since each reduced-form equation includes all of the parameters that will be needed for the market efficiency test, it is sufficient to determine the underlying parameters of any one of these equations. A consistent estimator of the parameters in any one of the equations involves estimating the VAR by OLS and then estimating the parameters of the chosen reduced-form equation conditional on the first-stage estimates of the VAR.

4.2 Estimating the VAR

On estimating the first-stage estimates of the VAR, not only must the exogenous variables specified in the model in Chapter Three be included, but also other variables that help predict the former ones should be included. There are numerous exogenous variables that may be important for the representative firm in forecasting the future value of each exogenous variable for corn and wheat industry. However, there are practical limitations in choosing the proper exogenous variables for the VAR model due to the limited number of observations.

The exogenous variables that were previously included in the system of corn market equations were the direct cost of producing corn, the direct carrying cost, the animal units of livestock fed annually and the brokerage fee. The series of direct carrying cost and brokerage fee could not be obtained directly. Therefore, they were estimated by using some related variables. For the direct carrying cost, the major expense was the interest cost. Warehousing fees and insurance premiums were also part of carrying cost, however they were considered to be more of fixed costs which are treated as the indirect carrying cost in here. This part would be estimated as a coefficient in the system. The direct carrying cost was calculated by using the
Production Credit Association (PCA) loan rate times cash prices. A good series could not be found for the brokerage fee. The only knowledge of brokerage fee was reported in one of the U.S.D.A. publications, January, 1973 to be equal to 0.6 cent per bushel for both corn and wheat. Therefore this value was assumed to be the brokerage fee in the year 1972. The proxies for the other periods were calculated by using the ratio of average hourly earning: brokerage, between the calculated year and the year 1972 multiplied by 0.6 cent. The details of the data were shown in the Appendix C.

Other exogenous variables which were included to help the prediction process of those specified above were government support price and fertilizer price index. Though government support price was not part of the cost incurred to any firm, it might have an indirect effect on those variables. Fertilizer prices affected the cost of production directly, so this variable was also added.

The exogenous variables that were previously included in the system of wheat market equations were the direct cost of producing wheat, the direct carrying cost, the per capita disposable personal income and the brokerage fee. The series of direct carrying cost and brokerage fee were calculated in the same manner as described above in relation to the corn market equations.

The other exogenous variables that were added to help in the prediction of the first ones were the government support price and the fertilizer price index. They were included for the same reason given above in the discussion of the corn market equations.

Since the vector autoregressive representation is possible only if the process is wide-sense stationary, all the series were filtered to remove nonstationary compo-
nents. Only a nonstationary mean was apparent, therefore the data used in this estimation process were detrended and the differences from mean series were used.

The proper lag length of the VAR model was selected based on the modified log likelihood ratio test (Sims, 1980). Different VAR orders were tried and pairwise selections were done. The log likelihood ratio test has asymptotically chi-squared distribution, where the chi-squared statistic is calculated as

\[ X^2(q) = (T - k) \log \text{det} D_r - \log \text{det} D_u \]

where \( \text{det} D_r \) = determinant of the estimated contemporaneous covariance matrix of the restricted model’s disturbances.

\( \text{det} D_u \) = determinant of the estimated contemporaneous covariance matrix of the unrestricted model’s disturbances.

\( T \) = number of observation.

\( k \) = correction factor.

\( q \) = number of restrictions.

In testing a lag length of order ‘\( n \)’ against a lag length of order ‘\( n - m \)’ with ‘\( p \)’ variables, the value of ‘\( k \)’ which is the number of correction factor will be equal to ‘\( n \)’ times ‘\( p \)’. The value of ‘\( q \)’ is the number of restrictions imposed which is equal to ‘\( m \)’ times ‘\( p^2 \)’.

Under the log likelihood ratio test, if the restricted model is not statistically different from the unrestricted model, the chi-squared statistic will be smaller than
Table 4.1: Testing lag length for corn

| Lag length | Log $|D_u|$ | Log $|D_r|$ | Chi-square | Significance level |
|------------|--------|--------|------------|--------------|
| 1 vs 2     | 64.13413 | 65.48265 | 51.24410 | 0.047        |
| 2 vs 3     | 62.07960 | 63.99300 | 59.31549 | 0.008        |
| 3 vs 4     | 59.95008 | 62.04271 | 50.22322 | 0.058        |
| 4 vs 5     | 58.67624 | 59.87408 | 20.36325 | 0.983        |

There are 6 exogenous variables included in the VAR process in corn and wheat equation. Hence, there are 36 restrictions for each pairwise selection. As Nickelsburg (1985) pointed out, the modified log likelihood ratio test proposed by Sims (1980) was biased towards large lag models. Therefore the 1% significance level was chosen as the criterion in selecting the appropriate lag length. The chi-squared statistic with 36 degree of freedom and 1% significance level has the value between 50.8922 and 63.6907. The only pair that the restriction does make statistically different is the second pair for corn and the first pair for wheat. Thus, the VAR(3) was selected as an appropriate model for the exogenous variables for the corn equation and the
Table 4.2: Testing lag length for wheat

| Lag length | $\log |D_u|$ | $\log |D_r|$ | Chi-square | Significance level |
|------------|------------|------------|------------|--------------|
| 1 vs 2     | 6.95965    | 8.52781    | 59.590     | 0.008        |
| 2 vs 3     | 5.06824    | 6.74441    | 51.961     | 0.041        |
| 3 vs 4     | 2.99911    | 5.02933    | 48.725     | 0.077        |
| 4 vs 5     | -0.54420   | 1.68728    | 37.935     | 0.381        |

VAR(2) was selected for the wheat equation.

The VAR(3) for corn can be put in a compact form as:

$$X_t = A(1)X_{t-1} + A(2)X_{t-2} + A(3)X_{t-3} + \epsilon_t$$

where $X_t$ = vector of exogenous variables in corn equation, i.e.,

$$X_t = (x_{1t} \ x_{2t} \ x_{3t} \ x_{4t} \ x_{5t} \ x_{6t})^T$$

where $x_{1t}$ is animal unit of livestock fed annually.

$x_{2t}$ is direct cost of production.

$x_{3t}$ is direct carrying cost.

$x_{4t}$ is cost of using futures market, which is brokerage fee.

$x_{5t}$ is fertilizer price index.

$x_{6t}$ is government support price.

$X_{t-1}$ = vector of one period lag exogenous variables.
\[ X_{t-2} = \text{vector of two period lag exogenous variables.} \]
\[ X_{t-3} = \text{vector of three period lag exogenous variables.} \]
\[ A(1) = \text{matrix of the VAR coefficients of the one period lag exogenous variables.} \]
\[ A(2) = \text{matrix of the VAR coefficients of the two period lag exogenous variables.} \]
\[ A(3) = \text{matrix of the VAR coefficients of the three period lag exogenous variables.} \]
\[ \epsilon_t = \text{disturbance terms.} \]

The VAR(2) for wheat can be put in a compact form as:

\[ X_t = A(1)X_{t-1} + A(2)X_{t-2} + \epsilon_t \]

where \( X_t = \text{vector of exogenous variables in corn and wheat equation, i.e.,} \)

\[ X_t = (x_{1t} \ x_{2t} \ x_{3t} \ x_{4t} \ x_{5t} \ x_{6t})' \]

where \( x_{1t} \) is per capita disposable personal income.
\( x_{2t} \) is direct cost of production.
\( x_{3t} \) is direct carrying cost.
\( x_{4t} \) is cost of using futures market, which is brokerage fee.
\( x_{5t} \) is fertilizer price index.
\( x_{6t} \) is government support price.
\( X_{t-1} = \) vector of one period lag exogenous variables.

\( X_{t-2} = \) vector of two period lag exogenous variables.

\( A(1) = \) matrix of the VAR coefficients of the one period lag exogenous variables.

\( A(2) = \) matrix of the VAR coefficients of the two period lag exogenous variables.

\( \epsilon_t = \) disturbance terms.

The estimated coefficients of the VAR(3) for corn industry are:
$$A(1) = \begin{bmatrix} .76526 & .00006 & -.0088 & -.00042 & -.82187 & .00003 \\ 47.56252 & -.17640 & 1.21796 & .01165 & 32700.18 & -.14926 \\ 22.29156 & -.09666 & .70332 & .03400 & -.443.8266 & .04396 \\ 284.5953 & .07120 & -.45648 & .30067 & -.3046.100 & -.10030 \\ -.00928 & -.00002 & .00002 & -.00003 & .93937 & .000002 \\ 246.5965 & .27953 & -.216640 & -.57073 & -.8178.665 & .91090 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} -.22992 & -.00012 & .00048 & .00003 & 3.34095 & .00006 \\ 1665.782 & .044403 & -.226988 & -.07345 & 4195.658 & -.02098 \\ 73.01956 & .03226 & -.30362 & .06120 & 1831.774 & -.03456 \\ -171.5948 & -.05067 & .33990 & .20568 & -.2718.134 & .03630 \\ .01511 & .000002 & -.00001 & -.000001 & -.30113 & -.000009 \\ 79.88010 & -.12809 & 1.81586 & .47590 & 18031.07 & .29383 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} .11875 & -.00005 & .00002 & .00003 & -.28337 & -.00010 \\ 105.7322 & .01774 & .96756 & -.40062 & -.3728.976 & .23754 \\ 117.6451 & .01375 & .06403 & .02366 & -.2183.838 & .02425 \\ 180.8626 & -.00994 & .51261 & -.09631 & -.10326.59 & .00246 \\ -.00989 & .0000006 & -.00001 & .00004 & -.11558 & -.000005 \\ 197.9192 & -.10235 & 1.41593 & -.41518 & -.20484.07 & -.43644 \end{bmatrix}$$
The estimated coefficients of the VAR(2) for wheat industry can be shown as:

\[
A(1) = \begin{bmatrix}
.76293 & .01060 & -.09848 & .02771 & 1.19176 & .00957 \\
5.75707 & .29359 & 1.35431 & .89211 & -14.10503 & -.09078 \\
1.43642 & .05208 & .67256 & -.03526 & -6.75281 & -.01159 \\
.67030 & .02095 & -.48834 & .34726 & 4.90953 & -.00619 \\
-.00256 & -.00099 & .03140 & .00425 & .66222 & .00092 \\
1.05925 & -.61274 & 2.20431 & .18230 & -2.39921 & .89859
\end{bmatrix}
\]

\[
A(2) = \begin{bmatrix}
-.10591 & -.02309 & -.00135 & .04455 & .06176 & -.00480 \\
-3.74048 & -.22332 & 1.80804 & .13338 & -14.89036 & .12528 \\
-1.27535 & -.03211 & .18841 & .17557 & -2.50463 & .02493 \\
.97902 & -.03658 & .36656 & -.15041 & -8.96000 & -.02883 \\
-.01635 & -.00198 & -.01031 & .00391 & -.13923 & -.00080 \\
.48091 & .18447 & -.05024 & .64761 & 30.46247 & -.06107
\end{bmatrix}
\]

4.3 Estimating the Structural Coefficients

Based upon Hansen and Sargent’s prediction formula, given in equation (30), the estimated VAR coefficients can be used to transform the infinite sums of conditional expectations that appear in the reduced-form equations into closed-forms according to Table 4.3 and Table 4.4.
Table 4.3: Closed-form prediction formula for VAR(2)

<table>
<thead>
<tr>
<th>Infinite form</th>
<th>Closed-form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_{t-1}X_{t+k}$</td>
<td>$\theta^{-1} [I - A_1 \theta - A_2 \theta^2]^{-1} - I]X_{t-1}$</td>
</tr>
<tr>
<td></td>
<td>$(I - A_1 \theta - A_2 \theta^2)^{-1}A_2X_{t-2}$</td>
</tr>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_{t-1}X_{t+k+1}$</td>
<td>$[\theta^{-2}((I - A_1 \theta - A_2 \theta^2)^{-1} - I]) - \theta^{-1}A_1]X_{t-1}$</td>
</tr>
<tr>
<td></td>
<td>$+\theta^{-1}((I - A_1 \theta - A_2 \theta^2)^{-1} - I]A_2X_{t-2}$</td>
</tr>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_tX_{t+k-1}$</td>
<td>$\theta(I - A_1 \theta - A_2 \theta^2)^{-1}X_t$</td>
</tr>
<tr>
<td></td>
<td>$+[\theta^2(I - A_1 \theta - A_2 \theta^2)^{-1}A_2 + I]X_{t-1}$</td>
</tr>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_tX_{t+k}$</td>
<td>$(I - A_1 \theta - A_2 \theta^2)^{-1}X_t$</td>
</tr>
<tr>
<td></td>
<td>$+\theta(I - A_1 \theta - A_2 \theta^2)^{-1}A_2X_{t-1}$</td>
</tr>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_tX_{t+k+1}$</td>
<td>$\theta^{-1}((I - A_1 \theta - A_2 \theta^2)^{-1} - I]X_t$</td>
</tr>
<tr>
<td></td>
<td>$(I - A_1 \theta - A_2 \theta^2)^{-1}A_2X_{t-1}$</td>
</tr>
<tr>
<td>$\sum_{k=0}^{\infty} \theta^k E_tX_{t+k+2}$</td>
<td>$[\theta^{-2}((I - A_1 \theta - A_2 \theta^2)^{-1} - I}) - \theta^{-1}A_1]X_t$</td>
</tr>
<tr>
<td></td>
<td>$+\theta^{-1}((I - A_1 \theta - A_2 \theta^2)^{-1} - I]A_2X_{t-1}$.</td>
</tr>
</tbody>
</table>
Table 4.4: Closed-form prediction formula for VAR(3)

<table>
<thead>
<tr>
<th>Infinite form</th>
<th>Closed-form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{k=0}^{\infty} \theta_k E_{t-1}X_{t+k} )</td>
<td>( (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} X_{t-1} )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} \theta X_{t-2} )</td>
</tr>
<tr>
<td></td>
<td>( + \theta (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 X_{t-3} )</td>
</tr>
<tr>
<td>( \sum_{k=0}^{\infty} \theta^k E_{t-1}X_{t+k+1} )</td>
<td>( \theta^{-1} [(I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} - I] X_{t-1} )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} (A_2 - \theta A_3) X_{t-2} )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 X_{t-3} )</td>
</tr>
<tr>
<td>( \sum_{k=0}^{\infty} \theta^k E_{t}X_{t+k} )</td>
<td>( \theta(I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} X_t )</td>
</tr>
<tr>
<td></td>
<td>( + \theta^2 (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_2 )</td>
</tr>
<tr>
<td></td>
<td>( + \theta^2 (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 + I] X_{t-1} )</td>
</tr>
<tr>
<td></td>
<td>( + \theta^2 (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 X_{t-2} )</td>
</tr>
<tr>
<td>( \sum_{k=0}^{\infty} \theta^k E_{t}X_{t+k+1} )</td>
<td>( \theta^{-1} [(I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} - I] X_t )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} (A_2 - \theta A_3) X_{t-1} )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 X_{t-2} )</td>
</tr>
<tr>
<td>( \sum_{k=0}^{\infty} \theta^k E_{t}X_{t+k+2} )</td>
<td>( \theta^{-1} [\theta^{-1} (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} - \theta^{-1} I - A_1] X_t )</td>
</tr>
<tr>
<td></td>
<td>( + \theta^{-1} [(I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} - I] A_2 )</td>
</tr>
<tr>
<td></td>
<td>( + (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 X_{t-1} )</td>
</tr>
<tr>
<td></td>
<td>( + \theta^{-1} [(I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} - I] A_3 X_{t-2} )</td>
</tr>
</tbody>
</table>
Thus, the reduced form equations of corn and wheat industry are expressed as functions of a finite number of current and past values of exogenous variables. The particular reduced-form equation which will be estimated in this study is the price equation, which is equation (19) from Chapter Three. This reduced-form equation can be expressed in terms of the closed form equation of the VAR(2) process for wheat industry as follows:

\[
P_t = \frac{1}{k(\frac{1}{a} + \frac{1}{b})} + \frac{1}{\frac{1}{c} + \frac{1}{k}} d_1(\frac{1}{a} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{\frac{1}{c} + \frac{1}{b} + \frac{1}{k}} \left( \frac{1}{k} \right)^2 \left[ \frac{1}{k} \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) \right] \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_2 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_3
\]

\[
+ \frac{2}{k} \left( \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} - \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_3 - \frac{1}{k c} U_2
\]

\[
\left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} \left( (I - A_1 \theta - A_2 \theta^2)^{-1} - I \right) - 2 \theta^2 \left( \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 \right)^{-1}
\]

\[
\cdot A_2 X_{t-2}
\]

\[
+ \left[ \frac{1}{c} + \frac{1}{k} \right] \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_2 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_3
\]

\[
+ \frac{2}{k} \left( \frac{1}{a} + \frac{1}{c} \right) U_4 \left[ \left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} - I \right] - \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_3 - \frac{1}{k c} U_2
\]

\[
\left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} \left( (I - A_1 \theta - A_2 \theta^2)^{-1} - I \right) - 2 \theta^2 \left( \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 \right)^{-1}
\]

\[
- \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_2 + \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) U_3 + \right.
\]

\[
2 \left( \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} A_2 + \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) U_3 - \frac{1}{k c} U_2
\]

\[
\left( I - A_1 \theta - A_2 \theta^2 \right)^{-1} A_2 + 2 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) U_4 \left( \theta^2 (I - A_1 \theta - A_2 \theta^2)^{-1} A_2 + I \right)
\]

\[
\left] X_{t-1}
\right] \]
The similar estimated equation for corn which was of the VAR(3) process could be expressed as follows:

\[ P_t = \theta P_{t-1} + \frac{1}{k} \left( \frac{\bar{a}}{c} + \frac{1}{b} \right) \theta - d_1 \left( \frac{\bar{a}}{c} + \frac{1}{b} \right) U_1 + \left( \frac{\bar{a}}{c} + \frac{1}{b} \right) U_2 \]

\[ + 2 \left( \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} A_2 X_t - 3 \]

\[ + \left[ \frac{\bar{a}}{c} + \frac{1}{b} \right] \theta \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} \right) \frac{1}{U_2} + \frac{1}{k} U_3 \right] \]

\[ + 2 \left( \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right) U_4 \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} A_2 X_t - 3 \]

\[ + \left[ \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right] \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} A_2 X_t - 3 \]

\[ - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) d_2 U_1 + \frac{1}{c} U_2 \left[ \theta^{-1} \left( \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \right) + I \right] \]

\[ + \left[ \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right] \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} A_2 \]

\[ - 2 \left( \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right) \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \theta A_2 + \theta^2 A_3 \]

\[ + \left[ \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right] \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[ - \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{c} \right) \left( - U_2 + \frac{1}{k} U_3 \right) \]

\[ + 2 \left( \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right) U_4 \theta \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} A_3 \]
\[-2 \left( \frac{1}{k} \frac{1}{a} - \frac{1}{c} \right) U_4 \theta^2 (I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3)^{-1} A_3 \right] X_{t-2} \]

\[+ \left[ \frac{1}{k} \frac{1}{a} \right] \left[ \left( \frac{1}{a} + \frac{1}{b} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} \right) \left( \frac{1}{c} U_2 + \frac{1}{k} U_3 \right) \right] \]

\[+ 2 \left( \frac{1}{k} \frac{1}{a} \right) U_4 \left[ \left( \frac{1}{a} + \frac{1}{b} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} \right) \left( \frac{1}{c} U_2 + \frac{1}{k} U_3 \right) \right] \]

\[+ \left( \frac{1}{k} \frac{1}{a} \right) \left[ \left( \frac{1}{a} + \frac{1}{b} \right) d_2 U_1 + \left( \frac{1}{a} + \frac{1}{b} \right) \left( \frac{1}{c} U_2 + \frac{1}{k} U_3 \right) \right] \]

\[-\frac{\theta \theta^{-1} I - A_1}{2} \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[+ \theta^{-1} A_1 - 2 \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[-\frac{\theta \theta^{-1} I - A_1}{2} \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[+ \theta^{-1} A_1 - 2 \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[-\frac{\theta \theta^{-1} I - A_1}{2} \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[+ \theta^{-1} A_1 - 2 \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[-\frac{\theta \theta^{-1} I - A_1}{2} \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[+ \theta^{-1} A_1 - 2 \left( \frac{1}{k} \frac{1}{a} \right) \theta^{-1} \left( I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \right)^{-1} \]

\[\theta \theta^{-1} I - A_1 \theta - A_2 \theta^2 - A_3 \theta^3 \left[ X_t \right] \left] + \epsilon_t \right. \]

where \( U_1 = (1 0 0 0 0 0) \)

\( U_2 = (0 1 0 0 0 0) \)

\( U_3 = (0 0 1 0 0 0) \)

\( U_4 = (0 0 0 1 0 0) \)
\[ A_1 = \text{matrix of the coefficients of the one period lag exogenous variables.} \]

\[ A_2 = \text{matrix of the coefficients of the two period lag exogenous variables.} \]

\[ A_3 = \text{matrix of the coefficients of the three period lag exogenous variables.} \]

\[ X_t = \text{the vector of exogenous variables, which includes those directly specified in the model and those incorporated to help the forecasting process of the former ones.} \]

\[ X_{t-1} = \text{vector of lag one period of exogenous variables.} \]

\[ X_{t-2} = \text{vector of lag two period of exogenous variables.} \]

\[ X_{t-3} = \text{vector of lag three period of exogenous variables.} \]

The matrix of coefficients obtained from the VAR process are treated as given in estimating the structural coefficients. Therefore the set of free parameters that needs to be estimated is \( \theta, \frac{1}{\delta}, \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, d_1, \) and \( d_2. \)

Equations (31) and (32) can be estimated alone to get the consistent estimates of the structural coefficients by applying nonlinear estimation techniques. The nonlinear procedures employ an iterative numerical scheme to search for the set of
appropriate parameters that will maximize the log likelihood function.

From the estimated equation, 'a' is the product of risk averse coefficient of hedgers, $\alpha$, and the variance of price, $\sigma_p^2$, and 'b' is the product of the risk averse coefficient of speculators, $\beta$, and the variance of price, $\sigma_p^2$. Therefore the free parameters that will be estimated are $\theta$, $k$, $\alpha$, $\beta$, $\sigma_p^2$, $c$, $d_1$ and $d_2$.

Following Bard (1974), when the covariance matrix is unknown, maximizing the log likelihood function is the same as maximizing $L(\theta)$ where

$$L(\theta) = \frac{TN}{2} \left[ \log \left( \frac{T}{2\pi} \right) - 1 \right] - \frac{T}{2} \log \det M(\theta)$$

where

$\begin{align*}
L(\theta) &= \text{log likelihood function to be maximized.} \\
T &= \text{number of time series observations in each equation.} \\
N &= \text{number of equations to be estimated.} \\
M(\theta) &= \text{moment matrix of the residuals which is equivalent to } \sum_{t=1}^{T} e_t e'_t \text{ where } e_t \text{ is the vector of residuals arranged by the sequence of equations within the same time.} \\
\theta &= \text{structural parameters to be estimated.}
\end{align*}$

However, maximizing the above equation is equivalent to minimizing:

$$S(\theta) = \frac{T}{2} \log \det M(\theta)$$

where $S(\theta)$ = objective function to be minimized.
In the case where there is only one equation to be estimated, det $M(\theta)$ is equal to $M(\theta)$ which is the sum of square residuals. Then the objective function reduces to minimizing:

$$S(\theta) = \frac{T}{2} \log M(\theta)$$

The objective function $S(\theta)$, can be minimized numerically via an acceptable gradient method. A gradient method is called an acceptable gradient method when the gradient direction is acceptable. The details of the nonlinear parameter estimation can be found in Bard (1974) and Gallant (1987).

After the starting value is specified, the selected iterative scheme will search for the parameters that will minimize the objective function. The $i^{th}$ iteration is found as:

$$\theta_{i+1} = \theta_i + \sigma_i v_i$$

where $\theta_{i+1} =$ the $i^{th}$ iteration, $i + 1^{st}$ estimated parameters.

$\theta_i =$ the $i^{th}$ iterate of the parameters.

$\sigma_i =$ step size.

$v_i =$ step direction.

For the gradient method the step direction, $v_i$, is acceptable if there exists a positive definite matrix, $R$, such that

$$v_i = -Rq_i$$

where $R =$ positive definite matrix.

$q_i =$ gradient vector.

The solution is found when the stationary point or the first order condition
analogous to the linear case, \( \frac{\partial s(\theta)}{\partial \theta} = 0 \), is satisfied: that is when \( \theta_{i+1} = \theta_i \). In practice, this condition is impossible to reach with this precision. The solution in this study will be obtained when the change in objective function with respect to the change in estimated parameter is close to zero. The difference of a small amount, known as \( \epsilon \), is allowed. The \( \epsilon \) can be assumed to be a small number, for example \( 10^{-4} \). Therefore the criterion for convergence can be summarized by two conditions. First, when \( |\frac{\partial s(\theta)}{\partial \theta}| < \epsilon_1 \) and second, when \( \theta_{i+1} - \theta_i < \epsilon_2 \), where \( \epsilon_i \) is the tolerance level. When talking about convergence, the problem of local minimum seems to be unavoidable. There is no guarantee that the global minimum will be found. One way of reducing the chance of accepting the local minimum is to try different starting values.

The estimation was done through the method of maximum likelihood\(^1\) estimation. The algorithm used in this paper to estimate the parameters was Davidon-

\(^1\)In fact the nonlinear least squares which employed the Gauss-Newton method had been tried, but the problem of nonpositive definite matrix was encountered. Though this problem had been corrected by using the Marquardt method, the estimated coefficients were unreasonable. The sizes of the estimated parameters were much different and unreliable. This may due to the problem of the ill-conditioned matrix used in this study together with the way of obtaining the direction matrix. The direction matrix obtained from Gauss-Newton method uses only the first derivative which neglects some terms according to the second order condition as opposed to the quasi-Newton method.

With advanced computerized programs, taking second derivative is not a major problem any more. Therefore the Newton algorithm which used the inverse of the hessian matrix as the direction matrix had been tried. Again, the nonpositive definite matrix due to the ill-conditioned matrix terminated the iteration process before the convergence could be reached. Thus, the quasi-Newton method was employed. Under this method the hessian matrix was updated to yield a positive definite matrix. With proper starting values, the convergent estimates could be obtained.
Table 4.5: Results of the parameter estimation for corn equation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>.5</td>
<td>.0040</td>
<td>4.010*</td>
</tr>
<tr>
<td>$\frac{1}{k}$</td>
<td>1</td>
<td>5.3297</td>
<td>56.429*</td>
</tr>
<tr>
<td>$\frac{1}{\alpha}$</td>
<td>.25</td>
<td>6.2166</td>
<td>60.598*</td>
</tr>
<tr>
<td>$\frac{1}{\beta}$</td>
<td>.25</td>
<td>5095.0137</td>
<td>1744.936*</td>
</tr>
<tr>
<td>$\frac{1}{\sigma_P^2}$</td>
<td>4</td>
<td>2991.4321</td>
<td>1337.055*</td>
</tr>
<tr>
<td>$\frac{1}{c}$</td>
<td>.25</td>
<td>105.2980</td>
<td>250.840*</td>
</tr>
<tr>
<td>$d_1$</td>
<td>1</td>
<td>56.0606</td>
<td>1370.459*</td>
</tr>
<tr>
<td>$d_2$</td>
<td>1</td>
<td>0.3221</td>
<td>7.645*</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.

Objective function = 597.611854

Fletcher-Powell (DFP), which is a quasi-Newton or variable metric method. It was executed using the canned GAUSS program.

The estimated structural coefficients for corn are shown in Table 4.5 and the estimated structural coefficients for wheat are shown in Table 4.6. All of the estimated parameters in corn and wheat equation were statistically significant different from zero with the asymptotic t-statistics greater than two. However, as mentioned before, the value of the asymptotic t-statistics were calculated from the given knowl-
Table 4.6: Results of the parameter estimation for wheat equation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.004842</td>
<td>0.0048</td>
<td>2.454*</td>
</tr>
<tr>
<td>$\frac{1}{k}$</td>
<td>0.052812</td>
<td>0.0528</td>
<td>4.720*</td>
</tr>
<tr>
<td>$\frac{1}{\alpha}$</td>
<td>44.439329</td>
<td>44.4393</td>
<td>112.032*</td>
</tr>
<tr>
<td>$\frac{1}{\beta}$</td>
<td>2.863717</td>
<td>2.8637</td>
<td>28.438*</td>
</tr>
<tr>
<td>$\frac{1}{\sigma^2_p}$</td>
<td>5.902665</td>
<td>5.9027</td>
<td>40.830*</td>
</tr>
<tr>
<td>$\frac{1}{c}$</td>
<td>0.928159</td>
<td>0.9281</td>
<td>14.820*</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.519598</td>
<td>0.5196</td>
<td>7.852*</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.793509</td>
<td>0.7935</td>
<td>13.323*</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.

Objective function = 282.441532
edge of the VAR coefficients, therefore these t-statistics will be overestimated due to the underestimated standard deviation.

The root of the characteristic equation was estimated to be 0.004. The indirect carrying cost was estimated to be $0.188 per bushel. The indirect production cost was estimated to be $0.009 per bushel.

One dollar increase in cash price of corn would cause the demand for corn to go down by 56.060 bushels. Elasticity of demand for corn calculated at mean value was equal to 0.022. One unit increase in animal unit of livestock fed would cause an increase in demand for corn 0.0086 bushels.\(^2\)

The estimated variance of corn cash price was equal to 0.000334 while the sample variance of corn cash price was equal to 0.002198. The difference was due to the estimation procedure. The variance in this paper was estimated as an endogenous variable which depended on other exogenous variables included in the system while the sample variance was estimated independently from other variables.

The risk aversion coefficient of hedgers (\(\alpha\)) was 0.161 compare to the risk aversion coefficient of speculators (\(\beta\)) which was estimated to be 0.002. The estimates showed that speculators in corn futures market were less averse to risk than hedgers. If these risk aversion coefficients were considered as risk premiums, these risk premiums were small or both hedgers and speculators. However the estimates were statistically significant different from zero, then this study confirm the existence of

\(^2\)In estimating corn equation each unit of corn was equal to \(10^8\) bushels. Since all prices per bushel were scaled by multiplying with \(10^8\), then prices were considered per \(10^8\) bushels as well. Therefore all estimated parameters that showed the relationship between quantity and price would not be affected by the scaling process. One animal unit of livestock fed was also equal to \(10^4\) units. Therefore the corresponding parameter estimated could be interpreted directly.
risk premiums in corn futures market with small amount of risk premiums. The estimate of risk premium of speculators was close to zero, this might suggest that speculators in corn futures market were more of risk neutral other than risk averter.

For wheat equation, the root of the characteristic equation was estimated to be 0.005 which was about the same as in corn equation. The indirect carrying cost was estimated to be $18,939 per thousand bushels or $0.019 per bushel. The indirect production cost was estimated to be $1.077 per thousand bushels or $0.001 per bushel.\(^3\)

One dollar increase in cash price of wheat would cause the demand for wheat to decrease by 0.5196 thousand bushels or 519.648 bushels. Elasticity of demand for wheat calculated at mean value was equal to 0.825. One unit of increase in U.S. per capita disposable personal income would cause an increase in total demand for U.S. wheat 793,500 bushels.

The estimated variance of wheat cash price was equal to 0.169. The sample variance of wheat cash price was 0.0028. The estimated variance of wheat in this paper differed from the sample variance due to the estimation procedure as what had been mentioned in estimating the variance of corn. The sample variance of wheat cash price was a little bit higher than corn, which suggested that wheat was a little bit more risky than corn.

The risk aversion coefficient of hedgers (\(\alpha\)) was 0.023 compare to the risk aversion coefficient of speculators (\(\beta\)) which was estimated to be 0.349. The estimates

\(^3\)In estimating wheat equation, each unit of wheat was equal to \(10^6\) bushels while all prices per bushel were scaled by multiplying with \(10^3\). Therefore one unit of price per bushel will affect the corresponding quantity of wheat \(10^3\) bushels. Therefore all of the estimated costs were costs per \(10^3\) bushels of wheat.
<table>
<thead>
<tr>
<th>Risk Aversion Coefficients</th>
<th>Corn</th>
<th>Wheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedgers ($\alpha$)</td>
<td>0.161</td>
<td>0.023</td>
</tr>
<tr>
<td>Speculators ($\beta$)</td>
<td>0.002</td>
<td>0.349</td>
</tr>
<tr>
<td>Variance of Cash Prices ($\sigma_p^2$)</td>
<td>0.000334</td>
<td>0.169</td>
</tr>
</tbody>
</table>

showed that speculators in wheat futures market were more averse to risk than hedgers, which seemed to controvert to the theoretical argument.

The comparison between the risk premiums of traders in corn and wheat futures market was shown in Table 4.7. It was noticeable that the maximum risk aversion coefficient of traders in wheat futures market was higher than the maximum risk aversion coefficient of traders in corn futures market. And the minimum of risk aversion coefficient of traders in wheat futures market was also higher than the minimum of risk aversion coefficient of traders in corn futures market. Therefore, if risk premiums in each futures market were considered as indicators of how risky that futures market was, wheat futures market was also considered to be more risky than corn futures market under this criterion.

In fact the estimated risk premiums obtained by this procedure were not much different from what Dusak had obtained under the CAPM framework. The beta-coefficients that Dusak found in corn futures market range from 0.007 to 0.038, and
in wheat futures market range from 0.028 to 0.098. However, none of her estimates were statistically significant different from zero. The risk premiums that Dusak had obtained were the average of the industry for each delivery period. Those risk premiums were not assigned to any group of traders according to their different attitude towards risk particularly. Only the minimum values of the estimates in this paper, which represented the risk premiums of only one side of traders with lower degree of risk aversion coefficients, were consistent with the risk premiums obtained under Dusak’s CAPM.

These groups of traders (speculators in corn futures market and hedgers in wheat futures market) were suspected to be more of risk neutral other than risk averter according to their value of risk premiums which were close to zero. However, what this paper found in addition were the risk premiums of another side of traders (hedgers in corn futures market and speculators in wheat futures market) which were higher than the risk premiums of their opposite side traders and statistically significant different from zero. Therefore the estimates of this paper were more consistent with Keynes’ theory.

The estimates of risk aversion coefficients in wheat futures market suggested that hedgers can be less averse to risk than speculators. This may be due to the restriction of the constructed model in this study that there is no pure speculator. The speculators in this paper are inventory holders which can be the exporters who try to avoid risk by having inventories on hand at the time of shipment. Hence speculators in here can be more averse to risk than hedgers. Therefore, in this case, speculators had to pay the risk premiums to hedgers instead.

One problem that was encountered in estimation was that the parameter esti-
mates were very sensitive to the starting value. Each starting value lead to different estimates. In addition, the different tolerance levels also yielded different estimates. The criterion used in deciding which estimates were going to be used other than the convergence results was the value of the objective function and the t-statistics of the estimated coefficient. Though the method used in estimation was the method of maximum likelihood, the maximizing process turned out to be equivalent to minimizing log of the sum of square residuals as mentioned before. In case that there were many sets of convergence results which yielded different estimates, the one with the lowest value of the objective function was selected.

Some of the convergence results other than the selected ones which were shown in Table 4.5 for corn and Table 4.6 for wheat are shown in Appendix D. Those were the results from the different starting values. More than half of the starting values that were tried for both corn and wheat yielded divergent results.

4.4 Conclusion

This chapter contains the results of the parameter estimates that will be used in testing the efficiency of corn and wheat futures markets. The estimates showed that hedgers in corn futures market were more averse to risk than speculators while the reverse held for wheat futures market. The estimated risk premiums were small in both markets, however all were statistically significant different from zero. The estimates also showed that the futures price was a biased predictor of future cash price, although the bias was not originated from the risk transferring per se. If the inverse of the indirect carrying cost and indirect production cost were zero, the futures price would be an unbiased predictor of future cash price, no matter how
large the risk averse coefficients of hedgers and speculators were. In such a situation, i.e., if the cost of using futures market was not equal to zero, the expected cash price and the current futures price were not going to be equal. The difference would be due to some proportion of the cost of using the futures market.

Based on the estimated results, a few questions could be answered. First, there existed risk premiums in corn and wheat futures market where those risk premiums were small. Second, the bias of futures price as predictor of future cash price did not depend on the risk transferring between hedgers and speculators. It depended on the inverse of the indirect carrying cost and the production cost which were estimated to be statistically significant different from zero in this study. Third, the estimated results in corn and wheat futures market showed that the futures price was a biased predictor of the future cash price though the bias was small.
5 ON TESTING FUTURES MARKET EFFICIENCY

In this chapter, efficiency tests on the corn and wheat futures markets will be performed. The test is based on the guideline that Fama (1970) has suggested. The equilibrium expected price model is assumed to be equation (27) from Chapter Three. That is

\[
\hat{P}_{t+1} = \frac{\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right)P_{t+1}^f}{\left(\frac{1}{a} + \frac{1}{b}\right)} - \frac{1}{\left(\frac{1}{a} + \frac{1}{b}\right)}P_t - \frac{1}{c}\frac{C_t+1}{\left(\frac{1}{a} + \frac{1}{b}\right)} - \frac{1}{k}\frac{K_t+1}{\left(\frac{1}{a} + \frac{1}{b}\right)} - \frac{1}{b}\frac{\hat{a}}{\left(\frac{1}{a} + \frac{1}{b}\right)}G_t
\]

(27)

where \( \hat{a} = \hat{a}\sigma_p^2 \) and \( b = \beta\sigma_p^2 \).

\( \hat{P}_{t+1} \) = conditional expectation of cash price at time \( t + 1 \), expected at time \( t \).

\( P_t \) = current cash price at time \( t \).

\( P_t^f(t + 1) \) = current futures price at time \( t \) for the delivery at time \( t + 1 \).
The estimates of \( \frac{1}{\alpha} \), \( \frac{1}{\beta} \), \( \frac{1}{\sigma_\beta^2} \), \( \frac{1}{k} \) and \( \frac{1}{\epsilon} \) were obtained in Chapter Four and will be plugged in the above equation for both corn and wheat equations. The criterion used in order to decide if futures market is efficient is based on the randomness of actual cash prices around the predicted values of cash prices calculated from the equation above. If the deviations are unsystematic, the efficiency of futures market is concluded. After combining all estimates that are needed for the efficiency test into equation (27), the predicted value of cash price for corn and wheat equation can be presented as:

**Corn**

\[
\hat{P}_{t+1} = 1.000007\hat{P}_t(t + 1) - 0.000003P_t - 0.000007C_{t+1} - 0.000003K_{t+1} + 0.997556G_t
\]

**Wheat**

\[
\hat{P}_{t+1} = 1.003513\hat{P}_t(t + 1) - 0.000189P_t - 0.003324C_{t+1} - 0.00019K_{t+1} - 0.882055G_t
\]

The above equations will be regarded as equilibrium expected price models. The efficiency of futures market will be concluded if the actual prices deviate randomly around this equilibrium expected prices. To decide on this, the randomness of the residuals which are the differences between the actual cash prices and their predicted values will be used as a criterion. The residuals are random if they are
serial uncorrelated or if they are independent.

There are many methods that can be used to test the randomness of the residuals. The first one is to do the residual plots. If the residuals scatter randomly around zero, the efficiency of futures market is concluded.

Though the residual plots can be used as a rough guide to randomness, a statistical test is still needed to confirm the conclusion. The test statistic that is used frequently for testing randomness is the Box-Pierce chi-squared statistic. This statistic can be found by computing the first $k$ residual autocorrelations as:

$$Q = n \sum_{k=1}^{K} r_k^2$$

where $n = \text{number of observations}$.

$r_k = \text{residual autocorrelation, } k^{th}\text{ order}$.

$K = \text{number of residual autocorrelation used in calculating the statistics}$.

Another chi-squared test that may be better than the Box-Pierce chi-squared statistics is the Ljung-Box chi-squared statistic. This chi-squared statistic has a distribution which is closer to chi-square when the sample size is moderate. It can be calculated as:

$$Q = n(n + 2) \sum_{k=1}^{K} (n - k)^{-1} r_k^2$$

where $n = \text{number of observations}$.

$r_k = \text{residual autocorrelation, } k^{th}\text{ order}$.

$K = \text{number of residual autocorrelation used in calculating the statistics}$.
For $K = 15$, the chi-squared statistic at 5% significance level has a value equal to 25.

Another test for randomness is the phase frequency test of Wallis and Moore (1941), also known as the difference-sign run test. This test is based on the frequency of plus and minus signs which represent the differences in signs of the data between two consecutive periods. If the number of times the sign changes is large the randomness of the data is indicated. Therefore the null hypothesis is on the randomness of the time series data. The test statistics for a sample size which is larger than 30 is:

$$Z = \frac{|h - \frac{2n-7}{3}|}{\sqrt{\frac{16n-29}{90}}}$$

where $h = \text{phase number, which is the number of changes in sign between the first sign and the last sign.}$

$n = \text{number of observations.}$

$Z = \text{standard normal statistic.}$

At the 5% significance level, the critical value of $Z$ is equal to 1.96. Therefore if the computed test statistics is lower than 1.96, random data are concluded at the 5% significance level.

Finally, the estimates of the first order serial correlation of the disturbance terms can be calculated.

The data used in testing the efficiency of futures market can be separated into two sets: the in sample data and the out of sample data.
Table 5.1: Box-Pierce chi-squared statistics

<table>
<thead>
<tr>
<th>Crop</th>
<th>Trading month</th>
<th>Delivery month</th>
<th>Box-Pierce chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>May</td>
<td>December</td>
<td>9.4102</td>
</tr>
<tr>
<td>Wheat</td>
<td>October</td>
<td>July</td>
<td>7.7483</td>
</tr>
</tbody>
</table>

5.1 In Sample Data

The corn cash prices used for the in sample data are the average monthly cash prices of no.3 yellow corn in December from 1934 to 1960 and of no.2 yellow corn in December from 1961 to 1985 at Chicago market. The cash prices for wheat are the average monthly cash prices per bushel of no.2 hard red winter wheat in July from 1934 to 1950 and of no.1, hard red winter, ordinary protein wheat in July from 1951 to 1985 at Kansas City market. The month end closing futures prices at the end of May for the delivery in December are used for corn and the month end closing futures prices at the end of October for the delivery in July are used for wheat. The data are from 1934 to 1985. The residual plots for corn are shown in Figure 5.1 and for wheat in Figure 5.2.

The residual plots did not show any obvious sign of serial correlation for either corn and wheat. Therefore, the randomness of the residuals for the in sample data is not ruled out for either market. The Box-Pierce chi-squared statistics are shown in Table 5.1 and Ljung-Box chi-squared statistics are shown in Table 5.2.

The Box-Pierce chi-squared statistics for both corn and wheat are lower than
Figure 5.1: The residual plots of in sample data for corn.
Figure 5.2: The residual plots of in sample data for wheat.
Table 5.2: Ljung-Box chi-squared statistics

<table>
<thead>
<tr>
<th>Crop</th>
<th>Trading month</th>
<th>Delivery month</th>
<th>Ljung-Box chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>May</td>
<td>December</td>
<td>11.2265</td>
</tr>
<tr>
<td>Wheat</td>
<td>October</td>
<td>July</td>
<td>9.4229</td>
</tr>
</tbody>
</table>

Table 5.3: Difference-sign run tests

<table>
<thead>
<tr>
<th>Crop</th>
<th>Trading month</th>
<th>Delivery month</th>
<th>Z-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>May</td>
<td>December</td>
<td>0.46</td>
</tr>
<tr>
<td>Wheat</td>
<td>October</td>
<td>July</td>
<td>1.37</td>
</tr>
</tbody>
</table>

25. Therefore the randomness of residuals is confirmed for both futures markets.

The Ljung-Box chi-squared statistics also have value lower than 25 for both corn and wheat. Therefore the randomness of residuals for corn and wheat futures market according to the in sample data is also confirmed under this statistical test based on the theoretical model used in this study.

The results of the difference-sign run tests for both markets are shown in Table 5.3. The Z-statistics for both corn and wheat have lower values than 1.96. Therefore the null hypothesis that the residuals are random cannot be rejected at 5% significance level.

The estimates of the first order serial correlation for the residuals in both equa-
Table 5.4: First order serial correlation of the residuals

<table>
<thead>
<tr>
<th>Crop</th>
<th>Trading month</th>
<th>Delivery month</th>
<th>Estimated coefficient</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>May</td>
<td>December</td>
<td>0.1436</td>
<td>0.745</td>
</tr>
<tr>
<td>Wheat</td>
<td>October</td>
<td>July</td>
<td>0.1413</td>
<td>1.030</td>
</tr>
</tbody>
</table>

None of the estimates of the first order serial correlation of the residuals are significantly different from zero. Therefore the randomness of the residuals is supported by this estimation.

5.2 Out of Sample Data

The data used for this part for cash prices and futures prices are not the ones used in estimating the structural coefficients. Those are the closing futures prices at the end of March, July, September and December for the delivery in September, March, May and July respectively for corn. And the closing futures prices at the end of January, April, June and August for the delivery in September, December, March and May respectively for wheat. The data are from 1934 to 1985.

The cash prices for corn and wheat are obtained the same way as the in sample data except they are the prices from different month. Those are the cash prices in March, May, July, September and December for corn and the cash prices in January, March, April, May, June, August, September and December for wheat. The difference between the quoted month and the delivery month are maintained to be approximately the same as the in sample data in order for the same amount
Table 5.5: Box-Pierce chi-squared statistics for corn

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Box-Pierce chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>September</td>
<td>16.536</td>
</tr>
<tr>
<td>July</td>
<td>March</td>
<td>5.811</td>
</tr>
<tr>
<td>September</td>
<td>May</td>
<td>5.979</td>
</tr>
<tr>
<td>December</td>
<td>July</td>
<td>10.008</td>
</tr>
</tbody>
</table>

of carrying cost, production cost and the brokage fee as the in sample data can be used. All data are detrended the same way as the data used in finding the structural coefficients, and the difference from mean is used.

The residual plots for corn are shown in Figure 5.3 to 5.6 and the residual plots for wheat are shown in Figure 5.7 to 5.10. The residual plots for corn and wheat against time did not show any particular pattern to indicate the serial correlation. The rough figures of residual plots for four different periods within a year suggested independent disturbance terms.

The Box-Pierce chi-squared statistics for corn equation are shown in Table 5.5 and Ljung-Box chi-squared statistics are shown in Table 5.6. Table 5.7 is the Box-Pierce chi-squared statistics for wheat equation and Table 5.8 is the Ljung-Box chi-squared statistics for the same crop.

All the chi-squared statistics for corn and wheat equation are lower than 25, therefore at 5% significance level the randomness of the residuals in both corn and
Figure 5.3: The residual plots for corn out of sample data for the futures price quoted in March for the delivery in September.
Figure 5.4: The residual plots for corn out of sample data for the futures price quoted in July for the delivery in March.
Figure 5.5: The residual plots for corn out of sample data for the futures price quoted in September for the delivery in May.
Figure 5.6: The residual plots for corn out of sample data for the futures price quoted in December for the delivery in July.
Figure 5.7: The residual plots for wheat out of sample data for futures price quoted in January for the delivery in September.
Figure 5.8: The residual plots for wheat out of sample data for the futures price quoted in April for the delivery in December.
Figure 5.9: The residual plots for wheat out of sample data for the futures price quoted in June for the delivery in March.
Figure 5.10: The residual plots for wheat out of sample data for the futures price quoted in August for the delivery in May.
Table 5.6: Ljung-Box chi-squared statistics for corn

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Ljung-Box chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>September</td>
<td>20.243</td>
</tr>
<tr>
<td>July</td>
<td>March</td>
<td>7.110</td>
</tr>
<tr>
<td>September</td>
<td>May</td>
<td>5.979</td>
</tr>
<tr>
<td>December</td>
<td>July</td>
<td>10.008</td>
</tr>
</tbody>
</table>

Table 5.7: Box-Pierce chi-squared statistics for wheat

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Box-Pierce chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>September</td>
<td>5.135</td>
</tr>
<tr>
<td>April</td>
<td>December</td>
<td>8.690</td>
</tr>
<tr>
<td>June</td>
<td>March</td>
<td>8.274</td>
</tr>
<tr>
<td>August</td>
<td>May</td>
<td>13.759</td>
</tr>
</tbody>
</table>
Table 5.8: Ljung-Box chi-squared statistics for wheat

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Ljung-Box chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>September</td>
<td>6.080</td>
</tr>
<tr>
<td>April</td>
<td>December</td>
<td>9.999</td>
</tr>
<tr>
<td>June</td>
<td>March</td>
<td>11.378</td>
</tr>
<tr>
<td>August</td>
<td>May</td>
<td>16.920</td>
</tr>
</tbody>
</table>

The wheat equation cannot be rejected in all tested periods based upon the equilibrium expected price model used in here.

The results of the difference-sign run tests are shown in Table 5.9 for corn and Table 5.10 for wheat. The run tests confirmed the randomness of the disturbance terms in corn equation, however the randomness did not hold for all periods in wheat futures market. The Z-statistic for June trading month for the delivery in March had value over two which rejected the randomness hypothesis of wheat futures market. Therefore according to the difference-sign run tests the efficiency of corn futures market could be confirmed while the efficiency of wheat futures market was still ambiguous.

The estimates of the first order serial correlation of the residuals for corn is shown in Table 5.11 and for wheat is shown in Table 5.12. The estimation of the first order serial correlation of the residuals in corn and wheat equation showed that none of the estimates are statistically significant different from zero. Therefore the
Table 5.9: Difference-sign run tests for corn

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Z-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>September</td>
<td>1.66</td>
</tr>
<tr>
<td>July</td>
<td>March</td>
<td>0.94</td>
</tr>
<tr>
<td>September</td>
<td>May</td>
<td>0.38</td>
</tr>
<tr>
<td>December</td>
<td>July</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 5.10: Difference-sign run tests for wheat

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Z-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>September</td>
<td>0.45</td>
</tr>
<tr>
<td>April</td>
<td>December</td>
<td>1.37</td>
</tr>
<tr>
<td>June</td>
<td>March</td>
<td>2.30*</td>
</tr>
<tr>
<td>August</td>
<td>May</td>
<td>0.68</td>
</tr>
</tbody>
</table>

*Indicates the Z-statistic which has value over two.
Table 5.11: First order serial correlation of the residuals for corn

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Estimated coefficient</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>September</td>
<td>0.1835</td>
<td>1.307</td>
</tr>
<tr>
<td>July</td>
<td>March</td>
<td>0.2710</td>
<td>1.637</td>
</tr>
<tr>
<td>September</td>
<td>May</td>
<td>0.0226</td>
<td>0.157</td>
</tr>
<tr>
<td>December</td>
<td>July</td>
<td>-0.1081</td>
<td>-0.760</td>
</tr>
</tbody>
</table>

Table 5.12: First order serial correlation of the residuals for wheat

<table>
<thead>
<tr>
<th>Trading month</th>
<th>Delivery month</th>
<th>Estimated coefficient</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>September</td>
<td>-0.0897</td>
<td>-0.637</td>
</tr>
<tr>
<td>April</td>
<td>December</td>
<td>0.2694</td>
<td>1.953</td>
</tr>
<tr>
<td>June</td>
<td>March</td>
<td>0.1868</td>
<td>1.107</td>
</tr>
<tr>
<td>August</td>
<td>May</td>
<td>0.0538</td>
<td>0.375</td>
</tr>
</tbody>
</table>
The randomness of the residuals is suggested for both corn and wheat equation according to the first order serial correlation.

In summary the results from all the statistical tests concluded the efficiency of corn and wheat futures market according to the efficiency criteria developed in this study. However, one of the test which was the difference-sign run test did not accept the efficiency hypothesis for the wheat futures market in all periods. Therefore the efficiency of wheat futures market cannot be concluded. As has been mentioned before, if the test is rejected the inefficiency market cannot be concluded, since the specified model may be inappropriate. Certainly we still cannot avoid the problem like Type II error which is accepting the false hypothesis as a true one. This problem may arise if the theoretical model used in this study is not true from start, then there is a chance to accept that futures market is efficient though in fact it is not.

One possible problem for the rejection of the efficient wheat futures market, for example, is that the acquired structural coefficients in estimating the wheat price equation are based on the local minimum of the objective function, rather than the global minimum. Another problem is those values are only asymptotically consistent estimates, not asymptotically efficient estimates. If efficient estimates could have been obtained, the results might have been different.
6 SUMMARY AND CONCLUSIONS

The risk premium obtained in most former studies have been based on the framework of the CAPM model. There is no direct relationship between traders' attitude towards risk and the risk premium obtained under the CAPM model. The study of risk premium in this paper renders another method of obtaining the risk premium which has direct meaning as the constant absolute risk aversion of traders. This meaning is based on the well known definition of the risk premium proposed by Pratt (1964) and Arrow (1965). Therefore the estimated coefficients that are obtained from this study will characterize the risk averse behavior of traders exactly according to Keynes' normal backwardation theory.

The estimated results from this empirical study showed that a risk premium does exist in corn and wheat futures market. And different group of traders had different risk aversion coefficients. It was not necessary, according to the estimated results from this study, that hedgers must be more averse to risk than speculators. This might due to the limited construction of the model here in that there is no pure speculator take part in this model. It might be more interesting if the model has been enlarged to take into account of pure speculators. However, it is beyond the scope of this study and will be left to further research. Though the estimated risk premiums in this paper are small, they are statistically significant different from
zero. Therefore this study supports the normal backwardation theory of Keynes on this particular point.

The estimated coefficients from corn and wheat equations showed that wheat futures market was more risky than corn futures market due to the higher variance of cash prices of wheat and the overall risk premiums of traders between these two futures market.

The empirical estimates shed some light on the argument regarding the bias of futures prices as predictors of future cash prices. The futures prices are downward biased predictors of future cash prices according to this study. However, due to the particular assumptions made in this study, the only prominent result will be the normal backwardation alone, there is no chance for the contango to exist at all if the influence of futures prices only is considered in predicting future cash prices.

Though Keynes' normal backwardation has been supported in certain points (i.e., the existence of risk premium, the downward biased predictor of futures price), not all of his claims have been supported by this study. If futures prices are considered alone in forecasting future cash prices, the risk transference between hedgers and speculators does not play a direct role in causing the bias. Though hedgers and speculators are equally averse to risk, so long as there exist indirect costs of production and/or indirect carrying costs, futures prices will always be downward biased predictors of future cash prices. Therefore it does not matter if hedgers or speculators will be more averse to risk, there is nothing to do with the biased predictor of futures prices. The bias is caused from the indirect cost of production and the indirect carrying cost, not the risk premium per se.

Moreover, not only should the futures price alone be used to predict future cash
prices. The current cash price, the direct cost of production, the direct carrying cost and the brokerage fee should also be used since all take part in predicting the future cash prices. The larger the risk aversion coefficients, the less will be the influence of futures prices as predictors of future cash prices. At the same time, the higher the risk aversion coefficients, the higher the influence of the past period cash prices, the direct carrying cost and the direct cost of production in lowering the predicted value of future cash prices. If the coefficient that characterize the risk aversion of hedgers is treated as indirect production cost and the coefficient that characterize the risk aversion of speculators is treated as indirect carrying cost, when the cost incurred by a producer is more than that incurred by an inventory holder, the brokerage fee will have a positive effect on the expected cash price. Therefore, if producers' cost is more than inventory holders' cost the futures price will have less influence in predicting future cash price. On the contrary if the inverse holds, the influence of futures price alone will be more.

The test on the efficiency of futures market in corn and wheat futures market is based on the equilibrium expected price model proposed in Chapter Three. The test was done on the in sample data and the out of sample data. The results for the in sample data confirmed the efficiency of futures market for both corn and wheat. For the out of sample data, the hypothesis that corn futures market is efficient is concluded based upon the model specified in this study. However, it was rejected for wheat futures market. Therefore this study confirmed that corn futures market is efficient while the efficiency of wheat futures market is still in question. What is suspected for wheat futures market is that the model used is inappropriate other than the market itself is inefficient.
The results found in this study, especially for the corn futures market yield another view that the efficiency of futures market has nothing to do with the biased predictor of futures price on future cash price nor the existence of risk premium. The results from corn futures market suggested both the existence of risk premium and also the biases in prediction by the futures price while corn futures market still remains efficient. Though the biases imputed in futures prices directly for both corn and wheat futures market were small, however, not only futures prices can be used as predictors of future cash prices alone, but also current cash prices, direct carrying cost, direct production cost and brokerage fee. Thus the influence of futures prices will be diluted by the influence of other variables that help predicting the future cash prices either in the positive direction or negative direction and still yield the efficiency of futures market.

However, this study is still not complete. If the coefficients were estimated via the system of equations where all the VAR coefficients were also treated as the free parameters together with the structural coefficients, the most desirable estimates could be obtained. This remaining part will be left for further research in the future.
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APPENDIX A. DERIVATION

8.1 Producers or Hedgers

If \( u^h_t = -e^{-\alpha \pi(h)} \)

Max \( E_{t-1}(u^h_t) = E_{t-1} \pi^h_t - \frac{1}{2} \alpha \sigma^2_{\pi(h)} \)

where \( \pi^h_t = P_t[q_t - x^{t-1}(t)] + P^f_{t-1}(t)x_{t-1}(t) \)

\( -C_t q_t - \frac{c}{2} q_t^2 - G_{t-1} x_{t-1}(t) \)

\( E_{t-1}(\pi^h_t) = [q_t - x^{t-1}(t)] E_{t-1}(P_t) + P^f_{t-1}(t)x_{t-1}(t) \)

\( -C_t q_t - \frac{c}{2} q_t^2 - G_{t-1} x_{t-1}(t) \)

\( \sigma^2_{\pi(h)} = E_{t-1}(\pi^h_t - E_{t-1}(\pi^h_t))^2 \)

\( = [q_t - x^{t-1}(t)]^2 E_{t-1}[P_t - E_{t-1}(P_t)]^2 \)

\( = [q_t - x^{t-1}(t)]^2 \sigma^2_{P} \)

Max \( E_{t-1}(u^h_t) = [q_t - x^{t-1}(t)] E_{t-1}(P_t) + P^f_{t-1}(t)x_{t-1}(t) \)

\( -C_t q_t - \frac{c}{2} q_t^2 - G_{t-1} x_{t-1}(t) \)

\( -\frac{1}{2} \alpha [q_t - x^{t-1}(t)]^2 \sigma^2_{P} \)
\[
\frac{\partial E_{t-1}(u_t^L)}{\partial q_t} = E_{t-1}(P_t) - C_t - c q_t - \alpha [q_t - x_{t-1}(t)] \sigma_p^2
\]

\[
= 0
\]

(1)

\[
\frac{\partial E_{t-1}(u_t^L)}{\partial x_{t-1}(t)} = P_{t-1}^f(t) - E_{t-1}(P_t) - G_{t-1}
\]

\[+ \alpha [q_t - x_{t-1}(t)] \sigma_p^2
\]

\[
= 0
\]

(2)

from (1) and (2), \[P_{t-1}^f(t) - C_t - c q_t - G_{t-1} = 0\]

\[
q_t = \frac{1}{c} P_{t-1}^f(t) - \frac{1}{c} C_t - \frac{1}{c} G_{t-1}
\]

(3)

and \[x_{t-1}(t) = \frac{1}{\alpha \sigma_p^2} P_{t-1}^f(t) - \frac{1}{\alpha \sigma_p^2} E_{t-1}(P_t)
\]

\[+ \frac{1}{\alpha \sigma_p^2} G_{t-1} + q_t
\]

\[= \left( \frac{1}{\alpha \sigma_p^2} + \frac{1}{c} \right) P_{t-1}^f(t) - \frac{1}{\alpha \sigma_p^2} E_{t-1}(P_t)
\]

\[- \frac{1}{c} C_t - \left( \frac{1}{\alpha \sigma_p^2} + \frac{1}{c} \right) G_{t-1}
\]

Let \[\alpha \sigma_p^2 = a,\]

then \[x_{t-1}(t) = \left( \frac{1}{a} + \frac{1}{c} \right) P_{t-1}^f(t) - \frac{1}{a} E_{t-1}(P_t)
\]

\[- \frac{1}{c} C_t - \left( \frac{1}{a} + \frac{1}{c} \right) G_{t-1}
\]

(4)

Assuming individual producers and hedgers are homogeneous, then aggregate supply and hedgers are:

\[S_t = \sum_{j=1}^n q_t j = n q_t \]

\[X_{t-1}(t) = \sum_{j=1}^n x_{t-1}(t) j = n x_{t-1}(t) \]
Aggregate Supply:

\[ S_t = n\left[ \frac{1}{c}P_{t-1}^f(t) - \frac{1}{c}C_t - \frac{1}{c}G_{t-1} \right] \]

Aggregate Hedging:

\[ X_{t-1}(t) = n\left[ \frac{1}{a} + \frac{1}{c} \right]P_{t-1}^f(t) - \frac{1}{a}E_{t-1}(P_t) - \frac{1}{c}C_t - \left( \frac{1}{a} + \frac{1}{c} \right)G_{t-1} \]

8.2 Inventory Holders or Speculators

If \( u_t^s = -e^{-\beta \pi(s)} \)

Max \( E_{t-1}(u_t^s) = E_{t-1}(\pi_t^s) - \frac{1}{2}\beta \sigma_{\pi(s)}^2 \)

where \( \pi_t^s = i_{t-1}P_t + y_{t-1}(t)P_t - i_{t-1}P_{t-1} \)

\[ -y_{t-1}(t)P_{t-1}(t) - K_{t}i_{t-1} \]

\[ -k\frac{2}{2}i_{t-1} - G_{t-1}P_{t-1}(t) \]

\[ E_{t-1}(\pi_t^s) = i_{t-1}[E_{t-1}(P_t) - P_{t-1}] \]

\[ +y_{t-1}(t)[E_{t-1}(P_t) - P_{t-1}^f(t)] \]

\[ -K_{t}i_{t-1} - k\frac{2}{2}i_{t-1} - G_{t-1}y_{t-1}(t) \]

\( \sigma_{\pi(s)}^2 = E_{t-1}[\pi_t^s - E_{t-1}(\pi_t^s)]^2 \)

\[ = (i_{t-1} + y_{t-1}(t))^2E_{t-1}[P_t - E_{t-1}(P_t)]^2 \]

\[ = (i_{t-1} + y_{t-1}(t))^2\sigma_P^2 \]

\( E_{t-1}(u_t^s) = i_{t-1}[E_{t-1}(P_t) - P_{t-1}] \)
\[
\begin{align*}
\frac{\partial E_{t-1}(u_t^g)}{\partial i_{t-1}} &= E_{t-1}(P_t) - P_{t-1} - K_t - k_i_{t-1} \\
&\quad - \beta [i_{t-1} + y_{t-1}(t)]\sigma_p^2 \\
&= 0 \\
\frac{\partial E_{t-1}(u_t^g)}{\partial y_{t-1}(t)} &= E_{t-1}(P_t) - P_{t-1}(t) - G_{t-1} \\
&\quad - \beta [i_{t-1} + y_{t-1}(t)]\sigma_p^2 \\
&= 0
\end{align*}
\]

From (5) and (6),
\[
P_{t-1}(t) - P_{t-1} - K_t - k_i_{t-1} + G_{t-1} = 0
\]

\[
i_{t-1} = \frac{1}{k} P_{t-1}(t) - \frac{1}{k} P_{t-1} - \frac{1}{k} K_t + \frac{1}{k} G_{t-1}
\]

And
\[
y_{t-1}(t) = \frac{1}{\beta \sigma_p^2} E_{t-1}(P_t) - [\frac{1}{\beta \sigma_p^2} + \frac{1}{k}] P_{t-1}(t) \\
\quad + \frac{1}{k} P_{t-1} + \frac{1}{k} K_t - [\frac{1}{\beta \sigma_p^2} + \frac{1}{k}] G_{t-1}
\]

Let \( \beta \sigma_p^2 = b, \)

then,
\[
y_{t-1}(t) = \frac{1}{b} E_{t-1}(P_t) - (\frac{1}{b} + \frac{1}{k}) P_{t-1}(t) + \frac{1}{k} P_{t-1} \\
\quad - \frac{1}{k} K_t - (\frac{1}{b} + \frac{1}{k}) G_{t-1}
\]

Assuming individual inventory holders and speculators are homogeneous, then aggregate inventory demand and speculations are:
Aggregate inventory holders:

\[ I_{t-1} = \sum_{j=1}^{n} i_{t-1,j} = n i_{t-1} \]

\[ Y_{t-1}(t) = \sum_{j=1}^{n} y_{t-1}(t)_j = n y_{t-1}(t) \]

Aggregate speculations:

\[ I_{t-1} = n \left[ \frac{1}{k} P_{t-1}^f(t) - \frac{1}{k} P_{t-1} - \frac{1}{k} K_t + \frac{1}{k} G_{t-1} \right] \]

8.3 Consumer Demand

Aggregate Consumer Demand:

\[ D_t = n [d_0 - d_1 P_t + d_2 X D_t + v_{5t}] \]

If we assume that \( n \) is large, though the number of individual in each group may not be equal; however, as \( n \) becomes larger, their ratio approaches unity. Therefore \( n \) can be cancelled out. And the number of the firm will be ignored in here. The equilibrium in each market can be found by treating \( n \) equal to unity and can be summarized as follows:

**Cash Market**

Supply:

\[ S_t = \frac{1}{c} P_{t-1}^f(t) - \frac{1}{c} C_t - \frac{1}{c} G_{t-1} + v_{1t} \]
Beginning Inventory:
\[ I_{t-1} = \frac{1}{k} P^f_{t-1}(t) - \frac{1}{k} P_{t-1} + \frac{1}{k} K_t + \frac{1}{k} G_{t-1} + v3t-1 \quad (10) \]

Ending Inventory:
\[ I_t = \frac{1}{k} P^f_t(t + 1) - \frac{1}{k} P_t - \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v3_t \quad (11) \]

Consumer Demand:
\[ D_t = d_0 - d_1 P_t + d_2 X D_t + v5_t \quad (12) \]

Market Clearing:
\[ S_t + I_{t-1} = D_t + I_t \quad (13) \]

**Futures Market**

Hedging:
\[ X_{t-1}(t) = \left( \frac{1}{a} - \frac{1}{c} \right) P^f_{t-1}(t) - \frac{1}{a} E_{t-1}(P_t) - \frac{1}{c} C_t \\
- \left( \frac{1}{a} + \frac{1}{c} \right) G_{t-1} + v2t-1 \quad (14) \]

Speculations:
\[ Y_{t-1}(t) = \frac{1}{b} E_{t-1}(P_t) - \frac{1}{b} + \frac{1}{c} P^f_{t-1}(t) + \frac{1}{k} P_{t-1} \\
+ \frac{1}{k} K_t - \left( \frac{1}{b} + \frac{1}{c} \right) G_{t-1} + v4t-1 \quad (15) \]

Market Clearing:
\[ X_{t-1}(t) = Y_{t-1}(t) \quad (16) \]

Substitute (9), (10), (11) and (12) into (13),
\[ \left( \frac{1}{c} P^f_{t-1}(t) - \frac{1}{c} C_t - \frac{1}{c} G_{t-1} + v1t \right) + \left( \frac{1}{k} P^f_{t-1}(t) - \frac{1}{k} P_{t-1} - \frac{1}{k} K_t + \frac{1}{k} G_{t-1} \right) \]
\[ + v_3 t - 1 = \left[ d_0 - d_1 P_t + d_2 X D_t + v_5 t \right] + \left[ \frac{1}{k} P^f_t (t + 1) - \frac{1}{k} P_t - \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v_3 t \right] \]

Rearrange terms and let \( v_5 t + v_3 t - v_1 t + v_3 t - 1 = \epsilon_t \). Then

\[
\frac{1}{k} P^f_t (t + 1) - \left( \frac{1}{c} + \frac{1}{k} \right) P^t_{t-1}(t) - (d_1 + \frac{1}{k}) P_t + \frac{1}{k} P_{t-1} =
\]

\[
- d_0 + d_2 X D_t + \frac{1}{c} C_t + \frac{1}{k} K_t + \frac{1}{k} K_{t+1}
\]

\[
+ \left( \frac{1}{c} - \frac{1}{k} \right) G_{t-1} + \frac{1}{k} G_t + \epsilon_t \]

Thus we get

\begin{equation}
\frac{1}{k} P^f_t (t + 1) - \left( \frac{1}{c} + \frac{1}{k} \right) P^t_{t-1}(t) - (d_1 + \frac{1}{k}) P_t + \frac{1}{k} P_{t-1} =
\end{equation}

Substitute (14) and (15) into (16),

\[
\frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) P^f_{t-1}(t) = \left( \frac{1}{a} + \frac{1}{b} \right) E_{t-1}(P_t) + \frac{1}{k} P_{t-1} + \frac{1}{c} C_t
\]

\[
+ \frac{1}{k} K_t + \left[ \frac{1}{c} \right] G_{t-1} + v_4 t - 1
\]

Thus we get

\[
P^f_{t-1}(t) = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}
\]

\[
\left[ \frac{1}{a} + \frac{1}{b} \right] E_{t-1}(P_t)
\]

\[
+ \frac{1}{k} P_{t-1} + \frac{1}{c} C_t + \frac{1}{k} K_t + \left[ \frac{1}{a} \right] G_{t-1} + v_4 t - 1 - v_2 t - 1
\]

And
\[ P_t^f(t+1) = \frac{1}{a+1+c+b+k} \left[ \left( \frac{1}{a} + \frac{1}{b} \right) E_t(P_{t+1}) + \frac{1}{k} P_t + \frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1} + \left( \frac{1}{a} + \frac{1}{c} \right) \right. \\
\left. - \left( \frac{1}{b} + \frac{1}{k} \right) [G_t + v_{4t} - v_{2t}] \right] \]  

(19)

Substitute (18) and (19) into (17), one gets
\[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) E_t(P_{t+1}) - \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{b} \right) E_{t-1}(P_t) + \left( \frac{1}{c} \right)^2 - \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) (d_1 + \frac{1}{k}) P_t \]
\[-\left( \frac{1}{c} \left( \frac{1}{c} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right) P_{t-1} = \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ d_0 + d_2 XD_t + \frac{1}{c} C_t \right. \\
\left. + \frac{1}{k} K_t - \frac{1}{k} K_{t+1} + \left( \frac{1}{c} - \frac{1}{k} \right) G_{t-1} \right. \\
\left. + \frac{1}{k} G_t + \epsilon_t \right] - \left( \frac{1}{a} + \frac{1}{c} \right) \left[ \left( \frac{1}{c} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \]
\[-\left( \frac{1}{b} + \frac{1}{k} \right) [G_t + v_{4t} - v_{2t}] \]
\+[\left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{c} + \frac{1}{k} \right) C_t + \frac{1}{k} K_t + \left( \frac{1}{a} + \frac{1}{c} \right) \\
\left. - \left( \frac{1}{b} + \frac{1}{k} \right) [G_{t-1} + v_{4t-1} - v_{2t} - 1] \right] \]

which can be written as
\[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) E_t(P_{t+1}) - \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{b} \right) E_{t-1}(P_t) - \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] P_t \]
\[+ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) P_{t-1} = \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ d_0 + d_2 XD_t + \frac{1}{c} C_t + \frac{1}{k} K_t - \frac{1}{k} K_{t+1} \right. \\
\left. + \frac{1}{k} \left( \frac{1}{c} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \\
\left. + \frac{1}{a} \left( \frac{1}{c} + \frac{1}{k} \right) G_t + \epsilon_t - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \]
\[-\left( \frac{1}{b} + \frac{1}{k} \right) [v_{4t} - v_{2t}] \]
\[\left. - \left( \frac{1}{c} + \frac{1}{k} \right) [v_{4t-1} - v_{2t-1}] \right] \]  

(20)
Take conditional expectation on both sides at time 't-1'

\[
\frac{1}{k}(\frac{1}{a} + \frac{1}{b})E_{t-1}(P_{t+1}) - \left[ d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{c}(\frac{1}{a} + \frac{1}{b} + \frac{1}{k}) + 2\frac{1}{k}(\frac{1}{a} + \frac{1}{b}) \right] E_{t-1}(P_t)
\]

\[
+ \frac{1}{k}(\frac{1}{a} + \frac{1}{b})P_{t-1} = -\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ d_0 + d_2 E_{t-1}(XD_t) + \frac{1}{c} E_{t-1}(C_t) + \frac{1}{k} E_{t-1}(K_t) \right]
\]

And,

\[
E_{t-1}(P_{t+1}) = \left[ d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{c}(\frac{1}{a} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{k}(\frac{1}{a} + \frac{1}{b}) \right] + 2 E_t(P_t) + P_{t-1}
\]

Let \[
\frac{d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{c}(\frac{1}{a} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{k}(\frac{1}{a} + \frac{1}{b}) + 2}{\frac{1}{k}(\frac{1}{a} + \frac{1}{b})} = \phi
\]

\[
(1 - \phi L + L^2)E_{t-1}(P_{t+1}) = \frac{1}{k}(\frac{1}{a} + \frac{1}{b}) \left[ -\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0 + d_2 E_{t-1}(XD_t)
\]

\[
+ \frac{1}{c} E_{t-1}(C_t) + \frac{1}{k} E_{t-1}(K_t) - \frac{1}{k} E_{t-1}(K_{t+1}) \right]
\]
Since all coefficients are assumed to be positive, thus
\[
\phi^2 - 4 > 0.
\]
And the roots of the characteristic equations are real and distinct,

Let \( \theta_1 + \theta_2 = \phi \)
\( \theta_1 \theta_2 = 1 \)
If \( \theta_1 = \theta \),
then \( \theta_2 = \theta^{-1} \)

\[
(1 - \theta L)(1 - \theta^{-1} L)E_{t-1}(P_{t+1}) = \frac{1}{k(a^2 + \frac{1}{b})} \left[ -(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})d_0 
+ d_2 E_{t-1}(XD_t) + \frac{1}{c} E_{t-1}(C_t) + \frac{1}{k} E_{t-1}(K_t)
- \frac{1}{k} E_{t-1}(K_{t+1}) - \frac{1}{k} E_{t-1}(C_{t+1})
+ \frac{1}{k} E_{t-1}(K_{t+1}) + \frac{1}{c} + \frac{1}{k} E_{t-1}(C_t)
+ \frac{1}{k} E_{t-1}(K_t) - 2\left(\frac{1}{k} + \frac{1}{c}\right) E_{t-1}(G_t)
+ 2(\frac{1}{k} - \frac{1}{c} - \frac{1}{b})G_{t-1}\right]
\]

\[
(1 - \theta L)E_{t-1}(P_{t+1}) = \frac{-\theta L^{-1}}{k(\frac{1}{a} + \frac{1}{b})(1 - \theta L^{-1})} \left[ -(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})d_0 
+ d_2 E_{t-1}(XD_t) + \frac{1}{c} E_{t-1}(C_t) + \frac{1}{k} E_{t-1}(K_t)\right]
\]
\begin{align*}
(1 - \theta L)E_t - 1(P_{t+1}) &= \frac{\theta L^{-1}}{k(a + b)} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0] \\
&+ d_2 E_t - 1(XD_{t+j}) + \frac{1}{c} E_t - 1(C_{t+j}) \\
&+ \frac{1}{k} E_t - 1(K_{t+j}) - \frac{1}{k} E_t - 1(K_{t+j+1}) \\
&+ \frac{1}{k} [\frac{1}{c} E_t - 1(C_{t+j+1}) + \frac{1}{k} E_t - 1(K_{t+j+1})] \\
&- \left( \frac{1}{a} + \frac{1}{k} \right) E_t - 1(C_{t+j}) + \frac{1}{k} E_t - 1(K_{t+j}) \\
&+ 2[\frac{1}{k} (\frac{1}{a} + \frac{1}{c})] E_t - 1(G_{t+j}) \\
&- 2[\frac{1}{k} (\frac{1}{c} + \frac{1}{k})] E_t - 1(G_{t+j-1}) + c_2 \theta_{t+1}^2 \\
\end{align*}

\begin{align*}
(1 - \theta L)E_t - 1(P_{t+1}) &= \frac{1}{k(a + b)} \sum_{j=1}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0] \\
&+ d_2 E_t - 1(XD_{t+j}) + \frac{1}{c} E_t - 1(C_{t+j}) \\
&+ \frac{1}{k} E_t - 1(K_{t+j}) - \frac{1}{k} E_t - 1(K_{t+j+1}) \\
&+ \frac{1}{k} [\frac{1}{c} E_t - 1(C_{t+j+1}) + \frac{1}{k} E_t - 1(K_{t+j+1})] \\
&- \left( \frac{1}{a} + \frac{1}{k} \right) E_t - 1(C_{t+j}) + \frac{1}{k} E_t - 1(K_{t+j}) \\
&+ 2[\frac{1}{k} (\frac{1}{a} + \frac{1}{c})] E_t - 1(G_{t+j}) \\
&- 2[\frac{1}{k} (\frac{1}{c} + \frac{1}{k})] E_t - 1(G_{t+j-1}) + c_2 \theta_{t+1}^2 \\
\end{align*}

\begin{align*}
(1 - \theta L)E_t - 1(P_t) &= \frac{\theta}{k(a + b)} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0] \\
\end{align*}
For a bounded solution, we need $c_2 = 0$.

Thus,

$$E_{t-1}(P_t) = \theta P_{t-1} + \frac{\theta}{k(a + b)} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) d_0 ight.\\ + d_2 E_{t-1}(X D_{t+j}) + \frac{1}{c} E_{t-1}(C_{t+j})\\ + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1})\\ + \frac{1}{k} \left( \frac{1}{c} E_{t-1}(C_{t+j+1}) + \frac{1}{c} E_{t-1}(K_{t+j+1}) \right)\\ - \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{c} E_{t-1}(C_{t+j}) + \frac{1}{c} E_{t-1}(K_{t+j}) \right)\\ + 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) E_{t-1}(G_{t+j})ight.\\ \left. - 2 \left( \frac{1}{k} a - \frac{1}{k} \frac{1}{c} b \right) E_{t-1}(G_{t+j-1}) \right]$$ \hspace{1cm} (21)

And by the same token

$$E_t(P_{t+1}) = \theta P_t + \frac{\theta}{k(a + b)} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) d_0 ight.\\ + d_2 E_t(X D_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1})\\ + \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2})$$
Substitute (21) and (22) into (20), we get

\[
\begin{align*}
&P_t = \\
&\left[\frac{1}{k} \left(\frac{1}{a} + \frac{1}{b}\right) \theta - d_1 \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) - \frac{1}{k} \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b}\right)\right] P_t = \\
&\left[\frac{1}{k} \left(\frac{1}{a} + \frac{1}{b}\right) \theta - \frac{1}{k} \left(\frac{1}{a} + \frac{1}{b}\right)\right] P_{t-1} \\
&\quad - \left[\frac{1}{a} + \frac{1}{k}\right] d_0 + d_2 XD_t + \frac{1}{c} C_t + \frac{1}{k} K_t - \frac{1}{k} K_{t+1} \\
&+ \frac{1}{k} \left(\frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1}\right) - \left(\frac{1}{c} + \frac{1}{k}\right) \left(\frac{1}{c} C_t + \frac{1}{k} K_t\right) \\
&+ 2 \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c} \right) G_{t+1} - 2 \left(\frac{1}{k} \left(\frac{1}{a} - \frac{1}{c} \right) G_{t-1}\right)\right] \\
&- \sum_{j=1}^{\infty} \theta^j \left[\left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right) d_0 + d_2 E_t XD_{t+j} + \frac{1}{c} E_t C_{t+j}\right] \\
&+ \frac{1}{k} E_t \left(K_{t+j} - K_{t+j+1}\right) + \frac{1}{k} E_t \left(C_{t+j+1}\right) \\
&+ \frac{1}{k} E_t \left(K_{t+j+1}\right) - \left(\frac{1}{c} + \frac{1}{k}\right) \left(\frac{1}{c} E_t C_{t+j} + \frac{1}{k} E_t C_{t+j}\right) \\
&+ 2 \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c} \right) E_t G_{t+j} - 2 \left(\frac{1}{k} \left(\frac{1}{a} - \frac{1}{c} \right) E_t G_{t+j-1}\right)\right] \\
&+ \frac{1}{k} E_{t-1} \left(C_{t+j} + \frac{1}{k} E_{t-1} \left(K_{t+j}\right) - \frac{1}{k} E_{t-1} \left(K_{t+j+1}\right)\right) \\
&+ \frac{1}{k} E_{t-1} \left(C_{t+j+1}\right) + \frac{1}{k} E_{t-1} \left(K_{t+j+1}\right) \\
&- \left(\frac{1}{c} + \frac{1}{k}\right) \left(\frac{1}{c} E_{t-1} C_{t+j} + \frac{1}{k} E_{t-1} C_{t+j}\right) \\
&+ 2 \left(\frac{1}{k} \left(\frac{1}{a} + \frac{1}{c}\right) E_{t-1} G_{t+j} - 2 \left(\frac{1}{k} \left(\frac{1}{a} - \frac{1}{c}\right) E_{t-1} G_{t+j-1}\right)\right]
\end{align*}
\]

(22)
\[
- \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \varepsilon_t + \frac{1}{k} (v4t - v2t) \right] \\
- \left( \frac{1}{c} + \frac{1}{k} \right) (v4t-1 - v2t-1) \\
\]

\[
\left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \theta - d_1 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \right] P_t = \\
\left[ \frac{1}{c} + \frac{1}{k} \right] \left( \frac{1}{a} + \frac{1}{b} \right) \theta - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \right] P_{t-1} \\
- \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ d_0 + d_2 E_t(XD_{t+j}) + \frac{1}{c} E_t(C_{t+j}) \right] \\
+ \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) \right] + \frac{1}{k} \left[ \frac{1}{c} E_t(C_{t+j+1}) \right] \\
+ \frac{1}{k} E_t(K_{t+j+1}) - \left( \frac{1}{c} + \frac{1}{k} \right) \left[ \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) \right] \\
+ 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right) E_t(G_{t+j}) - 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right) E_t(G_{t+j-1}) \\
+ \left( \frac{1}{c} + \frac{1}{k} \right) \theta \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ d_0 + d_2 E_{t-1}(XD_{t+j}) \right] \\
+ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) \right] \\
+ \frac{1}{k} \left[ \frac{1}{c} E_{t-1}(C_{t+j+1}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) \right] \\
- \left( \frac{1}{c} + \frac{1}{k} \right) \left[ \frac{1}{c} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) \right] + 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right) E_{t-1}(G_{t+j}) \\
- 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right) E_{t-1}(G_{t+j-1}) \\
- \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \varepsilon_t + \frac{1}{k} (v4t - v2t) \\
- \left( \frac{1}{c} + \frac{1}{k} \right) (v4t-1 - v2t-1) \right]
\]

Since \( \theta + \theta^{-1} = \phi \)

\[
\phi = \frac{d_1 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) + 2}{\frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right)}
\]

Then,
\[
\frac{1}{k}(\frac{1}{a} + \frac{1}{b})\theta + \frac{1}{k}(\frac{1}{a} + \frac{1}{b})\theta^{-1} = d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{k}(\frac{1}{a} + \frac{1}{b} + \frac{1}{c})
+ 2 \frac{1}{k}(\frac{1}{a} + \frac{1}{b})
= d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) + \frac{1}{k}(\frac{1}{a} + \frac{1}{b} + \frac{1}{c})
- \frac{1}{c} + \frac{1}{k}(\frac{1}{a} + \frac{1}{b})
\]

\[
\frac{1}{k}(\frac{1}{a} + \frac{1}{b})\theta - d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{k}(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) = \frac{1}{(\frac{1}{a} + \frac{1}{b})}\theta^{-1}
\]

And finally,

\[
\frac{1}{k}(\frac{1}{a} + \frac{1}{b})\theta - d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{k}(\frac{1}{a} + \frac{1}{b} + \frac{1}{c})\theta = \frac{1}{(\frac{1}{a} + \frac{1}{b})}\theta^{-1}
\]

The LHS divided by \(\theta\) is the coefficient of \(P_t\) and the RHS is the coefficient of \(P_{t-1}\) in (23).

Substitute (24) into (23), one gets the reduced form of price.

\[
P_t = \theta P_{t-1} + \frac{1}{k(\frac{1}{a} + \frac{1}{b})\theta - d_1(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{k}(\frac{1}{a} + \frac{1}{c} + \frac{1}{b})} \left[ \frac{(\frac{1}{a} + \frac{1}{b})}{1} \sum_{j=0}^{\infty} \theta j \left(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}\right)d_0 + d_2E_{t-1}(XD_{t+j}) + \frac{1}{c}E_{t-1}(C_{t+j}) + \frac{1}{k}E_{t-1}(K_{t+j}) - \frac{1}{k}E_{t-1}(K_{t+j+1}) \right]
\]
\[ + \frac{1}{k} \left[ \frac{1}{k} C_{t+j+1} + \frac{1}{c} E_{t-1}(K_{t+j+1}) \right] - \left( \frac{1}{k} - \frac{1}{c} \right) \left[ \frac{1}{k} C_{t+j} + \frac{1}{c} E_{t-1}(K_{t+j}) \right] \\
+ \frac{1}{k} E_{t-1}(K_{t+j}) + 2\left[ \frac{1}{k} \left( \frac{1}{k} + \frac{1}{c} \right) \right] E_{t-1}(G_{t+j}) - 2\left( \frac{1}{k} \frac{1}{c} a \right) \\
- \frac{1}{c b} E_{t-1}(G_{t+j-1}) \right] - \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_0 \right] \\
+ d_2 E_t(X D_{t+j}) + \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) \\
+ \frac{1}{k} \left[ \frac{1}{k} C_{t+j+1} + \frac{1}{c} E_t(K_{t+j+1}) \right] - \left( \frac{1}{k} + \frac{1}{c} \right) \left[ \frac{1}{k} E_t(C_{t+j}) \right] \\
+ \frac{1}{k} E_t(K_{t+j}) + 2\left[ \frac{1}{k} \left( \frac{1}{k} + \frac{1}{c} \right) \right] E_t(G_{t+j}) - 2\left( \frac{1}{k} \frac{1}{c} a \right) \\
- \frac{1}{c b} \left[ E_t(G_{t+j-1}) \right] - \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \epsilon_t \right] \\
+ \frac{1}{k} \left( v^4_t - v^2_t \right) - \left( \frac{1}{c} + \frac{1}{k} \right) \left( v^4_{t-1} - v^2_{t-1} \right) \right] \tag{25} \]

Substitute (21) into (18), the reduced form of futures price is obtained.

\[ P_{t-1}^f(t) = \left[ \frac{(a + b) \theta + \frac{1}{k}}{\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)} \right] P_{t-1} + \frac{\theta}{\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_0 + d_2 E_{t-1}(X D_{t+j}) + \frac{1}{c} E_{t-1}(C_{t+j}) \\
+ \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) \right] + \frac{\frac{1}{k} \left( \frac{1}{k} + \frac{1}{c} \right) \left[ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) \right] \\
+ \frac{1}{k} \left[ \frac{1}{k} C_{t+j+1} + \frac{1}{c} E_{t-1}(K_{t+j+1}) \right] - \left( \frac{1}{k} + \frac{1}{c} \right) \left[ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) \right] \\
+ \frac{1}{k} E_{t-1}(K_{t+j}) + 2\left[ \frac{1}{k} \left( \frac{1}{k} + \frac{1}{c} \right) \right] E_{t-1}(G_{t+j}) - 2\left( \frac{1}{k} \frac{1}{c} a \right) \\
- \frac{1}{c b} \left[ E_{t-1}(G_{t+j-1}) \right] - \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \epsilon_t \right] \\
+ \frac{1}{k} \left( v^4_{t-1} - v^2_{t-1} \right) \right] \tag{26} \]

By the same method, we also get

\[ P_{t}^f(t + 1) = \left[ \frac{(a + b) \theta + \frac{1}{k}}{\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)} \right] P_{t} + \frac{\theta}{\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)} \sum_{j=0}^{\infty} \theta^j \]
\[
\left[ \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right] d_0 + d_2 E_t(XD_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1})
\]
\[
+ \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2}) + \frac{1}{k} \left[ \frac{1}{c} E_t(C_{t+j+1}) \right] + \frac{1}{k} E_t(K_{t+j+1})
\]
\[
\frac{1}{k} E_t(K_{t+j+2}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) \right] + 2 \left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) E_t(G_{t+j+1}) - 2 \left[ \frac{1}{k} \frac{1}{a} - \frac{1}{k} \frac{1}{c} \right] E_t(G_{t+j}) \right]
\]
\[
+ \frac{1}{k} \left[ \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right] \left[ \frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1} + \left[ \frac{1}{a} + \frac{1}{c} \right]
\]
\[
- \left( \frac{1}{b} + \frac{1}{k} \right) \left[ G_t + \nu_4 t - \nu_2 t \right]
\]  

(27)

And the reduced form of current futures price can be found by substituting the reduced form of current cash price into (27).

\[
P_t^f(t+1) = \left[ \frac{(\frac{1}{a} + \frac{1}{b}) \theta + \frac{1}{k}}{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} \right] \theta P_{t-1} +
\]
\[
\left[ \frac{1}{c} + \frac{1}{b} \right] \left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \theta - d_1 \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \right]
\]
\[
\sum_{j=0}^{\infty} \theta_j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \left[ \frac{1}{a} \frac{1}{c} \right] \right] d_0 + d_2 E_{t-1}(XD_{t+j})
\]
\[
+ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \left[ \frac{1}{c} E_{t-1}(C_{t+j}) \right]
\]
\[
E_{t-1}(C_{t+j+1}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{k} E_{t-1}(C_{t+j}) \right] + 2 \left[ \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right] E_{t-1}(G_{t+j}) - 2 \left( \frac{1}{k} \frac{1}{a} - \frac{1}{k} \frac{1}{c} \right) E_t(XD_{t+j})
\]
\[
+ \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) + \frac{1}{k} \left[ \frac{1}{c} E_t(C_{t+j+1}) \right]
\]
\[
+ \frac{1}{k} E_t(K_{t+j+1}) - (\frac{1}{c} + \frac{1}{k}) \left[ \frac{1}{k} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) \right] + 2 \left[ \frac{1}{k} \frac{1}{a} + \frac{1}{c} \right] E_t(G_{t+j}) - 2 \left( \frac{1}{k} \frac{1}{a} - \frac{1}{k} \frac{1}{c} \right) E_t(G_{t+j+1})
\]
\[
\left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \epsilon_t + \frac{1}{k} (v^4_t - v^2_t) - \left( \frac{1}{c} + \frac{1}{k} \right) (v^4_{t-1} - v^2_{t-1}) \right] + \frac{\theta}{k (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) [d_0 + d_2 E_t(XD_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2})] + \frac{1}{c} E_t(C_{t+j+2}) + \frac{1}{k} E_t(K_{t+j+2}) - \left( \frac{1}{c} + \frac{1}{k} \right) \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) \right] 2 \left[ \frac{1}{k} \frac{1}{c} \right] \frac{1}{c} E_t(G_{t+j}) + \frac{1}{c} \frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1} + \frac{1}{a} \frac{1}{c} - \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v^4_t - v^2_t \right] \right]
\]

Substitute (27) into (11), we will get

\[
I_t = \frac{1}{k} \left[ \left( \frac{1}{a} + \frac{1}{b} \right) \theta + \frac{1}{k} \right] \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) - 1 \right] P_t - \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v^4_t
\]

\[
+ \frac{\theta}{k (\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} \sum_{j=0}^{\infty} \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) [d_0 + d_2 E_t(XD_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2})] + \frac{1}{c} E_t(C_{t+j+2}) + \frac{1}{k} E_t(K_{t+j+2}) - \left( \frac{1}{c} + \frac{1}{k} \right) \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) \right] 2 \left[ \frac{1}{k} \frac{1}{c} \right] \frac{1}{c} E_t(G_{t+j}) + \frac{1}{c} \frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1} + \frac{1}{a} \frac{1}{c} - \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v^4_t - v^2_t \right] \right]
\]

And substitute (25) into (29), the reduced form of inventory demand is ob-
\[
(1 + \ell + i \mathcal{D})^2 \mathcal{G} \left[ \left( \frac{q}{1} + \frac{\omega}{1} \right) \zeta + \left( (1 + \ell + i \mathcal{D})^2 \mathcal{G} \right) \frac{q}{1} \right] - \left( (1 + \ell + i \mathcal{D})^2 \mathcal{G} \right) \left[ \frac{q}{1} \right] \zeta + \left( (1 + \ell + i \mathcal{D})^2 \mathcal{G} \right) \frac{q}{1} + (1 + \ell + i \mathcal{D})^2 \mathcal{G} \frac{q}{1} + (1 + \ell + i \mathcal{D})^2 \mathcal{G} \frac{q}{1} + \ldots
\]

\[
\left[ (1 - \ell^2 a - 1 - i \mathcal{P} \mathcal{D} \mathcal{G} \frac{q}{1} + 0_p) \left( \mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1} \right) \right] \mathcal{D} \theta \left. \right|_{\infty}^{0} = \frac{\ell}{\theta} \left( \frac{\mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1}}{\mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1}} \right) \mathcal{D} \theta \left. \right|_{\infty}^{0}
\]

\[
\left[ (1 - \ell^2 a - 1 - i \mathcal{P} \mathcal{D} \mathcal{G} \frac{q}{1} + 0_p) \left( \mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1} \right) \right] \mathcal{D} \theta \left. \right|_{\infty}^{0} = \frac{\ell}{\theta} \left( \frac{\mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1}}{\mathcal{G} \frac{q}{1} + \frac{\omega}{1} + \frac{q}{1}} \right) \mathcal{D} \theta \left. \right|_{\infty}^{0}
\]
\begin{equation}
-2 \left( \frac{1}{k} \frac{1}{a} - \frac{1}{c} \frac{1}{b} \right) E_t(G_{t+j}) + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ \frac{1}{c} C_{t+1} + \frac{1}{k} K_{t+1} + \left[ \frac{1}{a} + \frac{1}{c} \right] - \left( \frac{1}{a} + \frac{1}{k} \right) G_t + v4_t - v2_t \right] \right. \\
\left. - \frac{1}{k} K_{t+1} + \frac{1}{k} G_t + v3_t \right) \tag{30}
\end{equation}

Substitute (25) into (12) we will get the reduced form of consumer demand.

\begin{equation}
D_t = d_0 - d_1 \theta P_{t-1} + d_2 XD_t \\
- \frac{d_1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right. \\
\left. \left[ \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right] \right. \\
\left. \sum_{j=0}^{\infty} \theta_j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) d_0 + d_2 E_{t-1}(XD_{t+j}) \\
+ \frac{1}{c} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) \right. \\
+ \frac{1}{k} \left[ \frac{1}{c} E_{t-1}(C_{t+j+1}) + \frac{1}{k} E_{t-1}(K_{t+j+1}) \right] - \left( \frac{1}{c} + \frac{1}{k} \right) E_{t-1}(C_{t+j}) \right. \\
+ \frac{1}{k} \left[ \frac{1}{k} E_{t-1}(K_{t+j+1}) \right] + \frac{1}{c} \left[ \frac{1}{c} + \frac{1}{k} \right] E_{t-1}(G_{t+j}) - \frac{1}{k} \frac{1}{c} a \\
- \frac{1}{c} \frac{1}{b} \frac{1}{k} \frac{1}{a} \frac{1}{c} (G_{t+j}) - 2 \frac{1}{k} \frac{1}{c} a \\
\right. \\
- \frac{1}{k} \frac{1}{c} \frac{1}{b} \frac{1}{k} \frac{1}{a} \frac{1}{c} (G_{t+j}) - \left( \frac{1}{c} + \frac{1}{k} \right) (G_{t+j}) - 2 \frac{1}{k} \frac{1}{c} a \\
- \frac{1}{k} \frac{1}{c} \frac{1}{b} \frac{1}{k} \frac{1}{a} \frac{1}{c} (G_{t+j}) - \left[ \left( \frac{1}{c} + \frac{1}{k} \right) (G_{t+j}) - \frac{1}{k} \frac{1}{c} \frac{1}{b} \frac{1}{k} \frac{1}{a} \frac{1}{c} (G_{t+j}) \right] \right. \\
\left. + v5_t \right) \tag{31}
\end{equation}

From (14), the aggregate hedging can be obtained as a function of current
cash price as follows:

\[ X_t(t+1) = \left[ \frac{1}{a} + \frac{1}{c} \right] \left[ \frac{(\frac{1}{a} + \frac{1}{b})\theta + \frac{1}{k}}{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} - \frac{1}{a} \right] P_t \\
+ \left[ \frac{1}{k}(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k}) - \frac{1}{k}(\frac{1}{a} + \frac{1}{b}) \right] \theta \sum_{j=0}^{\infty} q^j \\
\left[ \frac{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})}{(\frac{1}{a} + \frac{1}{c})} \right] \left[ d_0 + d_2 E_t(XD_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1}) \\
+ \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2}) + \frac{1}{k} \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) \\
\right] \\
+ \frac{1}{k} E_t(G_{t+j+1}) \left[ \frac{(\frac{1}{a} + \frac{1}{c})}{(\frac{1}{a} + \frac{1}{c})} \right] \left[ \frac{1}{a} C_{t+1} + \frac{1}{k} K_{t+1} + \left[ \frac{1}{a} + \frac{1}{c} \right] \\
- \frac{(\frac{1}{a} + \frac{1}{c})}{(\frac{1}{a} + \frac{1}{c})} \left[ \frac{1}{a} C_{t+1} + \frac{1}{a} C_{t+1} - \frac{(\frac{1}{a} + \frac{1}{c})}{(\frac{1}{a} + \frac{1}{c})} G_t + v2_t \\
- \frac{1}{c} C_{t+1} - \frac{(\frac{1}{a} + \frac{1}{c})}{(\frac{1}{a} + \frac{1}{c})} \right] \\
\right] + \frac{1}{k} \frac{1}{c} \theta \sum_{j=0}^{\infty} q^j \left[ \frac{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})}{(\frac{1}{a} + \frac{1}{c})} \right] \left[ \frac{1}{k} E_{t-1}(XD_{t+j}) \\
+ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \frac{1}{c} E_{t-1}(C_{t+j}) \\
+ \frac{1}{k} E_{t-1}(K_{t+j+2}) + \frac{1}{k} \frac{1}{c} E_{t-1}(G_{t+j}) + \right] \\
\]

Substitute the reduced formed of current cash price in the above equation, the reduced form of aggregate hedging can be formed as follows:

\[ X_t(t+1) = \left[ \frac{1}{a} + \frac{1}{c} \right] \left[ \frac{(\frac{1}{a} + \frac{1}{b})\theta + \frac{1}{k}}{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})} - \frac{1}{a} \right] \theta P_{t-1} + \\
\frac{1}{k} \frac{1}{c} \theta \sum_{j=0}^{\infty} q^j \left[ \frac{(\frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k})}{(\frac{1}{a} + \frac{1}{c})} \right] \left[ \frac{1}{k} E_{t-1}(XD_{t+j}) \\
+ \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \frac{1}{c} E_{t-1}(C_{t+j}) \\
+ \frac{1}{k} E_{t-1}(K_{t+j+2}) + \frac{1}{k} \frac{1}{c} E_{t-1}(G_{t+j}) + \right] \\
\]

\[ + \frac{1}{k} E_{t-1}(C_{t+j}) + \frac{1}{k} E_{t-1}(K_{t+j}) - \frac{1}{k} E_{t-1}(K_{t+j+1}) + \frac{1}{k} \frac{1}{c} E_{t-1}(C_{t+j}) \\
+ \frac{1}{k} E_{t-1}(K_{t+j+2}) + \frac{1}{k} \frac{1}{c} E_{t-1}(G_{t+j}) + \right] \\
\]
\[ E_{t-1}(G_{t+j-1}) - \sum_{j=0}^{\infty} \theta j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0 + d_2 E_t(XD_{t+j})] 
+ \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+1}) \right] 
+ \sum_{j=0}^{\infty} \theta j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0 + d_2 E_t(XD_{t+j})] 
+ \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) \right] 
+ \sum_{j=0}^{\infty} \theta j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right)[d_0 + d_2 E_t(XD_{t+j})] 
+ \frac{1}{c} E_t(C_{t+j}) + \frac{1}{k} E_t(K_{t+j}) - \frac{1}{k} E_t(K_{t+j+1}) \right] \]

\[ \theta \sum_{j=0}^{\infty} \theta j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \theta \sum_{j=0}^{\infty} \theta j \right] \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \theta \sum_{j=0}^{\infty} \theta j \right] \]
\[
-2 \left( \frac{1}{k} a - \frac{1}{c} b \right) E_t(G_{t+j}) + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ \frac{1}{c} C_{t+1} \right.
\]
\[+ \frac{1}{k} K_{t+1} + \left( \frac{1}{a} + \frac{1}{c} \right) - \left( \frac{1}{a} + \frac{1}{k} \right) \right] G_t + v4_t - v2_t \]
\[- \frac{1}{c} C_{t+1} - \left( \frac{1}{a} + \frac{1}{c} \right) G_t + v2_t \quad (32)
\]

By the same way from (15), substitute the value of \( E_t(P_{t+1}) \) from (22) and \( P_{t+1} \) from (27), aggregate speculation can be shown as function of current cash price as:

\[
Y_t(t+1) = \left[ \frac{1}{k} + \frac{1}{b} \theta - \left( \frac{1}{b} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] P_t
\]
\[+ \left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} \right) \frac{\theta}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \theta \sum_{j=0}^{\infty} \theta j
\]
\[\left[ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) \right] \left[ d_0 + d_2 E_t(XD_{t+j+1}) + \frac{1}{c} E_t(C_{t+j+1}) \right]
\[+ \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2}) + \frac{1}{k} \left( \frac{1}{c} \right) E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1})
\]
\[+ 2 \left[ \frac{1}{1} \left( \frac{1}{a} + \frac{1}{c} \right) \right] E_t(G_{t+j+1}) - 2 \left[ \frac{1}{1} \right] - \frac{1}{1} \left( \frac{1}{c} \right) E_t(G_{t+j+1})
\]
\[+ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \left[ \frac{1}{a} C_{t+1} + \frac{1}{k} K_{t+1} - \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v4_t \right]
\[- \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v4_t - v2_t \right] \right] \theta P_{t-1} +
\]
\[\frac{1}{1} - \left( \frac{1}{b} + \frac{1}{k} \right) \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) \right] \left[ \left( \frac{1}{k} + \theta \right) \right] \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) - \left( \frac{1}{b} + \frac{1}{k} \right) \left( \frac{1}{b} + \frac{1}{k} \right) \theta + \frac{1}{k}
\]
\[\left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \left[ \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{k} \right) \theta - d_1 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} + \frac{1}{k} \right) - \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \right]
\]
\[
\left\{ \begin{array}{l}
\frac{1}{\zeta} + \frac{1}{k} \theta \sum_{j=0}^\infty \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \left( d_0 + d_2 E_t(XD_t+j) \right) \\
+ \frac{1}{c} \left( \frac{1}{k} \right) E_t-1(C_t+j) + \frac{1}{k} \left( \frac{1}{c} \right) E_t-1(K_t+j) - \frac{1}{k} \left( \frac{1}{c} \right) E_t-1(K_t+j+1) + \frac{1}{k} \left( \frac{1}{c} \right) \right] \\
+ \frac{1}{k} E_t-1(C_t+j+1) + \frac{1}{k} E_t-1(K_t+j+1) - \left( \frac{1}{c} + \frac{1}{k} \right) \frac{1}{c} E_t-1(C_t+j) \\
+ \frac{1}{k} E_t-1(K_t+j) + 2 \left( \frac{1}{k} \right) \left( \frac{1}{c} + \frac{1}{k} \right) E_t-1(G_t+j) - 2 \left( \frac{1}{k} \right) \frac{1}{c} \\
\right. \\
\left. \right] - \sum_{j=0}^\infty \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \left( d_0 + d_2 E_t(XD_t+j) \right) \\
+ \frac{1}{c} E_t(C_t+j) + \frac{1}{k} E_t(K_t+j) - \frac{1}{k} E_t(K_t+j+1) + \frac{1}{k} \left( \frac{1}{c} \right) E_t(C_t+j) \\
+ \frac{1}{k} E_t(K_t+j) - \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{c} \right) E_t(C_t+j) + \frac{1}{k} E_t(K_t+j) + 2 \left( \frac{1}{k} \right) \frac{1}{c} \\
\right. \\
\left. \right] E_t(G_t+j) - 2 \left( \frac{1}{k} \right) \frac{1}{c} \frac{1}{b} E_t(G_t+j) - 1 \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) E_t \\
+ \frac{1}{k} \left( v_4 t - v_2 t \right) - \left( \frac{1}{a} + \frac{1}{c} \right) \left( v_4 t - v_2 t \right) \left( 1 \right) \right\} \\
+ \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \frac{1}{b} \frac{1}{k} \sum_{j=0}^\infty \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \left( d_0 + d_2 E_t(XD_t+j+1) \right) \\
+ \frac{1}{c} E_t(C_t+j+1) + \frac{1}{k} E_t(K_t+j+1) - \frac{1}{k} E_t(K_t+j+2) \\
+ \frac{1}{k} \left( \frac{1}{c} \right) \left( \frac{1}{k} \right) E_t(C_t+j+2) + \frac{1}{k} E_t(K_t+j+2) - \left( \frac{1}{c} + \frac{1}{k} \right) \left( \frac{1}{c} \right) E_t(C_t+j+1) \\
+ \frac{1}{k} E_t(K_t+j+1) + 2 \left( \frac{1}{k} \right) \left( \frac{1}{c} + \frac{1}{k} \right) E_t(G_t+j+1) \\
- 2 \left( \frac{1}{k} \right) \frac{1}{c} \frac{1}{b} E_t(G_t+j) \frac{1}{k} \left( \frac{1}{c} \right) C_t+1 + \frac{1}{k} K_t+1 \\
+ \left( \frac{1}{a} + \frac{1}{c} \right) \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v_4 t - v_2 t \right] \\
+ \left[ \frac{1}{k} \frac{1}{b} \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \right) \right] \theta^j \left[ \left( \frac{1}{a} + \frac{1}{c} \right) \left( d_0 + d_2 E_t(XD_t+j+1) \right) \\
+ \left( \frac{1}{k} \right) \frac{1}{k} \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right] \right\} 
\right\}
\]
\begin{equation}
\frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) - \frac{1}{k} E_t(K_{t+j+2}) + \frac{1}{k} \left[ \frac{1}{c} E_t(C_{t+j+2}) + \frac{1}{k} E_t(K_{t+j+2}) \right] - \left( \frac{1}{c} + \frac{1}{k} \right) \frac{1}{c} E_t(C_{t+j+1}) + \frac{1}{k} E_t(K_{t+j+1}) + 2 \left( \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) \right) E_t(G_{t+j+1}) - 2 \left( \frac{1}{k} a - \frac{1}{c} b \right) E_t(G_{t+j+1}) + \frac{1}{k} \left( \frac{1}{a} \right) \frac{1}{c} \left( \frac{1}{b} \right) \left( \frac{1}{k} \right) \left[ \frac{1}{c} C_{t+1} \right] + \frac{1}{k} K_{t+1} + \frac{1}{k} \left( \frac{1}{a} + \frac{1}{c} \right) G_t + v4_t - v2_t - \frac{1}{k} K_{t+1} - \left( \frac{1}{b} + \frac{1}{k} \right) G_t + v4_t \end{equation}
APPENDIX B. VARIABLE LIST

\( u_t^h \) = utility function of producer or hedger at time \( t \).
\( \alpha \) = constant absolute risk aversion of producers or hedgers.
\( \pi_t(h) \) = profit of producers' firm at time \( t \).
\( E_{t-1}(u_t^h) \) = expected utility of producer or hedger at time \( t - 1 \) for the profit to be realized at time \( t \).
\( \sigma_t^2(h) \) = one period variance of profit of producers' firm.
\( C_t \) = direct cost of production of producer's firm at time \( t \).
\( q_t \) = quantity of production of a commodity by producers' firm at time \( t \).
\( x_{t-1}(t) \) = quantity of commodity for short hedge in the futures market at time \( t - 1 \) for the delivery at time \( t \).
\( c \) = cost of producers' firm other than direct cost of production, is assumed to be quadratic.
\( G_{t-1} \) = cost of using futures market which is commission to broker's firm at time \( t - 1 \).
\( E_{t-1}(P_t) \) = expected cash price for the period \( t \).
\[ P_t = \text{cash price of commodity at time } t. \]
\[ P_{t-1}^f = \text{futures price at time } t - 1 \]
\[ \text{for the delivery at time } t. \]
\[ \sigma_p^2 = \text{one period variance of cash prices.} \]
\[ S_t = \text{aggregate supply which is total production of a commodity at time } t. \]
\[ X_{t-1}(t) = \text{aggregate short hedge by producers' firm at time } t - 1 \]
\[ \text{for the delivery at time } t. \]
\[ u_t^g = \text{utility function of inventory holder or speculator at time } t. \]
\[ \beta = \text{constant absolute risk aversion of inventory holder or speculator.} \]
\[ \pi_t(s) = \text{profit of inventory holder or speculator at time } t. \]
\[ E_{t-1}(u_t^g) = \text{expected utility of inventory holder or speculator} \]
\[ \text{at time } t - 1 \text{ for the profit to be realized at time } t. \]
\[ \sigma^2(s) = \text{one period variance of profit of inventory holder or speculator.} \]
\[ i_{t-1} = \text{amount of investment in inventory at time } t - 1. \]
\[ y_{t-1}(t) = \text{amount of speculation to buy in futures market at time } t - 1 \]
\[ \text{for the commodity to be delivered at time } t. \]
\[ K_t = \text{direct carrying cost of inventory holder's firm at time } t. \]
\[ k = \text{cost of inventory holder's firm other than direct carrying cost,} \]
\[ \text{assumed to be quadratic.} \]
\[ I_{t-1} = \text{aggregate inventory demand at time } t - 1. \]
\( Y_{t-1}(t) = \) aggregate speculation at time \( t - 1 \) for the commodity to be delivered at time \( t \).

\( D_t = \) aggregate consumer demand at time \( t \).

\( XD_t = \) exogeneous variable representing demand shifter at time \( t \).

\( v_{1_t} = \) supply disturbance at time \( t \).

\( v_{2_t} = \) disturbance in hedging at time \( t \).

\( v_{3_t} = \) disturbance in inventory demand at time \( t \).

\( v_{4_t} = \) disturbance in speculation at time \( t \).

\( v_{5_t} = \) disturbance in demand at time \( t \).

\( d_0, d_1, d_2 = \) parameters in the consumer demand equation.
The data used in this study are annual data. The reason was because production of corn and wheat were done annually. The available data were from 1934 to 1985.

Since the estimation was done annually on one equation, which was equation (19) in Chapter Three, data for the variables specified in that equation were necessary. Those variables that entered the corn and wheat equations were: cash prices, futures prices, the direct cost of production, the direct carrying cost, the cost of using futures market, the fertilizer price index and the government support price. The variable that only entered the corn equation was the animal unit of livestock fed annually. The variable that only entered the wheat equation was per capita disposable personal income.

The direct cost of production for both corn and wheat were the variable costs of production per bushel. The data used were collected from various sources. Those were the variable cost per acre from various issues of Agricultural Statistics, U.S.D.A., from Center for National Food and Agricultural Policy, Iowa State University, and from Cost of Producing Major Crops and Economic Indicators of the Farm Sector, U.S.D.A. The variable cost per bushel was found by dividing the variable cost per acre by the average yield per acre which was obtained from var-
ious issues of Agricultural Statistics, U.S.D.A. The missing data were calculated by using the ratio of the price index for the missing year to the price index of the available year multiplied the variable cost per acre of the available year. And the variable cost per bushel was calculated in the same way as mentioned before. The index used for calculating the variable cost of corn for the missing year is the Index of Total Cost per Unit of Production for cash grain for the corn belt farm. The index used for calculating variable cost of wheat for the missing year is the Index of Total Cost per Unit of Production for winter wheat farm. Those indices were obtained from costs and return, Agricultural Information Bulletin, U.S.D.A.

The data that had been calculated by this method were from the year 1947 to 1959 for both corn and wheat. The data from 1961 to 1974 of corn, from 1961 to 1965 and from 1969 to 1973 of wheat were provided by Center for National Food and Agricultural Policy (CNFAP). Those were the estimates done by Karl Skold by the method of backcasting from the available data in the later periods.

The data used to measure direct carrying costs per bushel were calculated by using the Interest Rate: Production Credit Associations Average Cost of Loans timed the cash prices of that particular crop. The cash prices used in calculating the direct carrying cost were the Average Cash Price Received by Farmers. Both data were collected from various issues of Agricultural Statistics, U.S.D.A.

The data on the brokerage fee which were the cost of using futures market were unavailable for most of the estimating periods. The only one number that could be found from the published document was the commissions to the brokerage firm in using futures market which were reported to be 0.6 cent per bushel for both
corn and wheat. These data were published in “Hedging Potential in Grain Storage and Livestock Feeding”, Agricultural Economic Report No. 238, E.R.S., U.S.D.A., January 1973. Therefore this amount was taken to be the brokerage fee in 1972. The brokerage fee per bushel for other years were calculated by using the ratio of average hourly earning: brokerage, between the calculated year and the year 1972 multiplied by 0.6 cent.

The data on average hourly earning, brokerage, were collected from Statistical Abstract of the U.S.: Nonagricultural Industries, Number of Employees, and Number, Hours, and Earnings of Production Worker, Production Workers, Average Hourly Earnings: Brokerage. However, the whole series of data were not completed. The data on the average hourly earning: brokerage, for the missing years which were from year 1934 to 1938, 1940 to 1942 and 1978 to 1985 were estimated. The estimation method was done by running OLS of average hourly earning: manufacturing and index of farm wage rate. Then using those estimated coefficients together with the average hourly earning, manufacturing and index of farm wage rate for the available years to get the estimates of the average hourly earning: brokerage for the missing years.

The fertilizer price index is the Price Paid by Farmers, Production Indexes: Fertilizer, which is collected from various issues of Agricultural Statistics, U.S.D.A. The government support price for corn and wheat were Support Price per Bushel: Target, collected from various issues of Agricultural Statistics, U.S.D.A.

The data on animal units of livestock fed annually were collected from various issues of Statistical Abstracts of the U.S., U.S. Department of Commerce, Bureau
of the Census. The data on per capita disposable income were Per Capita Income: Disposable Personal Income, collected from various issues of Statistical Abstract of the United States, U.S. Department of Commerce.

The corn cash prices used for both the in sample data and the out of sample data were collected from the various issues of Feed Situation, Feed Outlook and Situation, and Statistical Bulletin, U.S.D.A. Those are the average monthly cash price of no.3 yellow corn from 1934 to 1960 and of no.2 yellow corn from 1961 to 1985 from Chicago market. The collected data for the in sample data were the December cash prices and for the out of sample data were the cash prices in September, March, May and July.

The wheat cash prices used for both the in sample data and the out of sample data were the average monthly cash prices of no.2 hard winter wheat from 1934 to 1950 and of no.1, hard red winter, ordinary protein, from the Kansas City market. The data were collected from various issues of Statistical Bulletin and Wheat Situation. The in sample data were the data in July. The out of sample data were the data in September, December, March and May.

The corn futures prices were the month end closing futures prices in May for the delivery in December. The wheat futures prices were the month end closing futures prices in October for the delivery in July. The data for the corn futures prices for out of sample data were the month end closing futures prices in March, July, June and December for the delivery in September, March, May and July. The data for the wheat futures prices for out of sample data were the month end closing futures prices in January, April, June, and August for the delivery in September,
December, March and May. The data used for out of sample data were also from 1934 to 1985. The data were collected from various issues of Commodity Futures Statistics, Statistical Bulletin, U.S.D.A., The Wall Street Journal, and the Annual Report of the Board of Trade of the City of Chicago.
11 APPENDIX D. OTHER CONVERGENCE RESULTS

Based on different starting value the other estimates which were also the convergence results were shown as follows.
Table 11.1: Results of the parameter estimation for corn equation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.25</td>
<td>0.0040</td>
<td>4.06*</td>
</tr>
<tr>
<td>$\frac{1}{k}$</td>
<td>2.25</td>
<td>2.7719</td>
<td>5.65*</td>
</tr>
<tr>
<td>$\frac{1}{\alpha}$</td>
<td>0.25</td>
<td>0.0043</td>
<td>0.91</td>
</tr>
<tr>
<td>$\frac{1}{\beta}$</td>
<td>0.25</td>
<td>201.2844</td>
<td>275.87*</td>
</tr>
<tr>
<td>$\frac{1}{\sigma^2}$</td>
<td>0.25</td>
<td>60.3493</td>
<td>110.59*</td>
</tr>
<tr>
<td>$\frac{1}{c}$</td>
<td>2.25</td>
<td>55.0982</td>
<td>23.41*</td>
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<tr>
<td>$d_1$</td>
<td>1</td>
<td>29.2147</td>
<td>516.99*</td>
</tr>
<tr>
<td>$d_2$</td>
<td>1</td>
<td>0.1681</td>
<td>3.98*</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.

Objective function = 597.614772
Table 11.2: Results of the parameter estimation for wheat equation

<table>
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<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
</tr>
</thead>
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<tr>
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<td>2.1159</td>
<td>2.63*</td>
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<tr>
<td>$\frac{1}{\alpha}$</td>
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<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>$\frac{1}{\beta}$</td>
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<td>0.0684</td>
<td>0.77</td>
</tr>
<tr>
<td>$\frac{1}{\sigma_p^2}$</td>
<td>1.96</td>
<td>0.0947</td>
<td>0.58</td>
</tr>
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<td>$d_1$</td>
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<tr>
<td>$d_2$</td>
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<td>0.3525</td>
<td>0.72</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.

Objective function = 286.957369
Table 11.3: Results of the parameter estimation for wheat equation

<table>
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<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
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<td>$\frac{1}{\alpha}$</td>
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<tr>
<td>$\frac{1}{\beta}$</td>
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<td>0.6876</td>
<td>0.41</td>
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<tr>
<td>$\frac{1}{\sigma_p^2}$</td>
<td>.25</td>
<td>0.0896</td>
<td>0.29</td>
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<tr>
<td>$\frac{1}{c}$</td>
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<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>$d_1$</td>
<td>1</td>
<td>3.1755</td>
<td>4.91*</td>
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<tr>
<td>$d_2$</td>
<td>1</td>
<td>0.4479</td>
<td>0.71</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.
Objective function = 286.957369
Table 11.4: Results of the parameter estimation for wheat equation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Starting value</th>
<th>Coefficients</th>
<th>Asymptotic t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>.0049</td>
<td>.0048</td>
<td>1.13</td>
</tr>
<tr>
<td>$\frac{1}{k}$</td>
<td>.04</td>
<td>.0087</td>
<td>1.59</td>
</tr>
<tr>
<td>$\frac{1}{\alpha}$</td>
<td>1</td>
<td>5.1483</td>
<td>0.99</td>
</tr>
<tr>
<td>$\frac{1}{\beta}$</td>
<td>25</td>
<td>.3290</td>
<td>0.34</td>
</tr>
<tr>
<td>$\frac{1}{\sigma^2_p}$</td>
<td>5.76</td>
<td>5.5054</td>
<td>23.99*</td>
</tr>
<tr>
<td>$\frac{1}{c}$</td>
<td>1</td>
<td>.1527</td>
<td>2.92*</td>
</tr>
<tr>
<td>$d_1$</td>
<td>.5</td>
<td>.0853</td>
<td>1.16</td>
</tr>
<tr>
<td>$d_2$</td>
<td>.8</td>
<td>.1300</td>
<td>0.52</td>
</tr>
</tbody>
</table>

*Indicates the t-statistic which has value over two.

Objective function = 282.441583