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OPTIMAL PRICE AND QUANTITY OF REFURBISHED PRODUCTS

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Abstract

Many retail product returns can be refurbished and resold, typically at a reduced price. The price set for the refurbished products affects the demands for both new and refurbished products, while the refurbishment and resale activities incur costs. To maximize profit, a manufacturer in a competitive market must carefully choose the proportion of returned products to refurbish and their sale price. We model the sale, return, refurbishment and resale processes in an open queueing network and formulate a mathematical program to find the optimal price and proportion to refurbish. Examination of the optimality conditions reveals the different situations in which it is optimal to refurbish none, some, or all of the returned products. Refurbishing operations may increase profit or may be required to relieve a manufacturing capacity bottleneck. A numerical study identifies characteristics of the new product market and refurbished products that encourage refurbishing and some situations in which small changes in the refurbishing cost and quality provoke large changes in the optimal policy.

(REVERSE LOGISTICS, REFURBISHMENT; QUEUEING NETWORKS; OPTIMIZATION)

November, 2005

1. Introduction

Over $100 billion worth of products are returned from customers to retailers annually (Stock, Speh and Shear, 2002). The reasons for and time scales of these enormous returns are summarized in Table 1 (Silva, 2004 and Souza et al., 2005). Other than at the end of life, products are returned relatively soon after distribution. Dowling (1999) shows that up to 35 percent of new products are returned before the end of their life cycle. The value of these returns is considerable since they still preserve features and technologies of new products that are currently for sale. When returns come to manufacturers, the right decision has to be made to manage these returns profitably. Depending on their quality and the manufacturer's policy, some returns even qualify to be sold again as new products to regain the total margin. Products that have been used or have some defects will be either refurbished then resold whole or dismantled into parts that are either kept for service or sold.

Refurbished products are those that have been verified by the manufacturer to be as functional as new products. White and Naghibi (1998) described the refurbishment process as complying with the highest standards and giving careful attention to both the interior and the exterior of the product. Electronic products are subjected to rigorous electrical testing to ensure they meet all original manufacturing specifications. Examples of products that qualify for refurbishment are consumer-returned products, off-lease products, products with shipping damage, and over stocks (Silva, 2004 and Souza et al., 2005).

From the consumer perspective, buying refurbished products is an economical way to obtain goods that perform as well as new products. For the manufacturer, refurbished products broaden the market by drawing the consumer who is not willing to pay full price to purchase

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refurbished products for less. However, there may be an overlap between the markets for new and refurbished products. Consumers in this overlap market will choose between new or the refurbished product based on price and perceived quality.

In this paper, we study a manufacturer in a competitive market for new products, such as a producer of personal computers. New products are produced to order, but for a variety of reasons, some of them are returned soon after sale for a refund. One choice is whether to refurbish returns and offer them for sale, and if so, how many. Unlike the market for new products, where the manufacturer is a price-taker, we assume that because the manufacturer would be the only source for certified refurbished products, it is able also to choose the price at which to offer its inventory of refurbished items. This price must be chosen with care because it will play an important role in determining the demand for both new and refurbished products. In addition, there may be substantial costs associated with refurbishing products and holding them in inventory. Therefore, both the price and the quantity of refurbished products may have significant impacts on the manufacturer's total profit.

TABLE 1. Reasons for product returns

Reasons for product returns	Description	Length of time (approximate)
Customer satisfaction	The quality of product does not meet the customer's expectation. This category also includes miscellaneous reasons such as customers cannot use products, find a better price, over ordered or feel remorse.	Return period (30 days)
Evaluation product	Products that were reviewed and tested by editors or vendors.	Evaluation period (30 days)
Shipping damage	Products cannot be sold as new when their containers are damaged.	Shipping period (< 7 days)
Defective	The product cannot perform functions as described.	Warranty period (1 year)
End of lease	The product is returned after the end of the lease.	Lease period (varied)
End of life	The product is collected after it has been discarded.	Life cycle of product (varied)

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In order to model the significant uncertainties in the arrivals of demands for new and refurbished products, the time from sale to return, and the length of time that may be required to refurbish items, we model the (partially) closed loop supply chain as an open queueing network (Buzacott and Shanthikumar, 1993). Whereas some previous reverse logistics papers (Toktay et al., 2000, Bayindir et al., 2003, and Souza et al., 2002) treated new and remanufactured products as indistinguishable, we follow the market segmentation literature in assuming that refurbished products with lower perceived quality may nevertheless compete with new products on the basis of price (Arunkundram and Sundararajan, 1998; Debo, et al., 2005). The proportion of returns to refurbish and their price are decision variables in a nonlinear program with the objective to maximize the total profit.

In this context, we seek to discover how the optimal price and volume of refurbished products are affected by characteristics of the markets for new and refurbished products. Additional profit may be achieved by selling refurbished items but possible pitfalls include reducing the demand for more profitable new products or accumulating large inventories of refurbished products with deteriorating value. On the other hand, when there is insufficient capacity to meet the demand for new products, offering refurbished ones can take up some of the excess demand. The analysis will show that the optimal policy is discontinuous – it is optimal to refurbish either no returns, or a significant proportion (perhaps all) of them. Therefore, it is valuable to know the conditions under which small changes in the model parameters tip the balance for or against refurbishing.

The remainder of this paper is organized as follows: Section 2 contains a literature review of previous market segmentation studies and approaches to apply queueing networks in closed

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loop supply chains. Section 3 provides the mathematical model and Section 4 shows numerical examples that illustrate the implementation of the model and its results. Section 5 concludes with a description of future work. An Appendix contains proofs of the feasible region's convexity, details of the Karush-Kuhn-Tucker optimality conditions and the concavity of total profit with respect to the price of new products.

2. Literature review

The management of item returns has been studied in a variety of models. Fleischmann et al. (1997) review the quantitative models for reverse logistics in three main areas: distribution planning, inventory control, and production planning. Since then, much work has focused on these operational areas of closed loop supply chains, but the competition between new and remanufactured items is a relatively recent concern.

Fasano et al. (2002) use an optimization tool to determine if end-of-life IBM equipment should be sold as whole or dismantled for service parts. They assume that demand for refurbished products in a particular time period is limited, and that a machine will be refurbished only when there is demand and potential positive net revenue that results in a specified profit margin.

Arunkundram and Sundararajan (1998) consider the competition of used and new products in the electronic secondary markets. The paper shows the situation where the secondary market benefits the profitability of new product sales. Majumder and Groenevelt (2001) study the competition in a remanufacturing scheme between an original equipment manufacturer and a local remanufacturer. Their two-period game theoretic model finds the Nash equilibrium of the price and quantity for both competitors in different scenarios. Ferrer and Swaminathan (2005) extend the previous work by developing multi-period model to find the

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Nash equilibrium for duopoly case. From the remanufacturer perspective, Guide et al. (2003) propose an economic analysis for finding the optimal acquisition and selling prices, along with quantity of used product acquisitions in the cellular telephone industry. In more closely related work, Debo et al. (2005) study a monopolist's decision of whether to produce a remanufacturable product, where competition with third party remanufacturers may exist. The additional cost to make a product remanufacturable may be worthwhile if enough customers value the remanufactured product highly, but competition reduces the optimal level of remanufacturability. They also expose the role of new products as a source for products to be remanufactured. Ferguson and Toktay (2005) examine competition for remanufactured products in more detail, exploring strategies by which the manufacturer can exploit its access to used products to ward off third party remanufacturers.

Several types of queueing models have been applied in the remanufacturing environment. Toktay et al. (2000) construct a queueing network to simulate the entire supply chain of a single-use camera. The optimization model minimizes the costs of procurement, inventory, and lost sales for different policies. Bayindir et al. (2003) find the optimal probability of return for items sold. They assume that the returns are controllable and the manufacturer has infinite capacity. Unlike our model, the consumers are indifferent between the new and remanufactured products. Souza et al. (2002) simulate the remanufacturing facility as a $GI/G/1$ queueing network. The product returns have different grades that require different processing times. The model finds the optimal product mix for remanufacturing to maximize profit. Ketzenberg et al. (2003) compare two configurations of a remanufacturing process, mixed and parallel lines, in several scenarios by using $GI/G/c$ queueing network. Souza et al. (2005) analyze and suggest the

appropriate supply chain for commercial product returns for products with different decays in price.

This paper differs from the previous ones in that we explicitly consider the competition between new and refurbished products in the context of a competitive market for new products, and we jointly optimize the price and quantity of refurbished products from the manufacturer's perspective. The manufacturer has little control over the quantity and timing of product returns and may not have sufficient capacity to meet the demand for new products. The queuing network model allows for modeling a significantly variable time with customers, rather than the uniform one period assumed by Debo et al. (2005) and Ferguson and Toktay (2005), as well as other sources of variability. It also permits accounting of costs such as transportation, handling and inventory holding throughout a closed loop supply chain where new products are made to order while returned products are refurbished to stock. Analyzing numerical results allows us to assess the sensitivity of the objective function and decision variables to parameters concerning new products (i.e., price and backorder cost) and refurbished products (i.e., cost of refurbishing and perceived quality).

3. Model Formulation and Optimality Conditions

Our research is primarily motivated by manufacturers who produce and sell electronic products via an online store, e.g., Dell, Apple, or Gateway Computers. New products are produced to order while returned products will be refurbished and ready to ship to consumers immediately according to a make-to-stock policy. In this section, we first describe assumptions and function of demand for new and refurbished products. Next, the supply chain is simulated as

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an open queueing network. Finally, we present a nonlinear program for maximizing total profit and outline its optimality conditions.

3.1. *The Demand Function*

Demands for new and refurbished products are interdependent and can be described as functions of their prices and the quality of refurbished products. We assume the price of new products is an exogenous constant, as in a market where different brands of products have similar performance, so that a small change in price may cause a significant change in market share. On the other hand, as the sole source of manufacturer-certified refurbished products, the producer can control both their price and their supply. We focus on the market where consumers have declared interest in a specific brand and model of products but are still deciding to whether buying either new or refurbished or not. That is, we explicitly model only the internal competition between the new and the refurbished products. This competition can be viewed as imperfect or monopolistic since products are similar but one is still not a perfect substitute for the other (Nichols and Reynolds, 1971). The demand model is similar to similar to those of Arunkundram and Sundararajan (1998) and Debo et al. (2005). Although the price of new products is not a decision variable, it is varied in the numerical study to see its effect on the objective function and decision variables.

The market size of these consumers in a given study period is normalized to one. In this market, the valuations of consumers are uniformly distributed from 0 to 1. The consumer's willingness to pay or valuation of a product is directly proportional to its quality. If a consumer values the new product at υ, then that consumer values the refurbished product at $\delta\upsilon$. The parameter δ is the perceived quality factor of refurbished products, $0 < \delta < 1$. The perceived

quality includes many attributes such as technical specification, warranty period and physical appearance.

The price of new products (P_{new}) is scaled down to the same scale as the consumer's valuation ranging from 0 to 1. The value $P_{new} = 0$ corresponds to the minimum value of υ and $P_{new} = 1$ is the maximum possible value of υ. When we consider only new products in the market, if $P_{new} = 1$, no consumers in the market will buy the new product since no one has valuation higher than the price of new products. In contrast, when $P_{new} = 0$, all consumers in the market are willing to buy new products. The scaled price of refurbished products (P_{ref}) ranges from 0 to P_{new}. A consumer will buy the product that gives him the higher surplus, which is the difference between his valuation and the price of products. A consumer with valuation υ will choose to buy the new product when $\upsilon - P_{new} \geq 0$ and $\upsilon - P_{new} \geq \delta\upsilon - P_{ref}$, i.e., $\upsilon \geq \dfrac{P_{new} - P_{ref}}{1 - \delta}$.

On the other hand, he will buy the refurbished product when $\delta\upsilon - P_{ref} \geq 0$ and $\upsilon - P_{new} < \delta\upsilon - P_{ref}$ i.e. $\dfrac{P_{ref}}{\delta} \leq \upsilon < \dfrac{P_{new} - P_{ref}}{1 - \delta}$. If $\dfrac{P_{ref}}{\delta} = \dfrac{P_{new} - P_{ref}}{1 - \delta}$, i.e., $P_{ref} = \delta P_{new}$, there is no demand for refurbished products.

The proportion of consumers who will not buy any products is $\dfrac{P_{ref}}{\delta}$. The proportion of consumers with valuation less than $\dfrac{P_{new} - P_{ref}}{1 - \delta}$ but greater than or equal to $\dfrac{P_{ref}}{\delta}$ will buy refurbished products. The rest are willing to buy new products. The demands per unit time are scaled to represent the proportion of consumers in the market who are willing to buy products at the stated prices during the study period. The demand for new products is $\lambda_{new} =$

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$1 - \max\left(P_{new}, \dfrac{P_{new} - P_{ref}}{1 - \delta}\right)$ and the demand for refurbished products is $\lambda_{ref} =$

$\max\left(\dfrac{P_{new} - P_{ref}}{1 - \delta} - \dfrac{P_{ref}}{\delta}, 0\right)$. The sum $0 \le \lambda_{new} + \lambda_{ref} \le 1$ represents the proportion of all

consumers per unit time who will buy this product. The relationship between demand, price and

quality of refurbished products is illustrated in Figure 1. There are three different regions

separated by the two linear equations $P_{ref} = \delta P_{new}$ and $P_{ref} = P_{new} - (1 - \delta)$:

> *Region 1*: $P_{ref} \ge \delta P_{new}$. There is no demand for refurbished products and the demand for
>
> new products equals $\lambda_{new} = 1 - P_{new}$.
>
> *Region 2*: $P_{new} - (1 - \delta) < P_{ref} < \delta P_{new}$. There are demands for both new and refurbished
>
> products. We have $\lambda_{new} = 1 - \dfrac{P_{new} - P_{ref}}{1 - \delta}$ and $\lambda_{ref} = \dfrac{P_{new} - P_{ref}}{1 - \delta} - \dfrac{P_{ref}}{\delta}$.
>
> *Region 3*: $P_{ref} \le P_{new} - (1 - \delta)$. There is no demand for new products and the demand for
>
> refurbished products equals $\lambda_{ref} = 1 - \dfrac{P_{ref}}{\delta}$.
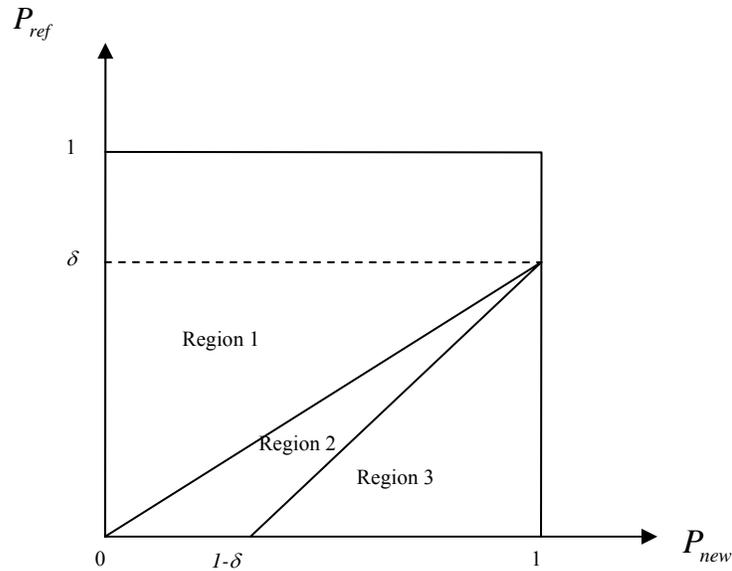
FIGURE 1. The relationship between demand, price and quality of refurbished products

We will consider only cases in which the price of refurbished products will affect the total profit. When P_{ref} is less than $P_{new} - (1 - \delta)$, the demand for new products will be zero. As a result, even though the demand for refurbished products is higher, there will be no supply of products to refurbish and the manufacturer cannot make profit from both types of products. Similarly, when the value of P_{ref} is higher than δP_{new}, there is no demand for refurbished products so that the higher value of P_{ref} will not affect the total profit. Therefore, only the values of P_{ref} in the range $[P_{new} - (1 - \delta), \delta P_{new}]$, i.e., Region 2 and its boundary, will be considered. The value of P_{ref} is also limited from above by the quantity of refurbished products supplied in order to satisfy guarantee a steady state in the queueing network, as discussed in Section 3.3 and Appendix 2.

3.2. *The Queueing Network Model*

The lowest tier of the supply chain including refurbishing is formulated as a queueing network model (Buzacott and Shantikumar, 1993). Similar approaches can be found in Toktay et al. (2000) and Souza et al. (2005). Figure 2 illustrates the whole system consisting of five stations. Each station represents a location or status of products.

According to the make-to-order policy, arrivals of the external demands for new products, at rate λ_{new}, cause materials to be released and authorize production to begin at the manufacturing site (station 1). New products are produced, and then distributed to consumers (station 2). A consumer receives and tries the product, and then decides whether to keep or to return it. The service time of station 2 represents the time the consumer keeps the product before returning it; if the consumer chooses not to return the product, the product exits the system. The manufacturer evaluates all returned products at the evaluation site (station 3) and decides the proportions of products to be refurbished at the refurbishing site (station 4) or dismantled. The dismantling process is done outside the system so those products to be dismantled exit the system. Although returned products at station 4 may have different qualities and some of them may be defective, we assume all returns can be refurbished and sold for the same price at station 5. We assume exponential service times at all stations. At station 5, the service time interval represents the interarrival time between demands for refurbished products, or its residual in case of a product arrival to an empty store. We assume no backorders for refurbished products; therefore, the demand arriving when there is no inventory will be considered lost.

We assume that products are manufactured, evaluated, refurbished and sold one at a time. Therefore, all stations except station 2 are considered to have single servers. Station 2 is

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represented as an infinite-server station because the make-to-order policy guarantees that all new

products produced will be used immediately by the consumers who order them.
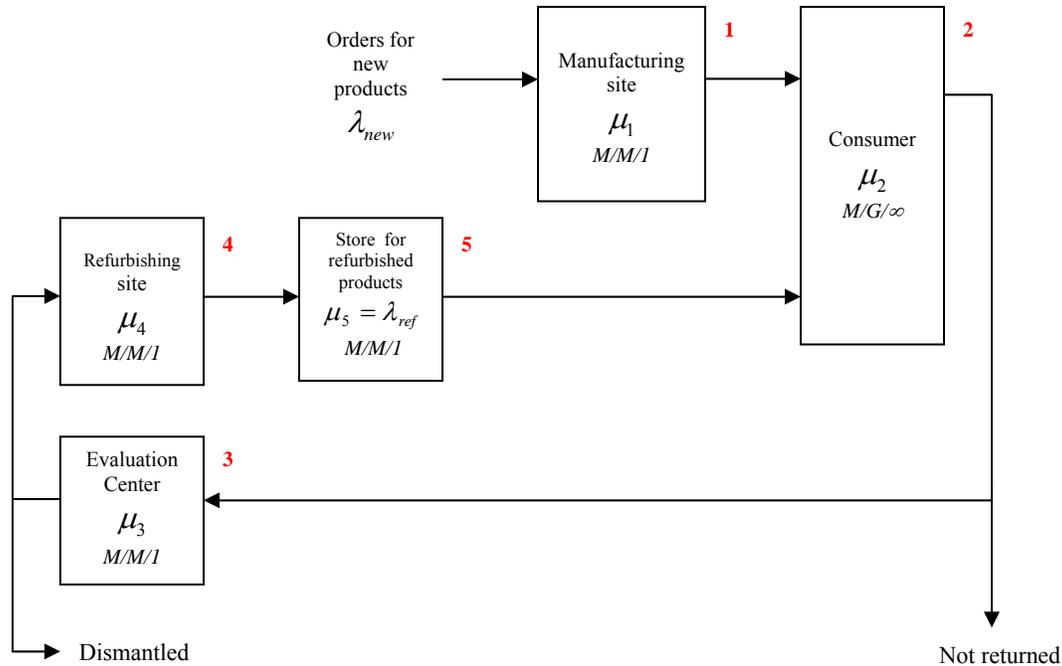

FIGURE 2. The open queueing network

3.3. *The Optimization Model and its Optimality Conditions*

The notation for the model is shown as follows.

P_{dis} = Scaled price of a dismantled product.

p_{ij} = The probability that products transfer from station i to station j, for $i, j = 0, \ldots, 5$,

where a subscript of 0 is used for transitions into or out of the system

p_{cr} = The probability that a consumer will return products to the manufacturer. To

simplify the constraints, we assume $0 < p_{cr} < 1$.

p_{mr} = The probability that the manufacturer will send returned products to be refurbished. This probability is a decision variable that indicates the proportion of product returns to be refurbished.

λ_i = The arrival rate at station i.

S_i = The service time at station i (random variable).

μ_i = Mean service rate of station i, $\mu_i = \dfrac{1}{E[S_i]}$, $i = 1, \ldots, 5$.

ρ_i = Utilization of station i, for $i = 1, 3, 4$ and 5, or the expected number of customers at station i for $i = 2$, $\rho_i = \dfrac{\lambda_i}{\mu_i}$.

γ = Required minimum ratio of the supply of refurbished items to their demand, $0 < \gamma < 1$.

N_i = Number of customers (random variable) at station i, $i = 1, \ldots, 5$.

c_{ij} = Scaled cost of transferring a product from station i to station j,

      $i, j = 1, \ldots, 5$.

c_{i0} = Scaled cost of transferring a product from station i to the world outside the system.

h_i = Cost per unit time per item being held in station i, $i = 1, \ldots, 5$.

In our queueing network model, $p_{23} = p_{cr}$ and $p_{34} = p_{mr}$. The routing matrix is

$$
\mathbf{P} \;=\; \begin{array}{c} \\ \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 0 & \;1 & \,2 & 3 & \;4 & \;5 \end{array} \\
\left[\begin{array}{cccccc}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1-p_{cr} & 0 & 0 & p_{cr} & 0 & 0 \\
1-p_{mr} & 0 & 0 & 0 & p_{mr} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]
\end{array}
$$

We assume a steady state exists, i.e., $\rho_i < 1$ for $i = 1, 3, 4, 5$. This assumption is reasonable when time to returns is relatively short compared with product life cycle. At station 1, $\rho_1 < 1$ means the manufacturing capacity exceeds the demand for new products. At station 3 (4), the inspection (refurbishing) rate must be larger than the arrival rate of returns (to be refurbished). The requirement that $\rho_5 < 1$ means that the production or supply rate of refurbished products does not exceed their demand rate, i.e., $\dfrac{p_{cr} p_{mr}}{1 - p_{cr} p_{mr}}\left(1 - \dfrac{P_{new} - P_{ref}}{1-\delta}\right) < \dfrac{P_{new} - P_{ref}}{1-\delta} - \dfrac{P_{ref}}{\delta}$. This

inequality can be rewritten as $P_{ref} < \dfrac{\delta P_{new} - p_{cr}\delta(1-\delta)p_{mr}}{1 - p_{cr}(1-\delta)p_{mr}}$ which is a more restrictive constraint

than δP_{new} as the upper bound of P_{ref}. We also require $\rho_5 \geq \gamma > 0$, i.e., $\lambda_5 \geq \gamma \lambda_{ref}$, so that demand for refurbished items is not created without also providing some supply of them. The traffic

equations in steady state, $\lambda_i = \lambda_0 p_{0i} + \sum_{j=1}^{n} \lambda_j p_{ji}$, $i = 1, \ldots, 5$, are used to find λ_i as follows: $\lambda_1 =$

λ_{new}, $\lambda_2 = \dfrac{\lambda_{new}}{1 - p_{cr} p_{mr}}$, $\lambda_3 = \dfrac{p_{cr}\lambda_{new}}{1 - p_{cr} p_{mr}}$, $\lambda_4 = \lambda_5 = \dfrac{p_{cr} p_{mr}\lambda_{new}}{1 - p_{cr} p_{mr}}$. Note that the availability of

products to refurbish is automatically constrained by production of new products, eliminating the need for an explicit constraint as in Ferguson and Toktay (2005) and Debo et al. (2005).

The service times in each station consist of:

S_1 = The time to manufacture a new product.

S_2 = The time a product is kept by a consumer before being returned for a refund. Its distribution is the same for both new and refurbished products.

S_3 = The time required to evaluate a return.

S_4 = The time to refurbish a returned product.

S_5 = The time between demands for refurbished products.

As the service time distributions at stations 1, 3, 4 and 5 are assumed to be exponential, the resulting queueing network is a Jackson network. Each of stations 1, 3, 4, 5 can be analyzed independently as an M/M/1 queueing system while station 2 is M/G/∞ (Buzacott and Shanthikumar, 1993). The expected number of customers at each station is (Gross and Harris, 1985):

$$E[N_i] = \frac{\rho_i}{1 - \rho_i} \text{ for } i = 1,3,4,\ 5, \text{ while } E[N_2] = \rho_2$$

$E[N_1]$ is the expected number of orders being processed or waiting to be processed by the manufacturer. $E[N_i]$ for $i = 2,\ldots, 5$ are the expected numbers of products with consumers at the evaluation center, the refurbishing site and in the inventory of refurbished product store, respectively. The cost per new product backorder per unit time is h_1 while h_i is the holding cost of products in station i, $i = 2,\ldots, 5$.

The cost of transferring a product from station i to station j may consist of handling, production and/or transportation cost. Table 2 shows the breakdown of c_{ij}.

TABLE 2. Breakdown of costs in c_{ij}.

	Handling	Production	Transportation
c_{12}	×	×	×
c_{23}	×		×
c_{20}	×		
c_{34}	×		×
c_{30}	×		×
c_{45}	×	×	
c_{52}	×		×

The total scaled profit is given by:

Total Profit $\quad$ = Total Revenue – Total Cost, where

Total Revenue $\quad$ = Revenue New + Revenue Refurbish + Revenue Dismantle

$$= P_{new}\lambda_{new}(1-p_{cr}) + P_{ref}\lambda_5(1-p_{cr}) + P_{dis}\lambda_3(1-p_{mr})$$

Total Cost $\quad$ = Handling, Production, and Transportation cost + Backorder cost

+ Holding cost

$$= \sum_j \sum_i c_{ij}\lambda_i p_{ij} + \sum_i h_i E[N_i]$$

Revenues are retained only for those products sold but not returned by the consumer. Note that, like the costs, the total revenue is also scaled. Let $f(P_{ref}, p_{mr})$ be the total profit as a function of the decision variables P_{ref} and p_{mr}. The optimization model is

Objective: Max $f(P_{ref}, p_{mr})$

Subject to:

$$P_{ref} \geq P_{new} - (1-\delta) \tag{1}$$

$$P_{ref} \leq \delta P_{new} \tag{2}$$

$$\rho_1 \leq 1 - \varepsilon_1 \tag{3}$$

$$\rho_3 \leq 1 - \varepsilon_3 \tag{4}$$

$$\rho_4 \leq 1 - \varepsilon_4 \tag{5}$$

$$\rho_5 \leq 1 - \varepsilon_5 \tag{6}$$

$$\rho_5 \geq \gamma \tag{7}$$

$$p_{mr} \leq 1 \tag{8}$$

$$p_{mr} \geq 0 \tag{9}$$

where ε_i, $i = 1, 3, 4, 5$, are some small positive constants. Constraints (1) and (2) come from the demand assumptions in Section 3.1. Inequalities (3) to (6) are the nonstrict inequality form of steady-state conditions, suitable for optimization software. A minimal service level at the refurbished products store is guaranteed by (7), while (8) and (9) are upper and lower bounds on the probability p_{mr}. Constraints (5) and (6) are immaterial when $p_{mr} = 0$ because there are no arrivals to stations 4 or 5.

In line with Souza et al. (2002), we assume the time to evaluate returned products is short so that the manufacturer should have ample capacity to handle all returned products; therefore, we will discard constraint (4). We also assume that the refurbishing site has enough capacity to process the maximum possible rate of incoming returned products, i.e., $\mu_4 > \dfrac{p_{cr}(1 - P_{new})}{1 - p_{cr}}$, which implies that (5) is less restrictive than (2). It can be shown in turn that (6) renders (2) redundant while (7) is more restrictive than (1). Therefore the feasible region is defined by (3), (6), (7), (8) and (9). To guarantee its convexity, we also assume $\gamma \leq \delta$. When demand of new products exceeds the manufacturing capacity, another upper bound on γ is required to guarantee

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that the feasible region is nonempty. More details about the constraints can be found in the Appendix. Since the feasible region is convex, any hill-climbing algorithm can correctly find local optima. However, because the total profit is not necessarily pseudoconcave over the entire feasible region, there may be multiple local optima. Although these local optimal solutions could lie on any point in the feasible region, a closer examination of the objective function shows that points on boundary of (3) and (6) do not yield the maximum total profit. Therefore (3) and (6) will not be active at an optimal solution. From the Karush-Kuhn-Tucker (KKT) optimality conditions, we find three possibilities for locally optimal solutions:

1) The solution lies on the minimal point $\left(P_{ref}, p_{mr}\right) = \left(\delta P_{new}, 0\right)$ when new product manufacturing capacity exceeds demand, or where (3) and (7) are both binding when it does not.

2) The solution is in the interior of the feasible region, i.e., $0 < p_{mr} < 1$.

3) The optimal solution lies on the boundary $p_{mr} = 1$.

The details of KKT conditions can be found in Appendix 2.

Points corresponding to case (1) can be identified simply, and correspond to not refurbishing, or refurbishing the minimum amount required to relieve the manufacturing capacity constraint. Points corresponding to case (2) can be found by a hill-climbing procedure from an initial value of p_{mr} between 0 and 1. The third case is simply optimizing P_{ref} at the point $p_{mr} = 1$. Finally, we can find the global optimum by comparing profits for the locally optimal points from cases 1, 2 and 3. In the following section, we will see sensitivity analysis and comparative statics from numerical results and discuss the conditions under which the different cases are optimal.

4. Numerical Results and Discussion

We designed a numerical study to explore the demand and cost characteristics of new and refurbished products that would encourage or discourage refurbishing and influence the price of refurbished products. Specifically, we study the impact of P_{new}, the backorder penalty for new products h_1, the perceived quality δ, and the cost of refurbishing c_{45}. Other parameter values were set to represent a real situation as closely as possible. The probability that a consumer would return a product was $p_{cr} = 0.25$, based on the $15 - 20\%$ rate of commercial returns for high-tech products (Toktay, 2003) plus additional returns from leasing and other sources. Given that the prices are normalized between 0 and 1, we set c_{12}, the manufacturing variable cost, to 0.25 so that $P_{new} \geq 0.3$ would provide a reasonable profit margin. Other costs were set in relation to c_{12}, as $c_{23} = 0$, $c_{20} = 0$, $c_{34} = 0.01$, $c_{30} = 0.02$, and $c_{52} = 0$. Holding costs were $h_2 = 0$ and $h_i = 0.00005$, $i = 3,\ldots,5$. These costs per unit time appear small because they are scaled twice, first by a price factor and second by a time factor; for instance, an item that cost \$500 to produce at the rate of 300 per unit time would have a holding cost of $(500/0.25)(300/0.6)h_i = \$50$ per month. Other combinations of cost and production rate would scale holding costs differently. The price of components of a dismantled product was $P_{dis} = 0.15$.

Given a minimum value of 0.3 for P_{new}, the demand rate $\lambda_{new} \leq 0.7$. The manufacturing rate μ_1 was set to 0.6, so that demand would be less than capacity at station 1 in most but not all cases. Correspondingly, μ_2, μ_3 and μ_4 were set to 0.006 and 0.6 and 0.3, respectively. The consumer evaluation rate μ_2 is much less than rates at other stations because the mean time products are held by consumers is very long compared with the time to manufacture them. Nevertheless, the consumer station has unlimited capacity due to its infinite number of servers.

As in Souza et al., (2002) the mean evaluation time should be quite short; however, we set $\mu_3 =$ 0.6 to reflect the fact that resources such as manpower may not be continuously available and to provide a nontrivial utilization for station 3. We also set the refurbishing rate μ_4 low enough for its utilization to be noticeable but not high enough for its utilization to constrain the optimal solution.

Taken together, the parameter values allowed examination of the tradeoffs between profits to be made from new and refurbished products and potential problems of long waits for new products or high inventories of refurbished products. The numerical example was solved by Mathematica (Wolfram Research, Inc., 2003) and LINGO (Lindo, Inc., 2004) software.

Figure 3 illustrates the three cases of global optimum by plotting the optimal profit as a function of p_{mr}. There are at most two local optima at each value of δ. As suggested by case 1 of the KKT conditions, $\partial f / \partial p_{mr} < 0$ for small values of p_{mr}, so that $\left(P_{ref}, p_{mr}\right) = \left(\delta P_{new}, 0\right)$ is a local optimum in each case. Therefore, a case 2 local optimum in the interior of the feasible region is separated from the case 1 solution by a significant margin, suggesting that it is never optimal to refurbish only a small fraction of returns. By a careful examination of the different components of profit, we observe that a small increase from $p_{mr} = 0$ has two negative effects on profit: (1) the demand for new products falls, causing all three types of revenue to decrease because λ_3 and λ_5 decrease in proportion to λ_{new}, and (2) the inventory cost at station 5 rises sharply because $P_{ref}^*\left(p_{mr}\right)$ close to δP_{new} creates little demand for refurbished items. For larger values of p_{mr} (with decreasing P_{ref}), the slope becomes positive as the rate of increase in total profit per new item produced exceeds the rate of decrease in λ_{new}. When the perceived quality is low, the slope is negative at high p_{mr} because at the correspondingly low values of P_{ref} it is

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more profitable to dismantle some items. Note that the optimal policy has this discontinuous

character even without fixed costs for setting up the refurbishment processes nor economies of

scale. Table 3 shows the comparisons to identify the global optimum in each case.
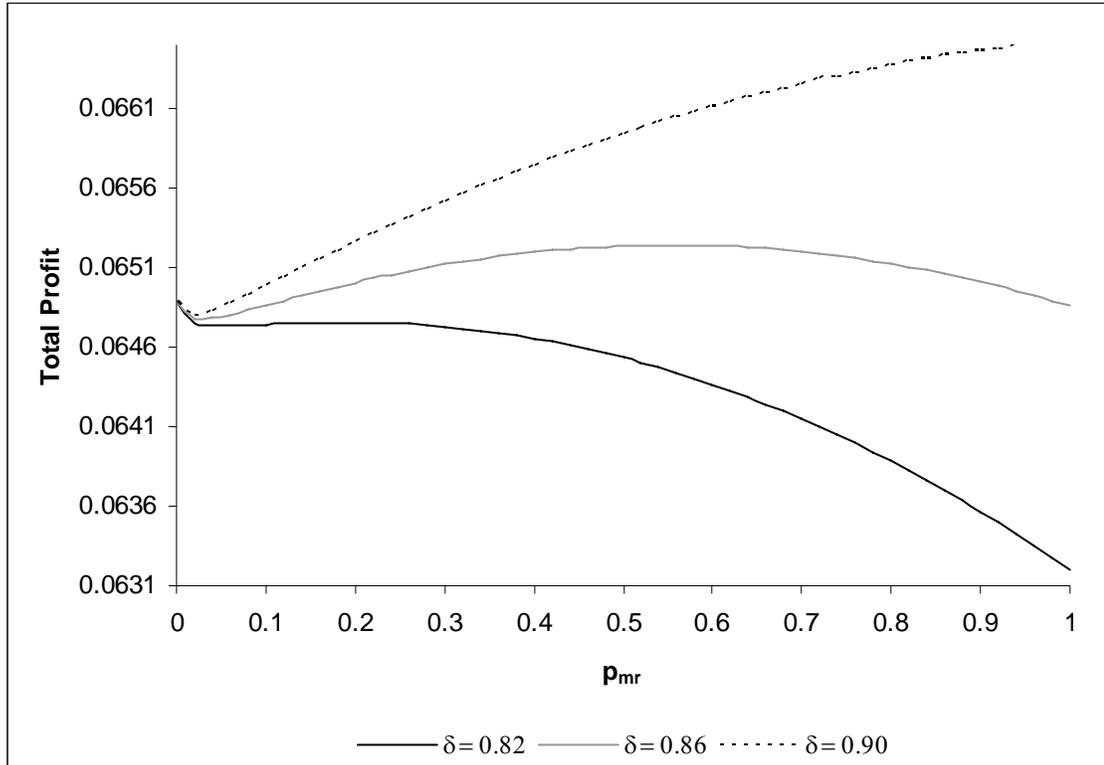
FIGURE 3 Total profit for different values of p_{mr} for $P_{new} = 0.45,\ c_{45} = 0.06,\ \text{h1} = 0.0001$ and

$\delta = 0.82, 0.86$ **and 0.90**

TABLE 3. Comparing local optima to find the global optimum.

| δ	Local optima $\left(P_{ref}, p_{mr} \right)$
0.82	**(0.3690,0)**, (0.3648,0.18)
0.84	(0.3870,0), **(0.3769,0.56)**
0.90	(0.4050,0), **(0.3918,1)**

4.1. *Cost and Quality of Refurbished Products*

Although the quality of refurbished products might be improved by investing in more costly refurbishing processes, this relationship is difficult to measure as the quality also depends on consumer perception. Figure 4 shows the effects of δ and c_{45} on the optimal objective function value and decision variables, when the two quantities are varied independently. Increasing δ increases the total profit, p^*_{mr}, and P^*_{ref} because refurbished products will enjoy higher demand and merit higher prices. The opposite effects result from increasing c_{45} because the higher cost reduces the profit from selling refurbished products. Note that a shift from $p_{mr} = 0$ to a positive value is accompanied by a discontinuous drop in P_{ref}. This is indicated by the discontinuous lines connecting the points in Figures 4b and 4c. In cases where $p^*_{mr} = 1$, P^*_{ref} does not necessarily increase with c_{45} because, while the profit margin for refurbished products decreases, profit from new products is unchanged. It may be more profitable overall to sacrifice profit from refurbished products in favor of higher demand for new products instead of increasing P_{ref} to recover the cost.

4.2. *Price and Backorder Cost for New Products*

The price of new products, determined in the competitive market, affects both the market share as measured by the demand rate and the marginal profit from new products. A low value of P_{new} creates a scenario where the manufacturer sells a high volume of products with a low profit margin. If the price is very low, the new product demand may exceed the manufacturing capacity. We varied P_{new} from 0.30, enough higher than the manufacturing cost to generate profit, to 0.95, less than one to guarantee some demand. Figure 5a shows the feasible region

when P_{new} is between 0.3 and $0.4 = 1 - \mu_1$. In this case, constraint (3) forms part of the feasible

region's boundary because any value of $P_{ref} \geq P_{new} - (1 - \delta)(1 - \mu_1)$ would cause demand for new

products to exceed the manufacturing capacity, resulting in unbounded backorders at the

manufacturing station. Instead, the price of refurbished products must be set low enough to

decrease the demand for new products and create demand for refurbished products instead.

Feasible values of p_{mr} do not include 0. When p_{mr} is large, P_{ref} is also bounded above by

constraint (6) to prevent an exploding inventory of refurbished products. When $P_{new} > 0.4$, the

manufacturing site has enough capacity to process all possible demands for new products. In this

case, p_{mr} may take on any value between 0 and 1 and only the stability of the queue of

refurbished inventory at station 5 places an upper bound of P_{ref}^* as shown in Figure 5b.
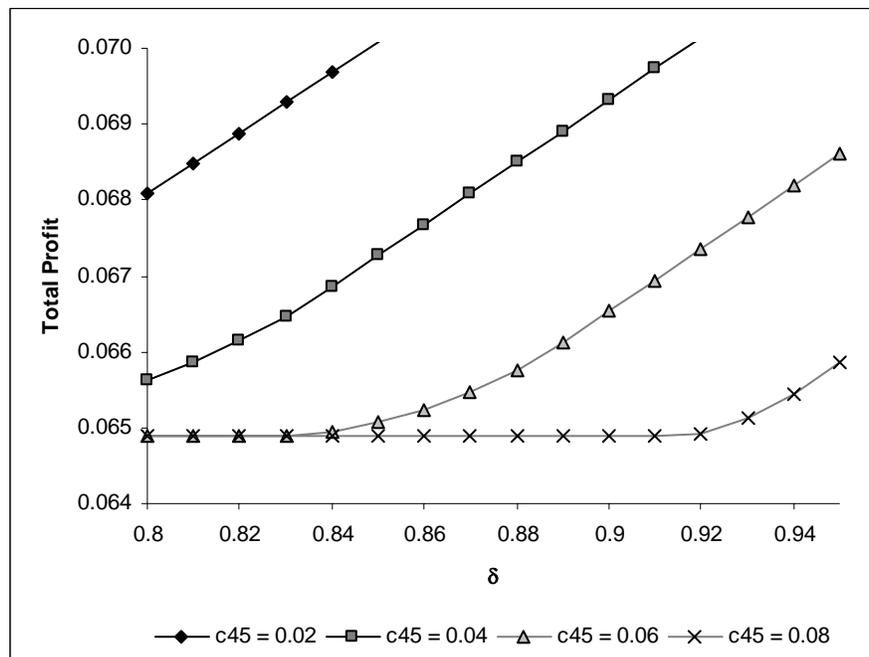
(a)
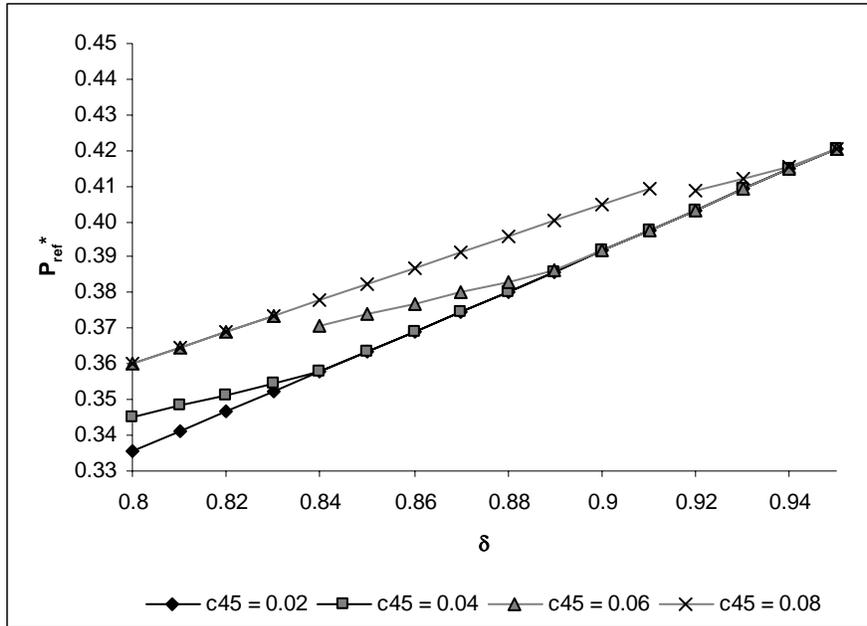
(b)
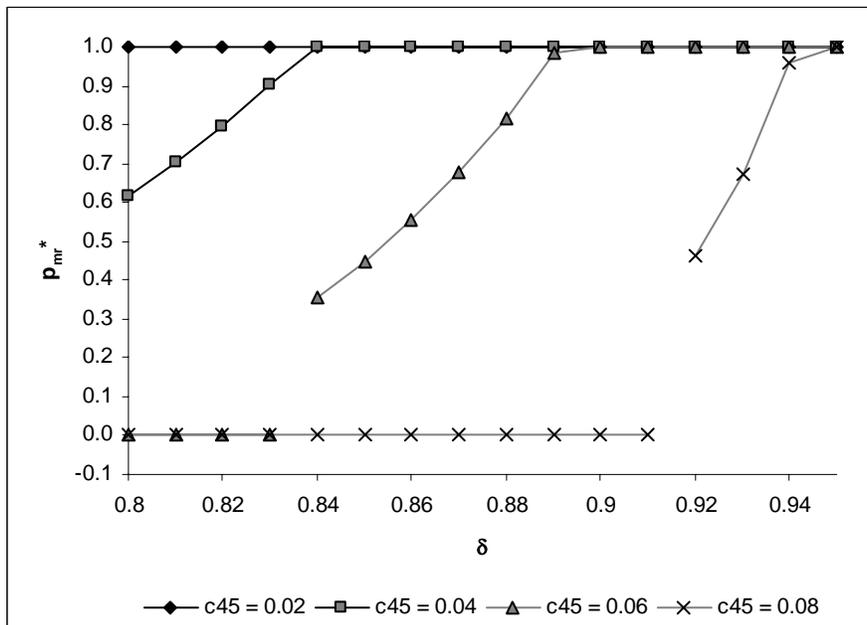
(c)

FIGURE 4. Total profit (a), P_{ref}^* (b) and p_{mr}^* (c) for different values of δ and c_{45} at $P_{new} = 0.45$ and $h_1 = 0.0001$
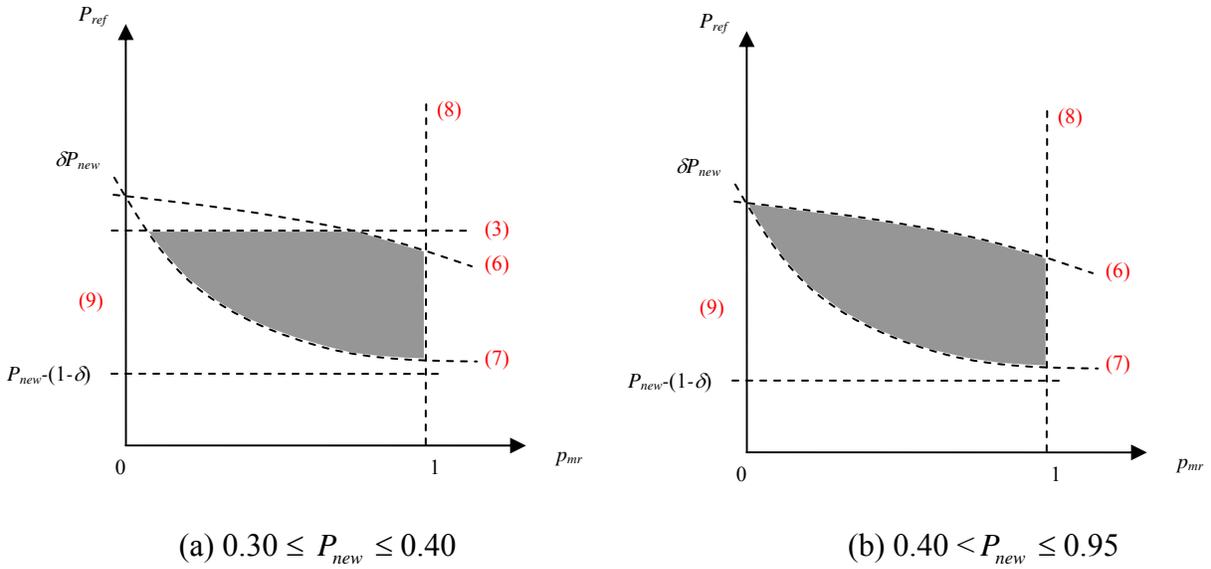
(a) $0.30 \leq P_{new} \leq 0.40$

(b) $0.40 < P_{new} \leq 0.95$

FIGURE 5. Feasible regions with two different sets of constraints depending on P_{new}.

(a) $h_1 = 0.00005$
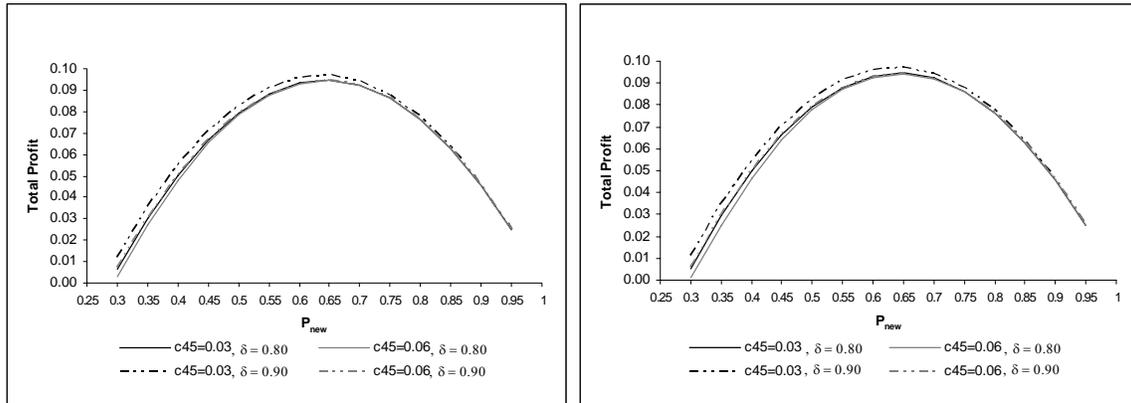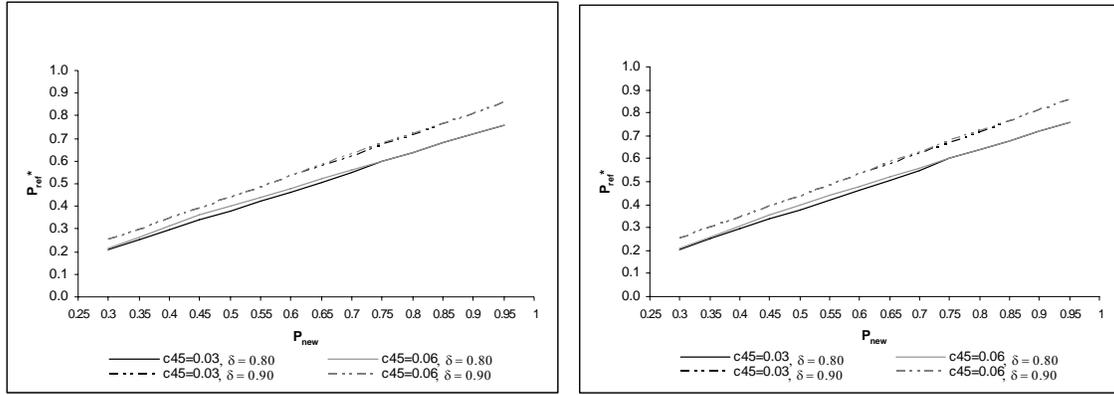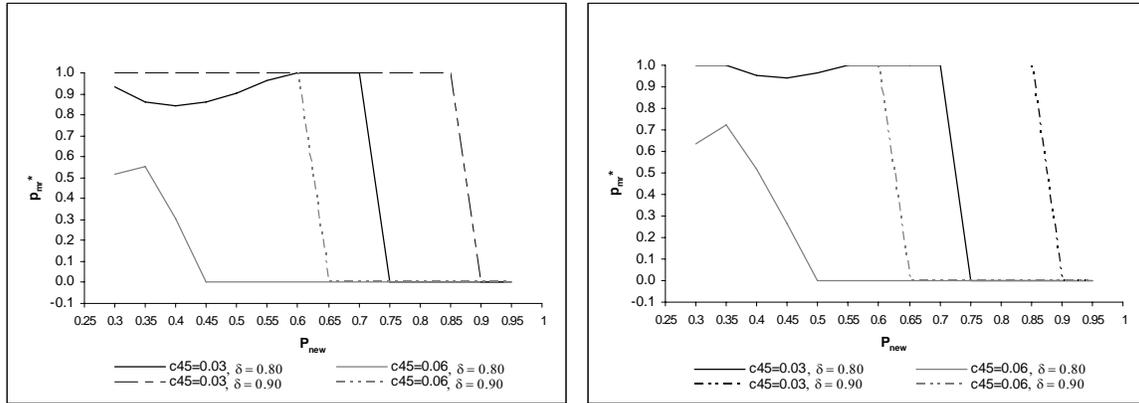
(b) $h_1 = 0.00020$

FIGURE 6. The optimal total profit

(a) $h_1 = 0.00005$ (b) $h_1 = 0.00020$

FIGURE 7. The optimal price for refurbished products.

(a) $h_1 = 0.00005$ (b) $h_1 = 0.00020$

FIGURE 8. The optimal proportion of returns to refurbish.

The cost of new product backorders is another important parameter that can be difficult to quantify. We expect that higher values would also encourage refurbishing as a way to increase customer satisfaction. Figures 6, 7 and 8 show total profit, P_{ref}^* and p_{mr}^* at different values of P_{new}, h_1, c_{45} and δ. The total profit is concave with respect to P_{new} (see proof in Appendix 3) such that the highest total profit is achieved under similar values of P_{new} for a variety of combinations of the other parameters. Figure 7 shows that P_{ref}^* predictably increases with P_{new}. When P_{new} is large, its increase reduces the optimal p_{mr} because of the higher profit margin for

new products; however, the optimal proportion to refurbish exhibits varied behavior when the price of new products is low, particularly when it is low enough that new product demand exceeds capacity. Moreover at low P_{new}, p_{mr}^* increases when backorder cost is higher (figure 8a, 8b) because refurbished products ease the demand for new products and subsequently lower the number of orders waiting in the manufacturing site. This effect is less significant as P_{new} increases because the number of orders waiting in the manufacturing site decreases. The effect of backorder cost on total profit and P_{ref}^* is not obviously seen because of the vast difference in total profit and P_{ref}^* for different P_{new}. Table 4 summarizes the effects of parameters P_{new}, h_1, δ and c_{45} on the total profit, p_{mr}^*, and P_{ref}^*. Generally, refurbishing is encouraged by high perceived quality achieved at a low refurbishing cost, and/or high backorder costs with a low price for new products.

TABLE 4. The effect of parameters P_{new}, h_1, δ and c_{45} on total profit, p_{mr}^*, and P_{ref}^*.

	Total Profit	p_{mr}^*	P_{ref}^*		Sensitivity of the optimal policy
			$0 < p_{mr} < 1$	$p_{mr} = 1$	
δ	pos.	pos.	pos.	pos.	pos.
c_{45}	neg.	neg.	pos.	neg.	pos.
P_{new}	concave	varies.	pos.	pos.	pos.
h_1	neg.	pos.	neg.	neg.	neg.

4.3. *Sensitivity of the Optimal Policy*

Given that it is optimal to refurbish either none or a significant proportion of the returned products, it is important to understand conditions under which small changes in parameters cause a shift from one paradigm to the other. Figure 9 shows the sensitivity of the optimal proportion to refurbish. The cost of refurbishing products c_{45} ranges from 0.02, or 8% of the manufacturing cost to 0.10, beyond which $p_{mr}^* = 0$ in most cases regardless of the quality. The quality parameter δ ranges from 0.80 to 0.95 to focus on commercial product returns, which still preserve most but not all of the perceived quality of new products. In Figure 9a, when the refurbishment cost and quality of refurbished products are both large, p_{mr}^* shifts abruptly from zero to one; in other words, the optimal policy is more sensitive to small changes in these parameters. The sensitivity is reduced for all cost and quality combinations when the backorder cost is higher as in Figure 9c. In Figures 9b and 9d, when P_{new} is 0.65 or near where the maximum profit can be achieved (from the previous section), p_{mr}^* is very sensitive to changes in refurbishment cost and quality of refurbished products and the backorder cost becomes almost irrelevant except at the lowest value of δ shown in Figure 9d. Table 4 summarizes these results. It is worthwhile to carefully assess the market for refurbished products before deciding whether to offer them. But when high demand for new products and stiff backorder penalties combine with low refurbishment cost and less perceived quality, refurbishment is less of an all-or-nothing proposition.
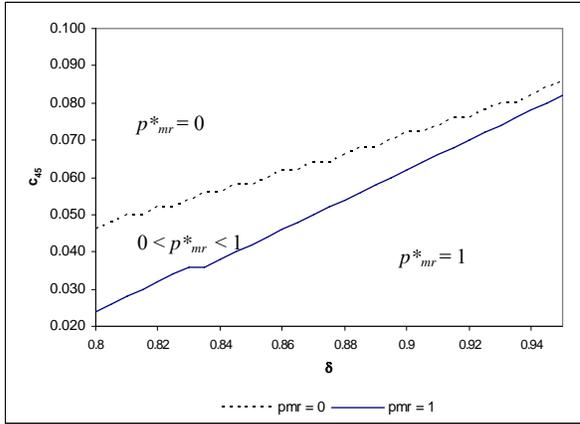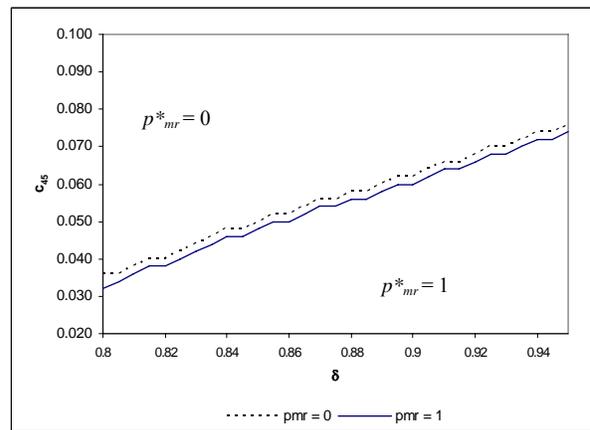
(a) $P_{new} = 0.45$ and $h_1 = 0.00005$
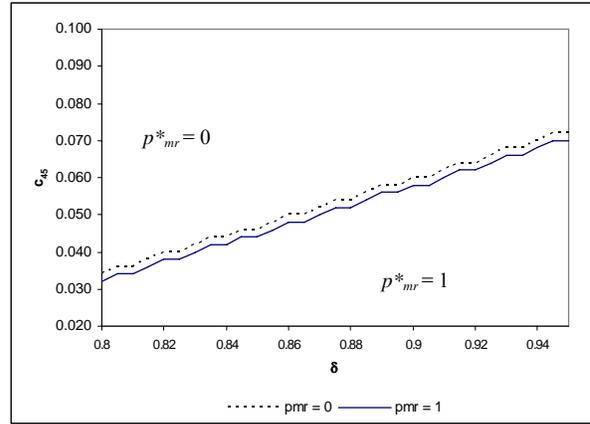
(b) $P_{new} = 0.65$ and $h_1 = 0.00005$
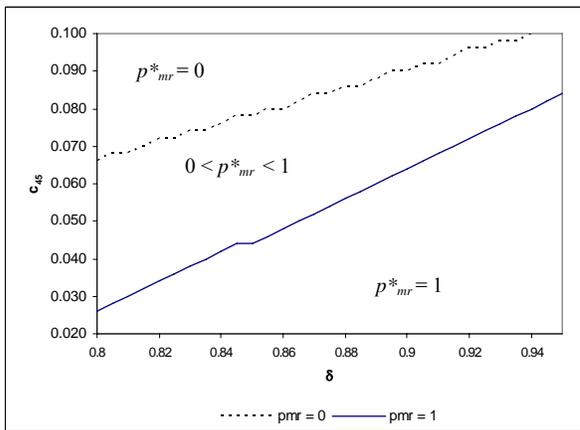
(c) $P_{new} = 0.45$ and $h_1 = 0.00020$

(d) $P_{new} = 0.65$ and $h_1 = 0.00020$

FIGURE 9. The position of p_{mr}^* with respect to different P_{new}, h_1, δ and c_{45}

5. Conclusions and Future Research

This model confirms the results of other studies in suggesting that, for a manufacturer in a competitive market, introducing refurbished products to the market can be profitable even when they potentially reduce the demand for new products. The optimization model sets the appropriate price of refurbished products, relative to the new product price, and the proportion of product returns to be refurbished. It also shows how opening up a market for refurbished products may be necessary to relieve a capacity constraint for manufacturing new products. The

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numerical study reveals that significant proportions of returns should be refurbished when demand for new products and high backorder penalties combine with low refurbishment costs and high perceived quality of the refurbished products.

Although the model was motivated by electronics manufacturers who refurbish the products internally in their own facilities, the model could also be applied in the situation where product returns are refurbished by an external subcontractor as long as the objective remains to maximize the total profit from selling new, refurbished and dismantled products.

The observed discontinuous structure for the optimal policy suggests interesting avenues for further research. The optimality conditions and numerical results suggest that when manufacturing capacity is sufficient to meet demand, then it is optimal to either not refurbish any returns or to refurbish a significant proportion of them. Under certain conditions, very small changes in conditions will cause the decision to swing from one extreme to the other. Moreover, this characteristic is observed in the absence of either fixed costs for setting up the refurbishment operations or other economies of scale. Further research is needed to understand the interactions among the different components of revenue and cost in this highly interdependent closed loop system. Additional sensitivity analysis could be performed to assess the effect of a processing or restocking fee for returns.

Other future work will relax some of the simplifying assumptions in this paper. First, although the exponential distribution has been widely used to represent interarrival times, it may not be a good representation of the service times at stations 1, 3 and 4 due to its high variability (Bitran and Tirupati, 1988). The general open queueing network presented in Bitran and Morabito (1994) can be employed to represent real situations more accurately. However, without the characteristics of the Jackson network, the stations cannot be analyzed

independently. Instead, a more complicated system of nonlinear equations can be used to approximate the system's performance. This modification will complicate both the objective function and the constraint set. Nevertheless, we expect the same characteristics of the optimal solution to hold; namely, that the optimal solution will occur in the interior of the feasible region with respect to price, and may lie on either boundary or in the interior with respect to proportion of returns to refurbish.

Variabilities in the quality of product returns and consumer perceptions of quality of refurbished products are also worth exploring. The refurbishment cost and processing time could both depend on the quality of product returns. Some returns might have as good as new quality and thus could be resold directly. On the other hand, some returns might be damaged so much that it is not worthwhile to refurbish them. With regard to quality, some consumers might be nearly indifferent between new and refurbished products while only new items would satisfy others. Appropriate modifications to the demand function could incorporate these differences.

APPENDIX

1. *Proof that the feasible region is convex*

Constraints (1) to (9) can be expressed as function of $g_k(P_{ref}, p_{mr}) - b_k \leq 0$, $k = 1,...,9$ as follows

$$-P_{ref} + P_{new} - (1-\delta) \leq 0 \tag{1a}$$

$$P_{ref} - \delta P_{new} \leq 0 \tag{2a}$$

$$P_{ref} - \left(P_{new} - (1-\delta)\left(1 - \left(1 - \varepsilon_1\right)\mu_1\right)\right) \leq 0 \tag{3a}$$

$$P_{ref} - \frac{(1-\delta)(1-\varepsilon_3)\mu_3 + p_{cr}(P_{new} - (1-\delta)(1+(1-\varepsilon_3)\mu_3 p_{mr}))}{p_{cr}} \le 0 \quad (4a)$$

$$P_{ref} - \frac{(1-\delta)(1-\varepsilon_4)\mu_4 + p_{cr}(P_{new} - (1-\delta)(1+(1-\varepsilon_4)\mu_4))p_{mr}}{p_{cr}p_{mr}} \le 0 \quad (5a)$$

$$P_{ref} - \frac{\delta(1-\varepsilon_5)P_{new} - p_{cr}\delta(1-\delta-\varepsilon_5 P_{new})p_{mr}}{1-\varepsilon_5(1-p_{cr}p_{mr}) - p_{cr}(1-\delta)p_{mr}} \le 0 \quad (6a)$$

$$\frac{\delta(\gamma P_{new} - p_{cr}(1-P_{new}(1-\gamma)-\delta)p_{mr})}{\gamma(1-p_{cr}p_{mr}) + p_{cr}p_{mr}\delta} - P_{ref} \le 0 \quad (7a)$$

$$p_{mr} \le 1 \quad (8a)$$

$$-p_{mr} \le 0 \quad (9a)$$

where ε_k, $k = 1, 3, 4, 5$ are some small positive constants.

Since

$$\frac{\partial}{\partial p_{mr}} \frac{\delta(1-\varepsilon_5)P_{new} - p_{cr}\delta(1-\delta-\varepsilon_5 P_{new})p_{mr}}{1-\varepsilon_5(1-p_{cr}p_{mr}) - p_{cr}(1-\delta)p_{mr}} = -\frac{p_{cr}(1-\varepsilon_5)(1-P_{new})(1-\delta)\delta}{(1-\varepsilon_5(1-p_{cr}p_{mr}) - p_{cr}p_{mr}(1-\delta))^2} < 0 \text{ and}$$

$$\frac{\partial}{\partial p_{mr}} \frac{\delta(\gamma P_{new} - p_{cr}(1-P_{new}(1-\gamma)-\delta)p_{mr})}{\gamma(1-p_{cr}p_{mr}) + p_{cr}p_{mr}\delta} = -\frac{p_{cr}\gamma\delta(1-P_{new})(1-\delta)}{((1-p_{cr}p_{mr})\gamma + p_{cr}p_{mr}\delta)^2} < 0, \ P_{ref} \text{ in the}$$

equality forms of (6) and (7) are both decreasing functions of p_{mr} starting from δP_{new} at $p_{mr} = 0$.

Therefore, constraint (1) is unnecessary since (6) is equally or more restrictive than (1). Also,

(7) is also more restrictive than (2), since at $p_{mr} = 1$ where P_{ref} is minimized along (7) we find

$$\lim_{\gamma \to 0^+} \frac{\delta(\gamma P_{new} - p_{cr}(1-P_{new}(1-\gamma)-\delta))}{\gamma(1-p_{cr}) + \delta p_{cr}} = P_{new} - (1-\delta).$$

Referring to section 3.3, feasible solutions are not limited by (4a) and (5a). Therefore,

only (3a), (6a), (7a), (8a) and (9a) can be boundaries of the feasible region. From Rardin (1998),

the feasible set defined by constraints $g_k(P_{ref}, p_{mr}) - b_k \le 0$, $k = 1,...,m$, is convex if each g_k is a

convex function. The functions g_k for $k = 2, 3, 8$ and 9 are linear and therefore convex. The

nonlinear function g_6 is a linear function of P_{ref} less a concave function of p_{mr} and therefore

convex. Similarly, since $\dfrac{\partial^2}{\partial p_{mr}^2} \dfrac{\delta\left(\gamma P_{new} - p_{cr}\left(1 - P_{new}(1-\gamma) - \delta\right)p_{mr}\right)}{\gamma\left(1 - p_{cr}p_{mr}\right) + p_{cr}p_{mr}\delta} =$

$-\dfrac{2p_{cr}^2\left(1 - P_{new}\right)\gamma(\gamma - \delta)(1-\delta)\delta}{\left(\gamma - p_{cr}p_{mr}(\gamma - \delta)\right)} \leq 0$ when $\gamma \leq \delta$, g_7 is a convex function of p_{mr} less a linear

function of P_{ref} and therefore also convex..

2. Derivation of KKT conditions

Since constraints (6) and (7) are nonlinear, the problem has a nonlinear objective function

and nonlinear inequality constraints. To help find its solution, we apply the Karush-Kuhn-

Tucker (KKT) optimality conditions (Bazaraa et al., 1993). Introducing the Lagrange multipliers

$u_k, k = 3,6,7,8,9$, the conditions for a maximization problem are:

1. The gradient of the Lagrangian function equals zero.

$$-\frac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial P_{ref}} + u_3 + u_6 - u_7 = 0$$

$$-\frac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial p_{mr}} + u_6 \frac{p_{cr}(1-\varepsilon_5)(1-P_{new})(1-\delta)\delta}{\left(1 - \varepsilon_5\left(1 - p_{cr}p_{mr}^*\right) - p_{cr}p_{mr}^*(1-\delta)\right)^2} - u_7 \frac{p_{cr}\gamma\delta(1-P_{new})(1-\delta)}{\left(\left(1 - p_{cr}p_{mr}^*\right)\gamma + p_{cr}p_{mr}^*\delta\right)^2} + u_8 - u_9 = 0$$

2. The constraints and multipliers satisfy complementary slackness conditions.

$$u_3\left(P_{ref}^* - P_{new} + (1-\delta)\left(1 - (1-\varepsilon_1)\mu_1\right)\right) = 0$$

$$u_6\left(P_{ref} - \frac{\delta(1-\varepsilon_5)P_{new} - p_{cr}\delta(1-\delta-\varepsilon_5 P_{new})p_{mr}}{1 - \varepsilon_5\left(1 - p_{cr}p_{mr}\right) - p_{cr}(1-\delta)p_{mr}}\right) = 0$$

$$u_7\left(\frac{\delta\left(\gamma P_{new} - p_{cr}\left(1 - P_{new}(1-\gamma) - \delta\right)p_{mr}\right)}{\gamma\left(1 - p_{cr}p_{mr}\right) + p_{cr}p_{mr}\delta} - P_{ref}\right) = 0$$

$$u_8\left(p_{mr}^* - 1\right) = 0$$

$$u_9\left(-p^*_{mr}\right) \quad = 0$$

3. The Lagrange multipliers are nonnegative.

$$u_k \geq 0, \, k = 3, 6, 7, 8, 9$$

These conditions are necessary for optimality if a set of constraint qualifications is satisfied. Winston (2004) provides a simple set of constraint qualifications: Let $\left(P^*_{ref}, p^*_{mr}\right)$ be an optimal solution. If all the constraints are continuous, and the gradients of all binding constraints at $\left(P^*_{ref}, p^*_{mr}\right)$ form a set of linearly independent vectors, then the KKT conditions must hold at $\left(P^*_{ref}, p^*_{mr}\right)$. These qualifications are clearly satisfied by our constraints because the only pair of gradients that are not linearly independent are those for (8) and (9); however, these two constraints cannot be binding simultaneously.

If $P_{new} \leq 1 - \mu_1$ then constraint (3a) places a more restrictive upper bound on P_{ref} than does (6a) in the interval $\dfrac{\gamma(P_{new} - (1 - \mu_1))}{p_{cr}(\gamma(P_{new} - (1 - \mu_1)) - \delta\mu_1)} \leq p_{mr} \leq \dfrac{P_{new} - (1 - \mu_1)}{p_{cr}((P_{new} - (1 - \mu_1)) - \delta\mu_1)}$. In this case, for the feasible region to be nonempty, the curves corresponding to (3a) and (7a) must cross within the bounds of the other constraints. This occurs when $\gamma < \dfrac{p_{cr}\delta\mu_1}{(1 - p_{cr})(1 - P_{new} - \mu_1)}$.

Therefore when the demand for new products exceeds the manufacturing capacity, the upper bound for γ is $\min\left(\delta, \dfrac{p_{cr}\delta\mu_1}{(1 - p_{cr})(1 - P_{new} - \mu_1)}\right)$. In the numerical examples, constraint (7) will be active only where its curve crosses the vertical axis or (3), as described in case 1 below.

Constraint (3a) and (6a) are equivalent to $\rho_i \leq 1 - \varepsilon_i$, $i = 1$ and 5. When constraints (3) and (6) are active, $\rho_i = 1 - \varepsilon_i$, $i = 1$ and 5, and the backorder cost at station 1 and holding cost at

station 5 are $h_i E[N_i] = h_i \left(\dfrac{\rho_i}{1-\rho_i} \right) = h_i \left(\dfrac{1-\varepsilon_i}{\varepsilon_i} \right)$, $i = 1$ and 5, respectively. Since ε_i is arbitrarily

small, the cost at station 1 and 5 will be too high for the maximum profit to be achieved.

Therefore (3) and (6) will not be active at optimal solution, i.e., $u_3 = u_6 = 0$.

The KKT conditions are reduced to the following set:

$$-\frac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial P_{ref}} - u_7 \quad = 0 \qquad (10a)$$

$$-\frac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial p_{mr}} - u_7 \frac{p_{cr}\gamma\delta(1-P_{new})(1-\delta)}{\left(\left(1-p_{cr}p_{mr}^*\right)\gamma + p_{cr}p_{mr}^*\delta\right)^2} + u_8 - u_9 \quad = 0 \qquad (11a)$$

$$u_7 \left(\frac{\delta\left(\gamma P_{new} - p_{cr}\left(1-P_{new}(1-\gamma)-\delta\right)p_{mr}\right)}{\gamma\left(1-p_{cr}p_{mr}\right) + p_{cr}p_{mr}\delta} - P_{ref} \right) \quad = 0 \qquad (12a)$$

$$u_8\left(p_{mr}^* - 1\right) \quad = 0 \qquad (13a)$$

$$u_9\left(- p_{mr}^*\right) \quad = 0 \qquad (14a)$$

$$u_k \quad \geq 0, \ k = 7, 8, 9$$

In terms of values of p_{mr}, three possible cases of local optima are:

1) The solution lies on the minimum p_{mr}: When $P_{new} > 1 - \mu_1$, the minimum is $p_{mr} = 0$

where (7) crosses the vertical axis; if $P_{new} \leq 1 - \mu_1$, the minimum $p_{mr} =$

$\dfrac{\gamma\left(1-P_{new}-\mu_1(1-\varepsilon_1)\right)}{p_{cr}\left(\delta\mu_1(1-\varepsilon_1) + \gamma\left(1-P_{new}-\mu_1(1-\varepsilon_1)\right)\right)} > 0$, where (3) and (7) cross. At this point,

$u_8 = u_9 = 0$, and $u_7 \geq 0$, conditions (10a) and (11a) imply that $\nabla f\left(P_{ref}, p_{mr}\right) \leq 0$.

2) The solution lies between the p_{mr} from case 1 and $p_{mr} = 1$. Assuming a minimal value

of γ, no constraints are active, and the KKT conditions imply that $\nabla f\left(P_{ref}, p_{mr}\right) = 0$, i.e.,

a stationary point.

3) The solution lies on $p_{mr} = 1$. At this point $\dfrac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial P_{ref}} \le 0$ while $\dfrac{\partial f\left(P_{ref}^*, p_{mr}^*\right)}{\partial P_{ref}} \ge 0$ for

small γ.

3. *Proof that the total profit is concave in P_{new}*

$$\frac{\partial^2}{\partial P_{new}^2}(\text{Total Profit}) \quad = \frac{\partial^2}{\partial P_{new}^2}(\text{Revenue} - \text{Total Cost})$$

$$= -\frac{2(1 - p_{cr})}{1 - \delta} - \frac{h_5 \partial^2 E[N_5]}{\partial P_{new}^2}, \text{ where}$$

$$\frac{\partial^2 E[N_5]}{\partial \rho_5^2} = \frac{2}{\left(1 - \rho_5\right)^2} + \frac{2\rho_5}{\left(1 - \rho_5\right)^3} \ge 0$$

$$\frac{\partial^2 \rho_5}{\partial P_{new}^2} = \frac{2 p_{cr} p_{mr}\left(1 - \frac{P_{new} - P_{ref}}{1 - \delta}\right)}{\left(1 - p_{cr} p_{mr}\right)\left(\frac{P_{new} - P_{ref}}{1 - \delta} - \frac{P_{ref}}{\delta}\right)^3 (1 - \delta)^2} + \frac{2 p_{cr} p_{mr}}{\left(1 - p_{cr} p_{mr}\right)\left(\frac{P_{new} - P_{ref}}{1 - \delta} - \frac{P_{ref}}{\delta}\right)^2 (1 - \delta)^2} \ge 0$$

Recall the following properties of convex functions (Floudas, 2000):

i) if $f(x)$ is convex, $-f(x)$ is concave,

ii) if $f_1(x), \ldots, f_n(x)$ be concave functions on a convex subset S of R^n, then $\displaystyle\sum_{i=1}^{n} f_i(x)$ is

concave, and

iii) if $f(x)$ is convex on a convex subset S of R^n, and $g(x)$ is an increasing convex

function defined on the range of $f(x)$ in R. Then, the composite function of $g(f(x))$ is

convex on S.

Since $E[N_5]$ is an increasing convex function of ρ_5 and ρ_5 is a convex function of P_{new}, $E[N_5]$ is a convex function of P_{new}. We can conclude that the total profit is a concave function of P_{new}.

References

ARUNKUNDRAM, R. AND A. SUNDARARAJAN (1998), "An economic analysis of electronic secondary markets: installed base, technology, durability and firm profitability," *Decision Support Systems*, 24, 3-16.

BAYINDIR, Z. P., N. ERKIP, AND, R. GÜLLÜ (2003), "A model to evaluate inventory costs in a remanufacturing environment," *International Journal of Production Economics*, 81-82, 597-607.

BAZARAA, M. S., H. D. SHERALI AND C. M. SHETTY (1993), *Nonlinear Programming Theory and Algorithms*. John Wiley & Sons, Inc., NY

BUZACOTT, J. A. AND J.G. SHANTHIKUMAR (1993), *Stochastic Models of Manufacturing Systems*. Prentice Hall, Englewood Ciffs, NJ.

BITRAN, R. G., AND R. MORABITO (1996), "Open Queueing Networks: Optimization and performance evaluation models for discrete manufacturing systems," *Production and Operations Management*, 5(2), 163-193.

BITRAN, R. G., AND D. TIRUPATI (1988), "Multiproduct Queueing Networks with Deterministic Routing: Decomposition Approach and the Notation of Interference," *Management Science*, 34(1), 75-100.

DEBO, L.G., L.B. TOKTAY and L. N. VAN WASSENHOVE (2005), "Market Segmentation and Production Technology Selection for Remanufacturable Products," *Management Science*, 51(8), 1193-1205.

DOWLING, M. (1999), "Getting Less in Return," *Catalog Age*. 16 (4), pp. 1,18.

FASANO, J. P., R. HALE, M. JACQUES, AND P. VEERAKAMOL (2002), "Optimizing reverse logistics scenarios: A cost-benefit study using IBM's WIP tools," *Proceedings of the IEEE International Symposium on Electronics and the Environment*, 306-311.

FERGUSON, M. E. AND L. B. TOKTAY (2005), "The Effect of Competition on Recovery Strategies," *Working paper*, Georgia Institute of Technology.

FERRER, G. AND J. M. SWAMINATHAN (2005), "Managing New and Remanufactured Products," Forthcoming in *Management Science*.

FLOUDAS, C. A. 2000, *Deterministic Global Optimization*, Kluwer Academic Publishers, Boston.

GUIDE V. D. R., R. H. TEUNTER, AND L. N. VAN WASSENHOVE (2003), "Matching Demand and Supply to Maximize Profits from Remanufacturing," *Manufacturing & Service Operations Management*, 5(4), 303-316.

GROSS D., AND C. M. HARRIS (1983), *Fundamentals of Queueing Theory*, John Wiley & Sons, New York USA.

KETZENBERG, M. E., G. C. SOUZA, AND V. D. R. GUIDE, JR. (2003), "Mixed Assembly and Disassembly Operations for Remanufacturing," *Production and Operations Management*, 12(3), 321-335.

LINDO, INC. (2004), *LINGO 9.0*, from http://www.lindo.com

MAJUMDER P. AND H. GROENEVELT (2001), "Competition in Remanufacturing," *Production and Operations Management*, 10(2), 125-141.

NICHOLS D. A. AND C. W. REYNOLDS (1971), *Principles of Economics*. Holt, Rinehart and Winston Inc. NY.

RARDIN R. L., (1998), *Optimization in Operations Research*. Prentice Hall, Englewood Ciffs, NJ.

SILVA R. (2004), "Buying Refurbished Products - What You Need to Know," from http://hometheater.about.com/cs/beforeyoubuy/a/aarefurbinfoa.htm.

SOUZA G. C., V. D. R. GUIDE JR., L. N. VAN WASSENHOVE AND J. D. BLACKBURN (2005), "Time Value of Commercial Product Returns," *Working paper*, University of Maryland, College Park.

SOUZA, G. C., M. E. KETZENBERG AND V. D. R. GUIDE, JR. (2002), Capacitated Remanufacturing With Service Level Constraints, *Production and Operations Management*, 11(2), 232-248.

STOCK, J., T. SPEH AND H. SHEAR (2002), "Many happy (product) returns," *Harvard Business Review*, July 2002, 16-17

TOKTAY, L. B., L. M. WEIN AND S. A. ZENIOS, (2000), "Inventory Management of Remanufacturable Products," *Management Science* 46 (11), 1412 – 1426.

TOKTAY, L. B., (2003), "Forecasting Product Returns," in: *Business Aspects of Closed-Loop Supply Chains* (V.D.R. Guide, Jr., L.N. Van Wassenhove, eds). Carnegie Mellon University Press, Pittsburgh, PA USA.

WHITE, D., M. NAGHIBI (1998), "What's the Difference Between Refurbished and Used Equipment?," *Evaluation Engineering*, 37(2), 40-41.

WINSTON, W. (2004), *Operations Research: Applications and Algorithms*. Belmont, CA: Brooks/Cole – Thomson Learning.

WOLFRAM RESEARCH, INC. (2003), *Mathematica 5.0* from http://www.wolfram.com.