Comparing Decision Making Using Expected Utility, Robust Decision Making, and Information-Gap: Application to Capacity Expansion for Airplane Manufacturing

Krishna Sai Varun Kotta
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Comparing Decision Making Using Expected Utility, Robust Decision Making, and Information-Gap: Application to Capacity Expansion for Airplane Manufacturing

by

Krishna Sai Varun Kotta

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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Program of Study Committee:
Cameron A. MacKenzie, Major Professor
Gary Mirka

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this thesis. The Graduate College will ensure this thesis is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

Ames, Iowa

2018

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I dedicate this thesis to my parents Mr. Srinath Kotta and Mrs. Annapurna Kotta and my sister Ms. Alekya Kotta, without whose support, this work would not have been possible. You have always believed in me and for that I will forever be thankful.
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</tr>
<tr>
<td>GBM</td>
<td>Geometric Brownian Motion</td>
</tr>
<tr>
<td>EU</td>
<td>Expected Utility</td>
</tr>
<tr>
<td>RDM</td>
<td>Robust Decision Making</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Auto Regressive Integrated Moving Average</td>
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<tr>
<td>ARMA</td>
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<td>MLE</td>
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I would like to take this opportunity to express my gratitude towards my Major professor Dr. Cameron A. MacKenzie. He has supported me all through my research, his insights and guidance provided me with a direction towards finishing my research.

I would also like to thank my committee member and Director of Graduate Education of Industrial Engineering Dr. Gary Mirka for showing support and encouragement towards my research.

I would also like to thank Ms. Deborah McDonough, for always answering my queries patiently and making my graduate life much easier at Iowa State University.

I additionally want to thank my family and friends who have shown incredible belief in me and supported me unconditionally towards finishing my research.
Airplane manufacturing industry is a low-volume high-value industry; however, there is a very high uncertainty associated with it. The industry has long lead times and capacity expansion for such an industry requires huge capital investments. Therefore, capacity planning requires accurate demand forecasting based on the historical data. Various demand forecasting models based on the forecasted demand can serve as an influential tool for the decision making. Based on the profit requirements, cost saving, and the risk attitude of a decision maker, he or she may choose a different strategy. This primary purpose of this research is to model the uncertainty and analyze different decision-making approaches for long-term capacity planning for painting the Boeing 737 airplanes.

The first part of the research focusses on identifying the underlying demand trends for the Boeing 737 and Boeing 777 airplane models based on the historical data. Probabilistic models were evaluated for the demand based on model assumptions and statistical analysis. The stochastic processes Brownian motion and a modified geometric Brownian motion were used to predict the demand for the Boeing 737 and Boeing 777 respectively for the next 20 years.

The second part of the research focusses on decision making based on the forecasted demand for the Boeing 737 airplanes. The decision is when to construct new hangars to paint new airplanes. Three decision-making approaches were applied to this decision: expected utility, robust decision making, and information gap. Since significant uncertainty exists with the number of airplanes, it is important to compare the decision-making methodologies for different risk tolerances, probabilities, and required profits. The circumstances and assumptions favoring each of the decision-making philosophy under deep uncertainty was discussed and, based on the simulation results, the optimal strategies for the capacity expansion were summarized.
CHAPTER 1. GENERAL INTRODUCTION

Airplanes have a huge variability in demand each year. This variability is an effect of various factors including seasonal demand, fuel prices, and increasing air travelers. However, the demand for commercial aircraft has largely increased during the past 30 years. Figure 1.1 represents the historical data for the orders of Boeing’s commercial airplanes (Boeing, 2018) for the years 2005-2017. Figure 1.2 represents the historical data for the orders of Airbus’s commercial airplanes (Airbus, 2018) for the years 2005-2017. Both the figures show a remarkably similar trend and a clear upward trend is noticeable from the year 2010.

![Boeing Commercial Airplane Orders](image)

Figure 1.1: *Historical orders for the Boeing Commercial airplanes*

Given the variability in the market for commercial aircraft, companies like Boeing will have to make strategic decisions about its future capacity in order to maximize its profits.
Expanding capacity may require significant capital investment. However, due to the uncertainty in the demand, long-term capacity planning involves considerable uncertainty. Due to the risks involved, capacity planning should be decided using strategic tools and not by gut feeling.

Figure 1.2: *Historical orders for the Airbus Commercial airplanes*

Long-term capacity planning deals with various strategic issues in major production facilities, this causes high uncertainty. Tackling such a complex problem is not possible with a short-term or medium-term capacity planning techniques. The capacity planning is an issue in many major industries and not airplanes alone. Previously, there have been a lot of papers published regarding the capacity planning. Capacity planning under uncertainty using an Asian approach was discussed to produce beneficial results with expanding the capacity under average demand uncertainty and being beneficial with reducing capacity under high stochastic uncertainty (Driouchi et al., 2006). A cautious approach to capacity planning has been
described to produce robustness to likely errors. The capacity expansion problem is cast into a deterministic framework to avoid complexities in the non-linear stochastic formulations. Based on the large data, the results from the nonlinear programming capacity planning model shows that with an increase in the caution against demand uncertainty, the variance of the total profit decreases (Paraskevopoulos et al., 1991). Courtney et al. (1997) argue that traditional approaches could be downright dangerous for strategic planning under uncertainty. A traditional approach could be viewed in a binary way by the executives and assume that the demand prediction is precise. Various research articles have been published previously regarding the capacity planning in industries other than airplanes. A Simulation model for capacity planning under uncertainty in the food industry has been discussed by Higgins et al. (2005). The simulation is also applicable in biomedicine field to support decision making (Groothuis et al., 2001). A comprehensive capacity planning model integrating statistics, financial models and simulation to support decision making were proposed by Nazzal et al. (2006). Eppen et al. (1989) developed a mixed linear programming model to solve the capacity planning issue of General Motors.

Even though there has been a significant amount of research done in the past regarding the capacity planning, most of it has been focused on the deterministic problems, short or medium-term planning. Research gaps have been found in the long-term capacity planning and with the introduction of uncertainty, the capacity planning poses a challenge. Graves (2008) discusses current practices and possible improvements to the practices that could present with tactical decisions to properly handle the uncertainty in production planning. Dixit et al., (1994) speaks about some best ways and mathematical tools to make investment decisions under uncertainty. A review by Mula et al. (2006) discusses the forthcomings of production planning
where uncertainty is accounted for and compares it to the production planning models without uncertainty. In the review, 87 citations related to production planning for uncertainty have been analyzed.

Multi-stage stochastic programming which is a prevalent method for capacity planning under uncertainty has been discussed by Chen et al. (2002), Ahmed et al. (2003) and Geng et al. (2009). There are certain risks involved in capacity planning such as financial risk, downside risk and worst-case revenue (Bonfill et al., 2004) to be considered in the two-stage stochastic programming model. Meighem (2003) put forth the idea of incorporating the game theory, utility theory, financial hedging and operational hedging to provide a financial model for capacity planning problem. Understanding these risks is very essential to make informed decisions for long-term capacity planning and this gap needs to be addressed for tackling the planning issue in the aviation industry.

Zhang (2017) initiated a collaborative research to overcome this challenge by developing a practical model for making the decision regarding the expansion of the painting capacity for Boeing. The model is designed to provide decision making for capacity expansion information 20 years into the future. The research demonstrates several approaches to long-term production planning. Boeing 737 and 777 models have been evaluated using a modified geometric motion method for forecasting demand and were compared to the autoregressive integrated moving average method. A second study in this research demonstrates a working decision-making model using 3 decision-making strategies (Expected utility, Robust Decision making and Information Gap) and the results from each were compared.

The current research focusses on the reviewing the previously discussed framework developed by Zhang (2017), and discussed the expected utility, robust decision making and
information gap decision-making strategies. The demand forecasting was validated with the updated orders data from Boeing (2018). The demand was observed to follow a Brownian motion trend for the 737 model and 777 model follows the modified geometric Brownian motion trend discussed by Zhang (2017). The forecast for demand from the Brownian motion model for the Boeing 737 model has been applied in the decision-making model. The variation among optimal alternatives from the expected utility, robust decision making, and information gap model have been discussed. The influence of the assumptions in the model and sensitivity of the model is also investigated in the current research.
CHAPTER 2. PROBABILISTIC METHODS FOR LONG-TERM DEMAND FORECASTING IN THE AVIATION INDUSTRY

2.1 Introduction

The increase in global population and globalization has led to an increase in air travel. However, the demand for the airplane manufacturing is highly uncertain over long-term. Airplane manufacturing industry is a low-volume high value manufacturing market. Hence any decision regarding the investment in such an industry is difficult due to the high capital requirement and varying demand. Airplane manufacturers need to capitalize on the opportunities in this uncertain market by making proper investment decisions. Therefore, Boeing is analyzing its capacity to manufacture airplanes to meet the demand requirements for the future. Since expansion requires large capital investments, the demand for the aircraft orders must be predicted for a long term.

While there are several approaches to forecasting, there is no perfect method and no precise forecast technique. DeCroix (2015) highlights that in an uncertain environment, the production can be optimized by either making improvements to the forecasting models or allowing for flexibility in production such as having flexible production lines or shared components, which would reduce the lead times and prove beneficial. Huh (2005) addresses the issue of highly volatile demand by developing a cluster-based algorithm that reduces variance along with the principles of maximum flow algorithm. Huang (2008) discusses Monte-Carlo simulation model to approximate stochastic process and handle path dependent relationship between successive demands. Several studies (Johnson et al., 1974; Caldeira et al., 1983; Geng et al., 2009) represent demand as a probability distribution to find optimal resource
allocation for production planning under uncertainty. However, with the rapidly changing market, simple demand prediction models may not fare well for demand prediction.

Forecasting models of demand include the autoregressive integrated moving average (ARIMA) method and probabilistic models such as Brownian motion and Geometric Browning Motion (GBM) (Zhang, 2017). Probabilistic forecasting models incorporate uncertainty in a better fashion than deterministic time series models (Gneiting et al., 2014). Zhang (2017) develops a probabilistic model with a modified Brownian motion and modified GBM to predict future demand for the Boeing 737 and 777 models respectively. The results were compared to an ARIMA model. This paper updates the Brownian motion model for the 737 and the modified GBM for the 777.

2.2 Demand Forecast Models

2.2.1 Brownian Motion (BM)

Brownian motion is a prevalent technique for probabilistic demand forecasting. Britannica defines the Brownian motion as any phenomenon in which some quantity is constantly undergoing small, random fluctuations. The amount of the randomness increases with increasing time. Brownian motion is also called as Weiner process in general. Durett (1998), describes the characteristics of the Brownian motion as follows:

1. Brownian motion has independent increments. The increments in a one-time interval are independent of increments in any other time.

2. Brownian motion has Gaussian increments. The increments follow a normal distribution with a mean ‘0’ and variance of ‘u’.

3. Brownian motion is a continuous process.
Brownian motion with drift assumes that the annual demand follows a normal distribution with mean $\mu t + b$ and variance $t \sigma^2$, where $\mu$ is the mean shift in demand, $t$ is the number of years from the current year, $b$ is the current demand, $\sigma^2$ is the variance of demand at time $t=1$. If the demand for the Boeing Airplanes follows Brownian motion, the variance in demand increases in each year. The general equation for demand at time $t$ according to Brownian motion is:

$$X(t) = \sigma B(t) + \mu t + b$$

(2.1)

where $B(t) \sim N(0, t)$ is a standard Brownian motion, meaning that it is distributed normally with a mean 0 and variance $t$. If demand follows the Brownian motion, the expected demand and variance of demand in each year are:

$$E[X(t)] = \sigma E[B(t)] + \mu t + b = \mu t + b$$

(2.2)

$$Var[X(t)] = \sigma^2 Var[B(t)] = \sigma^2 t$$

(2.3)

Therefore, we can see that $X(t) \sim N(\mu t + b, \sigma^2 t)$. A Quantile-Quantile (Q-Q) plot can be used to verify the normality assumptions for the Brownian motion. Q-Q plot displays the residuals for the observed data minus the mean versus the quantiles of the normal distributions. The data points should approximate the straight line on the Q-Q plot. Maximum likelihood estimation (MLE) method can be used to estimate the parameters for the model such as mean (i.e., drift) and variance for Brownian motion based on the historical data.

### 2.2.2 Geometric Brownian motion (GBM)

GBM is a continuous time stochastic process. While a Brownian motion process can take a positive or negative value, a random variable that follows a GBM process is always positive. GBM is widely preferred for predicting stock prices (Dunbar, 2016). The annual
demand in a GBM is $Y(t) = \exp (X(t))$ where the logarithm of ratio $\frac{Y(t+1)}{Y(t)}$ follows a normal distribution $N(\mu+b, \sigma^2)$ (Marathe et al., 2005). The Q-Q plot can be used to check the normality assumptions for GBM, if the ratio $\frac{Y(t+1)}{Y(t)}$ follows a lognormal distribution, the data points should approximate a straight line.

The autocorrelation function (ACF) calculates the correlation between demands of different years. The difference in years is the lag. The GBM is valid only if the correlation between the ratios $\frac{Y(t+1)}{Y(t)}$ and $\frac{Y(t+1+k)}{Y(t+k)}$ is not significant, where $k > 0$ represents the lag. The MLE method can be used to estimate the parameters for the model such as mean (i.e., drift) and variance for GBM based on the historical data.

### 2.2.3 Modified Geometric Brownian Motion

The variance in the GBM process can be very large. In the Brownian motion model, the variance increases by a factor of $\sqrt{1 + \frac{1}{t}}$ each year, while in the GBM process, the variance increases by a factor of $e^{\mu \sigma}$ approximately. Such a large variance can introduce unrealistic values for demand in the far-distant future. This is a challenge in using the GBM process for predicting the future demand of the Boeing Airplanes. Therefore, a new approach to GBM was developed by Zhang (2017) called the modified GBM. The modified GBM is based on the lag variable $k$. For the modified GBM, it is assumed that for each year $t$, the ratio $R(t) = \frac{Y(t)}{Y(0)}$ follows a lognormal distribution with mean $\mu t$ and variance $\sigma^2 t$. This assumption can reduce variance, which should result in more realistic demand values for the future. Determining the year to set for $t = 0$ for the modified GBM is critical. If this year is set far back in the past, the variance
will likely be too large, and if the current year is chosen, the variability for the next few years is too low and would exhibit too much certainty about demand.

2.3 Application: Demand Forecast for Boeing Airplanes

The demand for Boeing’s commercial airplane models 737 and 777 were predicted using Brownian motion and GBM. The Boeing 737 is a short-to-medium range twin jet narrow-body airliner. It has been manufactured since 1965, and it is one of the popular commercial airplanes from Boeing. The Boeing 777 is part of a family of long-range twin jet engine wide-body airplanes. The Boeing 777 has been manufactured since 1990, and demand for the airplane has steadily increased. The historical demand data (Boeing, 2018) for these airplane models were collected. The data was analyzed to determine the appropriate demand forecasting models for each of these commercial airplanes and the demand was predicted for the period of 20 years (2018-2037) into the future.

2.3.1 Boeing 737 Demand Prediction

The data obtained from the Boeing’s website for the historical orders of the Boeing 737 for the years 1965 – 2017 has been plotted. Figure 2.1 shows the demand for each year. The demand for the years during the initial few years of the Boeing 737 is very low and inconsistent as it was a new model in the line of Boeing Commercial Airplanes, and an increasing trend has been observed starting from the year 1984 onwards. Therefore, the current demand model will be considering 1984 as the starting point for demand. The increasing variation in demand seems to suggest a Brownian motion model may be appropriate. The difference in the demand between adjacent years is represented in Figure 2.2.

Figure 2.2 shows that the difference in demand seems to exhibit increasing variance, which suggests that Brownian motion may not be appropriate for this process.
Brownian motion assumes the increments each year are independent and follow a normal distribution. This assumption could be verified with a Q-Q plot for the change in the number of orders of the Boeing 737 model each year since 1984. Figure 2.3 represents the Q-
Q plot for the quantiles of the standard normal distribution vs the quantiles for the change in the demand. The differences show some normality but are also more narrowly tailed than a normal distribution.

![QQ Plot of Difference in Annual Orders vs Standard Normal](image)

**Figure 2.3:** *Q-Q plot for the difference in the annual orders of Boeing 737*

The baseline demand, drift ($\mu$) and sigma ($\sigma$) are obtained from the historical demand for the Boeing 737. Baseline demand is the last observed value of demand ($b = 843$). The drift is the mean value of the difference in the annual orders for adjacent years ($\mu = 21.57$). Sigma is the sample standard deviation of the difference in the annual orders for adjacent years ($\sigma = 200.83$). The parameters are represented in Table 2.1.

<table>
<thead>
<tr>
<th>Drift ($\mu$)</th>
<th>Sigma($\sigma$)</th>
<th>Baseline($b$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.57</td>
<td>200.83</td>
<td>843</td>
</tr>
</tbody>
</table>
Brownian motion could lead to a negative demand, which is impossible for demand. To avoid this, a truncated normal distribution was used by eliminating any negative values simulated from the normal distribution. The demand was predicted for a period of 20 years into the future. 100,000 replications were run, and 90% probabilistic intervals were established based on the 95th percentile and the 5th percentile values from the replications. The median value provides the predicted demand for the Boeing 737 airplanes. Figure 2.4 represents the predicted demand for the Boeing 737 commercial airplane with the 95th quantile (purple diamonds) and 5th quantile (yellow diamonds) and the median (red stars). According to the prediction, the median demand for the single-aisle 737 airplanes is expected to be 21,621 units for the next 20 years. The median forecast is 865 airplanes in 2018 and rises to about 1,305 airplanes in the year 2037. There is a 10% probability that demand could be less than 535 airplanes or greater than 1,193 airplanes in 2018. In 2037, there is a 10% probability that demand could be less than 241 or greater than 2,761 airplanes.

The variability in demand increases with respect to time, which indicates that more uncertainty exists about demand as we attempt to forecast further out into the future. In 2018, the range of the 90% probability interval is 658, and in 2037, the range increases to 2,520. Since a truncated normal distribution was used, the simulated 5th quantile of demand is between 241 and 535 for the 20 years. However, the 95th quantile for demand is assessed between 1,193 and 2,761, which is a large variability in the upper spectrum. This would represent a lot more 737s than Boeing has sold previously, but it seems possible.
2.3.2 Boeing 777 Demand Prediction

The data obtained from the Boeing’s website for the historical orders of the Boeing 777 for the years 1990-2017 is depicted in Figure 2.5. An important note is that the data for the number of orders for the Boeing 777 Airplane is missing in the observations for the year 1994 as the data was not found in Boeing (2018). The representation suggests an increasing trend in the orders of Boeing 777 model, but there is also an increasing variation in the number of orders.
Figure 2.5: Annual orders for Boeing 777

If demand follows a GBM process, then the ratio of orders for adjacent years should follow a lognormal distribution. A Q-Q plot of the logarithm of the ratio of demand for adjacent years $\frac{Y(t)}{Y(t+1)}$ is depicted in Figure 2.6. Since the orders for the year 1994 is missing and as the initial few years after introduction into the market serves as a transition period, the log ratio was calculated for the years from 1995-2017. The figure suggests the ratios exhibit heavier tails than a lognormal distribution, which may be due in part to only have 23 points for analysis.

The baseline for the GBM is calculated as the log of the ratio of the demand of the year 2017 to the demand of the previous year (2016). The MLE method was used on the data from 1995-2017 and it gives the values of the $\mu$ and $\sigma$ parameters for GBM. The results from the MLE method for the GBM are represented in Table 2.2 along with the inputs for the prediction
model using GBM. The parameters were used to run the prediction model for traditional GBM.  

*Figure 2.7* shows the prediction for the annual orders of the Boeing 777 airplanes.

![QQ Plot of Log Ratio of Successive Orders of 777 vs Standard Normal](image)

*Figure 2.6: Q-Q plot of the log of the ratio of successive orders of Boeing 777 airplane*

<table>
<thead>
<tr>
<th>Drift (µ)</th>
<th>Sigma(σ)</th>
<th>Baseline – log ratio (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0237</td>
<td>0.8912</td>
<td>0.416423</td>
</tr>
</tbody>
</table>
The demand was simulated for a period of 20 years into the future for the Boeing 777. 100,000 replications were run, and the median and 5th and 95th percentiles were estimated from the simulation. The simulation displays the 95th percentile (purple diamonds), the 5th percentile (yellow diamonds) and the median (red stars). *Figure 2.7* depicts the forecast, which is quite unrealistic. The expected number of orders for the Boeing 777 according to the GBM approach is 24,075 airplanes over the next 20 years starting from 88 orders in the year 2018 to 2,157 orders in the year 2037. That is about 2300% increase in the next 20 years. The 95th quantile varies from 384 orders in the year 2018 to 1.5 million orders in the year 2037. The GBM for the 777 produces results with extremely high uncertainty and some unrealistic values.
As discussed in the subchapter 2.2.3, the orders for the Boeing 777 could be predicted using modified GBM approach by verifying the modified GBM assumptions for the existing data.

The baseline for the modified GBM is calculated as the log of the ratio of the demand of the year 2017 to the demand of the initial year (1990). The MLE method was used on the data from 1990-2017 to find the values of the $\mu$ and $\sigma$ parameters for the modified GBM. The results from the maximum likelihood method for the modified GBM are represented in Table 2.3 along with the inputs for the modified GBM prediction model. The parameters were used to run the modified prediction model, Figure 2.8 shows the prediction for the annual orders of the Boeing 777 airplanes.

![Figure 2.8: Prediction of annual orders for Boeing 777 with 90% confidence over the next 20 years—modified GBM approach – (2017 as the baseline)](image-url)
Table 2.3: Parameters for the Modified GBM (Baseline as 2017) – Boeing 777

<table>
<thead>
<tr>
<th>Drift (µ)</th>
<th>Sigma(σ)</th>
<th>Baseline – log ratio (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0499</td>
<td>0.2028</td>
<td>0.3309</td>
</tr>
</tbody>
</table>

First, the demand for the Boeing 777 was forecasted using the modified GBM by setting 2017 as the year where \( t = 0 \) (Figure 2.8). The median number of orders of the Boeing 777 for the next 20 years is 1,372 airplanes. The median demand is 41 orders in 2018 and 106 orders in 2037. The 95th percentile is estimated at 57 orders in 2018 and increase to 471 orders in 2037. However, Boeing had 283 orders in 2014 and 194 orders in 2012. These values are way beyond the 95th percentiles for forecast before 2017. This suggests that the forecast is not as variable as the recent historical demand. Since the forecast is less variable than the historical demand, the forecast seems to exhibit too much certainty over what the future demand will be.

To allow for more variability in the forecast, the baseline for the demand could be shifted to a prior year. The year 2005 is selected at the point at which \( t = 0 \). Table 2.4 depicts the GBM parameters, which are equivalent to the previous model (Table 2.3) except for the baseline value.

Table 2.4: Parameters for the Modified GBM (Baseline as 2017) – Boeing 777

<table>
<thead>
<tr>
<th>Drift (µ)</th>
<th>Sigma(σ)</th>
<th>Baseline – log ratio (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0499</td>
<td>0.2028</td>
<td>0.7375</td>
</tr>
</tbody>
</table>

Figure 2.9 depicts the results of the simulation using 2005 as the baseline year. The median demand for the 777 airplanes is estimated at 3,725 units for the next 20 years. The median prediction is 111 airplanes for 2018 and increases to 287 airplanes in 2037. The 5th percentile is 34 airplanes in 2018 and is 44 airplanes in 2037. The 95th percentile is 373
airplanes in 2018 and increases to 1,885 airplanes in 2037. Figure 2.9 shows how 95th percentile matches pretty closely a large number of orders in 2012 and 2014. According to this modified GBM, there is a 50% probability that the demand for the Boeing 777 will exceed 111 in 2018 and a 5% probability the demand will exceed 373 airplanes. In 2037, there is a 50% probability that demand will exceed 287 orders and a 5% probability that demand will exceed 1,885 airplanes. 1,885 airplanes represent 6.5 times as many airplanes as Boeing has ever sold in a single year. It is an unlikely number, but it does not seem impossible. The modified GBM using 2005 as a baseline year seems to be a plausible forecast of demand for Boeing 777 airplanes.

![Demand Prediction using Modified GBM (Baseline 2005) - Boeing 777](image)

Figure 2.9: Prediction of annual orders for Boeing 777 with 90% confidence over the next 20 years—modified GBM approach – (2005 as the baseline)
2.4 Conclusions

This study analyzed various demand prediction models to fit the historical demand of the airplanes appropriately. The historical data for Boeing 737 model captures some of the characteristics of Brownian motion such as independent increments and increasing variance over time. However, the difference in demand between adjacent years (i.e., the increments) also exhibits increasing variance, which contradicts one of the Brownian motion assumptions. Nevertheless, modeling demand for the 737 as a Brownian motion seems to generate realistic results for the future. The drift ($\mu$) was found to be 21.57 and sigma ($\sigma$) was found to be 200.83 for the single-aisle Boeing 737 airplanes. These settings for the Brownian motion generate the demand for 20 years into the future. The predicted demand indicates that the median future demand begins with 865 airplanes in 2018 and rises to about 1,305 airplanes in the year 2037, and we could say with a 90% confidence that the demand for the year 2018 stays in between 535 and 1,193 airplanes and the demand for the year 2037 stays in between 241 and 2,761 airplanes.

A modified GBM was used to model demand for the Boeing 777 model. The year 2005 is chosen as the baseline year, and the forecast seems to produce realistic results. The actual demand for the years 2006-2017 lies within the 90% confidence limits which are in alignment with the assumption of the geometric Brownian motion trend. For the year 2006, the actual demand is 76 airplanes, and the 90% confidence interval is 44-86. For the year 2017, and the median predicted demand is 106 airplanes and the 90% confidence interval is 34-342 airplanes. The demand starts for the year 2018 is predicted to be in between 34-378 with a 90% confidence, and the demand for the year 2037 is predicted to be in between 44-1934 with a 90% confidence. Though the 95th percentile for future years is very large which may
overestimate the amount of demand for this airplane, the median demand seems to produce very realistic results.

The goal of this predicted demand from these models is to help Boeing make decisions about its capacity expansion. Since there is considerable uncertainty in the future demand, it is necessary to use decision-making methodologies that can incorporate that uncertainty. Three decision-making frameworks will be explored: expected utility, robust decision making, and information-gap.
CHAPTER 3. COMPARING DECISION MAKING MODELS FOR CAPACITY EXPANSION OF AIRPLANE MANUFACTURING UNDER UNCERTAINTY

3.1 Introduction

The demand for the Boeing 737 airplanes has an increasing trend, therefore it is very important to plan for the capacity expansion. Boeing currently has in-house capacity to paint the airplanes in 9 hangars. If the airplane demand increases over the in-house capacity, they out-source the painting of these additional airplanes. Outsourcing involves huge cost. To reduce the cost, an option is to build new hangars to increase the painting capacity. However, building new hangars needs a huge capital investment. This raises a decision-making problem for Boeing. Uncertainty around the estimation of the demand for the airplanes makes the decision making difficult.

This chapter extends the work of Minxiang Zhang, a master’s student who graduated in 2017. He wrote an explanation of the three decision-making models for uncertainty: expected utility (EU), robust decision making (RDM), and information gap (info-gap). His explanation of the decision-making models and the equations to calculate Boeing’s profit as a function of the number of painting hangars to be built are found in the appendix. This appendix is helpful in understanding the application section which begins this chapter. Equation numbers provided in the application section refer to equations in the appendix.

3.2 Application of Decision Making Models

3.2.1 Model Settings

Table 3.1 provides values for the parameters for the three models. The maximum number of hangars $h_{\text{max}}$ that the manufacturer can build over the next $T = 20$ years is 2, and the
The decision of how many hangars and in which year to construct each hangar is made in year 0. Equation A.11 gives the total number of strategies, \( N = 231 \). We use Monte Carlo simulation to generate the results for each decision-making model. Results for the EU and RDM model are based on 200,000 replications and results for the info-gap model are based on 100,000 replications. The info-gap model takes longer to run, so fewer replications are simulated. If no hangars are built, the average profit from equation A.9 with these model parameters is $8.9 million over 20 years.

Table 3.1: Model Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Model</td>
<td>1000</td>
<td>Price for 737</td>
</tr>
<tr>
<td>( b )</td>
<td>Model</td>
<td>800</td>
<td>Outsourcing Price</td>
</tr>
<tr>
<td>( d )</td>
<td>Model</td>
<td>104</td>
<td>Coefficient of Expected Production Function</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>Model</td>
<td>0</td>
<td>Coefficient of Expected Production Function</td>
</tr>
<tr>
<td>( d_r )</td>
<td>Model</td>
<td>0.05</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( e )</td>
<td>Model</td>
<td>30000</td>
<td>Cost of a new Hangar</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>Model</td>
<td>0</td>
<td>Coefficient of Fixed Cost Function</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>Model</td>
<td>9</td>
<td>Initial Number of Hangars</td>
</tr>
<tr>
<td>( h_{\text{max}} )</td>
<td>Model</td>
<td>2</td>
<td>Maximum New Hangars Allowed</td>
</tr>
<tr>
<td>( k )</td>
<td>Model</td>
<td>500</td>
<td>Coefficient of Variable Cost Function</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>Model</td>
<td>0</td>
<td>Coefficient of Variable Cost Function</td>
</tr>
<tr>
<td>( m )</td>
<td>EU</td>
<td>0.05</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( n )</td>
<td>Model</td>
<td>2</td>
<td>Maximum New Hangars Allowed</td>
</tr>
<tr>
<td>( p )</td>
<td>EU</td>
<td>0.6</td>
<td>Coefficient in Risk Tolerance Estimation</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Model</td>
<td>200.83</td>
<td>Standard Deviation of Demand</td>
</tr>
<tr>
<td>( T )</td>
<td>Model</td>
<td>20</td>
<td>Total Years</td>
</tr>
<tr>
<td>( \mu_{D0} )</td>
<td>Model</td>
<td>843</td>
<td>Coefficient of Expected Demand Function</td>
</tr>
<tr>
<td>( \mu_{D1} )</td>
<td>Model</td>
<td>21.57</td>
<td>Coefficient of Expected Demand Function</td>
</tr>
<tr>
<td>( w_1 )</td>
<td>Info-Gap</td>
<td>0.5</td>
<td>Weight Parameter for Trend</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>Info-Gap</td>
<td>0.2</td>
<td>Weight Parameter for Standard Deviation</td>
</tr>
</tbody>
</table>
3.2.2 Expected Utility

The first decision model selects the alternative that maximizes the expected utility, where it is assumed that the parameters for demand $\mu$ and $\sigma$ are known with certainty. The decision-making problem is constructed with a risk-averse exponential utility function. We assume that the decision maker is indifferent between earning the baseline profit of $8.9$ million (if no hangars are constructed) and a 0.6 probability of earning $9.9$ million and a 0.4 probability of earning $7.9$ million. From equation A.14, this point of indifference means the decision maker’s $r = 2.47$ million. After conducting 200,000 replications assuming no hangars are constructed, we arbitrarily assign a utility of 1 for the maximum profit and a utility of -1 for the minimum profit. Using these values for the utility function, we calculate the coefficients $a_1 = 1.0008$ and $b_1 = 0.0721$ based on equation A.12. These coefficients simply act as constants in the utility function and do not change what the optimal strategy is in the EU model.

After the calculating the expected utility based on the Monte Carlo simulation, we calculate the certainty equivalent according to equation A.15. The certainty equivalent for the strategy where no hangars are built in the next 20 years is $7.086$ million. Figure 3.1 depicts the certainty equivalent for each of the 231 strategies. The numbers plotted at the top of each curve represent the year in which a hangar is built where the number 21 means not to build a hangar. For example, 16, 21 means the first hangar is built in year 16 and the second hangar should never be built. The results from the simulation suggest that for the risk tolerance of $2.47$ million, the first hangar should be built between the years 16 and the second hangar should be built between the years 18 for the maximum certainty equivalence.
Figure 3.1: Certain equivalence for different strategies based on the expected utility model. (The numbers at the top of the curves represent that year in which the hangars should be built where the number 21 indicates a hangar should not be built.)

We analyze the sensitivity of this decision to the decision maker’s risk tolerance. The risk tolerance is varied from $1 million to $10 million, and the optimal strategy for different risk tolerances is depicted in Figure 3.2.
Figure 3.2 shows that as the decision maker becomes less risk averse (which corresponds to an increasing risk tolerance), he or she should build the hangars more quickly. If the decision maker’s risk tolerance is greater than or equal to $10 million or if he or she is risk neutral, the optimal strategy is to build the first hangar in Year 2 and second hangar in the Year 3. As the decision maker becomes more risk averse, he or she should be less willing to build a hangar in the early years because of the large initial cost of constructing the hangar and the uncertain demand for the airplanes. Due to the manner in which the cost of constructing new hangars is depreciated, building a hangar later has smaller upfront costs. If the risk tolerance is greater than $5 million, the first hangar should be built in years 2 or 3, and the
second hangar should be built in years 3-5. If the risk tolerance equals $4 million, the first hangar should be built in year 3 and the second hangar in year 9. The optimal strategies change drastically for risk tolerance values less than $4 million. If the decision maker is very risk-averse (risk tolerance equal to $1 million), he or she should never build either hangar because the certain cost of the hangar is too large relative to the uncertain demand for the airplanes.

### 3.2.3 Robust Decision Making

In RDM, the decision depends on the state set which contains uncertainty around parameters. The strategy set for EU and RDM are the same, but the state set is different. EU assumes that $\mu_{D1}$ and $\sigma_D$ are constant, but the RDM model allows $\mu_{D1}$ and $\sigma_D$ to take on a wide range of values. In this model we assume that $0 \leq \mu_{D1} \leq 43.14$. The minimum value assumes that there is no expected increase in demand from one year to the next, and the maximum value assumes the expected increase in demand in each year is twice as great as that of the EU model.

The standard deviation for demand in each year $\sigma_D$ ranges between 20.08 and 301.25. The minimum value assumes the standard deviation is one-tenth of $\sigma_D$ in the EU model, and the maximum value assumes the standard deviation is 1.5 times larger than $\sigma_D$ in the EU model. The state set for the RDM model is the combination of the $\mu_{D1}$ and $\sigma_D$ within the feasible region.

Theoretically, an infinite number of probability distributions could characterize the state space exist within the feasible region creating an infinitely large state set. Four probability distributions are assumed for $\mu_{D1}$ and $\sigma_D$: uniform, triangular, a right-skewed beta, and left-skewed-beta. Table 3.2 depicts the parameters of each of the four distributions. The parameters of the uniform distribution are the lower and upper bound. The parameters of the triangular distributions are the lower bound, mode, and upper bound. The four parameters of the beta
distribution are $\alpha$ (which increases the mean of the beta distribution), $\beta$ (which decreases the mean of the beta distribution), the lower bound, and upper bound. Since $\mu_{D1}$ and $\sigma_D$ do not need to follow the same distribution, there are a total of $2^4 = 16$ combinations of probability distributions (or states of the world) that this RDM model considers.

Table 3.2: Parameter for the probability distribution

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameters</th>
<th>$\mu_{D1}$</th>
<th>$\sigma_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>$\mu_{D1}$ ~ Unif(0,43.14)</td>
<td>$\sigma_D$ ~ Unif(100.42,301.25)</td>
<td></td>
</tr>
<tr>
<td>Triangular</td>
<td>$\mu_{D1}$ ~ Tri(0.21,43.14,57)</td>
<td>$\sigma_D$ ~ Tri(20.08,281.16,301.25)</td>
<td></td>
</tr>
<tr>
<td>Right-skewed beta</td>
<td>$\mu_{D1}$ ~ Beta(5,20,0,43.14)</td>
<td>$\sigma_D$ ~ Beta(5,9.58,20.08,301.25)</td>
<td></td>
</tr>
<tr>
<td>Left-skewed beta</td>
<td>$\mu_{D1}$ ~ Beta(5,1.25,0,43.14)</td>
<td>$\sigma_D$ ~ Beta(5,0.30,20.08,301.25)</td>
<td></td>
</tr>
</tbody>
</table>

The parameters of the distributions are selected in order to provide a viable comparison between the EU and RDM models. If a decision maker does not have any information about which state of the world or probability distribution is more likely, the decision maker could assume that each distribution is equally likely. With that assumption, the expected value of $\mu_{D1} = 21.57$ and $\sigma_D = 200.83$, which are the values of these parameters assumed in the EU model. The expected values of the uniform and triangular distributions equal those values; the expected values of the right-skewed beta distributions are $\mu_{D1} = 8.63$ and $\sigma_D = 116.48$; and the expected values of the left-skewed beta distributions are $\mu_{D1} = 34.51$ and $\sigma_D = 282.85$.

The RDM model can be evaluated for both risk-neutral and risk-averse attitudes. We calculate expected regret for each of the 16 possible states of the world. The regret for a given state and strategy is calculated according to equation A.16. The best and the worst regret for each strategy is obtained, and the expected regret is calculated according to equation A.17. The value of $z \in [0,1]$ represents the decision maker’s confidence that the state of the world corresponding to his or her minimum or best-expected regret will occur. Figure 3.3 depicts the
results of this simulation for a risk-neutral decision maker for different values of $z$. The figure recommends the years in which the first and the second hangars should be built to minimize the weighted average of the best and worst expected regret.

Figure 3.3: Optimal strategy for different $z$ in RDM model with a risk-neutral attitude

Regardless of the decision maker’s optimism, the RDM model recommends that a risk-neutral decision maker should build the first hangar between years 1 and 3 and build the second hangar between years 2 and 6. If the decision maker is more optimistic ($z \geq 0.85$), the decision maker should build the first hangar immediately (year 1) and build the second hangar in year 2. A more pessimistic decision maker should wait a little longer to build the hangars, and a very pessimistic decision maker ($z \leq 0.2$) should build the hangars in years 3 and 6. The more pessimistic decision maker should act more like a risk-averse decision maker in the EU model.
although even the most pessimistic decision maker builds the hangars much sooner than the most risk-averse decision maker in the EU model. This is because the RDM is based on minimizing expected regret rather than minimizing absolute regret (which would be more similar to extreme risk aversion). The optimistic decision maker in the RDM should act just as an expected-value, risk-neutral decision maker, at least in this instance.

Figure 3.4, Figure 3.5, and Figure 3.6 depict the effect that different risk attitudes have on the recommended strategy in the RDM. If risk attitude is incorporated into the RDM, regret (as calculated in equation A.16) is based on the utility for each profit rather than on profit itself. 200,000 simulations are used to calculate expected regret. Three different risk tolerance values are chosen: $2 million, $2.47 million, and $3 million. The figures depict the optimal strategy for different values of $z$.

Figure 3.4: Optimal strategy for different $z$ in RDM model with risk tolerance of $2$ million
Figure 3.5: *Optimal strategy for different $z$ in RDM model with risk tolerance of $2.47$ million*

Figure 3.6: *Optimal strategy for different $z$ in RDM model with risk tolerance of $3$ million*
Similar to the EU model, increasing risk aversion should, in general, incentivize the decision maker to build hangars later although the decision maker’s optimism plays a large role as well. Figure 3.4 suggests that for a risk tolerance of $2 million, the optimistic decision maker should build hangars in years 8 and 13, and the pessimistic decision maker should build hangars in years 18 and 20. If the risk-averse decision maker’s value for $z$ ranges between 0.7 and 0.85, no hangars should be built. Unless the decision maker is really optimistic, the optimal strategy according to the RDM model when the risk tolerance is $2$ million aligns closely with the EU model for the same risk tolerance.

Increasing the risk tolerance slightly from $2$ million to $2.47$ million—which is the original risk tolerance in the EU model—should influence the decision maker to build the hangars much earlier. If $z \leq 0.4$, the first hangar should be built in years 13 or 15, and the second hangar should be built in years 17 and 18. The decision maker becomes more optimistic, he or she should build the hangars more quickly with the first hangar built between years 6 and 12 and the second hangar between years 9 and 16. The EU model with the same risk tolerance recommends building the first hangar in year 15 and the second hangar in year 18, which is the optimal strategy for the most pessimistic decision maker in the RDM model.

If the risk tolerance is $3$ million, the optimal strategy follows a similar trend to that of the risk tolerances as the hangars should be built earlier as the decision maker’s optimism increases (Figure 3.6). If $z \leq 0.25$, the decision maker should build the first hangar in years 11 or 12 and the second hangar in year 16. If the decision maker is very optimistic ($z = 1$), the hangars should be built in years 5 and 7. For the same risk tolerance in EU model, it is
suggested to build the first hangar in the year 10 and the second hangar in the year 15, which corresponds to $z = 0.3$ in the RDM model.

### 3.2.4 Information-Gap

The info-gap model begins with the initial state parameters in the EU model, $\mu_{D1} = 21.57$ and $\sigma_D = 200.83$. The info-gap model continues to increase the variability $\alpha$ around the initial state until no strategy satisfies the required expected profit $p_c$ as discussed in the subsection A.2.3.3. The algorithm identifies the only strategy to satisfy the expected profit requirement for the largest possible $\alpha$ around both the drift $\mu_{D1}$ and standard deviation $\sigma_D$. Figure 3.7 displays the optimal strategy for different required expected profits.

![Figure 3.7: Optimal strategy according to the info-gap model for different expected profit requirements](image)

Figure 3.7: Optimal strategy according to the info-gap model for different expected profit requirements
As the required expected profit increases, the decision maker should build the hangars earlier. If the required expected profit is $9.0-9.1 million, the decision maker should build the hangars in years 2 and 3. This strategy is identical to a risk-neutral decision maker. As is always true with the info-gap model, there is a sufficiently high expected profit requirement at which point the decision maker should follow a decision rule to maximize his or her expected profit.

The relationship of variability $\alpha$ around the initial state and the required expected profit is represented in Figure 3.8 where the influence of $\alpha$ on the uncertainty in $\mu_{D1}$ and $\sigma_D$ is shown in equation A.18. The amount of uncertainty or variability allowed in the parameters decreases as the required expected profit increases. If the expected profit requirement is relatively small (around $8 million), the decision maker can allow that the two parameters have a large amount of uncertainty. A lot of uncertainty is required in order to eliminate strategies whose expected profit do not meet the requirement. For the very large expected profit requirements, the decision maker should assume that the initial state parameters have very little uncertainty if any. If the required expected profit is $9.2 million or more, no strategies satisfy that requirement which means the problem is infeasible according to the info-gap model.

As depicted in Figure 3.7, if the required profit is less than $8.2 million, the info-gap model allows for large uncertainty around the initial state. With such a large amount of uncertainty, the optimal strategy is not to build any hangars. This strategy is the same as an extremely risk-averse decision maker according to the EU model and a decision maker with a risk tolerance of $2 million and $0.7 \leq z \leq 0.85$ according to the RDM model. Decreasing the required expected profit, which increases the allowable uncertainty around the initial state, seems to have a similar effect as increasing the risk aversion of the decision maker. In this
situation, allowing for smaller mean values $\mu_{D1}$ and larger standard deviations $\sigma_D$ steadily decreases the expected profit. Building expensive hangars early is not as profitable.

Figure 3.8: Relationship between required profit and maximum $\alpha$

The info-gap theory has been criticized for overestimating the importance of the initial state while dealing with the situation in deep uncertainty (Sniedovich, 2008, 2012, 2014). To check the validity of the critique for this application, sensitivity analysis on the info-gap model is tested. The required expected profit is fixed ($p_c = 8.7$ million) and the standard deviation is fixed ($\sigma_D = 200.83$). Figure 3.9 displays the result of the info-gap model as the base-case mean drift changes, $13.6 \leq \mu_{D1} \leq 29.6$. The optimal strategy does not change as the initial $\mu_{D1}$ changes, and the results of the info-gap model do not appear to be very sensitive to the initial state of the mean drift. This could be because the current problem has a limited state space and
fairly well-defined uncertainty. If the model had larger uncertainties around several parameters, perhaps the results would be more sensitive to the initial states.

Performing sensitivity analysis on the base-case standard deviation while assuming the drift remains constant ($\mu_{D1} = 21.57$) reveals that the info-gap model may not always provide a useful analysis of the uncertainty. If the mean drift remains constant, the expected profit for each strategy remains relatively constant even as the uncertainty set around the standard deviation expands according to the info-gap model. Thus, if the required expected profit is less than $9.3$ million, multiple strategies satisfy this requirement no matter how large the uncertainty around $\sigma_D$ becomes. If the required expected profit is greater than $9.3$ million, the only strategy that satisfies that threshold is the strategy that maximizes the expected profit.

![Figure 3.9: Sensitivity of info-gap model by varying initial $\mu_{D1}$](image)
when there is no uncertainty around $\sigma_D$. Thus, conducting sensitivity analysis on $\sigma_D$ while keeping $\mu_{D1}$ constant reverts to a risk-neutral, expected profit strategy for this application.

### 3.3 Comparison between the decision-making models

The EU model assumes the drift parameter $\mu_{D1}$ and the standard deviation $\sigma_D$ are known. If the decision maker is uncertain about these parameters, the RDM and info-gap models provide two different methods to make decisions while accounting for this uncertainty. The RDM model requires that a decision maker identifies or assumes some possible distributions around those parameters. The info-gap model does not require a distribution around those parameters but gradually increases the uncertainty around those parameters.

The RDM model calculates regret based on profit, rather than a risk-averse utility function, generates results that are similar to a risk-neutral to a moderately risk-averse decision maker in the EU model. The optimistic decision maker in the RDM model should behave similarly to a risk-neutral decision maker in the EU model. The pessimistic decision maker in the RDM model should behave similarly to a moderately risk-averse decision maker in the EU model. If the RDM calculates regret based on a risk-averse utility function, the results are similar to an extremely risk-averse to a moderately risk-averse decision maker in the EU model. The risk-averse decision maker in the RDM model should follow a similar strategy as in the EU model for the same risk tolerance when the decision maker is fairly pessimistic ($z \leq 0.3$). If a risk-averse decision maker is fairly optimistic ($z \geq 0.7$) in the RDM model, the decision maker should build hangars more quickly than if he or she followed the EU model.

The info-gap model approaches the decision-making process differently than the EU and RDM models. The uncertainty set is not a fixed set in the info-gap model but is determined
by an uncertainty parameter $\alpha$. The EU and RDM models identify the optimal alternative principally based on avoiding the worst outcomes, but the info-gap model identifies the optimal alternative based on those alternatives that exceed a required expected profit. However, the results of the models are similar. A very high required expected profit in the info-gap model is identical to a risk-neutral decision maker in the EU model and an optimistic, risk-neutral decision maker in the RDM model. Decreasing the expected profit requirement is similar to decreasing the risk tolerance (i.e., increasing the risk aversion) in the EU model. Figure 3.2 and Figure 3.7 are remarkably similar. The EU model recommends not to build no hangars if the decision maker is very risk-averse (risk tolerance = $1$ million), and info-gap model recommends not to build any hangars if the decision maker does not require very large expected profits ($< 8.2$ million).

A risk-averse decision maker in the RDM (risk tolerance = $2$ million) should only build one hangar if $0.2 \leq z \leq 0.85$ and build the second hangar in year 20 if $z < 0.2$. The first hangar should be built between years 18 and 20 for those values of $z$. These recommend strategies are similar to the recommend strategies if the required profit in the info-gap model is less than $8.25$ million. Unless the decision maker is very optimistic, the very risk-averse RDM model and the info-gap model with the smallest required profits generate similar optimal strategies for this application.

Both the RDM model and the info-gap model can help the decision maker consider different trade-offs that are not immediately apparent in the EU model. Computing the optimal strategy for different values of $z$ is easy within the RDM model, and the decision maker can understand how the trade-off between planning for the worst case and planning for the best case should impact his or her decision. In the info-gap model, the required expected profit (or
more generally, the minimum required value of the objective function) correlates with how much uncertainty is allowed in the model. If the decision maker requires a larger expected profit, then the decision maker is essentially assuming the initial state of parameters has very little uncertainty. If the decision maker requires a smaller expected profit, then the decision maker can assume a lot more uncertainty exists around those parameters. The decision maker can choose the trade-off point between required expected profit and allowable uncertainty with which he or she is comfortable.

3.4 Conclusions

The goal of this analysis is to examine different decision-making methods when significant uncertainty exists in a long-term capacity problem. EU, RDM, and info-gap are the three decision-making methods selected for analysis. The models are applied to determining when to expand painting capacity for the Boeing 737 airplane model. Each decision-making method might be appropriate in different circumstances.

In the case where the uncertainty could be modeled as a probability distribution, the EU model works well. It is possible to assign a probability to any uncertainty which represents the beliefs of a decision maker according to the subjective probability theory. However, in cases where there are large uncertainty and variability around several parameters, it may be very challenging to assign probabilities. In such a case, the optimal strategy suggested by the EU model can vary a lot as the parameters change. For the painting capacity problem, the results of the EU model will not be consistent if the estimated parameters $\mu_D$ or $\sigma_D$ are different from those estimated by the demand prediction model. If the drift parameter is overestimated and very high demands are expected in the future, the EU model would suggest building the
hangars quickly and the hangars might not be utilized to the full capacity. The EU model could carry significant risks to the decision maker if the parameters are not properly estimated. If the parameters in the model are estimated with high confidence, the EU model provides a good method for calculating the optimal alternative while accounting for the risk attitude of the decision maker.

The RDM model can generate a trade-off curve between optimality and optimism, and the info-gap model generates a trade-off curve between optimality and uncertainty. The RDM model expands the single probability distribution in the EU model into a set of probability distributions. The RDM model considers the severity of the positive and negative outcomes by assigning a regret value to the outcomes. RDM focusses on robustness rather than optimality. Based on the decision maker’s optimism in obtaining the strategy for the best-case regret versus the worst-case regret, he or she can make an informed decision about how his or her optimism about the future scenarios should affect the optimal strategy.

Since RDM focuses on minimizing regret and consequently thinking about the best-case expected regret versus the worst-case expected regret, it may be difficult for a decision maker to understand how the regret translates to profit (which is the ultimate objective in this private-sector model). The info-gap model may be more intuitive than RDM because the former provides a direct relation between the required profit and the best strategy. Info-gap also allows for dynamically changing parameters ($\mu_D$ and $\sigma_D$), which may allow for more uncertainty in the model. Info-gap provides a means for selecting a strategy under significant uncertainty without constructing multiple probabilistic models, as required by RDM. The info-gap model enables a decision maker to a trade-off between uncertainty in the parameters and optimality according to an expected-value model. The info-gap model has been criticized
because it searches for local optimality. The results for info-gap could be misleading if there is uncertainty in the model or uncertainty around several parameters in the problem.

Although each decision-making requires different assumptions and has different definitions of an optimal decision, the decision-making models seem to result in similar recommendations for this application. The risk-neutral EU model, the very optimistic RDM model, and the info-gap model with a very large profit requirement all generate the same strategy. Increasing risk aversion in the EU model, increasing the pessimism in the RDM model, and decreasing the profit requirement in the info-gap model seem to have similar effects on the recommended strategy.

Each decision-making model requires different assumptions, and perhaps the decision maker should choose the model that best meets what he or she is willing to assume about the situation. If the decision maker is confident in his or her probabilistic model of the uncertainty and does not want to confuse his or her risk aversion with other factors (such as uncertainty around the probabilities), the EU model is the best decision-making model. If the decision maker is confident in the initial estimation of parameters but realizes these parameters carry uncertainty, the info-gap model is beneficial. A decision maker does not need to assume probability distributions about the uncertain parameters, which may be attractive for some people. If the decision maker prefers to view the future in the context of distinct scenarios (e.g., high growth, medium growth, no growth) and wishes to avoid extremely bad outcomes, the RDM model might be the most appropriate. The RDM model does require the decision maker to choose probability distributions for each state of the world. Before choosing a decision-making model, it is important to decide if optimality is required or a solution that avoids the worst outcomes is desirable. If optimality is most important, the EU model that defines
optimality as the maximizing the decision maker’s expected utility is the best. If avoiding the worst outcomes is most important, a more pessimistic RDM model or the info-gap model with a smaller threshold in the objective function is preferred.
CHAPTER 4. GENERAL CONCLUSIONS

This research discusses the demand forecasting and decision making for long-term capacity planning for the aviation industry. While the current discussion focuses on airplane manufacturing, the models presented in this research could be used for the production planning in several other manufacturing or industry environments.

The first part of the research, the historical data for the Boeing 737 and 777 were analyzed. The data for the Boeing 737 model follows a Brownian motion trend with a significant variability. The median demand for the Boeing 737 was predicted using a forecasting model based on the parameters from the Brownian motion trend. However, due to the large value of standard deviation, there is large uncertainty associated with the demand prediction. Therefore, an overview of demand prediction for the next 20 years has been presented with a 90% confidence interval and the observed results were realistic.

The Boeing 777 model seems to follow a geometric Brownian motion. However, the demand prediction with this GBM assumption produces very unrealistic results. Therefore, a modified approach for the GBM model was fitted to the Boeing 777 data. The baseline year for the demand prediction was adjusted to the year 2005, so the 90% demand prediction from the year 2006-2017 includes the actual demand values. With these assumptions, the demand was predicted for the Boeing 777 model for 20 years. The results seem realistic. Although the 95th quantile for the demand is large, it does not seem impossible. The predicted demand for the Boeing 777 also has a huge variance due to the large value of standard deviation.

The second part of the research takes into account the predicted demand for the Boeing 737 model in decision making for the capacity expansion problem. Even though the median demand was predicted, it cannot be argued that it is an accurate prediction because of the huge
uncertainty in the prediction. To address the uncertainty in demand, several decision-making methods were used to find the solution. The expected utility, robust decision making, and information gap are the decision-making methods that were applied to the Boeing 737 model. The EU model works well when the uncertainty could be modeled as probability distributions, therefore the EU model is highly sensitive to input probability distribution. The RDM and info-gap methods are designed for deep uncertainty, and the performance is fairly stable. A detailed analysis of each method is conducted in this research.

All the decision methods use the same strategy set, the EU model finds the optimal strategy for decision making by maximizing the utility function based on the risk attitude of the decision maker. The RDM model takes into account the risk tolerance of the decision maker while minimizing the actual regret of the strategies. The info-gap method is more intuitive as it provides a direct relation between the expected profit and the best strategy. However, as the required profit increases, the uncertainty allowed around the initial estimates is reduced.

Compared to the expected utility, the optimal strategies suggested by the RDM and info-gap model are more robust. However, the RDM method may require intensive computing power and advanced optimization algorithms if the state space is larger, and the info-gap has been criticized to be biased towards finding local optimality. Each of the decision-making methods has their advantages and disadvantages, and a comprehensive discussion of the situations in which each method would be best applicable has been provided in chapter 3.

The demand prediction and decision-making philosophies together provide an overall framework for the long-term capacity planning for the painting facilities of Boeing 737 model in this research. However, these models could be utilized in several other manufacturing applications. In future, this model could be developed further to take into account the dynamics
of changing demand, variation in the painting capacity, and macroeconomics for the airplane models and update the decision strategies every year. This dynamic model would be helpful in avoiding any adverse effects due to demand variation, thereby reducing the uncertainty in the problem, allowing for a better long-term capacity planning.
APPENDIX

A.1 Background and literature review

Uncertainty is a state of not being definitive, which involves imperfections and/or lack of knowledge. Uncertainty is commonly handled by assign probability distributions based on the beliefs or available data. Deep uncertainty is defined by Walker et al. (2013) as "Uncertainties that cannot be treated probabilistically include model structure uncertainty and situations in which experts cannot agree upon the probabilities." Deep uncertainties extending over time have significant risk associated with the decision-making processes, making it difficult to manage. The manufacturing engineering decisions are especially prone to such uncertainties (Applequist, 2000; Brouthers, 2003). Mathematic models and optimizations tools generally provide possible answers to these uncertainties by assuming probability distributions for the uncertain parameters. Stochastic programming is a prominent approach to finding the optimal alternatives (Infanger, 1992; Ahmed, 2000; Santoso, 2005).

Courtney (2001) categorizes uncertainty into 5 intermediate levels between complete certainty and total ignorance. Level 4 (multiplicity of futures) and level 5 (unknown future) are extreme uncertainties and it is very difficult to assign a probability distribution to the uncertainties. The demand prediction models discussed in Chapter 2, helps reduce the uncertainty for the long-term capacity planning. The demand prediction models help analyze the available historical data and reduce the uncertainty of the long-term capacity problem. The demand prediction models are not very accurate and often there is uncertainty around the estimated parameters. Therefore, this problem could be considered as a level 3 uncertainty problem. This chapter provides a comparison of different decision-making models to address the issue of capacity planning based on the predicted demand from Chapter 2.
Previously, several decision-making methods have been developed and proposed to deal with uncertainty, including expected utility (Fishburn, 1970; Rabin, 2000), prospect theory (Tversky, 1992), interval analysis (Moore, 1979, 2003), mean-variance analysis (Epstein, 1985), robust decision making (Lempert, 2003), information gap (Ben-Haim, 2004, 2006, 2015), preserving flexibility (Mandelbaum, 1990), and the precautionary principle (Steele, 2006). Most papers discuss one decision-making model and very little work has gone into exploring when these decision-making models produced different results and what assumptions are necessary to implement a specific decision-making method. Lempert et al. (2007) compared robust decision making, expected utility and precautionary methods under a hypothetical environment. Hall (2012) made a comparison between robust decision-making and Info-gap for climate policies problem. A similar comparison was done in water resource system planning (Matrosov, 2013). However, to the best knowledge of the authors, not much research has been done to compare the decision-making models in a manufacturing capacity planning setting. Zhang (2017) compared the expected utility model (EU), robust decision making (RDM) and information gap (Info-gap) model for Boeing 737 by assuming a Brownian motion trend for the demand of the airplanes, however, the parameters for the demand estimation are assumed. This paper addresses the incorporation of demand forecast into decision making and analyzes the results from the EU, RDM and Info-Gap model.

Expected utility is probably the oldest and still one of the most popular decision-making philosophies under uncertain circumstances. EU is an optimality theory that maximizes the decision makers expected utility while incorporating the risk attitude of the decision maker. EU requires a probability for each potential outcome, and the probabilities represent the decision maker’s subjective beliefs about the future. It can be tricky to ascertain the decision
maker’s utility function, but there are a few papers which provide some useful guidance (Samuelson, 1937; Parzen, 1962; Alt, 1971).

Bell (1982) argues that incorporating regret into expected utility theory would improve the quality of the decision-making. The idea of robust decision framework is first proposed by Jonathan Rosenhead (Mingers & Rosenhead, 2001). In 2003, RDM framework was developed (Lempert, 2003). RDM is designed for deep uncertainty where probability distributions could not be easily modeled. It does not rely on the prior probability distribution which is a key input parameter for most decision-making models (Lempert et al., 2007). Even if the decision maker believes the uncertainty can be described by a probability distribution, there may be uncertainty around the parameters informing the probability distribution. RDM provides a solution to incorporate uncertainty in the parameter estimation. Generating all the plausible scenarios remains a challenge in applying RDM. RDM resembles regret-based decision making in which the decision maker seeks to minimize the regret from a bad outcome.

The info-gap method also provides scope for incorporating severe uncertainty around the probability distribution in decision-making (Ben-Haim, 2006). Info-gap uses a state space instead of a probability distribution, it allows for uncertainty around the initial state by dynamically changing the uncertain parameters to obtain the maximum reward. Info-gap has been criticized to be not applicable under severe uncertainty (Sniedovich, 2007).

This research compares these different decision-making methods by utilizing a stochastic process over time and comparing the results obtained from EU, RDM, and Info-gap. Simulation is used to generate the results and compare the methods. It is found that EU method is the best method if the probability distribution can be assigned to represent uncertainty with a high confidence level. If we have little information about the initial state and there is deep
uncertainty associated with it, then RDM model is the best decision-making method, however, it requires more efforts on scenario exploration and computational optimization. Info-gap model has more practical application in the industry as it provides critical reward information for level 2 or level 3 uncertainty problems.

A.2 Decision-Making Models for Aviation Industry with Deep Uncertainty

In the aviation industry, building a new facility for assembling and painting aircraft is expensive. Therefore, capacity planning is an important strategic decision for manufacturers. Although it is evident from the demand prediction model that the demand for the Boeing 737 airplane is likely to increase in the future, there is still large uncertainty associated with the demand prediction. Several factors including economic growth rate, global competition, fuel price, and the currency exchange rate influence the demand. Considering the uncertainty in the demand, it is important for the manufacturer to decide if and when to build the hangars. In the following sections, different decision-making theories which captures the essential factors and variables in the decision problem have been discussed.

A.2.1 Capacity Planning Model with Uncertain Demand

Table A.1 and Table A.2 list all the notations used in this chapter. An airplane manufacturer can plan to construct hangars on an annual basis, and \( I(t) \) is defined as the number of new hangars at time \( t \) years, where \( t \) is an integer representing years into the future. In this model, it is only considered to build new hangars and removing hangars in not an option, therefore \( I(t) \geq 0, \forall t \).

Table A.1: Notification of functions
<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Model</td>
<td>Maximum production capacity</td>
</tr>
<tr>
<td>$D$</td>
<td>Model</td>
<td>Demand</td>
</tr>
<tr>
<td>$DC$</td>
<td>Model</td>
<td>Depreciation cost</td>
</tr>
<tr>
<td>$D\hat{C}$</td>
<td>Model</td>
<td>Fixed cost</td>
</tr>
<tr>
<td>$G$</td>
<td>Model</td>
<td>Profit function</td>
</tr>
<tr>
<td>$H$</td>
<td>Model</td>
<td>Capacity</td>
</tr>
<tr>
<td>$I$</td>
<td>Model</td>
<td>Investment decision</td>
</tr>
<tr>
<td>$M$</td>
<td>Model</td>
<td>Actual production</td>
</tr>
<tr>
<td>$MC$</td>
<td>Model</td>
<td>In-house painting cost</td>
</tr>
<tr>
<td>$OC$</td>
<td>Model</td>
<td>Outsourcing cost</td>
</tr>
<tr>
<td>$P_s$</td>
<td>RDM</td>
<td>Reward</td>
</tr>
<tr>
<td>$R$</td>
<td>Model</td>
<td>Revenue</td>
</tr>
<tr>
<td>$RT_s$</td>
<td>RDM</td>
<td>Regret</td>
</tr>
<tr>
<td>$\theta$</td>
<td>RDM</td>
<td>Probability distribution</td>
</tr>
<tr>
<td>$U$</td>
<td>EU</td>
<td>Utility function</td>
</tr>
<tr>
<td>$VC$</td>
<td>Model</td>
<td>Variable cost</td>
</tr>
</tbody>
</table>

Table A.2: Notation of variables and sets

<table>
<thead>
<tr>
<th>Notation</th>
<th>Type</th>
<th>Definition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Variable</td>
<td>Info-gap</td>
<td>Horizon of uncertainty</td>
</tr>
<tr>
<td>$CE$</td>
<td>Variable</td>
<td>EU</td>
<td>Certainty equivalent</td>
</tr>
<tr>
<td>$g$</td>
<td>Variable</td>
<td>EU</td>
<td>Additional profit beyond baseline</td>
</tr>
<tr>
<td>$n$</td>
<td>Variable</td>
<td>Model</td>
<td>Number of decision options</td>
</tr>
<tr>
<td>$N$</td>
<td>Variable</td>
<td>Model</td>
<td>Number of strategies</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Set</td>
<td>Info-gap</td>
<td>Uncertainty space</td>
</tr>
<tr>
<td>$r$</td>
<td>Variable</td>
<td>EU</td>
<td>Risk tolerance</td>
</tr>
<tr>
<td>$s$</td>
<td>Variable</td>
<td>Model</td>
<td>Strategy</td>
</tr>
<tr>
<td>$S$</td>
<td>Set</td>
<td>Model</td>
<td>Strategy set</td>
</tr>
<tr>
<td>$s_0$</td>
<td>Variable</td>
<td>EU</td>
<td>Default strategy</td>
</tr>
<tr>
<td>$t$</td>
<td>Variable</td>
<td>Model</td>
<td>Time</td>
</tr>
<tr>
<td>$\mathfrak{t}$</td>
<td>Set</td>
<td>Model</td>
<td>Time set</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>RDM</td>
<td>Probability distribution set</td>
<td></td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>Variable</td>
<td>Model</td>
<td>Expected annual production</td>
</tr>
<tr>
<td>$\mu_D$</td>
<td>Variable</td>
<td>Model</td>
<td>Expected annual demand</td>
</tr>
<tr>
<td>$x$</td>
<td>Variable</td>
<td>Model</td>
<td>State</td>
</tr>
<tr>
<td>$X$</td>
<td>Set</td>
<td>Model</td>
<td>State set</td>
</tr>
<tr>
<td>$z$</td>
<td>Variable</td>
<td>Model</td>
<td>Confidence level of best distribution</td>
</tr>
</tbody>
</table>
It is assumed that the manufacturer cannot influence the demand and for the year \( t \), it is represented as \( D(t) \). Based on the previous analysis of aviation industry in Chapter 2, the demand for the Boeing 737 airplanes follows Brownian motion trend. Mathematically it is represented as demand \( D(t) \sim \text{Nor} \left( \mu_D, \sigma_D^2 dt \right) \), where \( \mu_D = \mu_{D0} + \mu_{D1} t \), \( \mu_{D0} \) is the mean of demand at \( t=0 \), \( \mu_{D1} \) is the annual trend coefficient, \( \sigma_D^2 \) is the variance in demand at \( t = 1 \). The production of airplanes by the manufacturer is assumed to be equal to the demand of the airplanes at each time \( t \).

The manufactured planes could be painted in-house or outsourced. The revenue for the airplanes is calculated as:

\[
R(t) = a \times D(t) \tag{A.1}
\]

where \( a \) is the selling price of the aircraft. Boeing 737 is the only model of the aircraft considered with a fixed selling price without adjusting the inflation.

The number of hangars in the year \( t \) is defined as \( H(t) \). The hangar capacity can only change at the beginning of a year and remains constant through the rest of the year. The number of hangars I the year \( t \) is given by:

\[
H(t) = \sum_{i=1}^{t-1} I(i) + h_0 \tag{A.2}
\]

where \( h_0 \) is the number of hangars at time \( t = 0 \). \( H(t) \) is the total number of hangars up to time \( t \), therefore it never decreases if \( I(t) \geq 0, \forall t \).

Straight-line depreciation method was used to calculate the depreciation cost of the new hangars at time \( t \). The depreciation cost is given by \( DC(t) \):

\[
DC(t) = e \times dr \times \left[ H(t) - h_0 \right] \tag{A.3}
\]
where $e$ is the cost to build a new hangar and $dr \leq 1$ is the depreciation rate. The capital cost is considered to be depreciated evenly over time $t$, then $dr = \frac{1}{T}$ where $T$ is the total number of years in the problem. For this problem, it is considered that the hangars can be used beyond the total number of years examined in this problem. Therefore, the manufacturer will not be penalized for building a hangar in the year $T$ due to the depreciation factor. In reality, if a manufacturer builds a hangar in the year $T$, it would be used into the future years as well.

The maximum number of airplanes that can be painted in a year $t$ is $A(t)$ which is uncertain. $A(t)$ is assumed to follow a Gaussian distribution $A(t) \sim \text{Nor}(\mu_A, \sigma^2_A)$, where $\mu_{A(H)}$ is the average number of planes that could be painted given the number of hangars and $\sigma^2_A$ is the variance. The average number of planes painted in a year is $\mu_A = d^*H(t) + d_0$, where $d$ and $d_0$ are positive parameters. Given the demand and maximum capacity, the actual number of planes at time $t$ is given by:

$$M(t) = \min\{D(t), A(t)\} \tag{A.4}$$

Painting the airplanes in-house is cost-efficient and it also has lower lead times when compared to outsourcing them. However, if the actual demand exceeds the maximum capacity, the manufacturer will choose to outsource the painting operations. Assuming the outsourcing capacity to be infinite, the outsourcing cost can be written as:

$$OC(t) = b \times \max\{D(t) - M(t), 0\} \tag{A.5}$$

where $b$ is the cost of outsourcing. The in-house painting cost is decomposed into two parts: fixed Cost $FC(t)$ and the variable cost $VC(t)$. Fixed cost in the maintenance cost of the capital, which is based on the number of hangars, it is obtained by:

$$FC(t) = f \times H(t) + f_0 \tag{A.6}$$
where $f$ and $f_0$ are fixed-cost coefficients. Variable cost is the operational cost based on the number of jobs.

$$ VC(t) = k \ast M(t) + k_0 $$  \hspace{1cm} (A.7)

where $k$ and $k_0$ are coefficients. The total in-house painting cost is the sum of the fixed cost and variable cost:

$$ MC(t) = FC(t) + VC(t) = f \ast H(t) + k \ast M(t) + f_0 + k_0 $$  \hspace{1cm} (A.8)

The profit function $G(t)$ at time $t$ is expressed as:

$$ G(t) = R(t) - MC(t) - OC(t) - DC(t) = a \ast D(t) - f \ast H(t) - f_0 - k \ast M(t) - k_0 - b \ast max\{ [D(t) - M(t)], 0 \} - e \ast dr \ast [H(t) - h_0] $$  \hspace{1cm} (A.9)

The manufacturer has to choose an investment strategy $s$ in order to maximize the total profit over a period of $T$ years. An investment strategy $s$ is a unique collection of $I(\hat{t})$ where $\hat{t} = \{0, 1, 2, ....T\}$. The objective function is calculated as:

$$ \hat{G}(T, s) = \sum_{t=1}^{T} G(t, s) $$  \hspace{1cm} (A.10)

where $G(t,s)$ is the profit function in year $t$ given an investment strategy $s$.

A.2.2 Decision Space

The decision space depends on the number of alternatives available to the decision maker and the total number of years. If there are $n$ different alternatives in each year for $T$ years, the number of strategies in the decision space is $n^T$. With the increase in the number of alternatives or years, the number of strategies increases exponentially. Therefore, for this decision problem, it is assumed that the maximum number of hangars that could be built over
$T$ years is $h_{\text{max}}$ and the hangars are identical. These assumptions reduce the total strategies $N$ which could be calculated as:

$$N = \sum_{i=1}^{h_{\text{max}}} \binom{T + 1}{i}$$  \hspace{1cm} (A.11)

### A.2.3 Decision-making Models

The framework for the three decision-making models: Expected Utility (EU), Robust decision making (RDM), Information gap (Info-gap) is discussed in this section.

#### A.2.3.1 Expected Utility

Expected utility theory assumes a single decision exhibits a risk-averse or risk-neutral behavior. Due to the large uncertainty in this problem, a risk-averse decision would be a realistic approach. An exponential utility function is used to compute the utility of profit. The general form of the exponential utility function is:

$$U(g) = a_1 - b_1 \exp\left(- \frac{g}{r}\right)$$  \hspace{1cm} (A.12)

where $r > 0$ is the risk tolerance; $a_1$ and $b_1$ define the scale of utility function; $g = \hat{G}(T, s) - E[\hat{G}(T, s_0)]$ which is the additional profit over $T$ years given strategy $s$ after removing the baseline expected profit. The baseline profit $E[\hat{G}(T, s_0)]$ is the expected profit over $T$ years given strategy $s_0$ (no additional hangars). If the decision maker is indifferent between obtaining the expected baseline profit and a $p$ probability of gaining an additional $m$ million and $1 - p$ probability of losing $m$ million, then the decision maker’s risk tolerance $r$ can be calculated as:

$$\tilde{r} = \left(\frac{p}{1-p}\right)\frac{1}{m}$$  \hspace{1cm} (A.13)
\[ r = \frac{1}{\ln r} \quad (A.14) \]

The parameters \( a_1 \) and \( b_1 \) could be calculated by assuming values for the best case and worst case in the utility function. The decision maker should choose the strategy that maximizes the expected utility. The certainty equivalent (CE) is the profit equivalence based on the expected utility, it is calculated as the inverse of the utility function. The results of CE could be used for comparison and judgment. CE is calculated as:

\[ CE = U^{-1}(E[U(g)]) \quad (A.15) \]

### A.2.3.2 Robust Decision Making

RDM incorporates several uncertainties into the model to support decision making. Uncertainty is represented as “a set of multiple, plausible future states of the world” (Hall et al., 2012). RDM assumes three sets: strategy set \( S \), a plausible future state set \( X \), and a probability distribution set \( \Theta \). The expected regret of strategy \( s \in S \) contingent on distribution \( \theta_i(x) \in \Theta \) is given by (Lempert et al., 2007):

\[ \overline{RT}_{s,i} = \int_x RT_s(x) \theta_i(x) dx \quad (A.16) \]

where \( RT_s(x) = Max_{s'}[P_{s'}(x)] - P_s(x) \) is the regret of strategy \( s \) in the state \( x \) and \( i \) is the index of the probability distribution in the set \( \Theta \). The reward function \( P_s(x) \) is the expected utility of profit \( E[U(g)] \) given state \( x \).

Given a strategy \( s \), there is a probability distribution \( \theta_{\text{best}}(x) \) in the set \( \Theta \) which minimizes the expected regret \( \overline{RT}_{s,\text{best}} \). Similarly, a probability distribution \( \theta_{\text{worst}}(x) \) yields the maximum expected regret \( \overline{RT}_{s,\text{worst}} \). The true expected regret, given the true probability distribution, should lie in the interval \([\overline{RT}_{s,\text{best}}, \overline{RT}_{s,\text{worst}}]\). RDM suggests a way to a trade-off
between the optimal performance and model sensitivity. Mathematically, the trade-off is written as a weighted average of the best and worst expected regret:

\[ V_s = z \overline{RT}_{s,\text{best}} + (1 - z)\overline{RT}_{s,\text{worst}} \]  

(A.17)

where \( 0 \leq z \leq 1 \).

The parameter \( z \) can be interpreted as the level of confidence of the decision maker in the probability distribution \( \theta_{\text{best}}(x) \). According to RDM, the decision maker should select the strategy \( s \) that minimizes \( V_s \) given the value of \( z \). For example, if the decision maker has 100% confidence that \( \theta_{\text{best}}(x) \) is the exact representation of truth, then \( z = 1 \) and the result would be the same as the expected utility because, the expected regret of a strategy is the difference between its expected utility and maximum expected utility over all the strategies. Conversely, if \( z = 0 \), the decision maker believes there is a high uncertainty in the probability distribution over the future state \( X \) and should prepare for the worst case.

### A.2.3.3 Information Gap

The info-gap model treats the uncertainty as a family of nested sets (Ben-Haim, 2004), unlike the EU and RDM models which assume that the uncertainty can be measured with probabilities. Info-gap model utilizes a dynamic uncertainty \( \Phi \) and does not assume any probability distribution over the uncertainty set, whereas the RDM model tries to measure a fixed uncertainty set with a branch of plausible probability distributions. The dynamic uncertainty set \( \Phi \) is defined by a variable \( \alpha \). Given a fixed \( \alpha \), the set \( \Phi(\alpha, \hat{x}) \) states a degree of variability around \( \hat{x} \) which is interpreted as the most likely state. The parameter \( \alpha \) is called the “horizon of uncertainty” (Ben-Haim, 2015) and explains the variability of \( x \). the greater the
value of $\alpha$, the larger the size of the set $\Phi (\alpha, \hat{x})$ and higher the variation. If $\alpha = 0$, then there is no uncertainty in the model.

There are several types of uncertainty models for $\Phi (\alpha, \hat{x})$, and the fraction error model is one of the most common models (Hayes et al., 2013). The fraction error model creates an interval based on an initial estimation for each uncertain parameter ($\mu_{D1}, \sigma_D \in X$) in the demand model:

$$
\Phi (\alpha, \hat{x}) = \Phi (\alpha, (\mu_{D1}, \sigma_{D}))
$$

(A.18)

$$
= \{ (\mu_{D1}, \sigma_D): \left| \frac{\mu_{D1} - \hat{\mu}_{D1}}{\mu_{D1}} \right| \leq w_1 \alpha, \left| \frac{\sigma_D - \hat{\sigma}_D}{\sigma_D} \right| \leq w_2 \alpha \}
$$

where weight parameter $w_1, w_2 \in [0, 1]$ and $(\hat{\mu}_{D1}, \hat{\sigma}_D)$ are initial estimates. Therefore, $\Phi$ represents the uncertainty space for this problem.

The decision space for the info-gap is also defined by the strategy set $S$ similar to the RDM model. A reward function $P_s(x)$ measures the expected utility given the strategy $s$ and state $x$. The decision maker selects $p_c$, which is the minimum requirement for the reward function. In the painting decision problem, $p_c$ is the required profit. In the info-gap model, robustness is defined as the maximum $\alpha$ that still maintains the critical requirement for a strategy $s$, and opportuneness is defined as the minimum $\alpha$ (Ben-Haim, 2006). The opportuneness function focusses on sweeping success, which might not be appropriate for situation examined in this paper. Hence, we focus on the robustness function $\hat{\alpha}(s, p_c)$ which calculates the greatest level of uncertainty that satisfies the minimum profit requirement.

$$
\hat{\alpha}(s, p_c) = \max\{\alpha: \min_{x \in \Phi(\alpha, \hat{x})} P_s(x) \geq p_c\}
$$

(A.19)

The decision maker should select the strategy $s$ that meets the critical requirement with largest $\hat{\alpha}(s, p_c)$. 


REFERENCES


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